# NASA TECH BRIEF



## Controllability of Distributed-Parameter Systems

A mathematical study has been made concerning the controllability of distributed-parameter control systems, that is, control systems that can be described by partial differential equations. The purpose of this study was to formulate a general theory for control systems to include those that cannot be described by ordinary differential equations.

The study is presented in a report which summarizes the techniques applicable to control systems problems. The eigenvalue-eigenfunction expansion method for the solution of homogeneous boundary value problems (BVP) is used. Problems in which the control appears at the boundary are treated by converting the nonhomogeneous BVP to an equivalent homogeneous BVP by introducing generalized functions. The generalization of the concept of controllability of finite dimensional systems to infinite dimensional systems is given. The pseudoinverse of a linear operator is defined which is a generalization of that of a matrix for finite dimensional spaces. The pseudoinverse is then used to obtain minimum energy control for distributed-parameter systems. It is shown that this generalization includes results for finite dimensional systems which are available. In the infinite dimensional problem, it is necessary to solve for the eigenvalues and eigenfunctions of an integral operator. The necessary and sufficient conditions for the states which are reachable when the control is required to satisfy a norm constraint are given. The conditions are obtained by an application of the moment problem to distributed-parameter systems. These results are then used to obtain conditions for complete controllability.

**Note:**

A copy of the report may be obtained from:

Technology Utilization Officer
Marshall Space Flight Center
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**Patent status:**

No patent action is contemplated by NASA.

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