X-713-67-537

NASA TM X-63020

UNIFORM LOCAL RESPONSIVITY OF DETECTOR'S BY SCANNING

MITCHELL W. FINKEL

NOVEMBER 1967

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-713-67-537

UNIFORM LOCAL RESPONSIVITY
OF DETECTOR'S BY SCANNING

Mitchell W. Finkel

November 1967

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

CONTENTS

UNIFORM LOCAL RESPONSIVITY
OF DETECTOR'S BY SCANNING

by
Mitchell W. Finkel
Goddard Space Flight Center

INTRODUCTION

Transmittance and reflectance measurements of scattering materials or rough surfaces requires complete impartiality of the detector with respect to the distribution of the intercepted radiant flux. Only in a few special cases, such as a perfectly specular surface or for a uniform flux distribution as in the case of an integrating sphere, can this problem be ignored. The problem can often be minimized if some pre-selected area of the detector is consistently referred to. But this procedure is not always feasible with hemispherical or ellipsoidal collectors. Generally, the distribution of the incident and transmitted or reflected flux will be vastly dissimilar. Consequently, uniform local responsivity is a matter of paramount importance. Yet, polycrystalline as well as large area single-crystal detectors are notoriously nonuniform.[(1)] We need only compare the efficiency and spectral range of say an ellipsoidal reflectometer and the usual integrating sphere photometer to appreciate the importance of the problem. A method is outlined here that can effectively achieve uniform local responsivity without accepting the limitations inherent in such devices as diffusing screens, and averaging spheres usually employed to obviate this problem. The central concept rests upon our viewing the local responsivity of the detector and the intercepted radiant flux as two random variables. It will then be shown how scanning can make the detector a completely impartial arbiter.

SCANNING THEORY

In a very real sense every physical measurement is ultimately a random variable. We can cite scattering and the local responsivity of a typical lead-compound film detector as obvious examples. In both of these cases observable variations will exist which do not readily submit to an explicit description.[(2)] For example, in a given measurement, the spectral response, modulation frequency, signal-to-noise ratio, angular dependence, and detector linearity are factors which in concert tend to introduce a chance mechanism such that we cannot assign with assurance a definite set of values to the local responsivities of the detector. Consequently, we choose to view both the distribution of intercepted radiant flux and the variation of detector's local responsivity as two random variables which we will refer to respectively as ξ_P and ξ_R. Since the scanning process converts position fluctuations to time fluctuations, we can in what follows replace length by time as a parameter.

Now let us consider a narrow strip arbitrarily selected from a detector. Conceptually, we will view this strip as an array of N elemental detectors as shown in Figure 1. Similarly, we will view the flux intercepted by the strip as if composed of N elements. If we assume that the probability density, $P_{\xi}(x, t_1)$ is invariant with time, the average of a stationary random variable is given by,

$$\langle \xi \rangle = \int_{-\infty}^{\infty} x P_{\xi}(x)\, dx \quad . \tag{1}$$

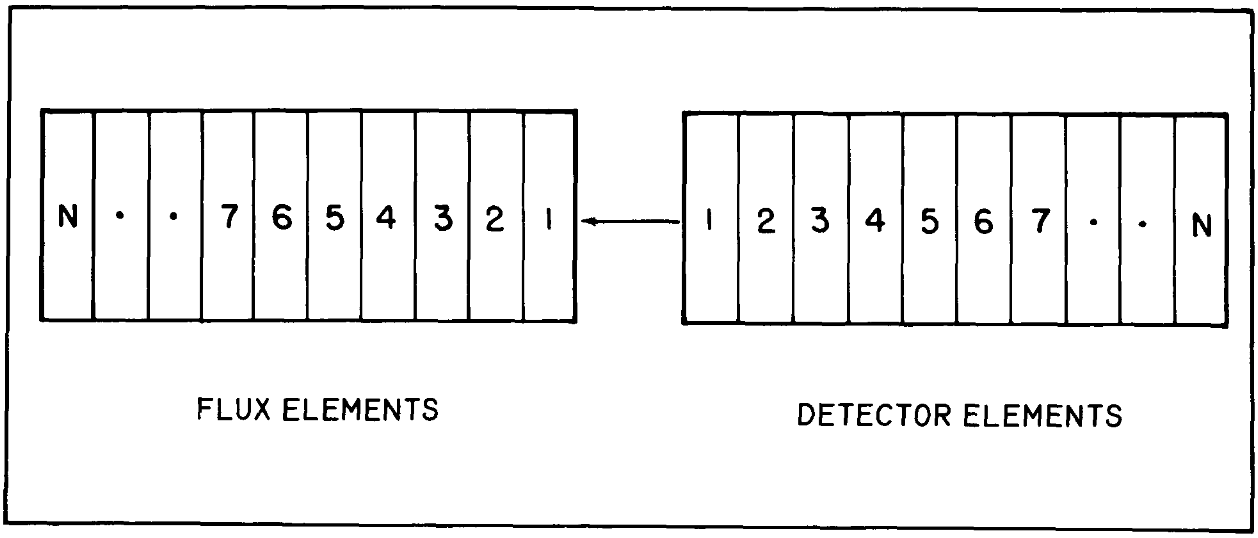

Figure 1—Conceptual division of intercepted flux and detector into N elements.

The mean of the product of two random variables can be expressed as,

$$\langle \xi_P \xi_R \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy\, P_{\xi_P \xi_R}(x, y; \tau)\, dx\, dy \quad , \tag{2}$$

where τ is a time displacement. If, however, the variables ξ_P, ξ_R are statistically independent, as they are assumed to be in our case, the conditional density $P_{\xi_P \xi_R}(x, y; \tau)$ can be restated as an unconditional density, and Equation (2) can be written as,

$$\langle \xi_P \xi_R \rangle = \int_{-\infty}^{\infty} x P_{\xi_P}(x)\, dx \int_{-\infty}^{\infty} y P_{\xi_R}(y)\, dy \tag{3}$$

Referring to (1), we have the important and quite desirable result,

$$\langle \xi_P \xi_R \rangle = \langle \xi_P \rangle \langle \xi_R \rangle \tag{4}$$

That is, the mean of the product of two statistically independent random variables is the product of their mean values.[3] We can illustrate the significance of (4) by considering an integrating sphere. Since the radiance of the sphere wall is perfectly uniform,

$$P_{\xi_P}(x) = P_{\xi_P}(x_i)\,\delta(x - x_i)$$

so that,

$$\langle \xi_P \rangle = \int_0^\infty x\,P_{\xi_P}(x_i)\,\delta(x - x_i)\,dx = x_i$$

Now, if a reflectance or transmittance measurement is made, we have since $\langle \xi_R \rangle = \langle \xi_R' \rangle$,

$$\frac{\langle \xi_P' \rangle \langle \xi_R' \rangle}{\langle \xi_P \rangle \langle \xi_R \rangle} = \frac{x_i'}{x_i'}$$

where ξ_P and ξ_P' refer to the incident and reflected or transmitted flux.

We will now show how scanning can determine $\langle \xi_P \rangle$ and $\langle \xi_P' \rangle$. As shown in Figure 2 the detector is slowly translated across the radiant flux. Following this process, we can write for the i^{th} strip,

$$
\begin{aligned}
V_{(1)\Delta t} &= P_{i1} R_{i1} \\
V_{(2)\Delta t} &= P_{i1} R_{i2} + P_{i2} R_{i1} \\
V_{(3)\Delta t} &= P_{i1} R_{i3} + P_{i2} R_{i2} + P_{i3} R_{i1} \\
\cdot &= \cdot\ \cdot\ \cdot \\
\cdot &= \cdot\ \cdot\ \cdot \\
\cdot &= \cdot\ \cdot\ \cdot \\
V_{(2N-2)\Delta t} &= P_{i(N-1)} R_{iN} + P_{iN} R_{i(N-1)} \\
V_{(2N-1)\Delta t} &= P_{iN} R_{iN}
\end{aligned}
$$

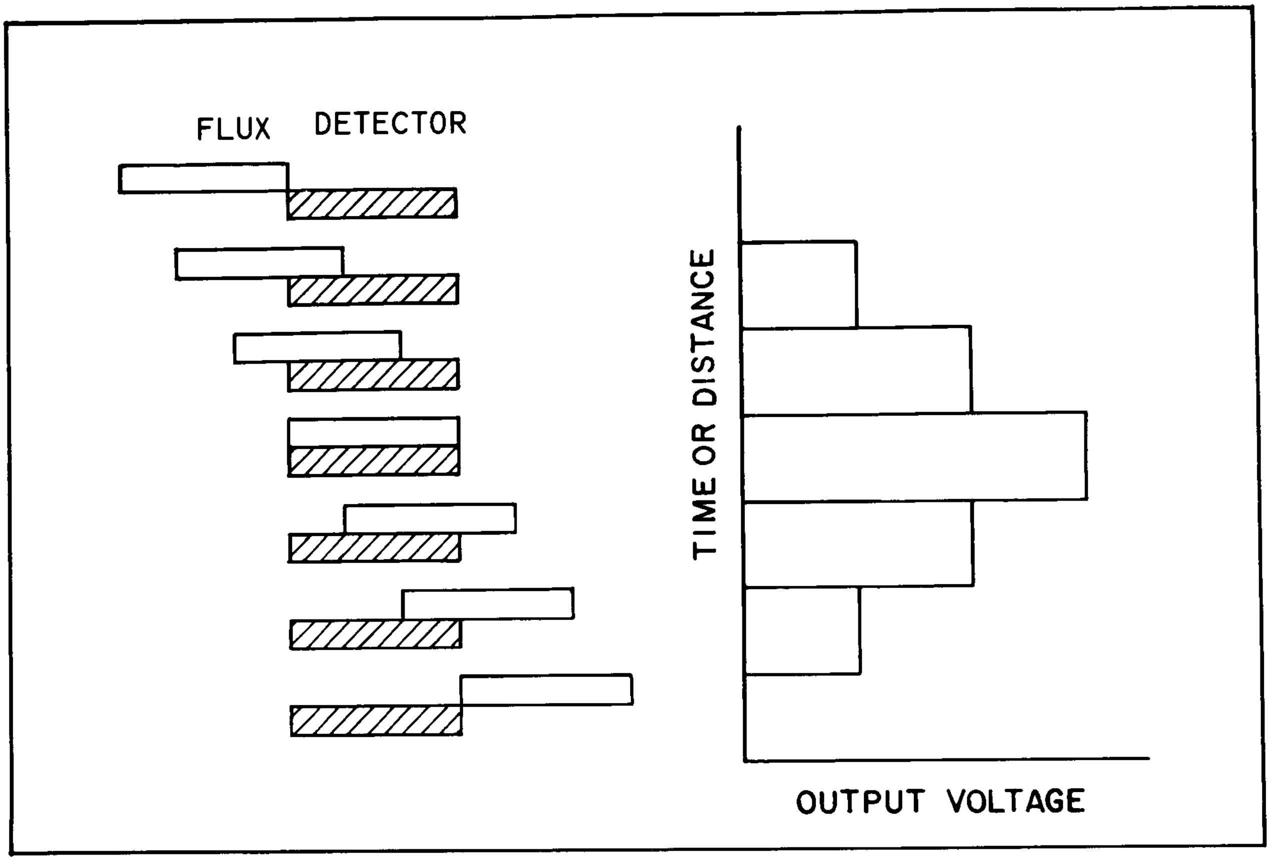

Figure 2—Flux-detector interaction and resulting output voltage.

where P is the radiant flux intercepted by the detector, R the responsivity, and V the voltage output. Collecting and rearranging terms we can also write:

$$\sum_{k=1}^{2N-1} V_{(k)\Delta t} = \begin{cases} P_{i1}\left(R_{i1} + R_{i2} + \cdots + R_{iN}\right) & + \\ P_{i2}\left(R_{i1} + R_{i2} + \cdots + R_{iN}\right) & + \\ \cdots & + \\ \cdots & + \\ \cdots & + \\ P_{iN}\left(R_{i1} + R_{i2} + \cdots + R_{iN}\right) & \end{cases}$$

or

$$\sum_{k=1}^{2N-1} V_{(k)\Delta t} = \left(P_{i1} + P_{i2} + P_{i3} + \cdots + P_{iN}\right)\left(R_{i1} + R_{i2} + R_{i3} + \cdots + R_{iN}\right) \tag{5}$$

Let us now consider the set of values $\left(R_{i1} + R_{i2} + \cdots + R_{iN}\right)$. Since the order of summation is not important, we can group all R_{ij}, values according to their magnitudes, $m\Delta x$ and collect all N_m

elements possessing this magnitude. Then,

$$\sum_{j=1}^{N} R_{ij} = \sum_{m=1}^{n} m \, \Delta x \, N_m$$

and

$$\sum_{j=1}^{N} R_{ij}/M_R = \sum_{m=1}^{n} m \, \Delta x \left(N_m / M_R \, \Delta x\right) \Delta x$$

where M_R is the total number of elements.

But $\left(N_m / M_R \, \Delta x\right)$ is an estimate of the probability density $P_\xi(x)$. It therefore follows that

$$\sum_{j=1}^{N} R_{ij}/M_R = \int_0^A x \, P_{\xi_R}(x) \, dx = \langle \xi_R \rangle \tag{6}$$

It is worth reiterating that

$$\sum_{j=1}^{N} R_{ij}$$

do not constitute a definite set of values. We have only to consider the spectral response of the detector to substantiate this point. Treating the P_{ij} values in a like manner we have,

$$\sum_{j=1}^{N} P_{ij}/M_P = \int_0^B y \, P_\xi(y) \, dy = \langle \xi_P \rangle \tag{6a}$$

We now will show how the scanning process can be used for a given ratio measurement. Let P_{ij} and P'_{ij} refer to the incident and reflected flux. Our purpose will be to determine the radiant reflectance, ρ. Concerning the number of elements M_P, M_P', M_R and M_R', obviously, $M_R' = M_R$. Now, suppose in a given measurement, $M_P' \neq M_P$, and further, $M_P' \neq M_R$ and $M_P \neq M_R$. We can then introduce N fictitious elements of zero magnitude such that $M_P = M_P' = M_R = M_R'$. Consequently,

$$\rho = \frac{\sum_{j=1}^{N} P'_{ij} \sum_{j=1}^{N} R'_{ij}}{\sum_{j=1}^{N} P_{ij} \sum_{j=1}^{N} R_{ij}} \tag{7}$$

But, since we can assume that

$$\sum_{j=1}^{N} R_{ij} = \sum_{j=1}^{N} R'_{ij}$$

$$\rho = \frac{\sum_{j=1}^{N} P'_{ij}}{\sum_{j=1}^{N} P_{ij}} = \frac{\langle \xi_P' \rangle}{\langle \xi_P \rangle} ,$$

This result is really not surprising. Were it not for the fact that statistically independent variables have zero correlation, we would certainly identify the process outlined as cross correlation. For essentially, cross-correlation is an averaging process in which one function may be thought of as the scanhing function of the other.

Ordinarily, each detector element is paired with but one flux element. Scanning, even if it involves a single pass, pairs each flux element with all the detector elements in a given strip. While it is reasonable to assume that the deviation between strips will be less than between individual elements, the fact remains that deviations will ordinarily exist. If we now consider the entire intercepted flux and detector, (7) becomes,

$$\frac{\sum_{i=1}^{K} P_i + \sum_{i=1}^{K} P_i \,\Delta R_i / \langle R \rangle}{\sum_{i=1}^{K} P_i + \sum_{i=1}^{K} P_i \,\Delta R_i / \langle R \rangle} \tag{8}$$

where

$$P_i = \sum_{j=1}^{N} P_{ij} , \qquad R_i = \sum_{j=1}^{N} R_{ij}$$

and ΔR_i is the fluctuation in the i^{th} strip about $\langle R \rangle$. If all the fluctuations, ΔR_i are small, or if they are equally likely in either directions, the error in our ratio measurement will be negligible.

EXPERIMENT

It is apparent from Equation (8) that the error for a given ratio measurement depends upon the fluctuations, ΔR_i. Consequently, detector orientation is an extremely critical factor since it is the simple t method of arranging the R_i values. This point is clarified by Figure 3 where we have plotted the average response of the rows and columns of our detector. Initially, a sensitivity contour was made, which, after appropriate normalization, was mapped as shown in Figure 4 for computational purposes. The detector was mounted on a traveling carriage as illustrated in Figure 5, and a scan rate of approximately 0.2 cm per minute adequately accommodated even the slowest response time of any component in our system. A typical response curve resulting from the signal-detector interaction is shown in Figure 6. The area under each response curve, which is proportional to

$$\sum_{i=1}^{K} P_i \sum_{i=1}^{K} R_i ,$$

was integrated with a planimeter.

Briefly, the procedure used to ascertain the value of the scanning process was as follows. A 180° ellipsoidal collector was used to measure the transmittance of a scattering material. Incident and transmitted flux measurements were made using scanning and non-scanning techniques. The results of these measurements were then compared to transmittance measurements made with a small averaging sphere which were accepted as a standard. On repeated measurements a 12% error for the non-scanning technique was fairly consistently reduced to about 4% for the scanning techniques.

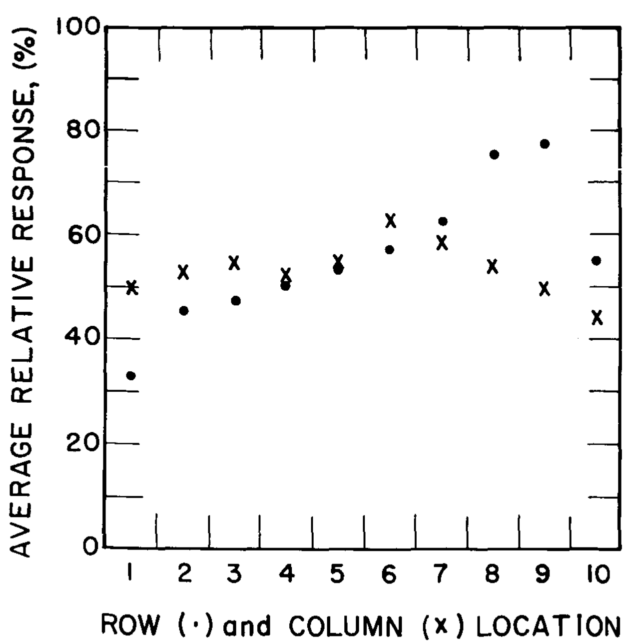

Figure 3—Average relative response of the rows and columns of the detector shown in Figure 4.

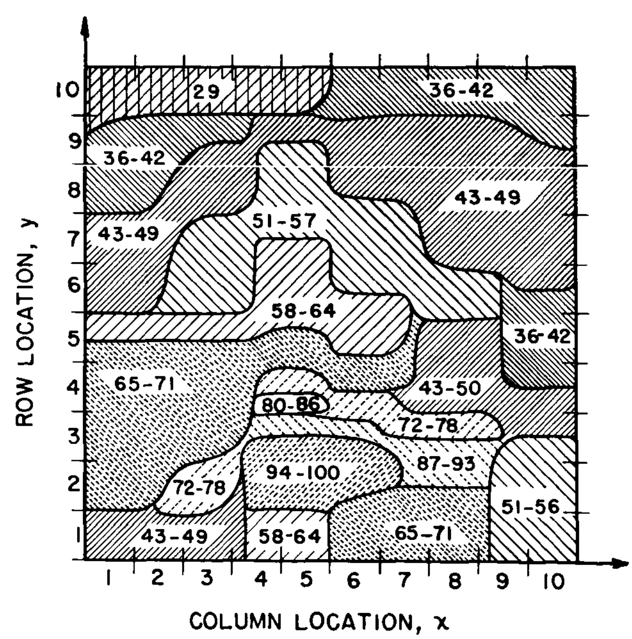

Figure 4—Range distribution of a typical lead sulfide detector (10 x 10 mm) responsivity.

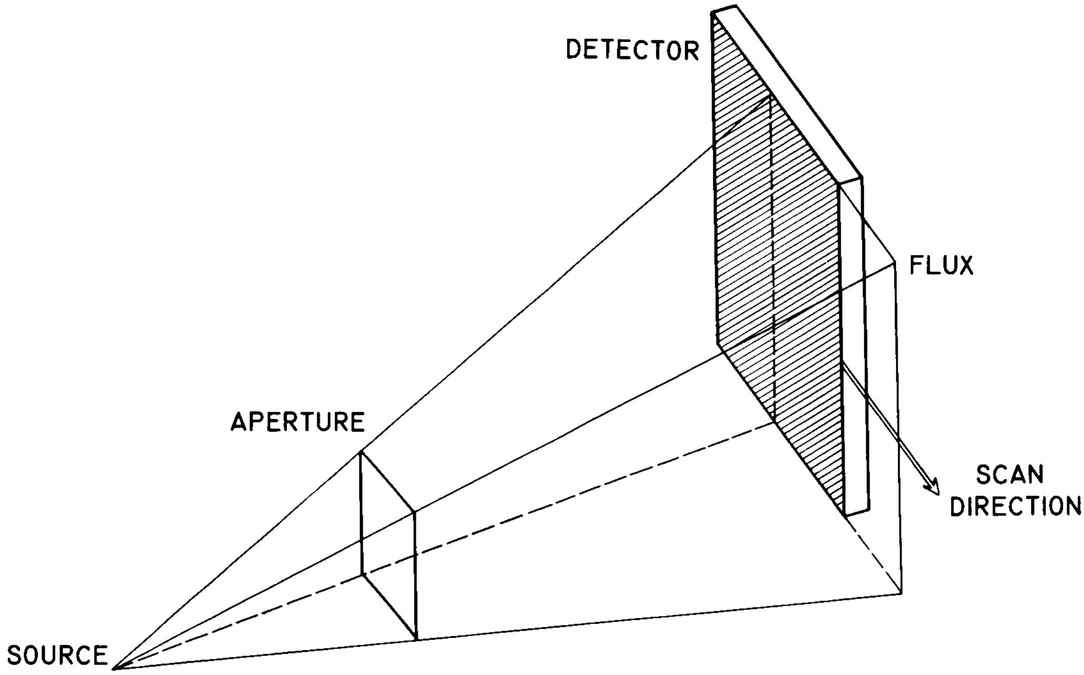

Figure 5—Schematic representation of the scanning process.

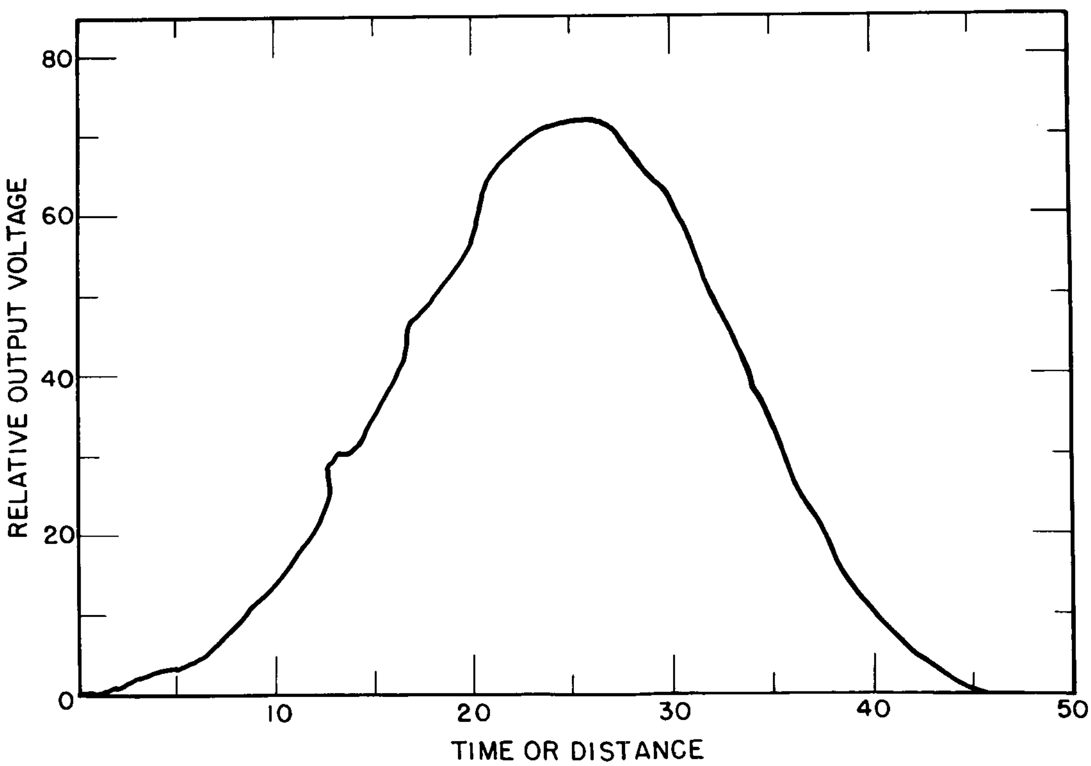

Figure 6—Typical response curve of the scanning process.

CONCLUSIONS

It appears that the error resulting from local nonuniform responsivity of the detector can be substantially reduced, provided the R_i values are reasonably well matched. This is a far less stringent requirement than complete uniformity. There is, of course, no reason to restrict this approach to a single element detector. Indeed, its most practical application would be with multi-element or mosaic type detectors. For now all the R_i values could be more satisfactorily matched. Furthermore, the null regions between individual elements would now become an integral part of each R_i. Returning to Figure 3, we could, by the simple expedient of masking off a few elements in each of the indicated columns, reduce the fluctuations, ΔR_i, so that, effectively, all $R_i = \langle R \rangle$ without seriously affecting the detectors responsivity. It should be noted that the response variation of our detector was about ±35%. Obviously, smaller errors result from smaller variations.

REFERENCES

1. P. W. Kruse, L. D. McGlauchlin, R. B. McQuistan, "Elements of Infrared Technology," (John Wiley and Sons, New York, 1962).

2. N. Arley, K. R. Buch, "Probability and Statistics," (John Wiley and Sons, New York, 1950).

3. Y. W. Lee, "Statistical Theory of Communication," (John Wiley and Sons, New York, 1961).