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ABSTRACT

A short discussion is given of the reasons for co-rotation of magnetospheres with their planets. It is shown that, if Jupiter's magnetosphere co-rotates, the plasma on any magnetic shell must tend to concentrate at those parts of the shell which are most distant from the rotational axis, because of the large centrifugal force. This would compress the plasma into a discus-shaped region about a planetary diameter thick, inclined to the equatorial plane by about 7°. The passage of this plasma over the satellite Io offers a new and attractive basis on which to explain the influence of Io on the decametric radiation of Jupiter. It is proposed that the maximum plasma frequency at Io's orbit is 40 MHz. Exploration of this possibility leads to a plasma temperature of 1800°K and a magnetic field of 30 gauss at Jupiter's equatorial surface, with the dipole displaced 0.4 planetary radius towards the north rotational pole. Some discussion of the difficulties of the theory is given and its future development is considered.

THE STRUCTURE OF JUPITER'S MAGNETOSPHERE AND THE EFFECT OF Io ON ITS DECAMETRIC RADIO EMISSION

1. INTRODUCTION

The radio emission from the planet Jupiter in the decametric wavelength range ($\lesssim$ 40 MHz) has been the subject of a great deal of speculation ever since its accidental discovery by Burke and Franklin (1955). The demonstration by Bigg (1964), that Jupiter's innermost Galilean satellite, Io, has a profound effect on the decametric radiation, added a bizarre note to the subject. Several good reviews have been published recently, e.g. those by Ellis (1965) and Warwick (1967). Most theories depend on the precipitation of electrons from the planet's magnetosphere into its ionosphere, but it is hard to see how Io's influence can extend over a large enough distance to account for the intensity of the radiation.

It is observed that the probability of receiving decametric radio noise from Jupiter shows two maxima if plotted as a function of the phase angle of Io from superior geocentric conjunction, one when the phase is 90° and the other when it is 240°. The theory described in this report grew out of an investigation of the idea that Io may penetrate the boundary of the Jovian magnetosphere at these two positions.

It is easy to show that this simple theory is untenable. Let us assume that the solar wind velocity remains constant as it streams outward to Jupiter's orbital distance of 5.2 AU; we shall use 500 km/sec as a typical value. We assume also that the particle density falls off in inverse proportion to the square of the distance, so that it will be about 1/25 that of the Earth's orbit; selecting 10 proton-electron pairs per cm^3 for the latter, the density at Jupiter's orbit will be $\frac{10}{25}$ cm^{-3} or approximately $4 \times 10^5 m^{-3}$. The kinetic energy density of the solar wind there will then be about 8×10^{-11} joule/m^3. If Io is to penetrate the magnetopause at the observed positions of maximum probability of radio reception, the stagnation point should lie about 5 equatorial planetary radii ($5r_o$) from the center of the planet. Thus, we require the magnetic energy density $B^2/2\mu_o$ to be 8×10^{-11} joules/m^3 at $5r_o$. The surface field at the equator of the planet is then given by $B_o = 5^3 B = 125 \times 2\mu_o \times 8 \times 10^{-11} = 1.8 \times 10^{-6}$ webers/m^2 or 0.018 Gauss, if we assume a dipole field. The field at $3r_o$ would then be 7×10^{-4} Gauss, which is far too small to contain the high energy electrons which almost certainly produce the decimetric radiation (Chang and Davis, 1962). Estimates based on "reasonable" magnetic field strengths place the magnetospheric stagnation point at $30r_o$ to $70r_o$ (e.g. Field, 1960; Ellis, 1963; Carr et al, 1966).

There is a point which deserves a little attention before we finally reject this theory. If the plasma in the magnetosphere co-rotates with the planet, the stagnation point in the solar wind is not the point of zero relative velocity. Axford (1963) has pointed out that in the Earth's magnetosphere this rotation would displace the point of zero relative velocity to the east of the stagnation point. In the case of Jupiter, a co-rotating point in the equatorial plane has a linear velocity of 500 km/sec at $40r_o$. Thus there may be no point of zero relative velocity at all if some of the earlier estimates of the size of the magnetosphere are correct. This may be important when discussing the shape of Jupiter's magnetospheric boundary and circulation inside it.

2. GRAVITATIONAL AND CENTRIFUGAL FORCES IN JUPITER'S MAGNETOSPHERE

The foregoing discussion leads us to suspect that the rapid rate of rotation of Jupiter may be important in determining the distribution of plasma in its magnetosphere. This has also been pointed out by Ellis (1965) and Melrose (1967). In this section we shall perform some preliminary calculations which confirm this suspicion.

Jupiter rotates extremely fast for so large a body. The period is just less than 10 hours, so that the angular velocity is $\Omega = 1.76 \times 10^{-4}$ rad/sec. Since the equatorial radius r_o is 71,350 km, points on the surface at the equator have a linear velocity of 12.5 km/sec due to the rotation. This is almost as large as the orbital velocity, 13.1 km/sec. The centripetal acceleration experienced by a particle on the surface is thus 2.2m/sec^2. Since the mass of the planet is 1.90×10^{27} kg, the gravitational acceleration at the surface on the equator is $g_o = 24.9 \text{ m/sec}^2$, whereas at the pole it is 28.5 m/sec^2, the polar radius being 66,600 km, or $\frac{14}{15}$ of r_o. Thus, even on the surface, at the equator, the centrifugal force is nearly 1/10 of the gravitational force. This is, of course, the reason for the pronounced oblateness of the planet. If the magnetosphere co-rotates, the distance r_e at which the gravitational and centrifugal forces are equal and opposite in the equatorial plane is $2.24\ r_o$. Outside this, the resultant force on the plasma, in a coordinate system rotating with the planet, is outward. It is obvious that this must have a profound effect on the plasma distribution at, for example, Io's orbit, which has a radius of $5.9\ r_o$. We should thus expect Jupiter's magnetosphere to be different in many respects from that of the Earth, for which r_e is 6.7 Earth radii.

3. MAGNETOSPHERIC CO-ROTATION

It is thus very important to know whether the magnetosphere of the planet will co-rotate or not, if we are to discuss intelligently the distribution of particles in it. The literature is far from clear on this question and there is still some doubt, both from the experimental and theoretical viewpoints, as to whether the outer parts of even our own magnetosphere co-rotate or not. No general review of this subject appears to have been published. Dungey (1958) and Hines (1964) have given short accounts of the physical principles involved.

If we regard the planet as a conducting sphere, of radius r_o, rotating with angular velocity $\vec{\Omega}$ in a magnetic field $\vec{B}$, the conduction electrons at the point $\vec{r}$ will experience a Lorentz force $-e(\vec{\Omega} \times \vec{r}) \times \vec{B}$ (where $-e$ is the charge on an electron). They will redistribute themselves so that the electrostatic field $\vec{E}$ due to the space charge, just cancels this. In the simplest case, where $\vec{B}$ is constant in time at each point, we can then write for the electrostatic potential Φ at the point $\vec{r}$:

$$-\vec{E} = \nabla\Phi = (\vec{\Omega} \times \vec{r}) \times \vec{B} \tag{1}$$

The solution of this equation gives, on substituting $r = r_o$, the electrostatic potential at the surface of the planet. If $\vec{B}$ varies with time, but in such a way that the variation could be attributed to rotation of the source of $\vec{B}$ with the planet, Backus (1956) shows that (1) must be replaced by

$$\Phi = (\vec{\Omega} \times \vec{r}) \cdot \vec{A} + C \tag{2}$$

where $\vec{A}$ is the (time-varying) vector potential of $\vec{B}$ and C is an arbitrary constant.

The potential so found at the surface of the planet can be used as the boundary condition at $r = r_o$ for the potential in the magnetosphere, which will be a solution of Poisson's equation

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_o} \tag{3}$$

where ρ is the charge density and ϵ_o the permittivity of free space. By solving this equation we could find the electric field $\vec{E} = -\nabla\Phi$ at all points in the magnetosphere. The plasma there must then react to this field by undergoing the well-known electric field drift, at a velocity $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$. It is not immediately obvious whether this drift velocity will be exactly that required to cause co-rotation or not. Nor is it clear how to solve (3), since ρ depends on the relative distribution of positive and negative ions in the magnetosphere, and this in turn depends on the field $\vec{E}$ and so on the rotation.

Most workers have avoided this difficulty by making assumptions about the conductivity of the plasma. The most direct attack on the problem is that of Hones and Bergeson (1965). They assume that the plasma is a perfect conductor along the lines of force, but has zero conductivity across the magnetic field. This implies that there is no component of the electric field along the lines of force, i.e. that in the magnetosphere

$$\vec{E} \cdot \vec{B} = 0 \tag{4}$$

or

$$\left(\nabla\Phi + \frac{\partial \vec{A}}{\partial t}\right) \cdot \nabla \times \vec{A} = 0\,. \tag{5}$$

They are able to solve this equation, subject to the boundary condition (2) at the surface of the sphere, for the special case in which the magnetic field is that of a dipole situated at the center of the sphere, inclined at an arbitrary angle to the axis of rotation and rotating with the sphere. The electrostatic potential is given by

$$\Phi = \frac{\mu_o M\Omega}{4\pi r} \sin\theta \; [\sin\theta \cos\gamma - \cos\theta \sin\gamma \cos(\phi - \Omega t)] \tag{6}$$

where r, θ, ϕ are the spherical polar coordinates of the point at which Φ is evaluated, γ is the angle between the rotational and dipole axes, M is the magnetic moment of the dipole and t is the time since the dipole axis was in the plane $\phi = 0$. From (6) Hones and Bergeson are able to show that the average drift velocity of a plasma particle is exactly that required for co-rotation, that the space charge required to maintain the electric field is negligible compared with the particle densities which exist in the Earth's magnetosphere, and that the currents due to the particle motions in the field of the rotating inclined dipole are not large enough to distort the dipole field appreciably.

Presumably, co-rotation begins to fail when the assumptions made in the above treatment fail:

(a) The plasma density may decrease to such a low value, that equation (4) no longer holds, so that components of the electric field can exist along the magnetic lines of force;

(b) The plasma may be so dense that it conducts well across the lines of force, so that (4) is again invalid and the magnetic field is excluded from the plasma;

(c) The currents due to differential motion of particles of different sign may distort the magnetic field appreciably;

(d) Near the outer boundary, equation (6) will no longer hold, since it will have to fit on to the magnetopause and interplanetary potential distribution instead of falling steadily to zero at $r = \infty$;

(e) The plasma density may become too low to maintain the space charge required to produce the electric field according to equation (3).

All these conditions except (b) are likely to be met in the outer parts of planetary magnetospheres, but it is very difficult to estimate just where they become important. Persson (1966) has suggested that (a) may be true over "vast regions" of the magnetosphere of the Earth. It is also well known that (c) applies to the Earth's magnetosphere during magnetic storms. (But see Hoffman and Bracken, 1967).

Ferraro has adopted a different approach. In his paper on the rotation of the sun (Ferraro, 1937), he showed that, if the plasma does rotate, Maxwell's equations, Ohm's law and the equation of continuity of mass flow combine to require that the angular velocity must be constant over the surface traced out by the complete revolution of a line of magnetic force about the axis of rotation; this is now known as the "law of iso-rotation". It is true, subject to the condition that no electric currents flow in the meridianal planes. Recently, Ferraro and Bhatia (1967) have extended the theory to planetary magnetospheres, assuming the conductivity of the plasma to be large <u>but isotropic</u>. They have concluded that if there is either no mass flow in the meridianal planes <u>or</u> no azimuthal component of the magnetic field, the iso-rotation law holds. They have used the mechanical force equation to derive expressions for the pressure (and thus number density), but these have only been solved in simple cases. In particular there is doubt as to what happens in the region where the centrifugal force exceeds that due to gravity.

Dungey (1958) has given a useful physical picture of the reason for isorotation. He points out that, as long as the magnetic field may be considered to be "frozen in" to the plasma, any differential rotation of plasma on the same line of force must tend to "wind up" the line, so that it might get wound several times round the axis. This would be a condition of increasing strain and would be resisted by the field. All the material on any line of force must obviously rotate with the same angular velocity if the line is to remain in a steady state.

Presumably, this law breaks down where the plasma is no longer dense enough (i.e. a good enough conductor) to hold the magnetic field frozen in. Breakdown will also occur if there is appreciable mass flow of plasma along the field lines in the meridianal planes, e.g. during the injection of new plasma or the ejection of old. Ferraro and Bhatia (1967) themselves suggest that the necessary condition for steady rotation is that "the magnetic pressure must exceed or be comparable with the centrifugal force."

We may note in passing a completely different approach, in which Vlasov and Khakimov (1964) have set up and solved the Vlasov equations for the distribution of electrons and protons in a magnetic dipole field, with rotation of the plasma, allowing for deformation of the magnetic field by the currents due to particle motions and for the electric field produced by the space charge. Unfortunately, their treatment leads to the result that the ratio of temperature to mass of the positive ions must equal that for negative ions. Since this would require the ion temperature in the terrestrial magnetosphere to be about 2000 times the electron temperature, it is clearly unacceptable. Presumably this condition results from the absence of a collision term in the Vlasov equation.

Rotating plasmas have been discussed from the point of view of single particle orbit theory by Longmire et al. (1959) and Northrop (1963), among others. The discussion by Boyer et al. (1958) of the thermonuclear device "Ixion" contains several points which are important in understanding magnetospheric corotation. In "Ixion", a cylinder of plasma is confined in a magnetic field, which is uniform except for stronger mirror fields at the ends of the cylinder. A radial electric field is applied, which sets the plasma into rapid rotation about the axis and hinders escape of particles through the mirror regions by adding the centrifugal force to the magnetic forces otherwise opposing escape. The reality of the rotation is confirmed by the Doppler shift of spectral lines and by the behaviour of the electrical parameters of the device as a whole. The rotational velocity is found to agree with the value calculated for the drift $\frac{\vec{E} \times \vec{B}}{B^2}$ produced by the applied fields. The authors point out, however, that the centrifugal force also produces a drift which is dependent on the mass and charge of the particles, unlike the electric field drift. As a result protons and electrons drift with different velocities and there is a net current flowing in the azimuthal direction, which causes the plasma as a whole to act as if it were diamagnetic. Thus, the magnetic field is reduced in the center of the cylinder; experiments whith a probe coil show that it is reduced almost to zero.

In a planetary magnetosphere we should expect, on the basis of the foregoing, that the plasma will rotate primarily because of the drift due to the electric field produced by the rotation of the planet in the magnetic field. We should expect as a first approximation to find the plasma co-rotating with the planet, at least in those regions where its conductivity is high along the lines of magnetic force. Beyond the radius where the gravitational and centrifugal forces balance one another, co-rotation, if it takes place at all, will require the magnetic pressure to be at least comparable with the centrifugal pressure. Charge and mass-dependent drifts, such as those due to the gravitational, centrifugal, pressure and magnetic field gradients, will act to produce a diamagnetic current flow in the plasma, which will tend to weaken the magnetic field in the regions close to the planet and increase it towards the outside of the region of trapped plasma. The net affect will be to distort the lines of force by moving them away from the planet, especially in regions where the plasma is most dense (c.f. Kendall et al., (1966); Hoffman and Bracken, (1967)). If plasma accumulates to such a density that the centrifugal pressure exceeds the magnetic pressure, we may perhaps expect it to break away from the field, possibly in a similar manner to that illustrated by Dungey (1958) Figure 5.3.

4. EQUILIBRIUM OF PLASMA IN A PLANETARY MAGNETOSPHERE

Let us set up a system of spherical polar coordinates which rotate with the planet. A particle of mass m which is at rest in this system at the point r, θ, ϕ outside the planet, experiences a gravitational force $\vec{f}_g = - m g_o \frac{r_o^2}{r^2} \hat{a}_r$ and a centrifugal force

$$\vec{f}_c = m\Omega^2 r \sin^2 \theta \, \hat{a}_r + m\Omega^2 r \sin\theta \cos\theta \, \hat{a}_\theta$$

as shown in Figure 1. These forces may be derived from a potential V,

$$\vec{f}_g + \vec{f}_c = - m \nabla V \tag{7}$$

where $$V = -g_o \frac{r_o^2}{r} - \tfrac{1}{2}\Omega^2 r^2 \sin^2\theta + \text{Constant}. \tag{8}$$

Here, $\hat{a}_r$ and $\hat{a}_\theta$ are unit vectors in the appropriate directions at r, θ, ϕ.

Positive ions of mass m_i and charge q_i will also experience a Lorentz force $q_i(\vec{E} + \vec{v}_i \times \vec{B})$, where $\vec{E}$, $\vec{v}_i$ and $\vec{B}$ are all measured in the rotating frame. We may note that this implies that $\vec{E}$ in the rotating frame can have no component perpendicular to the lines of magnetic force, for such a component would produce a drift of the plasma, with velocity $\frac{1}{B^2}(\vec{E} \times \vec{B})$, relative to this frame, which is contrary to the hypothesis that it is co-rotating with the planet. Therefore, in our present frame of reference

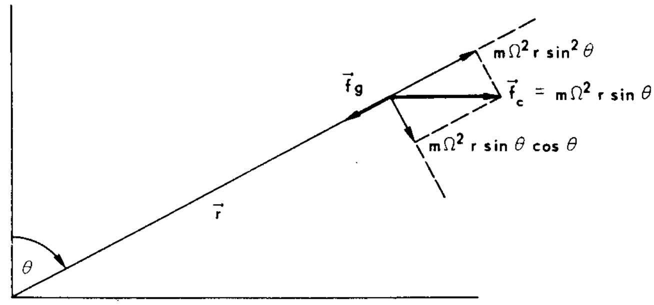

Figure 1

$$\vec{E} \times \vec{B} = 0. \tag{9}$$

The ions will move until equilibrium is attained, at which time the net force acting is zero. The ion pressure p_i will then be given by the condition

$$-\nabla p_i - N_i m_i \nabla V + N_i q_i \vec{E} + N_i q_i \vec{v}_i \times \vec{B} = 0 \tag{10}$$

where N_i is the number density of ions. Similarly, for electrons

$$-\nabla p_e - N_e m_e \nabla V + N_e q_e \vec{E} + N_e q_e \vec{v}_e \times \vec{B} = 0. \tag{11}$$

We now assume that the plasma consists only of protons and electrons, so that $q_i = -q_e = e$ and $m_e << m_i$. We also assume that the plasma is electrically neutral, so that $N_i = N_e = N$. This cannot be strictly true, because the gravitational and centrifugal fields will tend to separate the protons from the electrons and a space charge will be set up to produce an electric field to counteract the effect. However, the difference between N_e and N_i required to maintain this field is negligibly small compared with the quantities themselves (see, e.g. Alfvén and Fälthammar, 1963, p. 15) and we may justifiably regard them as equal.

Adding equations (10) and (11) under these conditions, we find

$$- \nabla(p_i + p_e) - Nm_i \nabla V + Ne(\vec{v}_i - \vec{v}_e) \times \vec{B} = 0. \tag{12}$$

If we assume a Maxwellian thermal plasma at temperature T, then

$$p_i = p_e = N k T \tag{13}$$

where k is Boltzmann's constant. Noting also that the current density is

$$\vec{J} = Ne(\vec{v}_i - \vec{v}_e) \tag{14}$$

(12) becomes

$$- \nabla(2N kT) - Nm_i \nabla V + \vec{J} \times \vec{B} = 0,$$

or if T is constant, dropping the subscript on m_i for convenience

$$2 k T N \nabla \left(\ln N + \frac{m V}{2kT} \right) = \vec{J} \times \vec{B}. \tag{15}$$

This is merely the equilibrium form of the hydromagnetic equation of motion (Alfvén and Fälthammar, 1963, p. 76), or the momentum equation for a conducting fluid (Holt and Haskell, 1965, p. 173) and could have been written down at once from either starting point. The derivation given, however, seems more illuminating as to the physics of the situation.

Strict co-rotation would require $\vec{v}_i = \vec{v}_e = 0$ in our equations. However, as we have already seen, the centrifugal, gravitational and field-gradient drifts will be different for protons and electrons, so that there will be a net current. This is discussed in Appendix A.

5. DISTRIBUTION ALONG A LINE OF MAGNETIC FORCE

If we take the scalar product of equation (15) with $\vec{B}$, we get

$$\nabla \left(\ln N + \frac{m V}{2kT} \right) \cdot \vec{B} = 0 .$$

Thus the gradient of $\ln N + \frac{m V}{2kT}$ is everywhere perpendicular to B, and therefore, along a line of force,

$$\ln N + \frac{m V}{2kT} = \text{Constant} . \qquad (16)$$

Substituting from equation (8) we have

$$\ln N = \frac{m g_o r_o^2}{2kTr} + \frac{m\Omega^2 r^2 \sin^2\theta}{4kT} + \text{Constant} . \qquad (17)$$

This is true along a line of magnetic force no matter what the shape of the magnetic field is like.

6. DISTRIBUTION IN AN ALIGNED DIPOLE FIELD

We now assume a dipole field, aligned along the rotational axis. Then the equation of a line of force is

$$r = L r_o \sin^2\theta \qquad (18)$$

where L is a constant which gives the distance from the axis, in equatorial planetary radii, at which the line of force cuts the equatorial plane. With this relation, we can reduce equation (17) to the form

$$\ln N = \frac{m g_o r_o^2}{2kTr} + \frac{m\Omega^2 r^3}{4kTL r_o} + \text{Constant}. \tag{19}$$

All that remains to be done before we can calculate the plasma number density along the line of force is to evaluate the constant.

An obvious method is to follow the approach of Angerami and Thomas (1965), by assuming that the value of N is known at some level in the ionosphere. The distances which we are considering are so large that we may consider the ionosphere as lying at radius r_o; let us assume that the plasma density there is N_o. Substitution in (19) and subtraction then leads at once to the equation

$$\ln N/N_o = \frac{m g_o r_o}{2kT}\left(\frac{r_o}{r} - 1\right) + \frac{m\Omega^2 r_o^2}{4kTL}\left(\frac{r^3}{r_o^3} - 1\right). \tag{20}$$

We assume a temperature of 150°K (Gross and Rasool, 1964) for this preliminary calculation.

It is now a simple matter to substitute the known values and calculate $\ln N/N_o$ for various values of L. The results for L = 6 are shown in Table 1.

Table 1

Plasma distribution along the line of magnetic force L = 6.

$\frac{r}{r_o}$	1	2	3	4	5	6
$\log_{10} N/N_o$	0	-144	-155	-101	+17	+206

The conclusion is inescapable, that Jupiter's ionosphere effectively ends long before a height of r_o above the surface is reached. This is not really surprising, for at a temperature of 150°K the scale height of atomic hydrogen is 50 km, so that at $\frac{r}{r_o} = 2$ we are 1400 scale heights above the surface. Clearly increasing the temperature, even by a factor of 100, will not materially affect our conclusion. We may deduce from Table 1 that the Angerami-Thomas method is inapplicable to Jupiter's magnetosphere, since any plasma there will tend to collect at the most distant parts of the lines of force and will have no connection with the ionospheric plasma.

This suggests that we should regard the thermal plasma as congregated round the furthest point of the line of force from the planet, which is the point of minimum potential energy according to equation (8), being kept there by the large centrifugal force and the magnetic field. Let us therefore suppose that $N = N_{max}$ at this point, where $r = L$. We may substitute these values in equation (19) and again eliminate the constant by subtraction, to obtain

$$\ln N/N_{max} = \frac{m g_o r_o}{2kT}\left(\frac{r_o}{r} - \frac{1}{L}\right) + \frac{m\Omega^2 r_o^2}{4kTL}\left(\frac{r^3}{r_o^3} - L^3\right). \qquad (21)$$

It is shown in Appendix B that this corresponds closely to a Gaussian distribution of the plasma about the point $r = L$ in the equatorial plane, the width of the distribution being proportional to the square root of the temperature T:

$$N = N_{max} \exp -\left(\frac{As^2}{2kT}\right) \qquad (22)$$

where

$$A = m\left(\frac{3}{2}\Omega^2 - \frac{g_o}{r_o L^3}\right) \qquad (23)$$

and s is the distance along the line of force from the equatorial plane. N is reduced to 1/e of N_{max} at

$$s = \sqrt{\frac{2kT}{A}} \tag{24}$$

Substitution of numerical values, retaining T = 150°K for the time being, gives s = 7500 km at L = 6. Thus, the effect of the centrifugal force is to compress the plasma into a thin "pancake", only about one tenth of the planetary diameter "thick" at L = 6.

7. RADIAL DISTRIBUTION

In order to produce a model of Jupiter's magnetosphere on the present basis, it is desirable to have some method of estimating N_{max}. Even if we could solve the equation of equilibrium perpendicular to the lines of force, obtained by taking the vector product of equation (15) with $\vec{B}$, we should still need a numerical estimate at one point. So far, it has not proved possible to solve this equation (cf. Ferraro and Bhatia, 1967). In the absence of a better method, we shall take the following simple way out. The magnetic field is the agency responsible for holding the plasma in against the centrifugal force, through the $\vec{J} \times \vec{B}$ term. (It should be noted that this implies that the behaviour of the plasma with respect to motion across the lines of magnetic force is not controlled by collisions, i.e.,the plasma is anisotropic enough to be considered "thermal" along the lines of force, but "trapped" against motion across them (cf. the "medium density" plasma of Alfvén and Fälthammar (1963)). We then suppose that the field will no longer be able to hold the particles in if the plasma density is so great that its rotational kinetic energy per unit volume exceeds the magnetic energy density. The limit thus occurs when $1/2 \; N_{max} \; m\Omega^2 r^2 = \frac{B^2}{2\mu_o}$, where μ_o is the permeability of free space. Introducing the dipole field $B = B_o \frac{r_o^3}{r^3}$, we find

$$N_{max} = \frac{B_o^2}{\mu_o m\Omega^2 r_o^2} \left(\frac{r_o}{r}\right)^8 . \tag{25}$$

This applies in the equatorial plane, where N_{max} occurs and B has its minimum value. Equation (25) has been derived by others, e.g. Hines (1964), on a different basis. It has been used to estimate the distance beyond which co-rotation may cease, or to a magnetospheric boundary due to rotation, but we shall regard it rather differently, as giving the maximum density of plasma which can co-rotate at a given radius r. We note that it is an inverse eighth power relation, and so will lead to a very rapid decrease of N_{max} with increasing r.

8. MAGNETOSPHERIC MODEL 1

In order to calculate the plasma density in our model, we require an estimate of the magnetic field B_o at the planet's equator. We shall arbitrarily assume $B_o = 1$ gauss ($= 10^{-4}$ weber/m^2), since the effect of changing B_o is easily visualized by noting from equation (25) that N_{max} is proportional to B_o^2. With this figure, and retaining $T = 150$°K for want of a better estimate, we arrive at the model shown in Figure 2. This shows a plasma distribution vastly different from that surrounding the Earth, and is more reminiscent of Saturn's rings than of a magnetosphere.

9. DISTRIBUTION IN TILTED DIPOLE FIELD

We now recall from the work of observers of Jupiter's decimetric radiation (Morris and Berge, 1962; Roberts and Komesaroff, 1965) that the magnetic dipole axis of Jupiter is inclined by an angle of 10° to its rotational axis. On the basis of our previous discussion, we would expect the plasma to congregate about the points on the lines of force of this tilted dipole at which the minimum potential energy occurs. At Io's orbit, the centrifugal term in equation (8) is 10 times the gravitational one; we may therefore get a good idea of the effect of the dipole tilt by merely moving the distributions in Figure 2 along the tilted lines of force to the points where $r \sin\theta$ is at its maximum, i.e. to the points furthest from the rotational axis.

The lines of force of the tilted dipole have the equation

$$r = L r_o \sin^2(\theta - 10°)$$

in the plane in which the dipole axis and the rotational axis lie. (Note that L is now the distance in units of r_o at which the line cuts the magnetic equatorial plane.) Thus,

$$r \sin\theta = L r_o \sin^2(\theta - 10°) \sin\theta .$$

Differentiation of this with respect to θ shows that the maximum occurs when

$$2 \tan \theta = \tan (\theta - 10°),$$

which is quickly solved to give $\theta \approx 96°40'$.

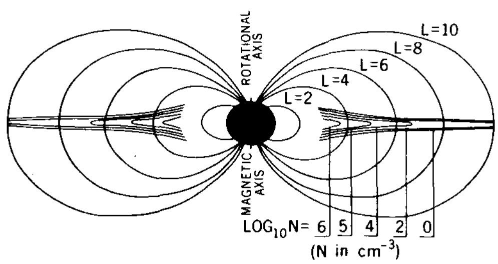

Figure 2

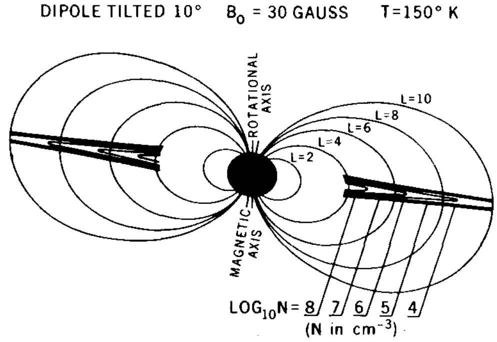

Figure 3

10. MAGNETOSPHERIC MODEL 2

Thus, the points of maximum plasma density will lie approximately in a plane tilted 6°40' to the equatorial plane of the planet. The resulting model is shown in Figure 3, where we have, however, taken B_o to be 30 Gauss in estimating N_{max}, for a reason which will become evident shortly.

The striking thing about Figure 3 is that it shows that, as the plasma disc co-rotates with the planet, it will sweep across Io twice per relative revolution. This offers the first direct clue as to a possible explanation of the effect of that satellite on the radio emission of the planet.

A prominent feature of the spectrograms of the Io-controlled emission from source B is a narrow-band emission which rises to a maximum frequency in the neighborhood of 40 MHz and then falls again (See e.g. Dulk, 1965, Figure 3). This offers a striking parallel to the behaviour of the plasma frequency in Io's vicinity on our model 2, which would also pass through a maximum as the disc swept over Io. This suggests that we should adjust the magnetic field so as to make the value of N_{max} at Io's orbital distance appropriate to this frequency. Using the well-known relation between electron density and plasma frequency, we find $N_{max} = 1.24 \times 10^4 \times (40)^2 = 2.0 \times 10^7$ electrons/cm^3. Equation (25) then requires us to set $B_o = 30$ Gauss, as in Figure 3.

11. PLASMA TEMPERATURE

With N_{max} adjusted to give the correct maximum frequency, we are in a position to estimate the plasma temperature, by matching the time scale of the observed frequency change to that expected for the passage of our model distribution over Io. To do this we require an expression for the distance s of Io from the plane of maximum plasma density, i.e. the distance CB in Figure 4.

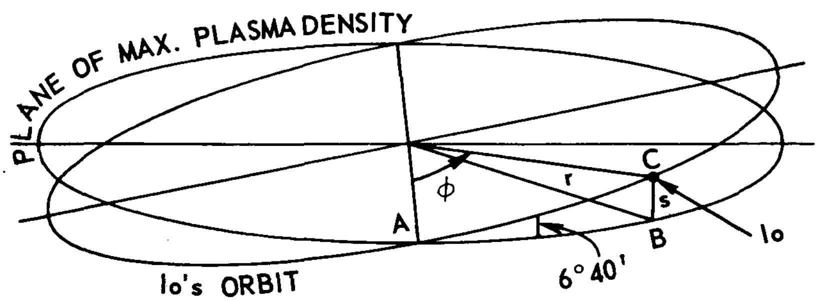

Figure 4

Let φ be the angle made by the radius vector to Io with that to the point A at which the locus of maximum plasma density crosses its orbit. In the spherical triangle ABC, angle, ABC is effectively a right angle, since we want to measure s along the line of force; angle BAC is 6° 40'. Then

$$\frac{\sin 6°40'}{\sin s/r} = \frac{1}{\sin \varphi}$$

whence

$$s \approx r \sin 6°40' \sin \varphi \approx 4.9 \times 10^4 \sin \varphi \text{ km}. \tag{26}$$

We may easily show from equation (22) that the plasma frequency varies with s as

$$f = 40 \exp\left(-1.34 \times 10^{-6} \frac{s^2}{T}\right) \text{MHz} \tag{27}$$

where the constants are appropriate to Io's orbit, L = 5.9. With (26) this gives:

$$f \approx 40 \exp\left(-3210 \frac{\sin^2 \varphi}{T}\right) \text{ MHz} \tag{28}$$

From the spectra, it takes about 50 minutes for f to change from f_{max} to 0.75 f_{max}. The plasma rotates with angular velocity 1.76×10^{-4} rad/sec and Io in the same direction with angular velocity 0.41×10^{-4} rad/sec. Thus φ in Figure 4 increases by 1.34×10^{-4} rad/sec, or from 0 to 23° in 50 minutes. Equation (28) then gives us $0.75 = \exp\left(-3210 \frac{\sin^2 23°}{T}\right)$, whence we find

$$T = 1800°\text{K}. \tag{29}$$

Figure 5 shows the frequency variation computed from equation (28) with T = 1800°K, compared with that actually observed during four well-defined storms from source B (Dulk, 1965). The agreement is good.

12. MAGNETOSPHERIC MODEL 3.

Recalculation of the data for Figure 3, using equation (29) and B_o = 30 gauss leads to the magnetospheric model shown in Figure 6. This has already been published in a brief note (Gledhill, 1967). It is worth noting that the temperature to which we have been led is comparable with that in the Earth's magnetosphere. We should also note that it is unlikely that the plasma density would actually reach values of 10^8 or 10^9 cm^{-3}, because diffusion across the field and recombination would then become important and these have not been considered in our model.

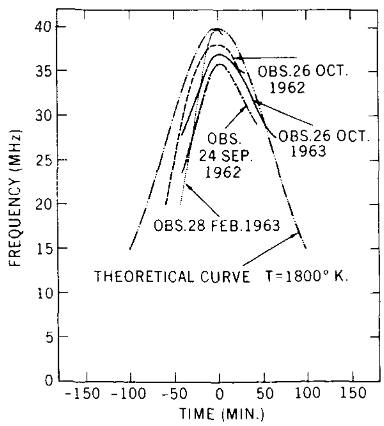

Figure 5

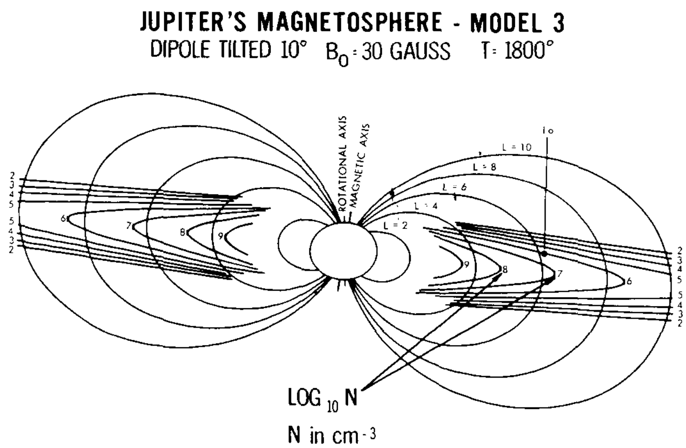

Figure 6

13. DISPLACED DIPOLE MODEL

It is obviously a requirement of our model in its present state that the maximum frequency of source B should occur when Io passes through the maximum density in the plasma plane. Since the magnetic dipole is tilted towards a system III longitude of 190° (Roberts and Komesaroff, 1965), this should occur when Io is above $\lambda_{III} = 100°$ and 280°. The dependence of the maximum frequency of emission on the position of Io in λ_{III} coordinates does not appear to have been studied in any detail, though Dulk (1965) has discussed the matter from a somewhat different point of view. Examination of a number of published spectrograms of the B source in fact shows that the maximum frequency is usually reached when Io is within 10° of $\lambda_{III} = 230°$. Spectrograms of source C also appear to show signs of a frequency maximum when Io is near the same value of λ_{III} as for source B. Source A spectra are less well-defined in respect of their maximum frequency, but a maximum at about $\lambda_{III} = 150°$ would be consistent with the spectra shown by Dulk (1965, Figure 4). These two positions are equally spaced on opposite sides of the plane of the inclined dipole, and are shifted 50° from the positions predicted by the model in its present form.

We may remove this discrepancy, however, if we assume that the dipole is not only tilted, but also is displaced from the center of the planet, O, (Figure 7), by a distance OD. P and G are the points of intersection of the displaced plasma plane with Io's orbit. We require $S\hat{O}P$ to be 40°, so that, since $OP = 5.9\ r_o$, $SO = 4.5\ r_o$. Thus, since $O\hat{S}D = 6°40'$, we find $OD = SO \tan 6°40' = 0.52\ r_o$. A more refined calculation, taking account of the effect of the gravitational field, which still acts toward the center of the planet and therefore no longer acts in the plane of the disc, and of the changed magnetic field in the disc due to the relative tilt, reduces this to $0.4\ r_o$. This is just at the limit of the displacement allowed by the precise observations of Roberts and Ekers (1965).

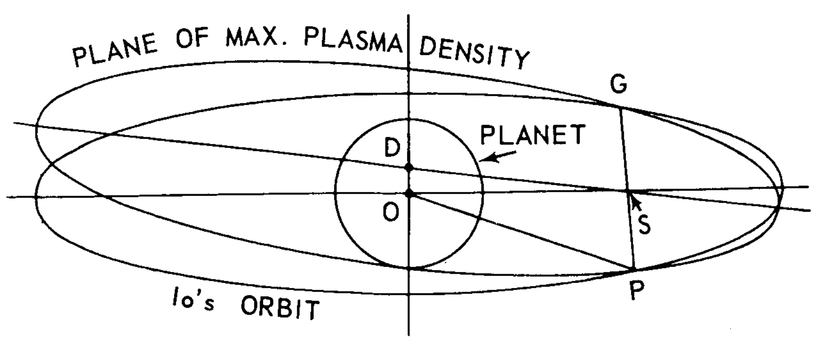

Figure 7

The plasma frequency predicted by this model at Io's orbit is shown in Figure 8, together with the frequencies actually observed during seven events recorded by the Boulder spectrograph. The agreement is reasonably satisfactory for so crude a model. If we take into account the observation by Roberts and Ekers (1966), that the dipole is shifted by 0.07 r_o from the planetary axis towards λ_{III} = 220°, we should expect a tendency for the plasma to concentrate in this direction, and thus for the peak at 230° to be higher than that at 150°. This is what is observed, as Figure 8 shows.

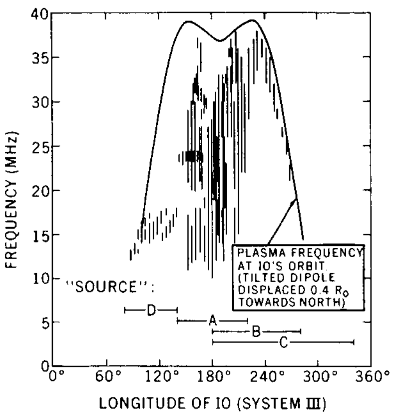

Figure 8

Figures 9 and 10 show schematically the relative configurations of Jupiter, Io, the plasma disc and the Earth when the various sources are observed to be active. It is seen that in every case the radio emission is received at the earth when Io is emerging from, or being overtaken by, the plasma disc.

14. COMMENTARY ON THE DISPLACED DIPOLE MODEL

The main point in favor of the model is that it offers a means by which Io can interact directly with the magnetospheric plasma at about the values of λ_{III} actually observed. It also predicts the correct type of maximum frequency variation, if the radiation is assumed to occur at or near the plasma frequency.

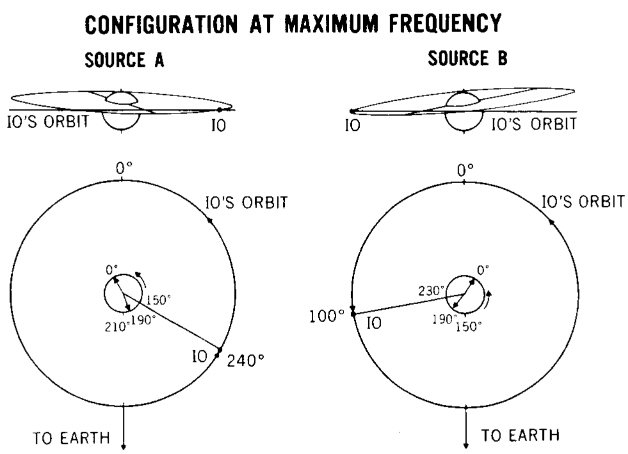

Figure 9

TYPICAL CONFIGURATIONS

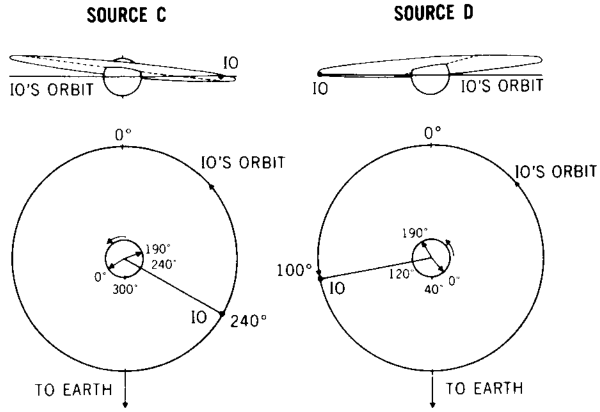

Figure 10

There is no immediate means of explaining the well-known "bursty" nature of the radiation observed at decameter wavelengths. The most reasonable basis seems to be to assume a "patchy" or turbulent plasma distribution. Alternatively, one may suppose that Io, by virtue of its gravitational, electric and/or magnetic fields, acts by detaching masses from the co-rotating plasma, and that these produce radiation as they move away from the planet, through the surrounding plasma.

The peculiar radiation pattern, by which radio noise is mostly observed when Io is around 90° and 240° from superior geocentric conjunction, is perhaps explainable on the basis of refraction of the waves by the plasma disc. Some very rough ray-tracing shows that there may be a cone above the disc, and one below, into which most of the noise is radiated. The cones could reasonably make angles of about 6° with the plane of maximum plasma density, more or less as required by Gulkis and Carr (1966). Further efforts at ray-tracing do not appear to be warranted until the radial distribution of the plasma is put on a sounder basis than the present very rough one.

It is to be expected that the diamagnetic nature of the plasma will result in a considerable distortion of the lines of magnetic force inside the disc. One would expect them to be bent so as to follow more closely the lines of constant plasma density in Figure 8. Some tendency to bend backwards, due to the tendency of outward moving plasma to conserve its angular momentum, is also to be anticipated. As a result, it appears to be impossible at present to make any valid predictions about the polarization and Faraday rotation produced by the model.

The weakest features of the model are undoubtedly the radial distribution and the relatively high plasma density required. We shall examine each of these a little more closely.

(a) The radial distribution

If the plasma were assumed to be perfectly conducting isotropic thermal plasma, it would exclude the magnetic field from its interior and there would be no force to hold it in against the centrifugal one. The model depends on the assumption that motion along the lines of force is sufficiently easy for thermal equilibrium to be maintained by collisions and the electric field to be removed by the high conductivity, while motion across the lines of force is sufficiently difficult to allow the plasma to be regarded as a non-conductor in that direction. Calculation on the basis of the model gives the electron-ion collision frequency at Io's orbit as 6.5 kc/sec, while the electron gyro-frequency is 65 kc/sec; an electron completes, on the average, 10 orbits round a line of force before suffering a collision. The plasma is thus of "medium density," in the sense of Alfvén and Fälthammar (1963). This is marginal for stability and would lead to fairly rapid diffusion across the field lines. This in turn would require a suitably strong source of replacement plasma to retain the high density required. It may also be noted that the electron gyrofrequency, ω_e, being proportional to B, decreases outwards as r^{-3}, while the electron-ion collision frequency, ν_{ei}, is roughly proportional to N and so varies as r^{-8}. Thus the ratio ν_{ei}/ω_e varies as r^{-5} and from this point of view the model distribution is reasonably stable outside Io's orbit, but would certainly not be so inside $4r_o$, where $\nu_{ei} > \omega_e/2\pi$, i.e. the plasma would take on "high density" characteristics.

The force holding the plasma is in fact the $\vec{J} \times \vec{B}$ force in equation (15) and the current density $\vec{J}$ is due to mass- and charge-dependent drifts as discussed in Appendix A. This azimuthal drift current must produce a magnetic field which opposes the applied dipole field inside the current ring and this effect will be most pronounced in the denser plasma. This is the field which produces the distortion of the lines of force already mentioned in Section 14. Rough calculations show that it would reduce the applied field very seriously inside $5r_o$, so that the model cannot be even approximately correct inside this distance.

Finally we may note that our assumption of a constant temperature is not necessary in view of the assumed difficulty of radial motion of electrons across the lines of force.

Quite obviously the problem of the radial distribution is a complex one, and demands serious study before the model can be regarded as even approximating to reality.

(b) The high plasma density

The density required at Io's orbit by the model is 2×10^7 electrons and the same number of protons per cm^3.

If we believe the model as far in as $5 r_o$ this rises to 8×10^7. Such high densities are not encountered in the Earth's magnetosphere and one instinctively shrinks from accepting them for Jupiter's. On the other hand we have no first-hand knowledge of the Jovian magnetosphere and, given the larger magnetic field and an adequate source, say from the solar wind, there seems to be no fundamental reason why high densities should not exist there.

Calculation suggests that a density of the order of 2×10^7 cm^{-3} over the volume involved should give out a detectable amount of radiation in the H_α line, though the detection may be difficult. It also appears that the radiation produced by free-free transitions may be detectable in the 6-20 cm wavelength range, though here again an examination of published papers suggests that it would be difficult to distinguish such radiation from the thermal component coming from the planet itself.

(c) High energy plasma

It has been assumed in the theory that only thermal energies are involved. The microwave observations show that there are electrons with energies of several MeV in the region $L = 1.5 - 3$ and it would be surprising if there were no particles further out with energies in excess of the thermal ones. Such particles, mirroring in the normal way but with their orbits modified by the centrifugal field, might offer a source for the radiation observed to come from the region of $L = 6.5$ by McAdam (1966). They would also serve to maintain conduction along the lines of force above and below the plasma disc, which may be necessary for its stability and corotation.

(d) Radiation mechanisms

No serious attention has yet been given to this aspect of the problem. It is well known that a plasma does not, in general, radiate electromagnetic waves at the plasma frequency. With respect to the Io-controlled emission, however, it may be significant that the streaming velocity of the co-rotating plasma past Io, 55 km/sec, is 11 times the velocity of sound, 5 km/sec; it is lower than the Alfvén velocity, which is 72 km/sec at the maximum of the plasma density at Io's orbit and is greater than this elsewhere at L = 5.9. Under these circumstances a modified sonic shock wave would be expected to precede Io in its orbit (since the plasma rotates faster than Io does). A mechanism such as that proposed by Tidman (1965) could then be invoked to account for the electromagnetic radiation. It remains to be seen whether it would be capable of explaining the observed intensity of the Jovian radiation.

That part of the radiation which is not controlled by Io may similarly be attributed to turbulence at, or the breaking away of clouds of plasma from, the boundary region between the co-rotating and the non-co-rotating plasma. This must, on the model, occur outside Io's orbit, and therefore, at a lower plasma frequency than that in Io's vicinity. Gruber (1967) has recently shown that, whereas Io exercises almost complete control at frequencies above 30 MHz, there is little evidence for such control below 25 MHz, except for a short range at 18-19 MHz. It may be significant that the higher range of Io-control centers on 36 MHz, which is the second harmonic of the 18 MHz band, as Gruber points out. This would seem to be consistent with Tidman's mechanism, which produces radiation at both the plasma frequency and its second harmonic. Much more work is necessary to examine this possibility.

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Appendix A.

Particle drifts in Jupiter's magnetosphere

The drift velocity (normal to $\vec{B}$) of the guiding center, produced by a force $\vec{F}$, is (Alfvén and Fälthammar, 1963, Ch. 2)

$$\vec{v} = \frac{1}{qB^2} \vec{F} \times \vec{B} \tag{A1}$$

where q is the charge on the particle. There will thus be the following drifts imposed on the ions in the Jovian magnetosphere:

(a) The pressure-gradient drift,

$$\vec{v}_{pi} = - \frac{1}{eB^2} \frac{1}{N_i} \nabla P_i \times \vec{B} = - \frac{kT}{eB^2} \nabla (\ln N) \times \vec{B}. \tag{A2}$$

(b) The gravitational drift

$$\vec{v}_{gi} = - \frac{m_i}{eB^2} g_o \frac{r_o^2}{r^2} \hat{a}_r \times \vec{B}. \tag{A3}$$

(c) The centrifugal drift

$$\vec{v}_{ci} = \frac{1}{eB^2} \frac{1}{2} m_i \Omega_i^2 \nabla (r^2 \sin^2 \theta) \times \vec{B}. \tag{A4}$$

(d) The magnetic field drift, produced by the gradient and curvature of the field lines:

$$\vec{v}_{Bi} = - \frac{1}{eB^2} \frac{m_i}{B} \left(\frac{1}{2} v_\perp^2 + v_\parallel^2 \right) \nabla B \times \vec{B} \tag{A5}$$

where $v_\perp$ and $v_\parallel$ are the components of $\vec{v}_i$ normal and parallel to $\vec{B}$ respectively.

(c) The electric field drift

$$\vec{v}_E = \frac{1}{B^2} \vec{E} \times \vec{B} \tag{A6}$$

where the subscript "i" is unnecessary since the velocity is independent of the charge and mass. There are corresponding expressions for the electrons.

It is instructive to evaluate these drift velocities for our model, at Io's orbit. The results are given in Table A1. All drift velocities are in the azimuthal direction. A positive sign indicates the direction of co-rotation and $\vec{B}$ has been taken as directed southwards at Io's orbit. In calculating the magnetic field drift, the second term, due to curvature of the lines of force, has been neglected. This is acceptable for the undistorted dipole field but may be unrealistic if the distortion is taken into account. The electric field used in computing the drift (e) was taken as that found by Hones and Bergeson (1965).

Table A1

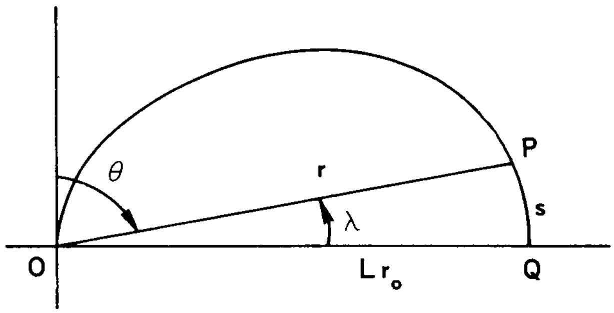
Drift	Proportional to	Metres/sec	
		Ions	Electrons
(a) Pressure gradient	r^2	$+ 2.2 \times 10^{-4}$	$- 2.2 \times 10^{-4}$
(b) Gravitational	r	$- 5.1 \times 10^{-4}$	$+ 2.8 \times 10^{-7}$
(c) Centrifugal	r^4	$+ 9.7 \times 10^{-3}$	$- 5.0 \times 10^{-6}$
(d) Magnetic	r^4	$+ 1.5 \times 10^{-3}$	$- 8.0 \times 10^{-7}$
(e) Electric	r	$+ 7.3 \times 10^{4}$	$+ 7.3 \times 10^{4}$

This illustrates clearly the overwhelming importance of the electric field in producing co-rotation and shows how closely one may approximate by regarding the electrons and ions as both co-rotating with the same velocity. The net drift velocity of the ions in the co-rotating frame is about 10^{-2} m/sec and that of the electrons is 2×10^{-4} m/sec. Thus, most of the current is due to the ion drift.

Appendix B.

Gaussian distribution of plasma

The diagram shows the line of force L, and the radius vector r to the point P. Provided that the latitude λ is small, we may write for the distance s along the line of force from Q to P.



$$s \approx L r_o \lambda . \tag{B1}$$

The equation of the line of force is

$$r = L r_o \sin^2 \theta = L r_o \cos^2 \lambda$$

$$\approx L r_o (1 - \lambda^2)$$

so that

$$\frac{r}{r_o} \approx L(1 - \lambda^2) \approx L\left(1 - \frac{s^2}{L^2 r_o^2}\right) \tag{B2}$$

using (B1).

Thus, in equation (21), we may write, since $\frac{s}{L r_o} << 1$,

$$\frac{r_o}{r} - \frac{1}{L} \approx \frac{s^2}{L^3 r_o^{\,2}} \tag{B3}$$

and

$$\left(\frac{r}{r_o}\right)^3 - L^3 \approx - \frac{3 s^2}{L^2 r_o^{\,2}} \tag{B4}$$

(21) then becomes

$$\ln N/N_{max} \approx \frac{m g_o s^2}{2kTL^3 r_o} - \frac{m\Omega^2 3 s^2}{4kTL^3}$$

$$\approx \frac{m}{2kT}\left(\frac{g_o}{r_o L^3} - \frac{3}{2}\,\Omega^2\right) s^2 . \tag{B5}$$

Then we can write

$$N = N_{max} \exp\left(- \frac{A s^2}{2kT}\right) \tag{B6}$$

where

$$A = m\left(\frac{3}{2}\,\Omega^2 - \frac{g_o}{r_o L^3}\right) \tag{B7}$$

which are (22) and (23).

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