GRAVITY GRADIENTS ON THE EARTH'S SURFACE AS DEDUCED FROM SATELLITE ORBITS

W. KÖHNLEIN

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TABLE OF CONTENTS

ii

LIST OF ILLUSTRATIONS

LIST OF TABLES

ABSTRACT

The variation of gravity on the earth's surface is computed in three mutually perpendicular directions: the horizontal anomalous variations along the geocentric latitude and longitude curves, and the vertical component along the plumb line. The numerical results obtained from Kozai's and Gaposchkin's latest harmonic coefficients indicate a correlation between the anomalous vertical gravity gradient and the earth's continental topography.

RÉSUMÉ

Les variations de la pesanteur à la surface de la terre sont calculées le long de trois axes trirectangles: les variations horizontales anormales, le long des courbes de latitude et longitude géocentriques, et la composante verticale, le long de la direction du fil à plomb. Les résultats numériques obtenus à partir des récents coefficients harmoniques de Kozai et Gaposchkin indiquent l'existence d'une corrélation entre le gradient vertical anormal de la pesanteur et la topographie des continents terrestres.

КОНСПЕКТ

Изменение силы тяжести на поверхности Земли высчитывается по трем взаимно перпендикулярным направлениям: горизонтальные аномальные изменения вдоль кривых геоцентрических широты и долготы, и вертикальной составляющей по отвесу. Цифровые результаты полученные по последним гармоническим коэффициентам Козая и Гапочкина указывают на связь между аномальным вертикальным градиентом силы тяжести и земной континентальной топографией.

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1. INTRODUCTION

Gravity gradients are widely used in geophysics for the analysis of complex geological structures. Mass defects, for example, are easily recognized in a gravity gradient field even in cases where purely gravimetric data sometimes fail to show a clear picture. We are trying to extend this procedure to a worldwide scale using as gravity information the zonal and nonzonal harmonic coefficients of the geopotential derived by Kozai (1964) and Gaposchkin (1967). We take as reference an ellipsoidal field of the same potential and of the same zonal coefficient of second degree.

We denote the geopotential as derived from artificial satellites by

$$U = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\, P_{nm}(\sin\phi)\right] + \frac{\omega^2 r^2}{2}\cos^2\phi \quad,$$

(1)

with

    GM = product of the gravitational constant and the mass of the earth,

    a = equatorial radius of the earth,

    r = geocentric radius,

    ϕ = geocentric latitude,

    λ = geocentric longitude,

C_{nm}, S_{nm} = harmonic coefficients,

$P_{nm}(\sin \phi)$ = Legendre associated functions,

ω = earth's angular velocity;

and, similarly, the ellipsoidal potential by

$$V = \frac{GM}{r_e}\left[1 + K_2\left(\frac{a}{r_e}\right)^2 P_{20}(\sin \phi) + K_4\left(\frac{a}{r_e}\right)^4 P_{40}(\sin \phi)\right] + \frac{\omega^2 r_e^2}{2}\cos^2\phi \quad , \tag{2}$$

with

r_e = geocentric ellipsoidal radius,

K_2, K_4 = ellipsoidal harmonic coefficients.

Hence we obtain the gravity at any point from

$$g = \left[\left(\frac{\partial U}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial U}{\partial \phi}\right)^2 + \frac{1}{r^2\cos^2\phi}\left(\frac{\partial U}{\partial \lambda}\right)^2\right]^{1/2} \quad , \tag{3}$$

and analogously for V. Of course, these expressions are valid, in a strict sense, only in free space while disturbances, introduced by topographic interference at sea level, are neglected.

To keep the analytical expressions to a minimum, we consider only the satellite potential U, and substitute for U the ellipsoidal potential V when necessary.

2. HORIZONTAL GRAVITY GRADIENT

The tangent plane (horizon) of the geoid (i.e., U = const at mean sea level) is obtained from equation (1):

$$\frac{\partial U}{\partial r} dr + \frac{\partial U}{\partial \phi} d\phi + \frac{\partial U}{\partial \lambda} d\lambda = 0 \quad ; \tag{4}$$

and the tangents of the geocentric latitude and longitude curves are

$$\frac{\partial U}{\partial r} dr + \frac{\partial U}{\partial \lambda} d\lambda = 0 \quad ,$$

$$\frac{\partial U}{\partial r} dr + \frac{\partial U}{\partial \phi} d\phi = 0 \quad . \tag{5}$$

If we introduce these expressions into the total differential of equation (3),

$$dg = \frac{\partial g}{\partial r} dr + \frac{\partial g}{\partial \phi} d\phi + \frac{\partial g}{\partial \lambda} d\lambda \quad , \tag{6}$$

we obtain the components of the horizontal gravity gradient along the geocentric latitude and longitude curves on the geoid:

$$\delta g_\lambda = \left(\frac{\partial g}{\partial \lambda} - \frac{\partial g}{\partial r} \frac{\partial U}{\partial \lambda} \bigg/ \frac{\partial U}{\partial r} \right) d\lambda \quad ,$$

$$\delta g_\phi = \left(\frac{\partial g}{\partial \phi} - \frac{\partial g}{\partial r} \frac{\partial U}{\partial \phi} \bigg/ \frac{\partial U}{\partial r} \right) d\phi \quad . \tag{7}$$

3

Because the arc lengths of the geocentric latitude and longitude curves can be written

$$\delta s_{\phi=\text{const}} = \left[r^2 \cos^2\phi + \left(\frac{\partial U}{\partial \lambda} \Big/ \frac{\partial U}{\partial r} \right)^2 \right]^{1/2} d\lambda \quad ,$$

$$\delta s_{\lambda=\text{const}} = \left[r^2 + \left(\frac{\partial U}{\partial \phi} \Big/ \frac{\partial U}{\partial r} \right)^2 \right]^{1/2} d\phi \quad ,$$

(8)

we obtain the variation of gravity along the latitude curves $\phi = \text{const}$,

$$\frac{\delta g}{\delta s}_{\phi=\text{const}} = \left[\frac{\partial g}{\partial \lambda} - \frac{\partial g}{\partial r} \left(\frac{\partial U}{\partial \lambda} \Big/ \frac{\partial U}{\partial r} \right) \right] \left[r^2 \cos^2\phi + \left(\frac{\partial U}{\partial \lambda} \Big/ \frac{\partial U}{\partial r} \right)^2 \right]^{-1/2} \quad , \quad (9)$$

and, similarly, the variation of gravity along the longitude curves $\lambda = \text{const}$,

$$\frac{\delta g}{\delta s}_{\lambda=\text{const}} = \left[\frac{\partial g}{\partial \phi} - \frac{\partial g}{\partial r} \left(\frac{\partial U}{\partial \phi} \Big/ \frac{\partial U}{\partial r} \right) \right] \left[r^2 + \left(\frac{\partial U}{\partial \phi} \Big/ \frac{\partial U}{\partial r} \right)^2 \right]^{-1/2} \quad . \quad (10)$$

Hence the total amount of the horizontal gravity gradient is readily obtained:

$$\left(\frac{\delta g}{\delta s} \right)_{\text{horiz}} \approx \left[\left(\frac{\delta g}{\delta s}_{\lambda=\text{const}} \right)^2 + \left(\frac{\delta g}{\delta s}_{\phi=\text{const}} \right)^2 \right]^{1/2} \quad . \quad (11)$$

The approximate sign corresponds only to the potential U, while any rotational symmetric potential such as V (substituted instead of U) satisfies equation (11) exactly; this is because the geocentric latitude and longitude curves in general do not intersect on the geoid perpendicularly. However, the deviation is so small that for practical purposes the equal sign can be used in equation (11).

4

3. VERTICAL GRAVITY GRADIENT

To obtain the vertical gravity gradient, we start from equation (6),

$$\frac{dg}{ds} = \frac{\partial g}{\partial r} \frac{dr}{ds} + \frac{\partial g}{\partial \phi} \frac{d\phi}{ds} + \frac{\partial g}{\partial \lambda} \frac{d\lambda}{ds} \quad , \tag{12}$$

and substitute for dr/ds, $d\phi/ds$, and $d\lambda/ds$ the expressions of the differential equation of the plumb line,

$$\frac{dr}{ds} = - \frac{\partial U}{\partial r} \Big/ g \quad ,$$

$$\frac{d\phi}{ds} = - \frac{\partial U}{\partial \phi} \Big/ g \, r^2 \quad , \tag{13}$$

$$\frac{d\lambda}{ds} = - \frac{\partial U}{\partial \lambda} \Big/ g \, r^2 \cos^2\phi \quad ,$$

which are easily derived by comparison of the corresponding terms in equations (14) and (15):

$$\frac{dx}{ds} = \cos \phi \cos \lambda \frac{dr}{ds} - r \sin \phi \cos \lambda \frac{d\phi}{ds} - r \cos \phi \sin \lambda \frac{d\lambda}{ds} \quad ,$$

$$\frac{dy}{ds} = \cos \phi \sin \lambda \frac{dr}{ds} - r \sin \phi \sin \lambda \frac{d\phi}{ds} + r \cos \phi \cos \lambda \frac{d\lambda}{ds} \quad , \tag{14}$$

$$\frac{dz}{ds} = \sin \phi \frac{dr}{ds} + r \cos \phi \frac{d\phi}{ds} \quad ,$$

and

$$\frac{dx}{ds} = -\cos\phi\cos\lambda\,\frac{1}{g}\,\frac{\partial U}{\partial r} + r\sin\phi\cos\lambda\,\frac{1}{g\,r^2}\,\frac{\partial U}{\partial\phi} + r\cos\phi\sin\lambda\,\frac{1}{g\,r^2\cos^2\phi}\,\frac{\partial U}{\partial\lambda} \quad,$$

$$\frac{\partial y}{ds} = -\cos\phi\sin\lambda\,\frac{1}{g}\,\frac{\partial U}{\partial r} + r\sin\phi\sin\lambda\,\frac{1}{g\,r^2}\,\frac{\partial U}{\partial\phi} - r\cos\phi\cos\lambda\,\frac{1}{g\,r^2\cos^2\phi}\,\frac{\partial U}{\partial\lambda} \quad, \qquad (15)$$

$$\frac{\partial z}{ds} = -\sin\phi\,\frac{1}{g}\,\frac{\partial U}{\partial r} - r\cos\phi\,\frac{1}{g\,r^2}\,\frac{\partial U}{\partial\phi} \quad,$$

with $dx^2 + dy^2 + dz^2 = ds^2$. Hence the vertical gravity gradient can be expressed as

$$\left(\frac{\delta g}{\delta s}\right)_{vert} = -\frac{1}{g}\left(\frac{\partial U}{\partial r}\,\frac{\partial g}{\partial r} + \frac{1}{r^2}\,\frac{\partial U}{\partial\phi}\,\frac{\partial g}{\partial\phi} + \frac{1}{r^2\cos^2\phi}\,\frac{\partial U}{\partial\lambda}\,\frac{\partial g}{\partial\lambda}\right) \quad. \qquad (16)$$

6

4. NUMERICAL RESULTS

As already stated, we based the numerical computations on Kozai's and Gaposchkin's latest harmonic coefficients. However, for comparison and to get an idea of the accuracy finally obtained, we also considered, where appropriate, King-Hele's (King-Hele and Cook, 1965; King-Hele, Cook, and Scott, 1965) zonal coefficients and Kaula's (1966) combination solution (obtained by merging satellite and surface gravity data).

The numerical results are presented in two steps:

A. We compute the horizontal and vertical gravity gradients of the ellipsoidal field, and

B. We add to these values corrections leading to the gravitational field under consideration. In this way we deal with only small numbers, which show immediately the deviations from an idealized field of a body with ellipsoidal equipotential surfaces.

4.1 The Ellipsoidal Gravity Gradient

If, in equation (2), we give the values for the equatorial radius and the harmonic coefficient of second degree, we can compute K_4 assuming an ellipsoidal field structure (Köhnlein, 1966); or, in detail,

$$a = 6\ 378\ 165\ m\ ,$$
$$K_2 = C_{20} = -\ 1082.645 \times 10^{-6} \quad (Kozai, 1964)\ ,$$
$$K_4 = 0.24 \times 10^{-5}\ ,$$

which means that the equipotential surface $V = $ const has at sea level a flattening of $f = 1/298.25$.

To obtain the variation of gravity, we only substitute V for U in equations (9), (10), (11), and (16). Hence we get the horizontal gravity gradient along V = const:

$$\frac{\delta g_e}{\delta s}\bigg|_{\phi=const} = 0 \quad , $$

$$\frac{\delta g_e}{\delta s}\bigg|_{\lambda=const} = \left[\frac{\partial g_e}{\partial \phi} - \frac{\partial g_e}{\partial r_e}\left(\frac{\partial V}{\partial \phi} \bigg/ \frac{\partial V}{\partial r_e}\right)\right]\left[r_e^2 + \left(\frac{\partial V}{\partial \phi} \bigg/ \frac{\partial V}{\partial r_e}\right)^2\right]^{-1/2} \quad , \tag{17}$$

and

$$\left(\frac{\delta g_e}{\delta s}\right)_{horiz} = \frac{\delta g_e}{\delta s}\bigg|_{\lambda=const} \quad , \tag{18}$$

i.e., the gravity of an equipotential ellipsoid varies only along the meridian. Table 1 gives the numerical values as a function of the geocentric latitude for the Northern Hemisphere. In the Southern Hemisphere, $(\delta g_e/\delta s)_{horiz}$ becomes negative, but the amounts remain unchanged (because of the direction in which the latitude is counted).

Table 1. Horizontal gravity gradients (ellipsoidal)

$\phi°$	$\left(\dfrac{\delta g_e}{\delta s}\right)_{horiz}$ (sec^{-2})
90	0
80	0.2767E-8*
70	0.5206E-8
60	0.7028E-8
50	0.8010E-8
40	0.8031E-8
30	0.7079E-8
20	0.5264E-8
10	0.2805E-8
0	0

*E-8 means $\times 10^{-8}$.

To obtain the vertical gravity gradient (perpendicular to V = const), we substitute V into (16):

$$\left(\frac{\delta g_e}{\delta s}\right)_{vert} = -\frac{1}{g_e}\left(\frac{\partial V}{\partial r_e}\frac{\partial g_e}{\partial r_e} + \frac{1}{r_e^2}\frac{\partial V}{\partial \phi}\frac{\partial g_e}{\partial \phi}\right) \quad , \tag{19}$$

which leads to negative values both for the Northern and Southern Hemispheres (see Table 2). The strongest variation occurs at the equator, and the amount of the gravity gradient monotonically decreases toward the poles.

Table 2. Vertical gravity gradients (ellipsoidal)

$\lvert \phi^\circ \rvert$	$\left(\dfrac{\delta g_e}{\delta s}\right)_{vert}$ $(\sec^{-2})$
90	-0.308337E-5
80	-0.308350E-5
70	-0.308388E-5
60	-0.308446E-5
50	-0.308518E-5
40	-0.308595E-5
30	-0.308667E-5
20	-0.308726E-5
10	-0.308764E-5
0	-0.308778E-5

4.2 The Horizontal Gravity Gradient

The components of the horizontal gravity gradient along U = const (with U = V) can be obtained from equations (9) and (10). However, if we at first subtract from $(\delta g/\delta s)_{\lambda=const}$ the ellipsoidal part $\delta g_e/\delta s_{\lambda=const}$ we obtain the components of the "anomalous" horizontal gravity gradient:

$$\frac{\delta g}{\delta s}_{\phi=const} = \left[\frac{\partial g}{\partial \lambda} - \frac{\partial g}{\partial r}\left(\frac{\partial U}{\partial \lambda}\bigg/\frac{\partial U}{\partial r}\right)\right]\left[r^2\cos^2\phi + \left(\frac{\partial U}{\partial \lambda}\bigg/\frac{\partial U}{\partial r}\right)^2\right]^{-1/2} \tag{20a}$$

9

and

$$\Delta \frac{\delta g}{\delta s}\Bigg|_{\lambda=const} = \left[\frac{\partial g}{\partial \phi} - \frac{\partial g}{\partial r}\left(\frac{\partial U}{\partial \phi}\bigg/\frac{\partial U}{\partial r}\right)\right]\left[r^2 + \left(\frac{\partial U}{\partial \phi}\bigg/\frac{\partial U}{\partial r}\right)^2\right]^{-1/2}$$

$$- \left[\frac{\partial g_e}{\partial \phi} - \frac{\partial g_e}{\partial r_e}\left(\frac{\partial V}{\partial \phi}\bigg/\frac{\partial V}{\partial r_e}\right)\right]\left[r_e^2 + \left(\frac{\partial V}{\partial \phi}\bigg/\frac{\partial V}{\partial r_e}\right)^2\right]^{-1/2} \quad . \quad (20b)$$

Hence the total amount of the horizontal anomalous gravity gradient is

$$\left(\Delta \frac{\delta g}{\delta s}\right)_{horiz} \approx \left[\left(\frac{\delta g}{\delta s}\bigg|_{\phi=const}\right)^2 + \left(\Delta \frac{\delta g}{\delta s}\bigg|_{\lambda=const}\right)^2\right]^{1/2} \quad . \quad (21)$$

This anomalous horizontal gravity gradient gives immediately the direction and magnitude of greatest variation in the gravity anomalies along $U = const$ referred to $V = const$. A vector map $10° \times 10°$ for the whole earth is shown in Figure 1, as obtained from Kozai's and Gaposchkin's coefficients. Besides areas of strong local disturbance, like the eastern part of the Antarctic $\left[(\Delta \delta g/\delta s)_{horiz} \approx 0.34 \times 10^{-9} \; \sec^{-2}\right]$, the northern part of India, etc., we also have a typical zone-oriented pattern, especially near the North and South Poles and along latitude $\phi = 60°$, etc., as can be seen in the figure. Hence it is natural to consider only the zonal coefficients C_{n0} in U of equation (1) and compare their deviations from the ellipsoidal part (see Figure 2). For comparison, we also included King-Hele's zonal coefficients, which give a somewhat smoother curve because of the lower degree (coefficients up to degree 9) considered. The maximum zonal anomaly of the horizontal gravity gradient approaches $0.08 \times 10^{-9} \; cm^{-2}$, i.e., only a quarter of the combined zonal plus nonzonal anomalous effects. At the poles the zonal anomaly is, of course, zero, because of the rotational symmetry of $\overline{U} = \overline{U}(C_{n0})$ with $\overline{U} = V$ linked to the ellipsoid.
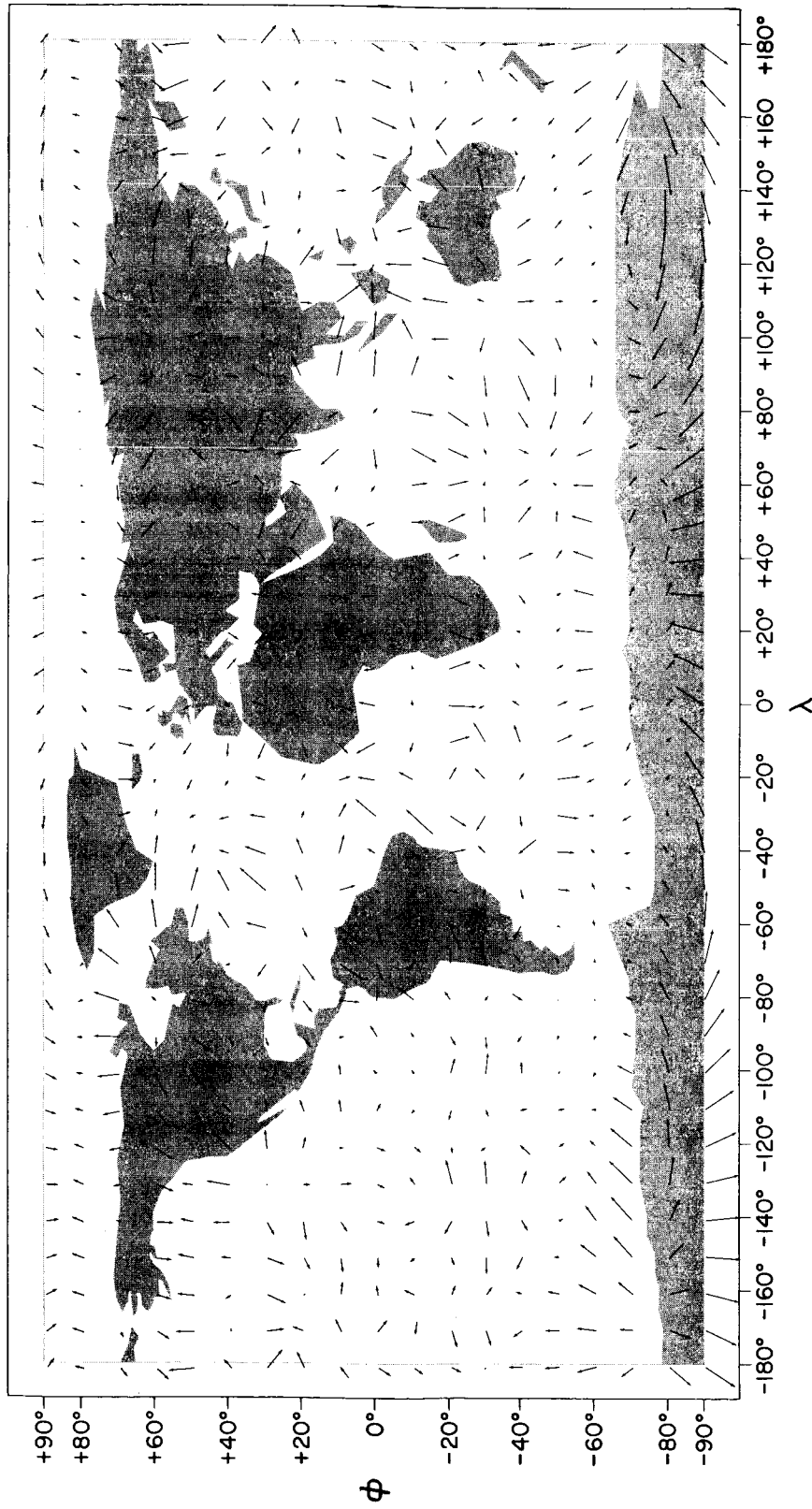
Figure 1. Anomalous horizontal gravity gradient $(\Delta \frac{\delta g}{\delta s})_{horiz} \times 10^{-11} \ sec^{-2}$.
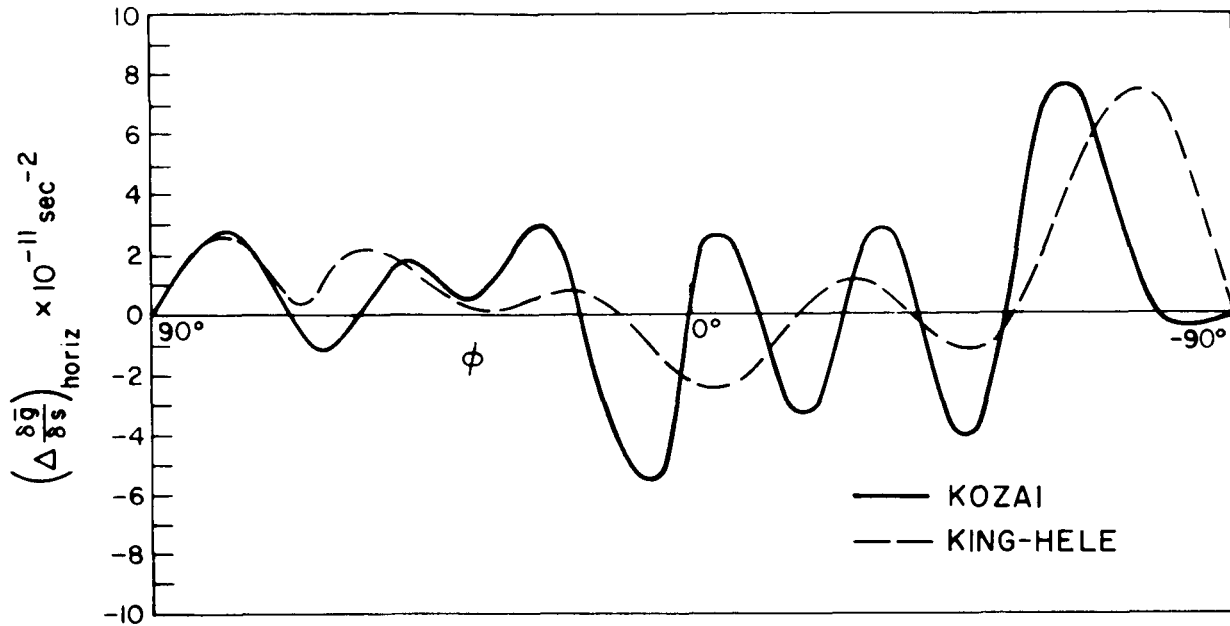
(f = 1/298.25)

11

Figure 2.   Anomalous horitzontal gravity gradient (zonal part).
($\overline{g}$ denotes the zonal part).

The overall results obtained from Kaula's coefficients are very similar to those obtained from the Kozai-Gaposchkin coefficients, although a direct comparison is somewhat difficult because of the different degrees and orders of harmonic terms used.

4. 3   The Vertical Gravity Gradient

The variation of gravity along the plumb line is obtained from equation (16). Similarly to the horizontal gradient, we consider only the anomalous effects relative to the ellipsoidal field, which lead to

$$\left(\Delta \frac{\delta g}{\delta s}\right)_{vert} = -\frac{1}{g}\left(\frac{\partial U}{\partial r}\frac{\partial g}{\partial r} + \frac{1}{r^2}\frac{\partial U}{\partial \phi}\frac{\partial g}{\partial \phi} + \frac{1}{r^2\cos^2\phi}\frac{\partial U}{\partial \lambda}\frac{\partial g}{\partial \lambda}\right)$$

$$+ \frac{1}{g_e}\left(\frac{\partial V}{\partial r_e}\frac{\partial g_e}{\partial r_e} + \frac{1}{r_e^2}\frac{\partial V}{\partial \phi}\frac{\partial g_e}{\partial \phi}\right) \quad . \tag{22}$$

If in equation (22) we substitute the potential $\overline{U} = \overline{U}(C_{n0})$ with $\overline{U} = V$ for U, we get the zonal part of the vertical variation. Figure 3 shows the anomalous zonal gravity gradient along the plumb line for the Kozai and King-Hele harmonic coefficients. Apart from the different degrees (14 and 9, respectively), we obtain quite different results around the North Pole, and then fairly good agreement along the meridian down to the South Pole (where the amplitudes are twice as large in King-Hele's case as compared to Kozai's). This spread gives an estimate of the accuracy at present obtained from satellite orbits. If we plot only the zonewise (latitude dependent) proportion between sea and continents (namely, the earth's topography) against Kozai's gravity gradient, we obtain roughly the dotted curve in Figure 3. The negative amplitude indicates the excess of water over land, and, analogously, a positive amplitude would mean that along a certain latitude the land masses dominate. The range is normalized and extends from -1 (only water) to 1 (only land). At the North Pole and at the South Pole both the solid and the dotted lines are clearly pointing in the same direction, and also the downward slope along the meridian seems to fit fairly well. The remaining discrepancy can be attributed : 1) to the uncertainty with which the vertical gradient can be currently determined, 2) the rough guess of land/sea distribution (ignoring elevations), and 3) other geophysical factors not considered. However, the overall correlation between the anomalous zonal gravity gradient along a plumb line and the mass distribution in the earth's crust is quite obvious from Figure 3.
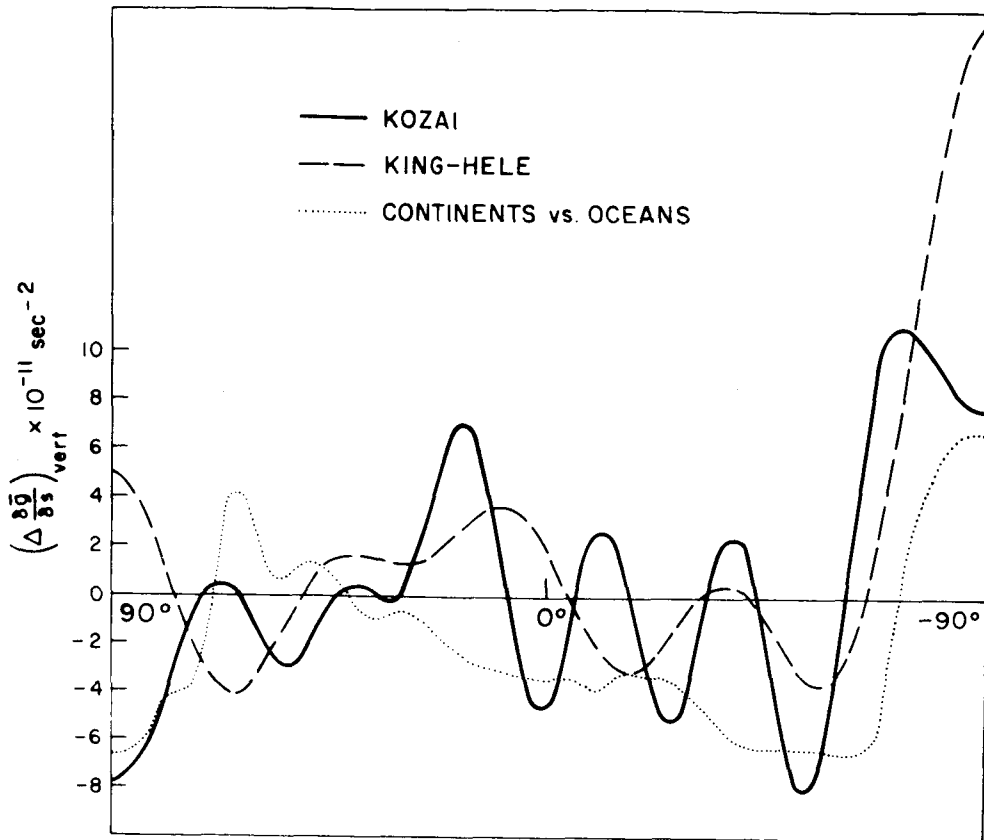
13

Figure 3. Anomalous gravity gradient along a plumb line (zonal part). ($\overline{g}$ denotes the zonal part).

Let us go one step further and consider the combined (zonal and nonzonal) anomalous vertical gravity gradient. Using equation (22) we obtain for the Kozai-Gaposchkin coefficients the number map shown in Figure 4. Each element gives the anomalous effect as a function of its position (ϕ, λ), relative to the ellipsoidal field in Table 2. If we superimpose the continent contours, we find the following characteristics:

A. Most of the continental area is covered with positive gravity gradient anomalies, i.e., the variation of gravity along the plumb line is smaller than the ellipsoidal part as already obtained for the zonal result (see Europe-Asia, America, Australia, etc.).

B. The negative disturbances of the continents are strongly correlated with the worldwide mountain chains, such as the Himalayas, the Rockies,

14

the Andes, etc. Hence, about 85% of the continental area shows its counter-part in the anomalous gravity gradient pattern.

Of course, we also have, similar to the zonal part, areas that fit neither the one nor the other pattern. For example, this is the case for the eastern part of the Antarctic, the western part of Greenland, etc. But all these areas have a high uncertainty as far as their gravity gradient anomalies are con-cerned. If we take Kaula's harmonic coefficients (obtained from a combina-tion of direct gravity measurements and satellite data), we can in such areas suddenly find the opposite results compared to the Kozai-Gaposchkin values. In brief, although the correlation between the continents and vertical gravity gradient anomalies is clearly visible from the most recent numerical analyses (Kozai, 1964; King-Hele and Cook, 1965; King-Hele et al., 1965; Gaposchkin, 1967; Kaula, 1966), we still need greater accuracies for more detailed investi-gations.
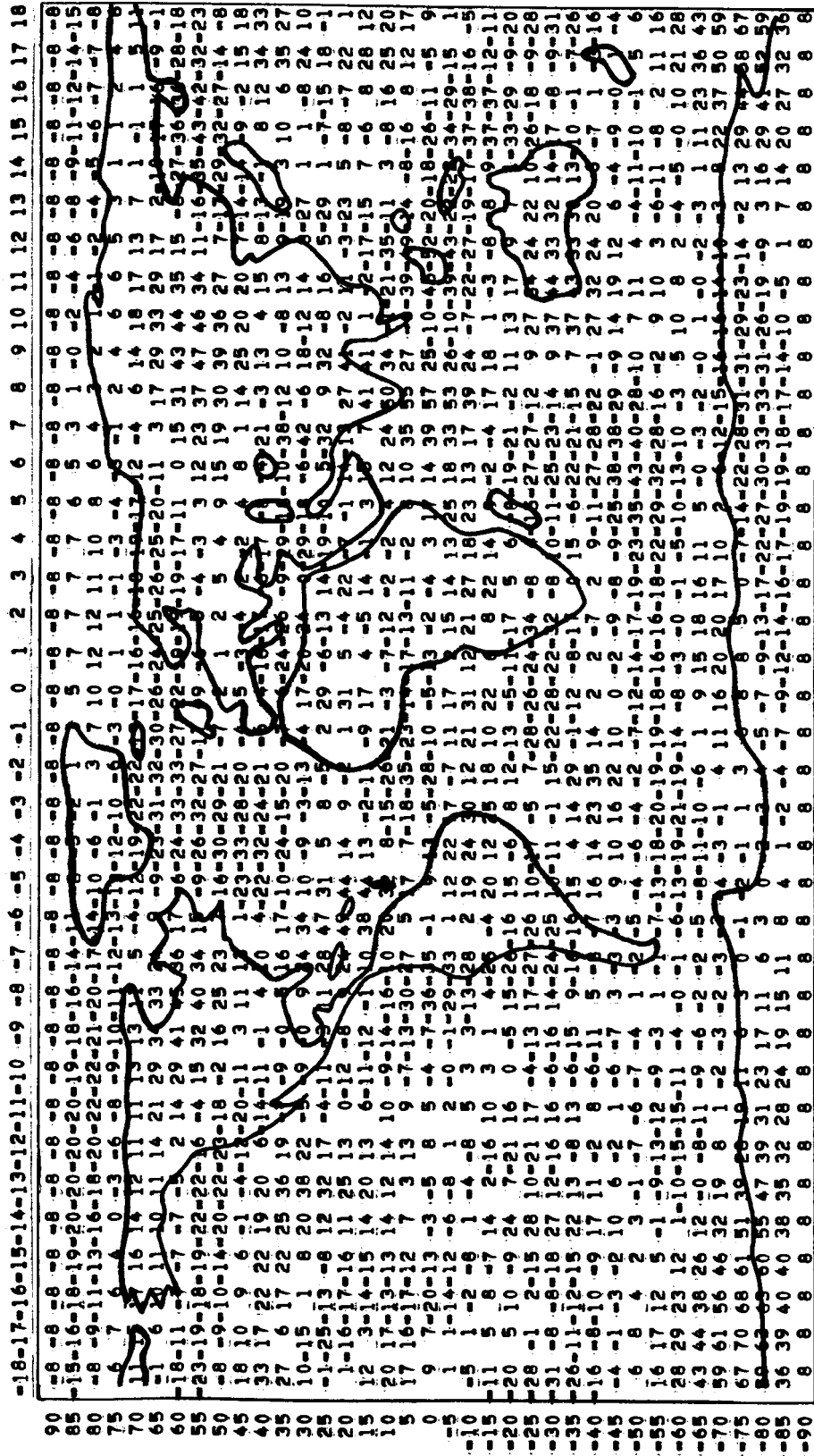
Figure 4. Anomalous variation of the gravity along a plumb line: $(\Delta \frac{\delta g}{\delta s})_{vert} \times 10^{-11} \text{ sec}^{-2}$ (first column at the left: geocentric latitude; first line at the top: 1/10 geocentric longitude).

$(f = 1/298.25)$

16

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BIOGRAPHICAL NOTE

WALTER KÖHNLEIN received his degrees from the Technological Institute in Munich, and the Technological Institute in Braunschweig, Germany, in 1958 and 1962, respectively.

He worked for a year at Ohio State University, Columbus, Ohio, before joining SAO in 1963. In 1965 he became an associate of the Harvard College Observatory.

Dr. Köhnlein's work at Smithsonian includes studies of the figure of the earth and its gravitational field by means of artificial satellites.

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