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STATION CONTACT TIMES FOR APOLLO ORBITS WITH VARIABLE LAUNCH AZIMUTHS

BY

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STATION CONTACT TIMES FOR APOLLO PARKING ORBITS WITH VARIABLE LAUNCH AZIMUTHS

SUMMARY

This paper presents graphs showing the times of contact of the Apollo spacecraft in its parking orbits. Orbit heights of 200 km ($\approx$110 nmi) and 300 km ($\approx$160 nmi) are considered.

It is hoped that these graphs will help solve some of the tracking and communication tasks connected with orbits that have launch azimuths which change continuously as the launch time is delayed. The graphs show the contact times for different ground stations for one to six parking orbits that have launch azimuths which vary continuously from 73° to 110°.

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STATION CONTACT TIMES FOR APOLLO PARKING ORBITS WITH VARIABLE LAUNCH AZIMUTHS

STATION CONTACT TIME

One of the major requirements for the Apollo ground tracking Network is to insure a large enough station contact time of the spacecraft in its first few (one to four) parking orbits. This is the time the spacecraft can be "seen" above a specified elevation angle ϵ (for instance $\epsilon = 0°$, $\epsilon = 5°$) from a ground station. In planning a network, the ground stations are located in a manner to fulfill requirements such as number of contacts per orbit and a certain station contact time Δt. (See Fig. 1, 2, 3 and 4.) From Fig. 1, 2, and 3 an easy estimate can be made of the times the spacecraft spent between the different stations (no contact) remembering that the period of one orbit is approximately 90 minutes. Under normal conditions, the station contact times are presented for a certain specified orbit in tabular form.

In the case of an infinite number of orbits with launch azimuths varying between certain values (see reference 1 and 2) ($\alpha = 73°$ continuous variable to $\alpha = 110°$, see Fig. 4) a tabular presentation is not practical.

In this paper the station contact times Δt for the proposed Apollo stations are given in graphical form thus making it easy to determine Δt for each parking orbit (see reference 2) and each launch azimuth to be considered for the Apollo missions. The graphs shown are valid for $\epsilon = 5°$ (tracking) and $\epsilon = 0°$ (communications) and are self explanatory.

ACKNOWLEDGMENT

The authors wish to acknowledge the excellent cooperation of Mr. Arthur Shapiro and his group in the preparation of this paper. The programming and computations were accomplished in an expeditious manner.

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2. Vonbun, F. O. "Parking Orbits and Tracking for Lunar Transfers," GSFC Report Number X-520-63-83 of June 7, 1962.

3. Reich, H. and Lyolpert, R., "Orbital Launch Escape Window Analysis," Northrop Corporation Report, NB 61-357, December 1961, pp 36.

4. Vonbun, F. O., Kahn, W. D. "Tracking Systems, Their Mathematical Models and Their Errors Part I – Theory," Technical Note D-1471 of October 1962, Appendix A, Equation (A-7).

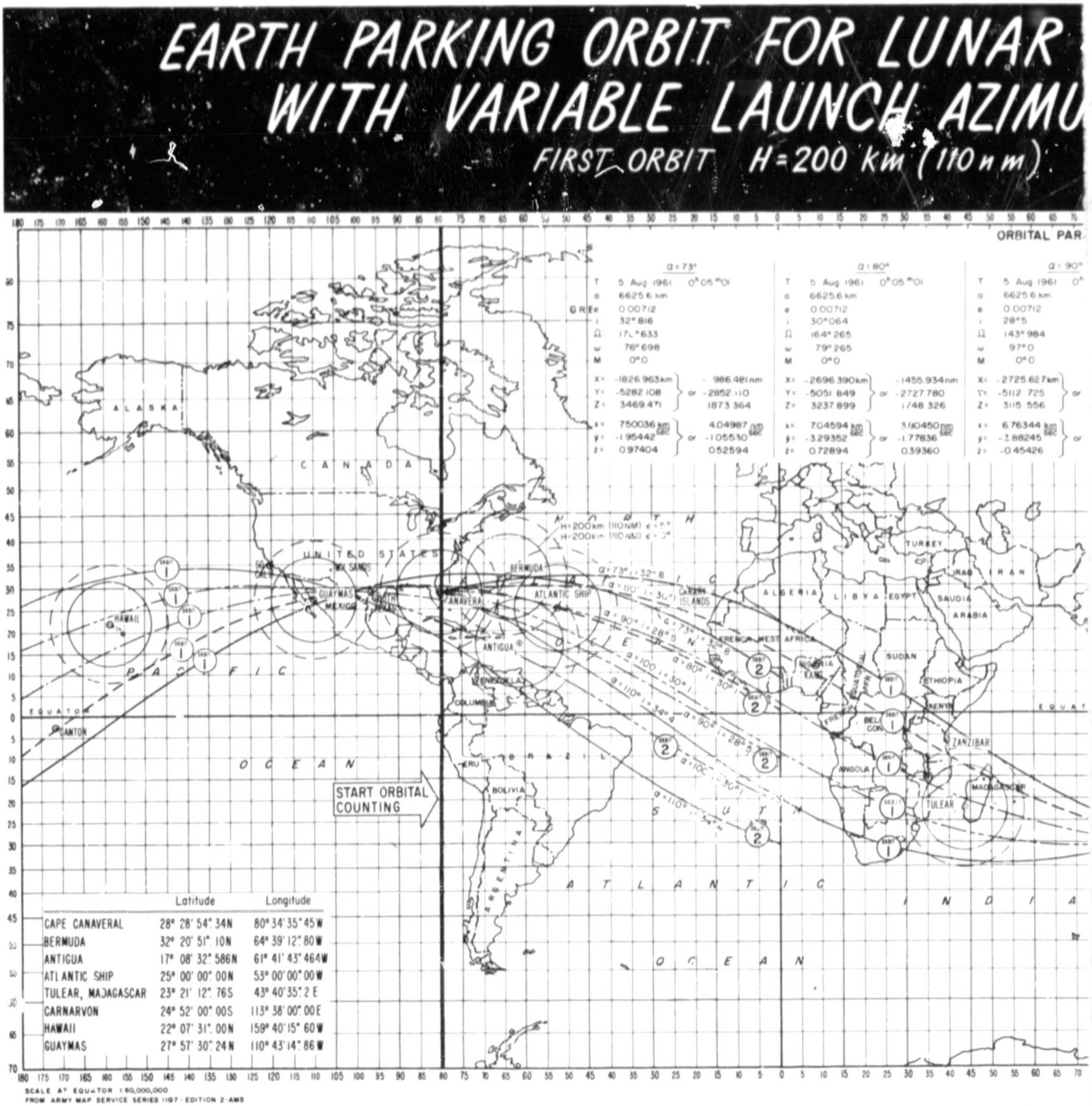
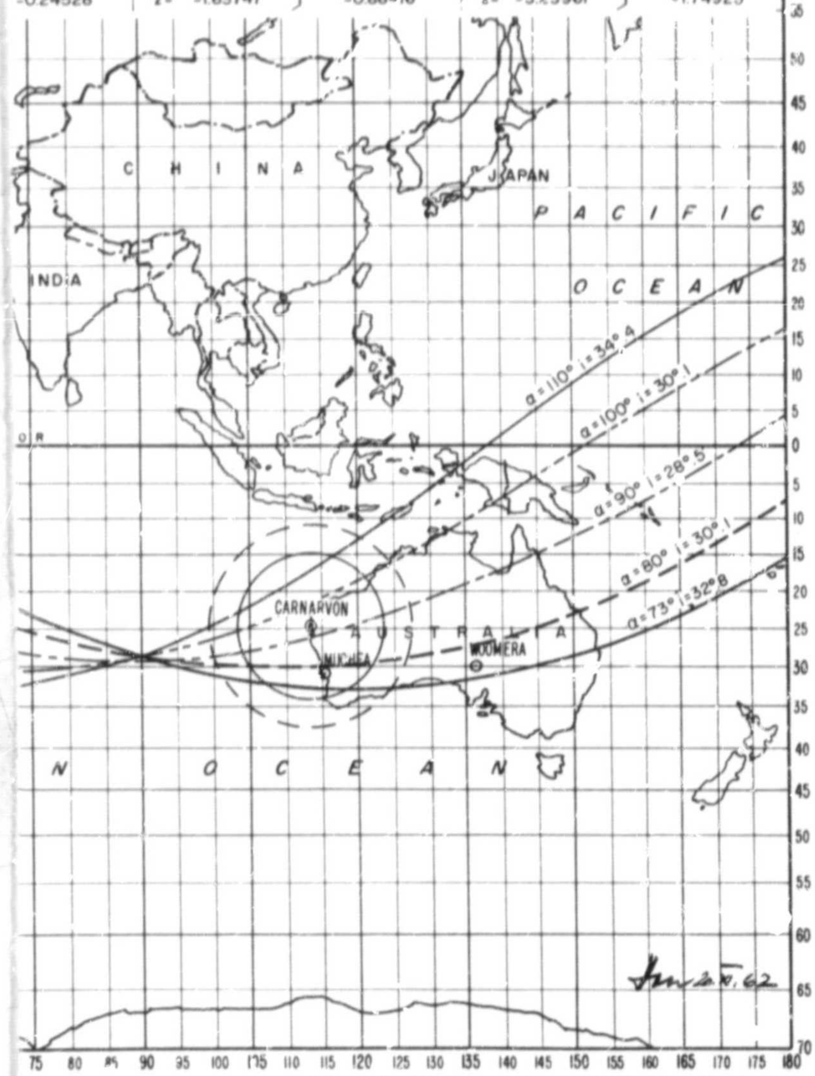


Figure 1 — Earth Parking Orbit for Lur
(α), First Orbi

MISSIONS
TH (α)

AMETERS

	$\alpha = 100°$		$\alpha = 110°$	
05.^{m}01	T: 5 Aug 1961 0^{h}05.^{m}01		T: 5 Aug 1961 0^{h}05.^{m}01	
	a: 6625.6 km		a: 6625.6 km	
	e: 0.00712		e: 0.00712	
	i: 30°.064		i: 34°.412	
	Ω: 123°.703		Ω: 106°.740	
	ω: 114°.735		ω: 137°.208	
	M: 0°.0		M: 0°.0	
-1471.721 nm	X= -2774.566 km	or -1498.146 nm	X= -2140.269 km	or -1155.653 nm
-2760.651	Y= -5159.276	-2785.786	Y= -5684.746	-3069.517
[illegible]82.266	Z= 2993.215	1616.207	Z= 2525.588	1363.708
3.65197 nm/sec	$\dot{x}$= 6.29039 km/sec	or 3.39654 nm/sec	$\dot{x}$= 6.05736 km/sec	or 3.27071 nm/sec
-2.09636	$\dot{y}$= -4.33286	-2.33956	$\dot{y}$= -3.71983	-2.00855
-0.24528	$\dot{z}$= -1.63747	-0.68416	$\dot{z}$= -3.23961	-1.74925



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ıar Missions with Variable Launch Azimuth
t h = 200km (110 nmi).

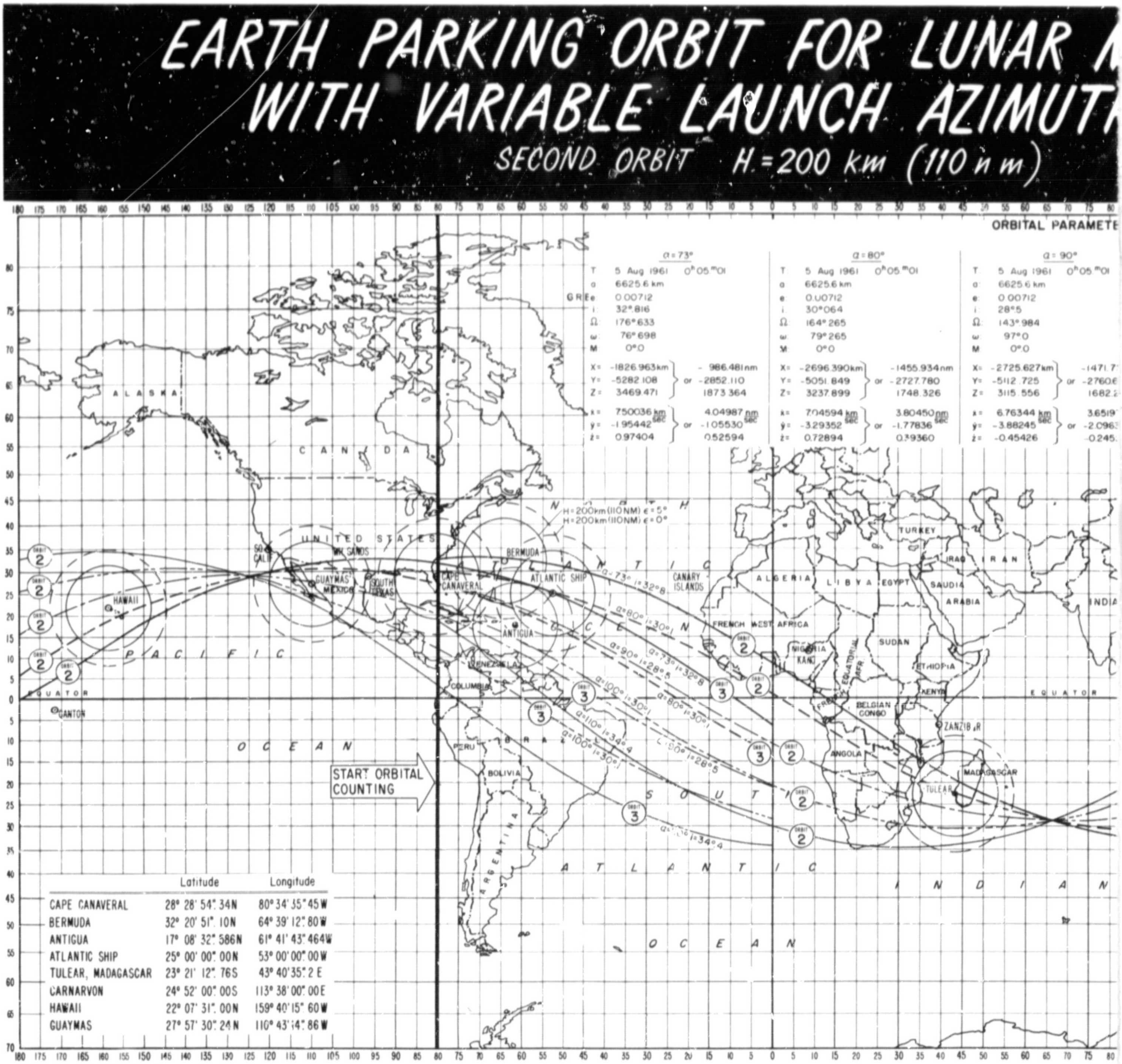
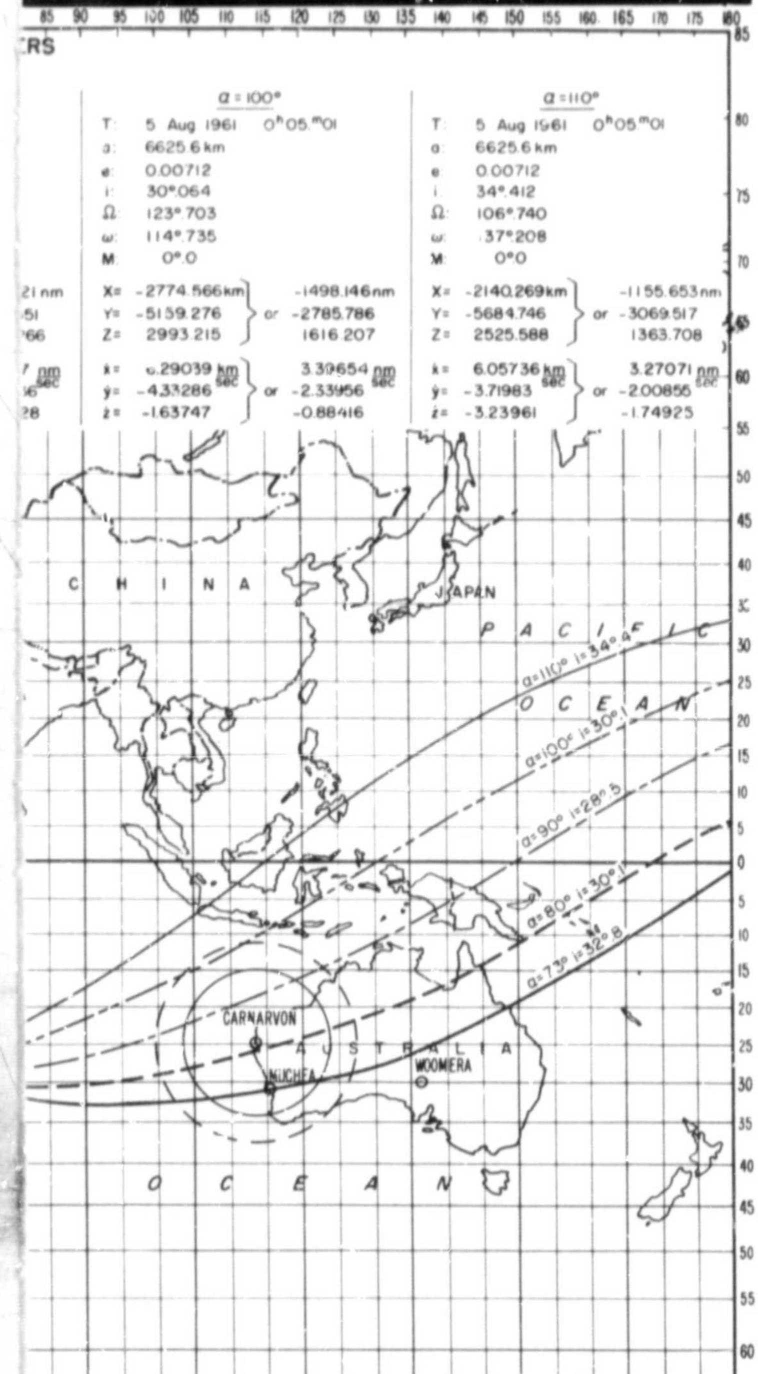


Figure 2 — Earth Parking Orbit for Lunar
(a), Second Orbit h =

4-1

MISSIONS
H (α)



Missions with Variable Launch Azimuth
: 200km (110 nmi).

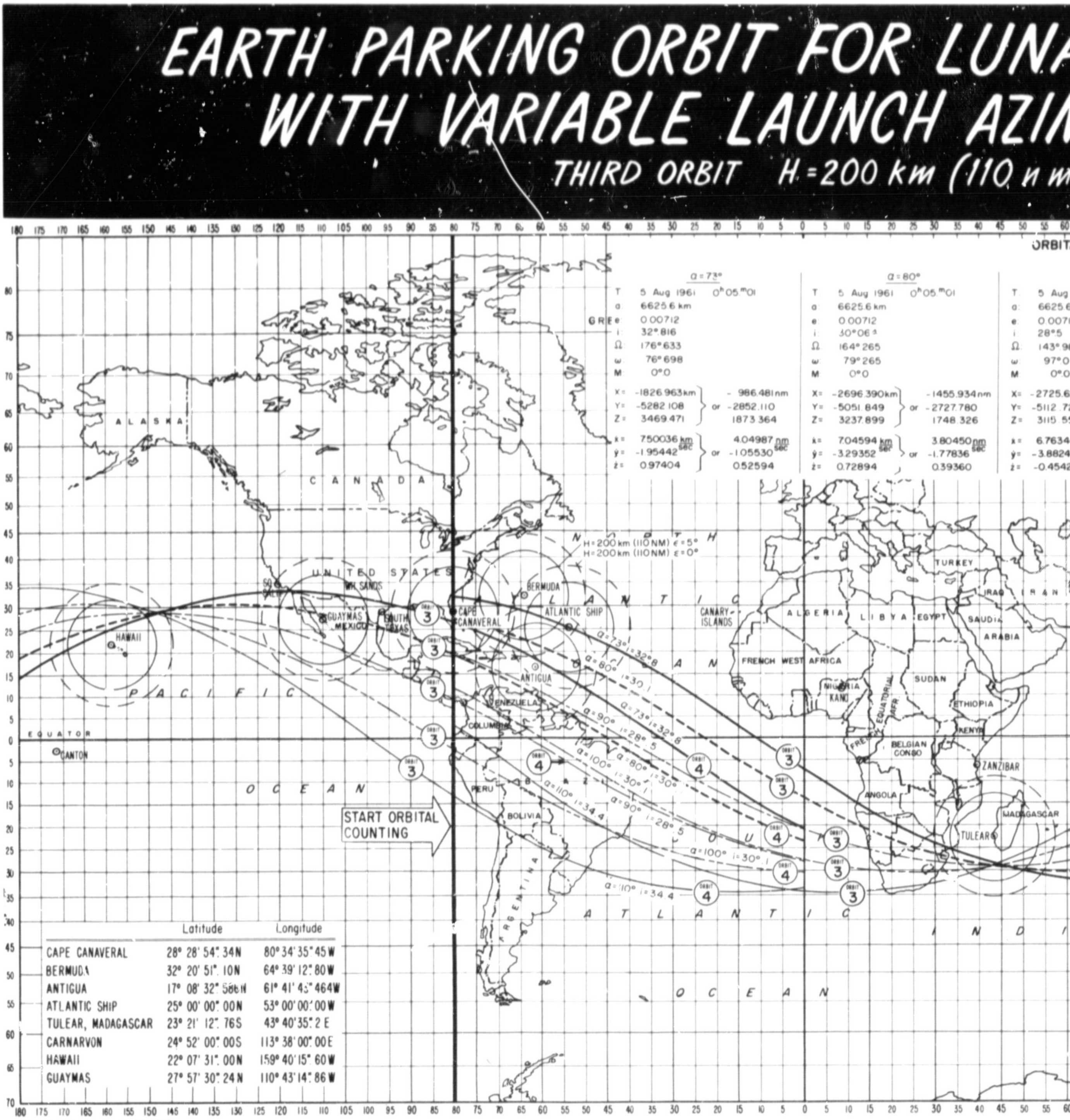
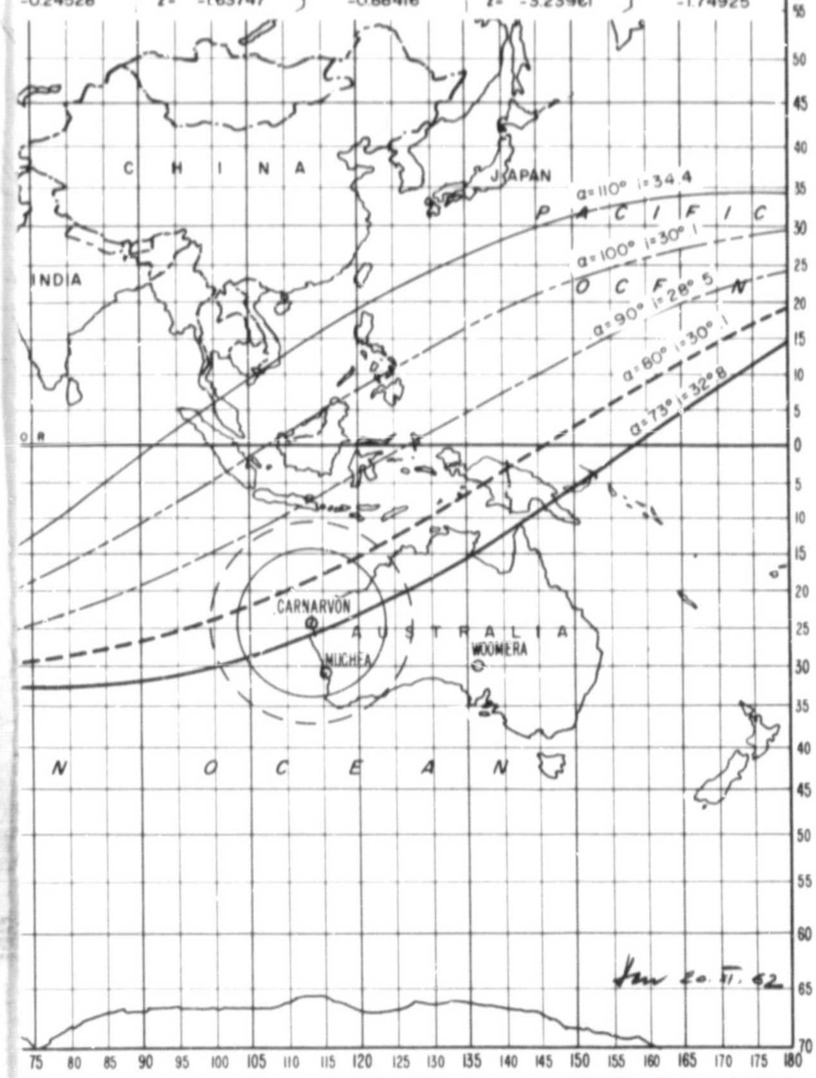


Figure 3 — Earth Parking Orbit
(α), Third C

MISSIONS
TH (α)

AMETERS

	$a = 100°$		$a = 110°$	
05.^{m}01	T: 5 Aug 1961 0^{h}05.^{m}01		T: 5 Aug 1961 0^{h}05.^{m}01	
	a: 6625.6 km		a: 6625.6 km	
	e: 0.007:2		e: 0.00712	
	i: 30°.064		i: 34°.412	
	Ω: 123°.703		Ω: 106°.740	
	ω: 114°.735		ω: 137°.208	
	M: 0°.0		M: 0°0	
-1471.721 nm	X= -2774.566 km	or -1498.146 nm	X= -2140.269 km	or -1155.653 nm
-2760.651	Y= -5159.276	or -2785.786	Y= -5684.746	or -3069.517
1682.266	Z= 2993.215	or 1616.207	Z= 2525.588	or 1363.708
3.65197 nm/sec	$\dot{x}$= 6.29039 km/sec	or 3.39654 nm/sec	$\dot{x}$= 6.05736 km/sec	or 3.27071 nm/sec
-2.09636	$\dot{y}$= -4.33286	or -2.33956	$\dot{y}$= -3.71983	or -2.00855
-0.24528	$\dot{z}$= -1.63747	or -0.88416	$\dot{z}$= -3.23961	or -1.74925



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h = 200km (110 nmi).

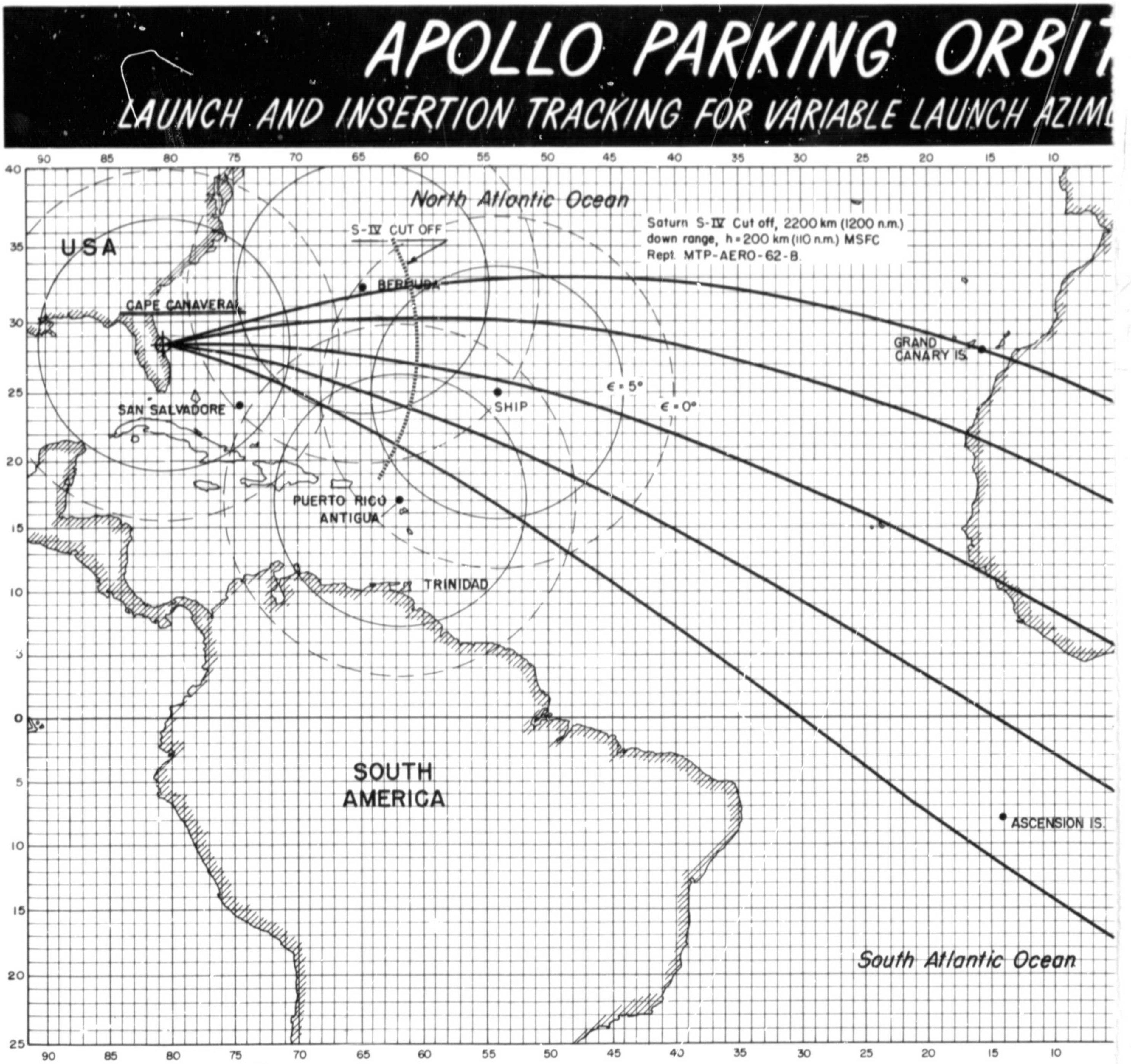


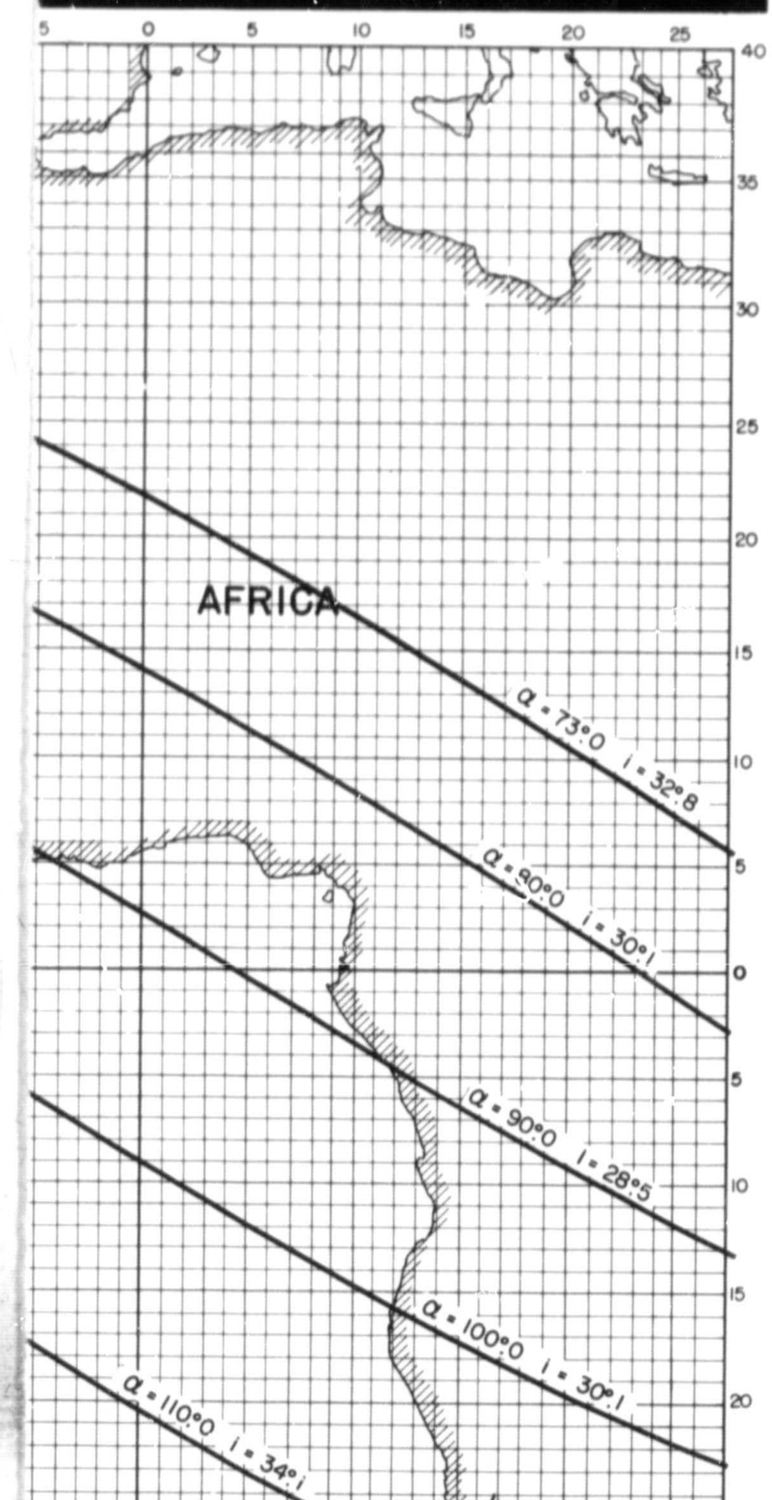
Figure 4 — Apollo Parking Orbits — Lau
Launch Azimuths α, h

6

6-1

TS

UTHS α, h=200 km(110 nm)



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nch and Insertion Tracking for Variable
= 200km (110 nmi).

, -2

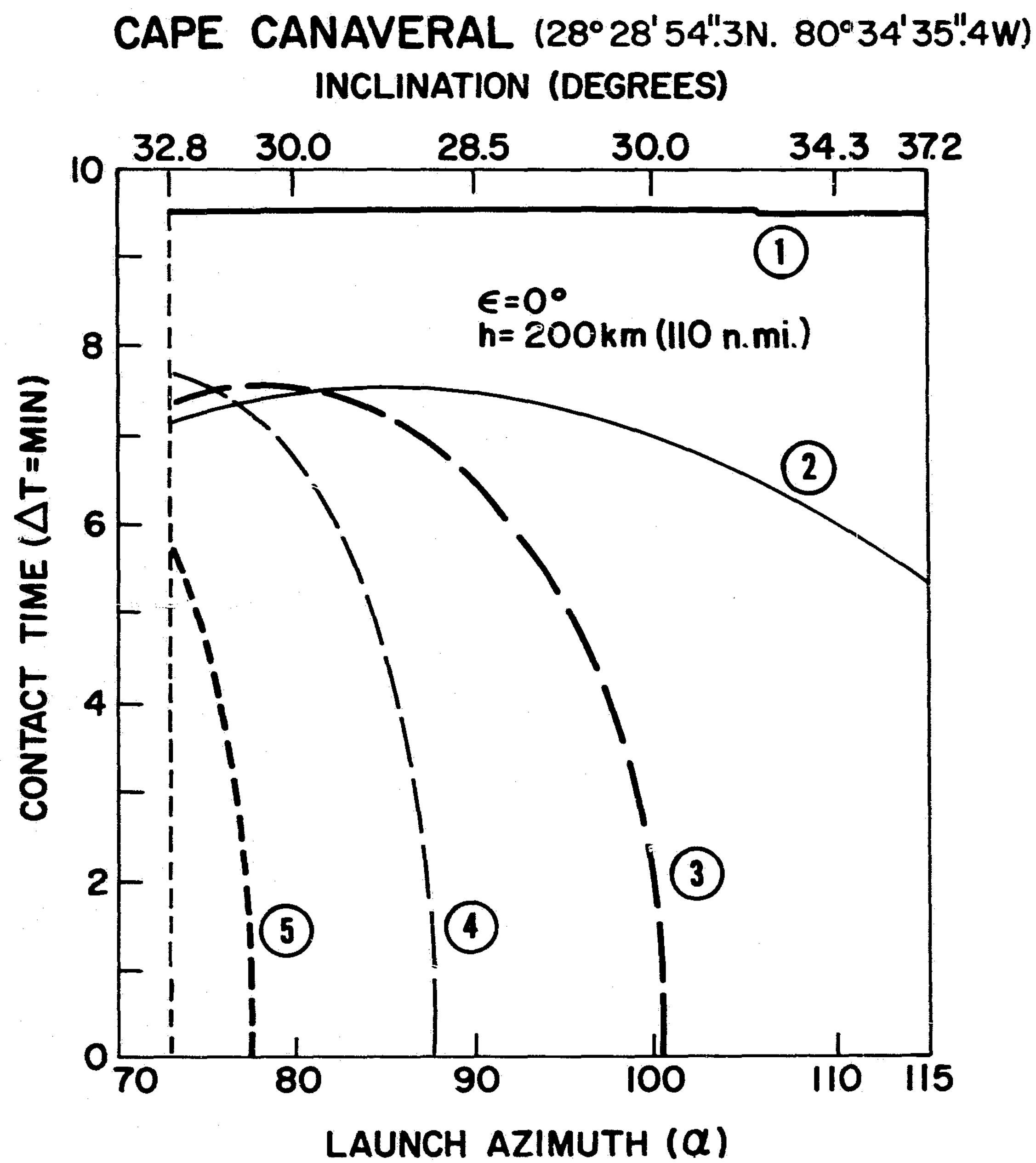

Figure 5a — Station contact time for Cape Canaveral for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

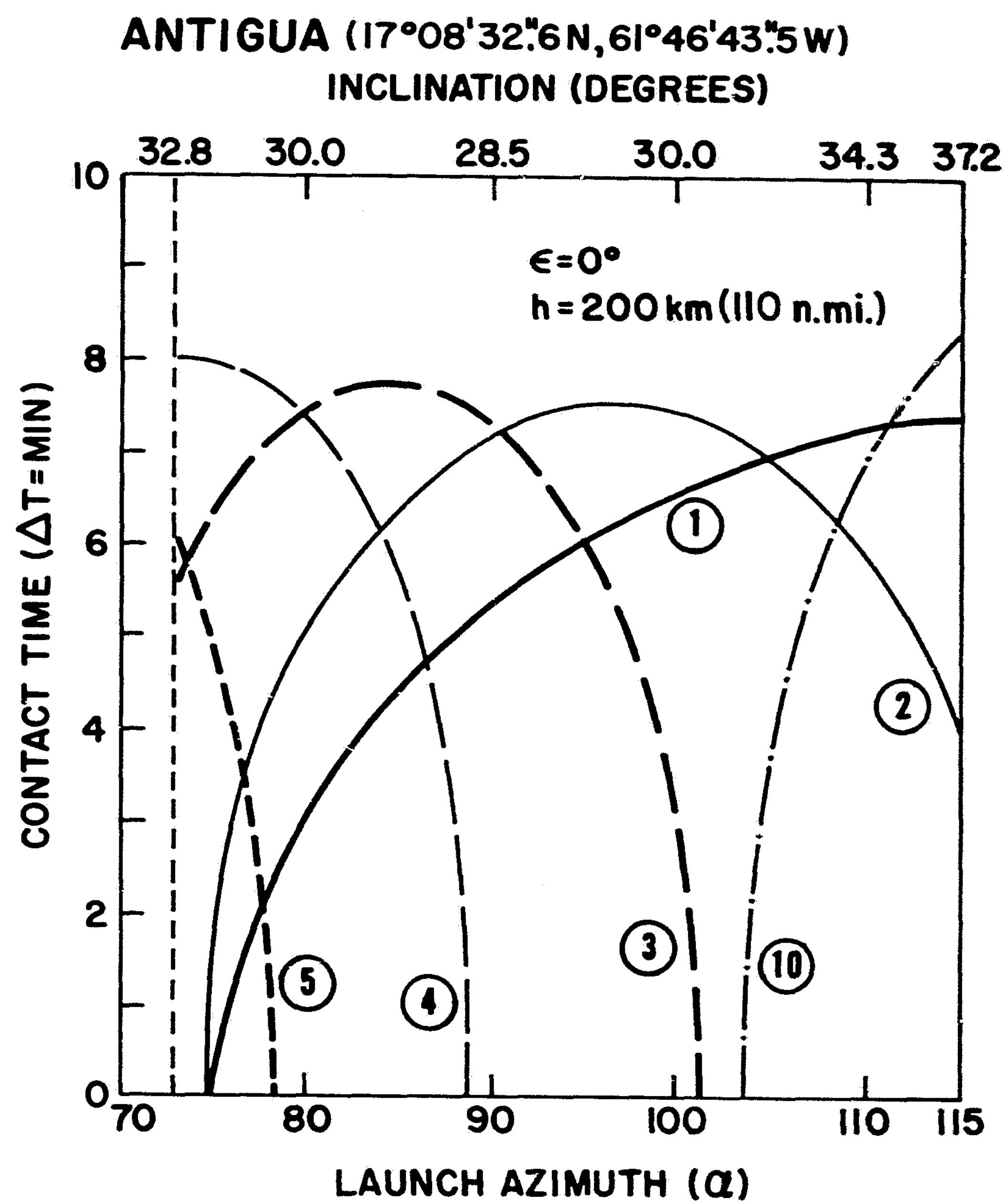

Figure 5b — Station contact time for Antigua for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

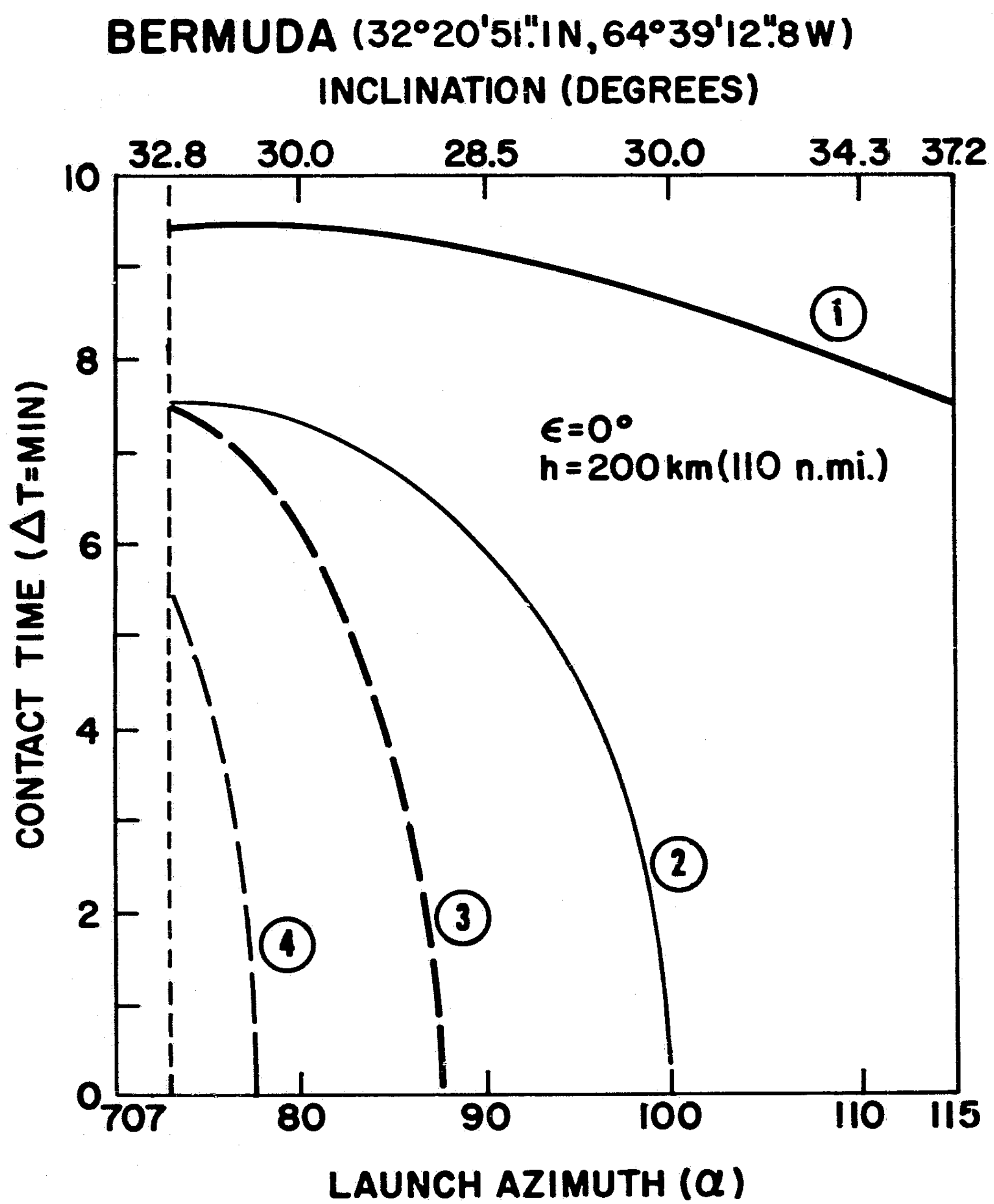

Figure 5c — Station contact time for Bermuda for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

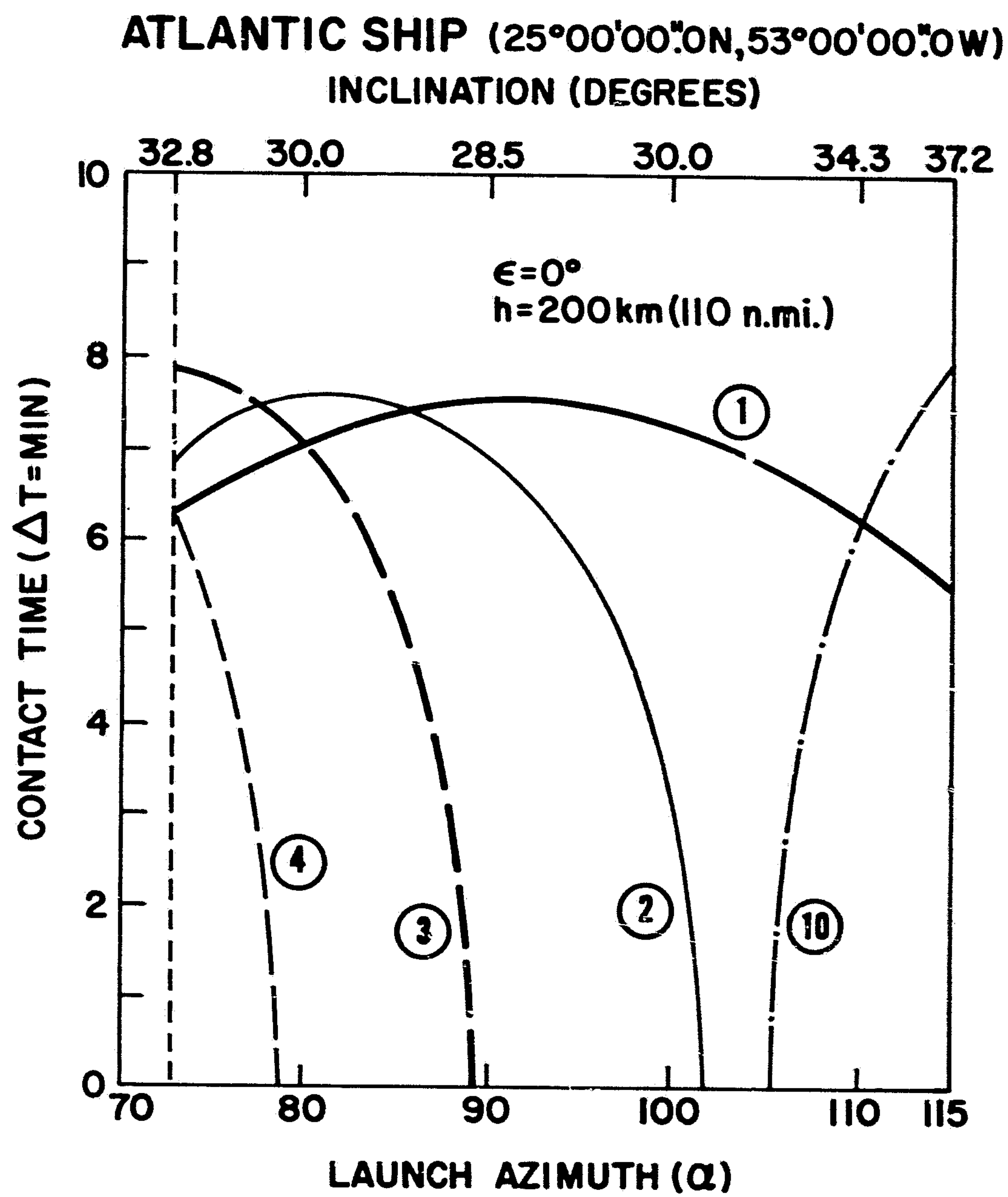

Figure 5d — Station contact time for Atlantic Ship for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

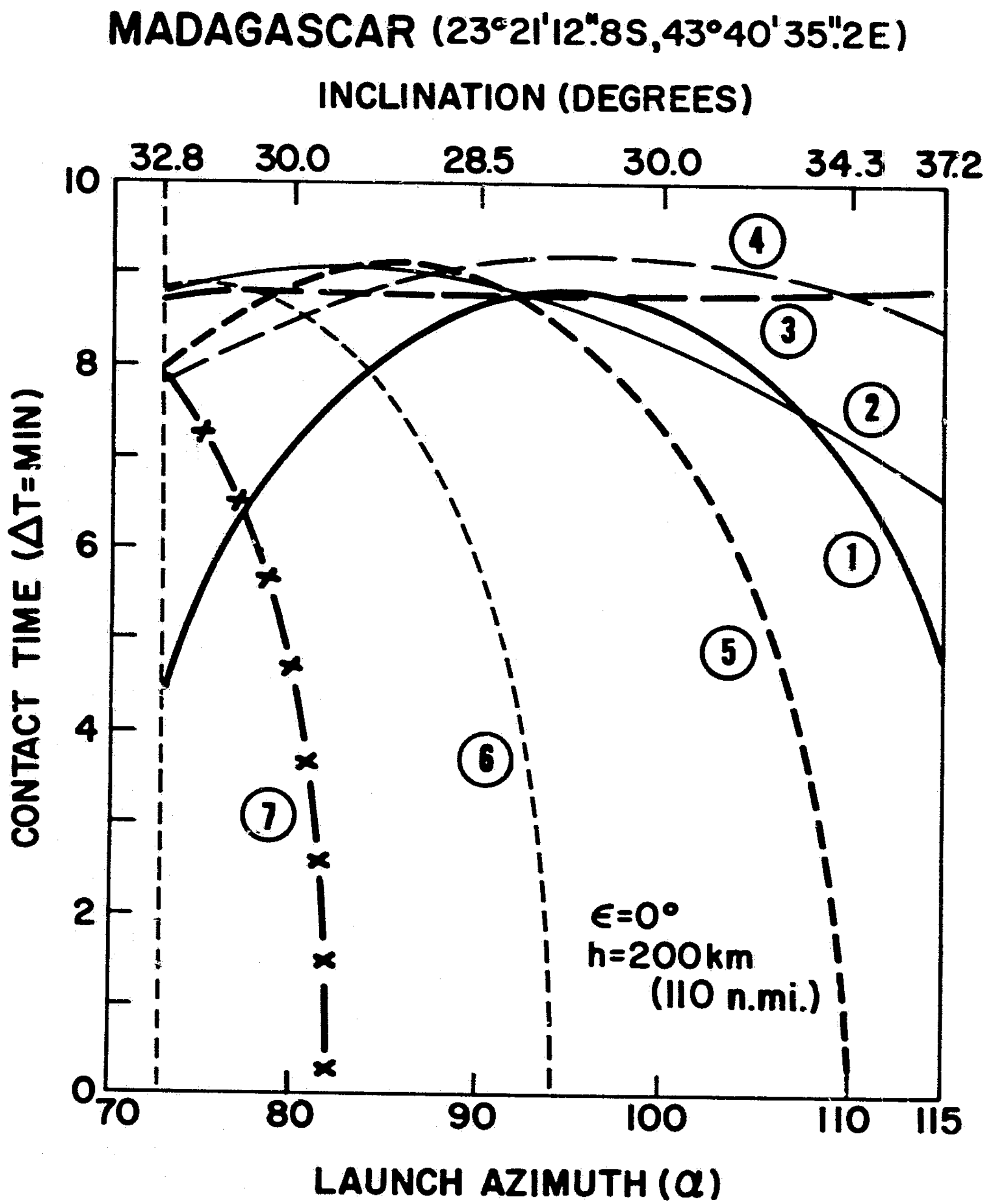

Figure 5e — Station contact time for Madagascar for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

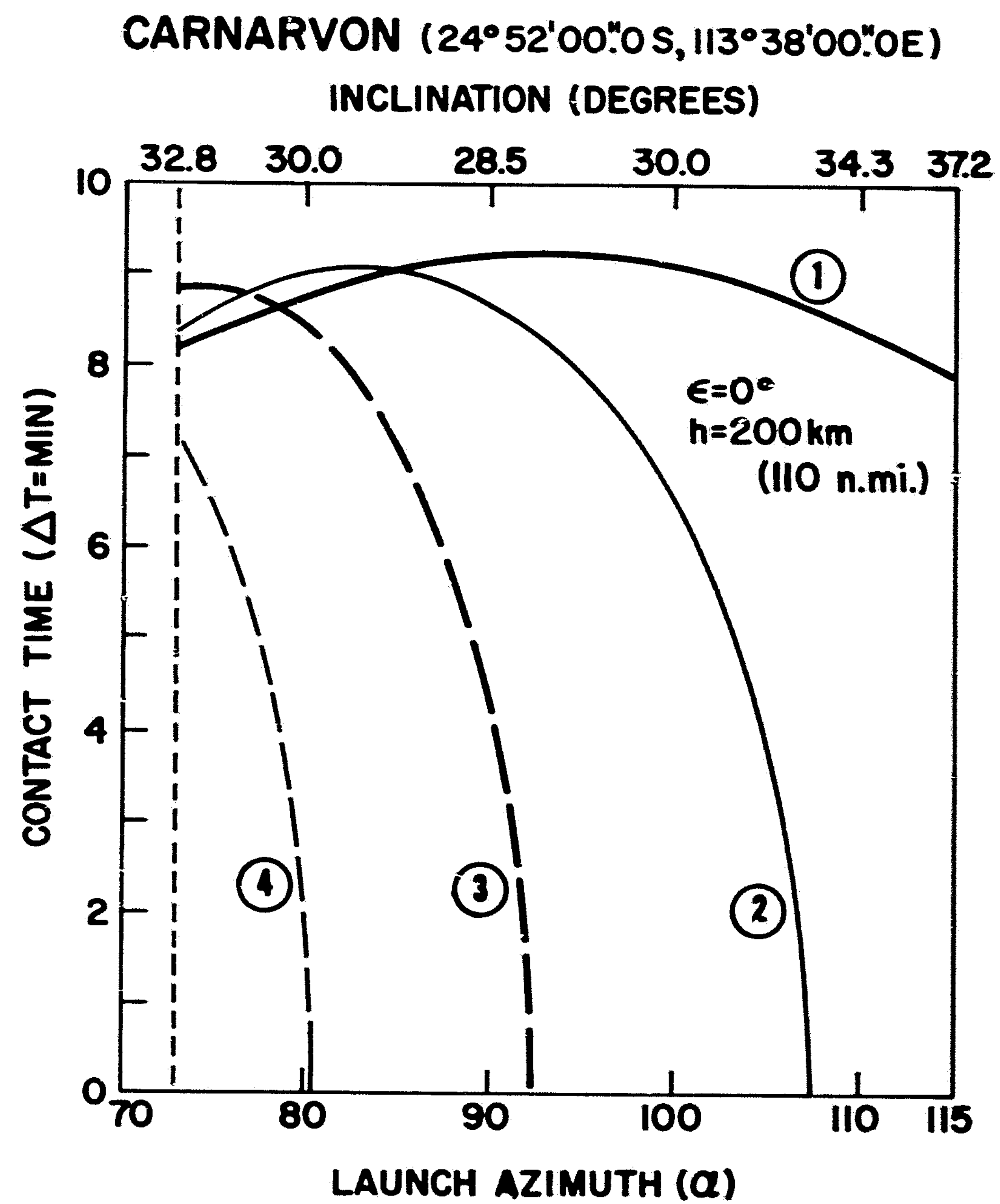

Figure 5f — Station contact time for Carnarvon for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

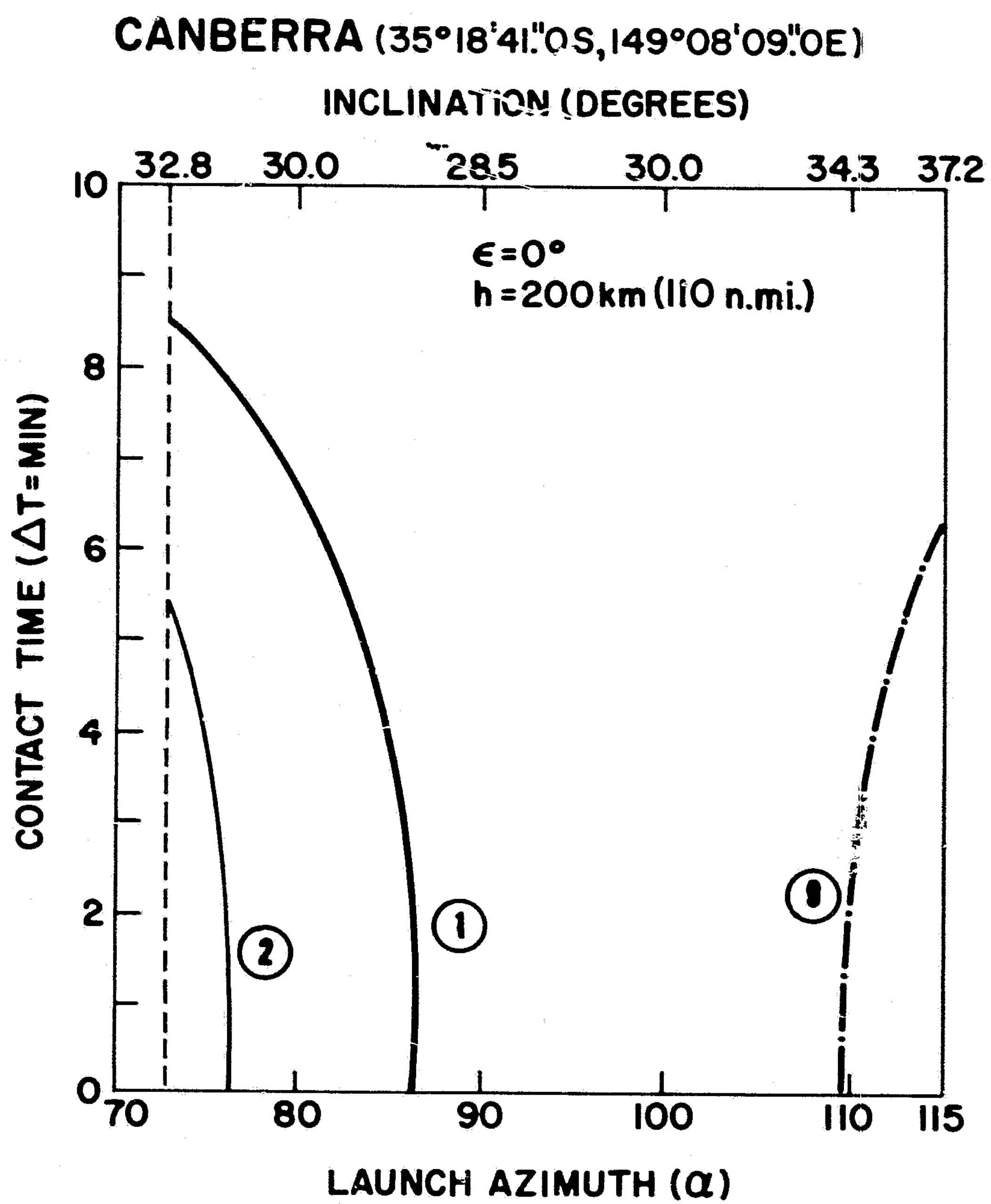

Figure 5g — Station contact time for Canberra for an elevation angle ϵ = 0° and an orbit height of h = 200km (110 n.mi.).

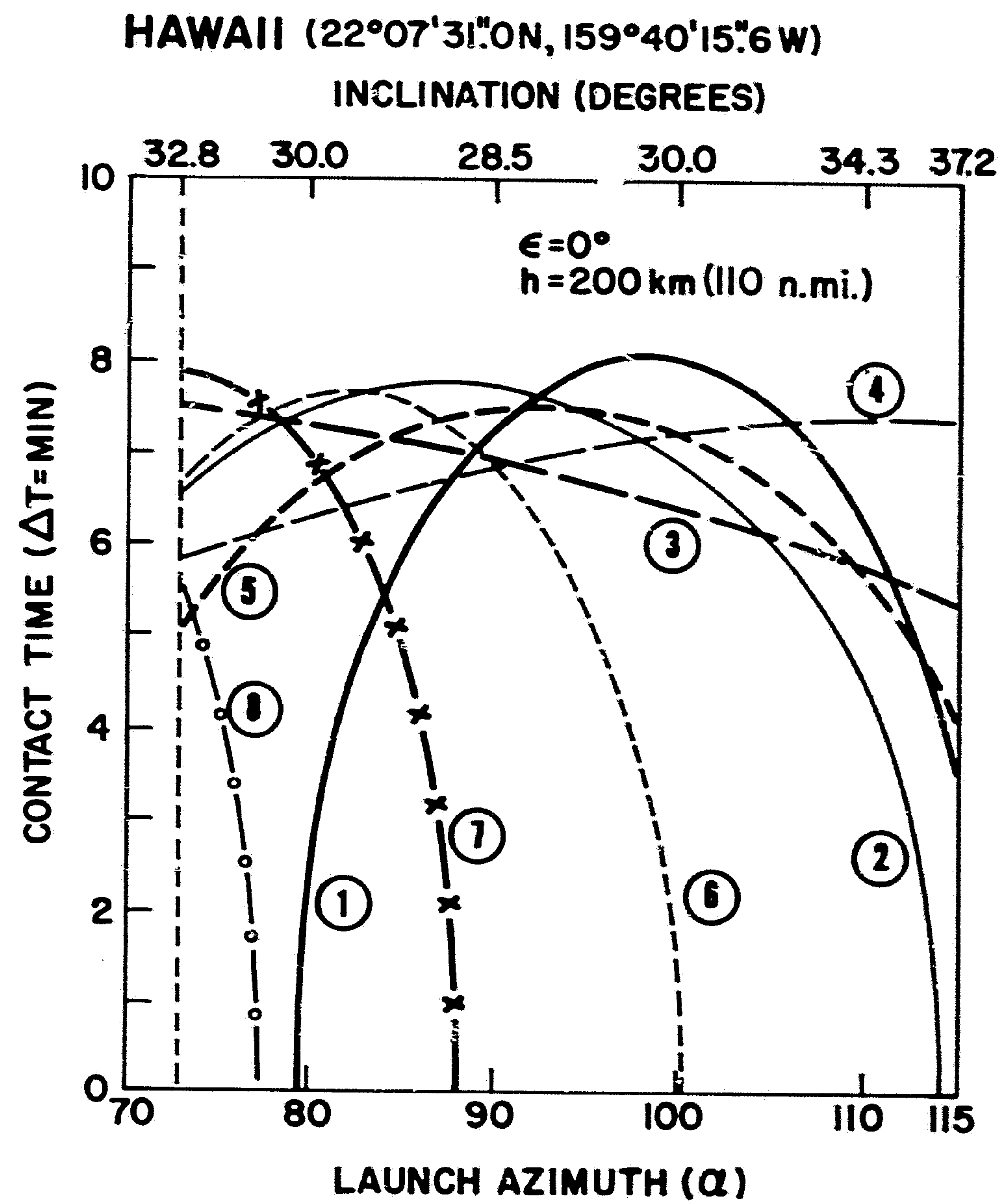

Figure 5h — Station contact time for Hawaii for an elevation angle ϵ = 0° and an orbit height of h = 200km (110 n.mi.).

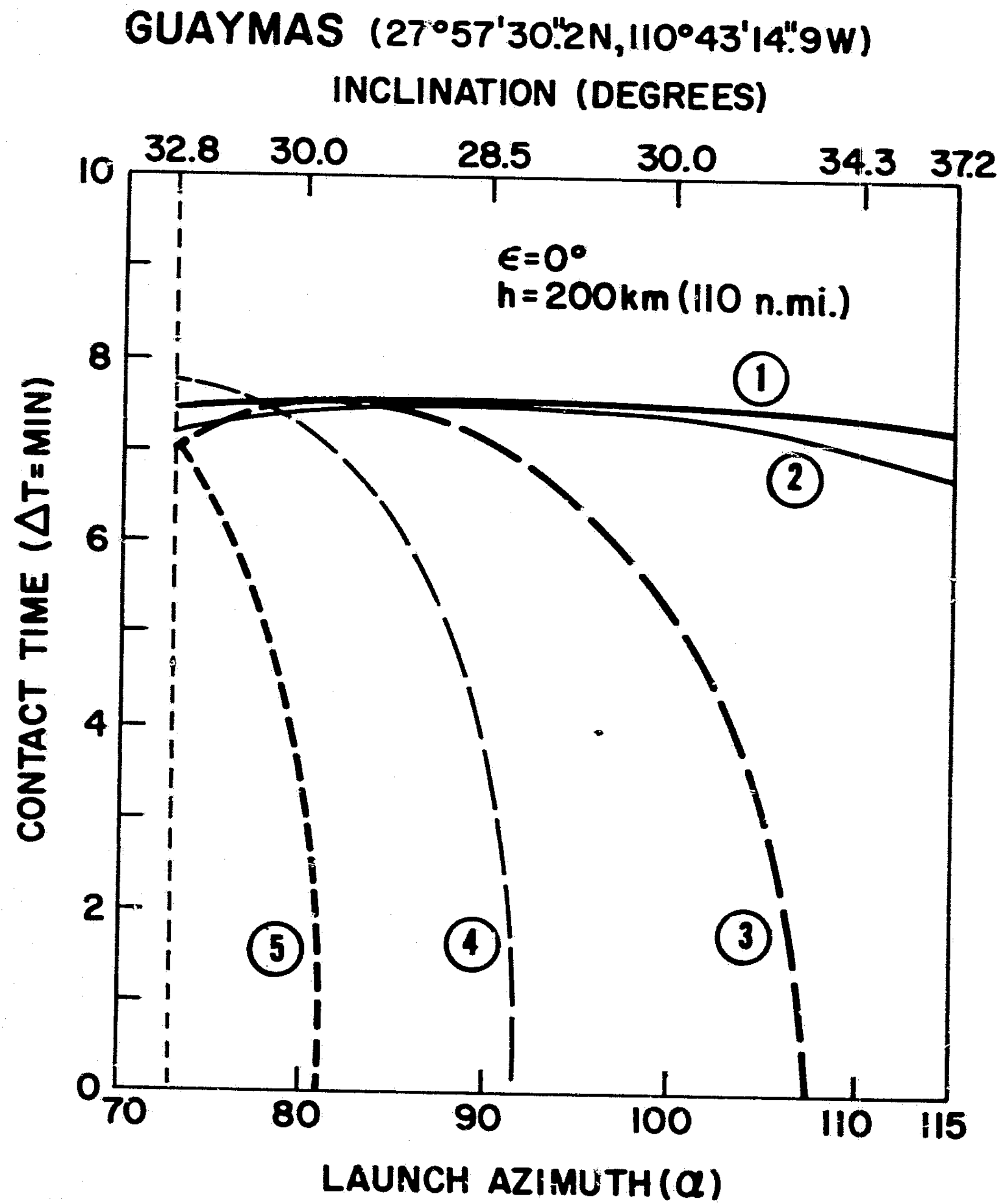

Figure 5i — Station contact time for Guaymas for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

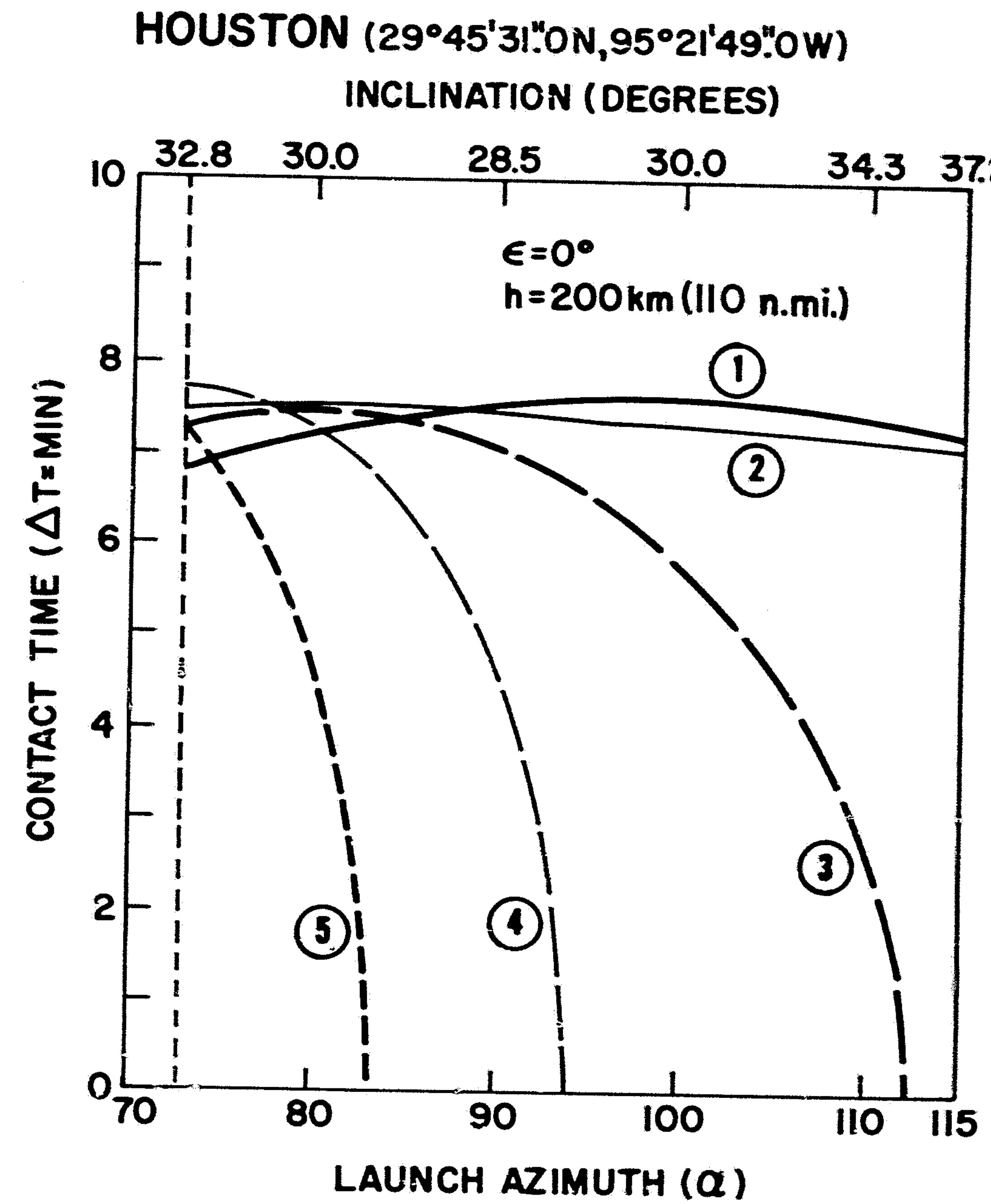

Figure 5j — Station contact time for Houston for an elevation angle $\epsilon = 0°$ and an orbit height of h = 200km (110 n.mi.).

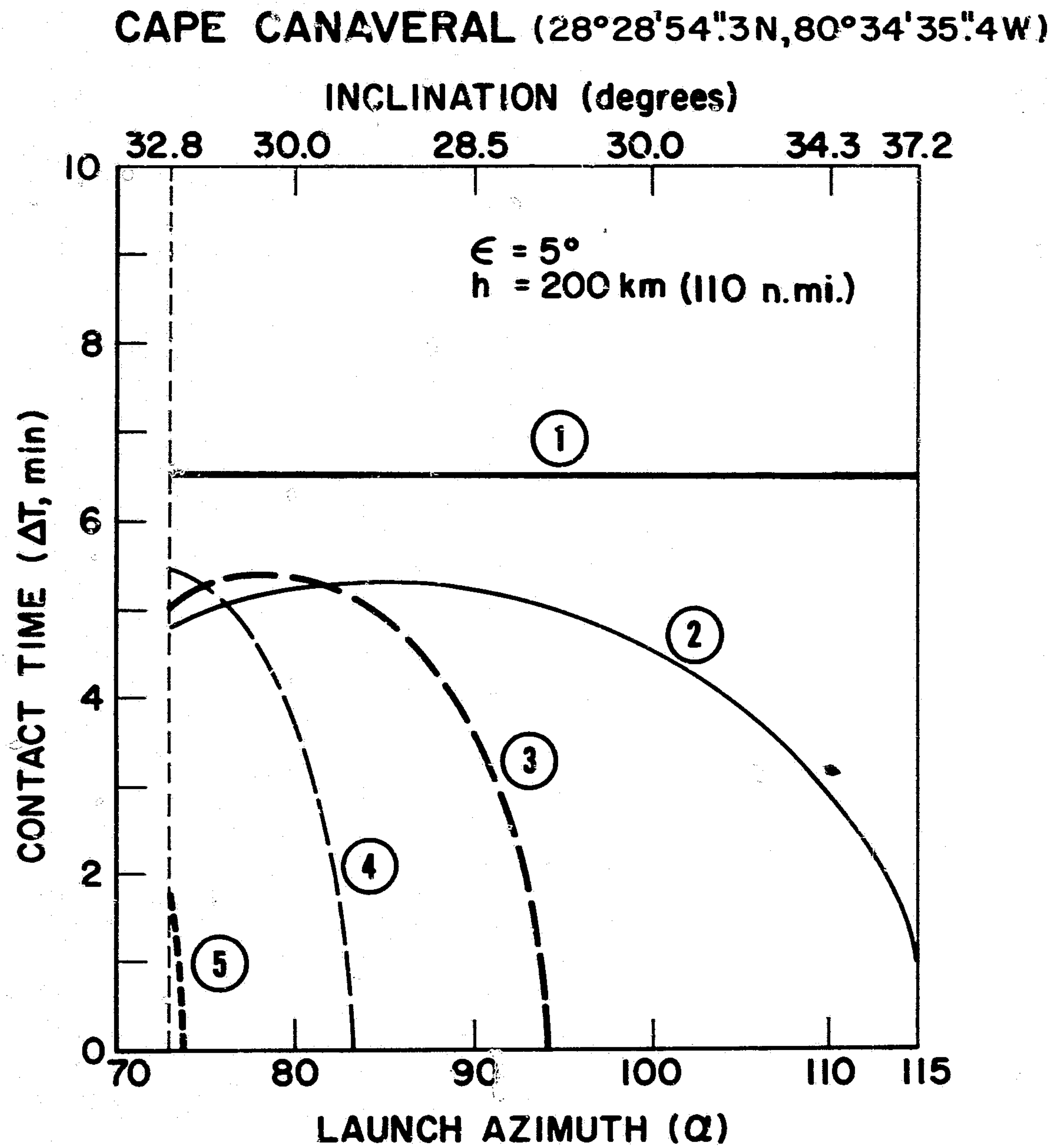

Figure 6a — Station contact time for Cape Canaveral for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

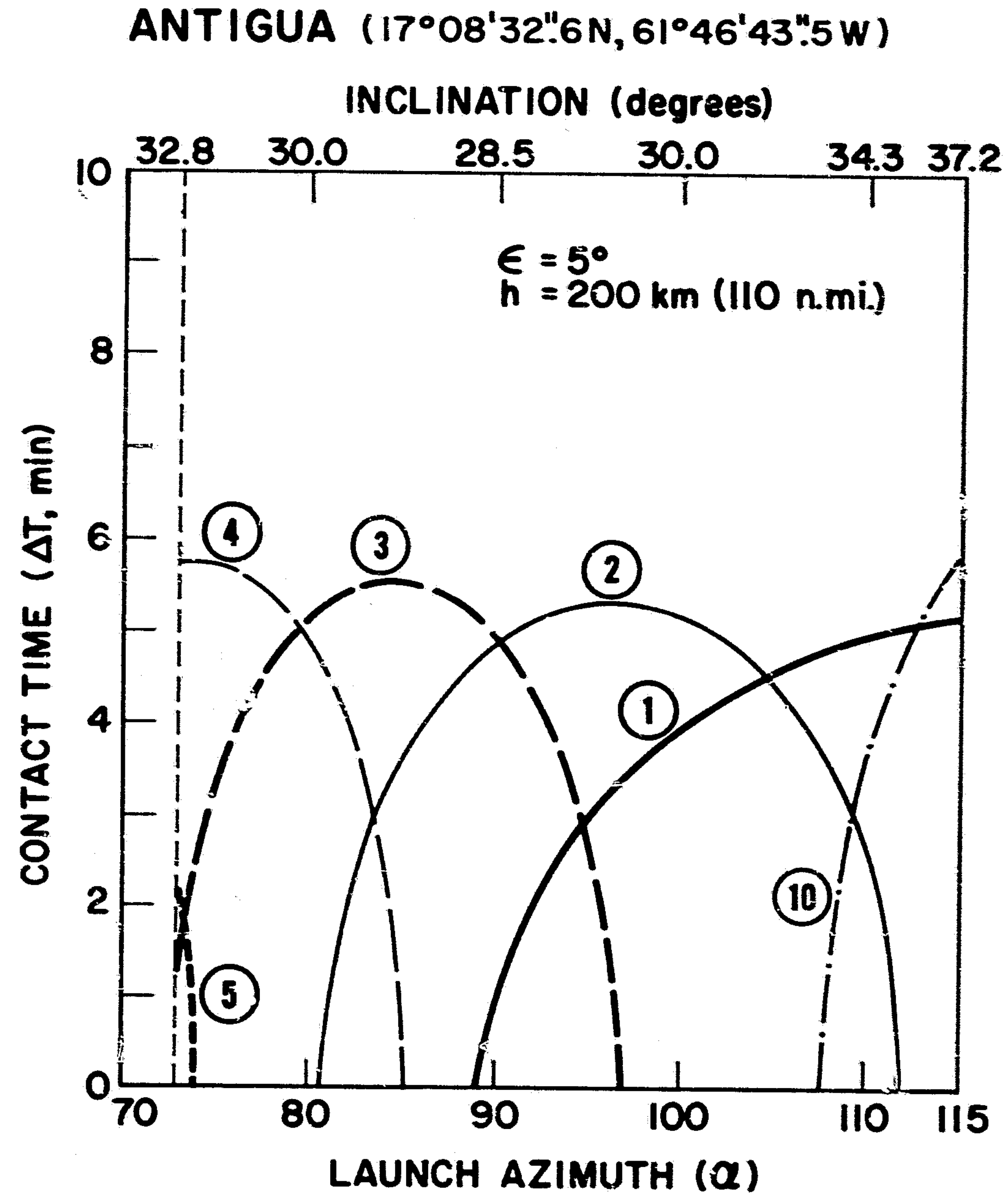

Figure 6b — Station contact time for Antigua for an elevation angle ϵ = 5° and an orbit height of h = 200km (110 n.mi.).

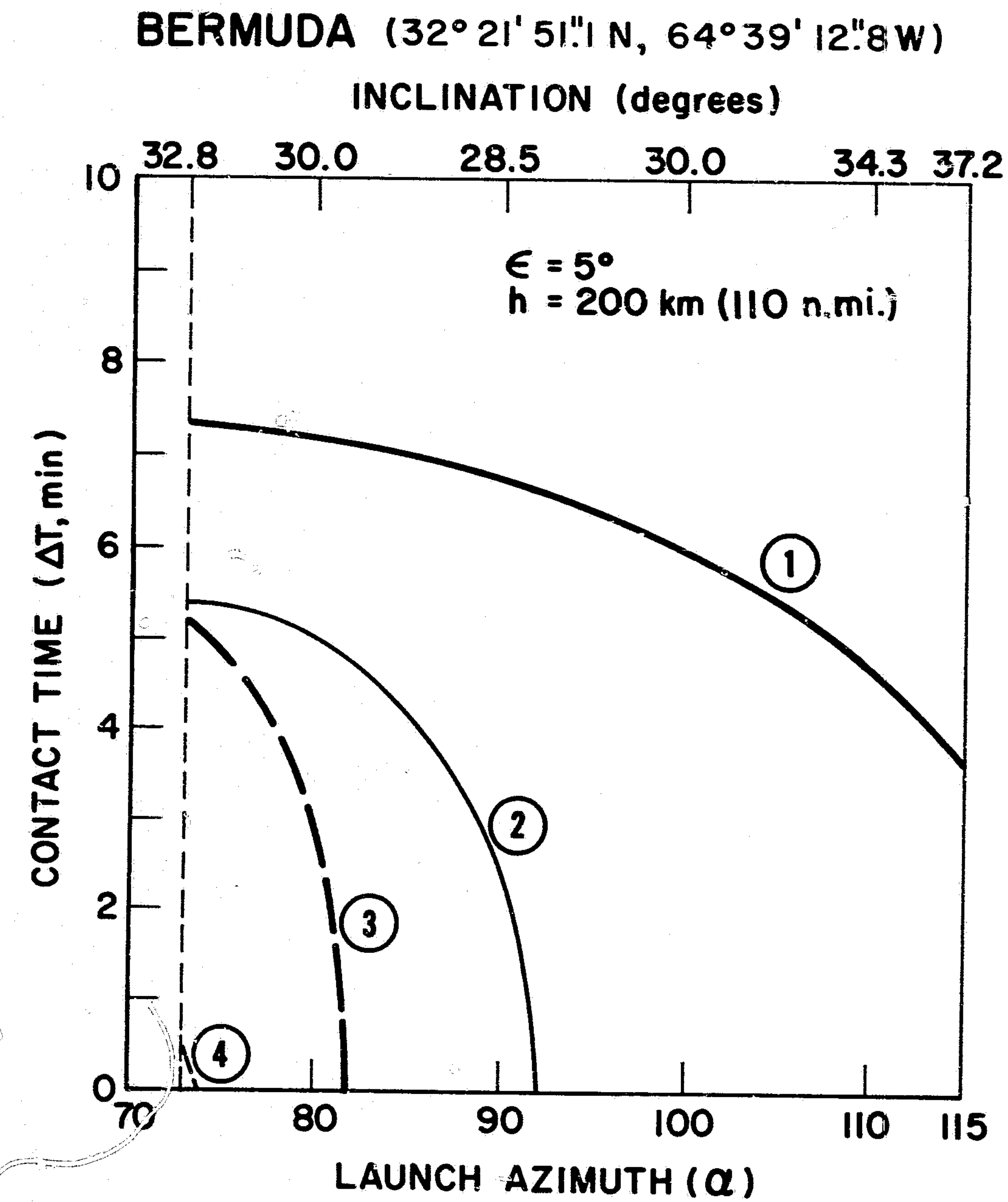

Figure 6c — Station contact time for Bermuda for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

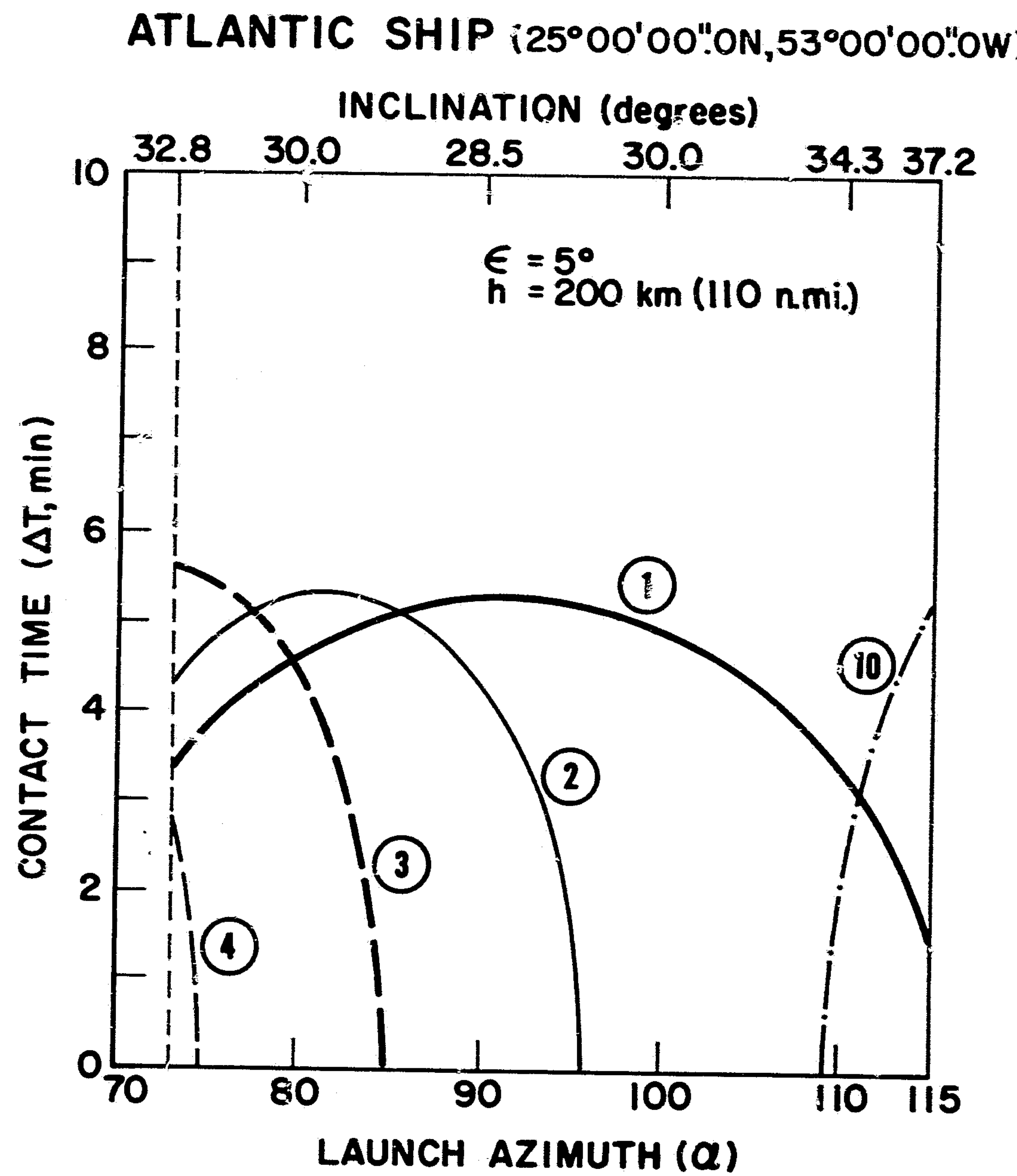

Figure 6d — Station contact time for Atlantic Ship for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

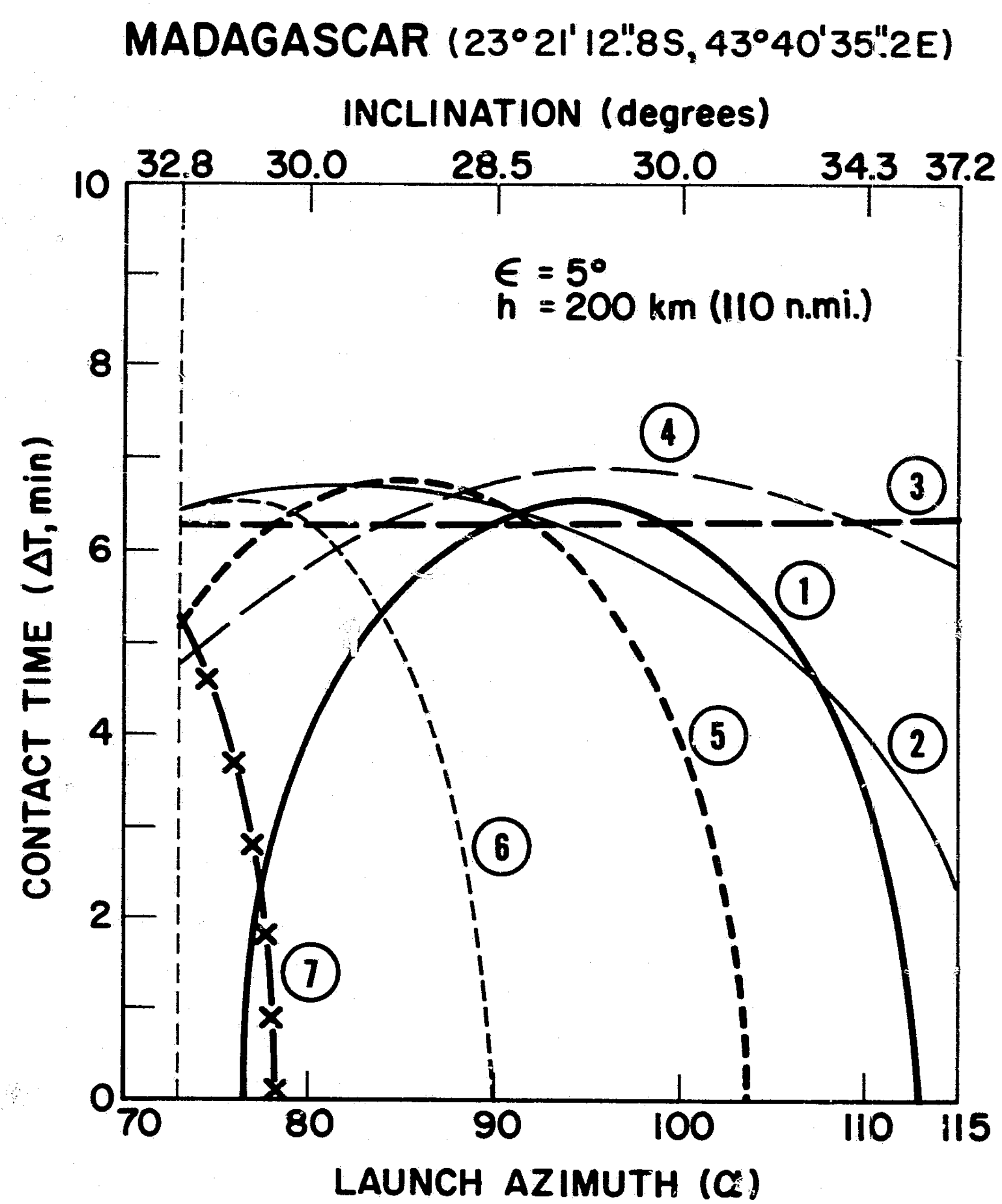

Figure 6e — Station contact time for Madagascar for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

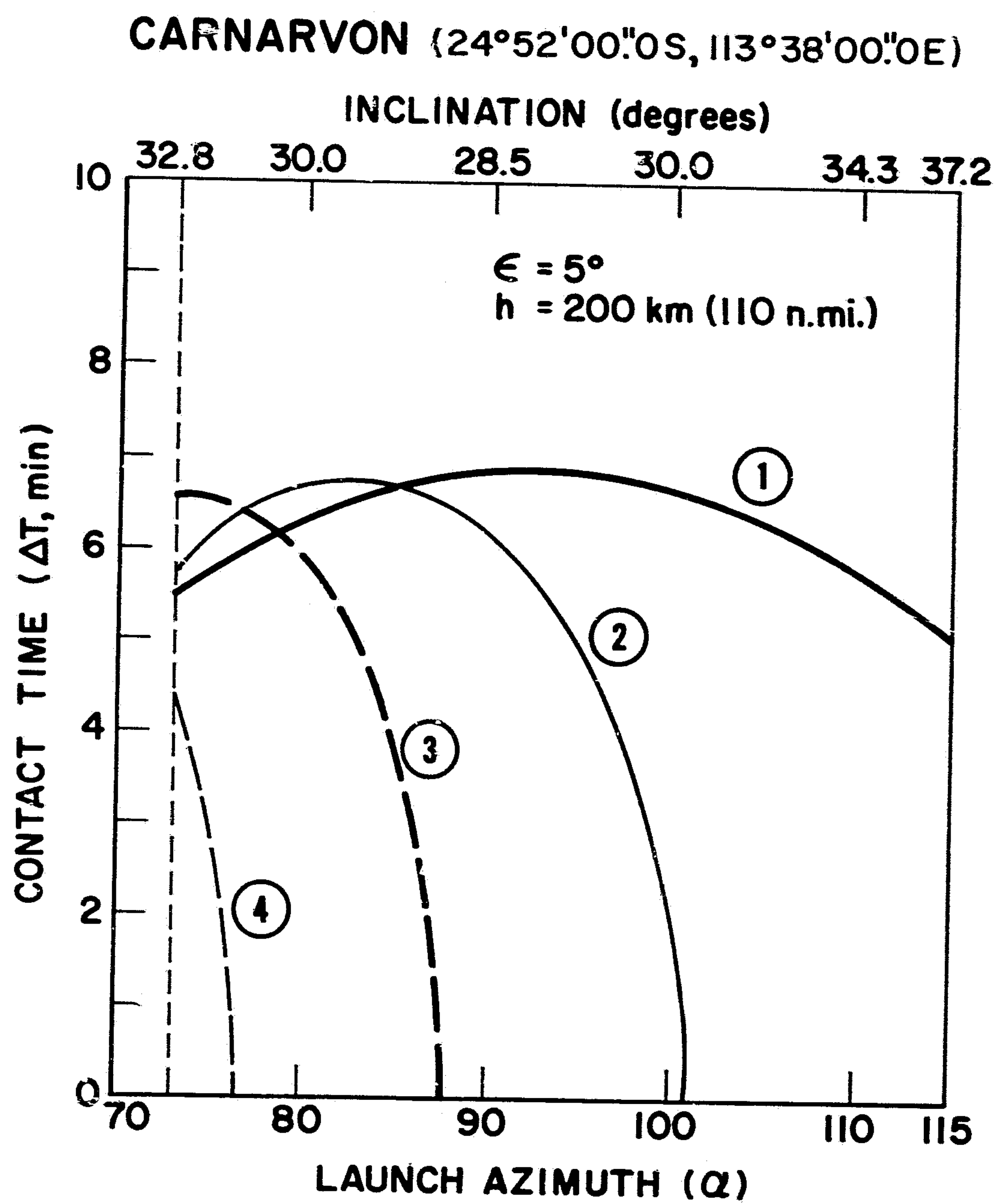

Figure 6f — Station contact time for Carnarvon for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

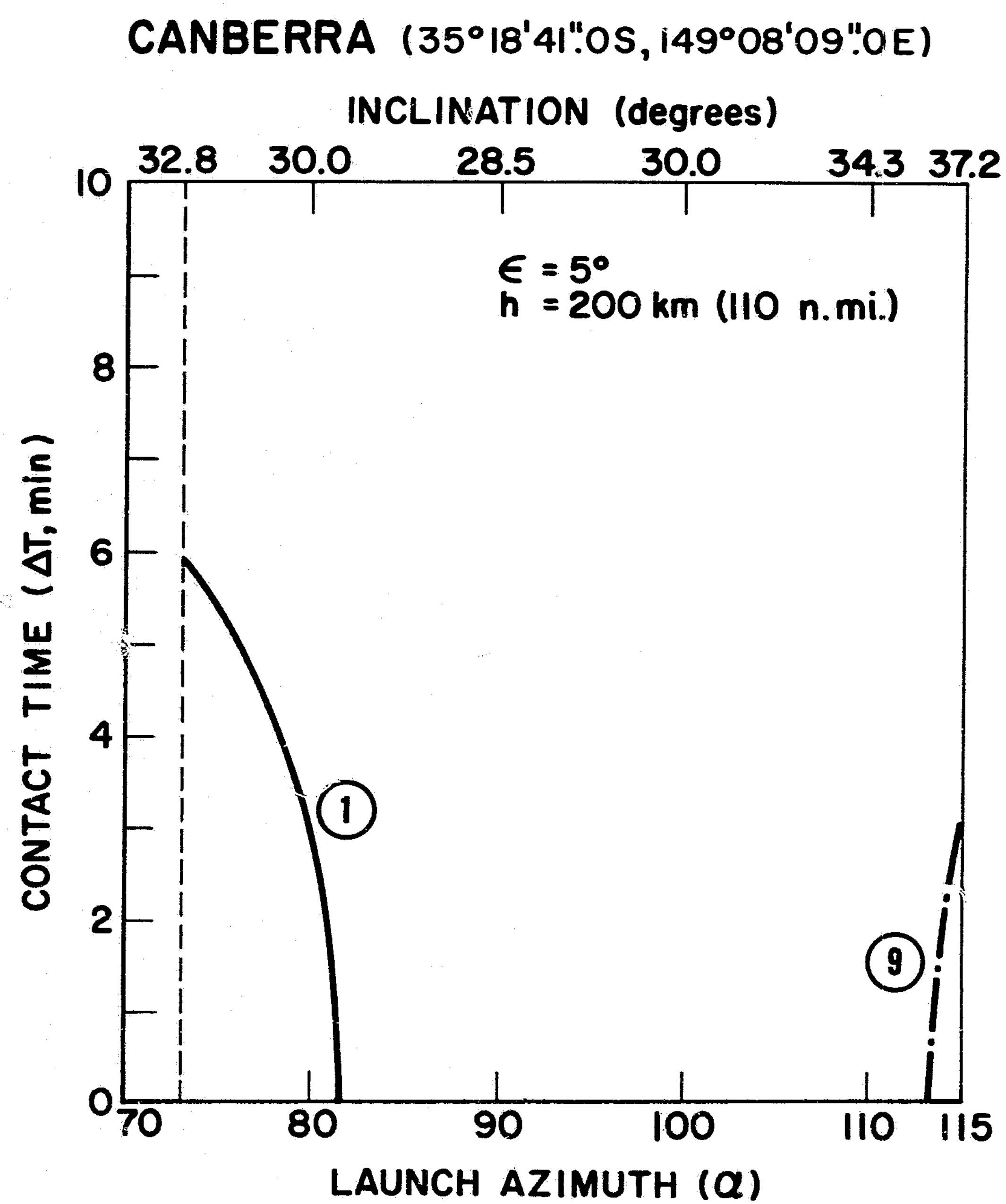

Figure 6g — Station contact time for Canberra for an elevation angle $\epsilon = 5°$ and on orbit height of h = 200km (110 n.mi.).

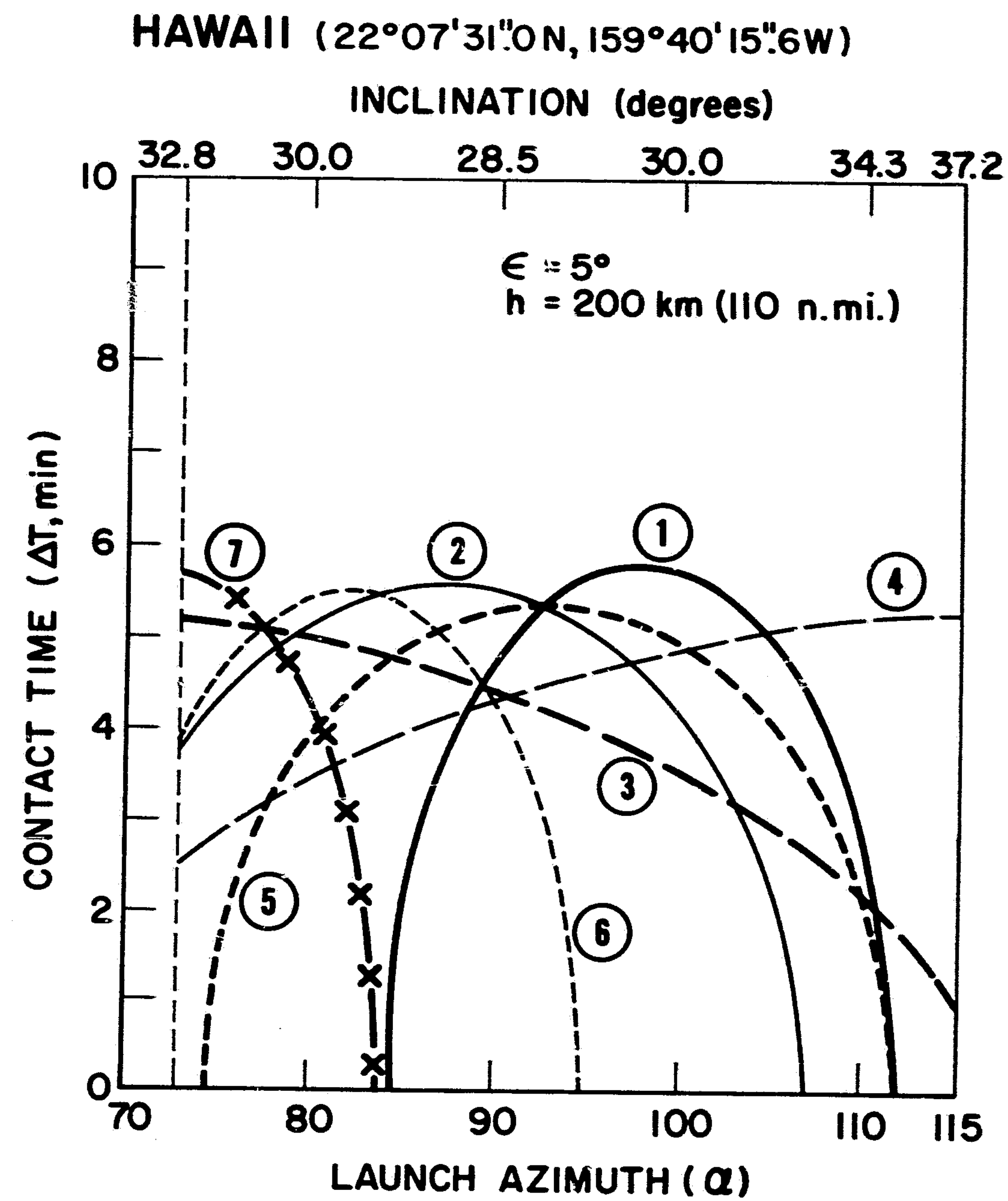

Figure 6h — Station contact time for Hawaii for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

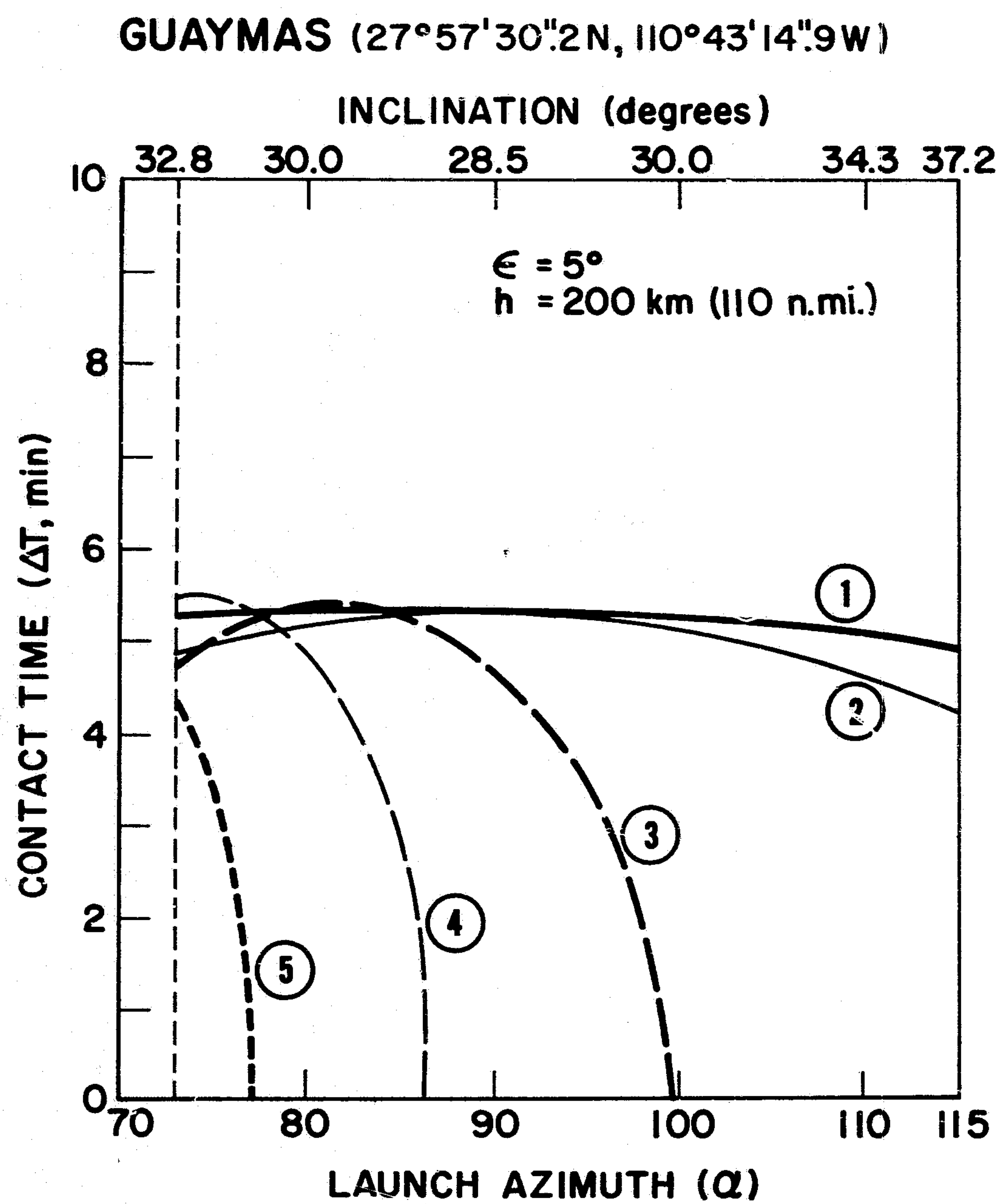

Figure 6i — Station contact time for Guaymas for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

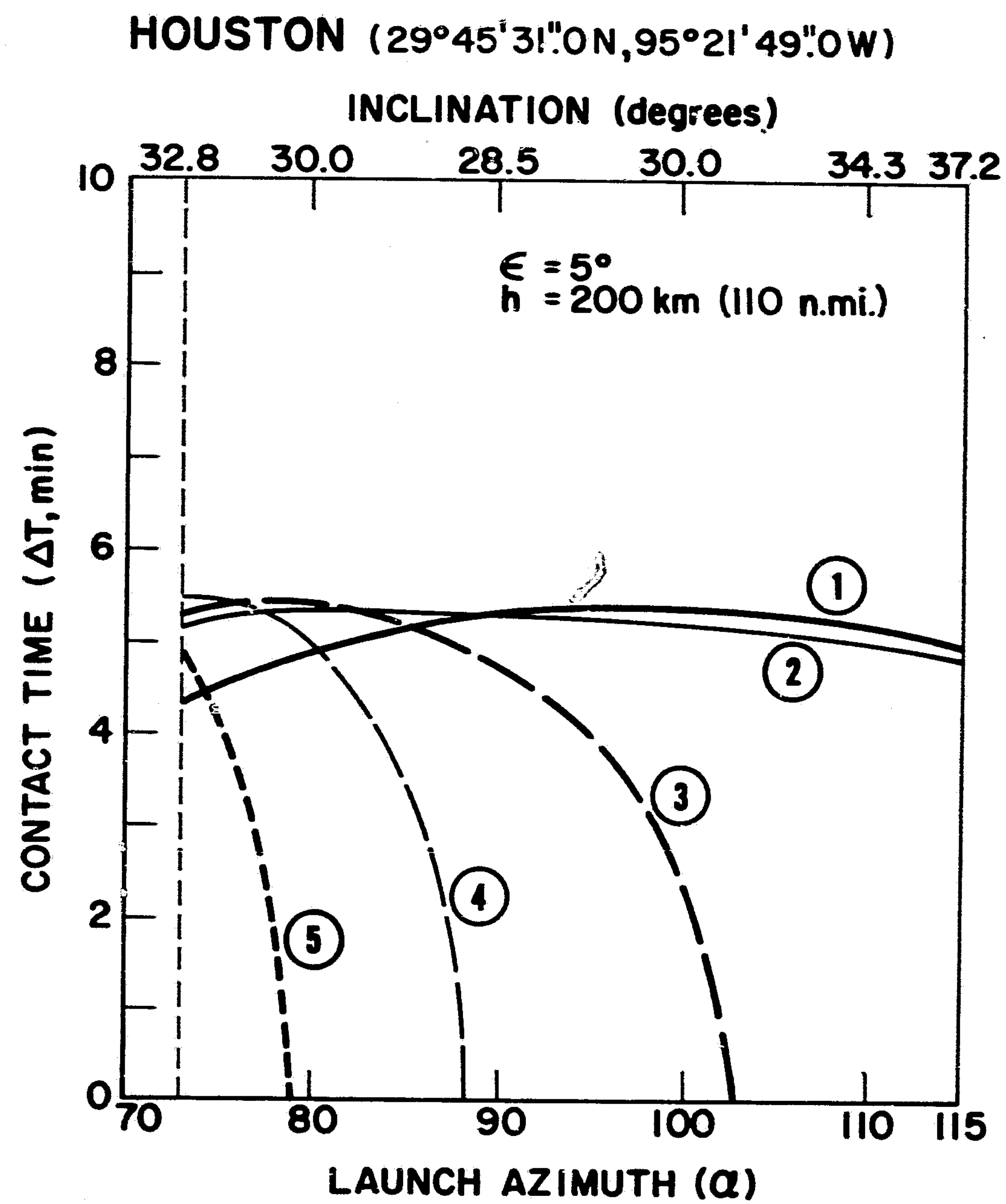

Figure 6j — Station contact time for Houston for an elevation angle $\epsilon = 5°$ and an orbit height of h = 200km (110 n.mi.).

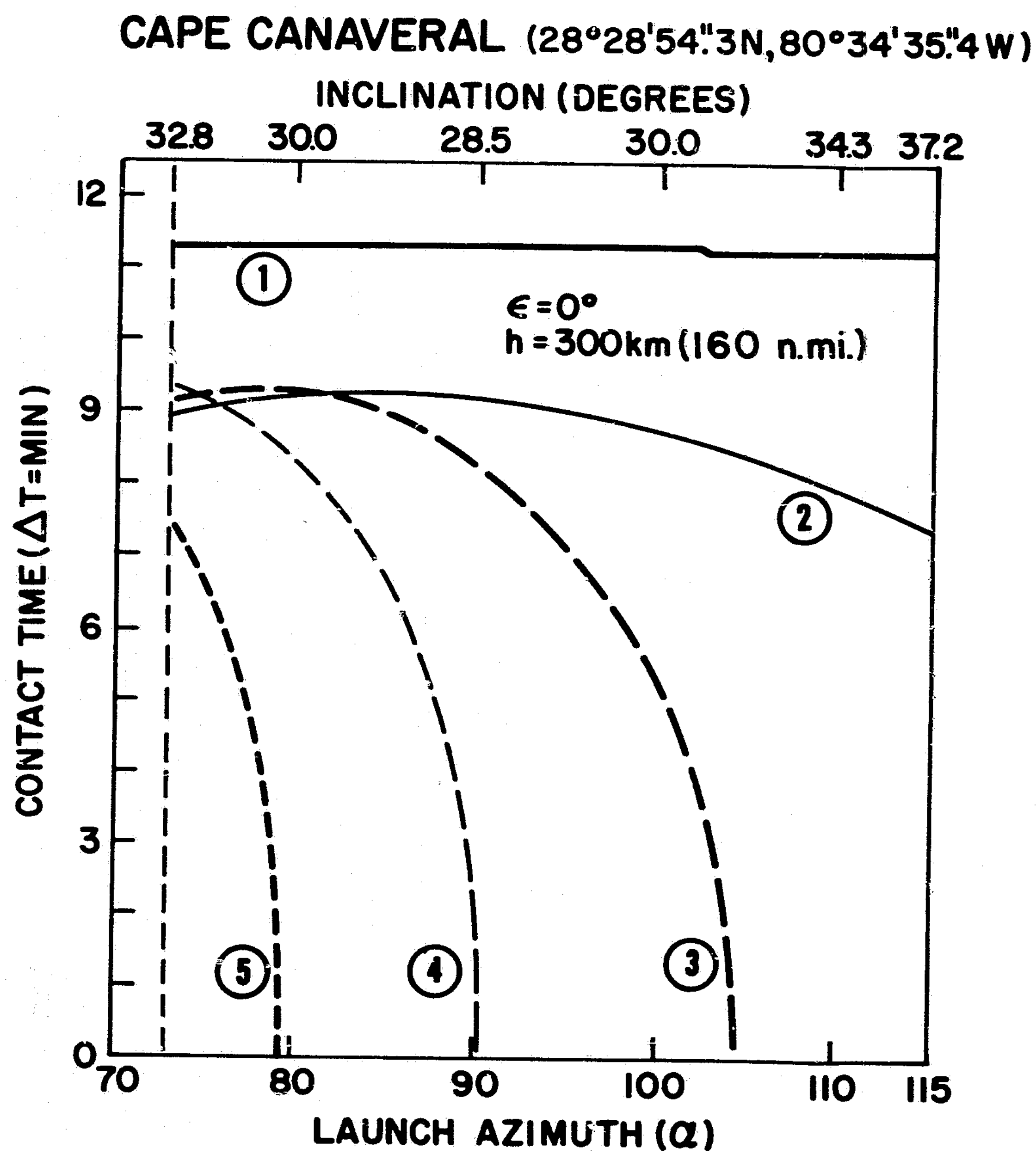

Figure 7a — Station contact time for Cape Canaveral for an elevation angle $\epsilon = 0^\circ$ and an orbit height of h = 300km (160 n.mi.).

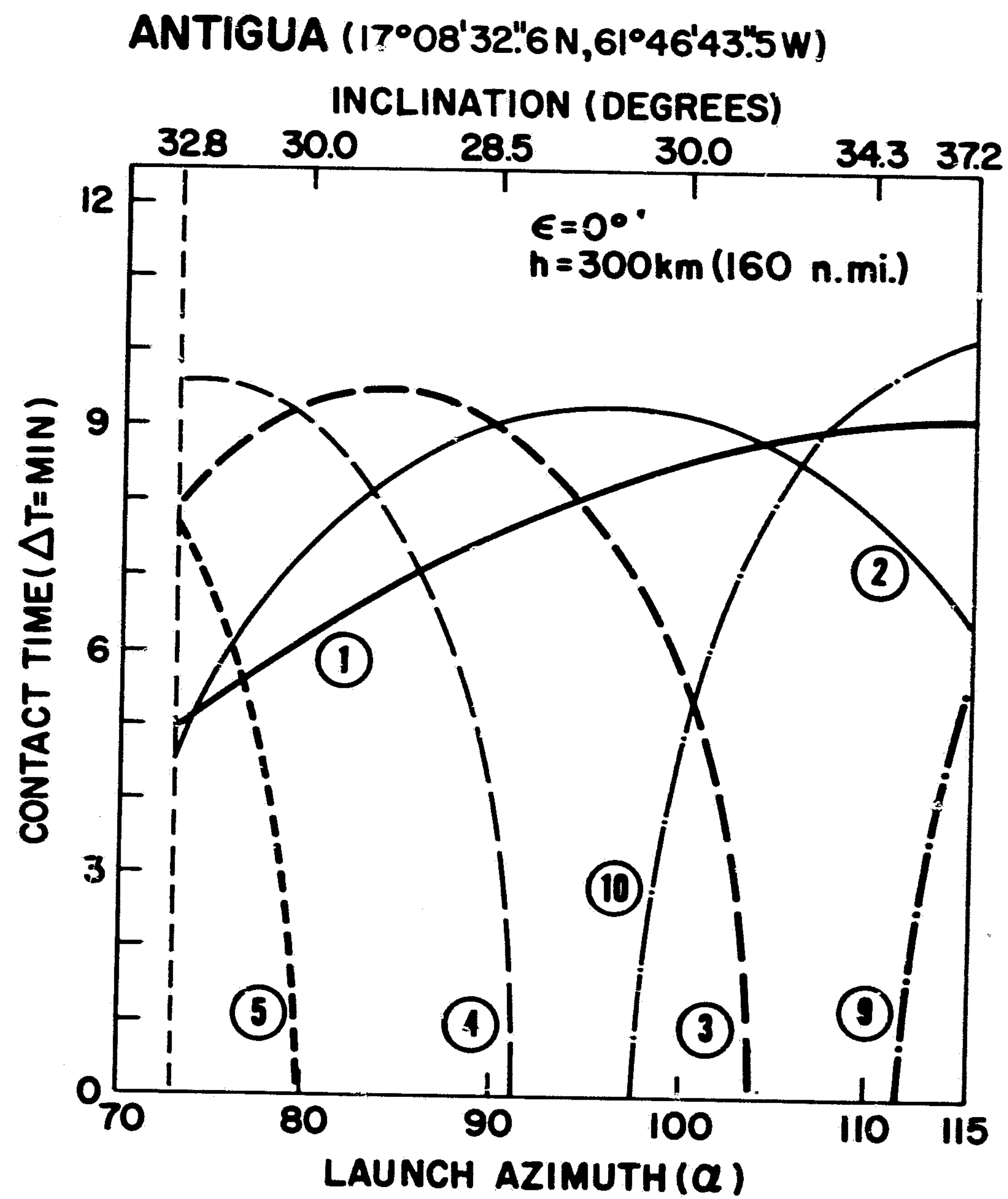

Figure 7b — Station contact time for Antigua for an elevation angle $\epsilon = 0^\circ$ and an orbit height of h = 300km (160 n.mi.).

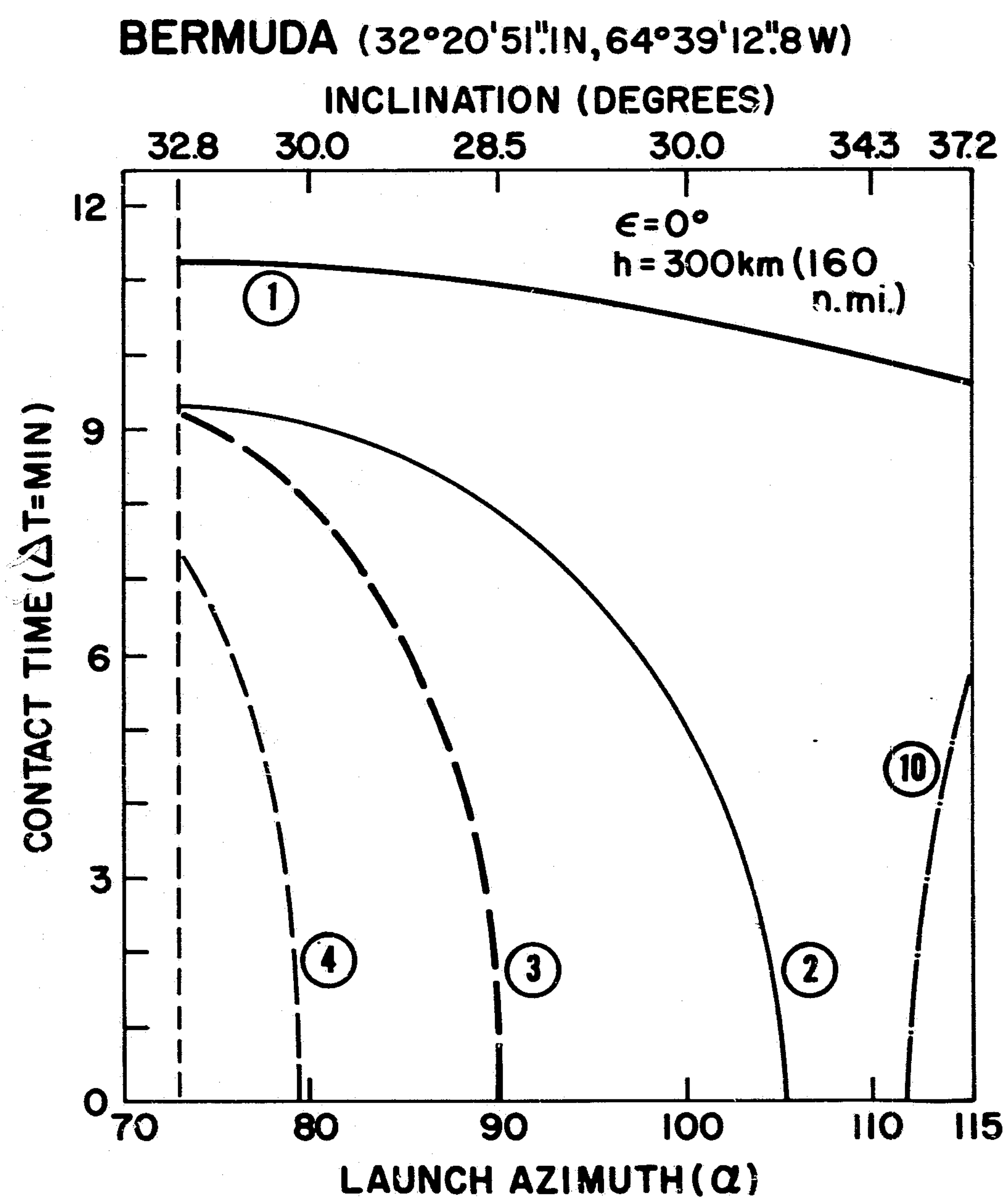

Figure 7c — Station contact time for Bermuda for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

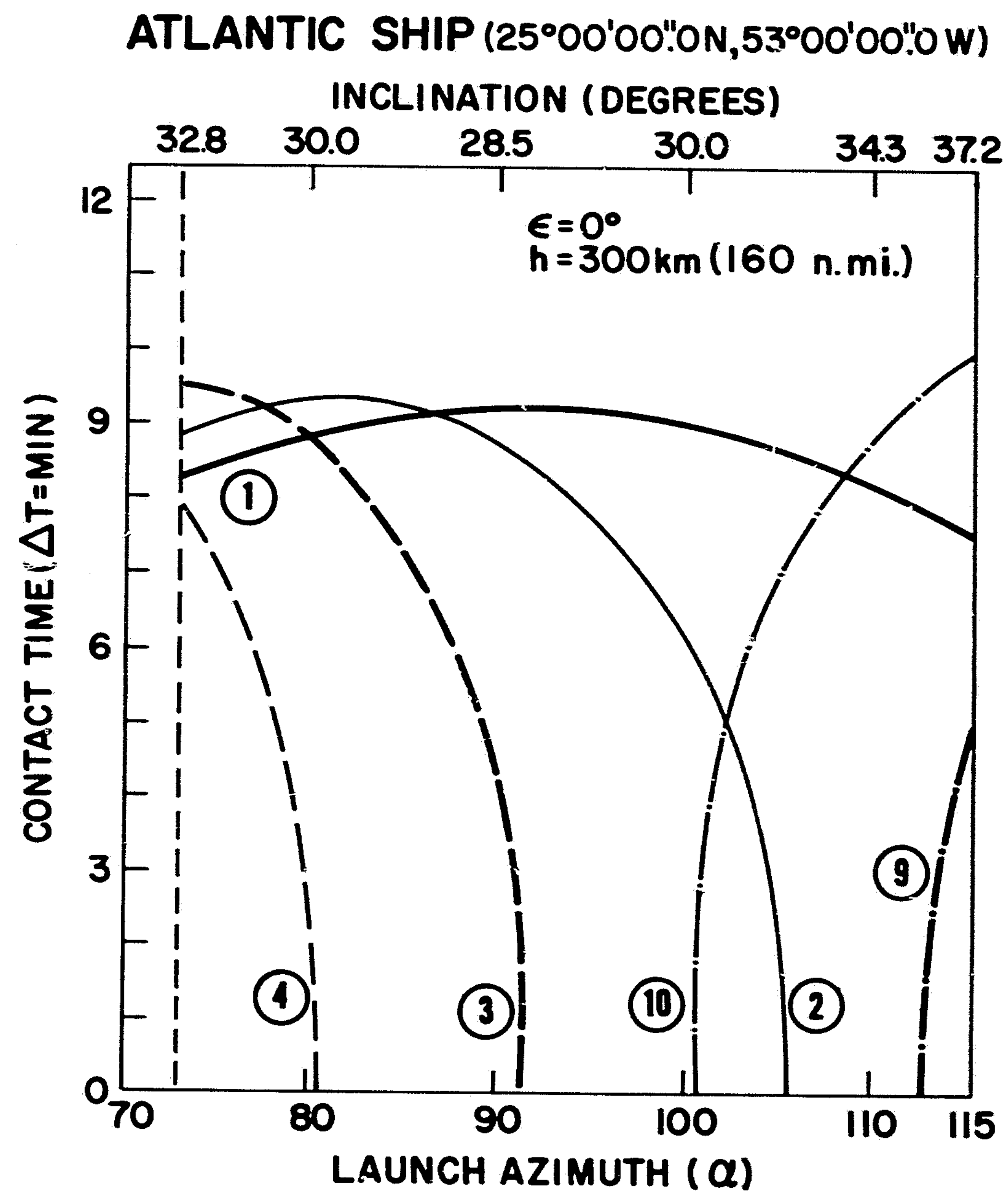

Figure 7d — Station contact time for Atlantic Ship for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

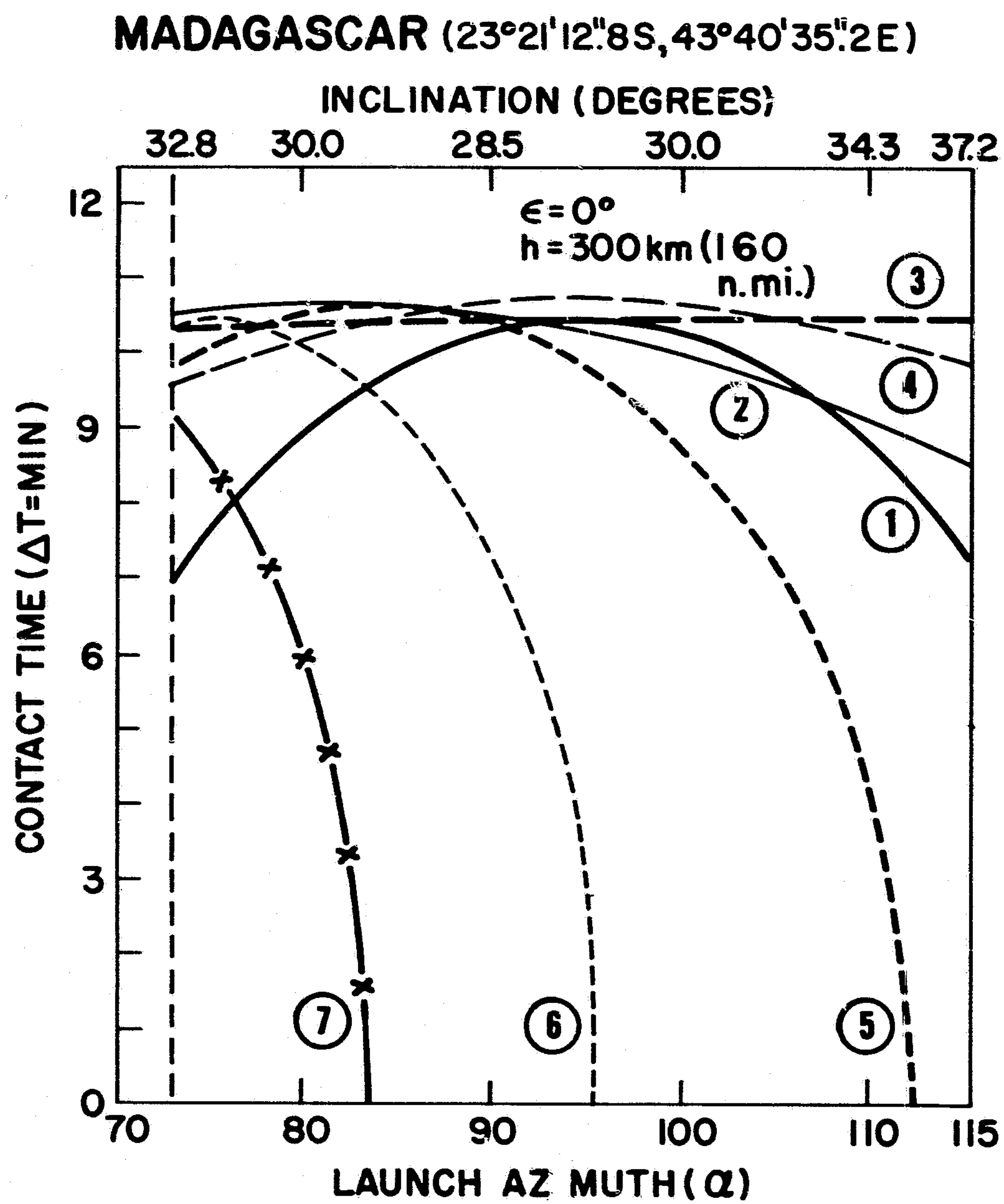

Figure 7e — Station contact time for Madagascar for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

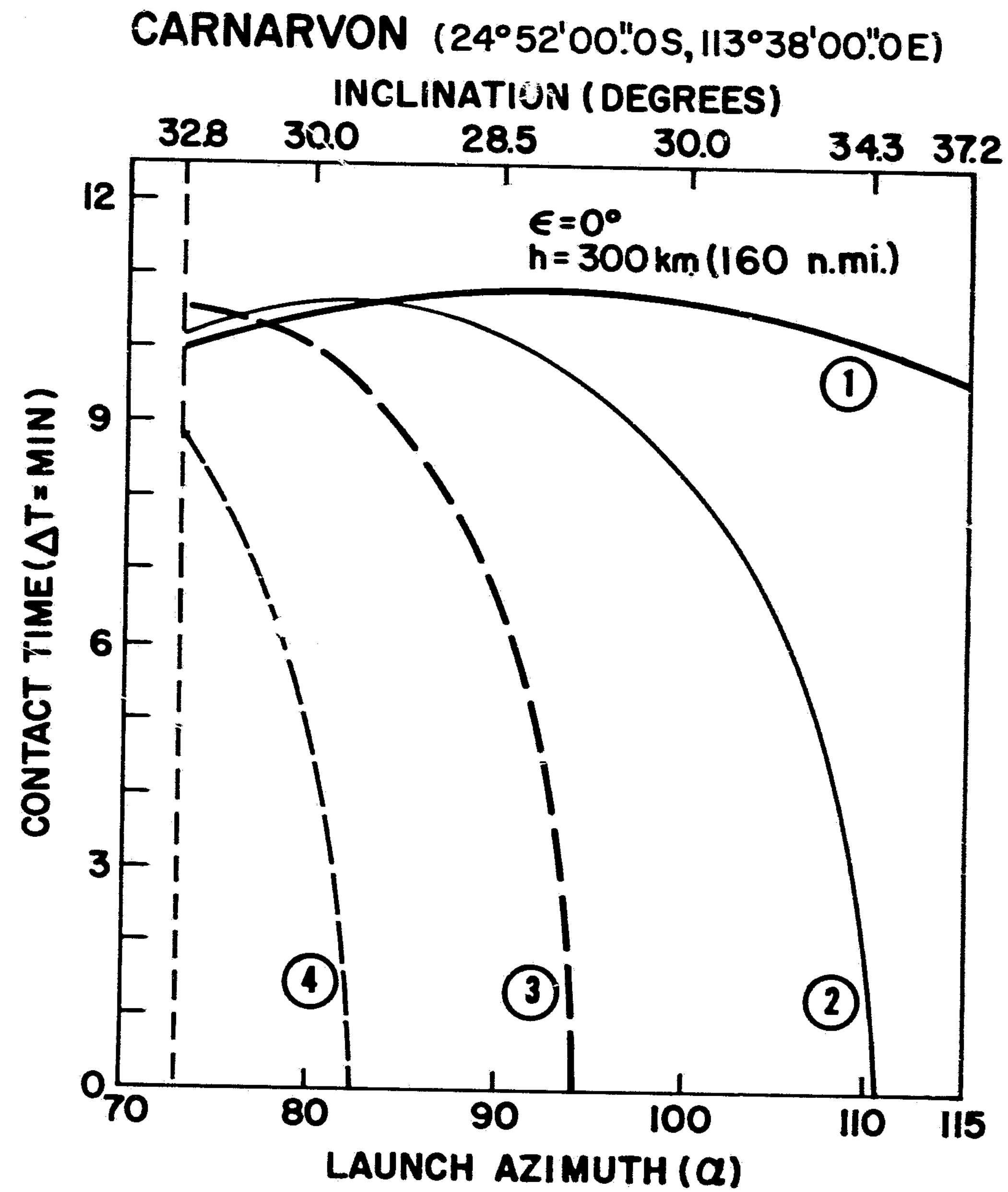

Figure 7f — Station contact time for Carnarvon for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

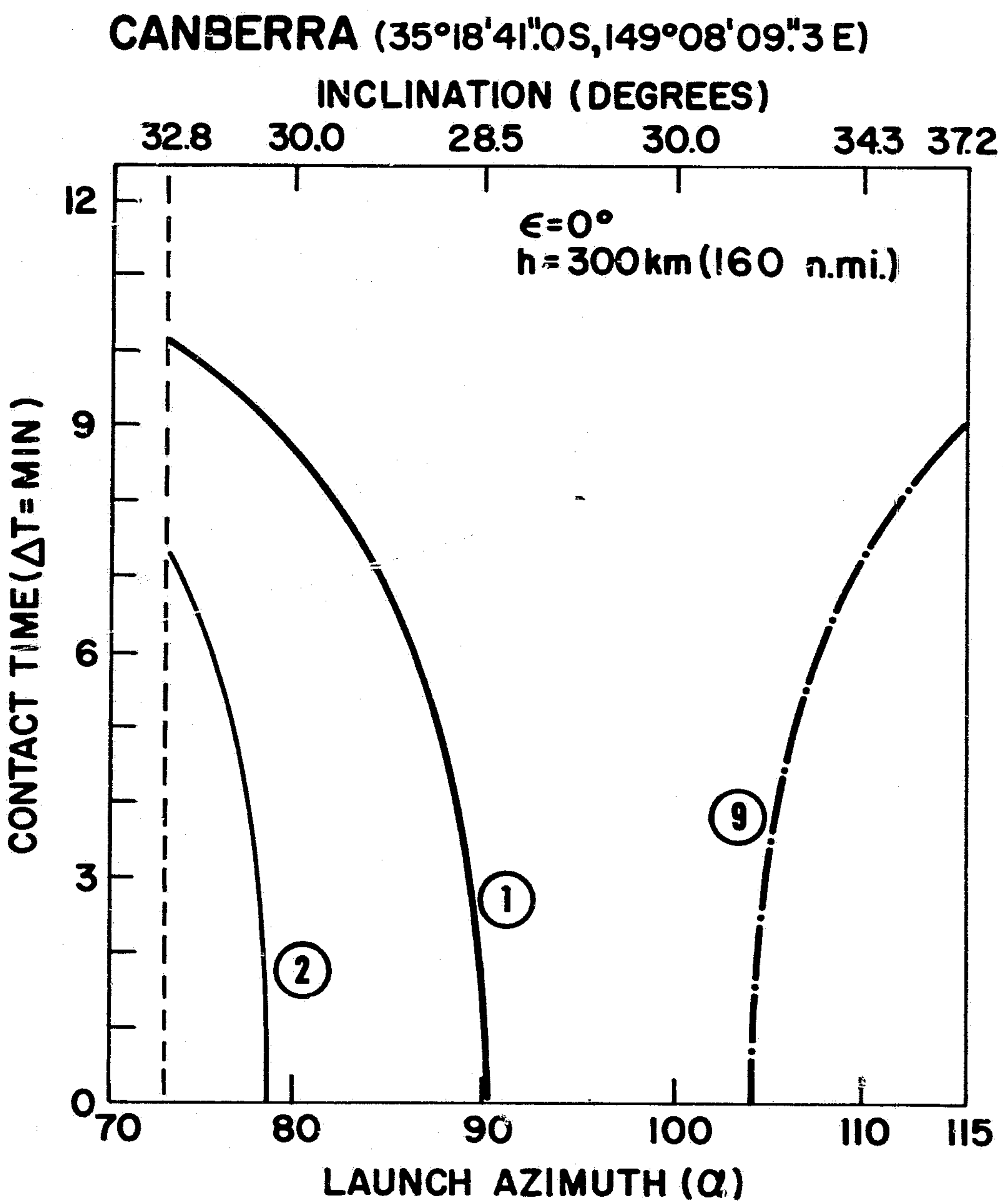

Figure 7g — Station contact time for Canberra for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

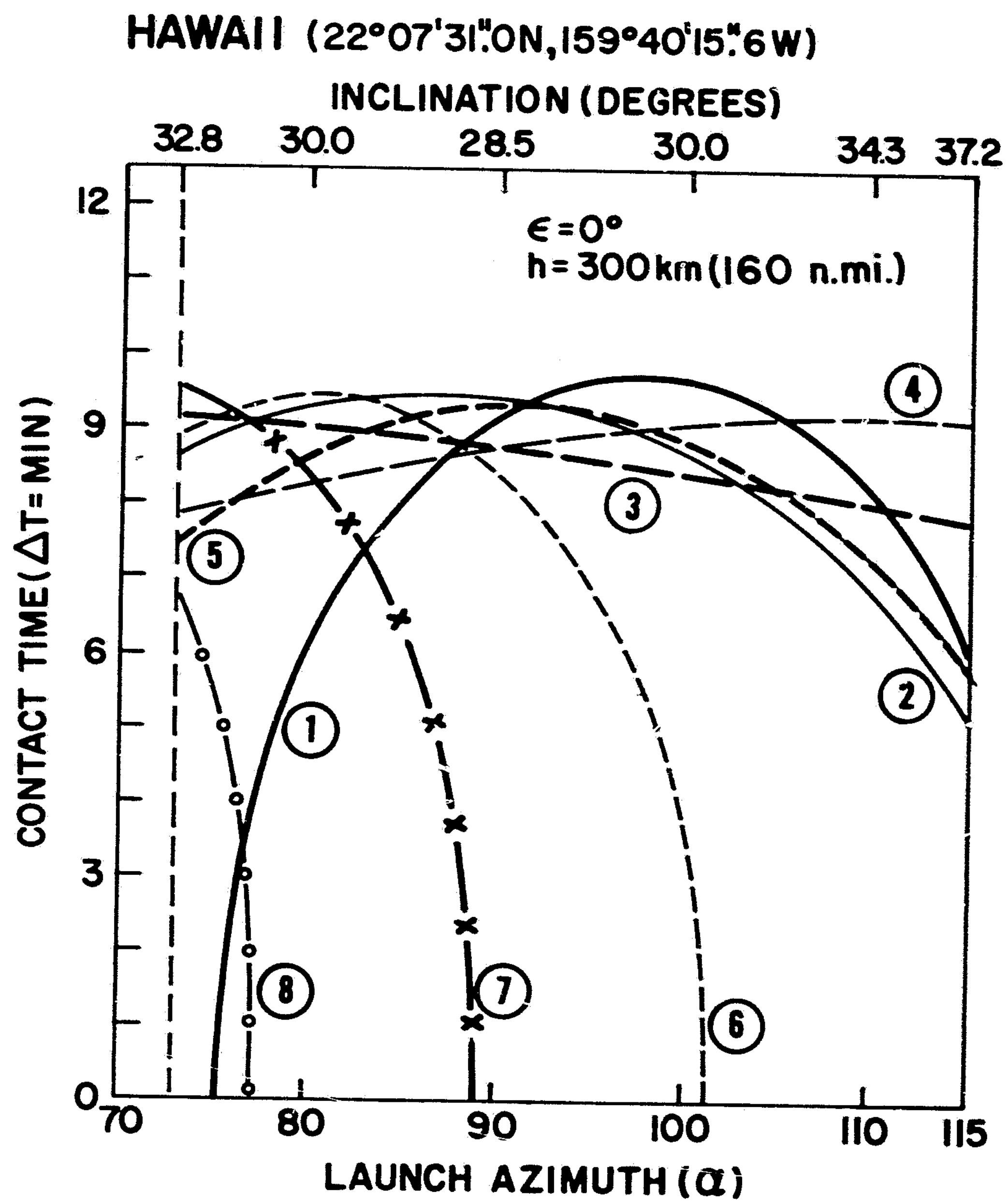

Figure 7h — Station contact time for Hawaii for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

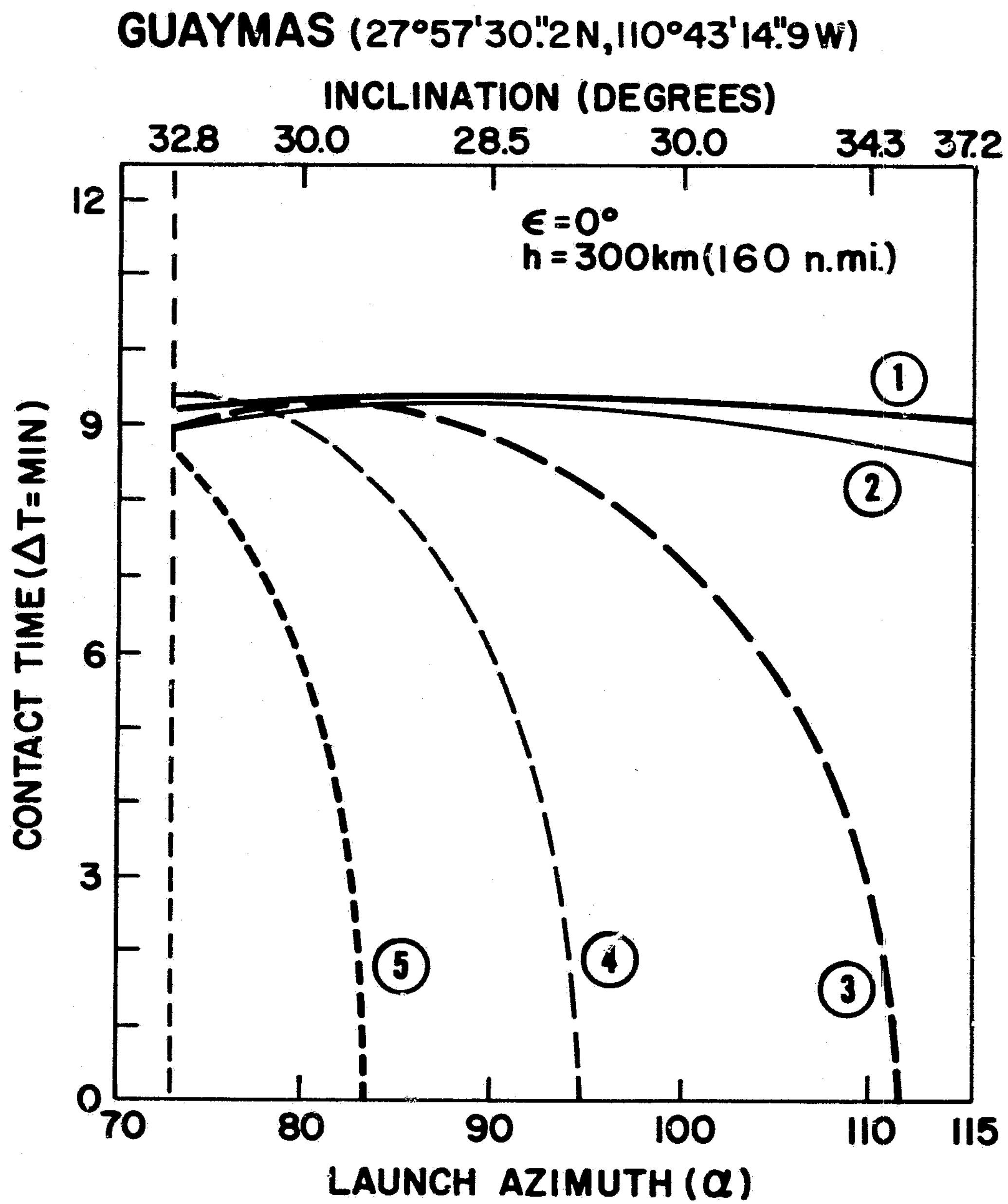

Figure 7i — Station contact time for Guaymas for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

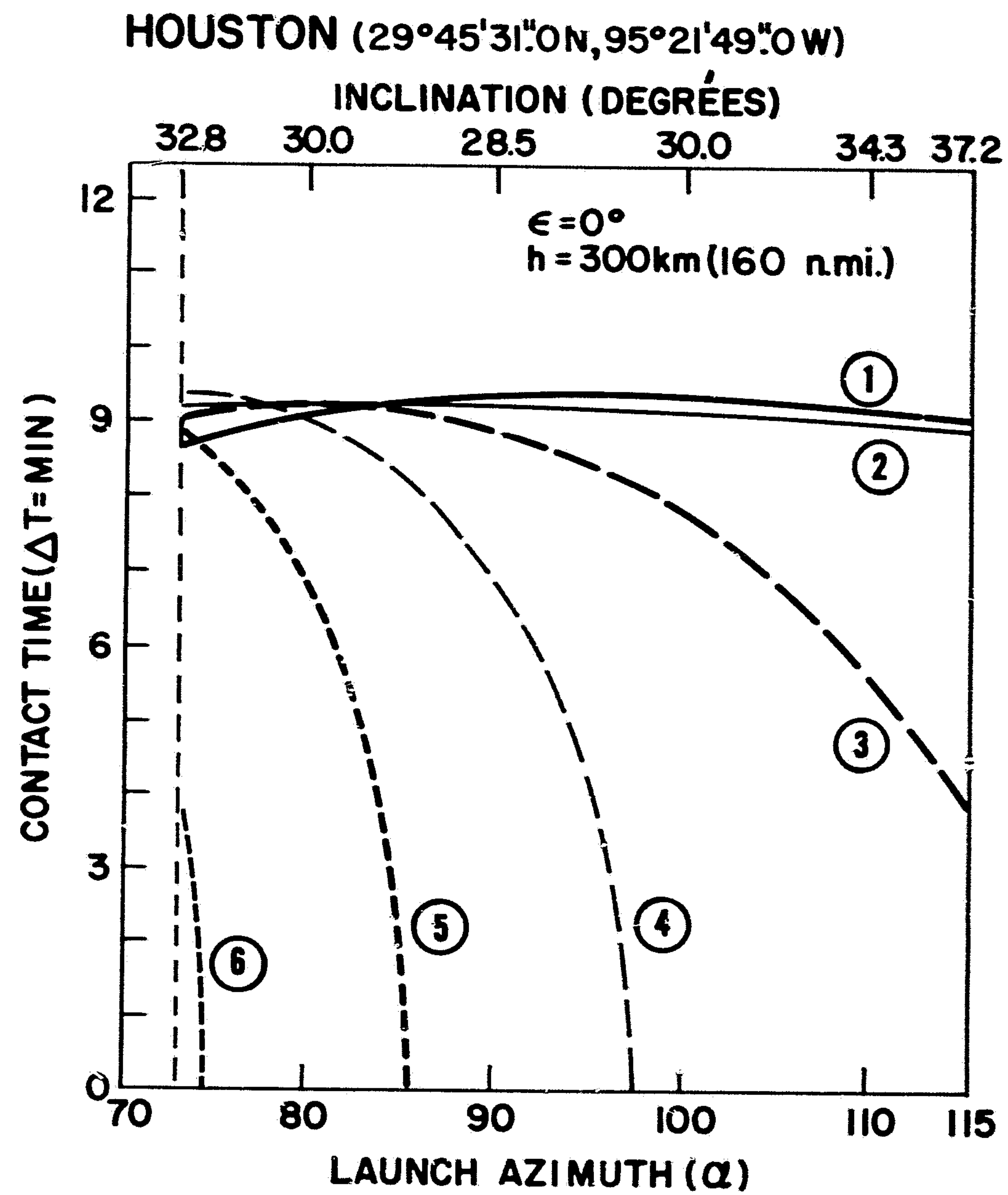

Figure 7j — Station contact time for Houston for an elevation angle $\epsilon = 0°$ and an orbit height of h = 300km (160 n.mi.).

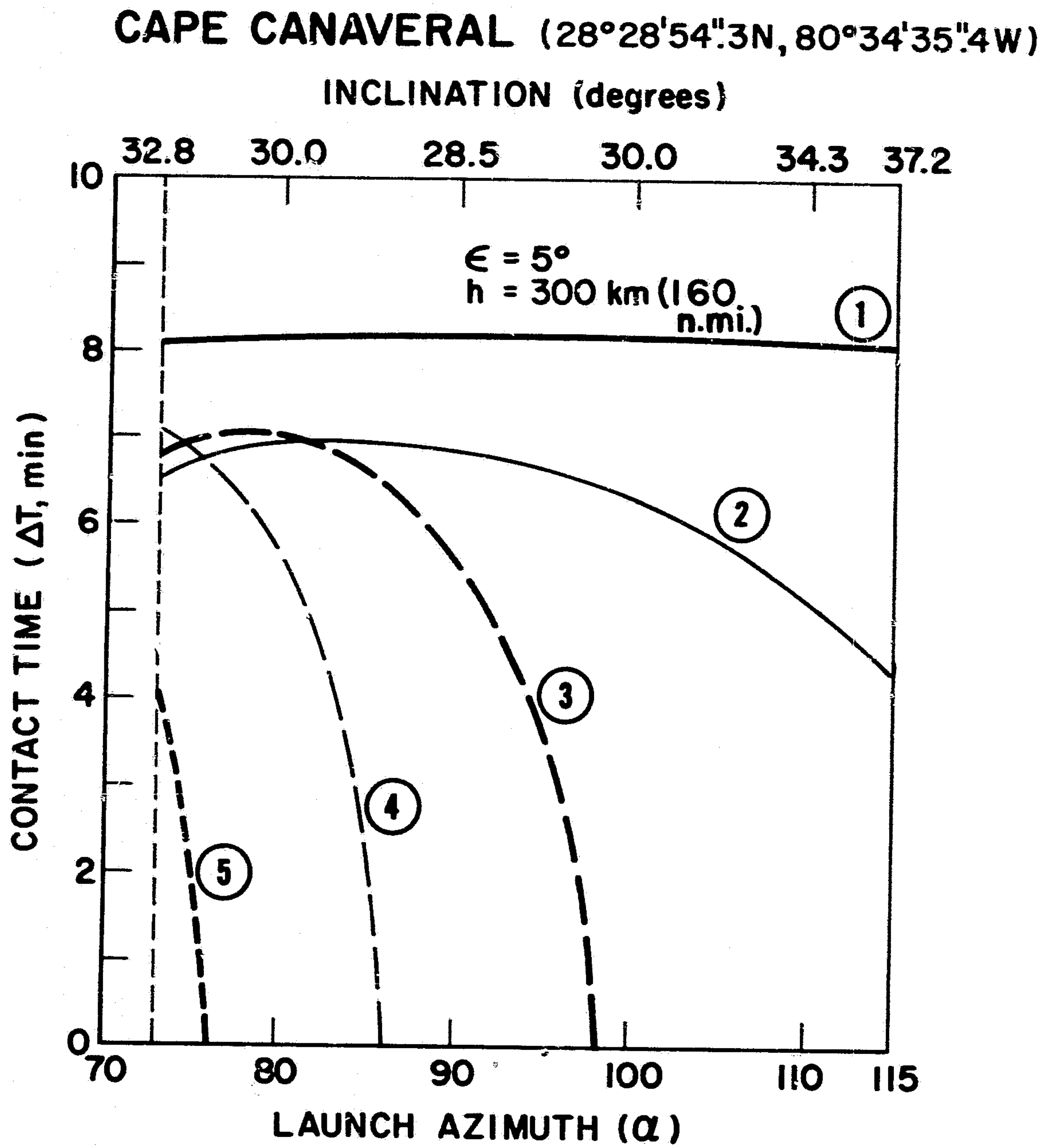

Figure 8a — Station contact time for Cape Canaveral for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

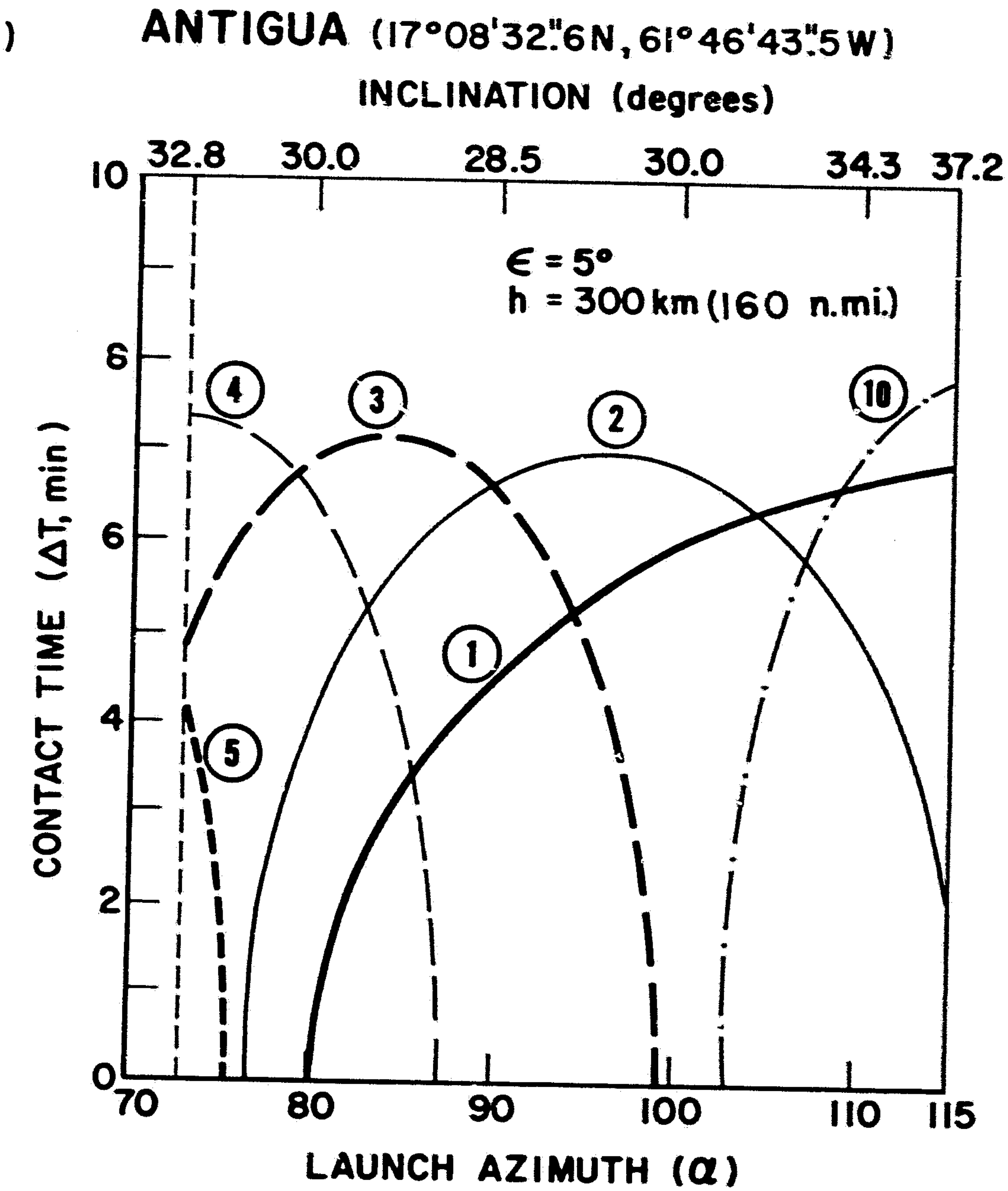

Figure 8b — Station contact time for Antigua for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

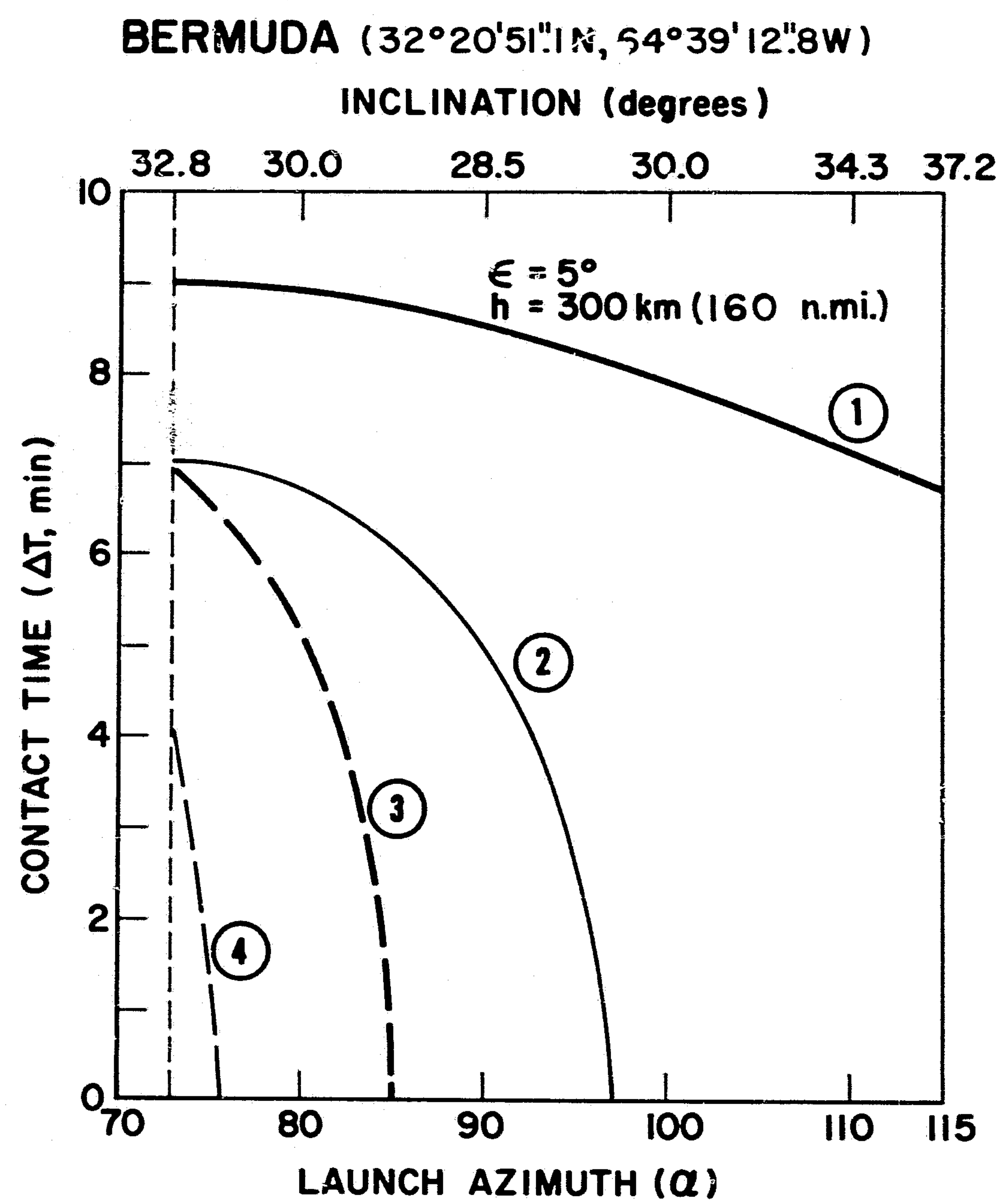

Figure 8c — Station contact time for Bermuda for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

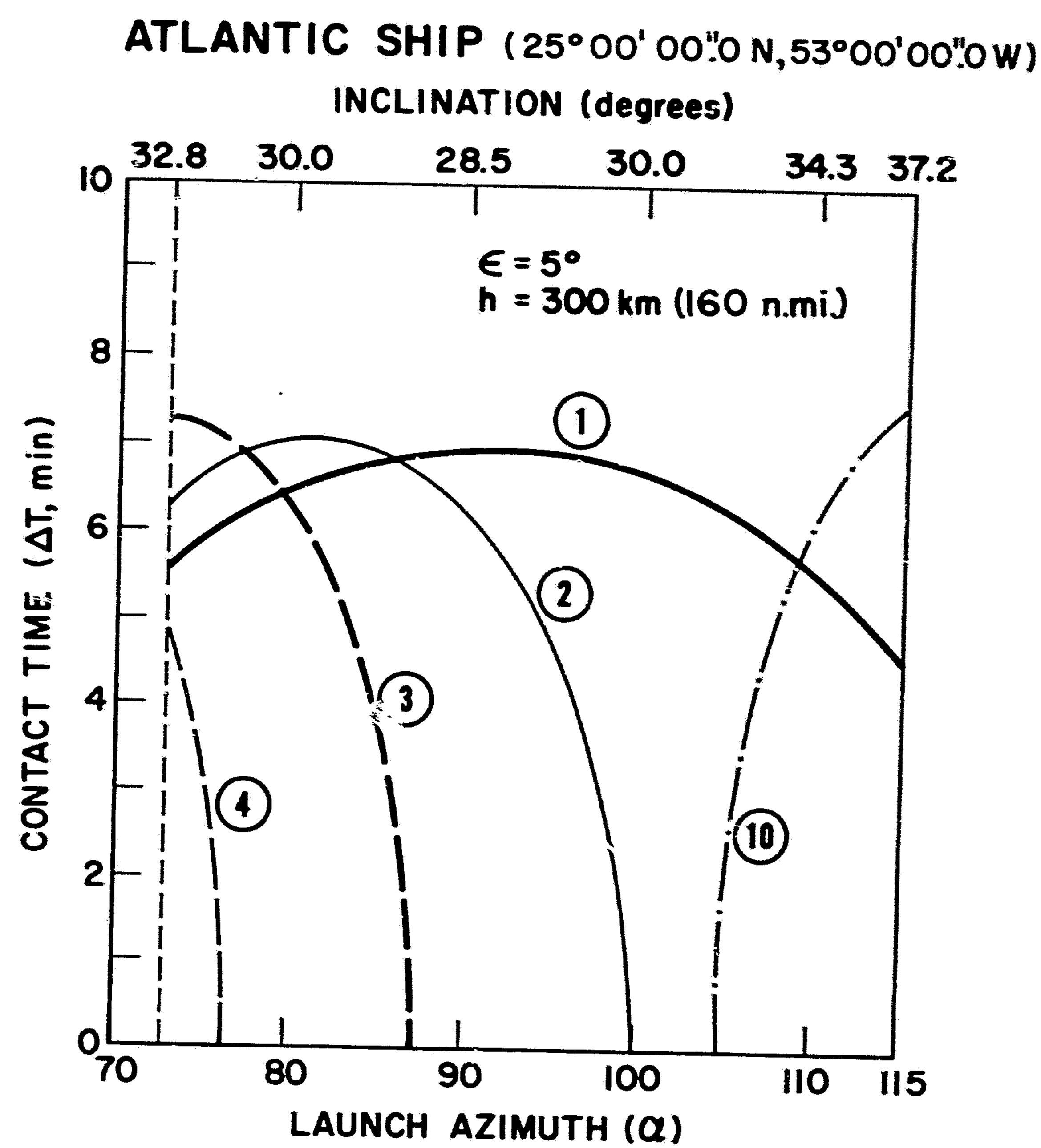

Figure 8d — Station contact time for Atlantic Ship for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

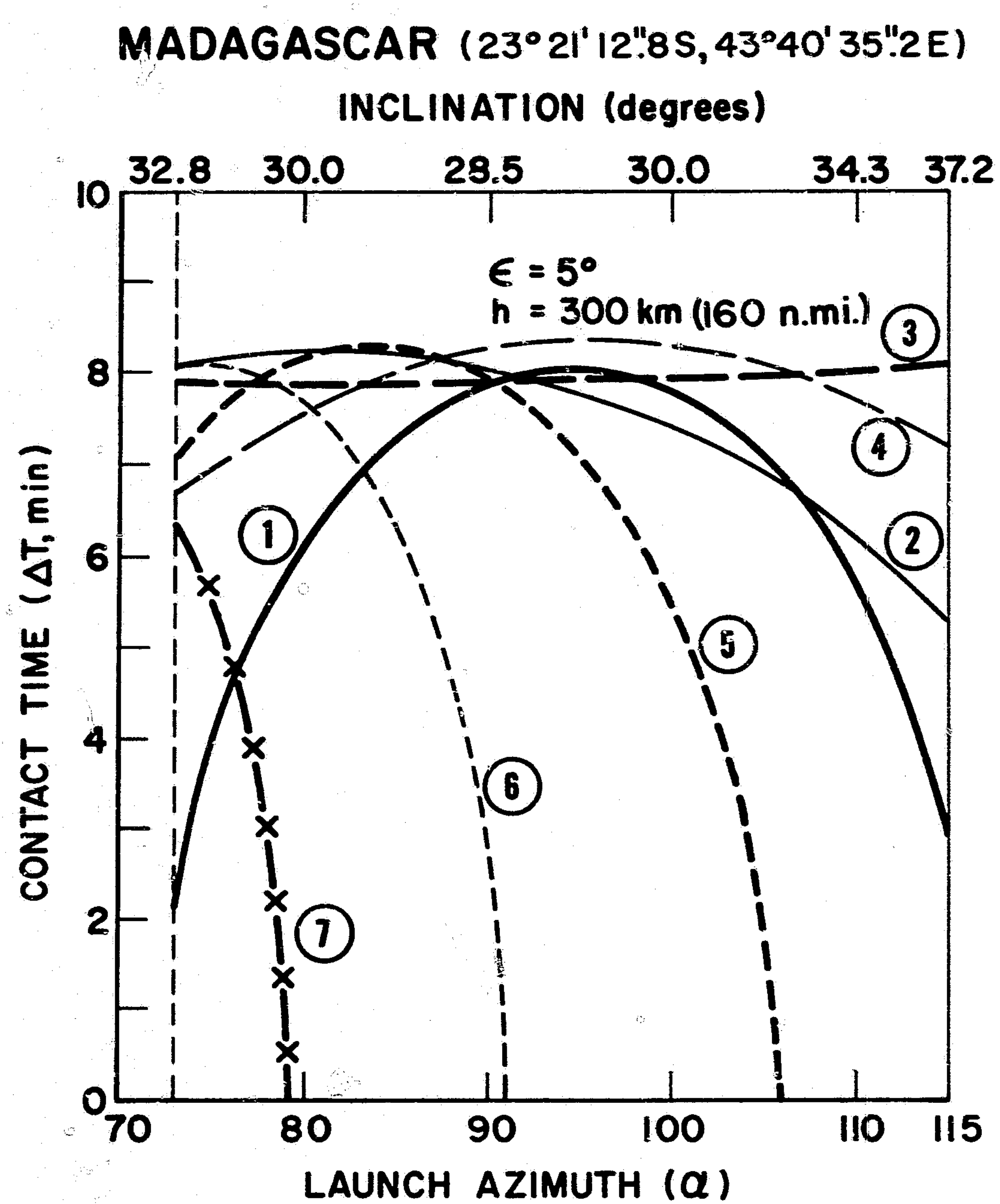

Figure 8e — Station contact time for Madagascar for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

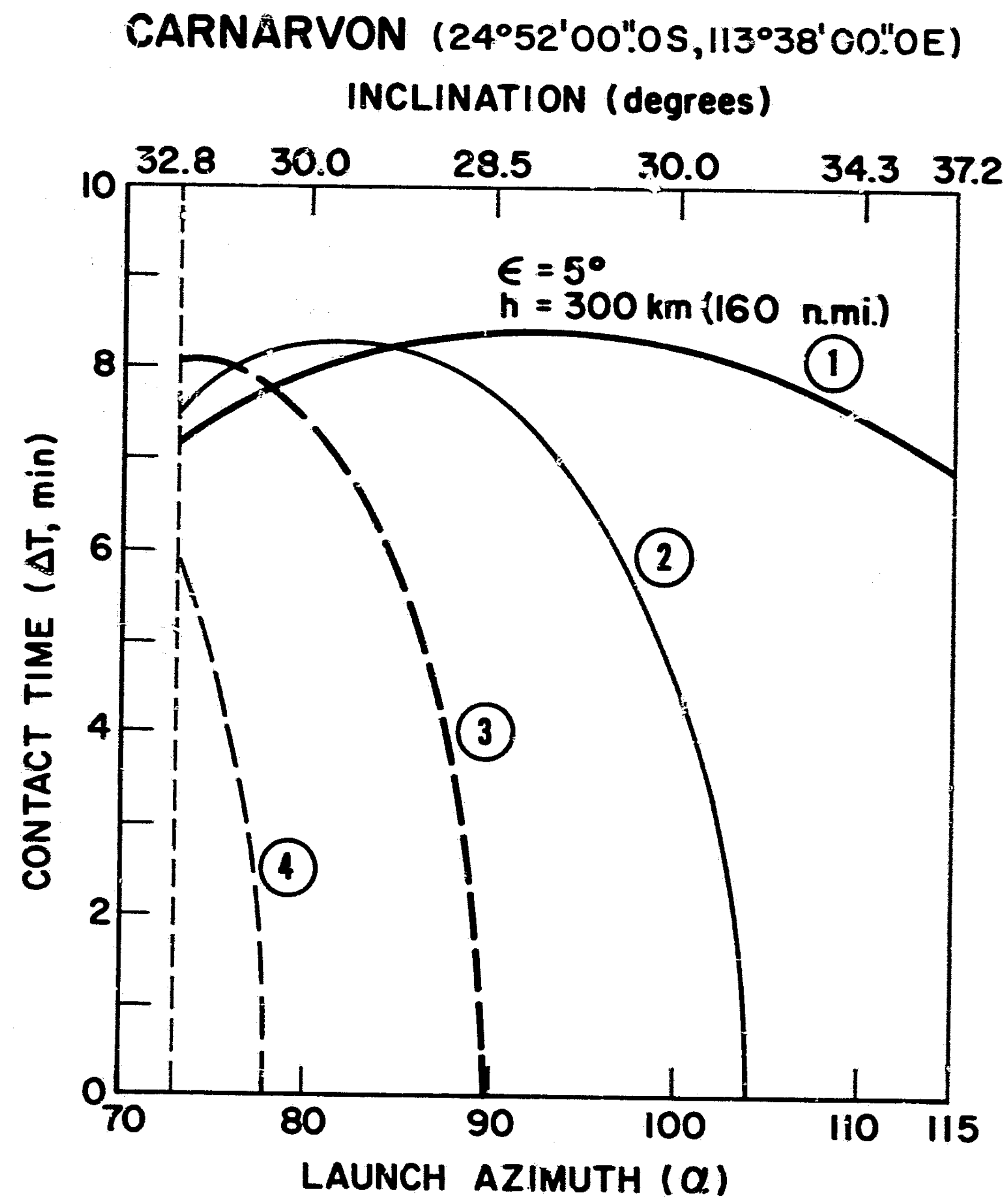

Figure 8f — Station contact time for Carnarvon for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

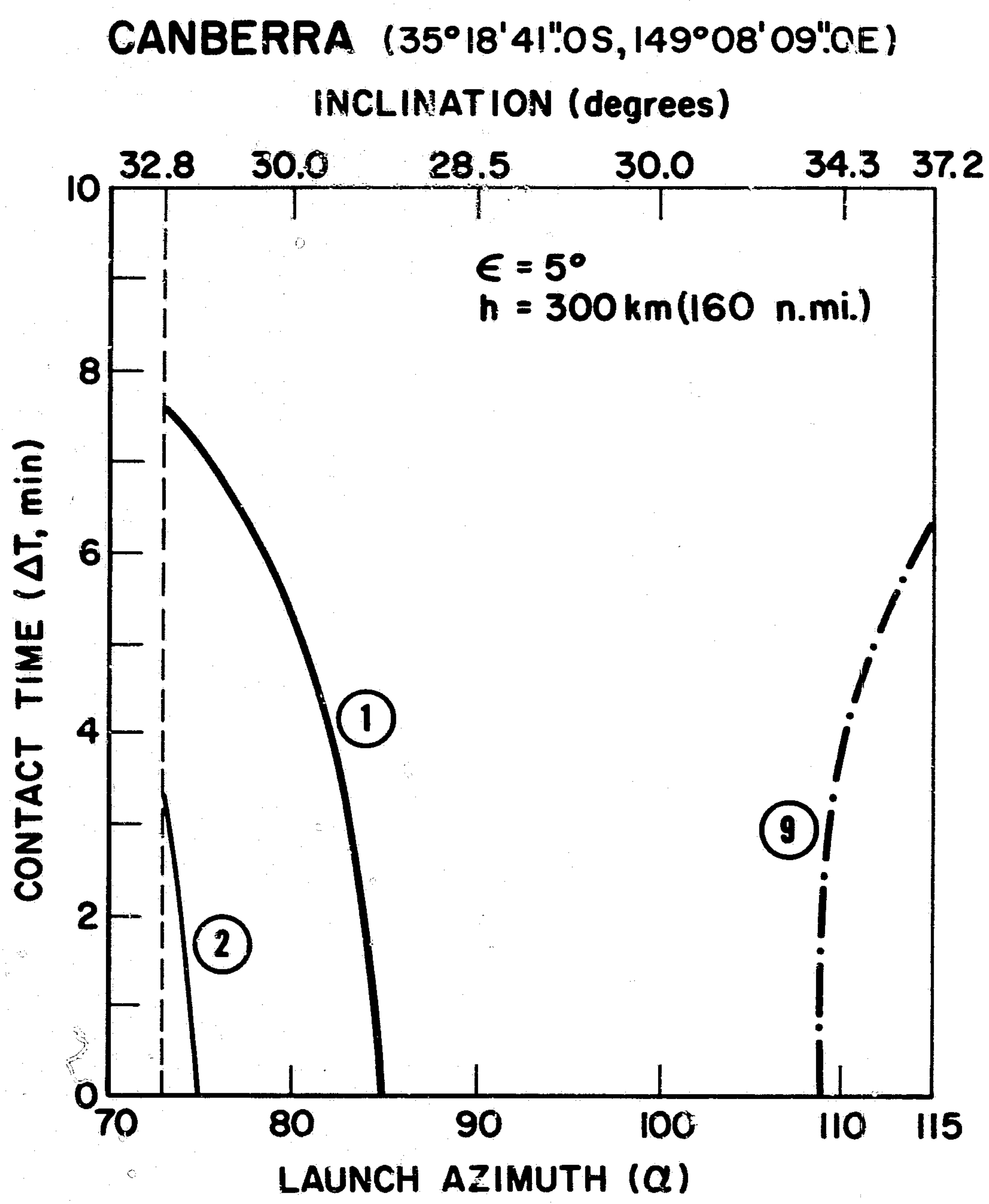

Figure 8g — Station contact time for Canberra for an elevation angle $\epsilon = 5°$ and an orbit height of $h = 300$km (160 n.mi.).

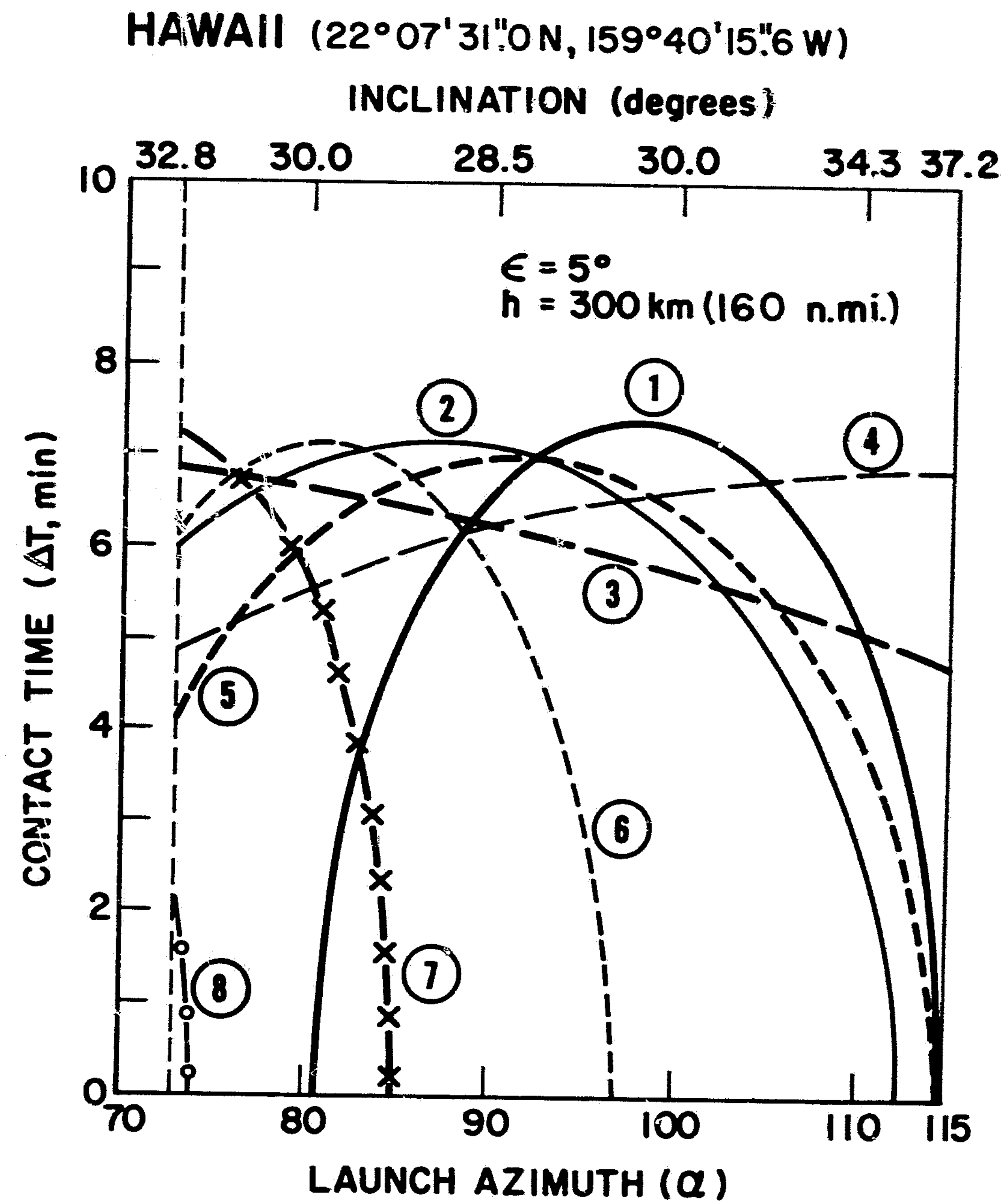

Figure 8h — Station contact time for Hawaii for an elevation angle $\epsilon = 5°$ and an orbit height of $h = 300$km (160 n.mi.).

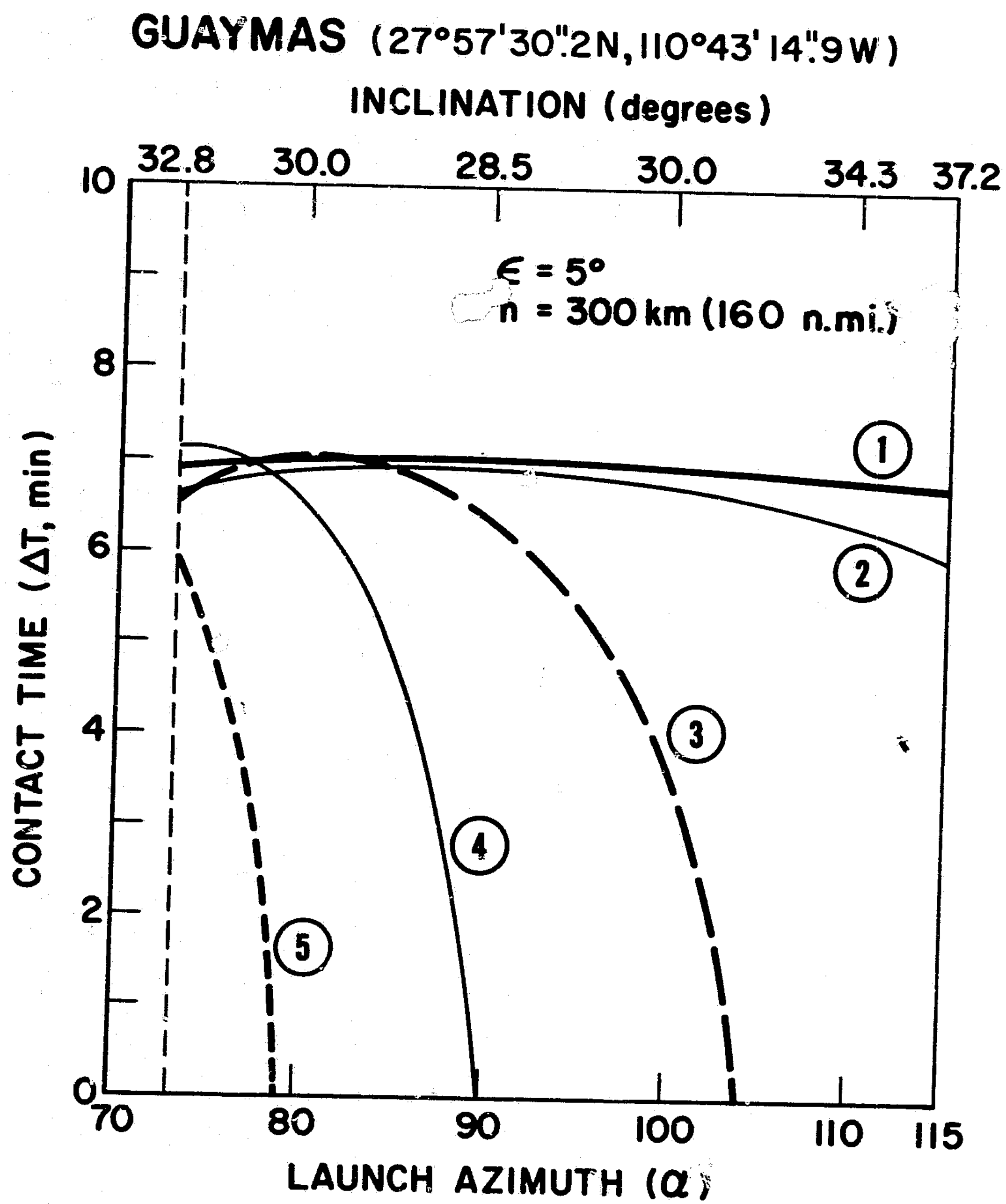

Figure 8i — Station contact time for Guaymas for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

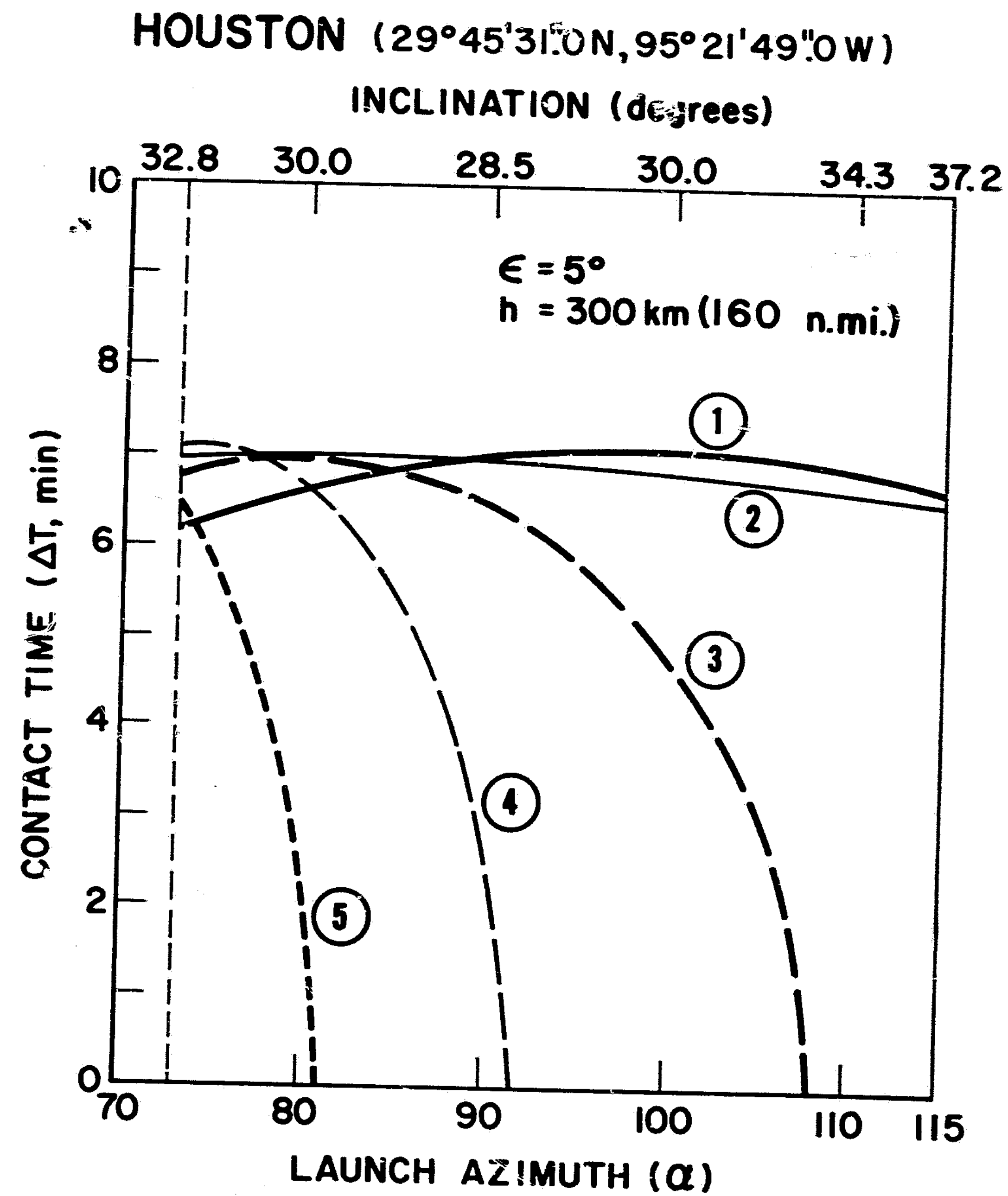

Figure 8j — Station contact time for Houston for an elevation angle $\epsilon = 5°$ and an orbit height of h = 300km (160 n.mi.).

Appendix - A

DERIVATION OF THE PERTINENT EQUATIONS FOR COMPUTING STATION CONTACT TIMES

The purpose of this Appendix is to describe how the data for the curves "Station Contact Times vs. Launch Azimuth" were obtained. For a given launch azimuth, the orbital elements

T :Epoch Time
a :Semi-major Axis of Orbit
e :Eccentricity of Orbit
i :Inclination of Orbit
Ω :Right Ascension of Ascending Node of Orbit
ω :Argument of Perigee
M :Mean Anomaly

are the input data for an orbit generator program which computes the instantaneous position and velocity vectors in the inertial coordinate system (see reference 4). By providing also the coordinates of the various tracking stations, this program computes range, azimuth, and elevation of the satellite with respect to the tracking stations. The latter computation is made whenever the satellite is above the tracking station's horizon. From this station data, the station contact – Δt would have to be hand-computed.

Therefore, a separate program was required, which would compute (1) new orbital elements whenever launch azimuth α_L was varied, (2) the station contact time for any ϵ, and (3) associate an orbit count with each satellite transit over a tracking station. A description of this program which was combined with the two programs mentioned above will now follow.

By solving spherical triangle S_1PS_2 of Fig. A1 and spherical triangle AS_2B of Fig. A2, the orbital elements which change due to varying launch azimuth α_L are obtained. To solve these spherical triangles, the following parameters are assumed known:

ϕ_L' : Geocentric Latitude of Launch Site
λ_L : Geocentric Longitude of Launch Site (positive eastward)
β : Burning Arc Generated by Booster from "Lift-off" up to Insertion into Orbit

α_L : Launch Azimuth (variable)
θ_{G_0} : Greenwich Sidereal Time at Zero Hours Universal Time (U.T.)
T : U.T. Time of Insertion into Orbit
a : Semi-major Axis of Orbit
e : Eccentricity of Orbit
M : Mean Anomaly

The following arguments are now to be determined. These are:

ϕ_i' : Geocentric Latitude at Time of Insertion into Orbit
λ_i : Geocentric Longitude at Time of Insertion into Orbit (counted positive eastward)
α_i : Azimuth of Satellite at Time of Insertion into Orbit

By solving spherical triangle $S_1 P S_2$ (see Fig. A1), formulae for evaluating $(\phi_i', \lambda_i, \alpha_i)$ are derived. These are:

$$\phi_i' = \sin^{-1}\left[\sin\phi_L' \cos\beta + \cos\phi_L' \sin\beta \cos\alpha_L\right] \tag{1}$$

$$-\frac{\pi}{2} \leq \phi_i' \leq \frac{\pi}{2}$$

$$\lambda_i = \lambda_L + \sin^{-1} \frac{\sin\alpha_L \sin\beta}{\cos\phi_i'} \tag{2}$$

$$0 \leq \lambda_i \leq 2\pi$$

and

$$\sin\alpha_i = \frac{\sin\alpha_L \cos\phi_L'}{\cos\phi_i'}$$

$$\cos\alpha_i = \frac{\sin\phi_i' \cos\beta - \sin\phi_L'}{\cos\phi_i' \sin\beta} \tag{3}$$

$$0 \leq \alpha_i \leq 2\pi$$

Because of the difference between geocentric latitude ϕ_i' and declination δ is at most 50'', it is assumed that

$$\delta \approx \phi_i' \tag{4}$$

Right ascension of a celestial object is defined as the local sidereal time minus the local hour angle, but when the object is on the meridian, the local hour angle is equal to zero degrees and the right ascension then equals the local sidereal time.

Then right ascension χ is given by

$$\begin{aligned} \chi &= \theta_G + \lambda_i \\ 0 &\leq \chi \leq 2\pi \end{aligned} \tag{5}$$

where

$$\theta_G = \theta_{G_0} + \left(\frac{15^\circ.04106864}{\text{hr}}\right) \times (\text{T in Hrs.})$$

By solving spherical triangle AS_2B (see Fig. A2) for (Ω, i, ω) use is made of the previously determined arguments $(\phi_i', \lambda_i, \alpha_i)$, that is

$$\begin{aligned} i &= \cos^{-1}\left[\sin \alpha_L \cos \phi_L'\right] \\ 0 &\leq i \leq \frac{\pi}{2} \end{aligned} \tag{6}$$

$$\begin{aligned} \sin\omega &= \frac{\sin\delta}{\sin i} \\ \cos\omega &= \cot i \cot \alpha_i \\ 0 &\leq \omega \leq 2\pi \end{aligned} \tag{7}$$

and

$$\sin(\chi - \Omega) = \sin \alpha_i \sin \omega$$

$$\cos(\chi - \Omega) = \frac{\cos \alpha_i}{\sin i} \tag{8}$$

$$\Omega = \chi - (\chi - \Omega)$$

$$0 \leq \Omega \leq 2\pi$$

The arguments (Ω, i, ω) are the variable orbital elements effected by varying launch azimuth α_L. The program automatically combines these elements with the invariant elements (T, a, e, M), to form the input to the orbit generator and local station prediction program. The latter program is then used for the automatic computation of station contact time. Thus, the modified program provides station contact time for each tracking station and for every value of launch azimuth α_L. Also corresponding to each station contact time Δt, the program provides an orbit count, that is, whenever the satellite traverses east of the 80th meridian W. longitude, the orbital count is advanced by one. The resulting orbit number is then associated with each computed station contact time.

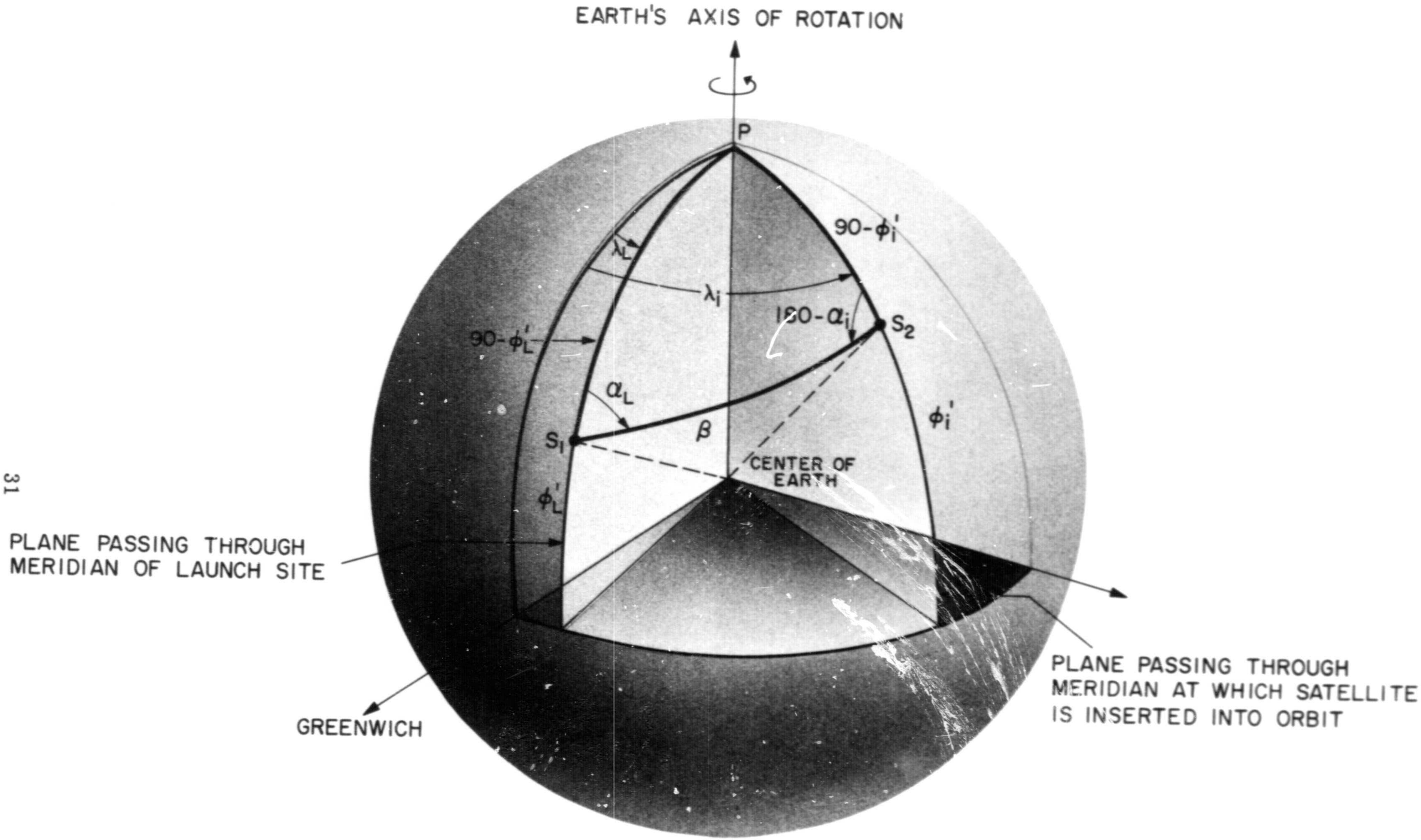

Figure A1 — Geometry of Satellite from "Lift off" to Insertion into Orbit.

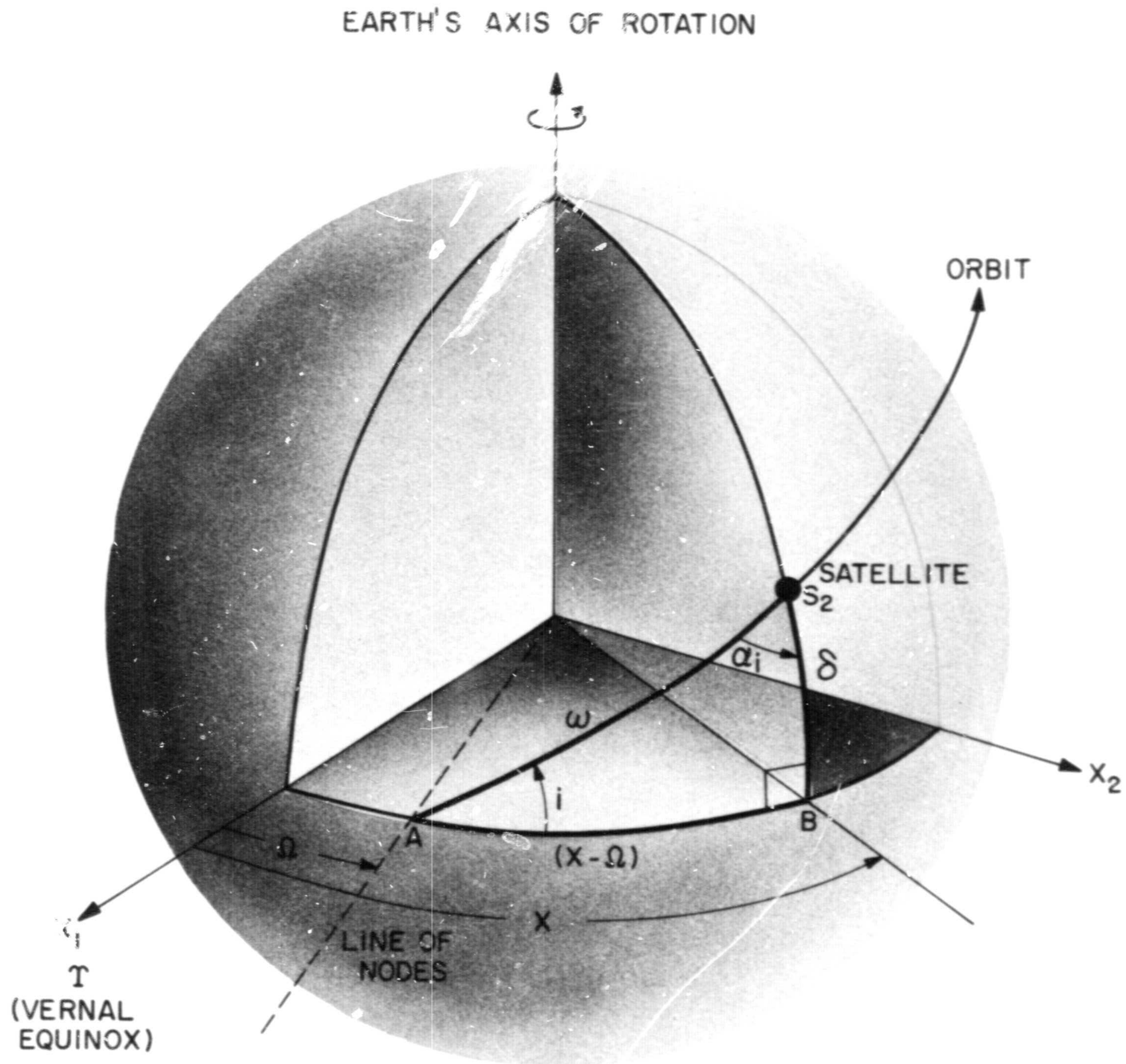

Figure A2 — Geometry of Inserting Artifical Satellite into Orbit at Perigee.