PRELIMINARY ANALYSIS OF THE EFFECTS OF PRESSURE SPACE CORRELATIONS ON THE VIBRATIONS OF APOLLO FLIGHT STRUCTURE
PRELIMINARY ANALYSIS OF THE EFFECTS OF PRESSURE SPACE CORRELATIONS ON THE VIBRATIONS OF APOLLO FLIGHT STRUCTURE

by
R.W. White
M.J. Crocker

Work Performed Under Contract NAS9-4305
Acoustic Study

Prepared by R.W. White
Prepared by M.J. Crocker

Approved by Kenneth McK. Eldred

May 1966
SUMMARY

This report contains a preliminary analytical investigation of the vibration response of Apollo skin structure to convected boundary layer turbulence, with emphasis on the variations of the response amplitudes with longitudinal and lateral correlation lengths.

The portion of the Apollo considered is the truncated conical Spacecraft Lunar Excursion Module Adaptor (SLA); and for purposes of this analysis, this structure is treated as an equivalent simply supported, uniform, flat, rectangular, honeycomb plate having essentially the same dynamic characteristics as the SLA. The response spectral density is computed for a pressure space correlation pattern having the functional form of an exponentially damped cosine along, and normal to, the flow axis. By including over 300 modes of the plate, a cursory parameter study is made to show how the response levels of the essentially discrete modes vary with longitudinal and lateral correlation lengths and convection velocities.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>vii</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 VIBRATION CHARACTERISTICS OF IDEALIZED APOLLO SKIN STRUCTURE</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Idealization of SLA Structure to Dynamically Equivalent Flat Plate</td>
<td>3</td>
</tr>
<tr>
<td>3.0 THE TURBULENT PRESSURE FIELD ENVIRONMENT</td>
<td>8</td>
</tr>
<tr>
<td>4.0 METHODS OF ANALYSIS OF FLAT PLATE RESPONSE TO TURBULENCE</td>
<td>10</td>
</tr>
<tr>
<td>4.1 Derivation of Equations for Response Spectral Density</td>
<td>10</td>
</tr>
<tr>
<td>4.2 Joint-Acceptance for Convected Boundary Layer Turbulence</td>
<td>15</td>
</tr>
<tr>
<td>4.3 Coincidence Conditions</td>
<td>18</td>
</tr>
<tr>
<td>4.4 Response Below Coincidence</td>
<td>22</td>
</tr>
<tr>
<td>4.5 Response Above Coincidence</td>
<td>23</td>
</tr>
<tr>
<td>4.6 Approximate Equation for RMS Deflection of a Single Mode</td>
<td>25</td>
</tr>
<tr>
<td>5.0 COMPUTATION AND ANALYSIS OF RESPONSE</td>
<td>27</td>
</tr>
<tr>
<td>6.0 CONCLUSIONS</td>
<td>31</td>
</tr>
<tr>
<td>APPENDIX A LOGIC FOR THE DIGITAL COMPUTER PROGRAM FOR STRUCTURAL RESPONSE TO BOUNDARY LAYER PRESSURE FLUCTUATIONS</td>
<td>32</td>
</tr>
<tr>
<td>APPENDIX B BIBLIOGRAPHY</td>
<td>38</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>40</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>41</td>
</tr>
<tr>
<td>TABLES</td>
<td>42</td>
</tr>
<tr>
<td>FIGURES</td>
<td>44</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Typical Overall SPL Time Histories Experienced on Large Rocket Vehicles (from Reference 1).</td>
</tr>
<tr>
<td>2</td>
<td>Saturn Launch Vehicles with Apollo Spacecraft (Extracted from Reference 6).</td>
</tr>
<tr>
<td>3</td>
<td>Details of Apollo Command Module, Service Module and LEM Adapter (Extracted from Reference 6).</td>
</tr>
<tr>
<td>4</td>
<td>General Configuration of SLA Skin Structure (Extracted from Reference 6).</td>
</tr>
<tr>
<td>5</td>
<td>Details of SLA Honeycomb Skin Construction (Extracted from Reference 6).</td>
</tr>
<tr>
<td>6</td>
<td>Non-Dimensionalized Fluctuating Pressure Spectra For a Number of Launch Vehicles (Extracted from Reference 4).</td>
</tr>
<tr>
<td>7</td>
<td>Maximum Fluctuation at any Mach Number versus Station.</td>
</tr>
<tr>
<td>8</td>
<td>Envelope of Maximum Fluctuation at any Station versus Mach Number.</td>
</tr>
<tr>
<td>9</td>
<td>Predicted Frequency Spectrum for Pressure Fluctuations on SLA.</td>
</tr>
<tr>
<td>10</td>
<td>Narrow-Band Space Correlation of the Wall Pressure Fluctuations at $M = 0.52$. Center Frequency cps: $\circ$, 1200; $\Delta$, 2400; $\Box$, 3600; $\Diamond$, 4800; $\nabla$, 6000.</td>
</tr>
<tr>
<td>11</td>
<td>Geometry of Plate and Direction of Convected Turbulent Flow Showing Convected Correlation Pressure Pattern.</td>
</tr>
<tr>
<td>12</td>
<td>Variation of Joint Acceptance at Resonance with Frequency.</td>
</tr>
<tr>
<td>13</td>
<td>Variation of Parameter $\Phi_y$ with Frequency and Mode Number $n$.</td>
</tr>
<tr>
<td>14</td>
<td>Variation of Parameter $\Phi_x$ with Frequency and Mode Number $m$.</td>
</tr>
<tr>
<td>15</td>
<td>Response of Plate in Frequency Range 0 $\rightarrow$ 25 cps.</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (continued)

Figure 16. Response of Plate in Frequency Range 0 → 250 cps. 
Figure 17. Response of Plate in Frequency Range 0 → 250 cps. 
Figure 18. Acceleration Response of Plate in Frequency Range 0 → 250 cps. 
Figure 19. Acceleration Response of Plate in Frequency Range 0 → 500 cps. 
Figure 20. Joint Acceptance for First Mode \( (f_{1,4}) \).

Figure 21. Variation of Plate Response with Longitudinal Exponential Decay Parameter \( k_1 \) and Correlation Length Parameter \( k_2 \). 
Figure 22. Variation of Plate Response with Longitudinal Exponential Decay Parameter \( k_1 \) and Correlation Length Parameter \( k_2 \). 
Figure 23. Variation of Plate Response with Longitudinal Exponential Decay Parameter \( k_1 \) and Correlation Length Parameter \( k_2 \). 
Figure 24. Variation of Plate Response with Longitudinal Exponential Decay Parameter \( k_1 \) and Correlation Length Parameter \( k_2 \). 
Figure 25. Variation of Plate Response with Longitudinal Correlation Length Parameter \( k_4 \). 
Figure 26. Plate Response with Joint Variation in Longitudinal Exponential Decay Parameter \( k_1 \) and Correlation Length Parameter \( k_2 \). 
Figure 27. Plate Response with Joint Variation in Lateral Exponential Decay Parameter \( k_3 \) and Correlation Length Parameter \( k_4 \). 
Figure 28. Variation of Plate Response with Lateral Decay Parameter \( k_3 \). 
Figure 29. Variation of Plate Response with Lateral Correlation Length Parameter \( k_4 \).
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE 1</td>
<td>Values of First 380 Resonance Frequencies of Plate and Joint - Acceptance for Each Frequency</td>
<td>42</td>
</tr>
<tr>
<td>TABLE 2</td>
<td>Calculated RMS Deflections for the Dominant Plate Modes</td>
<td>44</td>
</tr>
<tr>
<td>TABLE A1</td>
<td>Flow Chart of Computer Program</td>
<td>33</td>
</tr>
<tr>
<td>TABLE A2</td>
<td>The Listing of the Computer Program</td>
<td>34</td>
</tr>
</tbody>
</table>
**LIST OF SYMBOLS**

\( a \) = length of plate along the \( x \)-axis, in.
\( A \) = longitudinal pressure correlation decay parameter; eq. (35)
\( b \) = width of plate along the \( y \)-axis, in.
\( C_m \) = bending wave velocity component for \( m \)-elastic half waves along the \( x \)-axis, in/sec; eq. (27)
\( D \) = mean diameter of SLA = 17.8 ft.
\( E \) = Young's modulus of elasticity, \( \text{lb./in.}^2 \)
\( f \) = excitation and response frequency, cps
\( f_m \) = resonance frequency of the \( m \)-th longitudinal mode of plate, cps; eq. (29)
\( f_{mn} \) = resonance frequency of \( mn \)-mode of plate, cps; eq. (3)
\( F_{mn}(t, \omega) \) = generalized force for the \( mn \)-mode for harmonic excitation at frequency \( \omega \), lb; eq. (8)
\( g \) = acceleration of gravity = 386.4 in./sec.\(^2\)
\( h \) = overall thickness of honeycomb plate = 1.70 in.
\( h \) = location of neutral bending axis along \( z \)-axis of plate = 0.551 in.
\( H(\omega/\omega_{mn}) \) = dynamic magnification factor for the \( mn \)-mode; eq. (13)
\( I \) = moment of inertia of cross-section area per unit of length of the plate = 0.0287 in.\(^3\)
\( J_{mn}^2(\omega) \) = joint-acceptance of \( mn \)-mode at frequency \( \omega \); eq. (26)
\( k_i \) = pressure correlation parameters, \( i = 1, 2, 3, 4 \); eq. (43)
\( k_{mn} \) = generalized stiffness of the \( mn \)-mode, \( \text{lb./in.} \)
\( m \) = number of elastic half-waves of plate along \( x \)-axis
\( M \) = Mach number
\( M_o \) = total weight of plate, lb.
\( m_{mn} \) = generalized weight for the \( mn \)-mode, lb; eq. (11)
\( n \) = number of elastic half-waves of plate along \( y \)-axis
\[ P_{x,y} = \text{joint-acceptance parameters; eq. (26)} \]

\[ P(x,y,t;\omega) = \text{instantaneous harmonic pressure acting on plate at point } (x,y) \text{ at time } t \text{ and frequency } \omega, \text{ lb./in.}^2 \]

\[ q_{x,y} = \text{joint-acceptance parameters; eq. (26)} \]

\[ Q = \text{dynamic magnification factor at resonance} \]

\[ r_{x,y} = \text{joint-acceptance parameters; eq. (26)} \]

\[ R_p(x,y,x',y';\omega) = \text{pressure space cross-correlation function for points } (x,y) \text{ and } (x',y') \text{ at frequency } \omega; \text{ eq. (24)} \]

\[ S_p(\omega) = \text{power spectral density of pressure at frequency } \omega, \text{ (psi)}^2/\text{cps} \]

\[ S_w(x,y;\omega) = \text{power spectral density of plate deflection at point } (x,y) \text{ and frequency } \omega, \text{ in.}^2/\text{cps}; \text{ eq. (21)} \]

\[ S_{aw}(x,y;\omega) = \text{power spectral density of plate acceleration at point } (x,y) \text{ and frequency } \omega, \text{ (in./sec.}^2)\text{cps}; \text{ eq. (22)} \]

\[ t = \text{time, sec.} \]

\[ U = \text{free stream velocity of air flow} \]

\[ U_c = \text{turbulent boundary layer convection velocity} = 14,400 \text{ in./sec.} \]

\[ W_{mn}(x,y,t;\omega) = \text{instantaneous deflection of the mn-mode at time } t \text{ and frequency } \omega, \text{ in.; eq. (8)} \]

\[ x = \text{plate coordinate axis parallel to convected turbulent flow} \]

\[ \bar{x} = \text{nondimensional coordinate } = x/a \]

\[ y = \text{plate coordinate axis normal to convected turbulent flow direction} \]

\[ \bar{y} = \text{nondimensional coordinate } = y/b \]

\[ z = \text{plate coordinate normal to surface of plate} \]

\[ \gamma_x = \text{longitudinal pressure correlation length parameter; eq. (25)} \]

\[ \gamma_y = \text{lateral pressure correlation length parameter; eq. (25)} \]

\[ \delta_x = \text{longitudinal pressure space correlation decay rate; eq. (25)} \]

\[ \delta_y = \text{lateral pressure space correlation decay rate; eq. (25)} \]

\[ \Delta_{x,y} = \text{joint-acceptance parameters; eq. (26)} \]

viii
\( \xi \) = separation distance along y-axis = \( y - y' \), in.

\( \bar{\xi} \) = nondimensional form of \( \xi \) = \( \xi/b \)

\( \xi \) = separation distance along x-axis = \( x - x' \), in.

\( \bar{\xi} \) = nondimensional form of \( \xi \) = \( \xi/a \)

\( \xi_0 \) = first longitudinal zero crossing of pressure correlation pattern, in; eq. (33)

\( \xi_c \) = longitudinal pressure correlation length, in; eq. (34)

\( \xi_{mn} \) = generalized mass fraction for the \( mn \)-mode.

\( \theta(\omega/\omega_{mn}) \) = phase angle between the pressure excitation and the response for the \( mn \)-mode at frequency \( \omega \).

\( \lambda_m \) = bending wave length along x-axis for \( m \) elastic half waves along the x-axis; eq. (28)

\( \phi_{mn}(x,y) \) = mode shape of \( mn \)-mode of plate; eq. (2)

\( \mu \) = weight per unit area of plate = 2 lb./ft.\(^2\)

\( \phi_x \) = factor for longitudinal component of joint-acceptance; eq. (26)

\( \phi_y \) = factor for lateral component of joint-acceptance; eq. (26)

\( \omega \) = excitation and response frequency, rad./sec.

\( \omega_{mn} \) = resonance frequency of the \( mn \)-mode, rad./sec.

\( \Delta \omega \) = frequency bandwidth, rad./sec.

\( \| \) = denotes absolute value

\( \bar{\bar{\|}} \) = denotes time average value
1.0 INTRODUCTION

During its ascent through the atmosphere, the Saturn V launch vehicle with the Apollo Spacecraft will be subjected to severe fluctuating surface pressures which act as the major source of excitation for structural vibrations of the vehicle. These pressures are caused by acoustic noise from the engine exhaust, and unsteady aerodynamic flow over the vehicle, which flows are associated with such mechanisms as wakes, turbulent boundary layers, oscillating shocks and flow separation. Temporally, engine exhaust noise is the predominant source of these pressures in the first ten or twenty seconds of flight, while the unsteady aerodynamic flow dominates the pressure field during the remainder of the atmospheric flight phase, reaching a maximum environment near maximum dynamic pressure. Spatially, the maximum environment caused by engine noise occurs at the aft end of the vehicle in the vicinity of the engines, while the maximum aerodynamic environment occurs at the forward end of the vehicle. Predicted overall pressure levels for the vehicle are shown in Figure 1 for both the fore and aft ends; and it is seen that at the nose, the maximum aerodynamic environment is greater than the acoustic environment. Thus, with respect to the Apollo, the aerodynamic excitation at maximum dynamic pressure is the more severe environment.

During launch, the Apollo is equipped with a rocket escape tower which is situated forward of the Command Module. Because of its poor aerodynamic geometry, the air flow over this tower bathes the vehicle in a turbulent wake which, in addition to causing flow separation, ensures that the boundary layer over the vehicle is everywhere turbulent. Shock waves also form at the flares and shoulders of the vehicle, and during the transonic flight phase, these shock waves tend to oscillate and to progress slowly downward over the vehicle surface causing a severe unsteady pressure loading on the external skin. Although these aerodynamic phenomena are sometimes mutually coupled and sometimes coupled with the ensuing panel vibrations, first order estimates of their effects on structural response can be obtained by neglecting such coupling.

The oscillating shocks are found to be pseudo-discrete in the sense that the frequency spectrum of the surface pressure oscillations shows an energy concentration in a narrow frequency range while the amplitude excursion is random in time. Experimental evidence shows that such shocks are quick to couple with any forcing frequency present in the environment, including panel resonances. The response of panels to sinusoidally oscillating shocks is analyzed in Reference 2 and is not considered in this report.

This report is concerned primarily with the estimation of response levels of Apollo-like structures to turbulent boundary layer pressure fluctuations; and the pressure spectra and spatial correlation properties used in the analysis are typical of those which may occur for in-flight Apollo environments at maximum dynamic pressure. A limited study is also made to show how the response levels vary with longitudinal and lateral
correlation lengths and spatial correlation decay rates, so that the results can be extended to a degree in order to include structural response to separated flow pressure fluctuations. In particular, the turbulent boundary layer parameters are chosen on the basis of the predicted properties of the turbulent flow over the SLA. This section of the Apollo was chosen because the skin structure for this section is the most uniform and the most amenable to analysis — or, the most appropriate for the assumptions which must be made in the analysis.

In Reference 3, Wilby presents an analysis of the response of a simply supported, flat, rectangular plate to convected boundary layer turbulence; and since the complex equation for the joint-acceptance (the normalized mean square of the effective modal forcing function) has been worked out for such a structure and loading, it is desirable to idealize the SLA skin structure to a simply supported, flat, rectangular plate having essentially the same dynamic characteristics as the SLA. The details associated with this simplification are found in Section 2.0 of this report, where the resonance frequencies for several hundred modes are tabulated.

A discussion of the turbulent boundary layer properties and appropriate parameters used in the analysis is presented in Section 3.0 of this report. A summary of turbulence pressure spectra has been developed by Lowson in Reference 4; and, based on laboratory experimental data, Maestrello in Reference 5 has developed an empirical expression for the space correlation function for homogeneous boundary layer turbulence. These two important quantities are discussed in Section 3.0.

The general analytical procedure for predicting vibration response of plates to convected boundary layer turbulence is presented in Section 4.0, along with a summary of the necessary equations for joint-acceptance and deflection PSD. Using a digital computer program of these expressions, plate deflection spectra were computed for the predicted boundary layer turbulence and for variations in the space-correlation patterns associated with this turbulence. The results of these computations and an interpretive discussion are presented in Section 5.0. The primary results obtained are the relative response levels of the various modes for a given space correlation pattern, and the sensitivity of certain of the individual modal responses to longitudinal and lateral correlation lengths.

Pertinent conclusions obtained from the theoretical study and recommendations for further work in this problem area are discussed in Section 6.0.
2.0 VIBRATION CHARACTERISTICS OF IDEALIZED APOLLO SKIN STRUCTURE

In this section, the general configuration of the Apollo Spacecraft is presented showing the necessary geometry and dimensions. The skin panels of the SLA are shown in detail, as this structure is chosen for analysis as being representative of Apollo structure. The SLA shell is idealized to a uniform, flat, rectangular plate, simply supported along the four edges. The dynamic characteristics of this flat plate can be made approximately equivalent to those of the SLA by proper choice of the plate modes; and this choice is made so that the nodal pattern of the plate is similar to that of the SLA. A table of resonance frequencies for the plate is presented.

2.1 Idealization of SLA Structure to Dynamically Equivalent Flat Plate

A sketch of the Apollo Command Module, Service Module, and Spacecraft Lunar Excursion Module Adaptor (SLA) mounted on the Saturn launch vehicles is shown in Figure 2; and a more detailed sketch of the Apollo Spacecraft is shown in Figure 3. The section of the structure under study in this report is the skin of the SLA, which is shown in detail in Figures 4 and 5. (Figures 2 through 5 were extracted from Reference 6.)

Geometrically, the SLA is a truncated conical shell with a slant angle of slightly more than nine degrees. This shell is formed of two sets of panels, four panels per set. For this study, the vibration of the entire SLA is examined by considering the shell to be cut longitudinally, and when "unwrapped" to be approximately a rectangular panel. The upstream circumference is approximately 604 in. and the downstream circumference is approximately 816 in. The overall slant height of the SLA is 349 in. Therefore, the equivalent flat plate is assumed to have a length, along the flow axis, of 349 in., while the width of the plate is \((604 + 816) / 2\) or 710 in. A diagram of this plate is shown in the following sketch.

![Diagram](image)

\[a = 349 \text{ in.}\]

\[b = 710 \text{ in.}\]

\[h = 1.7 \text{ in.}\]
The skin is constructed from aluminum alloy (2024T-81) honeycomb material having an overall thickness of 1.7 in. and face sheet thicknesses of 0.015 in. and 0.032 in. The following sketch shows a typical cross-section of the honeycomb skin.

![Sketch of honeycomb skin](image)

The location $\bar{h}$ of the neutral bending axis is easily obtained from the geometry of the above sketch, and is given by the equation

$$\bar{h} = \frac{(0.032) \cdot (0.16) + (0.015) \cdot (1.693)}{0.047}$$

$$= 0.551 \text{ in.}$$

The moment of inertia of cross-section area $I$ about the neutral axis is given by:

$$I = (0.015) \cdot \left[ 1.693 - 0.551 \right]^2 + (0.032) \cdot \left[ 0.535 \right]^2$$

$$= 0.0287 \text{ in.}^3 \text{ (per unit length)}$$

From Reference 6, the mass/unit area $\mu$ of the material is approximately 2 lb./ft.$^2$ or 0.0139 lb./in.$^2$.

In an approximate manner, the modes of vibration of the SLA consist of two independent parts, namely, circumferential bending (ring) modes and longitudinal bending (beam) modes. These two components are shown diagrammatically in the sketch below for the fundamental SLA mode.
For this mode, the equivalent mode of the flat plate is as shown below.

For a simply supported flat plate, this mode shape is given by the equation

\[ \phi_{14}(x, y) = \sin\left(\pi \frac{x}{a}\right) \cdot \sin\left(4\pi \frac{y}{b}\right) \]  

(1)
For higher order modes of the plate, the mode shapes are

\[ \phi_{m,n}(x, y) = \sin(m \pi \frac{x}{a}) \sin(n \pi \frac{y}{b}) \]  

(2)

where

\[ m = \text{number of elastic half-waves along the } x\text{-axis} \]
\[ n = \text{number of elastic half-waves along the } y\text{-axis}. \]

Unlike a normal flat plate in which \( m \) and \( n \) can assume all integer values (\( m, n = 1, 2, 3, \ldots \)), the ring component of the SLA modes must have an even number of elastic half waves, which for dynamic similarity between SLA and plate requires that \( n \) must be even, starting with \( n = 4 \), so that \( n = 4, 6, 8, 10, \ldots \). The quantity \( m \) can assume all positive integer values.

Since the radius of curvature of the SLA shell is large compared with the shell thickness, effects of curvature on the resonance frequencies are neglected. Thus, the resonance frequency \( f_{m,n} \) of the \( m,n \) mode of the plate is given by the expression

\[ f_{m,n} = \frac{1}{2\pi} \sqrt{\frac{EIg}{\mu}} \left[ \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right] \text{ cps} \]  

(3)

where

\[ E = \text{Young's modulus of elasticity} \]
\[ = 10^7 \text{ lbs./in.}^2 \text{ for aluminum} \]
\[ I = \text{Moment of inertia of cross-section area, per unit length} \]
\[ = 0.0287 \text{ in.}^3 \]
\[ g = \text{Acceleration of gravity} \]
\[ = 386.4 \text{ in./sec.}^2 \]
\[ \mu = \text{Weight per unit area of plate, lb./in.}^2 \]
\[ = 0.0139 \text{ lb./in.}^2 \]
a = Length of plate
    = 349 in.

b = Width of plate
    = 710 in.

Substituting these parameter values into (3) gives

\[ f_{m,n} = 1.15 \left[ m^2 + 0.242 n^2 \right], \text{ cps} \]  \hspace{1cm} (4)

This frequency equation was numerically evaluated for:

\[
\begin{align*}
    m &= 1, 2, 3, \ldots, 19, 20 \\
    n &= 4, 6, 8, \ldots, 38, 40
\end{align*}
\]

and the results are presented in Table 1.

The modal density of this plate can be readily determined from inspection of Table 1. In the 5 – 10 cps band there are two resonances. In each .10 cps bandwidth, from 10 cps to 250 cps, there are four to eight resonances with an average of 5.7 modes per band in this range. In total, there are 139 modes within the frequency range from 5 cps to 250 cps.
### 3.0 THE TURBULENT PRESSURE FIELD ENVIRONMENT

In order to compute the power spectrum of the panel response to random pressure fluctuations, it is necessary to determine

i) The frequency spectrum of the forcing field,

ii) The overall level of the pressure fluctuations, and

iii) The longitudinal and lateral narrow band space correlations of the turbulent pressure field.

Since the Apollo capsule and SLA are in the wake flow from the escape tower, the boundary layer is thicker than would be normally expected, and it is found that the pressure fluctuations are increased in magnitude. Some areas of local separation are also probably caused by this wake flow.

Few flight measurements of pressure fluctuations have been made on the SLA. Thus the frequency spectrum is estimated from the nondimensional empirical curve given in Figure 6, extracted from Reference 4. This empirical curve was derived from measured data on several different missiles, including the Saturn I and is non-dimensionalized against vehicle diameter and free-stream velocity. In order to obtain a pressure power spectrum from Figure 7, it is necessary to choose representative values of the vehicle diameter \( D \), free-stream velocity \( U \), and an overall pressure level. The mean diameter for the SLA may be taken as \( D = (16.0 + 19.5)/2 = 17.8 \text{ ft.} \). The velocity at which maximum pressure fluctuations occur is approximately \( 1500 \text{ ft.}/\text{sec.} \). The overall sound pressure levels likely to be experienced by the SLA may be deduced from flight test data extrapolated from Saturn I flights (Reference 7). Unfortunately most of these data have been obtained for the Apollo sections immediately ahead of the SLA. Thus, use of the recommended testing levels given for these sections by Lowson, in Reference 7, are expected to be conservative by about 5 dB. Figures 7 and 8, from Reference 7, give the maximum pressure levels at any station versus Mach number. Using the data in Figures 7 and 8, and incorporating the 5 dB reduction factor, a reasonable pressure level for the SLA might be 160 dB. (It should be made clear that these levels are due to all sources such as wake flow, oscillating shocks, separated flows, and turbulent boundary layer pressure fluctuations). With \( D = 17.8 \text{ ft.}, \ U = 1500 \text{ ft.}/\text{sec.}, \) and an overall pressure level of 160 dB, the nondimensional empirical pressure spectrum in Figure 6 is converted to the true pressure spectrum shown in Figure 9. Both octave band and spectral density levels are presented.

It is much more difficult to predict the space correlations of the pressure field. Measurements in turbulent boundary layers at subsonic speeds are available; however, detailed measurements at supersonic speeds are very scarce and for separated flow, almost non-existent. In Figure 10, extracted from Reference 5, Maestrello
has shown that the measured narrow band longitudinal space correlation can be accurately represented by the empirical equation

\[ R_p(\xi, 0, 0, \omega) = e^{-0.1(|\xi|/U_c)} \cos(\xi \omega/U_c) \]  

(5)

where \( \xi \) denotes separation distance \((x - x')\) along the flow axis, and \( U_c \) is the mean eddy convection velocity.

The pressure field is less correlated in the lateral than the longitudinal directions and the authors find that the equation

\[ R_p(0, \xi, 0, \omega) = e^{-2.0(|\xi|/U_c)} \cos(2.0 \xi \omega/U_c) \]  

(6)

is a fairly good fit to Maestrello’s experimental results. In (7), \( \xi \) denotes the lateral separation distance \((y - y')\). The product of these two results is used to give the spatial cross correlation:

\[ R_p(\xi, \xi, 0; \omega) = R_p(\xi, 0, 0; \omega) \cdot R_p(0, \xi, 0; \omega) \]  

(7)

The measurements made by Maestrello correspond to a Mach number of 0.52, while the Mach number range of interest for the SLA is \( M = 0.8 \rightarrow 1.5 \). Since no other measured space correlation data are available, it is necessary in this analysis to assume that (5) and (6) are valid for the higher Mach number range. Further, it should be noted that Maestrello’s data are basically applicable to an attached turbulent boundary layer. Some of the flow over the SLA will undoubtedly be separated.

The mean velocity for convection of eddies downstream in the boundary layer was assumed to be 0.8 of the free stream velocity \( U \). That is the convection velocity

\[ U_c = 0.8 U = 1200 \text{ ft./sec.} \]

With these uncertainties regarding the space correlation function, it is appropriate to determine the sensitivity of panel response to pressure correlation lengths in order to assess the validity of the computed results obtained in this report. Thus, the response calculations to be discussed in the remaining sections are made for a range of cross correlation lengths covering several orders of magnitude. Data are not readily available for correlation patterns in separated flow; however, it is expected that the pressure fluctuations will be more correlated.
4.0 METHOD OF ANALYSIS OF FLAT PLATE RESPONSE TO TURBULENCE

The equations necessary for the computations of the response of a flat plate to convected boundary layer turbulence are discussed in this section of the report. These equations can be found, in part, in Reference 3; however, for sake of completeness and to add to the clarity of the discussion, a brief derivation is presented herein. Expressions are obtained for the power spectral density of the deflection and acceleration; and, the complex expression for joint-acceptance, which contains typographical errors in Reference 3, is presented in its correct form without derivation. The greatest response of the plate, per unit input, is expected to occur at coincidence, where the convection velocity equals the bending wave propagation velocity; and special consideration is given to the response equations for this condition. Consideration is also given to the panel response in the two regimes above and below coincidence. At the lower resonance frequencies, the response occurs in discrete modes; and introducing realistic assumptions on the joint-acceptance, an approximate, simple expression is derived for the RMS deflection of a single mode.

In the analysis to follow, the primary assumptions made are:

1.) the modes of vibration of the plate are uncoupled.
2.) the pressure field over the plate has a spectral density which is independent of position on the plate.
3.) the pressure field is homogeneous along the two principal axes of the plate so that the spatial correlation function along each axis is dependent only upon separation distance along that axis and independent of position along the axis.
4.) the space correlation function along each plate axis can be represented by an exponentially damped cosine.
5.) the response is a linear function of the excitation level.
6.) the resonance dynamic magnification factor is the same for all modes.

The geometry of the plate, the direction of the convected turbulent flow, and the pressure correlation pattern are shown in Figure 11.

4.1 Derivation of Equations for Response Spectral Density

With the assumption that the individual modes of the plate are uncoupled, each mode can be treated independently as a single degree of freedom system. In this sense, each mode will have an effective mass, a resonance frequency, and, for a given pressure distribution over the plate, each mode will have an effective or generalized force. For sinusoidal excitation of frequency \( \omega \), the steady state deflection, \( W_{mn}(x, y; \omega) \), at point \((x, y)\) of the \(m, n\) mode is
\[ W_{m,n}(x, y, t; \omega) = \phi_{m,n}(x, y) \cdot \frac{\mathcal{F}_{m,n}(t; \omega)}{K_{m,n}} \cdot H(\omega/\omega_{m,n}) \cdot e^{i\theta(\omega)} \] (8)

where

\[ \phi_{m,n}(x, y) = \text{normalized (to unity) deflection shape of the } m,n \text{ mode} \]
\[ = \sin m\pi \frac{x}{a} \cdot \sin n\pi \frac{y}{b} \text{ for a simply-supported plate} \]

\[ \mathcal{F}_{m,n}(t; \omega) = \text{generalized force of the } m,n \text{ mode} \]
\[ = \int_{a}^{b} \int_{0}^{b} P(x, y, t; \omega) \cdot \phi_{m,n}(x, y) \cdot dx \cdot dy, \text{ lb.} \] (9)

\[ P(x, y, t; \omega) = \text{sinusoidal pressure acting at point } (x, y), \text{ lb./in.}^2 \]

\[ K_{m,n} = \text{generalized stiffness of the } m,n \text{ mode} \]
\[ = M_{m,n} \omega^2 / g, \text{ lb./in.} \] (10)

\[ M_{m,n} = \text{generalized weight of the } m,n \text{ mode} \]
\[ = \xi_{m,n} M_o / g, \text{ lb.} \] (11)

\[ \xi_{m,n} = \text{generalized mass fraction of the } m,n \text{ mode} \]
\[ = \frac{1}{4} \text{ for all modes of a simply-supported plate} \]

\[ M_o = \text{total weight of panel, lb.} \]
\[ = ab\mu \] (12)

\[ a, b = \text{edge dimensions of plate} \]
\[ \mu = \text{weight per unit area of plate, lb./in.}^2 \]

\[ \omega_{m,n} = \text{resonance frequency of the } m,n \text{ mode, rad./sec.} \]

\[ H(\omega/\omega_{m,n}) = \text{single degree of freedom dynamic magnification factor of the } m,n \text{ mode} \]
\[ = \left[ \left( 1 - \left( \frac{\omega}{\omega_{m,n}} \right) \right)^2 + \frac{1}{Q^2} \left( \frac{\omega}{\omega_{m,n}} \right)^2 \right]^{-1/2} \] (13)
\( Q \) = dynamic magnification factor at resonance.
(assumed to be equal for all modes)

\( \theta(\omega) \) = phase angle between the pressure excitation and the response

\[
= -\tan^{-1}\left[ \frac{1}{Q}\left(\frac{\omega}{\omega_{mn}}\right) \right]
\]

The mean square value of the modal deflection in (8) is

\[
W_{mn}^2(x, y, t; \omega) = \frac{\phi_{mn}^2(x, y) \cdot H^2(\omega/\omega_{mn}) \cdot F_{mn}(t; \omega)}{K_{mn}^2}
\] (15)

where the mean-square value of the generalized force is

\[
F_{mn}(t; \omega) = \int_a^b \int_a^b P(x, y, t; \omega) \cdot P(x', y', t; \omega) \cdot \phi_{mn}(x, y) \cdot \phi_{mn}(x', y') \, dx \cdot dy
\] (16)

If the applied pressures contain several discrete frequency components, then the total mean-square deflection due to all frequency components is the summation of the mean square deflection due to each frequency component. Further, if the frequency content of the fluctuating pressures is limited to a very narrow band, \( \Delta \omega \), such that \( \omega - \Delta \omega \leq \omega \leq \omega + \Delta \omega \), then the dynamic magnification factor \( H(\omega/\omega_{mn}) \) can be considered to be the same for all components in the \( \Delta \omega \) band. Under this assumption, only the mean-square deflection

\[
W_{mn}^2(x, y, t; \omega)
\]
and the pressure cross-correlation term

\[ P(x, y, t; \omega) \cdot P(x', y', t; \omega) \]

will vary with frequency in the band. Neglecting rigorous mathematical arguments, it is plausible that these two discrete frequency quantities can be replaced by their narrow band random equivalents which contain all frequency components in the \( \Delta \omega \) band, their equivalents having the form

\[ W_{mn}^2(x, y, t; \omega, \Delta \omega), \quad P(x, y, t; \omega, \Delta \omega) \cdot P(x', y', t; \omega, \Delta \omega) \]

Assuming next that the magnitudes of these quantities are proportional to \( \Delta \omega \), so that spectral densities of each are certain to exist, then these narrow band quantities can be replaced by the deflection power spectral density \( S_{w_{mn}}(x, y; \omega) \) and by the pressure cross power spectral density \( S_P(x, y, x', y'; \omega) \) respectively. Thus, using (16), (15) can be rewritten in a form to give the deflection spectral density of the \( mn \) mode.

\[
S_{w_{mn}}(x, y; \omega) = \frac{\phi_{mn}^2(x, y) \cdot H^2(\omega/\omega_{mn})}{\mathcal{K}^2_{mn}} \cdot \int \int \int \int \int S_P(x, y; x', y'; \omega) \cdot \phi_{mn}(x, y) \cdot \phi_{mn}(x', y') \cdot dx \cdot dy \cdot dx' \cdot dy'
\]

(17)

It is convenient now to express the cross spectrum as the product of a pressure spectrum level \( S_P(\omega) \) and a normalized (to unity) cross correlation function \( R_p(x, y; x', y'; \omega) \) where it is assumed that the pressure spectrum level is uniform over the surface of the plate. Making this substitution in (17) and normalizing the range of integration gives

\[
S_{w_{mn}}(x, y; \omega) = \frac{\phi_{mn}^2(x, y) \cdot H^2(\omega/\omega_{mn}) \cdot S_P(\omega) \cdot J^2_{mn}(\omega) \cdot a^2 \cdot b^2}{\mathcal{K}^2_{mn}}
\]

(18)
where
\[ J_{mn}^2(\omega) = \text{joint-acceptance of the } mn \text{ mode} \]
\[ = \int_0^1 \int_0^1 \int_0^1 \int_0^1 R_p(x, y; x', y'; \omega) \cdot \phi_{mn}(x, y) \cdot \phi_{mn}(x', y') \cdot dx \cdot dy \cdot dx' \cdot dy' \]

(19)

However, from (10), (11), and (12), the generalized stiffness \( \kappa_{mn} \) is
\[ \kappa_{mn} = \left( \frac{\mu}{g} \right) \cdot ab \cdot \xi_{mn} \omega_{mn}^2 \]
so that (18) can be rewritten as
\[ \frac{S_{w_{mn}}(x, y; \omega)}{S_p(\omega)} = \left( \frac{g}{\mu} \right)^2 \cdot \frac{\phi_{mn}^2(x, y) \cdot H^2(\omega/\omega_{mn}) \cdot J_{mn}^2(\omega)}{\xi_{mn}^2 \omega_{mn}^4} \]

(20)

Summing expressions of the form (20) over all modes of the plate gives the total power spectral density of the plate deflection at point \((x, y)\), namely
\[ \frac{S_w(x, y; \omega)}{S_p(\omega)} = \left( \frac{g}{\mu} \right)^2 \sum_{m,n} \frac{\phi_{mn}^2(x, y) \cdot H^2(\omega/\omega_{mn}) \cdot J_{mn}^2(\omega)}{\xi_{mn}^2 \omega_{mn}^4} \]

(21)

The power spectral density of acceleration response, \( S_{aw} (x, y; \omega) \) can be obtained from the corresponding deflection by the expression
\[ S_{aw} (x, y; \omega) = \omega^4 S_w (x, y; \omega) \]

14
Introducing the assumption of homogeneity on the $x$ and $y$ components of the pressure field, the narrow band pressure space cross correlation function $R_p(x, y; x', y', \omega)$ appearing in (19), which is an even function, can be replaced by the simpler form

$$R_p(x, y; x', y'; \omega) = R_p(\xi, \zeta; \omega)$$  \hspace{1cm} (23)$$

where

$$\begin{align*}
|\xi| &= |x - x'| \\
|\zeta| &= |y - y'|
\end{align*}$$

so that the cross correlation, at a given frequency, is uniform over the plate and depends only upon the separation distances along the $x$ and $y$ axes.

From (5), (6), and (7), it is clear that (23) can be written in the functional form

$$R_p(x, y; x', y'; \omega) = e^{-\delta_x |\xi|} \cos \gamma_x \xi e^{-\delta_y |\zeta|} \cos \gamma_y \zeta$$  \hspace{1cm} (24)$$

where, for a turbulent boundary layer

$$\begin{align*}
\delta_x &= 0.10 \frac{w_0}{U_c} \\
\delta_y &= 2 \frac{w_0}{U_c} \\
\gamma_x &= \frac{w_0}{U_c} \\
\gamma_y &= 2 \frac{w_0}{U_c} \\
\bar{\xi} &= \xi/a = \bar{x} - \bar{x}' \\
\bar{\zeta} &= \zeta/b = \bar{y} - \bar{y}' \\
\bar{x} &= x/a \\
\bar{y} &= y/b
\end{align*}$$  \hspace{1cm} (25)$$
When the mode shapes and expression (24) are substituted into (19), and the necessary integrations performed, Wilby, in Reference 3, shows that the expression for joint-acceptance of the \( mn \) mode of the plate becomes

\[
J_{mn}^2(\omega) = 16 \Phi_x \Phi_y / (mn\pi)^2
\]

\[
\Phi_x = \frac{1}{\Delta_x^2} \left\{ p_x \left[ 1 - (-1)^m \cdot e^{-\delta x \cdot \cos \gamma_x} \right] + 4 (-1)^m q_x \cdot e^{-\delta x} \cdot \sin \gamma_x + \frac{2m}{r} \Delta \right\}
\]

\[
\Phi_y = \frac{1}{\Delta_y^2} \left\{ p_y \left[ 1 - (-1)^n \cdot e^{-\delta y \cdot \cos \gamma_y} \right] + 4 (-1)^n q_y \cdot e^{-\delta y} \cdot \sin \gamma_y + \frac{2n}{r} \Delta \right\}
\]

\[
\Delta_x = \left[ 1 + \left( \frac{\delta_x}{mn\pi} \right)^2 + \left( \frac{\gamma_x}{mn\pi} \right)^2 \right] - 4 \left( \frac{\gamma_x}{mn\pi} \right)^2
\]

\[
\Delta_y = \left[ 1 + \left( \frac{\delta_y}{mn\pi} \right)^2 + \left( \frac{\gamma_y}{mn\pi} \right)^2 \right] - 4 \left( \frac{\gamma_y}{mn\pi} \right)^2
\]

\[
p_x = \left[ 1 + \left( \frac{\delta_x}{mn\pi} \right)^2 - \left( \frac{\gamma_x}{mn\pi} \right)^2 \right] - 4 \left( \frac{\gamma_x}{mn\pi} \right)^2 \left( \frac{\delta_x}{mn\pi} \right)^2
\]

\[
p_y = \left[ 1 + \left( \frac{\delta_y}{mn\pi} \right)^2 - \left( \frac{\gamma_y}{mn\pi} \right)^2 \right] - 4 \left( \frac{\gamma_y}{mn\pi} \right)^2 \left( \frac{\delta_y}{mn\pi} \right)^2
\]

\[
q_x = \left( \frac{\delta_x}{mn\pi} \right) \left( \frac{\gamma_x}{mn\pi} \right) \left[ 1 + \left( \frac{\delta_x}{mn\pi} \right)^2 - \left( \frac{\gamma_x}{mn\pi} \right)^2 \right]
\]

\[
q_y = \left( \frac{\delta_y}{mn\pi} \right) \left( \frac{\gamma_y}{mn\pi} \right) \left[ 1 + \left( \frac{\delta_y}{mn\pi} \right)^2 - \left( \frac{\gamma_y}{mn\pi} \right)^2 \right]
\]

\[
r_x = \left( \frac{\delta_x}{mn\pi} \right) \left[ 1 + \left( \frac{\delta_x}{mn\pi} \right)^2 + \left( \frac{\gamma_x}{mn\pi} \right)^2 \right]
\]

\[
r_y = \left( \frac{\delta_y}{mn\pi} \right) \left[ 1 + \left( \frac{\delta_y}{mn\pi} \right)^2 + \left( \frac{\gamma_y}{mn\pi} \right)^2 \right]
\]
For convenient reference later in the analysis, the joint acceptance at the resonance frequency of each mode listed in Table I was numerically evaluated using a digital computer program of the equations (25) and (26). Values of $a = 349$ in, $b = 710$ in, and $U_c = 14,400$ in./sec. were used in the computations. The results are listed in Table I also.

In Reference (5), Maestrello's experimental data on lateral correlation of the turbulent boundary layer pressures shows no definite zero crossing; and thus, it might be sufficiently accurate to approximate (6) by an exponential function only. This is equivalent to setting $\gamma_y = 0$. Introducing this assumption into the joint-acceptance expression (26), the equations for $\Delta_y$, $p_y$, $q_y$, $r_y$, and $\Phi_y$ simplify to the following:

$$\Delta_y = p_y = \left[ 1 + \left( \frac{\delta_y}{mn} \right)^2 \right]^{1/2}$$

$$q_y = 0$$

$$r_y = \left( \frac{\delta_y}{mn} \right) \left[ 1 + \left( \frac{\delta_y}{mn} \right)^2 \right]^{1/2}$$

and hence, noting from Section 2.0 that $n$ is an even integer,

$$\Phi_y = \frac{-\delta_y}{1 - e^{\delta_y}} + \frac{\delta_y/2}{1 + \left( \frac{\delta_y}{mn} \right)^2}$$

(26a)

Exponential Lateral Correlation
4.3 Coincidence Conditions

Physically, the narrow band space correlation function \( R_\rho (\xi, \zeta; \omega) \), is a measure of the time average value of the relative phase between pressures acting at two points \((x, y)\) and \((x', y')\) which are separated by component distances \(\xi\) and \(\zeta\). This implies that the pressure acting at any point \((x', y')\) within the central region of positive correlation shown in Figure (11) will, over a long time average, be in-phase for an \( R_\rho \) fraction of this time with the pressure acting at the center \((x, y)\) of the region. Thus, the central region of the positive correlation is often referred to as the correlation area, and its component dimensions as correlation lengths.

When the pressure correlation lengths are equal to the bending half-wave lengths of the plate for a particular mode, the pressures are, on the average, spatially correlated with the bending deflection shape of that mode. This condition results in a maximum joint-acceptance for the mode; and when this condition also occurs at or near the resonance frequency of the mode, the modal response is a maximum, thus producing what is commonly called coincidence. It can be shown that wave length matching at resonance for a finite plate causes the bending wave propagation velocity to be equal to the surface pressure convection velocity. The latter is often used as the basic definition of coincidence.

Because of the two-dimensional character of the plate mode shapes, it is possible to have coincidence along the two principle plate axes. All considerations of coincidence in this analysis are restricted, however, to the flow axis for the reasons that

a) simultaneous wave matching along two axes is unlikely,

b) the high rate of spatial decay of the lateral pressure correlation would minimize the effect of any such coincidence,

c) the lowest coincidence frequency occurs for the lowest order lateral deflection shape,

d) the coincident mode, or near coincident mode, corresponds to a large number \(m\) of elastic half waves.

If \(C_m\) denotes the bending wave velocity of the plate along the flow axis, and if \(U_c\) is the pressure convection velocity along this axis, then at coincidence,

\[
U_c = C_m = \lambda_m m
\]  

(27)
where

\[ \lambda_m = \text{bending wave length of the } m\text{-th longitude mode of the plate} \]
\[ = 2 \frac{a}{m}, \text{ in.} \quad (28) \]
\[ f_m = \text{resonance frequency of the } m\text{-th longitudinal mode of the plate, cps} \]
\[ = 1.15 m^2 \quad \text{from (4)} \quad (29) \]

Combining (27), (28), and (29) gives the following approximate equation for the integer \( m \), and the equation for the coincidence resonance frequency \( f_m \):

\[ m = \frac{U_c}{2.30a} \]

**Coincidence Frequency**

\[ f_m = 0.217 \left( \frac{U_c}{a} \right)^2 \]

(30)

With a convection velocity of \( U_c = 14,400 \text{ in./sec.} \), and a plate length of \( a = 349 \text{ in.} \), then the nearest integer \( m \) is \( m = 18 \) and the corresponding coincidence frequency \( f_m = 360 \text{ cps} \).

Consider next the joint-acceptance at resonance for the coincident mode. From (27) and (28), the coincident mode resonance frequency \( f_m \) is

\[ f_m = \frac{mU_c}{2a} \quad \text{or} \quad \omega_m = \frac{m\pi U_c}{a} \]

(31)

Thus, from (25) and (31), the expressions for \( \gamma_x \) and \( \delta_x \) are:

\[ \gamma_x = \frac{m\pi}{10} \]

\[ \delta_x = \frac{m\pi}{10} \]

(32)
It is interesting to note that the coincidence value of $\gamma_x = m \pi$ ensures that the pressure correlation length is equal to a bending half-wave length of the coincident mode. Thus, this value is in agreement with the physical concept discussed above, where the maximum joint-acceptance is associated with maximum phase correlation between the pressure field and the mode shape. The proof is simple enough. From (24), the value of $\xi \equiv \xi_0$ for the first zero crossing of the correlation function $R_{P}$ along the flow axis is

$$\xi_0 = \frac{a}{\gamma_x} = \frac{a}{2m} \quad (33)$$

However, the pressure correlation length, $\xi_c$, along the flow axis is the distance between two successive zero crossings, so that

$$\xi_c = 2 \xi_0 = \frac{a}{m} = \text{length of bending half wave} \quad (34)$$

The joint-acceptance of the coincident mode at resonance can be obtained by substituting (32) into (26). For purposes of generality, $\gamma_x$ is set equal to $m \pi$ in the substitution; however, the ratio $\delta_x/m \pi$, which is equal to 0.10 in (32), is set equal to an arbitrary, but small, parameter $A$ so that

$$A \equiv \frac{\delta_x}{m \pi} \ll 1 \quad (35)$$

The advantage in this operation is to allow for simplification of the final result to pressure waves which have no longitudinal exponential spatial decay. Solving (26) for $\Delta_x, p_x, q_x, r_x$, and $\phi_x$ for this special coincidence case gives

$$\Delta_x = 4 \left[ 1 + \frac{A^2}{2} \right]^2 - 4 \approx 4A^2$$

$$p_x = A^4 - 4A^2 \approx -4A^2$$

$$q_x = A^3$$

$$r_x = A (1 + A^2) \approx A$$

$$\phi_x \approx \left[ \frac{1}{16A^4} - 4A^2 \left\{ 1 - e^{-m \pi A} \right\} + 4m \pi A^3 \right]$$
or
\[ \Phi_x = \frac{1}{4A^2} \left[ 1 + \frac{\sin \pi \delta}{\pi \delta} \frac{1}{\sinh A} + \frac{\sin \pi \delta}{\pi \delta} \right] \]

Coincidence for Non-decaying Longitudinal Pressure Field

If the pressure waves have no longitudinal decay in spatial correlation, similar to acoustic waves, the quantity \( A \) becomes zero and \( \Phi_x \) reduces to
\[ \Phi_x = \frac{(m\pi)^2}{8} \]

Coincidence for Non-decaying Longitudinal Pressure Field

If the pressure field consists of plane acoustic waves propagating along the \( x \)-axis, the lateral correlation factor, \( \exp(-\delta \phi_x \cos \gamma \phi_y) \), is equal to unity so that \( \delta = \gamma \phi_y = 0 \). Inspection of (26) for \( \Phi_y \) shows that for the modes of the plate considered in this analysis, \( n = 4, 6, 8, 10, \ldots \), \( \Phi_y = 0 \), and there is no other excitation, theoretically.

If the lateral component of the pressure field is coincident in the same manner as the longitudinal component of the pressure field, with no spatial decay in space correlation, then \( \Phi_y \) is
\[ \Phi_y = \frac{n\pi^2}{8} \]

Coincidence for Non-decaying Lateral Pressure Field

In the case where both axes are simultaneously coincident with a non-decaying pressure correlation, (37) and (38) can be substituted into (26) to give the following
\[ J^2 \left( \alpha_{mn} \right) = \frac{1}{4} \]

Simultaneous Coincidence for the \( x \) and \( y \) axes for a Non-decaying Pressure Field

It is interesting to note that this value of joint-acceptance is that given by Powell in Reference (8) for the case of perfect acoustic-bending wave length matching for a pinned-pinned beam.

More realistically, the joint-acceptance for the longitudinal coincident mode of the plate for actual homogeneous boundary layer turbulence, must be computed using (36) for \( \Phi_x \), and the general expression for \( \Phi_y \) and \( J^2 \) in (26). A reasonably accurate simplification could be made in the expression for \( \Phi_y \) by assuming that the lateral correlation function is non-periodic and exhibits only an exponential decay with distance \( \xi \).
4.4 Response Below Coincidence

It was shown in Section 4.3 that $\gamma_x \equiv \omega a/U_c = m\pi$ and $\delta_x = \omega a/10 U_c = m\pi/10$ correspond to longitudinal coincidence for the $m$-th longitudinal mode, the coincidence frequency being $\omega = \omega_m = m\pi U_c/a$. For a given mode $m$, values of the frequency $\omega < m\pi U_c/a$ are referred to as below coincidence frequencies; and modes having resonance frequency components $\omega_m < m\pi U_c/a$ are referred to as below coincidence modes. For these modes, the bending wave propagation velocity, $C_m$, is less than the turbulence pressure convection velocity $U_c$. This is easily shown by considering only the longitudinal component of the mode, for which the bending wave velocity is, by (28)

$$C_m = \frac{\omega_m}{2\pi} = 2 \frac{a}{m} \cdot \frac{\omega_m}{2\pi} \leq \frac{a}{m} \cdot \frac{mU_c}{a} = U_c$$

At resonance for the $m$-th longitudinal mode, the parameter $\gamma_x$ is, using (40),

$$\gamma_x = \frac{\omega m a}{U_c} = m\pi \left( \frac{C_m}{U_c} \right)$$

so that for modes which are below coincidence, $\gamma_x/m\pi < 1$. For those modes having resonance frequencies well below coincidence, $\gamma_x/m\pi \ll 1$, and the expressions (26) for the joint-acceptance amplify considerably. Noting from (25) that $\delta_x/m\pi \ll \gamma_x/m\pi \ll 1$, the quantities $\Delta_x$, $p_x$, $q_x$, $r_x$, and $\Phi_x$ become

$$\Delta_x \approx 1 - 2 \left( \frac{\gamma_x}{m\pi} \right)^2$$
\[ p_x \approx 1 - 2 \left( \frac{\gamma_x}{m \pi} \right)^2 \]
\[ q_x \approx \left( \frac{\delta_x}{m \pi} \right) \left( \frac{\gamma_x}{m \pi} \right) \]
\[ r_x \approx \left( \frac{\delta_x}{m \pi} \right) \]

and hence, retaining only first order terms so that \( \Delta_x = p_x \approx 1, q_x \approx 0 \), then

\[ \Phi_x \approx 1 - (-1)^m \left( -\delta_x \cos \gamma_x + \frac{\delta_x}{2} \right) \tag{42} \]

It is to be noted that this expression for \( \Phi_x \) depends only upon whether \( m \) is an even or an odd integer. With \( U_c = 14,400 \ \text{in./sec.} \) and \( a = 349 \ \text{in.} \), the expressions (25) for \( \gamma_x \) and \( \delta_x \) are

\[ \gamma_x = \frac{\omega a}{U_c} = 0.152 \ \text{f} \tag{43} \]
\[ \delta_x = 0.10 \ \frac{\omega a}{U_c} = 0.0152 \ \text{f} \]

A graph of (42) is presented in Figure 14 for a frequency range of 0 cps to 500 cps, for \( m \) equal to an even and an odd integer.

4.5 Response Above Coincidence

Above coincidence modes are those modes for which the bending wave velocity \( C_{mn} \) is greater than the pressure convection velocity, \( U_c \). Thus, from (41) it is seen that at the resonance frequencies of such modes, having \( \gamma_x / m \pi \gg 1 \), and for
those modes having \( \gamma_x / m \pi > 1 \), the joint-acceptance simplifies considerably. Noting that \( \delta_x / m \pi \ll \gamma_x / m \pi \), the expressions (26) for \( \Delta_x, p_x, q_x, r_x \) and \( \phi_x \) reduce to the following:

\[
\Delta_x \approx \left( \frac{\gamma_x}{m \pi} \right)^4 - 4 \left( \frac{\gamma_x}{m \pi} \right)^2 \approx \left( \frac{\gamma_x}{m \pi} \right)^4
\]

\[
p_x \approx \left( \frac{\gamma_x}{m \pi} \right)^4
\]

\[
q_x \approx r_x \approx 0
\]

so that \( \phi_x \) becomes

\[
\phi_x \approx \left( \frac{m \pi}{\gamma_x} \right)^4 \left[ 1 - (-1)^m \cdot e^{-\delta_x} \cdot \cos \gamma_x \right]
\]

which from (43) becomes

\[
\phi_x \approx \left( \frac{20.6 m}{\pi} \right)^4 \left[ 1 - (-1)^m \cdot e^{-0.0152 f} \cdot \cos(0.152 f) \right]
\]

From (45), it is seen that \( \phi_x \) decreases with increasing frequency by the fourth power of frequency for a given mode \( m \). This feature distinguishes the above coincident modes from the below coincident modes. Thus, the resonance excitation of the above coincidence modes decreases rapidly, as does the response.

24
Approximate Equation for RMS Deflection of a Single Mode

It is shown numerically in Section 5 that the joint-acceptance is a reasonably constant function of frequency over the response bandwidth, \( \omega_{mn}/Q \), for several of the modes. This fact can be used to obtain an approximate, yet convenient, expression for the rms deflection due to a single mode of vibration, if in addition, the reasonable assumption is made that the pressure spectral density \( S_p(\omega) \) is also constant over the band. Replacing \( J^2_{mn}(\omega) \) and \( S_p(\omega) \) by their constant values \( J^2_{mn}(\omega_{mn}) \) and \( S_p(\omega_{mn}) \) respectively, and integrating the \( mn \) term of (21) from \( f = 0 \) to \( \infty \) gives the mean-square value \( W^2_{mn}(x, y; \omega) \) of the \( mn \) mode, namely

\[
W^2_{mn}(x, y; \omega) = \int_0^\infty S_w(x, y; \omega) \cdot df
\]

\[
 \approx \left( \frac{q}{\mu} \right)^2 \frac{S_p(\omega_{mn}) \cdot \phi^2_{mn}(x,y) \cdot J^2_{mn}(\omega_{mn})}{2\pi \ell^2_{mn} \omega^3_{mn}} \int_0^\infty H^2 \left( \frac{\omega}{\omega_{mn}} \right) \cdot d \left( \frac{\omega}{\omega_{mn}} \right)
\]

However, it can be shown, using contour integration in the complex plane, that,

\[
\int_0^\infty H^2 \left( \frac{\omega}{\omega_{mn}} \right) \cdot d \left( \frac{\omega}{\omega_{mn}} \right) = \frac{\pi}{2} Q
\]

Thus, the mean-square deflection of the \( mn \) mode is
The PSD of the deflection \( S_w(x, y; \omega) \) at resonance for the \( mn \) mode can be obtained from (21) and is

\[
S(x, y; \omega_{mn}) = \left( \frac{g}{\mu} \right)^2 \frac{S_p(\omega_{mn}) \cdot \phi^2_{mn}(x, y) \cdot Q \cdot J^2_{mn}(\omega_{mn})}{4 \xi^2 \omega^3_{mn}} \cdot \frac{4Q}{\omega_{mn}}
\]

(41)

From (40) and (41), the approximate mean-square response is given by the expression

\[
W^2_{mn}(x, y; \omega) \approx \frac{\omega_{mn}}{4Q} \cdot S(x, y; \omega_{mn})
\]

(42)
5.0 **COMPUTATION AND ANALYSIS OF RESPONSE**

A digital computer program was made of the frequency equation (4), the deflection and acceleration response equations (21) and (22), and of the joint-acceptance equations (25) and (26). The program logic is presented in Appendix A. This program was first used to generate a summary list of the various plate resonance frequencies and joint-acceptance values given in Table I.

Ratios of the deflection and pressure power spectra were computed as functions of frequency for the points \( x/a = 0.5, y/b = 0.125 \), and \( x/a = 0.25, y/b = 0.125 \). These results are presented in Figures 15 through 17. An expanded frequency scale from \( f = 0 \) to \( 25 \text{ cps} \) is used in Figure 15 in order to show the detailed nature of the modal deflection. This graph shows the essentially discrete modal character of the response frequency scales of \( f = 0 \) to \( 250 \text{ cps} \), which are used in Figures 16 and 17. These graphs show that the first mode, \((m,n) = (1,4)\) has the greatest deflection response per unit of pressure excitation, and that the deflections for higher modes drop off rapidly with increasing frequency. This rapid decrease in deflection PSD is due, in part, to the increasing stiffness of the plate with the order of the mode being excited, and hence with frequency; this stiffness being controlled by the quantity \( \omega^4_{mn} \) appearing in equation (21). A curve of \( 1/\omega^4 \) passing through the response peak of the \((1,4)\) mode is included in Figure 17 to show the general effect on the deflection of this increasing stiffness. The decreasing joint-acceptance, shown in Figure 12, is primarily responsible for the additional rate of decrease of response amplitude with frequency.

Using the equation (42) for the approximate mean-square deflection per mode, the rms deflections for the \((1,4),(1,6),(2,4),(3,4),\) and \((3,6)\) modes were computed and the results are listed in Table II. The greatest rms deflection occurs for the \((1,4)\) mode, and this deflection (0.298 in.) is 17.5% of the plate thickness of 1.7 inch.

The acceleration spectral density for the above two points on the plate are presented in Figures 18 and 19 for frequencies up to 250 cps and 500 cps respectively. Figure 18 shows the essentially flat nature of the resonant acceleration response from 5 to 50 cps with a lower but flat response from 50 cps to 250 cps. Longitudinal coincidence of the plate was estimated in Section 4.3 to occur at about 375 cps, and for this reason the acceleration spectral density in Figure 19 is carried out to 500 cps so that the transition through and above coincidence is included. Figure 19 shows that above this predicted coincidence frequency, the response begins to drop off. This result is consistent with the decrease (by the fourth power of frequency) predicted in Section 4.5 for the above-coincidence modes.

In Section 4.5, it was assumed that the joint-acceptance is essentially constant over the frequency bond of the structural resonance. In support of this assumption for the highest response mode, the joint-acceptance for the \((1,4)\) mode is shown as a function of frequency in Figure 20. Clearly, \(J_{14}^2(\omega)\) is nearly constant in the neighborhood of \( f_{14} = 5.6 \text{ cps} \).
In using Maestrello's pressure correlation data, it is recognized that these data were obtained for flow conditions which may be different from those which occur in flight, so that the actual correlation lengths and spatial decay rates may differ from those employed in the above calculations. For this reason, it is desirable to test the sensitivity of the responses of the various modes to the correlation parameters. To do this, the following spatial correlation function was introduced into the computer program:

\[
R_p(\xi, \eta; \omega) = e^{-k_1 \xi_x |\xi|} \cos k_2 \gamma_x \xi_x \cdot e^{-k_3 \xi |\xi|} \cos k_4 \gamma_y \xi_y
\]  

In the remaining Figures (21 through 29), the responses of the first few major modes are shown as functions of the constants \(k_1, k_2, k_3, \text{ and } k_4\).

In Figure 21, the resonant response of the \((1, 4)\) mode is studied while the longitudinal correlation length parameter, \(k_2\), is varied from 0 to 20, while the longitudinal pressure decay parameter, \(k_1\), is given the discrete values \(k_1 = 0, 0.1, 1.0, 3.0, \text{ and } 10.0\). The axial correlation length, \(\xi_c\), as a function of \(k_2\), is

\[
\xi_c = \xi_c \cdot \frac{\alpha \pi}{k_2 \gamma_x} = \frac{\pi U_c}{\omega_{1,4} k_2} = \frac{U_c}{2 k_2 f_{1,4}}
\]

which for \(U_c = 14,400 \text{ in./sec.}, f_{1,4} = 5.6 \text{ cps}\) gives

\[
\xi_c = \frac{1290}{k_2} \text{ in.}
\]

For \(k_2 = 1.0\), the correlation length of 1290 inches is much greater than the panel length of 349 inches. Increasing \(k_2\) decreases the correlation length; and as seen by Figure 21, this decreases the response of the fundamental mode. From the joint-acceptance curve given by Powell in Reference (8), it is expected that a minimum excitation of the \((1, 4)\) mode will occur when the correlation length is only one-third the bending half wave length (349 in.) along the flow axis. A minimum response should occur, therefore, at \(k_2 = 1290/(349/1) = 11.1\), and this is seen to be the case in Figure 21.
The parameter \( k_1 \) controls the spatial correlation decay along the flow axis. For \( k_1 = 0 \), there is no correlation decay similar to the case of single frequency acoustic waves. For such excitation, cancellation between pressure and elastic waves due to wave mismatch, is sharper than for the exponentially decaying turbulent pressure field; and this is evidenced also in Figure 21.

The computer data presented in Figure 21 for the (1, 4) mode therefore shows certain distinct similarities to acoustic excitation as discussed by Powell in Reference (8), with the major difference coming from the exponential decay of the spatial correlation function. Further evidence of this can be found in Figures 22, for the (1, 6) mode, Figure 23, for the (3, 4) mode, and Figure 24 for the (2, 4) mode.

It should be noted, however, that for the (1, 4) mode, the sensitivity of the response to changes in \( k_1 \) depends upon the value \( k_2 \), the greatest sensitivity occurring for \( k_2 = 11 \) and 18. For \( k_2 = 1.0 \), which was used in the analysis, Figure 21 shows that changes in \(|\xi|\) should have only a slight effect on response. This is clearly in evidence in Figure 25, where \( k_2 = 2.0 \) and \( k_2 \) is varied.

The first negative peak of the longitudinal pressure correlation function occurs when

\[
\bar{\xi} = \frac{\pi}{k_2 \gamma_x}
\]

The value of the longitudinal correlation factor at this minimum is

\[
e^{-k_1 \delta_x |\xi|} e^{-\pi(k_1/k_2)(\delta_x/\gamma_x)} = e^{-(\pi/10)(k_1/k_2)}
\]

Thus, as the longitudinal correlation length is decreased by increases in the parameter \( k_2 \), the height of the negative correlation peak remains unchanged and equal to \( \exp(-0.314) \), then \( k_1 = k_2 = k \). The resulting changes in the response of the (1, 4) mode with changes in \( k \) are quite different from those shown in Figure 21, as can be seen in Figure 26.
The sensitivity of the (1,4) mode to changes in the lateral correlation length and lateral correlation decay rate are shown in Figures 27 through 29. This sensitivity is not large when the lateral correlation lengths are decreased and when the decay rate is increased. However, if the lateral correlation pattern is altered in such a manner that the pressures tend to be correlated over greater lateral distances, the response level decreases sharply. This is expected since the lateral mode shape consists of an even number of elastic half-waves, giving rise to cancellation of the input over adjacent lateral half-waves.
6.0 CONCLUSIONS

A preliminary estimate has been made in this report of the vibration response characteristics of SLA-like structure to convected boundary layer turbulence. The analysis is quite approximate in the sense that the SLA was represented as a simply supported, flat rectangular plate having the same cross-sectional dimensions and mass density as the SLA. The dynamic properties of the plate were made as equivalent to those of the SLA as could be done within this structural idealization by choosing only those resonance frequencies and modal patterns that are expected to occur on a pinned edge supported cylinder having a radius equal to the mean radius of the truncated conical SLA. All stiffness effects due to curvature were neglected. Further, approximations have been made regarding the uniformity of the SLA shell, the applicability of Maestrello's boundary layer correlation pattern to unsteady aerodynamic flow during flight, linearity of the structure, and uniformity of the surface pressure field.

Within these limitations, the primary results obtained are:

1. The greatest response, per unit of pressure excitation level, occurs in the fundamental (1, 4) mode at 5.6 cps.

2. Certain higher order modes have response levels which are of the same order of magnitude as that of the fundamental mode; and when the pressure spectrum level is applied to the response spectral densities, higher order modes could dominate the response spectrum.

3. The average acceleration spectral density (taken about midway between the resonant peaks and antiresonant troughs) tends to be constant with frequency below the (bending velocity-convection velocity) coincidence, frequency, and to drop off sharply with frequency above coincidence.

4. No significant increase in acceleration spectral density occurred at coincidence.

5. For nominal changes in the longitudinal pressure correlation pattern, the responses of the (1, 4) and (1, 6) modes experience little change, while the responses of the (3, 4) and (2, 4) modes could be significantly effected.

6. The general changes in the responses with changes in the pressure correlation pattern have strong resemblances to the sensitivity of panel vibrations to acoustic pressure correlations, although the sensitivity is much less for boundary layer turbulence.

7. For a spectral density level of 110 dB, at 5 to 6 cps, the fundamental mode of vibration rms response level is about 17.5% of the thickness (1.7 inches) of the plate.
APPENDIX A

LOGIC FOR THE DIGITAL COMPUTER PROGRAM FOR STRUCTURAL RESPONSE TO BOUNDARY LAYER PRESSURE FLUCTUATIONS

A computer program was written to calculate the displacement power spectral density for the equivalent SLA panel as given by Equation (21). The flow chart is given in Table A1, and the listing of the computer program is given in Table A2. The modal contributions of the first 380 modes were calculated and summed to give the displacement spectral density for each point against frequency.

The program was modified slightly to give the effect upon the displacement for the first few resonances as various correlation parameters were varied.
TABLE A 1. Flow Chart of Computer Program
PROGRAM APOLLO2
C RESPONSE OF SLA HONEYCOMB PANELS TO BOUNDARY LAYER NOISE
DIMENSION SW(1900),OMA(1880)
COMMON AJ(29,20),AX,AY,BX,RY,PD,H,PLX,PLY,XX,Y
1,MI,JP,P1,PI,R,0
PI=3.14159265
R=36,
E=1,0,N=07
REAL(X0,Y0,1)011 M,K
N=(N/2)+2
1 READ(60,1000)UC,TMU,TL,X,Y,OMA,OMAMAX,PLX,PLY,0
OMA=OMA+PI
OMAMAX=OMAMAX+PI
WRITE(61,1000)UC,TMU,TL,X,Y,OMA,OMAMAX,PLX,PLY,0
OMA(1)=0,0
K=1
10 K=K+1
AX=0,1*OMA(K)/UC
AY=OMA(K)/UC
AY=AY=2,0*AX
CALL JMN
SW(K)=0,0
DO 100 I=1,M
AN=FLOAT(I)
DO 100 J=1,N
AN=2,0*FLOAT(J)*2
W=PI**2*(AN/PLX)**2+(AN/PLY)**2)*SQRTF(6*TI*E/TMU)
PSI=6*(AN/PI)**2+(AN/PI)**2)*TI*E/TMU
W21=0,0/(6*AN/(OMA(K)/H)**2)*(OMA(K)/H)**2/1000
TEMP=W**2*PSI**2*AJ(I,J)*H2/(TII*W**2*AN**4*(I,0,4,0)**2)
DO TO 90,1000,1543TF(I)
90 WRITE(61,11002) TEMP,1,J
100 SW(K)=SW(K)*TEMP
OMA(K+1)=OMA(K)+OMA
WRITE(61,1000) SW(K),OMA(K)
IF(OMA(K),0,OMAMAX)200,10
200 CONTINUE
DO 150 I=1,K
OMA(I)=OMA(I)/196,5
150 SW(I)=LOG(SW(I)+1,0E+00),43429448
WRITE(61,11003) SW(K),OMA(K)
CALL PLOTCON
CALL LOGAXIS(U,0,6,0,1,0,1,0,6)
CALL REALAXIS(R,0,0,0,0,0,0,0,0)
I=3
DO 170 J=1,K
CALL PLOT(OMA(J),SW(J),1)
170 I=2
CALL PLOT(0,0,0,0,-3)
WRITE(59,1004)
STOP
1000 FORMAT(5E16,8)
1001 FORMAT(5E13)
1002 FORMAT(5X21THE VALUE OF W/H FOR M=17,8H AND N=12)
1003 FORMAT(5X,6HSW/SPE16,8,5X6HOMEGA=E16,8/)
1004 FORMAT(5X,7HTHE END/)END

3200 FORTRAN DIAGNOSTIC RESULTS - FOR APOLLO2

TABLE A 2. The listing of the Computer Program
3200 FORTRAN (2,1) / /

SURROUNTE JMN
DIMENSION XX(20), YY(20)
COMMON AJ(20,20), AX, AY, AY, PX, PY, X, Y
M, N, EI, J, N
AUXL = AX*PLY
RXLX = BX*PLY
DO 10 I=1, M
 ROL = FLOAT(I)***P1
 A2 = AUXL/ROL***2
 B2 = AUXL/ROL***2
 DX = (1,0+A2-B2)**2-4,0*B2
 PX = (1,0+A2-B2)**2-4,0*A2+B2
 GX = AUXL/ROL*(AXLX/ROL)*(1,0+A2-B2)
 VX = AUXL/ROL*(1,0+A2+B2)
 GO TO (5,10), SSWTCHF(2)
 5 WRITE(6,1,0) UX, PX, GX, VX, I
 10 XX(I) = 1,0/(I***2)*(PX*(1,0-I-1,0)**2*EXP(-AUXL)*COS(AUXL))
 1 = 4,0*GX*(1,0)**2*EXP(-AUXL)*SIN(AUXL+BOL/2,0+VX+DX)
 AYL = AYL*PLY
 WYLY = BY*PLY
 DO 20 I=1, N
 J = I+2
 ROL = FLOAT(I+2)**2*P1
 A2 = AYL/ROL***2
 B2 = AYL/ROL***2
 DY = (1,0+A2-B2)**2-4,0*B2
 PY = (1,0+A2-B2)**2-4,0*A2+B2
 GY = AYL/ROL*(AYLY/ROL)*(1,0+A2-B2)
 VY = AYL/ROL*(1,0+A2+B2)
 GO TO (15,20), SSWTCHF(2)
 15 WRITE(6,10) UY, PY, GY, VY, I
 20 YY(I) = 1,0/(I***2)*(PY*(1,0-I-1,0)**2*EXP(-AYLY)*COS(AYLY))
 1 = 4,0*GY*(1,0)**2*EXP(-AYLY)*SIN(AYLY+BOL/2,0+VY+DY)
 DO 30 J=1, M
 A = FLOAT(I)
 DO 30 J=1, N
 AN = FLOAT(J+2)
 AJ(I,J) = 16,0*XX(I)*YY(J)/*\((A+AN)**2*2)**2
 GO TO (25,30), SSWTCHF(2)
 25 WRITE(6,1002) AJ(I,J), I, J
 30 CONTINUE
RETURN
1000 FORMAT(5X11H0, PY, GY, VY, / (4F16,8, I3))
1001 FORMAT(5X11H0, PX, GX, VX, / (4F16,8, I3))
1002 FORMAT(5X, 12H0M1, I, J, / (E16,8, 2I3))
END

NO ERRORS

TABLE A 2 (Continued)

35
SUBROUTINE LOGAXIS(XLEN, YLEN, XSCALE, YSCALE, IEMP)
C XLEN= DISTANCE FROM ORIGIN ALONG X-AXIS TO BE PLOTTED
C YLEN= DISTANCE FROM ORIGIN ALONG Y-AXIS TO BE PLOTTED
C XSCALE= LENGTH OF MAJOR DIVISION OF X-AXIS, NON-ZERO
C YSCALE= LENGTH OF MAJOR DIVISION OF Y-AXIS, NON-ZERO
C IEMP IS CODE WORD FOR PAGE TICS (IEMP=0 NOT DRAWN, #1 DRAWN)
C SET PEN POINT AT LOCATION ORIGIN IS DESIRED
DIMENSION TICA(9)
IA=1
TICK(1)= .1459
TICK(2)= .1969
TICK(3)= .1549
TICK(4)= .2218
TICK(5)= .391
TICK(6)= .5979
TICK(7)= .5229
TICK(8)= .999
STOP=XLEN/XSCALE
SIGN=1,
IF(STOP .EQ. 0.) 997
997 ISTOP=IFIX(STOP)
A=FLOAT(ISTOP)
STOP=A*XSCALE
IF(IEMP .EQ. 0.) 999, 998
998 CALL PLOT(0.,1.,3)
CALL PLOT(-1.5,0.,3)
CALL PLOT(-1.5,1.,2)
CALL PLOT(0.2,0.,3)
CALL PLOT(0.5,1.,2)
CALL PLOT(0.5,2.1)
CALL PLOT(-1.5,1.,3)
CALL PLOT(-1.5,2.,2)
CALL PLOT(0.,2.2,3)
C READY TO PLOT AXIS
C MAKE SURE PEN POINT IS NOT DRIED UP
999 CALL PLOT(0.,0.,3)
CALL PLOT(0.,0.,2)
CALL PLOT(0.,0.,3)
CALL PLOT(0.,0.,2)
CALL PLOT(XLEN,0.,2)
X=STOP
DO 2 J=1,ISTOP
CALL PLOT(X,0.07,3)
CALL PLOT(X,-.07,2)
DO 1 I=1,IA
X0=X*(TICK(1)*XSCALE)
CALL PLOT(X0,-.03,3)
1 CALL PLOT(X0,.03,2)
2 X=X*XSCALE
3 CALL PLOT(0.,0.,3)
CALL PLOT(0.,YLEN,2)
STOP=YLEN/YSCALE
IF(STOP .LT. 0.) 4,5
SIGN=-SIGN

TABLE A 2 (Continued)

36
TABLE A 2. (Continued)
APPENDIX B

BIBLIOGRAPHY


REFERENCES


7. Lowson, M.V., "Prediction of Apollo Capsule Acoustic Environment", Private Communication, April, 1966

ACKNOWLEDGEMENTS

The authors would like to thank Mr. P. Rhodes of the Wyle Computation Staff who wrote the computer program and the flow chart given in Appendix A.
<table>
<thead>
<tr>
<th>$f(n, m)$</th>
<th>$J_{mn}^1$</th>
<th>$J_{mn}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0004</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0008</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

**Table 1:** Values of First 360 Resonance Frequencies of Plate and Joint-Acoustic For Each Frequency
<table>
<thead>
<tr>
<th>( f(m,n) ) (cps)</th>
<th>( n )</th>
<th>( j^2_{mn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**TABLE 1 (Concluded)**

---

43
<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency f cps</th>
<th>Plate Pos. x; y</th>
<th>$S_{w}/S_{p}$ in. $^2$/psi$^2$</th>
<th>$S_{p}$ dB; (psi)$^2$/cps</th>
<th>$W_{mn}^2$ in.$^2$</th>
<th>$W_{mn}$ rms in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 4</td>
<td>5.6022</td>
<td>a/2; b/8</td>
<td>$3.0 \times 10^5$</td>
<td>110; $8.4 \times 10^{-7}$</td>
<td>0.0889</td>
<td>0.298</td>
</tr>
<tr>
<td>1, 6</td>
<td>11.1659</td>
<td>a/2; b/8</td>
<td>$6.2 \times 10^3$</td>
<td>121; $1.06 \times 10^{-5}$</td>
<td>0.0461</td>
<td>0.215</td>
</tr>
<tr>
<td>3, 4</td>
<td>14.8128</td>
<td>a/2; b/8</td>
<td>$4.0 \times 10^2$</td>
<td>123; $1.5 \times 10^{-5}$</td>
<td>0.00558</td>
<td>0.075</td>
</tr>
<tr>
<td>3, 6</td>
<td>20.3765</td>
<td>a/2; b/8</td>
<td>$3.2 \times 10^1$</td>
<td>124; $2.02 \times 10^{-5}$</td>
<td>0.000826</td>
<td>0.029</td>
</tr>
<tr>
<td>2, 4</td>
<td>9.0562</td>
<td>a/4; b/8</td>
<td>$7.5 \times 10^3$</td>
<td>120; $8.4 \times 10^{-6}$</td>
<td>0.0360</td>
<td>0.19</td>
</tr>
</tbody>
</table>

$$W_{mn}^2 = \frac{\omega_{mn}}{4Q} \left( \frac{S_{w}}{S_{p}} \right) S_{p}$$

Overall rms displacement (For $x = a/2; y = b/8$)

$$W_o \approx 0.41 \text{ in.}$$

**TABLE 2. Calculated RMS Deflections for the Dominant Plate Modes**
Figure 1: Typical Overall SPL Time Histories Experienced on Large Rocket Vehicles (From Reference 1).
SATURN APOLLO LAUNCH VEHICLE CONFIGURATIONS

Figure 2. Saturn Launch Vehicles with Apollo Spacecraft
(Extracted from Reference 6)

46
Figure 4. General Configuration of SLA Skin Structure (Extracted from Reference 6)
Figure 5. Details of SLA Honeycomb Skin Construction (Extracted from Reference 6)
Figure 6. Non-Dimensionalized Fluctuating Pressure Spectra
For a Number of Launch Vehicles
(Extracted from Reference 4.)
Figure 7. Maximum Fluctuation at any Mach Number versus Station.
Figure 8. Envelope of Maximum Fluctuation at any Station versus Mach Number
Figure 9. Predicted Frequency Spectrum for Pressure Fluctuations on SLA
Figure 10. Narrow-Band Space Correlation of the Wall Pressure Fluctuations at $M = 0.52$. Center Frequency cps: $\circ$, 1200; $\Delta$, 2400; $\square$, 3600; $\diamondsuit$, 4800; $\blacktriangle$, 6000.
Figure 11. Geometry of Plate and Direction of Convected Turbulent Flow
Showing Convected Correlation Pressure Pattern
Figure 12. Variation of Joint Acceptance at Resonance with Frequency.

---

mode number = 1
peaks for frequency above coincidence
empirical envelope
Figure 13. Variation of Parameter $\phi_y$ with Frequency and Mode Number $n$. 
Figure 14. Variation of Parameter $\Phi_x$ with Frequency and Mode Number $m$. 

Longitudinal Space Correlation Parameter $\Phi_x$
Figure 15. Response of Plate in Frequency Range 0 → 25 cps.
Figure 16. Response of Plate in Frequency Range 0 → 250 cps.

\( x/a = 0.5 \)
\( y/b = 0.125 \)
Figure 17. Response of Plate in Frequency Range 0 → 250 cps.
Figure 18. Acceleration Response of Plate in Frequency Range 0 $\rightarrow$ 250 cps.
Figure 19. Acceleration Response of Plate in Frequency Range 0 → 500 cps.
Figure 20. Joint Acceptance for First Mode ($f_{1,4}$).
Fig. 5.1. Variation of Plate Response with Longitudinal Exponential Decay Parameter $k_1$ and Correlation Length Parameter $k_2$.

$(m,n) = (1,4)$
$k_3 = k_4 = 1.0$
$x/a = 0.50$
$y/b = 0.125$
$\omega = \omega_{14}$
Figure 22. Variation of Plate Response with Longitudinal Exponential Decay Parameter $k_1$ and Correlation Length Parameter $k_2$. 

$(m,n) = 1,6$
$k_3 = k_4 = 1.0$
$x/a = 0.50$
$y/b = 0.125$
$\omega = \omega_{14}$
Figure 23. Variation of Plate Response with Longitudinal Exponential Decay Parameter $k_1$ and Correlation Length Parameter $k_2$. 

- $(m,n) = (3,4)$
- $k_3 = k_4 = 1.0$
- $x/a = 0.50$
- $y/b = 0.125$
- $\omega = \omega_{3,4}$
Figure 24. Variation of Plate Response with Longitudinal Exponential Decay Parameter $k_1$ and Correlation Length Parameter $k_2$.

\begin{align*}
(m, n) &= 2, 4 \\
k_3 = k_4 &= 1.0 \\
x/a &= 0.25 \\
y/b &= 0.125 \\
\omega &= \omega_{24}
\end{align*}
Figure 25. Variation of Plate Response with Longitudinal Correlation Length Parameter $k_1$.
Figure 26. Plate Response with Joint Variation in Longitudinal Exponential Decay Parameter $k_1$ and Correlation Length Parameter $k_2$. 

Longitudinal Parameter, $k = k_1 = k_2$

(m,n) = 1,4

$k_1 = k_2 = k$

$k_3 = k_4 = 1.0$

$x/a = 0.5$

$y/b = 0.125$

$\omega = \omega_{14}$
Figure 77. Plate Response with Joint Variation in Lateral Exponential Decay Parameter $k_3$ and Correlation Length Parameter $k_4$. 

- $(m,n) = 1,4$
- $k_1 = k_2 = 1.0$
- $k_3 = k_4 = k$
- $x/a = 0.5$
- $y/b = 0.125$
- $\omega = \omega_{14}$
Figure 28. Variation of Plate Response with Lateral Decay Parameter $k_3$. 

\[
\frac{S_w(k, \gamma)}{S_p} \sim \frac{2}{\omega^2} 
\]

\[
(m, n) = 1, 4 \\
k_1 = k_2 = 1.0 \\
k_4 = 0 \\
x/a = 0.5; \gamma/b = 0.125 \\
\omega = \omega_{1, 4}
\]
Figure 29. Variation of Plate Response with Lateral Correlation Length Parameter $k_4$. 

\begin{align*}
(m,n) &= 1, 4 \\
k_1 &= k_2 = k_3 = 1.0 \\
x/a &= 0.5 \\
y/b &= 0.125 \\
\omega &= \omega_{1,4}
\end{align*}