NASA TECH BRIEF

Concept for Simplified Serial Digital Decoder

The problem:

To develop a method of decoding the first order Reed–Muller codes. The decoding apparatus should function as a maximum-likelihood exhaustive-search decoder, and should be a modular implementation to accommodate codes of any length.

The solution:

A modular decoder that lends itself best to special purpose digital equipment using sequential access memories.

How it's done:

The decoder considers a code or matrix. Multilevel symbol confidence levels are considered in a serial decoding scheme. The structure of the dictionary is as follows:

$$A = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} A & A \\ A & \overline{A} \end{bmatrix}; \quad N = \begin{bmatrix} (N-1) & (N-1) \\ (N-1) & \overline{(N-1)} \end{bmatrix}$$

The decoding problem is best described by an example.

a,	b,	c,	d		
+1	+1	+1	+1	=	a + b + c + d
+1	−1	+1	−1	=	a − b + c − d
+1	+1	−1	−1	=	a + b − c − d
+1	−1	−1	+1	=	a − b − c + d

In the exhaustive search through the original dictionary a received word (a, b, c, d) is multiplied symbol by symbol with each entry in this dictionary. The result is shown to the right of each entry in the dictionary. A search is then performed between the resultant correlation values. The largest value indicates the word most likely to have been transmitted. For k variables (information bits) the original dictionary contains 2^k code words each including 2^k symbols. A maximum likelihood exhaustive search through this dictionary requires 2^{2k} additions and sorting of 2^k correlation values. However, it can be shown that the original dictionary, or matrix, can be factored into the product of k different matrices. Each of these matrices has only 2^{k+1} nonzero entries and is distinct from each of the other k−1 matrices. Thus, the decoding scheme requires only $k \cdot 2^k$ additions. The same number of resultant values (2^k) have to be sorted for the final result.

The decoding scheme, applied to the previous example, is shown below:

a,	b,	c,	d		
+1	+1	0	0	=	a + b
+1	−1	0	0	=	a − b
0	0	+1	+1	=	c + d
0	0	+1	−1	=	c − d

a + b	a − b	c + d	c − d		
+1	0	+1	0	=	(a + b) + (c + d)
0	+1	0	+1	=	(a − b) + (c − d)
+1	0	−1	0	=	(a + b) − (c + d)
0	+1	0	−1	=	(a − b) − (c − d)

This technique is continued. For k = 3 factor matrices are shown below:

$$
\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{array}
\qquad \mathbf{1}
$$

$$
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1
\end{array}
\qquad \mathbf{2}
$$

$$
\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1
\end{array}
\qquad \mathbf{3}
$$

Each factor matrix can be implemented by a digital module, the block diagram of which is shown below.

The value of i ranges from 1 through k. The signals W_{i-1} and $\overline{W}_{i-1}$ are the outputs of the ith stage of a digital counter, the first stage, W_0, of which receives a new count pulse at the start of each new symbol of the received code word.

Finally, if the modules are cascaded in increasing order of i, the output of the ith module decodes a code arising from i variables. Thus, if a decoder for k variables exists and it becomes necessary to decode a code arising from k + 1 variables, only one new stage, corresponding to i = k + 1, and one stage to the W counter need be added.

Notes:

1. This development was designed, constructed, and approved for the Mariner project.
2. Inquiries concerning this development may be directed to:
   Technology Utilization Officer
   NASA Pasadena Office
   4800 Oak Grove Drive
   Pasadena, California 91103
   Reference: B68-10045

Patent status:

This invention is owned by NASA, and a patent application has been filed. Royalty-free, nonexclusive licenses for its commercial use will be granted by NASA. Inquiries concerning license rights should be made to NASA, Code GP, Washington, D.C. 20546.

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