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A METHOD OF STRESS ANALYSIS FOR SEMI-TOROIDAL TANKS

PART II

Prepared under Contract No. NAS 8-20073 by
Murray F. Burnette

BROWN ENGINEERING COMPANY, INC.

For

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama November 6, 1967

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ABSTRACT

The linear membrane equations and hoop compression parameters are developed for semi-toroidal and modified ogival tanks. A general computer program for designing these tanks, the classification of toroidal loadings, and a summary of the state-of-the-art on toroidal shells are also included.

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LIST OF SYMBOLS

a, b, c, b_1, c_1, ℓ, ℓ_o, ℓ_i	Dimensional quantities indicated in Figure 1 or Figure 2
$\overline{a}$	Nondimensional ratio, see Figure 9
E	Young's modulus
H_1	Height of fluid surface in the inner cylinder
H_2	Height of fluid surface between the inner and outer cylinder
K	Buckling coefficient used in Equation A-41
n	Number of equivalent "g" loads due to vertical acceleration
N_ϕ	Meridional force
N_θ	Hoop force
$\overline{p}$	Axial load per unit of circumference
p_1	Total internal pressure inside the inner cylinder
p_2	Total internal pressure between the inner and outer cylinder
p_{01}	Uniform gas pressure inside the inner cylinder
p_{02}	Uniform gas pressure between the inner and outer cylinder
r_1	Principal radius of curvature in the meridional plane
r_2	Principal radius of curvature in the plane perpendicular to the meridian
R	Radius of circular toroidal shell (see Figure 8)

S_ϕ	Meridional stress
S_θ	Hoop stress
t	Required thickness at a given point in a semi-toroidal or modified ogival tank
$\bar{u}, \bar{w}, V$	Expressions for displacements, see Figure 9b
v, w, V	Expressions for displacements, see Figure 9a
W_p	Partial weights of fluid in bulkheads, see Appendix Paragraph A. for location
x, x_1, z, z_1	Coordinates shown in Figures 1 and 2
z'	Coordinate of arbitrary point of stress measured from the bottom toroidal crown
γ_1	Specific weight of fluid in the inner cylinder
γ_2	Specific weight of fluid between the inner and outer cylinder
ϵ	Strain
ν	Poisson's ratio
ϕ	Reference angle, see Figures A.7, A.8, and 8

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SUMMARY

This report completes a study sponsored by the George C. Marshall Space Flight Center, R-P&VE-S, of the application of the toroidal configuration as liquid propellant containers in transtage and upper stage designs. The NASA technical representatives were Mr. F. Boardman and Mr. C. Bianca of R-P&VE-SSV.

Results previously presented in "A Method of Stress Analysis for Semi-Toroidal Tanks - Part I," J. A. Baird and M. F. Burnette, Technical Note R-140, Brown Engineering Company, Huntsville, Alabama, and "Semi-Toroidal Stage Optimization Study (S-II Stage)," P. M. Crossfield and M. F. Burnette (NAS8-20073, C1-SAA-026C, C1-SSP-020P), Brown Engineering Company, Huntsville, Alabama, June 3, 1966, are also included in this study.

The linear membrane equations and hoop compression parameters are developed in this report for the semi-toroidal tank (Figure 1) and the modified ogival tank (Figure 2). A general computer program for designing these tanks, the classification of toroidal loadings, and the summarizing of the state-of-the-art are also contained in this report.

SECTION I. INTRODUCTION

The toroidal shape has been considered as a potentially optimum configuration for advanced rocket engine concepts, space station applications, and liquid propellant containers in certain transtage and upper stage designs. It is the latter application that resulted in the publication of References 1 and 2 and this report.

A new concept in missile design generally introduces analytical as well as design problems. As more analytical studies were requested by design, it became apparent that parallel efforts were necessary to develop approximate methods for initial design and "exact" analytical methods for final design. A general review of all available literature on toroidal shells, including such classical publications as References 3 and 4, indicated a lack of complete solutions for toroidal shells of arbitrary shapes.

The linear membrane equations developed in Reference 1 and the extension of these equations to a more general loading condition and to the analysis of the modified ogival configuration (Figure 2) in this report were a result of an effort to develop approximate analytical procedure for the initial design in Reference 2. It has since been discovered that under certain loadings these "approximate" equations are very suitable for final design. Because of the need to analyze many configurations in Reference 2, a computer program based on Reference 1 and this report was developed and is listed in the Appendix.

With respect to possible buckling under internal loading, a general derivation of loadings and geometrical parameters that cause hoop compression in elliptical toroidal and modified ogival bulkheads is demonstrated in Section III.

Also included in this study is a discussion of discontinuity stresses as they occur in the semi-toroidal and modified ogival tanks. The simplicity of these stresses is of particular interest.

It is customary to determine the state-of-the-art in the introduction of a report; however, this will be reserved for Section VI. In this section, general toroidal loadings and support conditions will be classified along with the discussion of available solutions to these landings. Also the application of Reference 1 and this present study to the general solutions will be shown.

As a general supplement to the overall toroidal analysis, geometrical parameters have been derived in the Appendix for the elliptical toroidal, modified ogival, and oblate spheroidal shell configurations.

As an additional supplement, linear deformation expressions for the toroidal shape are developed in Section V. It is the singularities of these expressions at the toroidal crowns that lead directly to the nonlinear analyses of Section VI.

SECTION II. EXTENSIONS TO THE LINEAR MEMBRANE ANALYSIS

A. Unlike Fluids at Different Heights in the Inner and Outer Cylinders of the Semi-Toroidal Tank

The possibility of unlike liquids at different heights in the inner and outer cylinders of the semi-toroidal tank (Figure 1) was not taken into account in Reference 1. This important addition will now be included.

In this report, the semi-toroidal tank is considered to be loaded by an internal pressure p_1 in the inner cylinder and an internal pressure p_2 in the outer cylinder. Both p_1 and p_2 are total pressures which are created by fluids of specific weights γ_1 and γ_2 topped by uniform gas pressure p_{01} and p_{02}; the fluids are also assumed to be subjected to a vertical acceleration n times that of gravity. If the height of the fluid surfaces are designated by H_1 and H_2 (Figure 1), then the pressures at z' are given by

$$p_1 = p_{01} + n\,\gamma_1\,(H_1 - z'), \tag{1}$$

$$p_2 = p_{02} + n\,\gamma_2\,(H_2 - z'). \tag{2}$$

If

$$z' > H_1, \quad p_1 = p_{01}. \tag{1a}$$

If

$$z' > H_2, \quad p_2 = p_{02}. \tag{2a}$$

In order to allow a complete analysis, a list of the appropriate meridional and hoop force equations for the semi-toroidal tank will now be provided. The expressions for the liquid weights, W_p, are defined in the Appendix.

1. Upper Elliptical Toroidal Bulkhead

$$N_\phi = \frac{\left[p_2\,x(2a + x) \mp (n\,W^{\pm}_{p_1}/\pi)\right]\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}}{2\,c\,x\,(a + x)} \qquad \text{(From Ref. 1)} \tag{3}$$

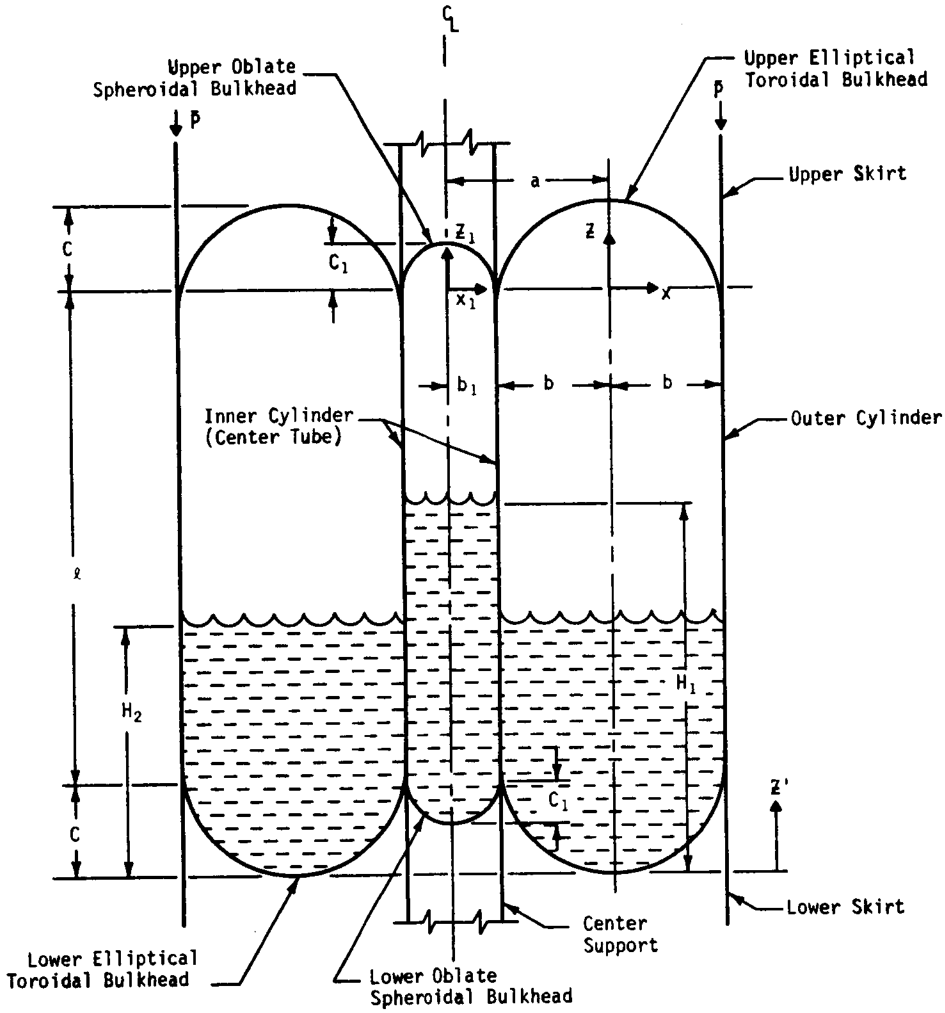


Figure 1. Cross Section of Semi-Toroidal Tank

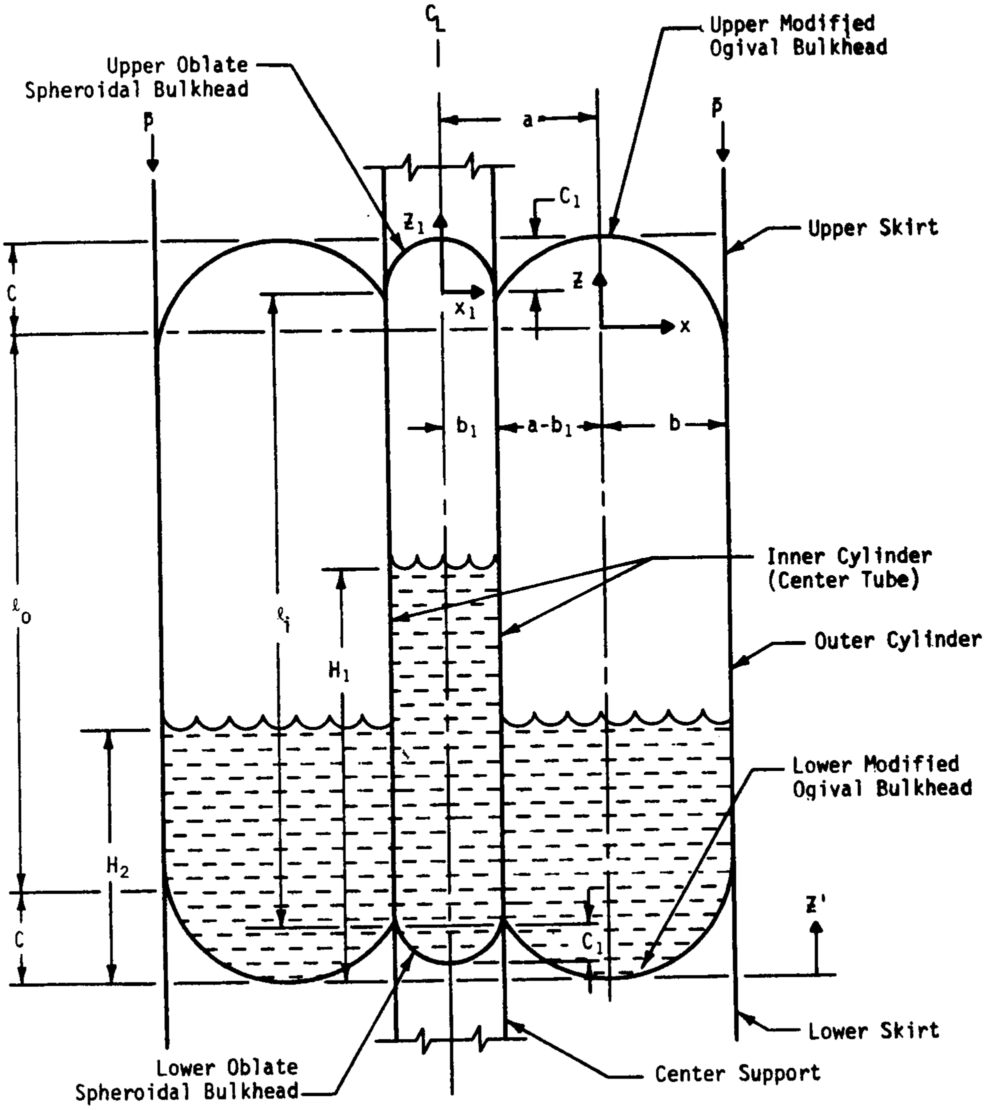


Figure 2. Cross Section of Modified Ogival Tank

$$N_\theta = \frac{2\,p_2\,x(a + x)\left[b^4 - x^2\,(b^2 - c^2)\right] - b^4\left[p_2\,x(2a + x) \mp (n\,W_{p_1}^{\pm}/\pi)\right]}{2\,c\,x^2\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}} \tag{4}$$

(From Ref. 1)

When x is positive, the expression for $W_{p_1}^{+}$ and the minus sign should be used; and when x is negative, $W_{p_1}^{-}$ and the plus sign should be used. When x is zero, the expression for N_θ becomes indeterminate. In the limit, the value of N_θ is given by

$$N_\theta = \frac{b^2}{2\,c}\,p_{02} \tag{5}$$

(From Ref. 1)

2. Lower Elliptical Toroidal Bulkhead

$$N_\phi = \frac{\left[p_2\,x(2a + x) \pm (n\,W_{p_2}^{\pm}/\pi)\right]\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}}{2\,c\,x\,(a + x)} \tag{6}$$

(From Ref. 1)

$$N_\theta = \frac{2\,p_2\,x(a + x)\left[b^4 - x^2\,(b^2 - c^2)\right] - b^4\left[p_2\,x(2a + x) \pm (n\,W_{p_2}^{\pm}/\pi)\right]}{2\,c\,x^2\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}} \tag{7}$$

(From Ref. 1)

When x is positive, the expression for $W_{p_2}^{+}$ and the plus sign should be used; and when x is negative, $W_{p_2}^{-}$ and the negative sign should be used. When x is zero, the expression for N_θ becomes indeterminate. In the limit, the value of N_θ is given by

$$N_\theta = \frac{b^2}{2\,c}\,p_2 \tag{8}$$

(From Ref. 1)

3. Upper and Lower Oblate Spheroidal Bulkheads

$$N_\phi = \frac{\left[p_1 \mp (n\,W_p/\pi\,x_1^2)\right]\left[b_1^4 - x_1^2\,(b_1^2 - c_1^2)\right]^{1/2}}{2\,c_1} \tag{9}$$

(From Ref. 1)

The minus sign and W_{p_3} are used for the upper bulkhead and the plus sign and W_{p_4} for the lower bulkhead.

$$N_\theta = \frac{p_1\left[b_1^4 + 2\,x_1^2\,(c_1^2 - b_1^2)\right] \pm \left[(n\,W_p/\pi\,x_1^2)\right]b_1^4}{2\,c_1\left[b_1^4 + x_1^2\,(c_1^2 - b_1^2)\right]^{1/2}} \tag{10}$$

(From Ref. 1)

The plus sign and W_{p_3} are used for the upper bulkhead and the minus sign and W_{p_4} for the lower bulkhead.

4. Inner Cylinder

$$N_\phi = \frac{p_1'(a-b)^2 + p_2'b(2a-b) - (n/\pi)(W_{p_1}^{-} + W_{p_3}^{+})}{2(a-b)} \quad \text{(From Ref. 1)} \tag{11}$$

Where prime (') indicates pressure evaluated at $z' = \ell + c$,

$$N_\theta = (p_1 - p_2)(a - b) \quad \text{(From Ref. 1)} \tag{12}$$

5. Outer Cylinder

$$N_\phi = \frac{p_2'b(2a+b) - (n\, W_{p_1}^{+}/\pi)}{2(a+b)} + \bar{p} \quad \text{(From Ref. 1)} \tag{13}$$

$$N_\theta = p_2\,(a + b) \quad \text{(From Ref. 1)} \tag{14}$$

6. Upper Skirt

$$N_\phi = \bar{p} \quad \text{(From Ref. 1)} \tag{15}$$

The circumferential load, N_θ, is zero.

7. Center Support

$$N_\phi = \frac{n\, W_{p_5}}{2\,\pi\,(a - b)} \quad \text{(From Ref. 1)} \tag{16}$$

The circumferential load, N_θ, is zero.

8. Lower Skirt

$$N_\phi = \frac{n\, W_{p_6}}{2\pi\,(a + b)} + \bar{p} \quad \text{(From Ref. 1)} \tag{17}$$

The circumferential load, N_θ, is zero.

B. The Linear Membrane Analysis of Modified Ogival Tank

The membrane analysis of the modified ogival tank (Figure 2) requires revisions to Equations (11) and (12) of the inner cylinder (center tube) analysis and Equation (16) of the center support analysis.

The equations for the inner cylinder (center tube) are

$$N_\theta = (p_1 - p_2)(b_1) \tag{18}$$

and

$$N_\phi = \frac{p_1'' b_1^2 + p_2'' (a^2 - b_1^2) - \frac{n}{\pi}(W_{\bar{p}_1} + W_{p_3})}{2\, b_1} \tag{19}$$

where double prime ('') indicates pressure evaluated at $z' = c + \frac{\ell_o + \ell_i}{2}$ for the modified ogival tank.

The equation for the center support is

$$N_\phi = \frac{n\, W_{p_5}}{2\pi\, b_1} \tag{20}$$

The remainder of the modified ogival membrane equations is identical to those of the semi-toroidal tank.

SECTION III. HOOP COMPRESSION EFFECTS

A. Uniform Internal Pressure - Upper and Lower Heads

The compressive hoop stresses that occur in elliptical toroidal and modified ogival bulkheads under uniform internal pressure with respect to buckling are of great interest. An analytical approach to the phenomenon of hoop compression has been partially completed by Reference 5. In this report, the effects of uniform pressure on elliptical toroidal shells with parameters $b < c$ (Figures 1 and 2) were indicated. This derivation will now be presented so as to provide full continuity to the study of these effects.

From equilibrium conditions it is obvious that N_ϕ must always be positive. This is not true with respect to membrane Equations (4) and (7) for the N_θ forces; thus the possibility of buckling necessitates a detailed study of these two expressions. We write for the upper head

$$N_\theta = \frac{2\, p_2\, x(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - b^4 \left[p_2\, x(2a + x) \mp (n\, W^{\pm}_{p_1}/\pi)\right]}{2\, c\, x^2 \left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}} \qquad (4)$$

and for the lower head

$$N_\theta = \frac{2\, p_2\, x(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - b^4 \left[p_2\, x(2a + x) \pm (n\, W^{\pm}_{p_2}/\pi)\right]}{2\, c\, x^2 \left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}} \qquad (7)$$

Equations (4) and (7) allow for both uniform internal pressure and internal liquid effects. For uniform pressure only both equations may be reduced to an identity by allowing the liquid effects to go to zero. The result is

$$N_\theta = \frac{2\, p_2\,(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p_2\, b^4\,(2a + x)}{2\, c\, x\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}}. \qquad (21)$$

This is identical to results of Reference 5.

We first look at Equation (21) with the following restrictions; $a \geq b$, $b < c$, and $x = (+)$. Equation (21) may be arranged as

$$N_\theta / p_2 = \frac{\overset{①}{b^4\,(2a + 2x)} + \overset{②}{2(a + x)\left[- x^2 b^2 + x^2 c^2\right]} - \overset{③}{b^4\,(2a + x)}}{\underset{④}{2\, c\, x\left[b^4 - x^2\,(b^2 - c^2)\right]^{1/2}}} \qquad (22)$$

For $a \geq b$, $b < c$, and $x = (+)$, we see that term 1 > 3 and terms 2 and 4 are always positive; thus Equation (22) is always positive for $x = (+)$ and $b < c$.

For $x = (-)$ and $b < c$ we take terms 1, 2, and 3 of Equation (22) and rearrange them as follows:

$$① + ② - ③ = x\left[2x^2(c^2 - b^2) + 2ax(c^2 - b^2) + b^4\right] \tag{23}$$

We can immediately note that for all $x = (-)$ the denominator of Equation (22) is negative, and the sign of the equation for N_θ is entirely dependent on the bracket terms in Equation (23). Dropping the x, we write

$$\left[2x^2(c^2 - b^2) + 2ax(c^2 - b^2) + b^4\right] = \begin{matrix}(+)\\ 0\\ (-)\end{matrix} \tag{24}$$

If Equation (24) changes sign from (+) to (-) when $x < 0$ then hoop compression occurs at some $a/b \geq 1$ and $b/c < 1$ when the equation is equal to zero. Rearranging Equation (24) and setting equal to zero we get the quadratic equation

$$x^2 + ax + \frac{b^2}{2\left(\frac{c^2}{b^2} - 1\right)} = 0 \tag{25}$$

Solving for x,

$$x = \frac{-a \pm \{a^2 - 2b^2/[(c^2/b^2) - 1]\}^{\frac{1}{2}}}{2} . \tag{26}$$

We can now easily deduce that if the terms under the radical become negative the roots of x would become imaginary, but Equation (25) must have real roots in our case in order for hoop compression to exist. Thus the limiting b/c for a given a/b may be expressed as

$$a^2 = 2b^2/\left[(c^2/b^2) - 1\right] .$$

Rearranging this equation,

$$\left(\frac{a}{b}\right)^2 = \frac{2}{\left(\frac{c}{b}\right)^2 - 1} . \tag{27}$$

At the lower boundary of $a/b = 1.0$, the resulting b/c ratio that limits hoop compression effects determined from Equation (27) is equal to 0.577. As the b/c ratio approaches 1.0 the a/b ratio approaches infinity as the upper bound.

Solutions to Equation (27) are plotted as the left half of the curve on Figure 3. This part of the curve represents toroidal shells with $b/c < 1$ and approaching 1 as a limit from the left.

Once more, we will look at Equation (22) with the following restrictions; $a \geq b$, $b > c$, and $x = (-)$. We note again that the denominator is negative for all $x = (-)$, and that the sign of the equation for N_θ is entirely dependent on the bracket terms in Equation (23). We write again

$$\left[\overset{①}{2x^2(c^2 - b^2)} + \overset{②}{2ax(c^2 - b^2)} + \overset{③}{b^4}\right] = \begin{matrix}(+)\\ 0\\ (-)\end{matrix} \quad . \tag{24}$$

For the conditions $a \geq b$, $b > c$, and $x = (-)$, we see that term 1 is negative for all $x = (-)$ and terms 2 and 3 are positive for all $x = (-)$. Since $2 \geq 1$ obviously no hoop compression can occur in the inner half of the shell, $x = (-)$, when $b > c$.

Assume now $a \geq b$, $b > c$, and $x = (+)$. Rewrite Equation (24) as

$$\overset{①}{(2x^2 + 2ax)}\overset{②}{(c^2 - b^2)} + \overset{③}{b^4} = \begin{matrix}(+)\\ 0\\ (-)\end{matrix} \quad . \tag{28}$$

Term 2 is a constant and is always negative; term 3 is a constant and is always positive. Term 1 is a variable with values from $0 \leq x \leq b$. If compression is to exist, terms 1 x 2, which are a maximum at $x = b$, must overcome term 3. Therefore, the critical b/c ratio at a given a/b occurs at $x = b$ and Equation (28) equal to zero.

We rewrite Equation (28) as

$$\lim_{x \to b} \left| (2b^2 + 2ab)\left(\frac{c^2}{b^2} - 1\right) + b^2 = 0 \right| \; .$$

Finally,

$$\left(\frac{c}{b}\right)^2 = 1 - \frac{b^2}{2b^2 + 2ab} \quad .$$

Let $a/b = K$, then

$$\left(\frac{c}{b}\right)^2 = 1 - \frac{b^2}{2\,b^2 + 2\,K\,b^2} = 1 - \frac{1}{2 + 2\,K}\,. \tag{29}$$

From Equation (29) we see that as $a/b \rightarrow \infty$, b/c approaches 1 as an upper bound. At the lower bound of $a/b = 1.0$, the resulting b/c ratio that limits hoop compression is equal to 1.155.

Solutions to Equation (29) are plotted as the right half of the curve on Figure 3. This part of the curve represents toroidal shells with $b/c > 1$ and approaching 1 as a limit from the right.

Some comments about Figure 3 are appropriate at this time. Two sets of geometric parameters (a/b and b/c) are necessary to determine if hoop compression occurs in elliptical toroidal bulkheads under uniform internal pressure; this differs from the one parameter (b/c) that determines these effects in oblate spheroidal bulkheads. We also note that when $b/c < 1$, compression occurs in the inner half of the shell, $x = (-)$, only; when $b/c > 1$, the reverse is true and compression occurs only in the outer half of the shell, $x = (+)$. We see also that circular toroidal shells are always in tension from a uniform internal pressure; this is not true for liquid loading as we shall demonstrate later.

Equations (27) and (29) are also applicable to modified ogival bulkheads. The upper bound in this case is $a/b = 1.0$ where the modified ogival shell converts to an elliptical toroid, and $a/b = 0$ where the modified ogival shell becomes an oblate spheroid.

Solutions to Equations (27) and (29) are plotted in Figure 4 as the left half and the right half of the curve, respectively. Note that the curves in Figures 3 and 4 are tangent to one another at the transition value of $a/b = 1.0$. A very interesting phenomenon occurs as $a/b \rightarrow 0$ and the modified ogival shape in effect becomes an oblate spheroid. As the inner half of the bulkhead disappears, i.e., $a/b \rightarrow 0$, any tendency for hoop compression to occur in this half at b/c ratios less than one approaches zero. Simultaneously, at the higher b/c ratios where hoop compression occurs in the outer half of the shell only, the curve approaches a limit of 1.414, the exact results of an oblate spheroid under uniform internal pressure.

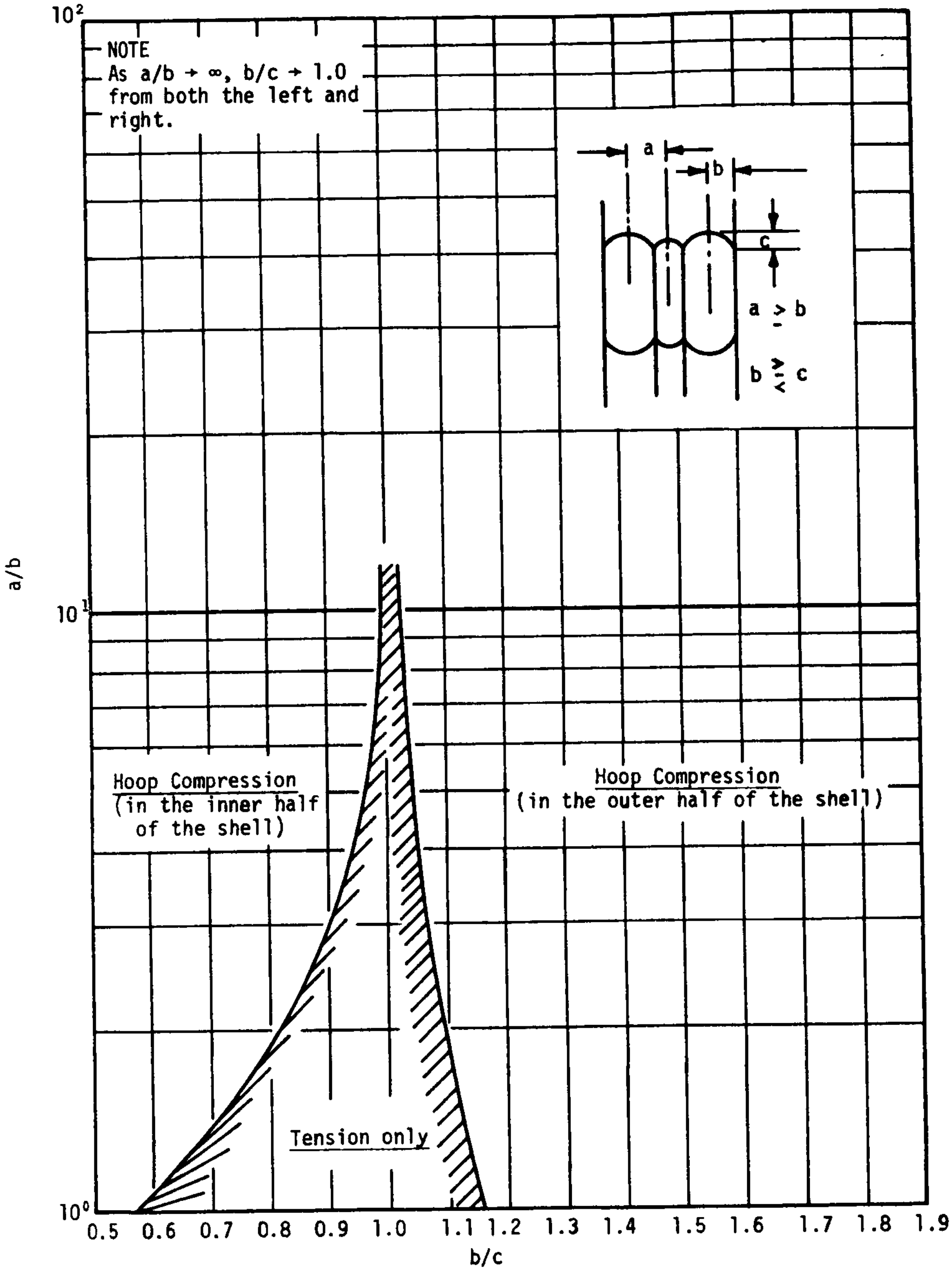


Figure 3. Hoop Compression Effects in Elliptical Toroidal Bulkheads Under Uniform Internal Pressure

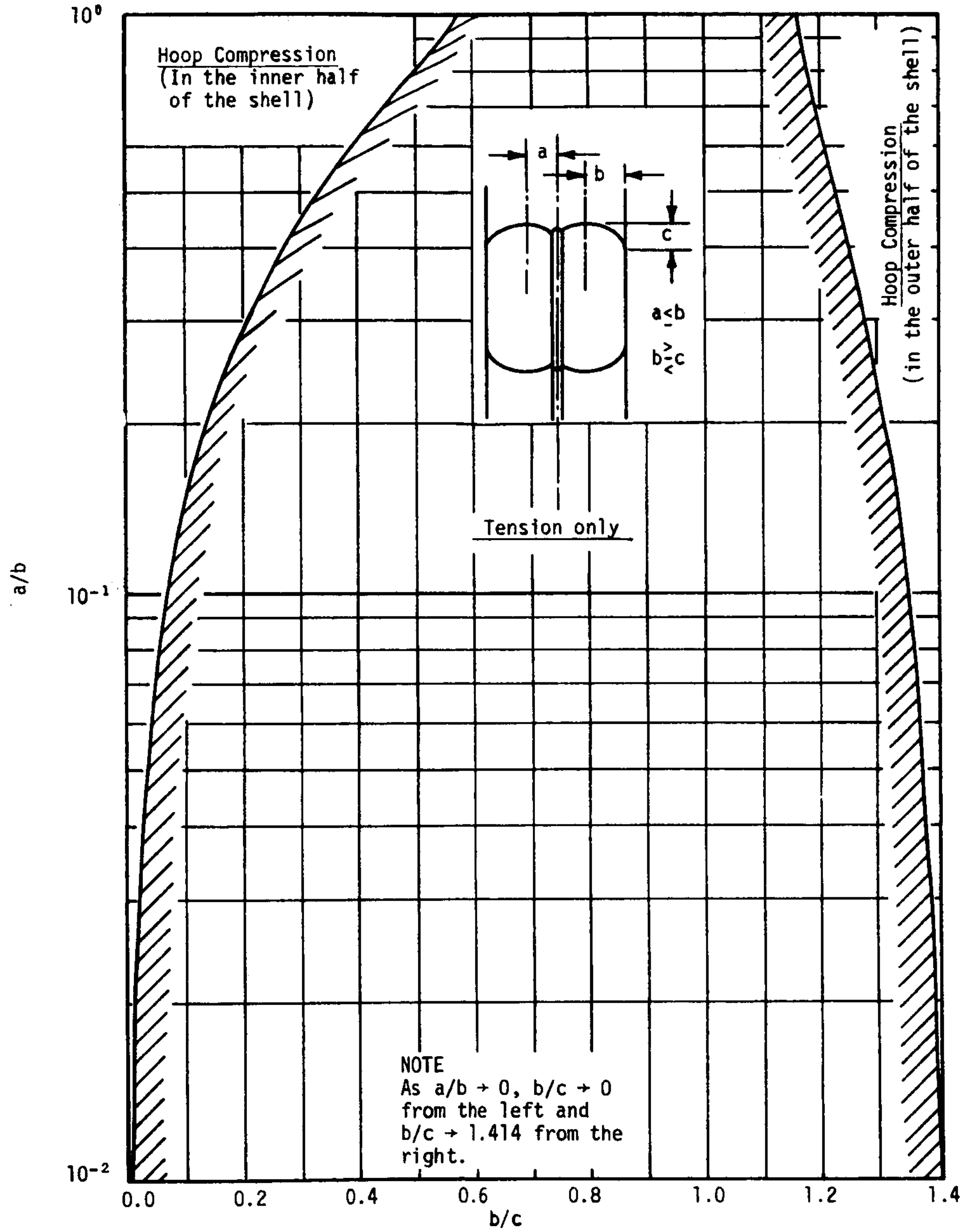


Figure 4. Hoop Compression Effects in Modified Ogival Bulkheads Under Uniform Internal Pressure

B. Liquid Loading - Upper Head

Elliptical toroidal and modified ogival bulkheads, with support conditions as indicated in Figures 1 and 2, are subjected to hoop compression forces from liquid loading which are in addition to those caused by uniform internal pressures. The liquid loading stresses are, in general, not related to the uniform pressure stresses with respect to geometrical parameters or location of occurrence in the shell. It, therefore, becomes necessary to undertake a separate investigation of these effects.

We will first analyze Equation (4) for the upper head with the following restrictions; the head is full, no gas pressure is present, $a \gtrless b$, $b < c$, and $x = (+)$. Multiplying the numerator and denominator by $1/x$, we get

$$\underset{(+x)}{N_\theta} = \frac{2p_2\,(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p^2 b^4(2a + x) + \dfrac{n\, b^4\, W^+_{p_1}}{\pi x}}{2\,c\,x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}} \qquad (30)$$

Additional rearranging provides

$$\underset{(+x)}{\frac{N_\theta}{p_2}} = \frac{\overset{(1)}{b_4(2a + 2x)} + \overset{(2)}{2(a + x)\left[- x^2b^2 + x^2c^2\right]} - \overset{(3)}{b^4(2a + x)} + \overset{(4)}{\dfrac{n\, b^4\, W^+_{p_1}}{\pi\, x p_2}}}{\underset{(5)}{2\,c\,x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}}} \qquad (31)$$

In this case, terms 2, 4, and 5 are always positive and term 1 is always > 3. Thus for the conditions $a \gtrless b$, $b < c$, and $x = (+)$, hoop compression cannot exist in the +x region for an upper head.

It is very easy to extend this proof to the case of $a \gtrless b$, $b = c$ (circular shape), and $x = (+)$. In Equation (31), term 2 goes to zero while remaining conditions prevail as previously noted; therefore, for $a \gtrless b$, $b = c$, and $x = (+)$, hoop compression is nonexistant in the +x region for an upper head.

We now examine Equation (31) for $a \gtrless b$, $b > c$, and $x = (+)$. As before terms 4, 5, and (1 - 3) are always positive, but term 2 in this case is negative for all $b > c$ values. The numerator of Equation (31) may be rearranged as follows:

$$\text{①}\quad\text{②}\quad\text{③}\quad\text{④}$$

$$b^4x + \frac{nb^4\,W^+_{p_1}}{\pi x p_2} + 2ax^2(c^2 - b^2) + 2x^3(c^2 - b^2) \quad . \tag{32}$$

Let $x = b$, $p_2 = n\,\gamma_2\,(H_2 - z') = n\,\gamma_2\,(h_1 - h_5) = n\,\gamma_2\,c$

where $h_1 = c$ and $h_5 = 0$.

$$\text{①}\quad\text{②}\quad\text{③ \& ④}$$

$$b_5 + \frac{b^3\,W^+_{p_1}}{\pi\,\gamma_2\,c} + \left[2\,a\,b^2c^2 + 2\,b^3c^2 - 2\,a\,b^4 - 2\,b^5\right] \quad . \tag{33}$$

The point $x = b$ has proven to be the critical point of hoop compression in this case. Terms 3 and 4 are proportionally a larger absolute value at $x = b$ (due to the terms x^2 and x^3, respectively) than term 1. Also term 2 does not increase proportionally because of the terms x and p_2 in the denominator.

Applying the value of $W^+_{p_1}$ found in the Appendix, Equation (33) reduces to

$$2\,a\,c^2b^2 + 2\,b^3c^2 - .429\,ab^4 - .333\,b^5 \quad . \tag{34}$$

Setting Equation (34) equal to zero, we may then obtain critical values of b/c and a/b at which hoop compression occurs in upper heads in the +x regions. Figures 5 and 6 are solutions to this equation and are self explanatory.

We will now investigate Equation (4) with respect to the inner half (the -x regions) of an upper head.

$$\underset{(-x)}{N_\theta} = \frac{2\,p_2\,(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p_2b^4(2a + x) - \dfrac{n\,b^4\,W^-_{p_1}}{\pi x}}{2\,c\,x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}} \tag{35}$$

The following assumptions will be applied to the preceding equation; the upper head is full, no gas pressure is present, $a \gtreqless b$, $b = c$ (circular shape), and $x = (-)$. Rearranging Equation (35) similar to Equation (31) we get

$$\text{①}\quad\text{②}\quad\text{③}\quad\text{④}$$

$$\frac{N_\theta}{p_2} = \frac{b^4(2a + 2x) + 2(a + x)\left[- x^2b^2 + x^2c^2\right] - b^4(2a + x) - \dfrac{n\,b^4\,W_{p_1}}{\pi x p_2}}{\underbrace{2\,c\,x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}}_{\text{⑤}}} \quad . \tag{36}$$

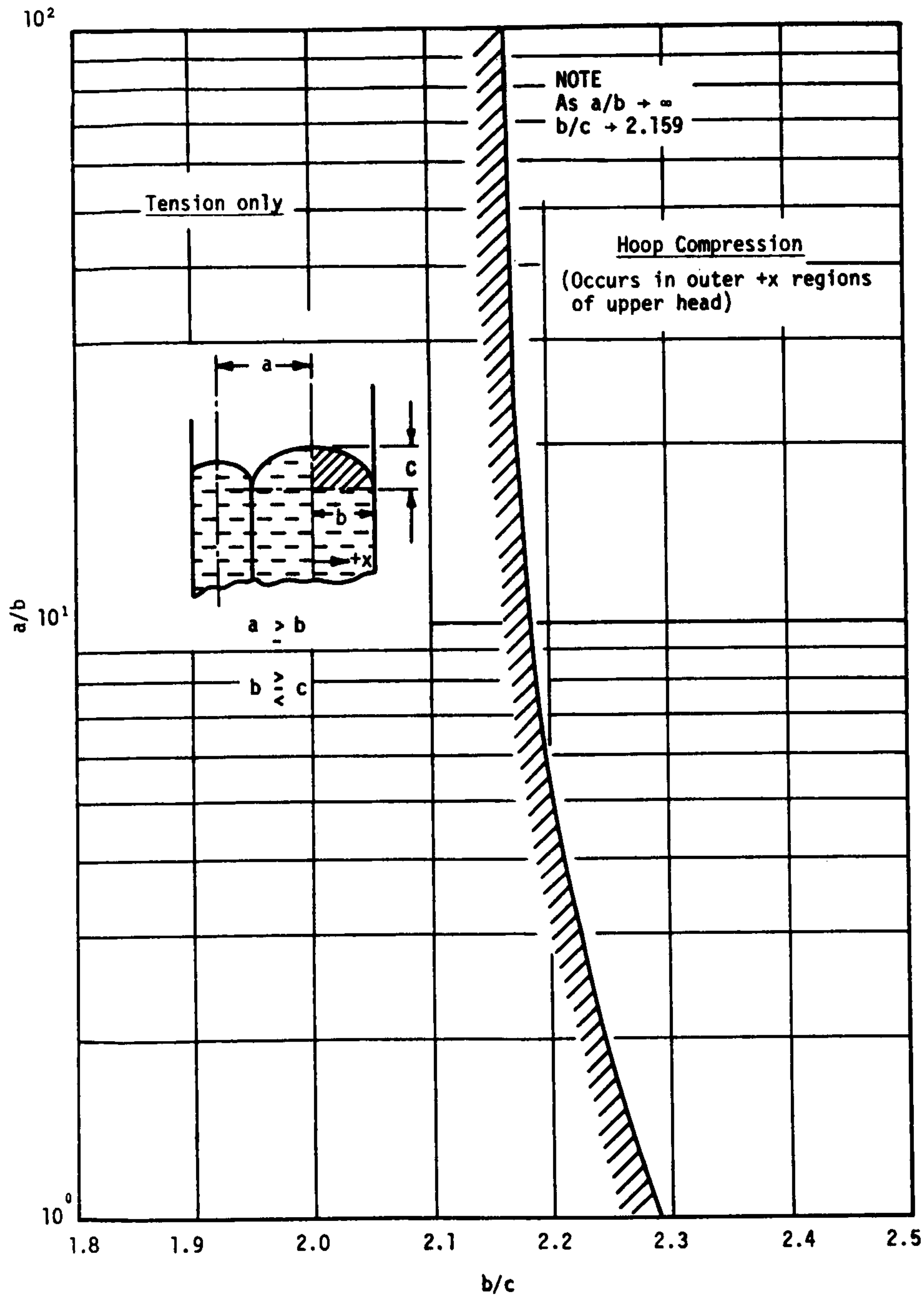


Figure 5. Hoop Compression Effects in the +X Regions for Upper Elliptical Toroidal Bulkheads Under Full Liquid Head

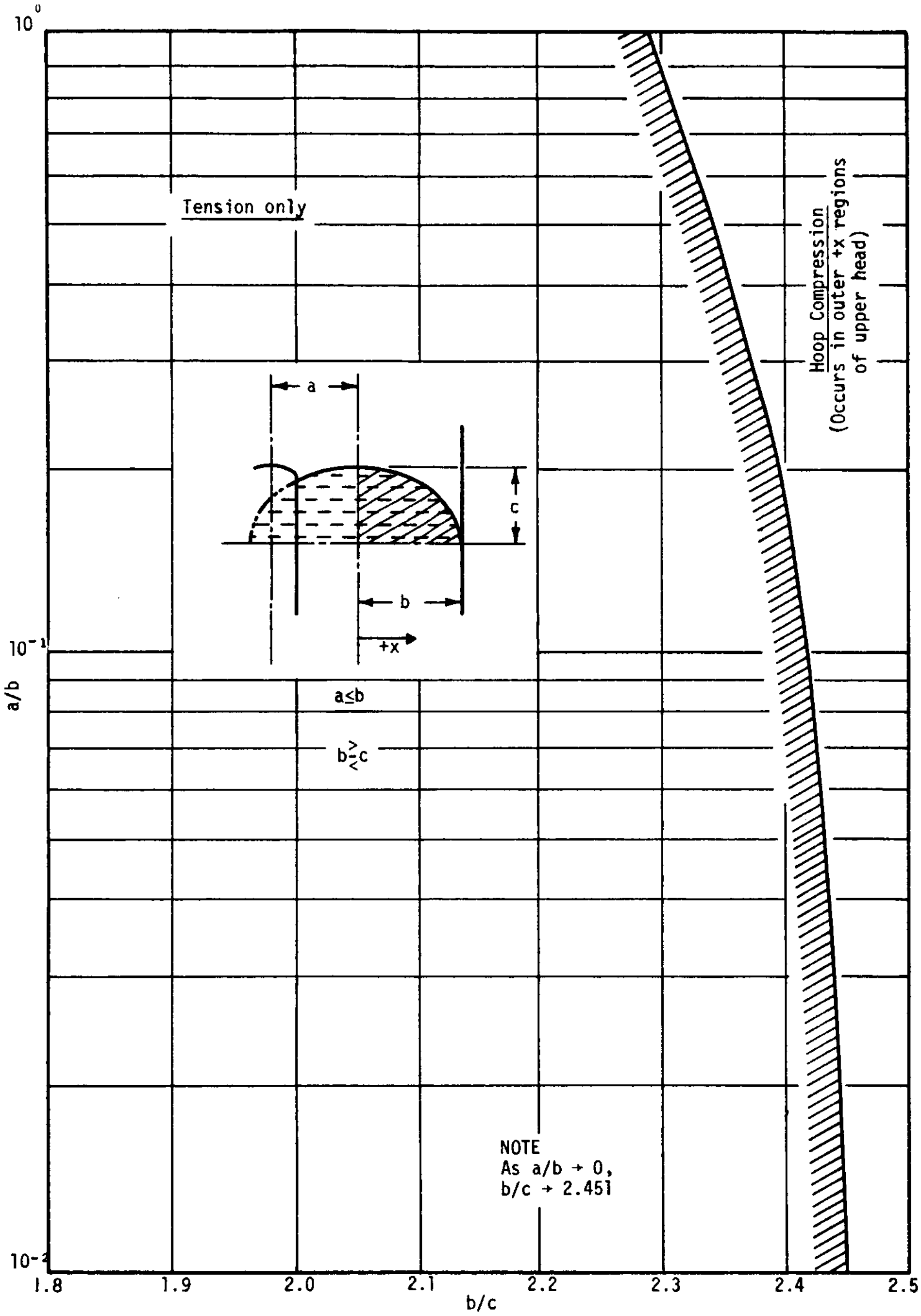


Figure 6. Hoop Compressive Effects in the +X Regions for Upper Modified Ogival Bulkheads Under Full Liquid Head

Applying the above restrictions, we get

$$\frac{N_\theta}{p_2} = \frac{\overset{\textcircled{1}}{b^4(2a + 2x)} + \overset{\textcircled{2}}{0} - \overset{\textcircled{3}}{b^4(2a + x)} - \overset{\textcircled{4}}{\dfrac{n\, b^4\, \bar{W}_{p_1}}{\pi x p_2}}}{\underset{\textcircled{5}}{2\, b^3\, x}} \tag{37}$$

Equation (37) can be further reduced to

$$\frac{N_\theta}{p_2} = \frac{b}{2} - \left(\frac{b}{2}\right)\left(\frac{n\, W_{p_1}}{\pi\, x^2 p_2}\right) \tag{38}$$

It can be shown that term 3 is always greater than 1 for all values of $-x$; thus, N_θ must always be negative for an upper bulkhead under full liquid head with no gas pressure present.

If the restriction $b = c$ is replaced by $b < c$, we see that term 2 in Equation (36) merely strengthens the solution to the circular shape; thus N_θ again must always be negative for all values of $-x$. If $b > c$ is used, the hoop compressive forces are relieved by term 2, but generally compressive forces still exist for reasonable values of $b > c$. In fact, for values of b/c to the right of the curves in Figures 5 and 6, hoop compressive forces may exist in both halves of the head.

C. Liquid Loading - Lower Head

We now examine Equation (7) to determine possible hoop compression effects in lower heads under fluid loading. Once more, multiplying the numerator and denominator by $1/x$, we have

$$\underset{(+x)}{N_\theta} = \frac{\overset{\textcircled{1}}{2p_2(a + x)\left[b^4 - x^2(b^2 - c^2)\right]} - \overset{\textcircled{2}}{p_2 b^4(2a + x)} - \overset{\textcircled{3}}{\dfrac{n\, b^4\, \overset{+}{W}_{p_2}}{\pi x}}}{\underset{\textcircled{4}}{2\, c\, x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}}} \tag{39}$$

and

$$N_{\theta\,(-x)} = \frac{2p_2}{} \; \frac{(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p_2 b^4(2a + x) + \dfrac{n\, b^4\, \bar{W}_{p_2}}{\pi x}}{2\, c\, x\left[b^4 - x^2(b^2 - c^2)\right]^{1/2}} \quad . \quad (40)$$

The lower head analysis is different in many respects to the upper head. The terms multiplied by p_2 in Equations (39) and (40) can become very dominant since the height of liquid above the lower head may vary, whereas term 3 is still only concerned with the actual weight of liquid in the head. If term 3 is eliminated from these equations, e.g., a very long tank with small heads, we then have in effect the membrane equations for pressure only (uniform or variable). On the other hand, for a toroidal vessel with no cylindrical portion, term 3 can become the principle part of the equations.

We will first assume the liquid level at a depth "c" (see Figures 1 and 2) in the lower head (basically the same effects occur with partially filled heads at less critical stresses). For Equation (39), the critical value occurs at $x = +b$ where $\overset{+}{W}_{p_2}$ is a maximum and $p_2 = 0$. As terms 1 and 2 are equal to zero, the shape of the head now has no effect and hoop compression occurs in the outer half (+x region) in every case. The exact opposite can be said of equation (40); hoop compression can never occur in the inner half (-x region) for any $b \geq c$; however, for $b < c$ compression can occur in the inner half of the shell at low values of $-x$.

As the fluid level is increased above the top of the head, terms 1 and 2 of Equations (39) and (40) will react on the bulkhead as an internal pressure, and Figures 3 and 4, which are independent of the quantity of pressure, will determine the effects of these two terms. For heads whose geometrical parameters lie to the right of the "tension only" area of Figures 3 and 4, the rise of the fluid level continues to add additional hoop compression stresses to those caused by term 3 in the outer half of the shell. For heads whose geometrical parameters lie in the "tension only" area, additional liquid height tends to overcome the hoop compressive stresses caused by term 3. At a particular height dependent upon geometrical parameters, hoop compressive forces will be completely eliminated and any additional liquid weight above this value merely causes additional tension forces in the head. For any given case, this height can be solved by the following expression:

$$2p_2(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p_2 b^4(2a + x) = \frac{n\, b^4\, W^{+}_{p_2}}{\pi x}, \qquad (41)$$

where $p_2 = n\,\gamma_2\,(H_2 - c)$. For heads whose geometrical parameters lie to the left of the "tension only" area, the hoop compressive forces caused by term 3 in the outer half of the shell are again relieved; but in this case hoop compressive forces can then appear in the inner half of the shell or be added to those already existing in the inner half of the shell. We note for this last case that the disappearance of hoop compression in outer half of the shell does not necessarily occur at the same time hoop compression forces begin in the inner half of the shell. A general solution to the following equation will determine at what height hoop compressive forces will start in the inner half of the head;

$$2p_2(a + x)\left[b^4 - x^2(b^2 - c^2)\right] - p_2 b^4(2a + x) = -\frac{n\, b^4\, W^{-}_{p_2}}{\pi x}, \qquad (42)$$

where again $p_2 = n\,\gamma_2\,(h_2 - c)$.

D. Summary

As the explanation of hoop compression effects in elliptical toroidal and modified ogival heads under internal loading was quite extensive, the results of this analysis is presented in Table 1. The shape of the tank involved and the ratio of any combined liquid and/or uniform internal pressure loadings will determine whether the hoop compressions described in Table 1 are critical with respect to the buckling of a bulkhead.

A similar treatment of hoop compressive effects is given in Reference 3 for oblate spheroidal heads.

TABLE 1. - HOOP COMPRESSIVE EFFECTS IN ELLIPTICAL TOROIDAL AND MODIFIED OGIVAL SHELLS

Case No.	Loading	Bulkhead	Parameters	Comments
1	Uniform Internal Pressure	Upper and Lower	$a \gtreqless b$, $b \gtreqless c$	Hoop compression may occur in either the inner half or outer half of the shell. See Figures 3 and 4.
2	Liquid Loading (full head)	Upper	$a \gtreqless b$, $b>c$	Hoop compression may occur in the outer half of the shell; see Figures 5 and 6. Hoop compression will also generally exist in the inner half of the shell for reasonable values of b>c and may occur simultaneously with compressive forces in the outer half. At very high values of b>c, the compressive forces in the inner half will disappear and those in the outer half will increase.
3	Liquid Loading (full head)	Upper	$a \gtreqless b$, $b \overline{<} c$	Hoop compression will always occur for all values of -x in the inner half of the shell. Compressive forces can never occur for any value of +x in the outer half of the shell.
4	Liquid Loading (partially filled to full head, no liquid head above top of bulkhead)	Lower	$a \gtreqless b$, $b>c$	Hoop compression will always occur for higher values of +x in the outer half of the shell; it can never occur in the inner half of the shell.
5	Liquid Loading (partially filled to full head, no liquid head above top of bulkhead)	Lower	$a \gtrless b$, $b<c$	Hoop compression will always occur for higher values of +x in the outer half of the shell. If the values of b/c lie to the left of the "tension only" area of Figures 3 and 4, hoop compression will also occur in the inner half of the shell at lower values of -x.
6	Liquid Loading (full head, liquid head exists above top of bulkhead)	Lower	$a \gtreqless b$, b/c values to the right of the "tension only" areas of Figures 3 and 4.	The rise of the liquid level adds additional hoop compressive forces to those already existing in Case 4 in the outer half of the shell. As in Case 4, these compression forces can never exist in the inner half of the shell.
7	Liquid Loading (full head, liquid head exists above top of bulkhead)	Lower	$a \gtreqless b$, b/c values in the "tension only" area of Figures 3 and 4.	Additional liquid height will overcome the compression of Cases 4 and 5 in the outer half at a level dependent upon geometrical parameters. As in Case 6, hoop compression can never exist in the inner half.
8	Liquid Loading (full head, liquid head exists above top of bulkhead)	Lower	$a \lessgtr b$, b/c values to the left of the "tension only" area of Figures 3 and 4.	Additional liquid height will overcome the compression of Case 5 in the outer half at a level dependent upon geometrical parameters. However, the liquid rise will add to those existing in the inner half of the shell.

SECTION IV. DISCUSSION OF DISCONTINUITY STRESSES

One distinct advantage the semi-toroidal or modified ogival tank has is that the discontinuity stresses require only the normal two degrees of freedom. Thus, discontinuity stresses present no problem in toroidal tanks as long as they are located away from the crown area (nonlinear crown effects will be discussed in Section VI of the report).

The equations developed in Reference 1 for determining discontinuity stresses are correct for an "ideal" semi-toroidal tank (Figure 7). In an actual case, a more refined analysis would have to be made, and the equations of Reference 1 would have to be modified to allow for several free bodies at each joint. Several computer programs are now available to assist in such an analysis.

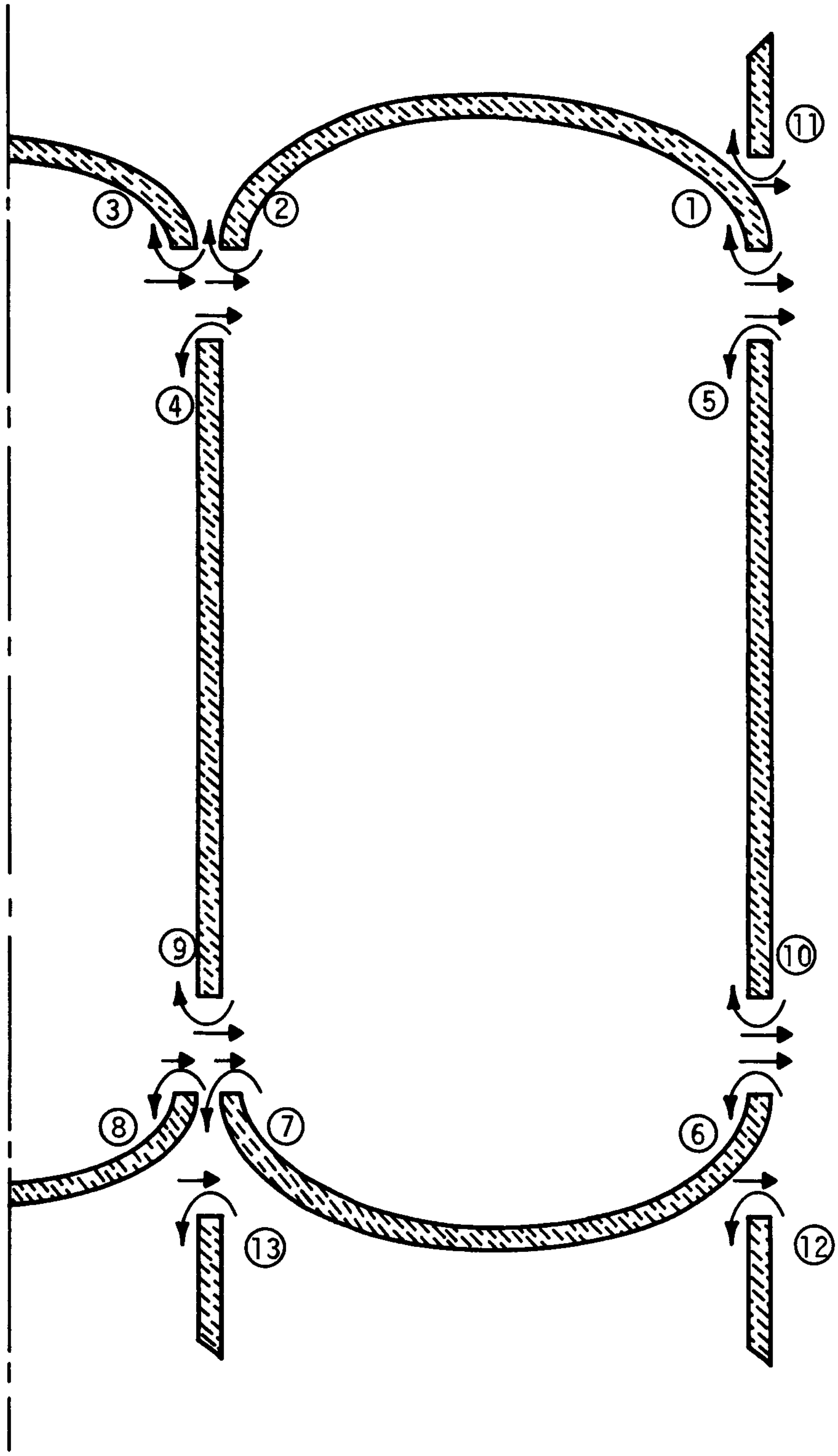


Figure 7. Components of Semi-Torus showing Discontinuity Shears and Moments

SECTION V. GENERAL DEFORMATION EXPRESSIONS (LINEAR MEMBRANE ANALYSIS)

A. Circular Toroidal Shell Under Uniform Pressure

The circular toroidal shell under uniform pressure is the basic shape and loading which will be investigated first with respect to displacement characteristics. The equations of this report yield the following force and geometrical expressions for this configuration.

$$r_1 = R \tag{A-31}$$

$$r_2 = \frac{(a + x)R}{x} \tag{A-34}$$

$$\sin \phi = \frac{x}{R} \tag{A-33}$$

$$N_\phi = \frac{p(2a + x)R}{2(a + x)} \tag{43}$$

$$N_\theta = \frac{pR}{2} \tag{44}$$

where Equations (43) and (44) are determined from Equations (3), (4), (6), and (7) with $b = c = R$ (Figure 8), $p = p_{02}$, and $W_{\bar{p}_1}^{+}$, $W_{\bar{p}_2}^{+} = 0$.

The derivation of the linear membrane deformation equations in Reference 4 for a shell of revolution loaded symmetrically with respect to its axis of revolution is based on the following assumptions:

- The thickness of the shell, t, is considered small in comparison with other dimensions of the shell.

- The thickness of the shell, t, is considered small in comparison with its radii of curvature; i.e., $t/r_1 << 1$, $t/r_2 << 1$.

- Small deflections prevail; i.e., equations of equlibrium may be obtained with sufficient accuracy from the undeflected state.

- All bending effects are negligible and may be neglected; thus, extensional strains only are considered.

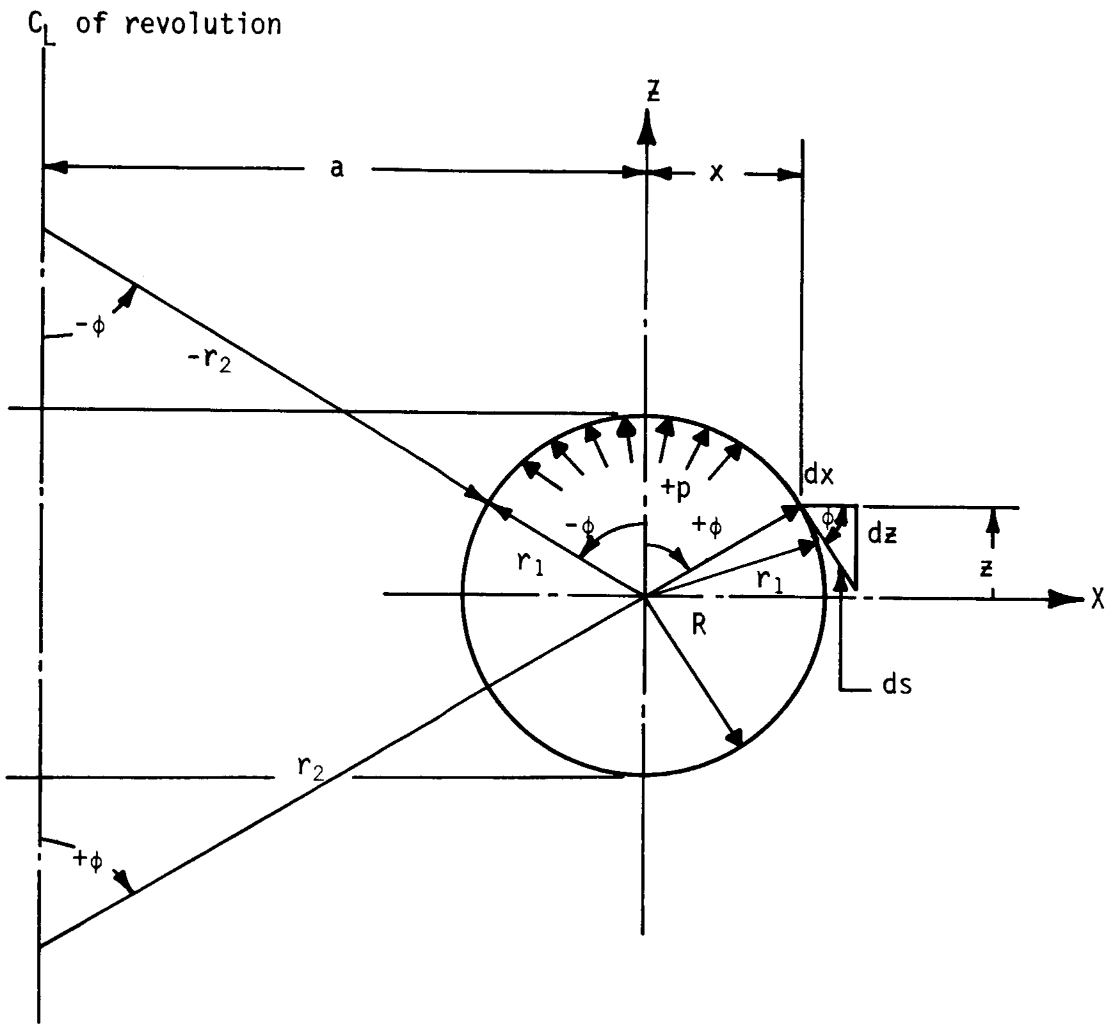


Figure 8. Circular Toroidal Shell under Uniform Pressure

Figure 9 indicates a deflected element of a toroidal shell. From Reference 4, the expressions for the small displacements v, w, and V are

$$v = \sin\phi \left[\int \frac{f(\phi)}{\sin\phi}\, d\phi + C \right] \qquad (45)$$

where $f(\phi) = \frac{1}{Et}\left[N_\phi (r_1 + \nu r_2) - N_\theta (r_2 + \nu r_1)\right]$ and C is a constant of integration determined from support conditions,

$$w = v \cot\phi - \epsilon_\theta r_2 \qquad (46)$$

and

$$V = \frac{\cot\phi}{r_1\, Et} \left[(r_1 + \nu r_2)N_\phi - (r_2 + \nu r_1)N_\theta\right] - \frac{1}{r_1}\frac{d}{d\phi}\left[\frac{r_2}{Et}(N_\theta - \nu N_\phi)\right]. \qquad (47)$$

Since the displacement equations are derived as a function of ϕ, we will change Equations (A-31), (A-34), (43), and (44) accordingly through the use of (A-33).

$$r_1 = R \qquad (A\text{-}31)$$

$$r_2 = \frac{(a + R\sin\phi)}{\sin\phi} \qquad (A\text{-}34a)$$

$$N_\phi = pR\left(\frac{a + \frac{1}{2} R \sin\phi}{a + R\sin\phi}\right) \qquad (43a)$$

$$N_\theta = \frac{pR}{2} \qquad (44)$$

The displacement convention of this report (Figure 9b) has been changed from that of Reference 4 since $\bar{u}$ can be used directly in discontinuity analyses, and horizontal and vertical deflections ($\bar{u}$ and $\bar{w}$) are more convenient in the nonlinear analyses of Section VI.

Another substitution which will assist in later nondimensional analyses of Section VI is to assume

$$a = \bar{a} R$$

where $\bar{a}$ is the ratio of the two dimensions a and R as described in Figure 7.

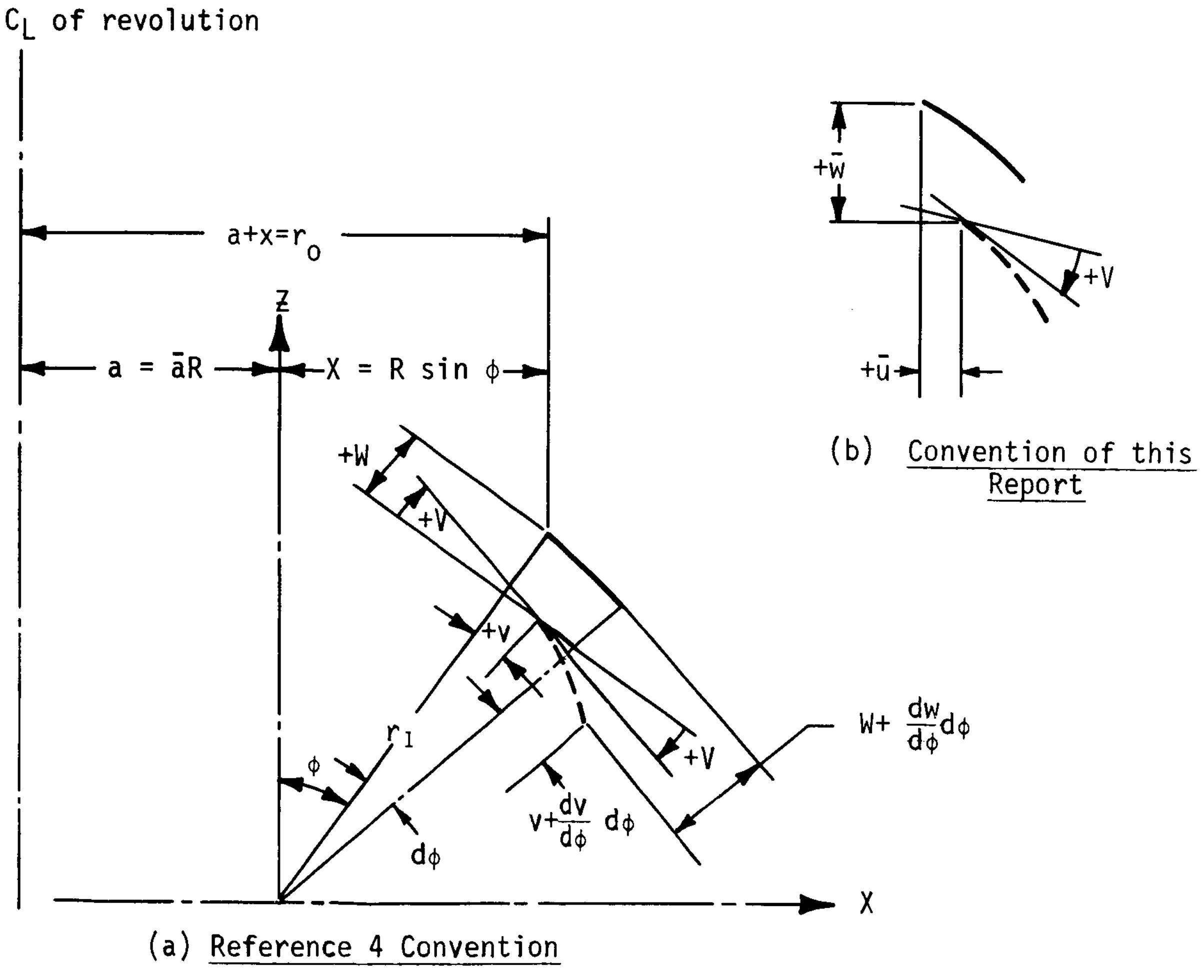


Figure 9. Deflected Element of Toroidal Shell

We rewrite Equations (A-31), (A-34a), (43a), and (44).

$$r_1 = R \tag{A-31}$$

$$r_2 = \frac{R(\bar{a} + \sin\phi)}{\sin\phi} \tag{A-34b}$$

$$N_\phi = pR\left(\frac{\bar{a} + \frac{1}{2}\sin\phi}{\bar{a} + \sin\phi}\right) \tag{43b}$$

$$N_\theta = \frac{pR}{2} \tag{44}$$

From trigonometric and strain relations we see that

$$\bar{u} = v\cos\phi - w\sin\phi = \epsilon_\theta R(\bar{a} + \sin\phi) \tag{48}$$

and

$$\bar{\bar{w}} = w\cos\phi + v\sin\phi. \tag{49}$$

Using the constitutive relationship

$$\epsilon_\theta = \frac{1}{Et}(N_\theta - \nu N_\phi)$$

we determine

$$\bar{u} = \frac{pR^2}{2\,Et}\left[\bar{a}(1 - 2\nu) + (1 - \nu)(\sin\phi)\right]. \tag{50}$$

Rearranging Equation (48), we have

$$-w = \frac{\epsilon_\theta R(\bar{a} + \sin\phi)}{\sin\phi} - \frac{v\cos\phi}{\sin\phi}.$$

Therefore, from Equation (49)

$$\bar{\bar{w}} = -\epsilon_\theta R(\bar{a} + \sin\phi)\frac{\cos\phi}{\sin\phi} + v\frac{\cos^2\phi}{\sin\phi} + v\sin\phi$$

which reduces to

$$\overline{w} = -\overline{u} \cot \phi + \left(\int \frac{f(\phi)}{\sin \phi} \, d\phi + C \right) .$$

Substituting Equation (48) for $\overline{u}$ and the expression for $f(\phi)$ in Equation (44), we have

$$\overline{w} = \frac{pR^2}{Et} \left\{ -\frac{1}{2} \, \overline{a} (1 - 2\nu) \cot \phi - \frac{1}{2} (1 - \nu) \cos \phi \right.$$

$$\left. + \frac{-\overline{a}^2 (1 - 2\nu)}{2} \int \frac{d\phi}{\sin^2 \phi \, (\overline{a} + \sin \phi)} + \nu \, \overline{a} \int \frac{d\phi}{\sin \phi \, (\overline{a} + \sin \phi)} + C . \right\}$$

Integrating the above expression results in the following equation.

$$\overline{w}(\phi) = \frac{pR^2}{Et} \left\{ \frac{1}{2} \ln \left| \tan \frac{\phi}{2} \right| - \frac{1}{2} (1 - \nu) \cos \phi \right.$$

$$\left. - \frac{1}{(\overline{a}^2 - 1)^{1/2}} \arctan \left[\frac{1 + \overline{a} \tan \frac{\phi}{2}}{(\overline{a}^2 - 1)^{1/2}} \right] + C \right\}$$

Now we know from symmetry that $\overline{w}(\pi/2) = \overline{w}(-\pi/2) = 0$. Utilizing these boundary conditions to solve for C, we arrive at the final form of the equation.

$$\overline{w} = \frac{pR^2}{Et} \left\{ \frac{1}{2} \ln \left| \tan \frac{\phi}{2} \right| - \frac{1}{2} (1 - \nu) \cos \phi \right.$$

$$- \frac{1}{(\overline{a}^2 - 1)^{1/2}} \arctan \left[\frac{1 + \overline{a} \tan \frac{\phi}{2}}{(\overline{a}^2 - 1)^{1/2}} \right] \tag{51}$$

$$\left. \pm \frac{1}{(\overline{a}^2 - 1)^{1/2}} \arctan \left[\left(\frac{\overline{a} + 1}{\overline{a} - 1} \right)^{1/2} \right]^{\pm 1} \right\}$$

where the upper sign holds for $+\phi$'s and the lower sign holds for $-\phi$'s.

To determine the angle of rotation of a tangent to the meridian at a given point on the toroidal shell, Equation (47) is used. Direct substitution of Equations (A-31), (44), (A-34b), and (43b) and differentiation with respect to ϕ results in the following expression.

$$V = \frac{pR}{Et} \left[\frac{\bar{a}}{2(\bar{a} + \sin \phi)} \right] \cot \phi \tag{52}$$

Rewriting Equations (50), (51), and (52), we have for a circular toroidal shell under uniform pressure (linear membrance analysis) the following displacements expressions as defined by Figure 9.

$$\bar{u} = \frac{pR^2}{2Et} \left[\bar{a}(1 - 2\nu) + (1 - \nu)(\sin \phi) \right] \tag{50}$$

$$\bar{w} = \frac{pR^2}{Et} \left\{ \frac{1}{2} \ln \left| \tan \frac{\phi}{2} \right| - \frac{1}{2}(1 - \nu) \cos \phi - \frac{1}{(\bar{a}^2 - 1)^{1/2}} \arctan \left[\frac{1 + \bar{a} \tan \frac{\phi}{2}}{(\bar{a}^2 - 1)^{1/2}} \right] \right.$$

$$\left. \pm \frac{1}{(\bar{a}^2 - 1)^{1/2}} \arctan \left[\left(\frac{\bar{a} + 1}{\bar{a} - 1} \right)^{1/2} \right]^{\pm 1} \right\} \begin{matrix} \text{Upper Sign } +\phi \\ \text{Lower Sign } -\phi \end{matrix} \tag{51}$$

$$V = \frac{pR}{Et} \left[\frac{\bar{a}}{2(\bar{a} + \sin \phi)} \right] \cot \phi \tag{52}$$

We note that the equations for the deflection $\bar{w}$ and the rotation V become discontinuous at $\phi = 0°$. At first glance, the singularity of these displacements at toroidal crowns would appear to invalidate the linear membrane solution; however, it will be shown in Section VI that this solution may be considered approximately correct for the N_ϕ, N_θ, and $\bar{u}$ terms for the entire shell. The displacements $\bar{w}$ and V are also approximately correct except near the crown.

It is these particular singularities that has led to the more "exact" nonlinear solution as the linear membrane assumptions described in this section appear to break down at least in the neighborhood of the shell crowns.

B. Other Shapes and Loadings

Similar displacement equations for toroidal shells supported as in Reference 1 and this report may be calculated in the same manner as above for both liquid and pressure loading. These equations will all have similar singularities at the crowns but may be considered approximately correct elsewhere.

SECTION VI. NONLINEAR APPROACH FOR "EXACT" TOROIDAL SOLUTIONS

A. General Toroidal Loadings, Support Conditions and Ranges of Solutions

We have assumed thus far in our linear membrane analysis that the only loads acting on the toroidal tank are uniform pressure and a fluid loading which are supported by membrane forces alone. This particular fluid loading requires both a center support and a lower skirt which have nearly the same stiffness (Figures 1 and 2).

In actuality, the toroidal shell is capable of resisting varied loadings in a manner which is unique in shell structures and in many ways unique in missile design. To better define these general loadings and support conditions, the following summary is presented.

1. Uniform Internal Pressure Loading

   No support is required.

2. Axial Load

   Loads are applied uniformly around the inner radius (radius b_1 in Figures 1 and 2) and reacted uniformly around the outer radius (radius a + b in Figures 1 and 2) or vice versa. The support condition is implicit in the loading.

3. Fluid Loading

   Loads are applied by filling the tank with liquid. The support conditions vary and can be defined as follows:

   (a) Support on both inside and outside radii.

   (b) Support on outside radius only.

   (c) Support on inside radius only.

   (d) Support on both inside and outside radii, but differential support deflections occur. This loading is the general load condition with loadings (b) and (c) as the limiting conditions and loading (a) as a special case.

There are three ranges of "exact" solutions for each of the three types of loadings and an approximate solution for loadings 1 and 3a. The ranges of solutions are

- Nonlinear membrane range (Range 1, exact solution),
- Transition range (Range 2, exact solution),
- Linear bending range (Range 3, exact solution),
- Linear Membrane Range (Range 4, approximate solution).

An explanation of the "Ranges of Solutions" will follow.

The loadings defined so far by Reference 1 and this report can be classified as toroidal loading 1, range 4, and toroidal loading 3a, range 4.

B. "Exact" Solution of Circular Toroidal Shell Under Uniform Pressure (Loading 1)

In Section V we noted that the linear membrane force equations for the circular toroidal shell under loading 1 (Equations (43b) and (44)) provided a continuous solution to the stress problem; however, when these force equations were used to solve the linear membrane deformation equations (Equations (50), (51), and (52)), singularities developed at the toroidal crowns in Equations (51) and (52). The incompatibility of the deflection $\overline{w}$ and the rotation V at the toroidal crown appears to nullify the concept of the ideal membrane; whereas, all the conditions for an ideal membrane exist.

The first investigation of this phenomenon was carried out in Reference 6. This report concluded that no suitable membrane solution existed, and that additional bending moments and shears must be added to the toroidal crown to provide compatibility. Classical publications such as References 3 and 4 have agreed with Reference 6 that bending stresses must invariably arise at the toroidal crowns.

The first paper to dispute the linear bending solution was given by Jordon (Ref. 7). He contended that this solution seems to admit a discontinuity exists at the toroidal crown, but such discontinuity effects usually involve abrupt changes of curvature, thickness, temperature, or stiffness. No such discontinuities exist at the toroidal crowns, and also in a very thin toroidal membrane, as examined in Reference 7, no appreciable bending or shear stiffness is present.

Jordon contends that the answers for a very thin pressurized toroidal membrane lie in a nonlinear membrane analysis (Range 1); i. e., the equilibrium equations are derived from the deformed shell. He then proceeds to develop differential equations based on the deformed shape, and solving these equations numerically, a compatible stress and deflection picture results for the very thin pressurized torus. The solutions of Reference 7 were later verified by an asymptotic approximation of the same basic problem in Reference 8.

The singularities of the linear membrane deformation equations now become obvious. The toroidal shell acts nonlinearly upon pressurization; i. e., it does not remain circular and the horizontal tangent to the toroidal crown no longer remains at $x = 0$ (Figure 10).

As the radius of curvature of the toroidal shell changes in the deformed shape (particularly in the crown area), membrane walls with normal thicknesses tend to develop bending stresses in addition to nonlinear membrane stresses. Reissner (Ref. 9) was the first to characterize the full range of solutions for the circular toroid under uniform internal pressure. He recognized the nonlinear membrane solution, range 1 (Ref. 7 and 8) as the lower bound and the linear bending solution, range 3 (Ref. 10 by Clark utilizes linear bending only to provide compatibility at the toroidal crown) as the upper bound. He then denotes the range between the upper and lower bounds in which both nonlinear membrane theory and the linear bending theory are both applicable as the transition range, range 2. Reissner also develops parameters to determine which range is valid for a particular problem.

Additional work in the transition range has been performed by References 11, 12, and 13. In these reports asymtotic approximations allow the linear membrane solution to become asymptotic to the nonlinear membrane-linear bending solution at the toroidal crown. References 12 and 13 will be discussed in more detail in Paragraph C of this section.

Investigation has proven that the stresses determined from "exact" analyses do not control design; therefore, for the final design of a circular

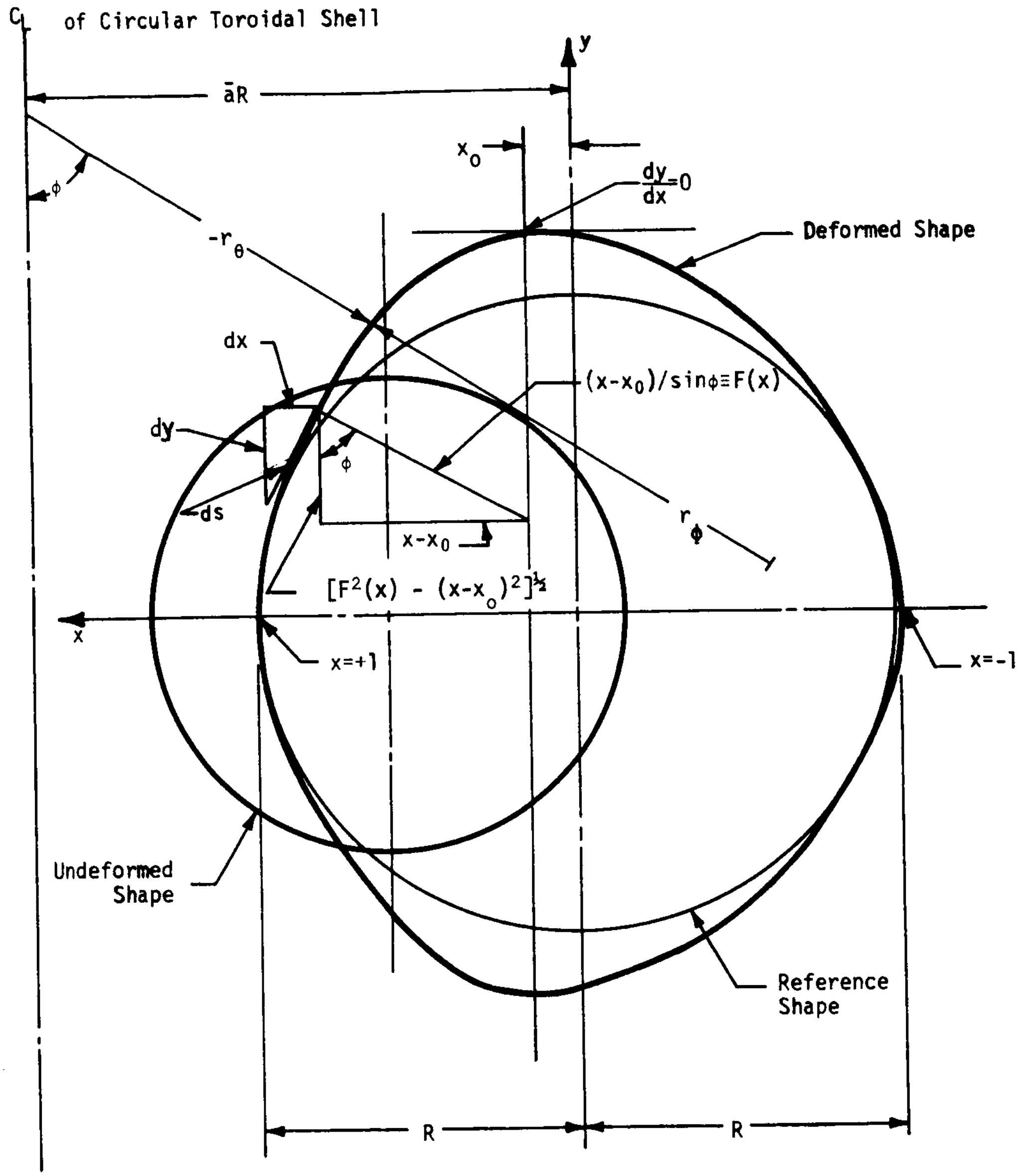


Figure 10. Circular Toroidal Shell under Uniform Pressure (Loading 1)

toroidal shell under loading 1, the equations of Reference 1 and this report are sufficient. However, nonlinearities cannot be ignored in loading 1 as they are necessary to complete the stress and deflection picture and also for the derivation of solutions to toroidal loading 2.

C. "Exact" Solution of Circular Toroidal Shell Under Axial Load (Loading 2)

As described in Paragraph A of this section, the term axial load denotes a uniform load applied around the inner radius and reacted uniformly around the outer radius or vice versa (Figure 11). This load may be thought of as an engine thrust load or the support of a spheroidal tank hung at its inner radius (Figure 12). This particular configuration would be ideal for a transtage where the overall length of the stage must be kept short but full utilization of fuel storage space is required.

The most startling aspect of the toroidal shell was first described by Jordan in Reference 7. He stated that the pressurized torus membrane is capable by adjusting its shape of transferring shear loads from the inner to the outer radius even though the membrane itself has no shear stiffness of its own. In essence, the pressure rather than the shear strength of the material will take care of the shear load. This solution of loading 2 represents a range 1 solution.

In References 12 and 13, he continues this analysis through the transition range. In this range, the shear load is resisted by both the pressure stiffness and bending stiffness of the shell. Reference 10 derives the linear bending range. An asymptotic series solution is used in References 10, 12, and 13.

The unique aspect of the toroidal shell resistance to loading 2 is that it is basically a nonlinear reaction and the shell resists the load by its tendency to roll in or out upon the application of the load. There is no solution in the linear membrane theory for this type of load reaction.

D. "Exact" Solution of Circular Toroidal Shell Under Fluid Loading (Loading 3)

No "exact" solutions have been developed for any of the fluid loadings on circular toroidal shells. An approximate solution to loadings 3b, 3c,

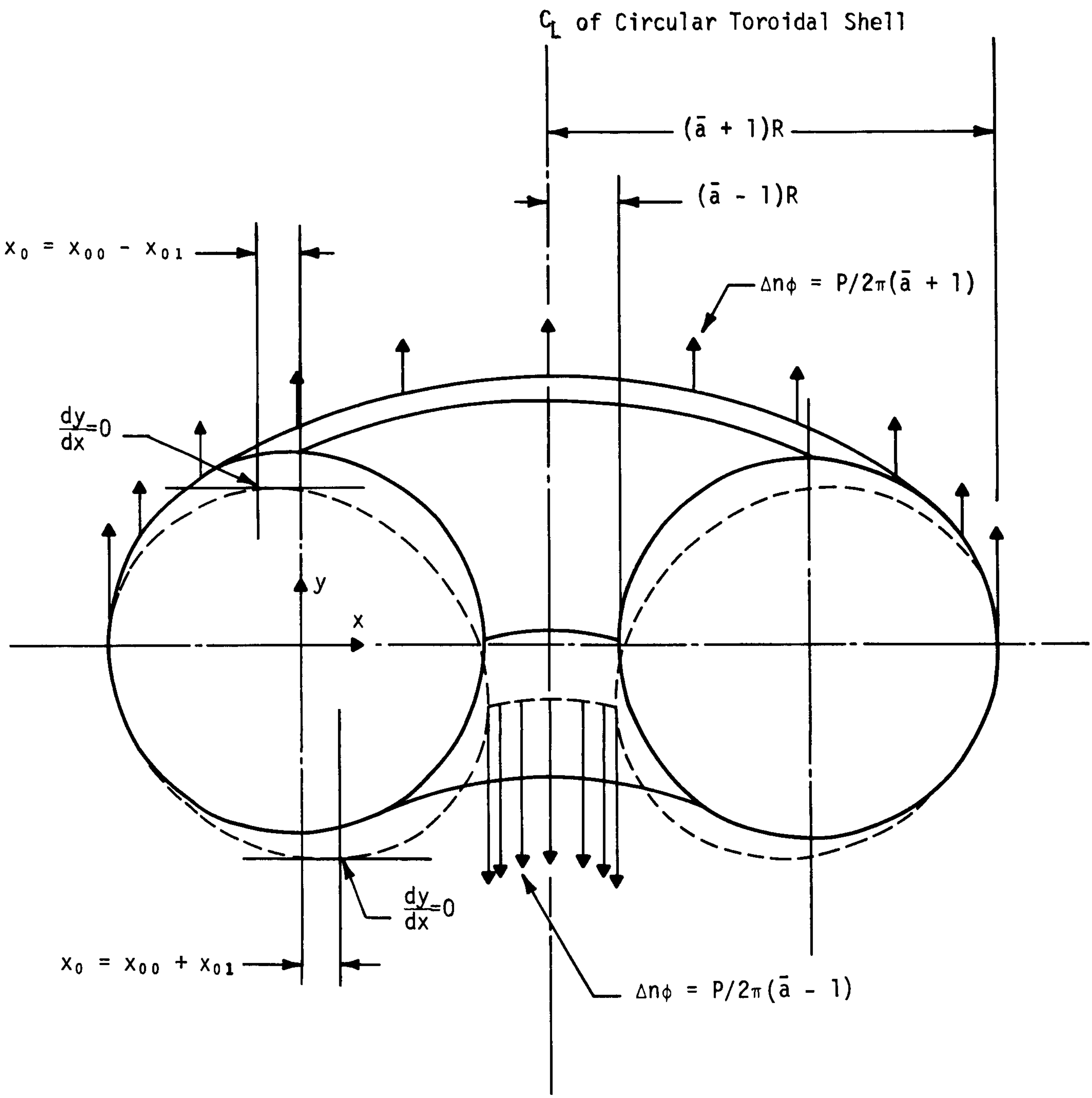


Figure 11. Circular Toroidal Shell under Axial Load (Loading 2)

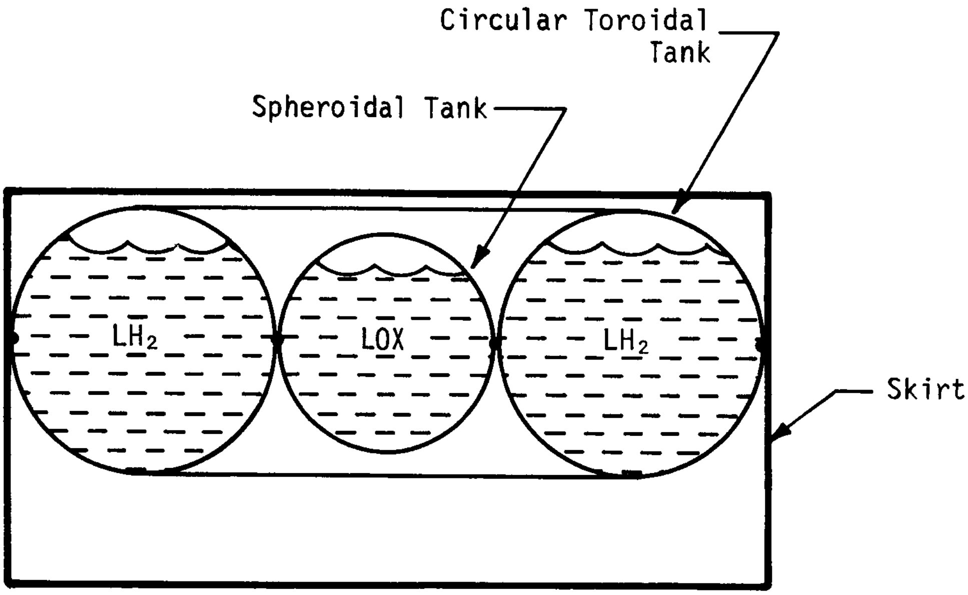


Figure 12. Possible Transtage Design, Circular Toroidal Tank Subjected to Loadings 2 and 3b

and 3d is now possible from References 7, 12, and 13. With respect to loading 3a, the nonlinear crown effects are very similar to loading 1 and do not control design; therefore, the equations of this report and Reference 1 are valid for final design.

E. Mathematical Formulation of an "Exact" Solution

In Paragraph A of this section we noted the varied loading conditions and ranges of solutions that are possible for a toroidal shell. As one might expect, mathematical difficulties arise as an exact solution is approached.

A complete derivation of Reference 7 was performed by the author along with a principal amount of the derivations of References 11, 12, and and 13. Considerable knowledge of the "exact" approach and its effects was gained and general extensions to these theories now are possible. As to an extensive presentation of nonlinear technical data at this time, another report of at least the size of this publication would be necessary; therefore, a cursory examination will be made now to allow the reader to become familiar with exact techniques. A summary of "exact" and "approximate" solutions will be presented in Paragraph F of this section for clarity.

As mentioned in Paragraph B of this section, Reference 7 was the first paper to present the toroidal crown problem as a nonlinear membrane solution. The clarified essentials of this report will now be presented.

The analysis is first made nondimensional by the introduction of the reference length R in Figure 10. A second order error is also introduced by assuming the radius of the deformed shape at the x axis as equal to R. From basic shell relationships, the following nondimensional equilibrium and displacement equations for the deformed shape are derived.

$$n_\theta = \left[(\bar{a} - x)/(\sin\phi)\right] \left[(R/r_1)\, n_\phi - 1\right] \quad \text{(From Ref. 7)} \qquad (53)$$

$$n_\phi = \left[(x - x_0)/\sin\phi\right] \left\{(2a - x - x_0)/\left[2(\bar{a} - x)\right]\right\} \quad \text{(From Ref. 7)} \qquad (54)$$

$$u = (n_\phi - \nu\, n_\theta)k \quad \text{(From Ref. 7)} \qquad (55)$$

$$v = (\bar{a} - x)\,(n_\theta - \nu\, n_\phi)k \quad \text{(From Ref. 7)} \tag{56}$$

$$k = \frac{pR}{Et} \quad \text{(From Ref. 7)} \tag{57}$$

where n_θ is the hoop stress function, n_ϕ is the meridional stress function, u is the strain of the meridional element ds, v is the increase of the radius $(\bar{a} - x)$ of the horizontal circle, and k is a reference strain.

The report then states why linear membrane theory does not provide a continuous solution through the crown area; the linear theory does not allow an unsymmetrical shell with respect to the y-axis or a provision for a shift in the position of the horizontal tangent $\left(\frac{dy}{dx} = 0\right)$ from $x = 0$. The author then states that two additional degrees of freedom are provided in going from a linear to a nonlinear approach and in the use of the open parameter x_0 (Figure 10); thus, a membrane solution to the toroidal shell should exist.

As a result of analytical procedures, the primary unknown in the solution of this problem is the shape function F(x) (Figure 10). The equilibrium and displacement equations can then be modified to read

$$2n_\theta = F(x) - (2\bar{a} - x - x_0)\, F'(x) \quad \text{(From Ref. 7)} \tag{58}$$

$$2n_\phi = \left[1 + (\bar{a} - x_0)/(\bar{a} - x)\right] F(x) \quad \text{(From Ref. 7)} \tag{59}$$

$$\frac{F^2(x)}{F^2(x) - (x - x_0)^2} = \left[1 + 2\,(u + \nu')\right] \left(\frac{ds*}{dx*}\right)^2 * \quad \text{(From Ref. 7)} \tag{60}$$

where the prime indicates differentiation with respect to x.

The following assumptions, which were later proven correct, are the most important with respect to a general solution of the formulation. If the deformed shape is a circle, we would obtain $F(x) \equiv 1$ and $x_0 = 0$. The assumptions are then made that

$$F(x) \equiv 1 - kf \quad \text{(From Ref. 7)} \tag{61}$$

where

$$f = 0(1), \; x_0 = 0(k).$$

* $\frac{ds*}{dx*}$ represents the original undeformed shape.

This allows the deformed shape to vary only slightly from the reference shape. The substitution of the above expression for F(x) into the equilibrium and displacement equations and the neglecting of terms of order $0(k^2)$ results in the following differential equation.

$$2fx^2 + kG(f', f'') = (1 - x^2)\alpha + 2x(x_0/k) \tag{62}$$

(From Ref. 7)

where

$$G(f', f'') \equiv (1 - x^2)(\beta f' - \gamma f'') - (x/2)\ \Big[2\gamma f' - (1 + x)(\gamma f')_{x=-1} - (1 - x)(\gamma f')_{x=-1}\Big]$$

The numerical solutions of the function f and parameter x_0 determine all stresses and deformations of the circular toroid under loading 1, range 1.

The solution to the circular toroid under loading 2, range 1 (Figure 11) is also obtained from the preceding differential equation by allowing the total load P applied at the inner and outer rim to act as a Δn_ϕ force to the n_ϕ force due to pressure alone. This results in new values of x_0 (Figure 11). In Figure 11, x_{00} is caused by pressure and x_{01} is caused by the axial load P.

In References 12 and 13, Jordan develops the more complete differential equation

$$fx^2 - \left\{ k\ m(f) - \frac{\bar{t}^2}{\bar{\nu}}\ b(xf) \right\}\ (1 - x^2) = \frac{k\bar{a}}{2(\bar{a} + x)}\ (1 - x^2) + xx_0 \tag{63}$$

where m(f) and b(f) are the contributions of both pressure (membrane) stiffness and bending stiffness. Thus with References 7, 12, and 13 one is able to solve the circular toroidal shell under loadings 1 and 2, ranges 1, 2, and 3.

F. Summary

At this time only circular toroidal bulkheads have been investigated in the "exact" range; however, it is possible to extend References 12 and 13 to include other shapes.

The effect of separating the toroidal bulkheads by cylindrical sections as in the semi-toroidal tank (Figure 13) should not affect the "exact" toroidal analyses. The only appreciable effect of the cylindrical portions would be basic discontinuity effects that occur locally at the bulkhead-cylindrical junctures.

To summarize the various "exact" and "approximate" solutions and also determine the validity of these solutions, the following outline is presented (the loadings and ranges of solutions are designated in Paragraph A. of this section).

1. Circular Toroidal Configuration

a. Loadings and Ranges

(1) Loading 1, Ranges 1, 2, and 3 (Exact Solution Range). Completely solved by References 7, 12, and 13 but not in readily usable form. The stresses determined by these analyses do not control design.

(2) Loading 1, Range 4 (Approximate Solution). Completely solved by Reference 1 and this report. Computer program available. Valid for final design.

(3) Loading 2, Ranges 1, 2, and 3, (Exact Solution Range). Completely solved by References 7, 12, and 13. Not in readily useable form.

(4) Loading 2, Range 4 (Approximate Solution). No linear membrane solution is possible.

(5) Loading 3a, Ranges 1, 2, and 3 (Exact Solution Range). No "exact" solution available but nonlinear crown effects are very similar to loading 1 and do not control design.

(6) Loading 3a, Range 4 (Approximate Solution). Completely solved by Reference 1 and this report. Computer program available. Valid for final design.

(7) Loadings 3b, 3c, 3d, Ranges 1, 2, and 3 (Exact Solution Range). Approximately solved by References 7, 12, and 13. Not in readily useable form.

(8) Loadings 3b, 3c, 3d, Range 4 (Approximate Solution). No linear membrane solution is possible.

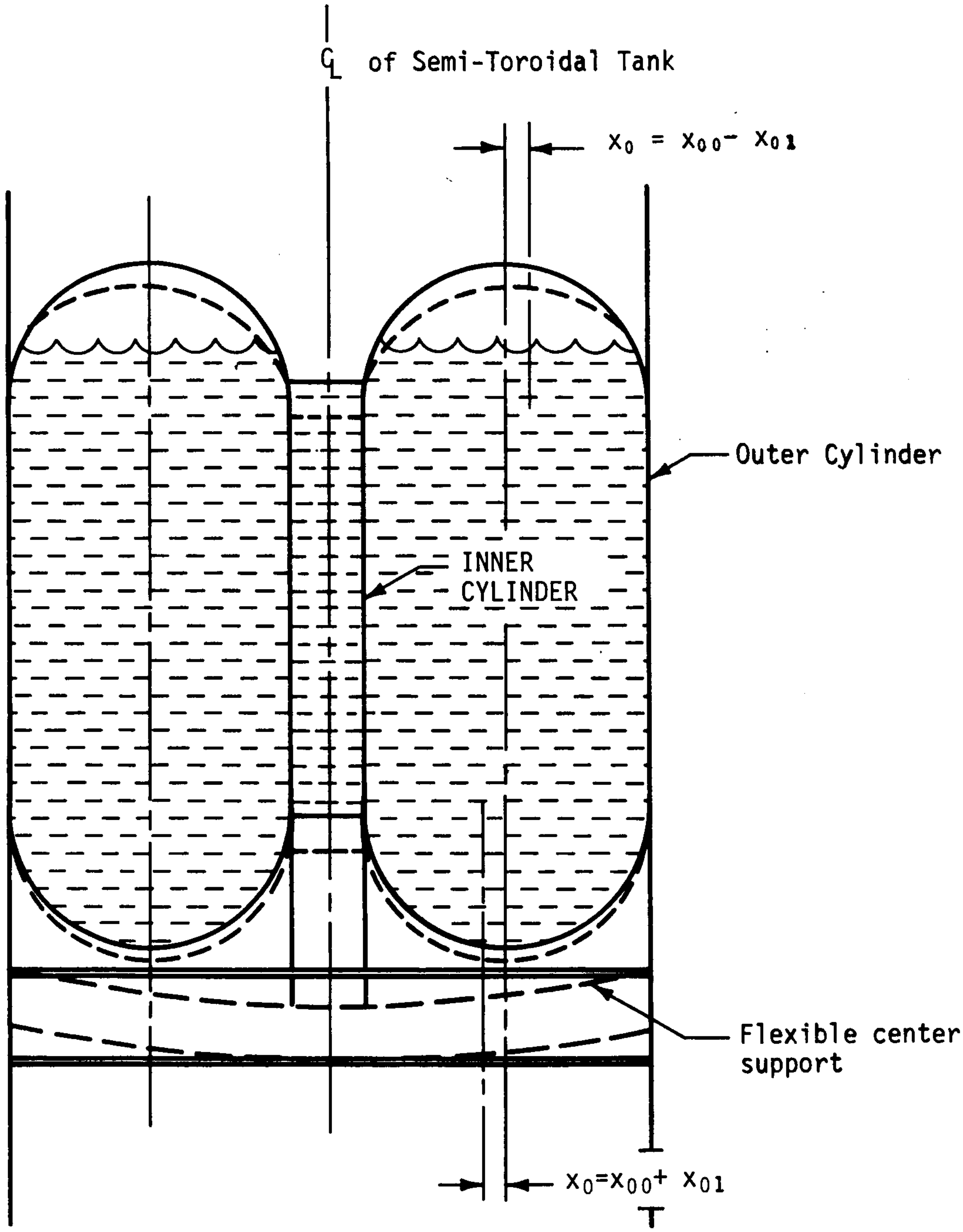


Figure 13. Semi-Toroidal Tank under Fluid Loading (Loading 3d)

2. Semi-Toroidal Configuration (Elliptical Toroidal and Modified Ogival Bulkheads)

a. Loadings and Ranges

(1) Loading 1, Ranges 1, 2, and 3 (Exact Solution Range). Partially solved by References 7, 12, and 13 for the toroidal heads only. The stresses determined by these analyses do not control design.

(2) Loading 1, Range 4 (Approximate Solution). Completely solved by Reference 1 and this report. Computer program available. Valid for final design.

(3) Loading 2, Ranges 1, 2, and 3 (Exact Solution Range). Partially solved by References 7, 12, and 13 for the toroidal heads only.

(4) Loading 2, Range 4 (Approximate Solution). No linear membrane solution is possible.

(5) Loading 3a, Ranges 1, 2, and 3 (Exact Solution Range). No solutions available. Nonlinearities are similar to loading 1 and do not control design.

(6) Loading 3a, Range 4 (Approximate Solution). Completely solved by Reference 1 and this report. Computer program available. Valid for final design.

(7) Loadings 3b, 3c, and 3d, Ranges 1, 2, and 3 (Exact Solution Range). Approximately solved by References 7, 12, and 13. Not in readily usable form.

(8) Loadings 3b, 3c, and 3d, Range 4 (Approximate Solution). No linear membrane solution is possible.

SECTION VII. CONCLUSIONS AND RECOMMENDATIONS

With the completion of this report, the present study of the toroidal configuration is terminated with the exception of the planned testing of a semi-toroidal and circular toroidal test tank.

Several important milestones were reached during this investigation. With respect to original work, the derivation of the equations which handle both uniform pressure and certain fluid effects simultaneously, the determination of hoop compression parameters and the development of the general computer program are the main accomplishments. Also, the classification of general toroidal loadings and ranges of solutions and the summarizing of the state-of-the-art is important.

From a design standpoint, Reference 2 indicated that the semi-toroidal configuration compares very favorably with existing configurations such as the common bulkhead concept. Reference 2 was conducted to establish a minimum weight semi-toroidal (Figure 14) or modified ogival (Figure 15) stage configuration based on existing S-II stage (Figure 16) design criteria. There was no intention of replacing the S-II stage, but to provide a proven design with which to compare these new stage concepts. Figure 17 indicates the final optimized semi-toroidal configuration that resulted from this study. Figure 18 shows the "ideal" weights of the various configurations studied. Since no discontinuity effects, thermal stresses, weld lands, etc. were considered, the "ideal" weight was increased 15 percent. The final results indicated an S-II structural weight of 51 607 pounds as compared to the semi-toroidal stage weight of 46 600 pounds. Figure 19 compares the lengths of the toroidal configurations to the present S-II stage.

Analytically, there is still a great deal to be done; however, the work accomplished by Jordan (Ref. 7, 12, and 13) should be rated as the best with respect to the development of the necessary theories to handle still pressing engineering problems. Perturbations upon the asymptotic analyses of these reports should help clarify such problems as vibration, buckling under internal and external loads, and stiffening on the general toroidal configuration.

This present study of the toroidal configuration has been, through necessity, a mixture of both analysis (Ref. 1 and this report) and design (Ref. 2). Perhaps this is an advantage with respect to a new configuration in that it tends to temper analytical judgement with practical considerations. The author feels a good balance of both has been accomplished.

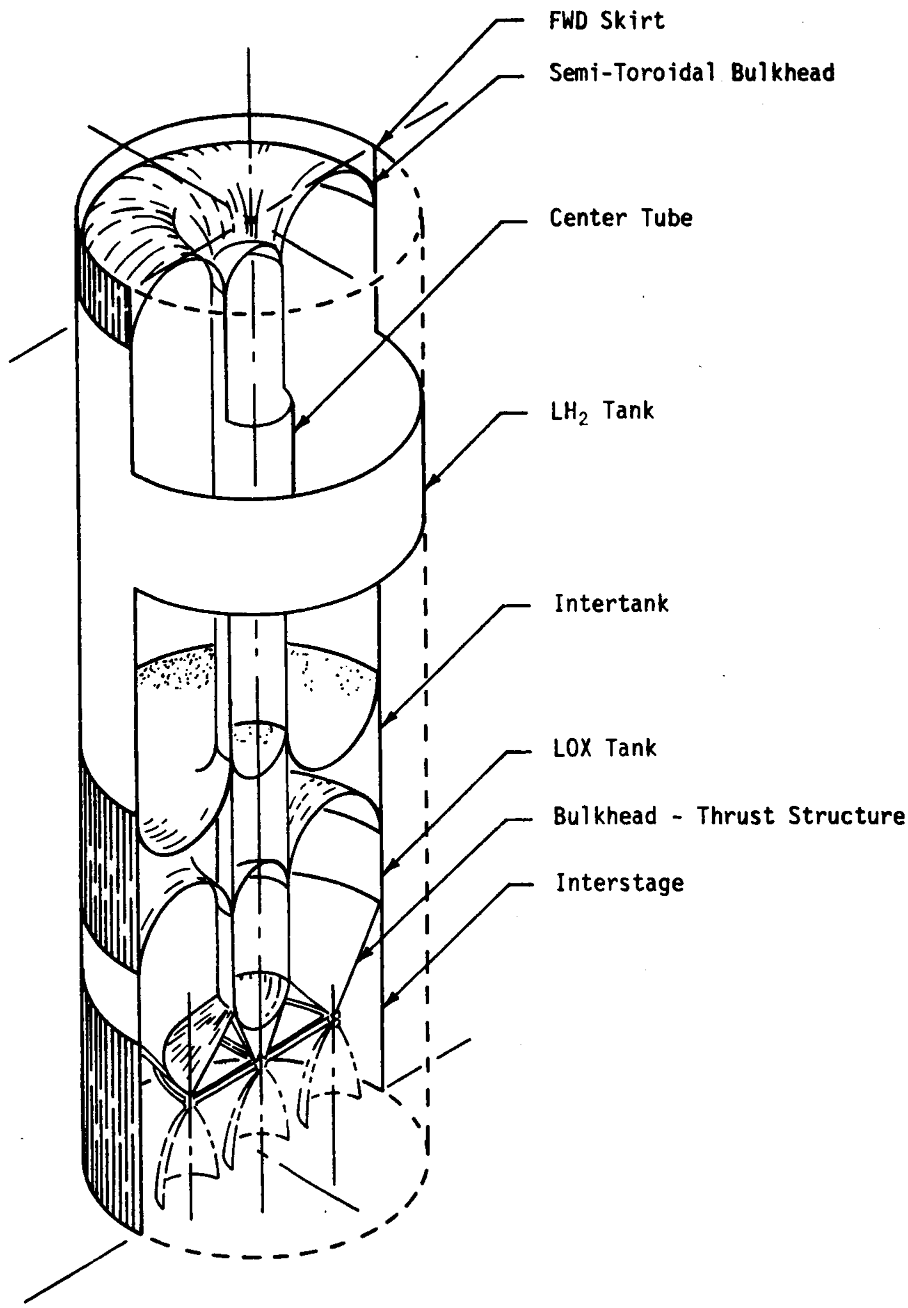


Figure 14. Semi-Toroidal Stage Configuration

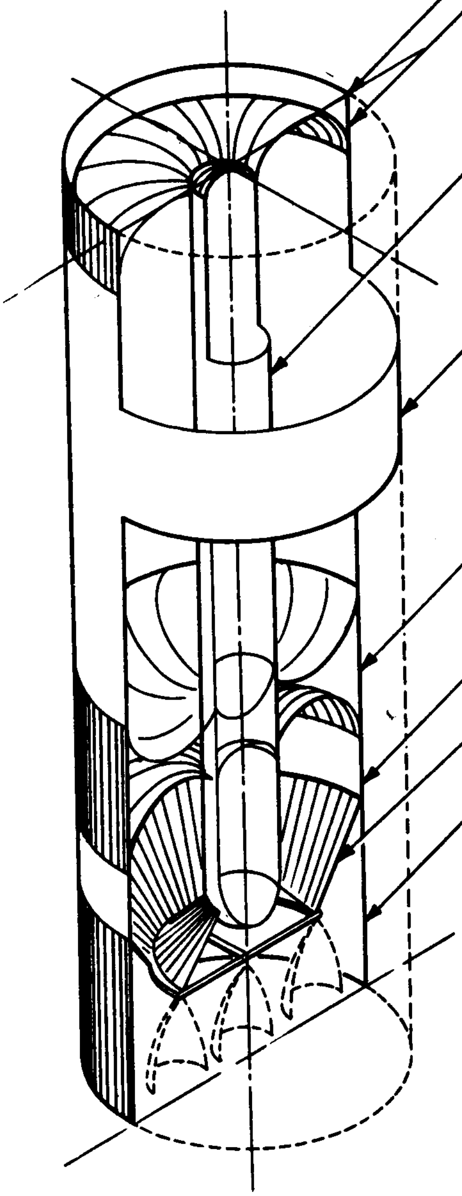
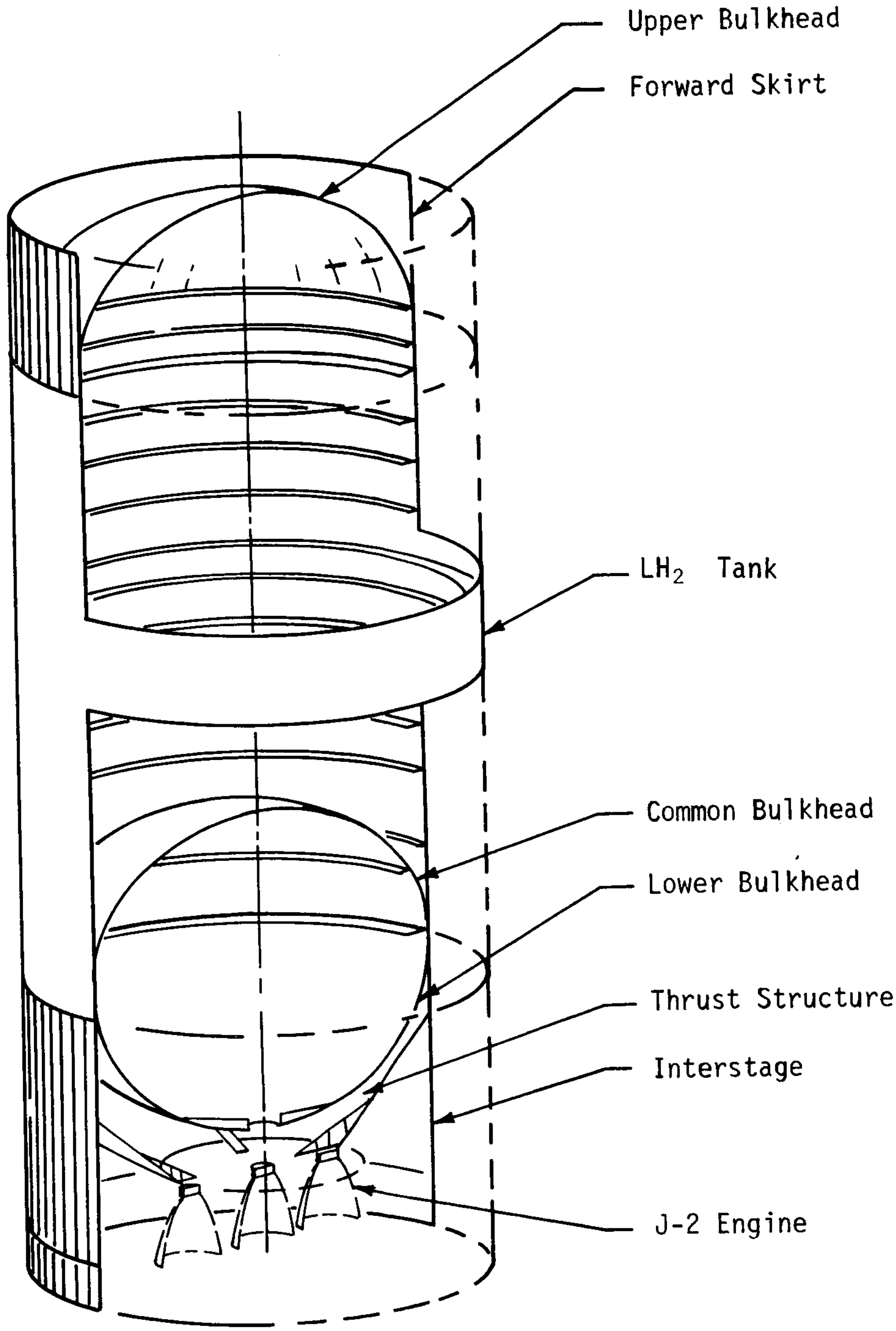
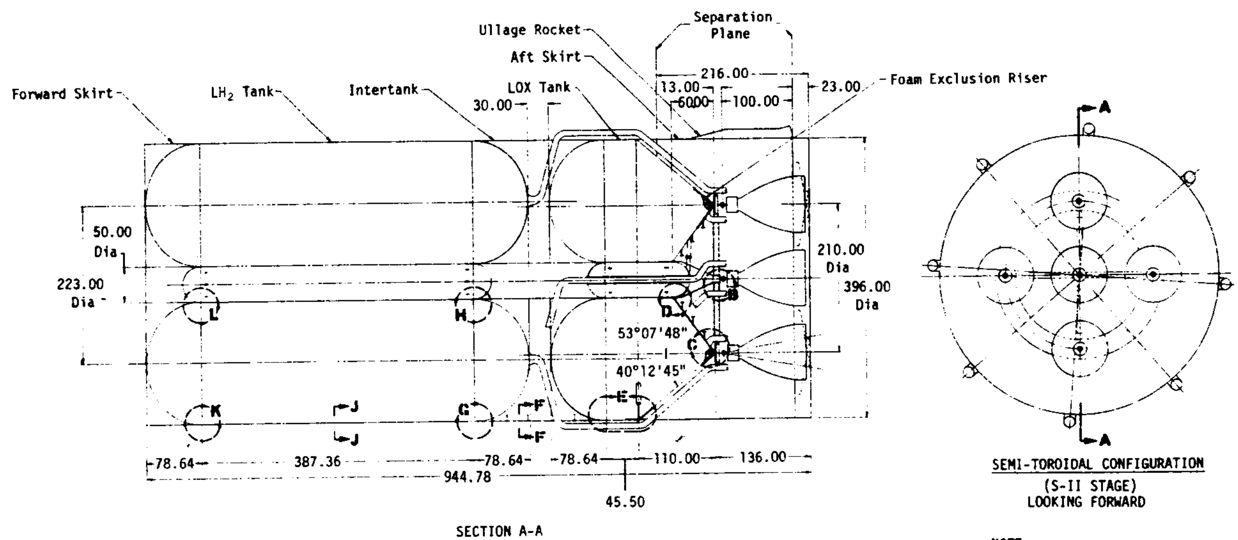


Figure 15. Modified Ogival Stage Configuration



S-II STAGE

Figure 16.



NOTE
THE OPTIMUM SEMI-TOROIDAL STAGE CONFIGURATION SHOWN IS BASED ON THE PRESENT S-II STAGE DESIGN REQUIREMENTS

Figure 17. Optimum Weight Semi-Toroidal Configuration

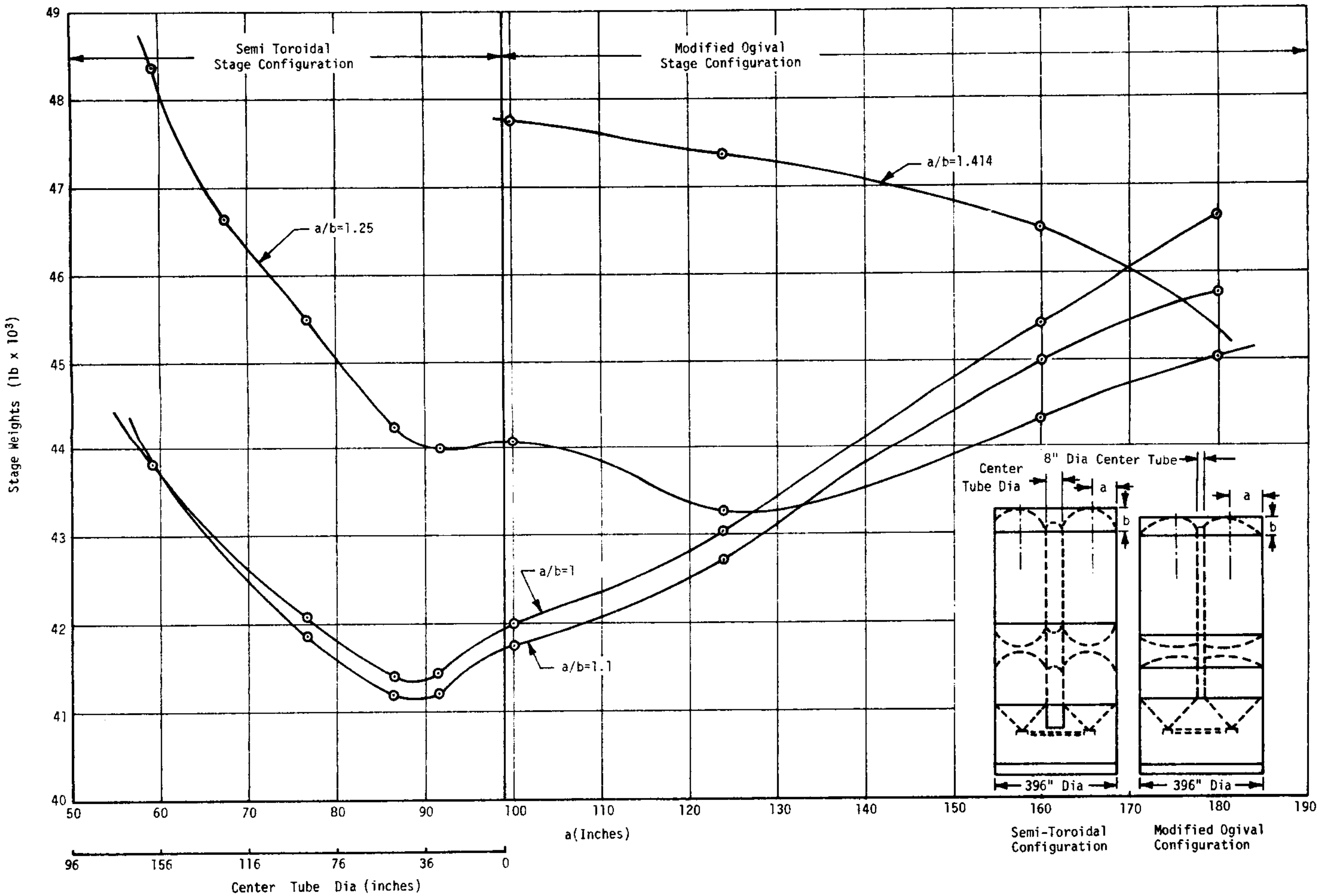


Figure 18. Vehicle Stage Weights Semi-Toroidal and Modified Ogival Configurations

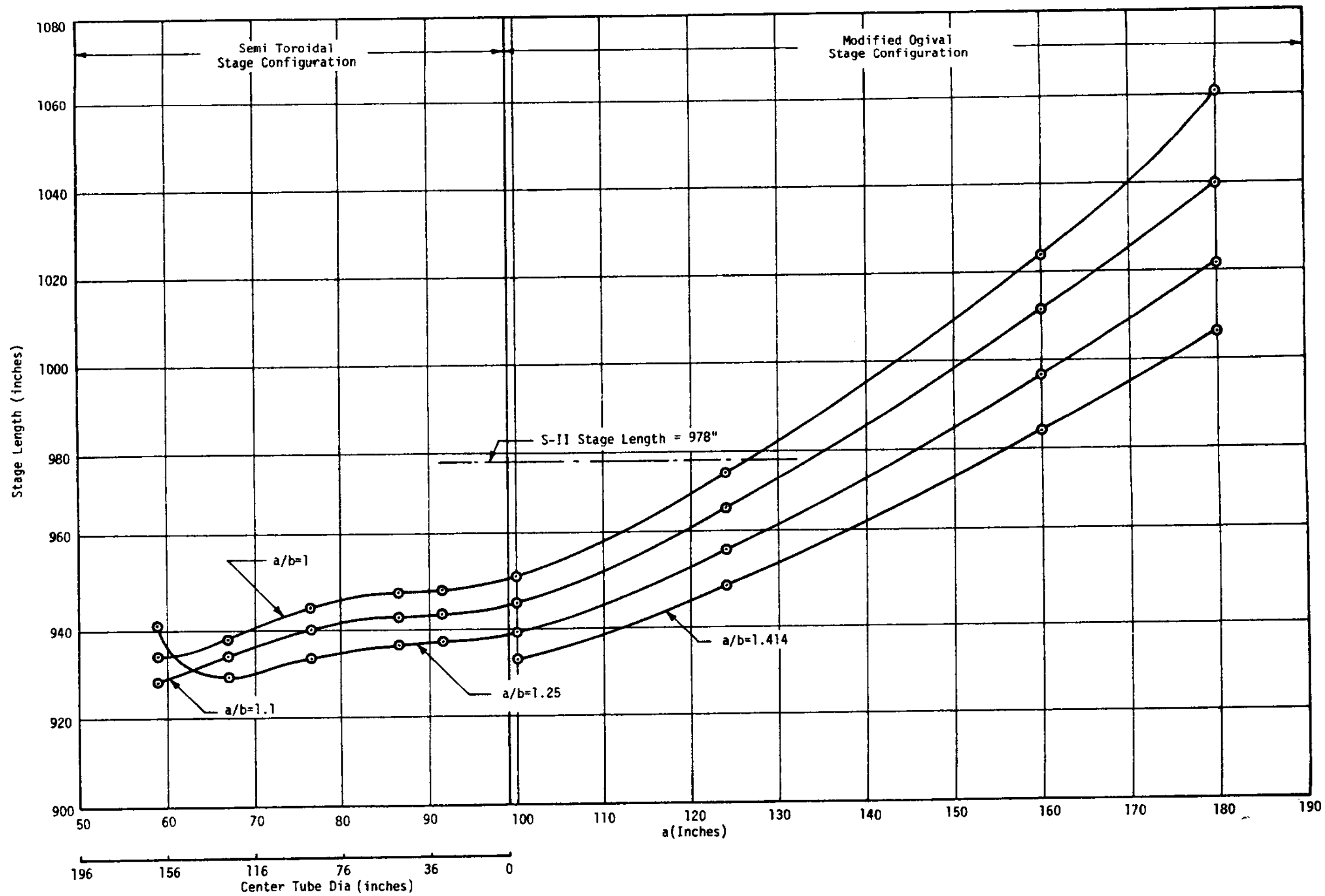


Figure 19. Vehicle Length Semi-Toroidal and Modified Ogival Stage Configuration

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APPENDIX

A. Computation of Partial Weights of Fluids

<u>For the upper eliptical toroidal bulkhead</u>

(Ref. Figure A.1, page 56)

$W^{\pm}_{p1}$ - Let $h_1 = H_2 - (\ell + c)$, $h_5 = z' - (\ell + c)$

<u>For the upper modified ogival bulkhead</u>

(Ref. Figure A.3, page 58)

W^{+}_{p1} - Let $h_1 = H_2 - (\ell_o + c)$, $h_5 = z' - (\ell_o + c)$

W^{-}_{p1} - Let $h_1 = H_2 - \left(\frac{\ell_o + \ell_i}{2} + c\right)$, $h_5 = z' - \left(\frac{\ell_o + \ell_i}{2} + c\right)$

$$W^{\pm}_{p1} = \frac{2ab\pi\gamma_2}{c}\int_{h_5}^{h_1} \left(c^2 - z^2\right)^{1/2} dz \pm \frac{\pi b^2\gamma_2}{c^2}\int_{h_5}^{h_1} (c^2 - z^2)\, dz$$

$$W^{\pm}_{p1} = \left\{ \frac{2ab\pi\gamma_2}{c} \left[\frac{h_1}{2}\left(c^2 - h_1^2\right)^{1/2} + \frac{c^2}{2}\sin^{-1}\left(\frac{h_1}{c}\right)\right] \pm \frac{\pi b^2\gamma_2}{c^2}\left(c^2 h_1 - \frac{1}{3}h_1^3\right)\right\}$$
$$- \left\{ \frac{2ab\pi\gamma_2}{c} \left[\frac{h_5}{2}\left(c^2 - h_5^2\right)^{1/2} + \frac{c^2}{2}\sin^{-1}\left(\frac{h_5}{c}\right)\right] \pm \frac{\pi b^2\gamma_2}{c^2}\left(c^2 h_5 - \frac{1}{3}h_5^3\right)\right\} \qquad \text{(A-1)}$$

The plus sign is used for W^{+}_{p1} and the minus sign for W^{-}_{p1} .

<u>For the lower elliptical toroidal bulkhead</u>

(Ref. Figure A.2, page 56)

$W^{\pm}_{p2}$ - Let $h_2 = H_2$ for $H_2 \leqq z'$, $h_2 = z'$ for $H_2 > z'$

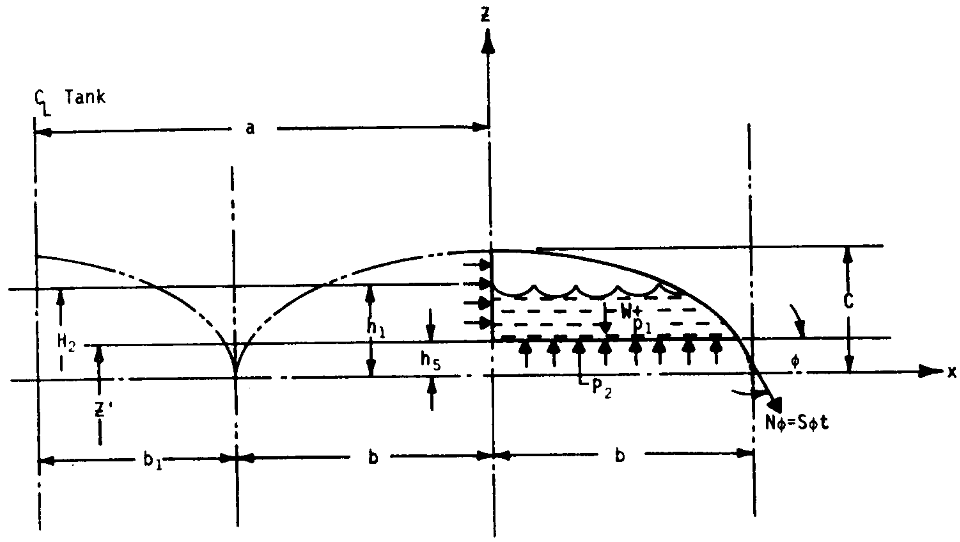


Figure A.1. Free Body of Portion of Upper Elliptical Toroidal Bulkhead

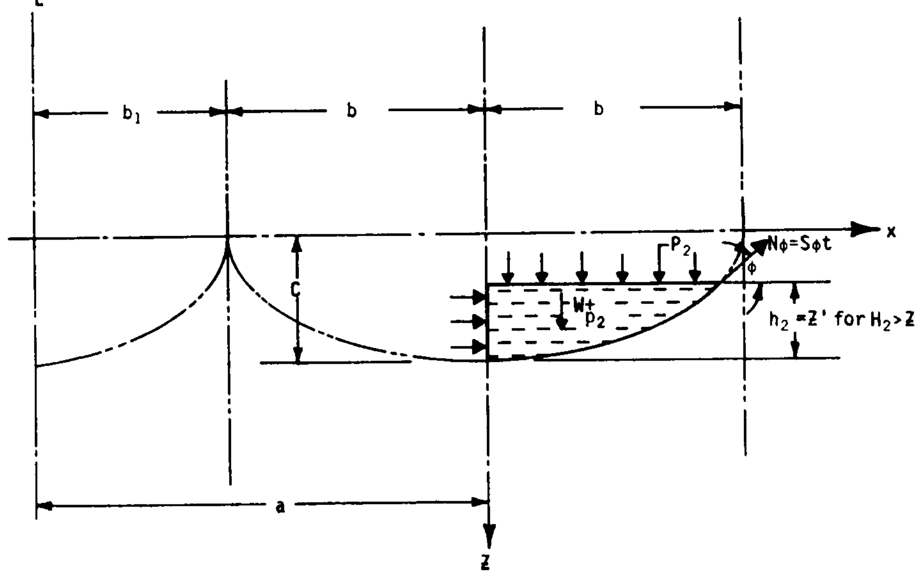


Figure A.2. Free Body of Portion of Lower Elliptical Toroidal Bulkhead

For the lower modified ogival bulkhead

(Ref. Figure A. 4, page 58)

W^{+}_{p2} - Let $h_2 = H_2$ for $H_2 \leqq z' \leqq c$, $h_2 = z' \leqq c$ for $H_2 > z'$

W^{-}_{p2} - Let $h_2 = H_2$ for $H_2 \leqq z' \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$,

$h_2 = z' \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $H_2 > z'$

$$W^{\pm}_{p2} = \frac{2ab\pi\gamma_2}{c}\int_{c-h_2}^{c} \left(c^2 - z^2\right)^{1/2} dz \pm \frac{\pi b^2\gamma_2}{c^2}\int_{c-h_2}^{c} (c^2 - z^2)\, dz$$

$$W^{\pm}_{p2} = \frac{2ab\pi\gamma_2}{c}\left[\frac{c^2\pi}{4} - \frac{c - h_2}{2}\left(2\,ch_2 - h_2^2\right)^{1/2} - \frac{c^2}{2}\sin^{-1}\left(\frac{c - h_2}{c}\right)\right]$$

$$\pm \frac{\pi b^2\gamma_2}{c^2}\left[-\frac{1}{3}c^3 + c^2h_2 + \frac{1}{3}(c - h_2)^3\right] \qquad \text{(A-2)}$$

The plus sign is used for W^{+}_{p2} and the minus for W^{-}_{p1} .

For the upper oblate spheroidal bulkhead

(Ref. Figure A. 5, page 60)

W_{p3} (semi-toroidal tank) - Let $h_3 = H_1 - (\ell + c)$, $h_6 = z' - (\ell + c)$

W_{p3} (mod. ogival tank) - Let $h_3 = H_1 - \left(\frac{\ell_o + \ell_i}{2} + c\right)$,

$h_6 = z' - \left(\frac{\ell_o + \ell_i}{2} + c\right)$

$$W_{p3} = \frac{\pi b_1^2\gamma_1}{c_1^2}\int_{h_6}^{h_3} (c_1^2 - z_1^2)\, dz_1 = \frac{\pi b_1^2\gamma_1}{c_1^2}\left[\left(c_1^2 h_3 - \frac{1}{3}h_3^3\right) - \left(c_1^2 h_6 - \frac{1}{3}h_6^3\right)\right]$$

(A-3)

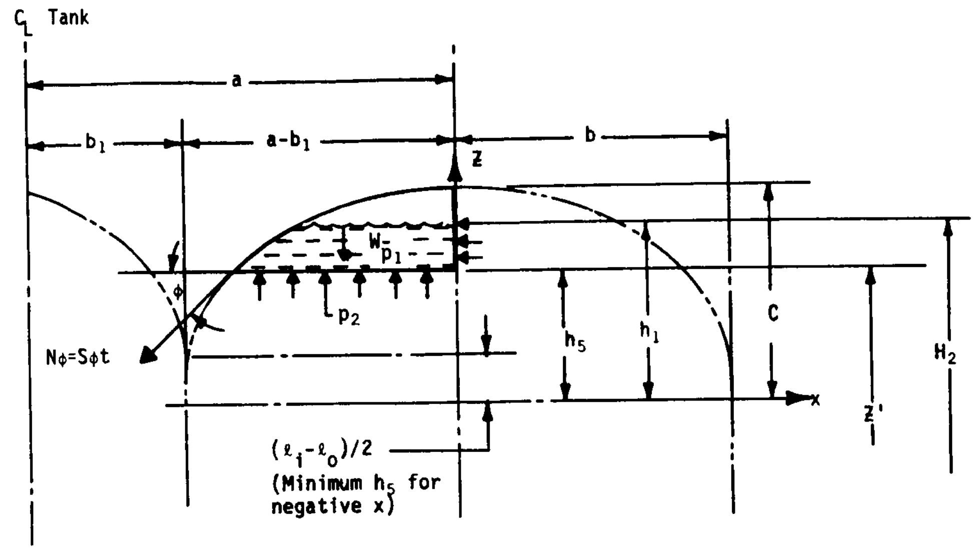


Figure A.3. Free Body of Portion of Upper Modified Ogival Bulkhead

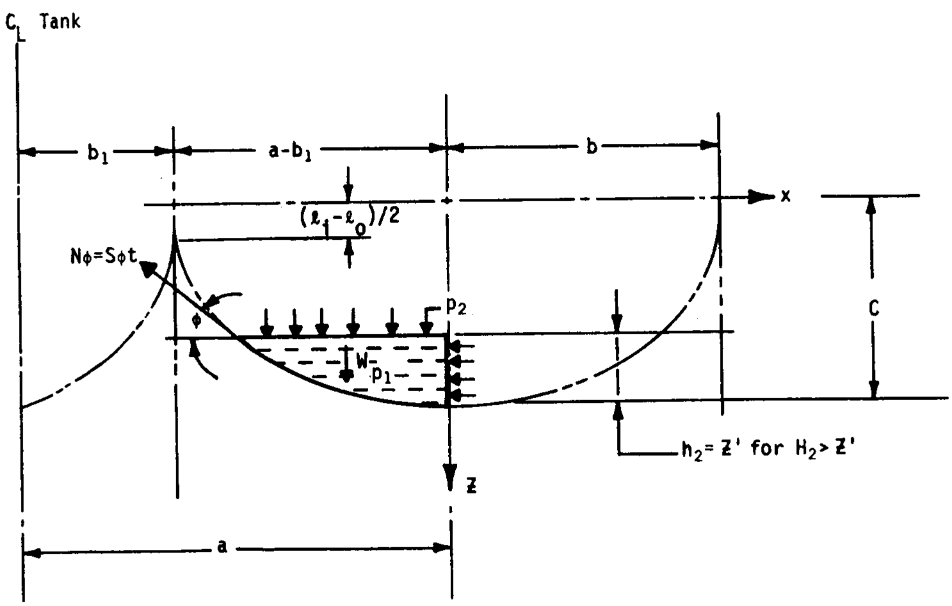


Figure A.4. Free Body of Portion of Lower Modified Ogival Bulkhead

For the lower oblate spheriodal bulkhead

(Ref. Figure A.6, page 60)

W_{p4} (semi-toroidal tank) - Let $h_4 = H_1 - (c - c_1)$ for $H_1 \leqq z'$,

$$h_4 = z' - (c - c_1) \text{ for } H_1 > z'$$

W_{p4} (modified ogival tank) - Let $h_4 = H_1 - \left(c - c_1 - \frac{\ell_i - \ell_o}{2}\right)$

$$\text{for } H_1 \leqq z \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right),$$

$$h_4 = z' - \left(c - c_1 - \frac{\ell_i - \ell_o}{2}\right) \text{ for } H_1 > z' \text{ and } z' \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$$

$$W_{p4} = \frac{\pi b_1^2 \gamma_1}{c_1^2} \int_{c_1 - h_4}^{c_1} (c_1^2 - z_1^2)\, dz_1 = \frac{\pi b_1^2 \gamma_1}{c_1^2} \left[-\frac{1}{3} c_1^3 + c_1^2 h_4 + \frac{1}{3} (c_1 - h_4)^3 \right] \tag{A-4}$$

To determine the weight W_{p5} carried by the center support, it is necessary to consider several cases of fluid height.

For $H_1 \leqq c$, $H_2 \leqq c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} \tag{A-5}$$

$h_4 = H_1 - (c - c_1)$ for W_{p4}

$h_2 = H_2$ for $W_{\bar{p}2}$

For $\ell + c \geqq H_1 > c$, $H_2 \leqq c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi \gamma_1 b_1^2 (H_1 - c) \tag{A-6}$$

$h_4 = c_1$ for W_{p4}

$h_2 = H_2$ for $W_{\bar{p}2}$

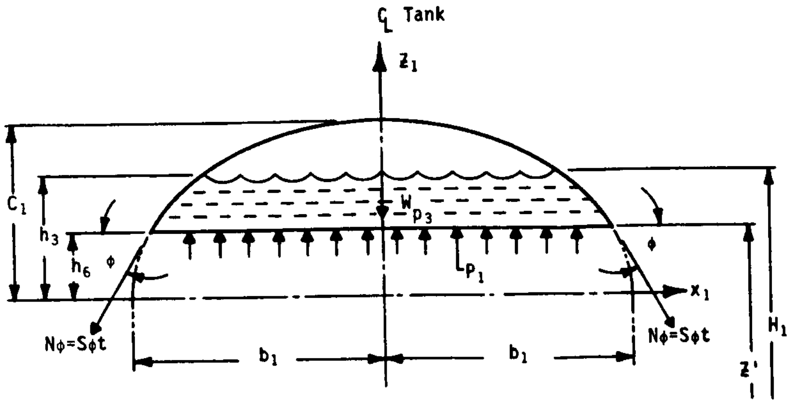


Figure A.5. Free Body of Portion of Upper Oblate Spheroidal Bulkhead

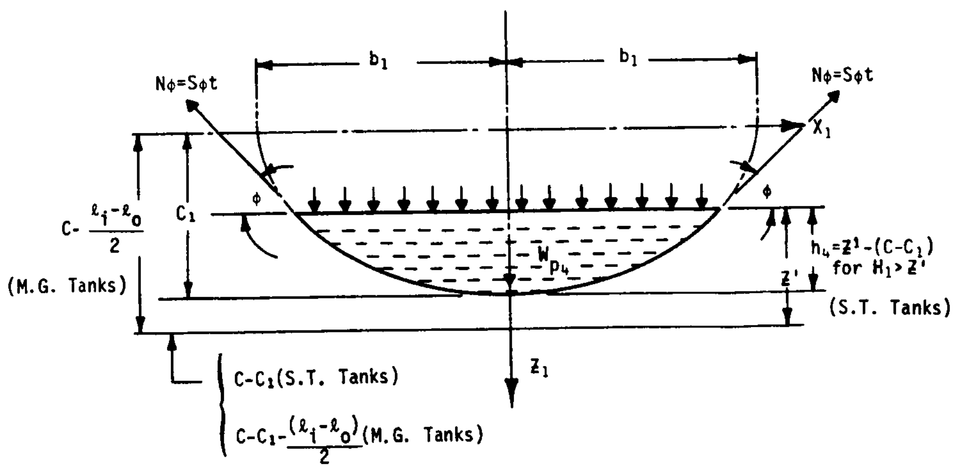


Figure A.6. Free Body of Portion of Lower Oblate Spheroidal Bulkhead

For $H_1 > \ell + c$, $H_2 \leqq c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2 \ell + W_{p3} \qquad \text{(A-7)}$$

$h_4 = c_1$ for W_{p4}

$h_2 = H_2$ for $W_{\bar{p}2}$

$h_3 = H_1 - (\ell + c)$ and $h_6 = 0$ for W_{p3}

For $H_1 \leqq c$, $\ell + c \geqq H_2 > c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_2 (a^2 - b_1^2)(H_2 - c) \qquad \text{(A-8)}$$

$h_4 = H_1 - (c - c_1)$ for W_{p4}

$h_2 = c$ for $W_{\bar{p}2}$

For $\ell + c \geqq H_1 > c$, $\ell + c \geqq H_2 > c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2 (H_2 - c) + \pi\gamma_2 (a^2 - b_1^2)(H_2 - c) \qquad \text{(A-9)}$$

$h_4 = c_1$ for W_{p4}

$h_2 = c$ for W_{p2}

For $H_1 > \ell + c$, $\ell + c \geqq H_2 > c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2 \ell + \pi\gamma_2 (a^2 - b_1^2)(H_2 - c) + W_{p3} \qquad \text{(A-10)}$$

$h_4 = c_1$ for W_{p4}

$h_2 = c$ for $W_{\bar{p}2}$

$h_3 = H_1 - (\ell + c)$ and $h_6 = 0$ for W_{p3}

For $H_1 \leqq c$, $H_2 > \ell + c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_2 (a^2 - b_1^2)\ell + W_{\bar{p}1} \qquad \text{(A-11)}$$

$h_4 = H_1 - (c - c_1)$ for W_{p4}, $h_2 = c$ for $W_{\bar{p}2}$

$h_1 = H_2 - (\ell + c)$ and $h_5 = 0$ for $W_{\bar{p}1}$

For $\ell + c \geqq H_1 > c$, $H_2 > \ell + c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2 (H_1 - c) + \pi\gamma_2 (a^2 - b_1^2)\,\ell + W_{\bar{p}1} \qquad \text{(A-12)}$$

$h_4 = c_1$ for W_{p4}, $h_2 = c$ for $W_{\bar{p}2}$

$h_1 = H_2 - (\ell + c)$ and $h_5 = 0$ for W_{p1}

For $H_1 > \ell + c$, $H_2 > \ell + c$ (semi-toroidal tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2\,\ell + \pi\gamma_2 (a^2 - b_1^2)\,\ell + W_{\bar{p}1} + W_{p3} \qquad \text{(A-13)}$$

$h_4 = c_1$ for W_{p4}, $h_2 = c$ for $W_{\bar{p}2}$

$h_1 = H_2 - (\ell + c)$ and $h_5 = 0$ for $W_{\bar{p}1}$

$h_3 = H_1 - (\ell + c)$ and $h_6 = 0$ for W_{p3}

For $H_1 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $H_2 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$ (mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} \qquad \text{(A-14)}$$

$h_4 = H_1 - \left(c - c_1 - \frac{\ell_i - \ell_o}{2}\right)$ for W_{p4}

$h_2 = H_2$ for $W_{\bar{p}2}$

For $\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right) \geqq H_1 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $H_2 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$

(mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\bar{p}2} + \pi\gamma_1 b_1^2 \left[H_1 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \qquad \text{(A-15)}$$

$h_4 = c_1$ for W_{p4}, $h_2 = H_2$ for $W_{\bar{p}2}$

For $H_1 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$, $H_2 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$ (mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_1 b_i^2 \ell_i + W_{p3} \tag{A-16}$$

$h_4 = c_1$ for W_{p4}, $h_2 = H_2$ for $W_{\overline{p2}}$

$h_3 = H_1 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_6 = 0$ for W_{p3}

For $H_1 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $\left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \geqq H_2 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$

(mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_2\left(a^2 - b_i^2\right)\left[H_2 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \tag{A-17}$$

$h_4 = H_1 - \left(c - c_1 - \frac{\ell_i - \ell_o}{2}\right)$ for W_{p4}

$h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

For $\left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \geqq H_1 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $\left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \geqq H_2 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$

(mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_1 b_i^2\left[H_1 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] + \pi\gamma_2\left(a^2 - b_i^2\right)\left[H_2 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \tag{A-18}$$

$h_4 = c_1$ for W_{p4}

$h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

For $H_1 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$, $\left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \geqq H_2 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$

(mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_1 b_i^2 \ell_i + \pi\gamma_2(a^2 - b_i^2)\left[H_2 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] + W_{p3} \quad \text{(A-19)}$$

$h_4 = c_1$ for W_{p4}, $h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

$h_3 = H_1 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_6 = 0$ for W_{p3}

For $H_1 \leqq \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $H_2 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$(mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_2 (a^2 - b_i^2)\, \ell_i + W_{\overline{p1}} \quad \text{(A-20)}$$

$h_4 = H_1 - \left(c - c_1 - \frac{\ell_i - \ell_o}{2}\right)$ for W_{p4}

$h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

$h_1 = H_1 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_5 = \frac{\ell_i - \ell_o}{2}$ for $W_{\overline{p1}}$

For $\left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] \geqq H_1 > \left(c - \frac{\ell_i - \ell_o}{2}\right)$, $H_2 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ (mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_2 (a^2 - b_i^2)\, \ell_i + \pi\gamma_1 b_i^2 \left[H_1 - \left(c - \frac{\ell_i - \ell_o}{2}\right)\right] + W_{\overline{p1}} \quad \text{(A-21)}$$

$h_4 = c_1$ for W_{p4}

$h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

$h_1 = H_2 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_5 = \frac{\ell_i - \ell_o}{2}$ for $W_{\overline{p1}}$

For $H_1 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$, $H_2 > \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ (mod. ogival tank):

$$W_{p5} = W_{p4} + W_{\overline{p2}} + \pi\gamma_2 (a^2 - b_1^2)\,\ell_i + \pi\gamma_1 b_1^2 \ell_i + W_{\overline{p1}} + W_{p3} \qquad \text{(A-22)}$$

$h_4 = c_1$ for W_{p4}

$h_2 = \left(c - \frac{\ell_i - \ell_o}{2}\right)$ for $W_{\overline{p2}}$

$h_1 = H_2 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_5 = \frac{\ell_i - \ell_o}{2}$ for $W_{\overline{p1}}$

$h_3 = H_1 - \left[\ell_i + \left(c - \frac{\ell_i - \ell_o}{2}\right)\right]$ and $h_6 = 0$ for W_{p3}

Determine the lower skirt weight, W_{p6}

For $H_2 \leqq c$ (semi-toroidal and mod. ogival tank):

$$W_{p6} = W_{p2}^{+} \qquad \text{(A-23)}$$

$h_2 = H_2$ for W_{p2}^{+}

For $\ell + c \geqq H_2 > c$ (semi-toroidal tank),

For $\ell_o + c \geqq H_2 > c$ (mod. ogival tank):

$$W_{p6} = W_{p2}^{+} + \pi\gamma_2\, b\,(2a + b)\,(H_2 - c) \qquad \text{(A-24)}$$

$h_2 = c$ for W_{p2}^{+}

For $H_2 > (\ell + c)$ (semi-toroidal tank):

$$W_{p6} = W_{p2}^{+} + \pi\gamma_2 b\,(2a + b)\,\ell + W_{p1}^{+} \qquad \text{(A-25)}$$

$h_2 = c$ for W_{p2}^{+}

$h_1 = H_2 - (\ell + c)$ and $h_5 = 0$ for W_{p1}^{+}

For $H_2 > (\ell_O + c)$ (mod. ogival tank)

$$W_{p6} = W_{p2}^{+} + \pi\gamma_2 b\,(2a + b)\,\ell_O + W_{p1}^{+} \tag{A-26}$$

$h_2 = c$ for W_{p2}^{+}

$h_1 = H_2 - (\ell_O + c)$ and $h_5 = 0$ for W_{p1}^{+}

B. Elliptical Toroidal and Modified Ogival Shell Geometry

The equation of the elliptical toroidal shell and the modified ogival shell (Figure A.7) is

$$\frac{x^2}{b^2} + \frac{z^2}{c^2} = 1\,. \tag{A-27}$$

Solving for z,

$$z = \pm(c/b)\left(b^2 - x^2\right)^{1/2}.$$

The present derivations will be concerned with +z only because of symmetry. Differentiate z with respect to x.

$$z = +\left(\frac{c}{b}\right)\left(b^2 - x^2\right)^{1/2}$$

$$\frac{dz}{dx} = \frac{-cx}{b\left(b^2 - x^2\right)^{1/2}}\,, \quad \left(\frac{dz}{dx}\right)^2 = \frac{c^2x^2}{b^2(b^2 - x^2)}$$

$$\frac{d^2z}{dx^2} = -\frac{cb}{\left(b^2 - x^2\right)^{3/2}}$$

The derivation of the general expression for $\sin\phi$ is necessary for the solution of the equilibrium equations (Figure A.7).

$$\sin\phi = \frac{dz}{ds} = \frac{dz}{\left(dz^2 + dx^2\right)^{1/2}} = \frac{dz/dx}{\left[1 + (dz/dx)^2\right]^{1/2}}$$

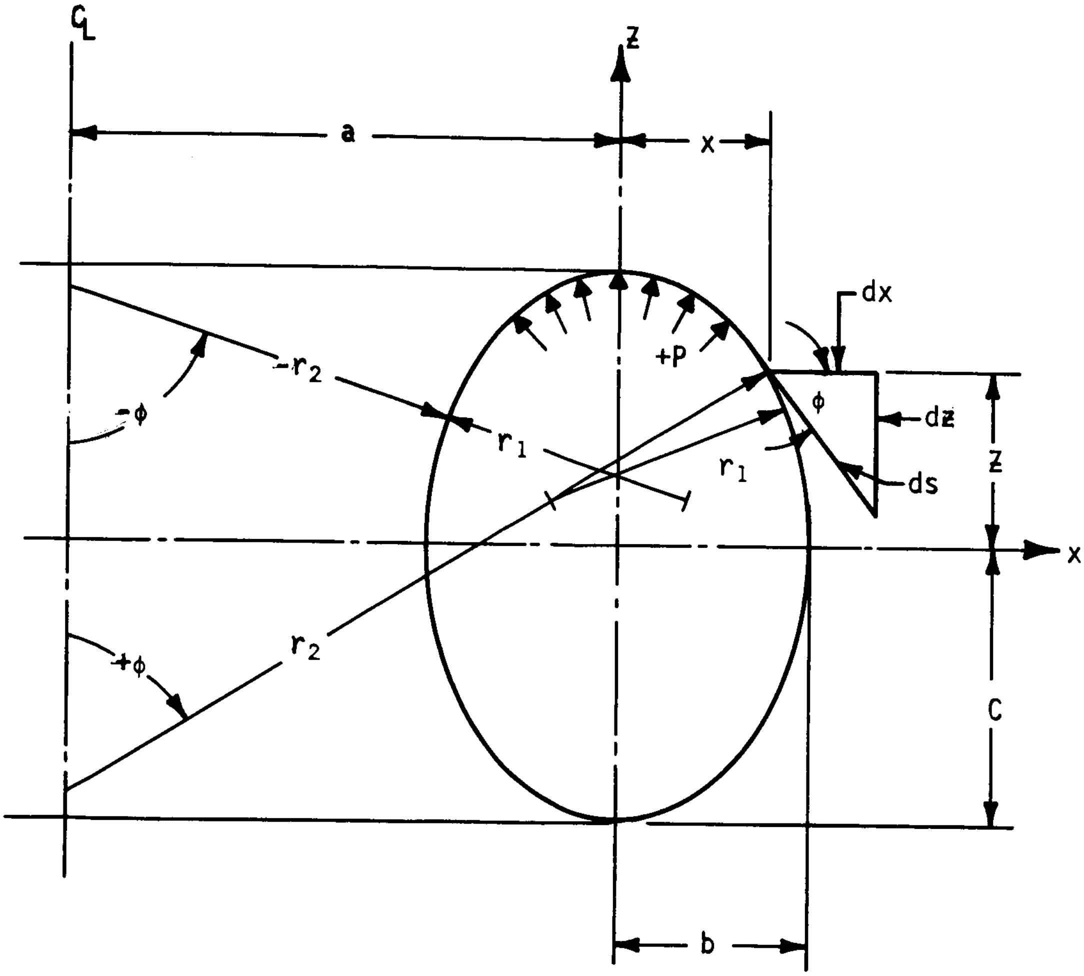


Figure A.7. Geometric Parameters of the Elliptical Toroidal and Modified Ogival Shells

Substituting in the appropriate expressions, sin ϕ is reduced to the following equation.

$$\sin \phi = \frac{-cx}{\left[b^4 - x^2 (b^2 - c^2)\right]^{1/2}}$$

To conform to the coordinate system and the sign convention of the equilibrium Equations (3) and (6), the sign of sin ϕ will be changed to positive for +x. Finally,

$$\sin \phi = \frac{cx}{\left[b^4 - x^2 (b^2 - c^2)\right]^{1/2}} . \tag{A-28}$$

With respect to the general shell equation,

$$N_\theta = r_2\left(p - \frac{N\phi}{r_1}\right), \tag{A-29}$$

interval pressure p is considered always positive, thus r_1 and r_2 are positive only when they are on the same side of the surface as the pressure (Figure A. 7).

From the calculus

$$r_1 = \frac{\left[1 + (dz/dx)^2\right]^{3/2}}{(d^2z/dx^2)} .$$

This reduces to the following expression:

$$r_1 = -\frac{\left[b^4 - x^2(b^2 - c^2)\right]^{3/2}}{b^4 c}$$

In this analysis r_1 will always be positive; therefore, we will change the sign of r_1 to positive.

$$r_1 = \frac{\left[b^4 - x^2 (b^2 - c^2)\right]^{3/2}}{b^4 c} . \tag{A-30}$$

In a circular toroidal shell $b = c = R$, thus Equation (A-30) results in

$$r_1 = R \tag{A-31}$$

From Figure A. 7

$$r_2 = \frac{a + x}{\sin \phi} = \frac{(a + x)\left[b^4 - x^2 (b^2 - c^2)\right]^{1/2}}{cx} . \tag{A-32}$$

Note that the above expression is positive for $+x$ and negative for $-x$; this conforms to the sign convention indicated in Figure A. 7.

For a circular toroidal shell Equation (A-28) reduces to

$$\sin \phi = \frac{x}{R} \tag{A-33}$$

where $b = c = R$; thus for a circular toroidal shell

$$r_2 = \frac{(a + x) R}{x} . \tag{A-34}$$

The geometric parameters derived in Paragraph B of this Appendix are basically identical to those in Reference 5.

C. Oblate Spheroidal Shell Geometry

We note that $\sin \phi$ is the same for the oblate spheroidal shell as for the elliptical thoroidal shell (Figure A. 8). The only basic difference between the elliptical toroidal bulkhead and the oblate spheroidal bulkhead with respect to geometric parameters is that $a = 0$ for the oblate spheroid. Therefore,

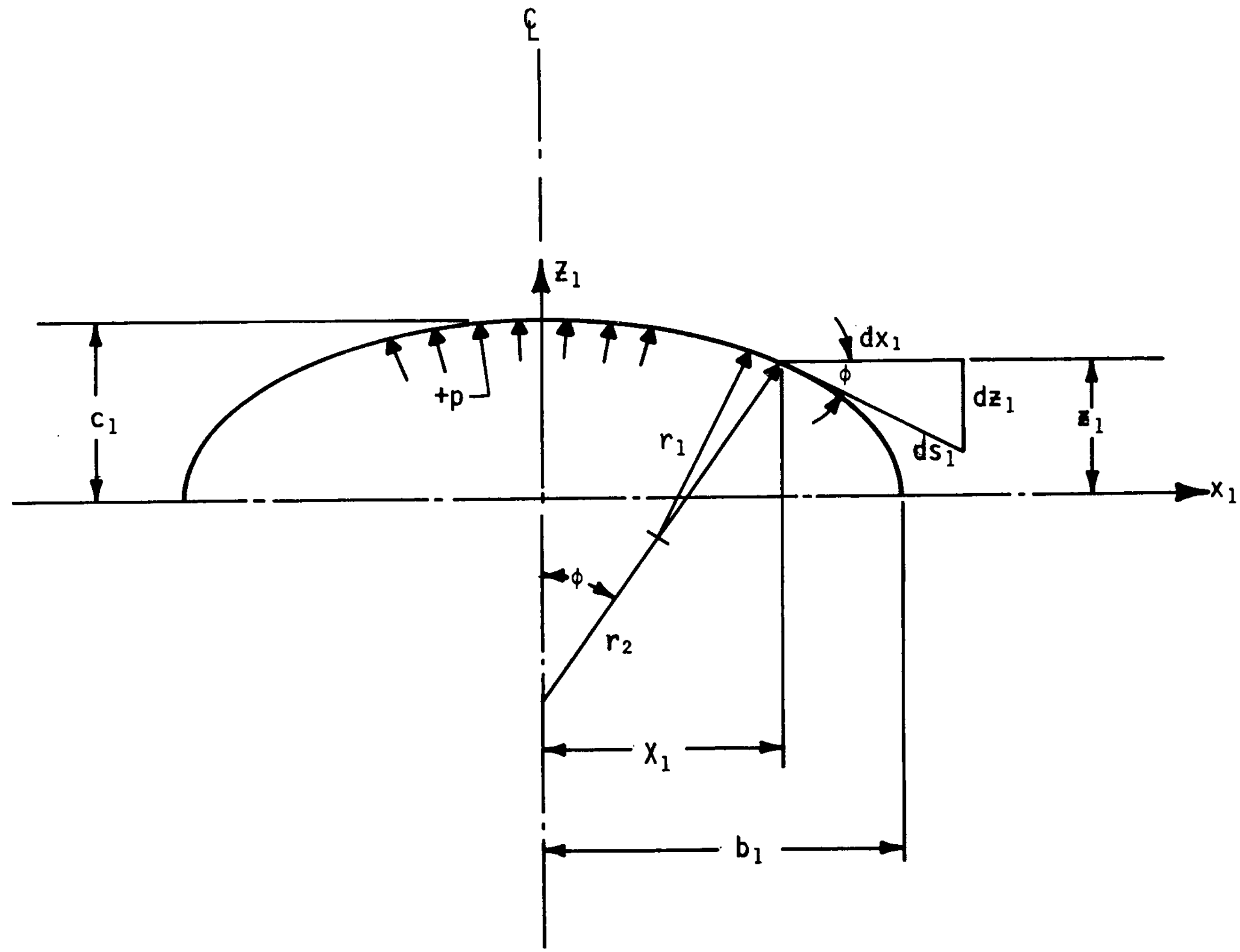


Figure A.8. Geometric Parameters of the Oblate Spheroidal Shell

$$\sin\phi = \frac{c_1 x_1}{\left[b_1^4 - x_1^2 (b_1^2 - c_1^2)\right]^{1/2}} , \qquad \text{(A-35)}$$

$$r_1 = \frac{\left[b_1^4 - x_1^2 (b_1^2 - c_1^2)\right]^{3/2}}{b_1^4 c_1} , \qquad \text{(A-36)}$$

$$r_2 = \frac{\left[b_1^4 - x_1^2 (b_1^2 - c_1^2)\right]^{1/2}}{c_1} . \qquad \text{(A-37)}$$

For a spheroidal bulkhead $b_1 = c_1 = R_1$, and Equations (A-35), (A-36), and (A-37) reduce to the following expressions.

$$\sin\phi = \frac{x_1}{R_1} , \qquad \text{(A-38)}$$

$$r_1 = R_1 , \qquad \text{(A-39)}$$

$$r_2 = R_1 . \qquad \text{(A-40)}$$

D. Computer Program

1. Abstract

This computer program is written in FORTRAN II for the GE235 computer. Utilizing the equations of Reference 1 and this report, the program calculates the linear membrane forces at any point in a semi-toroidal or modified ogival tank under varying internal liquid and/or uniform pressure loadings and the total liquid reaction to the center support and lower skirt. It also computes ideal thicknesses for both the upper and lower bulkheads at a predetermined number of points. For critical meridional or hoop tension forces, the thicknesses are based on an allowable uniaxial stress; for critical hoop compression forces, approximately required thicknesses are calculated by the following equation.

$$t_c = \left(\frac{N\theta \, R_2}{KE} \right)^{\frac{1}{2}} \quad \text{(A-41)}$$

Weights of the bulkheads are also computed, including both a monocoque weight and a reduced weight that is determined by substituting approximate waffle pattern weights for the hoop compression areas of the head.

We note also that this program can be used for the analysis of a cylindrical tank with oblate spheroidal heads provided dummy toroidal data is inputed with γ_2 and p_{O2} equal to zero.

2. Notation

Following is a comparison of text and program notation. Definitions are included if necessary.

Text	Program	Definition
a	CUL	(")
a	CLL	(")
b_1	RI	(")
a + b	RO	(")
b	A	(")
c	B	(")
c_1	CILOX	(")
ℓ or ℓ_O	FL	(")
—	WLH2	(#) Weight of liquid (limit) reacted through the center tube from the center support reaction of another tank.

Text	Program	Definition
n	ACCEL	
P_{02}	PUO	(#/"^{2}) Ullage pressure (limit) in the outer cylinder region.
γ_2	GAMMO	(#/"^{3}) Unit weight of liquid (limit) in the outer tank.
γ_1	GAMMI	(#/"^{3}) Unit weight of liquid (limit) in the inner tank.
—	FACSA	Factor of safety.
P_{01}	PUI	(#/"^{2}) Ullage pressure (limit) in the inner cylinder region.
H_2	HOPO	(")
H_1	HOPI	(") H_1 is measured from bottom of toroidal bulkhead. HOPI is measured from bottom of spheroidal bulkhead.
E	E	(#/"^{2})
–	FTU	(#/"^{2}) Uniaxial tension allowable (ultimate and cryogenic).
–	DEN	(#/"^{3}) Density of material used in tank.
–	XXROU	Number of divisions in the distance 2A made for point calculations on the toroidal or ogival bulkhead. Use even floating point number.
–	XXRIU	Number of divisions in the distance RI made for point calculations on the oblate spheroidal bulkhead. Use even or odd floating point number.
K	FK	
–	WAF	Approximate waffle pattern weight reduction factor for the hoop compression areas of the bulkheads.
	TMIN	(") Minimum specified thickness (see Paragraph 3)

Text	Program	Definition
$\overline{p}$	FNC	(#/")
–	XXFLO	Number of divisions in the distance FL for point calculations on the outer cylinder and center tube for the semi-toroidal tank and on the outer cylinder for the modified ogival tank.
–	XXFLI	Number of divisions for point calculations on the center tube (over the length ℓ_i in Figure 2) for the modified ogival tank (XXFLI = 0. for the semi-toroidal tank).
–	TRANS	(See Paragraph 3)
–	NCASE	(See Paragraph 3)
x or x_1	X	(")
z or z_1	Y	(")
N_ϕ	FPHI	(#/")
N_θ	FTHETA	(#/")
t	T	(") Required thickness at a given point in a toroidal, modified ogival, or oblate spheroidal head.
–	TAVG	(") Average thickness of bulkhead element between two given points in a toroidal, modified ogival, or oblate spheroidal head.
–	WTELE	(#) Weight of bulkhead element between two given points in a toroidal, modified ogival, or oblate spheroidal head.
-	XFLO, XFLI	(") The interval of stress readout for the outer cylinder measured from the top of the cylinder and the center tube measured from the top of the tube.

Sign Convention:

Tension Force (+)
Compression Force (-)
Geometric Convention - Same as Reference 1 and this report.

3. Restrictions and Limitations

- NCASE must be a fixed point integer equal to the number of cases being run.
- TRANS must have the value 1. or the value 2.; if not, the program will stop.
- TRANS = 1. if minimum thickness (TMIN) is specified.
- TRANS = 2. if minimum thickness (TMIN) is not specified.
- Because of mathematical procedures, the maximum value of the liquid level (HOPO) is 0.001" from the top of the toroidal or modified ogival heads.
- If the forces FPHI = FTHETA = 0., the thickness, T, at a given point in a toroidal, modified ogival, or oblate spheroidal bulkhead is printed out as TMIN.
- The program assumes equal pressure throughout the tank if GAMMO = GAMMI; therefore, causing an overpressure on the upper oblate spheroidal bulkhead (the top of toroidal or modified ogival bulkhead is assumed above or equal to the top of the oblate spheroidal bulkhead). If GAMMO = GAMMI but the inner cylinder and outer cylinder are not interconnected for equalized pressure effects, allow GAMMO and GAMMI to vary by 0.00001 #/"^{3} in the input data.
- This program is restricted to toroidal loadings 1 and 3a (see Section VI).

4. Input

The input data and corresponding column location on each input card are of the following order.

The first input card is the NCASE card. This card gives the number of cases to be run as a fixed point integer with the last integer in card column 5. The NCASE card is followed by NCASE sets of input data consisting of four cards each. For each set of input data the cards are of the following form.

Col.	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Card 1	CUL	CLL	RI	RO	A	B	CILOX	FL
Card 2	WLH2	ACCEL	PUO	GAMMO	GAMMI	EACSA	PUI	HOPO
Card 3	HOPI	E	FTU	DEN	XXROU	XXRIU	FK	WAF
Card 4	TMIN	FNC	XXFLO	XXFLI	TRANS			

5. Listing

The listing, including subroutines, is given on the following pages.

```
C MEMBRANE STRESS ANALYSIS OF SEMI-TOROIDAL AND MODIFIED OGIVAL TANKS
C       JOB NO. 566440      SPONSOR BURNETTE     PROGRAMMER WAUGH
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA,E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2,XA
      COMMON  AA,AC,BC,CA,C2R2,C,DELTA,DELT,FACT,FC,FCS,FLAM,FLAP
      COMMON FLB,FLIM,FLIP,FLI,FPHICS,FPHICT,FPHIOC,FPHUOU,FPHUTU,FTHECS
      COMMON  FTHECT,FTHELS,FTHEOC,FTHUAU,FTHUMU,FTHUOU,FTHUTU,GG,GI,GO
      COMMON  G,HAPO,HEK,HEP,HIK,HIPPI,HIPPY,HIP,HOK,HOPI,HOPO,HOP,PGA
      COMMON  PGACFL,PGAC,PGARFL,PGAR,PTI,PTIT,PTO,PTOT,PUI,PUO,R2I,R2
      COMMON R3,RI2,S2,SIPPY,SY,TAVGI,TAVG,TAVGWI,TAVGW,TCOMPI,TCOMP,TO1
      COMMON TRANS,TTENI,TTEN,TWA1,TWA2,UMU,WP1DN,WP1ITH,WP1UO,WP1UP,WW
      COMMON WP2ITH,WP3OSH,WP4OSH,WP5,WP6,WP7,WTELEI,WTELE,WTELWI,WTELW
      COMMON WTERM,WTTI,WTTIW,WTTO,WTTOW,WWW,XA2,XFLI,XFL,XLANGO,XROU
      COMMON XLONGI,XLONGO,XLONG,XXFL,FPHILS,TO2,PAP,POP,XRAU,PTA,PTAT
 1000 READ 198,NCASE
      DO 999 NC=1,NCASE
      READ 200,CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL,PUO,GAMMO,GAMMI,
     1FACSA ,PUI,HOPO,HOPI,E,FTU,DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,
     2XXFLI,TRANS
      NTRANS = TRANS
      AA=A**2
      AC=A-CUL
      BA=B/A
      CA=CUL+A
      C=CUL
      C2=CILOX**2
      DELT=RI/XXRIU
      DELTA=2.*A/XXROU
      RI2=RI**2
      C2R2=CUL**2-RI2
      FACT=FACSA*RI2  /(2.*CILOX)
      FC=FL+CILOX
      FLB=FL+B
      GG=GAMMO-GAMMI
```

```
      GI=GAMMI
      GO=GAMMO
      JROU=XXROU
      JR =JROU/2+1
      XXFL=XXFLO
      PI=3.1415927
      PRINT 202
      PRINT 203
      PRINT 204
      PRINT 205,CUL,CLL,RI,RO,A
      PRINT 206
      PRINT 207,B,CILOX,FL,WLH2,ACCEL
      PRINT 208
      PRINT 209,PUO,GAMMO,GAMMI,FACSA ,PUI
      PRINT 210
      PRINT 211,HOPO,HOPI,E,FTU,DEN
      PRINT 212
      PRINT 213,XXROU,XXRIU,FK,WAF,TMIN
      PRINT 260
      PRINT 261,     FNC,XXFLO,XXFLI ,TRANS
      PRINT 201
C
C     UPPER TORUIDAL AND OBLATE SPHEROIDAL BULKHEADS, SEMI-TOROIDAL
C     AND MODIFIED OGIVAL CONFIGURATIONS
      GO TO (302,301),NTRANS
  302 PRINT 259
  301 PRINT 214
      PRINT 215
      PRINT 216
      J=1
      JPOP=1
      WITO=0.
      WITOW=0.
      IF( AC )6000,6000,6001
```

```
6000 XROU=0.
     SIPPY = 0.
     BC=B-CILOX
     GO TO 1
6001 XROU=AC+RI
     SIPPY=RA*SQRTF[AA-[CUL-RI]**2]
     SY=SIPPY
     BC=B-CILOX-SIPPY
     POP=XROU/DELTA
     IF[POP-1.]2802,2802,2801
2801 JPOP=POP+1.
     J=JPOP
2802 PAP=J-1
     XRAU=DELTA*PAP
   1 XA=XROU-A
     XA2=XA**2
     HIPPY=RA*SQRTF[AA-XA2]
     NGO=1
     XLONGO=FLB+HIPPY
     XLANGO=FLB+B
     IF[HOPO-XLONGO]5,5,6
   5 IF[J-[JR       ]]651,652,651
 651 WP1ITH=0.
   2 PTO=PUO
     IF[J-[JR       ]]702,30,704
   6 IF[XLONGO-XLANGO]653,652,652
 653 HIP=HOPO-FLB
   9 HEP=XLONGO-FLB
   3 PTO=          PPT[PUO,GO,HOPO,XLONGO,0.]
     IF[J-[JR       ]]700,30,50
 700 WP1UP=          WP1[HIP,-1.]
     WP1DN=          WP1[HEP,-1.]
     WP1ITH=WP1UP-WP1DN
 702 FPHUTU=          FPH[WP1ITH,PTO]
     FTHUTU=          FTH[WP1ITH,PTO]
     GO TO 70
```

```
  652 PTO=PUO
   30 FPHUTU=(PTO*AA   )/B*FACSA
      FTHUTU=.5*FPHUTU
      GO TO 70
   50 WP1UP=          WP1(HIP,+1.)
      WP1DN=          WP1(HEP,+1.)
      WP1ITH=WP1UP-WP1DN
  704 FPHUTU=          FPH(-WP1ITH,PTO)
      FTHUTU=          FTH(-WP1ITH,PTO)
   70 IF(FTHUTU)71,705,81
   71 R2=((CUL+XA)*SQRTF(AA**2-XA2*(AA-B**2)))/(B*XA)
      R3=ABSF(R2)
 2071 FTHUAU=ABSF(FTHUTU)
      TCOMP=SQRTF((FTHUAU*R3)/(FK*E))
      TTEN=FPHUTU/FTU
      GO TO(800,850),NTRANS
  800 IF(TCOMP-TTEN)801,801,802
  801 IF(TTEN-TMIN)706,706,72
  802 IF(TCOMP-TMIN)706,706,73
  850 IF(TCOMP-TTEN)72,72,73
   72 TO1=TTEN
      TWA1=TO1
      GO TO 74
   73 TO1=TCOMP
      TWA1=TO1*WAF
   74 X1=XA
      Y1=HIPPY
      IF(J-JPOP)100,100,101
  705 IF(FPHUTU-FTHUTU)706,706,83
  706 TO1=TMIN
      TWA1=TO1
      GO TO 74
   81 IF(FPHUTU-FTHUTU)82,83,83
   82 TTEN=FTHUTU/FTU
      GO TO(801,72),NTRANS
```

```
   83 TTEN=FPHUTU/FTU
      GO TO(801,72),NTRANS
  101 TAVG=AVG(T01,T02)
      TAVGW=AVG(TWA1,TWA2)
      WTELE=WTE(C,TAVG)
      WTELW=WTE(C,TAVGW)
      WTTO=WTTO+WTELE
      WTTOW=WTTOW+WTELW
      PRINT 250,TAVG,WTELE
  100 PRINT 251,X1,Y1,FPHUTU,FTHUTU,T01
      T02=T01
      TWA2=TWA1
      X2=X1
      Y2=Y1
      IF(  AC )6002,6002,6003
 6003 IF(J-JPOP)2803,2803,6002
 2803 XROU=XRAU+DELTA
      GO TO 6600
 6002 XROU=XROU+DELTA
 6600 J=J+1
      GO TO (6004,6104),NGO
 6004 IF(J-(JROU+1))1,1,11
   11 PRINT 252,WTTO
      PRINT 253
      PRINT 254
      PRINT 255,WTTOW
      K=1
      WTTI=0.
      WTTIW=0.
      XRIU=0.
      PRINT 214
      PRINT 256
      PRINT 216
      S2=SIPPY*2.
      FCS=FC+S2
```

```
 401 HIPPI=CILOX/RI*SQRTF(RI2-XRIU**2)
     NGO=1
     XLONGI=FCS+HIPPI
     IF(HOPI-XLONGI)402,402,403
 402 HIK=0.
 405 HEK=0.
 408 PTI=PUI
     GO TO 410
 403 HIK=HOPI-FCS
 406 HEK=XLONGI-FCS
 409 PTI=           PPT(PUI,GI,HOPI,XLONGI,0.)
 410 XLONG=XLONGI+B-CILOX-SIPPY
     PTO=           PPT(PUO,GO,HOPO,XLONGI,BC)
     IF(XRIU)411,411,412
 411 IF(       GG    )450,451,450
 450 FPHUOU=PTI*FACT
     FTHUOU=FPHUOU
     GO TO 500
 451 IF(HOPO-XLONG)450,450,461
 461 FPHUOU=PTO*FACT
     FTHUOU=FPHUOU
     GO TO 500
 412 WP3OSH=             WP3(HIK,HEK)
     IF(GG            )452,453,452
 452 FPHUOU=             FPHOU(PTI,WP3OSH)
     FTHUOU=             FTHOU(PTI,WP3OSH)
     GO TO 500
 453 IF(HOPO-XLONG)452,452,462
 462 FPHUOU=             FPHOU(PTO,WP3OSH)
     FTHUOU=             FTHOU(PTO,WP3OSH)
 500 IF(FTHUOU) 501,530,502
 501 R2I=SQRTF(RI2**2*HIPPI**2+C2**2*XRIU**2)/C2
2501 FTHUMU=ABSF(FTHUOU)
     TCOMPI=SQRTF((FTHUMU*R2I)/(FK*E))
     TTENI=FPHUOU/FTU
     GO TO(810,840),NTRANS
```

```
810 IF(TCOMPI-TTENI)811,811,812
811 IF(TTENI-TMIN)531,531,503
812 IF(TCOMPI-TMIN)531,531,504
840 IF(TCOMPI-TTENI)503,503,504
503 T01=TTENI
    TWA1=T01
    GO TO 505
504 T01=TCOMPI
    TWA1=T01*WAF
505 X1=XRIU
    Y1=HIPPI
    IF(K-1)550,550,551
530 IF(FPHUOU-FTHUOU)531,531,507
531 T01=TMIN
    TWA1=T01
    GO TO 505
502 IF(FPHUOU-FTHUOU)506,506,507
506 TTENI=FTHUOU/FTU
    GO TO(811,503),NTRANS
507 TTENI=FPHUOU/FTU
    GO TO(811,503),NTRANS
551 TAVGI=AVG(T01,T02)
    TAVGWI=AVG(TWA1,TWA2)
    WTELEI=WTE(0.,TAVGI)
    WTELWI=WTE(0.,TAVGWI)
    WTTI=WTTI+WTELEI
    WTTIW=WTTIW+WTELWI
    PRINT 250,TAVGI,WTELEI
550 PRINT 251,X1,Y1,FPHUOU,FTHUOU,T01
    T02=T01
    TWA2=TWA1
    X2=X1
    Y2=Y1
    K=K+1
    XRIU=XRIU+DELT
    GO TO (599,2599),NGO
```

```
  599 IF(XRIU-RI) 401, 401, 600
 2599 IF(XRIU-RI)2401,2401,2600
  600 PRINT 257,WITI
      PRINT 253
      PRINT 254
      PRINT 258,WIIIW
C
C    CENTER TUBE AND OUTER CYLINDER LOADS,SEMI-TOROIDAL AND
C    MODIFIED OGIVAL CONFIGURATIONS
      PRINT 1201
      PRINT 214
      PRINT,1215
      PRINT 1216
      XFL=0.
      XFLI=0.
      PTII=PUI
      FLIM = FLB
      IF(  AC )6006,6006,6007
 6006 FLIP=FC
      SIPPY=0.
      G=B
      GO TO 1001
 6007 SIPPY=SY
      G=B-SIPPY
      S2=SIPPY*2.
      FLIP=FC+S2
 1001 FLAP=FLIP-XFLI
      FLAM=FLIM-XFL
      IF(HOPO-G)1100,1100,1101
 1100 WP1UP=0.
      WP1DN=0.
      WP1UO=0.
```

```
      PTO=PUO
      PTA=PUO
      PTOT=PUO
      PTAI=PUO
3001  IF (HOPI-CILOX)3300,3300,3301
3301  IF(HOPI-FLIP)3002,3002,3003
3002  WF30SH=0.
      IF(HOPI-FLAP)3004,3004,3005
3300  WF30SH=0.
3004  PTI=PUI
      GO TO 1106
3003  HIK=HOPI-FLIP
      HEK=0.
      WF30SH=            WP3(HIK,HEK)
      IF(GG)3050,3051,3050
3051  IF(HOPO-(HOPI+EC))3050,3050,3052
3052  PTII=PTAI
      PTI=PTA
      GO TO 1106
3050  PTII=           PPT(PUI,GI,HOPI,FLIP,0.)
3005  PTI=            PPT(PUI,GI,HOPI,FLAP,0.)
      GO TO 1106
1101  PTO=            PPT(PUO,GO,HOPO,FLAM,0.)
      IF(AC)8050,8050,8051
8050  PTA=PTO
      GO TO 8052
8051  PTA=PPT(PUO,GO,HOPO,FLAP,BC)
8052  HIP=HOPO-FLIP
      IF(HIP)1102,1102,1103
1102  WF10P=0.
      WF10N=0.
      WF10D=0.
      PTOT=PUO
      PTAI=PUO
```

```
      IF(HOPO-FLAM)1104,1104,9003
 1104 PIO=PUO
 9004 PTA=PUO
      GO TO 3001
 1103 IF(HOPO-(FLIP+BC))9000,9000,9001
 9000 PTAI=PUO
      GO TO 9002
 9003 IF(HOPO-(FLAP+BC))9004,9004,3001
 9001 HEP=SIPPY
      WF1UP=           WP1(HIP,-1.)
      WP1DN=WP1(HEP,-1.)
 9002 WP1UO=WP1(HIP,+1.)
      PTOI=          PPT(PUO,GO,HOPO,FLIM,0.)
      IF(AC) 8060,8060,8061
 8060 PTAT=PTOT
      GO TO 3001
 8061 IF(HOPO-(FLIP+BC))3001,3001,8062
 8062 PTAT=PPT(PUO,GO,HOPO,FLIP,BC)
      GO TO 3001
 1108 FPHICT=((PTIT*RI2+PTAT*C2R2-(WLH2+WP1UP-WP1DN+WP3OSH)/PI)/(2.*RI))
     1*FACSA
      FTHECT=RI*(PTI-PTA)*FACSA
      FPHIOC=(((PTOT*A*(2.*CUL+A)-WP1UO/PI)/(2.*CA))+FNC)*FACSA
      FTHEOC=PIO*CA*FACSA
      FLI=FL
      IF(SIPPY)7000,7000,7001
 7001 FLI=FL+S2
      IF(XFL-FL)7002,7002,7003
 7002 PRINT 1216,XFLI,FPHICT,FTHECT,XFL,FPHIOC,FTHEOC
      GO TO 7004
 7003 PRINT 1216,XFLI,FPHICT,FTHECT
 7004 XFLI=XFLI+FLI/XXFLI
      XFL=XFL+FL/XXFL
      IF(XFLI-FLI)1001,1001,1299
 7000 PRINT 1216,XFL,FPHICT,FTHECT,XFL,FPHIOC,FTHEOC
      XFL=XFL+FL/XXFL
```

```
      XFLI=XFL
      IF(XFL-FL)1001,1010,1299
 1010 IF(FL)1299,1299,1001
 1299 CONTINUE
C
C     LOWER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS, SEMI-TOROIDAL
C     AND MODIFIED OGIVAL CONFIGURATIONS
      PRINT 2201
      GO TO(2302,2301),NTRANS
 2302 PRINT 259
 2301 PRINT 214
      PRINT 2215
      PRINT  216
      J=1
      JPOP=1
      WITO=0.
      WITOW=0.
      IF(  AC )6100,6100,6101
 6100 XROU=0.
      GO TO 2001
 6101 XROU=AC+RI
      POP=XROU/DELTA
      IF(POP-1.)2806,2806,2805
 2805 JPOP=POP+1.
      J=JPOP
 2806 PAP=J-1
      XRAU=DELTA*PAP
 2001 XA=XROU-A
      XA2=XA**2
      HIPPY=BA*SQRTF(AA-XA2)
      NGO=2
      XLONGO=B-HIPPY
 2750 IF(HOPO)2005,2005,2751
 2005 PIO=PUO
 2651 WP2ITH=0.
      IF(J-JR) 2703,2030,2704
```

```
2751 PTO=PPT(PUO,GO,HOPO,XLONGO,0.)
     IF(XLONGO) 2030,2030,2756
2756 IF(HOPO-XLONGO)2752,2752,2753
2752 HIP=HOPO
2002 PTO=PUO
     GO TO 2702
2753 HIP=XLONGO
2702 IF(J-(JR        ))2700,2030,2050
2700 WP2ITH=WP2(HIP,-1.)
2703 FPHUTU=FPH(-WP2ITH,PTO)
     FTHUTU=            FTH(-WP2ITH,PTO)
     GO TO 70
2030 FPHUTU=(PTO*AA  1/8*FACSA
     FTHUTU=.5*FPHUTU
     GO TO 70
2050 WP2ITH=WP2(HIP,1.)
2704 FPHUTU=FPH(WP2ITH,PTO)
     FTHUTU=            FTH(WP2ITH,PTO)
     GO TO 70
6104 IF(J-(JRUU+1))2001,2001,2011
2011 PRINT 2252,WTTO
     PRINT 253
     PRINT 254
     PRINT 2255,WTTOW
     K=1
     WTTI=0.
     WTTIW=0.
     XRIU=0.
     PRINT 214
     PRINT 2256
     PRINT  216
2401 HIPPI=CILOX/RI*SQRTF(RI2-XRIU**2)
     NGO=2
     XLONGI=CILOX-HIPPI
     PTI=PUI
2760 IF(HOPI)2761,2761,2762
```

```
 2761 WP40SH=0.
      IF(XRI(I)2450,2450,2452
 2762 IF(HOPI-XLONGI)2763,2763,2764
 2763 HIK=HOPI
      WP40SH=          WP4(HIK)
      GO TO 2452
 2764 HIK=XLONGI
 2409 PTI=          PPI(PUI,GI,HOPI,XLONGI,9.)
 2410 HAPO=HOPO-R+CILOX
      PTO=          PPI(PUO,GO,HOPO,XLONGI,RC)
      IF(XRI(I)2411,2411,2412
 2411 IF(     GG      )2450,2451,2450
 2450 FPHUOU=PTI*FACT
      FTHUOU=FPHUOU
      GO TO 500
 2451 IF(HAPO-CILOX)2450,2450,2461
 2461 FPHUOU=PTO*FACT
      FTHUOU=FPHUOU
      GO TO 500
 2412 WP40SH=          WP4(HIK)
      IF(     GG      ) 2452,2453,2452
 2452 FPHUOU=FPHOU(PTI,-WP40SH)
      FTHUOU=FTHOU(PTI,-WP40SH)
      GO TO 500
 2453 IF(HAPO-CILOX)2452,2452,2465
 2465 FPHUOU=FPHOU(PTO,-WP40SH)
      FTHUOU=FTHOU(PTO,-WP40SH)
      GO TO 500
 2600 PRINT 2227,WTTI
      PRINT 253
      PRINT 254
      PRINT 2226,WTTIW
C
C     CENTER SUPPORT AND LOWER SKIRT LOADS, SEMI-TOROIDAL AND MODIFIED
C     OGIVAL CONFIGURATIONS
      PGAR=PI*GAMMI*ACCEL*RI2
```

```
      PGARFL=PGAR*FLI
      PGAC=PI*GAMMU*ACCEL*C2R2
      PGACFL=PGAC*FLI
      IF(  AC )8000,8000,8001
 8000 FLIP=FC
      GO TO 8002
 8001 SIPPY=SY
      S2=SIPPY*2.
      FLIP=FC+S2
 8002 IF(HOPO)4600,4600,4301
 4600 WP2ITH=0.
      WW=WP2ITH+WLH2
 3299 IF(HOPI)1500,1500,1501
 1500 WP5=WW
      GO TO 1350
 4301 G=G-SIPPY
      IF(HOPO-G)4302,4302,4303
 4302 HOP=HOPO
      WP2ITH=WP2(HOP,-1.)
      WW=WP2ITH+WLH2
      GO TO 3299
 1501 IF(HOPI-CILOX)5302,5302,5303
 5302 HOK=HOPI
      WP4OSH=WP4(HOK)
      WP5=WW+WP4OSH
      GO TO 1350
 5303 HOK=CILOX
      WP4OSH=WP4(HOK)
      WWW=WW+WP4OSH
      IF(HOPI-FLIP)5304,5304,5305
 5304 WP5=WWW+PGAR*(HOPI-CILOX)
      GO TO 1350
 5305 HIK=HOPI-FLIP
      HEK=0.
      WP3OSH=WP3(HIK,HEK)
      WP5=WWW+PGARFL+WP3OSH
      GO TO 1350
```

```
 4303 HOP=G
      WP2ITH=WP2(HOP,-1.)
      WW =WP2ITH+WLH2
      IF(HOPO-FLIM)2299,2299,4305
 2299 IF(HOPI)6500,6500,6501
 6500 WP5=WW+PGAC*(HOPO-G)
      GO TO 1350
 6501 IF(HOPI-CILOX)6302,6302,6303
 6302 HOK=HOPI
      WP4OSH=WP4(HOK)
      WWW=WW+WP4OSH
      WP5=WWW+PGAC*(HOPO-G)
      GO TO 1350
 6303 HOK=CILOX
      WP4OSH=WP4(HOK)
      WTERM=WW+WP4OSH+PGAC*(HOPO-G)
      IF(HOPI-FLIP)2304,2304,2305
 2304 WP5=WTERM+PGAR*(HOPI-CILOX)
      GO TO 1350
 2305 HEK=0.
      HIK=HOPI-FLIP
      WP3OSH=WP3(HIK,HEK)
 2306 WP5=WTERM+PGARFL+WP3OSH
      GO TO 1350
 4305 HIP=HOPO-FLIM
      HEP=SIPPY
      WP1UP=WP1(HIP,-1.)
      WP1UN=WP1(HEP,-1.)
      UMD=WP1UP-WP1UN
 4299 IF(HOPI)4500,4500,4501
 4500 WP5= WW+PGACFL+UMD
      GO TO 1350
 4501 HOK=HOPI
      WP4OSH=WP4(HOK)
      IF(HOPI-CILOX)7302,7302,7303
 7302 WP5=WW+WP4OSH+PGACFL+UMD
      GO TO 1350
```

```
 7303 HOK=CILOX
      WP4OSH=WP4(HOK)
      WTERM=WW+WP4OSH+UMD+PGACFL
      IF(HOPI-FLIP)4304,4304,6305
 4304 WP5=WTERM+PGAR*(HOPI-CILOX)
      GO TO 1350
 6305 HIK=HOPI-FLIP
      HEK=0.
      WP3OSH=WP3(HIK,HEK)
      WP5=WTERM+PGARFL+WP3OSH
 1350 WP5=WP5*FACSA
      FPHICS=WP5/(2.*PI*RI)
      FTHECS=0.
      PRINT 1219,WP5,FPHICS,FTHECS
      IF(HOPO)1700,1700,1800
 1700 WP6=0.
      GO TO 1450
 1800 IF(HOPO-B)1402,1402,1403
 1402 HOP=HOPO
      WP2ITH=WP2(HOP,1.)
      WP6=WP2ITH
      GO TO 1450
 1403 HOP=B
      WP2ITH=WP2(HOP,1.)
      PGA=PI*GAMMO*ACCEL*A*(2.*CLL+A)
      IF(HOPO-FLIM)1404,1404,1405
 1404 WP6=WP2ITH+PGA*(HOPO-B)
      GO TO 1450
 1405 HIP=HOPO-FLIM
      WP1UP=          WP1(HIP,+1.)
      WP6=WP2ITH+PGA* FL+WP1UP
 1450 WP6=WP6*FACSA
      FPHILS=WP6/(2.*PI*CA)+FNC*FACSA
      FTHELS=0.
      WP7=WP5+WP6-WLH2*FACSA
      PRINT 1220,WP6,FPHILS,FTHELS
```

```
      PRINT 1221,WP7
  999 CONTINUE
      PRINT 3000
      CALL EXIT
  198 FORMAT(5I5)
  200 FORMAT (8F10.3)
  201 FORMAT (//17X46HUPPER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS/16X
     1       48HSEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS/////)
  202 FORMAT(1H1,10X67HMEMBRANE STRESS ANALYSIS OF SEMI-TOROIDAL AND MOD
     1IFIED OGIVAL TANKS//////)
  203 FORMAT (35X10HINPUT DATA///)
  204 FORMAT (13X3HCUL,13X3HCLL,14X2HRI,14X2HRO,15X1HA)
  205 FORMAT (5(8XF8.3)//)
  206 FORMAT (15X1HB,11X5HCILOX,14X2HFL,12X4HWLH2,11X5HACCEL)
  207 FORMAT (3(8XF8.3),7XF9.1,9XF7.3//)
  208 FORMAT (13X3HPUO,11X5HGAMMU,11X5HGAMMI,10X6HFACSA ,13X3HPUI)
  209 FORMAT (8XF8.3,2(8XF8.5),10XF6.3,8XF8.3//)
  210 FORMAT (12X4HHOPO,12X4HHOPI,15X1HE,13X3HFTU,13X3HDEN)
  211 FORMAT (2(8XF8.3),6XF10.0,9XF7.0,11XF5.3//)
  212 FORMAT (11X5HXXROU,11X5HXXRIU,14X2HFR,13X3HWAF,12X4HTMIN)
  213 FORMAT (2(10XF6.2),3(10XF6.3)/////)
  214 FORMAT (32X11HOUTPUT DATA)
  215 FORMAT (29X23HUPPER TOROIDAL BULKHEAD///)
  216 FORMAT (10X1HX,10X1HY,8X4HFPHI,6X6HFTHETA,10X1HT,7X4HTAVG,7X5HWIEL
     1E/)
  250 FORMAT (5(XF11.3,F12.3)
  251 FORMAT (2F11.3,2F12.1,F11.3)
  252 FORMAT (///41HMONOCOQUE WEIGHT (UPPER TOROIDAL BLKHD.)=F12.1)
  253 FORMAT (///78HREDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIM
     1ATE WAFFLE PATTERN WEIGHTS)
  254 FORMAT (54H(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD)
  255 FORMAT (/47HAPPROX. REDUCED WEIGHT (UPPER TOROIDAL BLKHD.)=F12.1//
     1///)
  256 FORMAT (24X32HUPPER OBLATE SPHEROIDAL BULKHEAD///)
  257 FORMAT (///48HMONOCOQUE WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)=F12.
     11)
```

```
 258 FORMAT (/54HAPPROX. REDUCED WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)=
    1F12.1/////)
 259 FORMAT(51H TRANS=1. FOR SPECIFIED MIN. THICKNESS (INPUT TMIN)////)
 260 FORMAT (          13X3HFNC,11X5HXXFLO,11X5HXXFLI,11X5HTRANS)
 261 FORMAT (        6XF10.1,3(10XF6.2)          /////)
1201 FORMAT(///15X52HGENERAL LOADS FOR THE CENTER TUBE AND OUTER CYLIND
    1ER/17X48HSEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS/////)
1215 FORMAT (12X17HCENTER TUBE LOADS,23X20HOUTER CYLINDER LOADS///)
1216 FORMAT (8X4HXFLI,8X4HFPHI,6X6HFTHETA,13X4HXFLO,8X4HFPHI,6X6HFTHETA
    1/)
1218 FORMAT (F11.3,F12.1,F12.1,5XF11.3,F12.1,F12.1)
1219 FORMAT (/////30HTOTAL CENTER SUPPORT REACTION=F11.0//23HFPHI (CENT
    1ER SUPPORT) =F10.1//25HFTHETA (CENTER SUPPORT) =F10.1/////)
1220 FORMAT (37HTOTAL LIQUID REACTION (LOWER SKIRT) =F11.0//20HFPHI (LO
    1WER SKIRT) =F10.1//22HFTHETA (LOWER SKIRT) =F10.1)
1221 FORMAT (/////20HTOTAL WEIGHT LIQUID=F11.0////)
2201 FORMAT (1H1,17X46HLOWER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS/
    115X 48HSEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS/////)
2215 FORMAT (29X23HLOWER TOROIDAL BULKHEAD///)
2252 FORMAT (///41HMONOCOQUE WEIGHT (LOWER TOROIDAL BLKHD.)=F12.1)
2255 FORMAT (/47HAPPROX. REDUCED WEIGHT (LOWER TOROIDAL BLKHD.)=F12.1//
    1///)
2256 FORMAT (24X32HLOWER OBLATE SPHEROIDAL BULKHEAD///)
2257 FORMAT (///48HMONOCOQUE WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)=F12.
    11)
2258 FORMAT (/54HAPPROX. REDUCED WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)=
    1F12.1/////)
3000 FORMAT(1H1,10X,14HEND OF PROGRAM)
     END
*    FORTRAN
     FUNCTION AVG(T1,T2)
     KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
     KOMMON GAMMO,GAMMI,FACSA ,                E,FTU
     KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
     KOMMON PI,XRIU,X1,X2,Y1,Y2
     AVG=(T1+T2)/2.
```

```
      RETURN
      END
*     FORTRAN
      FUNCTION FPH(W,PT)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLM2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                  E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2,XA
      FPH=FACSA *((PT*XA*(2.*CUL+XA)+W/PI)*SQRTF(A**4
     1-XA**2*(A**2-B**2)))/(2.*B*XA*(CUL+XA))
      RETURN
      END
*     FORTRAN
      FUNCTION FPHOU(PI,WP)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLM2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                  E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      X=XRIU**2
      RI2=RI**2
      RI4=RI2**2
      FPHOU=FACSA*((PI-WP/(PI*X))*SQRTF(RI4-X*(RI2-CILOX**2)))/(2.*
     1 CILOX)
      RETURN
      END
*     FORTRAN
      FUNCTION FTH(W,PT)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLM2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                  E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2,XA
      XX=XA**2
```

```
      PX=PI*XA
      A4=A**4
      AXB=A4-XX*(A**2-B**2)
      T=2.
      FTH=((T*PX*(CUL+XA)*AXB-A4*(PX*(T*CUL+XA)+W/PI))
     1/(T*B*XX*SQRTF(AXB)))*FACSA
      RETURN
      END
*     FORTRAN
      FUNCTION FTHOU(PT,WP)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                   E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,ENC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      RI2=RI**2
      RI4=RI2**2
      CR=CILOX**2-RI2
      XR2=XRIU**2
      T=2.
      FTHOU =FACSA*(PT*(RI4+T*XR2*CR)+WP/(PI*XR2)*RI4)
     1/(T*CILOX*SQRTF(RI4+XR2*CR))
      RETURN
      END
*     FORTRAN
      FUNCTION PPT(PO,G,HO,XL,BC)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                   E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,ENC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      PPT=PO+ACCEL*G*(HO-XL-BC)
      RETURN
      END
```

```
*     FORTRAN
      FUNCTION WP1(H,P)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      B2=B**2
      H2=H**2
      SBH=SQRTF(B2-H2)
      APGA=A*PI*GAMMO*ACCEL
      WP1=CUL*APGA/B*(H*SBH+B2   *ATANF(H/SBH))+P*A*APGA/B2*H*(B2-H2/3.)
      RETURN
      END
*     FORTRAN
      FUNCTION WP2(HP,P)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      T=2.
      TH=3.
      APGA=A*PI*GAMMO*ACCEL
      B2=B**2
      BHP=B-HP
      WP2=T*CLL*APGA/B*(B2*PI/4.-BHP/T*SQRTF(T*B*HP-
     1HP**2)-B2/T* ATANF(BHP/SQRTF(B2-BHP**2)))+P*A*APGA/B2*
     2(-B*B2/TH+B2*HP+BHP**3/TH)
      RETURN
      END

*     FORTRAN
      FUNCTION WP3(HI,HE)
      KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                E,FTU
      KOMMON DEN,XXROU,XXRIU,FK,WAF,TMIN,FNC,XXFLO,XXFLI
```

```
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      C2=CILOX**2
      WP3=PI*RI**2/C2*GAMMI*ACCEL*[[C2*HI-HI**3/3.]-[C2*HE-HE**3/3.]]
      RETURN
      END

*     FORTRAN
      FUNCTION WP4[HK]
      KOMMON COL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                 E,FTU
      KOMMON DEN,XXROJ,XXRIU,FK,WAF,TMIN,FOC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      C2=CILOX**2
      WP4=PI*RI**2/C2*GAMMI*ACCEL*[-CILOX*C2/3.+C2*HK+[CILOX-HK]**3/3.]
      RETURN
      END

*     FORTRAN
      FUNCTION WTE[C,T]
      KOMMON COL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
      KOMMON GAMMO,GAMMI,FACSA ,                 E,FTU
      KOMMON DEN,XXROJ,XXRIU,FK,WAF,TMIN,FOC,XXFLO,XXFLI
      KOMMON PI,XRIU,X1,X2,Y1,Y2
      XX=X1-X2
      WTE=2.*PI*[C+X2+XX/2.]*SQRTF[[Y1-Y2]**2+XX**2]*DEN*T
      RETURN
      END
```

6. Output Examples

Following are two examples of program output. Example 1 is typical output data for a semi-toroidal tank; Example 2 is typical output data for a modified ogival tank.

EXAMPLE 1 - OUTPUT DATA FOR SEMI-TOROIDAL TANK

MEMBRANE STRESS ANALYSIS OF SEMI-TOROIDAL AND MODIFIED OGIVAL TANKS

INPUT DATA

CUL	CLL	RI	RO	A
51.063	51.063	7.313	94.813	43.750
B	CILOX	FL	WLH2	ACCEL
30.940	8.000	60.240	0.	1.000
PUO	GAMMO	GAMMI	FACSA	PUI
32.000	0.03613	0.	1.000	0.
HOPO	HOPI	F	FTU	DEN
122.119	0.	10600000.	45000.	0.102
XXRUU	XXRIU	FK	WAF	TMIN
44.00	3.00	0.179	0.670	0.312
FNC	XXFLO	XXFLI	TRANS	
0.	30.00	0.	2.00	

UPPER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA
UPPER TOROIDAL BULKHEAD

X,	Y	FPHI	FTHETA	T	TAVG	WTELE
-43.750	0.	5612.3	1633.5	0.125	0.115	5.788
-41.761	9.222	4759.5	1613.4	0.106	0.100	2.742
-39.773	12.890	4203.6	1594.0	0.093	0.089	2.356
-37.784	15.597	3811.4	1575.3	0.085	0.081	2.205
-35.795	17.789	3518.9	1556.7	0.078	0.076	2.142
-33.807	19.639	3291.7	1538.0	0.073	0.071	2.122
-31.818	21.236	3109.5	1518.5	0.069	0.067	2.126
-29.830	22.633	2959.6	1498.2	0.066	0.064	2.146
-27.841	23.867	2833.5	1476.9	0.063	0.062	2.176
-25.852	24.960	2725.6	1454.2	0.061	0.060	2.213
-23.864	25.932	2631.8	1430.1	0.058	0.058	2.255
-21.875	26.795	2549.1	1404.5	0.057	0.056	2.301

-19.886	27.559	2475.2	1377.2	0.055	0.054	2.349
-17.898	28.233	2408.5	1348.2	0.054	0.053	2.400
-15.909	28.822	2347.7	1317.3	0.052	0.052	2.452
-13.920	29.332	2291.6	1284.4	0.051	0.050	2.505
-11.932	29.767	2239.6	1249.4	0.050	0.049	2.559
-9.943	30.130	2190.8	1212.3	0.049	0.048	2.614
-7.955	30.424	2144.7	1172.9	0.048	0.047	2.669
-5.966	30.651	2101.0	1131.0	0.047	0.046	2.724
-3.977	30.812	2059.1	1086.7	0.046	0.045	2.780
-1.989	30.908	2018.7	1039.6	0.045	0.044	2.835
-0.000	30.940	1979.6	989.8	0.044	0.044	2.891
1.989	30.908	1941.6	937.0	0.043	0.043	2.947
3.977	30.812	1904.3	881.1	0.042	0.042	3.003
5.966	30.651	1867.6	821.9	0.042	0.041	3.059
7.955	30.424	1831.3	759.1	0.041	0.040	3.115
9.943	30.130	1795.2	692.5	0.040	0.039	3.173
11.932	29.767	1759.3	621.9	0.039	0.039	3.231
13.920	29.332	1723.4	546.9	0.038	0.038	3.290

15.909	28.822	1687.3	467.1	0.037	0.037	3.351
17.898	28.233	1650.9	382.3	0.037	0.036	3.415
19.886	27.559	1614.1	291.9	0.036	0.035	3.482
21.875	26.795	1576.7	195.3	0.035	0.035	3.555
23.864	25.932	1538.7	92.0	0.034	0.037	4.029
25.852	24.960	1499.9	-18.7	0.041	0.074	8.345
27.841	23.867	1460.1	-137.9	0.107	0.125	14.997
29.830	22.633	1419.3	-266.4	0.144	0.158	20.125
31.818	21.236	1377.2	-405.6	0.172	0.184	25.177
33.807	19.639	1333.7	-557.0	0.195	0.206	30.737
35.795	17.789	1288.5	-722.5	0.216	0.225	37.516
37.784	15.597	1241.5	-904.5	0.234	0.243	46.958
39.773	12.890	1192.4	-1105.8	0.251	0.259	63.571
41.761	9.222	1140.8	-1329.8	0.267	0.273	154.859
43.750	0.010	1086.4	-1567.6	0.280		

MONOCOQUE WEIGHT (UPPER TOROIDAL BLKHD.)= 501.3

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (UPPER TOROIDAL BLKHD.)= 367.8

OUTPUT DATA
UPPER OBLATE SPHEROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
0.	8.000	0.	0.	0.312		
					0.312	0.604
2.438	7.542	0.	0.	0.312		
					0.312	2.124
4.875	5.963	0.	0.	0.312		
					0.312	7.850
7.313	0.	0.	0.	0.312		

MONOCOQUE WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)= 10.6

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)= 10.6

GENERAL LOADS FOR THE CENTER TUBE AND OUTER CYLINDER
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA

CENTER TUBE LOADS			OUTER CYLINDER LOADS		
XFLI	FPHI	FTHETA	XFLO	FPHI	FTHETA
0.	5612.3	-242.2	0.	1086.4	3140.0
2.008	5612.3	-242.7	2.008	1086.4	3146.9
4.016	5612.3	-243.3	4.016	1086.4	3153.8
6.024	5612.3	-243.8	6.024	1086.4	3160.6
8.032	5612.3	-244.3	8.032	1086.4	3167.5
10.040	5612.3	-244.8	10.040	1086.4	3174.4
12.048	5612.3	-245.4	12.048	1086.4	3181.3
14.056	5612.3	-245.9	14.056	1086.4	3188.2
16.064	5612.3	-246.4	16.064	1086.4	3195.0
18.072	5612.3	-247.0	18.072	1086.4	3201.9
20.080	5612.3	-247.5	20.080	1086.4	3208.8
22.088	5612.3	-248.0	22.088	1086.4	3215.7
24.096	5612.3	-248.6	24.096	1086.4	3222.5
26.104	5612.3	-249.1	26.104	1086.4	3229.4
28.112	5612.3	-249.6	28.112	1086.4	3236.3
30.120	5612.3	-250.1	30.120	1086.4	3243.2
32.128	5612.3	-250.7	32.128	1086.4	3250.1
34.136	5612.3	-251.2	34.136	1086.4	3256.9
36.144	5612.3	-251.7	36.144	1086.4	3263.8
38.152	5612.3	-252.3	38.152	1086.4	3270.7
40.160	5612.3	-252.8	40.160	1086.4	3277.6
42.168	5612.3	-253.3	42.168	1086.4	3284.5
44.176	5612.3	-253.9	44.176	1086.4	3291.3
46.184	5612.3	-254.4	46.184	1086.4	3298.2
48.192	5612.3	-254.9	48.192	1086.4	3305.1

50.200	5612.3	-255.5	50.200	1086.4	3312.0
52.208	5612.3	-256.0	52.208	1086.4	3318.8
54.216	5612.3	-256.5	54.216	1086.4	3325.7
56.224	5612.3	-257.0	56.224	1086.4	3332.6
58.232	5612.3	-257.6	58.232	1086.4	3339.5
60.240	5612.3	-258.1	60.240	1086.4	3346.4

LOWER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA
LOWER TOROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
-43.750	0.	6333.7	1858.7	0.141		
					0.130	6.534
-41.761	9.222	5374.4	1836.0	0.119		
					0.112	3.097
-39.773	12.890	4749.4	1816.1	0.106		
					0.101	2.662
-37.784	15.597	4308.7	1797.0	0.096		
					0.092	2.493
-35.795	17.789	3980.3	1778.0	0.088		
					0.086	2.423
-33.807	19.639	3725.4	1758.4	0.083		
					0.081	2.402
-31.818	21.236	3521.1	1737.8	0.078		
					0.076	2.408
-29.830	22.633	3353.0	1716.0	0.075		
					0.073	2.432
-27.841	23.867	3211.9	1692.8	0.071		
					0.070	2.467

-25.852	24.960	3091.1	1667.8	0.069	0.068	2.510
-23.864	25.932	2986.0	1640.9	0.066	0.065	2.559
-21.875	26.795	2893.4	1612.1	0.064	0.063	2.612
-19.886	27.559	2810.7	1581.0	0.062	0.062	2.668
-17.898	28.233	2736.0	1547.8	0.061	0.060	2.726
-15.909	28.822	2667.8	1512.1	0.059	0.059	2.786
-13.920	29.332	2604.9	1473.9	0.058	0.057	2.848
-11.932	29.767	2546.4	1433.1	0.057	0.056	2.910
-9.943	30.130	2491.5	1389.5	0.055	0.055	2.973
-7.955	30.424	2439.6	1343.1	0.054	0.054	3.036
-5.966	30.651	2390.2	1293.8	0.053	0.053	3.099
-3.977	30.812	2342.8	1241.3	0.052	0.052	3.163
-1.989	30.908	2297.0	1185.5	0.051	0.051	3.226
-0.000	30.940	2252.6	1126.3	0.050	0.050	3.290
1.989	30.908	2209.2	1063.5	0.049	0.049	3.353
3.977	30.812	2166.6	996.9	0.048	0.048	3.416
5.966	30.651	2124.6	926.3	0.047	0.047	3.480

7.955	30.424	2083.0	851.5	0.046	0.046	3.543
9.943	30.130	2041.6	772.2	0.045	0.045	3.607
11.932	29.767	2000.2	688.1	0.044	0.044	3.672
13.920	29.332	1958.7	598.8	0.044	0.043	3.738
15.909	28.822	1916.9	504.0	0.043	0.042	3.806
17.898	28.233	1874.7	403.2	0.042	0.041	3.877
19.886	27.559	1832.0	296.0	0.041	0.040	3.952
21.875	26.795	1788.6	181.7	0.040	0.039	4.032
23.864	25.932	1744.3	59.6	0.039	0.059	6.351
25.852	24.960	1699.1	-71.1	0.079	0.106	11.970
27.841	23.867	1652.7	-211.4	0.132	0.150	17.952
29.830	22.633	1605.1	-362.4	0.168	0.182	23.166
31.818	21.236	1555.9	-525.6	0.196	0.208	28.458
33.807	19.639	1505.1	-702.7	0.219	0.230	34.371
35.795	17.789	1452.4	-896.1	0.240	0.250	41.653
37.784	15.597	1397.5	-1108.7	0.259	0.268	51.877
39.773	12.890	1340.1	-1344.3	0.277	0.285	70.004
41.761	9.222	1279.8	-1608.8	0.293	0.302	170.940

43.750	0.010	1216.1	-1923.1	0.310

MONOCOQUE WEIGHT (LOWER TOROIDAL BLKHD.)= 564.5

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (LOWER TOROIDAL BLKHD.)= 414.5

OUTPUT DATA
LOWER OBLATE SPHEROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
0.	8.000	0.	0.	0.312		
					0.312	0.604
2.438	7.542	0.	0.	0.312		
					0.312	2.124
4.875	5.963	0.	0.	0.312		
					0.312	7.850
7.313	0.	0.	0.	0.312		

MONOCOQUE WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)= 10.6

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)= 10.6

TOTAL CENTER SUPPORT REACTION= 33148.

FPHI (CENTER SUPPORT) = 721.4

FTHETA (CENTER SUPPORT) = 0.

TOTAL LIQUID REACTION (LOWER SKIRT) = 77248.

FPHI (LOWER SKIRT) = 129.7

FTHETA (LOWER SKIRT) = 0.

TOTAL WEIGHT LIQUID= 110396.

EXAMPLE 2 - OUTPUT DATA FOR MODIFIED OGIVAL TANK

MEMBRANE STRESS ANALYSIS OF SEMI-TOROIDAL AND MODIFIED OGIVAL TANKS

INPUT DATA

CUL	CLL	RI	RO	A
78.000	78.000	4.000	198.000	120.000
H	CILOX	FI	WLH2	ACCEL
120.000	2.000	100.000	0.	1.000

PUO	GAMMO	GAMMI	FACSA	PUI
65.350	0.03613	0.03613	1.000	65.350
HOPO	HOPI	E	FTU	DEN
339.999	292.934	12500000.	90000.	0.102
XXROU	XXRIU	FK	WAF	TMIN
60.00	2.00	0.250	0.670	0.001
FNC	XXFLO	XXFLI	TRANS	
0.	5.00	10.00	2.00	

UPPER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA
UPPER TOROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
-74.000	94.467	80574.6	3925.5	0.895		
					0.753	6.083
-72.000	96.000	55024.5	3923.2	0.611		
					0.498	12.571

-68.000	98.874	34583.2	3919.1	0.384	0.336	12.362
-64.000	101.509	25821.5	3915.5	0.287	0.260	12.450
-60.000	103.923	20953.1	3912.4	0.233	0.216	12.627
-56.000	105.132	17854.6	3909.9	0.198	0.186	12.847
-52.000	108.148	15709.1	3907.8	0.175	0.166	13.092
-48.000	109.982	14135.5	3906.3	0.157	0.150	13.356
-44.000	111.642	12932.0	3905.2	0.144	0.138	13.636
-40.000	113.137	11981.7	3904.5	0.133	0.129	13.930
-36.000	114.473	11212.5	3904.2	0.125	0.121	14.238
-32.000	115.655	10577.0	3904.4	0.118	0.115	14.559
-28.000	116.688	10043.2	3905.0	0.112	0.109	14.893
-24.000	117.576	9568.5	3906.1	0.107	0.104	15.240
-20.000	118.322	9196.7	3907.5	0.102	0.100	15.602
-16.000	118.929	8855.5	3909.4	0.098	0.097	15.979
-12.000	119.398	8555.8	3911.6	0.095	0.094	16.372
-8.000	119.733	8290.5	3914.3	0.092	0.091	16.781
-4.000	119.933	8054.0	3917.5	0.089	0.088	17.208
0.	120.000	7842.0	3941.0	0.087	0.086	17.654

4.000	119.933	7650.8	3925.0	0.085	0.084	18.121
8.000	119.733	7477.6	3929.4	0.083	0.082	18.610
12.000	119.398	7320.0	3934.3	0.081	0.081	19.124
16.000	118.929	7176.1	3939.6	0.080	0.079	19.665
20.000	118.322	7044.1	3945.4	0.078	0.078	20.234
24.000	117.576	6922.7	3951.7	0.077	0.076	20.836
28.000	116.688	6810.7	3958.5	0.076	0.075	21.474
32.000	115.655	6707.1	3965.9	0.075	0.074	22.152
36.000	114.473	6611.1	3973.8	0.073	0.073	22.874
40.000	113.137	6521.8	3982.3	0.072	0.072	23.646
44.000	111.642	6438.7	3991.4	0.072	0.071	24.475
48.000	109.982	6361.2	4001.2	0.071	0.070	25.368
52.000	108.148	6288.7	4011.7	0.070	0.069	26.335
56.000	106.132	6220.9	4022.9	0.069	0.069	27.388
60.000	103.923	6157.4	4035.0	0.068	0.068	28.541
64.000	101.509	6097.8	4047.9	0.068	0.067	29.812
68.000	98.874	6041.8	4061.8	0.067	0.067	31.225
72.000	96.000	5989.2	4076.8	0.067	0.066	32.807

76.000	92.865	5939.7	4093.0	0.066	0.066	34.601
80.000	89.443	5893.2	4110.5	0.065	0.065	36.658
84.000	85.697	5849.4	4129.7	0.065	0.065	39.054
88.000	81.584	5808.2	4150.6	0.065	0.064	41.897
92.000	77.045	5769.5	4173.8	0.064	0.064	45.355
96.000	72.000	5733.1	4199.6	0.064	0.064	49.695
100.000	65.332	5698.9	4228.8	0.063	0.063	55.384
104.000	59.867	5667.0	4262.5	0.063	0.063	63.339
108.000	52.307	5637.2	4302.6	0.063	0.062	75.699
112.000	43.081	5609.6	4352.8	0.062	0.062	99.389
116.000	30.725	5584.4	4423.0	0.062	0.062	241.013
120.000	0.	5562.2	4620.2	0.062		

MONOCOQUE WEIGHT (UPPER TOROIDAL BLKHD.)=   1496.3

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAFL FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (UPPER TOROIDAL BLKHD.)=   1496.3

OUTPUT DATA
UPPER OBLATE SPHEROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
0.	2.000	264.8	264.8	0.003	0.003	0.004
2.000	1.732	238.7	183.7	0.003	0.011	0.054
4.000	0.	132.4	-264.7	0.018		

MONOCOQUE WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)= 0.1

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (UPPER OBLATE SPHEROIDAL HEAD)= 0.0

GENERAL LOADS FOR THE CENTER TUBE AND OUTER CYLINDER
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA

CENTER TUBE LOADS

OUTER CYLINDER LOADS

XFLI	FPHI	FTHETA
0.	49820.1	0.
28.893	49820.1	0.
57.787	49820.1	0.
86.680	49820.1	0.
115.574	49820.1	0.
144.467	49820.1	0.
173.360	49820.1	0.
202.254	49820.1	0.
231.147	49820.1	0.
260.040	49820.1	0.
288.934	49820.1	0.

XFLO	FPHI	FTHETA
0.	5562.2	13797.7
20.000	5562.2	13940.8
40.000	5562.2	14083.9
60.000	5562.2	14227.0
80.000	5562.2	14370.0
100.000	5562.2	14513.1

LOWER TOROIDAL AND OBLATE SPHEROIDAL BULKHEADS
SEMI-TOROIDAL AND MODIFIED OGIVAL CONFIGURATIONS

OUTPUT DATA
LOWER TOROIDAL BULKHEAD

X	Y	FPHI	FTHETA	T	TAVG	WTELE
-74.000	94.467	95295.8	4623.5	1.059		
					0.891	7.195
-72.000	95.000	65082.1	4625.8	0.723		
					0.589	14.870
-68.000	98.874	40912.5	4660.0	0.455		
					0.397	14.626
-64.000	101.509	30555.1	4663.6	0.340		
					0.308	14.734
-60.000	103.923	24801.8	4666.6	0.276		
					0.255	14.949
-56.000	105.132	21141.1	4669.2	0.235		
					0.221	15.214

-52.000	108.148	18607.1	4671.2	0.207	0.196	15.510
-48.000	109.982	16749.1	4672.8	0.186	0.178	15.829
-44.000	111.642	15328.4	4673.9	0.170	0.164	16.166
-40.000	113.137	14206.9	4674.6	0.158	0.153	16.520
-36.000	114.473	13299.1	4674.8	0.148	0.144	16.890
-32.000	115.655	12549.2	4674.6	0.139	0.136	17.276
-28.000	116.688	11919.2	4674.0	0.132	0.129	17.677
-24.000	117.575	11382.5	4673.0	0.126	0.124	18.094
-20.000	118.322	10919.7	4671.5	0.121	0.119	18.527
-16.000	118.929	10516.5	4669.7	0.117	0.115	18.978
-12.000	119.398	10162.1	4667.4	0.113	0.111	19.446
-8.000	119.733	9848.1	4664.7	0.109	0.108	19.934
-4.000	119.933	9567.8	4661.6	0.106	0.105	20.442
0.	120.000	9316.1	4658.0	0.104	0.102	20.973
4.000	119.933	9088.8	4654.1	0.101	0.100	21.526
8.000	119.733	8882.4	4649.7	0.099	0.098	22.105
12.000	119.398	8694.2	4644.8	0.097	0.096	22.712
16.000	118.929	8521.8	4639.5	0.095	0.094	23.350

20.000	118.322	8363.2	4633.6	0.093	0.092	24.020
24.000	117.576	8216.8	4627.3	0.091	0.091	24.727
28.000	116.688	8081.2	4620.5	0.090	0.089	25.475
32.000	115.655	7955.2	4613.2	0.088	0.088	26.268
36.000	114.473	7837.8	4605.3	0.087	0.086	27.112
40.000	113.137	7728.1	4596.8	0.086	0.085	28.012
44.000	111.642	7625.3	4587.6	0.085	0.084	28.976
48.000	109.982	7528.7	4577.9	0.084	0.083	30.014
52.000	108.148	7437.8	4567.4	0.083	0.082	31.135
56.000	106.132	7351.9	4556.1	0.082	0.081	32.354
60.000	103.923	7270.7	4544.1	0.081	0.080	33.687
64.000	101.509	7193.7	4531.1	0.080	0.080	35.153
68.000	98.874	7120.6	4517.2	0.079	0.079	36.780
72.000	95.000	7051.0	4502.3	0.078	0.078	38.601
76.000	92.865	6984.6	4486.1	0.078	0.077	40.662
80.000	89.443	6921.1	4468.5	0.077	0.077	43.022
84.000	85.697	6860.3	4449.4	0.076	0.076	45.769
88.000	81.584	6802.0	4428.4	0.076	0.075	49.027

92.000	77.045	6745.8	4405.3	0.075	0.075	52.985
96.000	72.000	6691.7	4379.5	0.074	0.074	57.950
100.000	65.332	6639.5	4350.3	0.074	0.073	64.458
104.000	59.867	6588.8	4316.6	0.073	0.073	73.560
108.000	52.307	6539.5	4276.5	0.073	0.072	87.707
112.000	43.081	6491.4	4226.2	0.072	0.072	114.850
116.000	30.725	6444.0	4156.0	0.072	0.071	277.639
120.000	0.	6396.5	3958.9	0.071		

MONOCOQUE WEIGHT (LOWER TOROIDAL BLKHD.)= 1753.5

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (LOWER TOROIDAL BLKHD.)= 1753.5

OUTPUT DATA
LOWER OBLATE SPHEROIDAL BULKHEAD

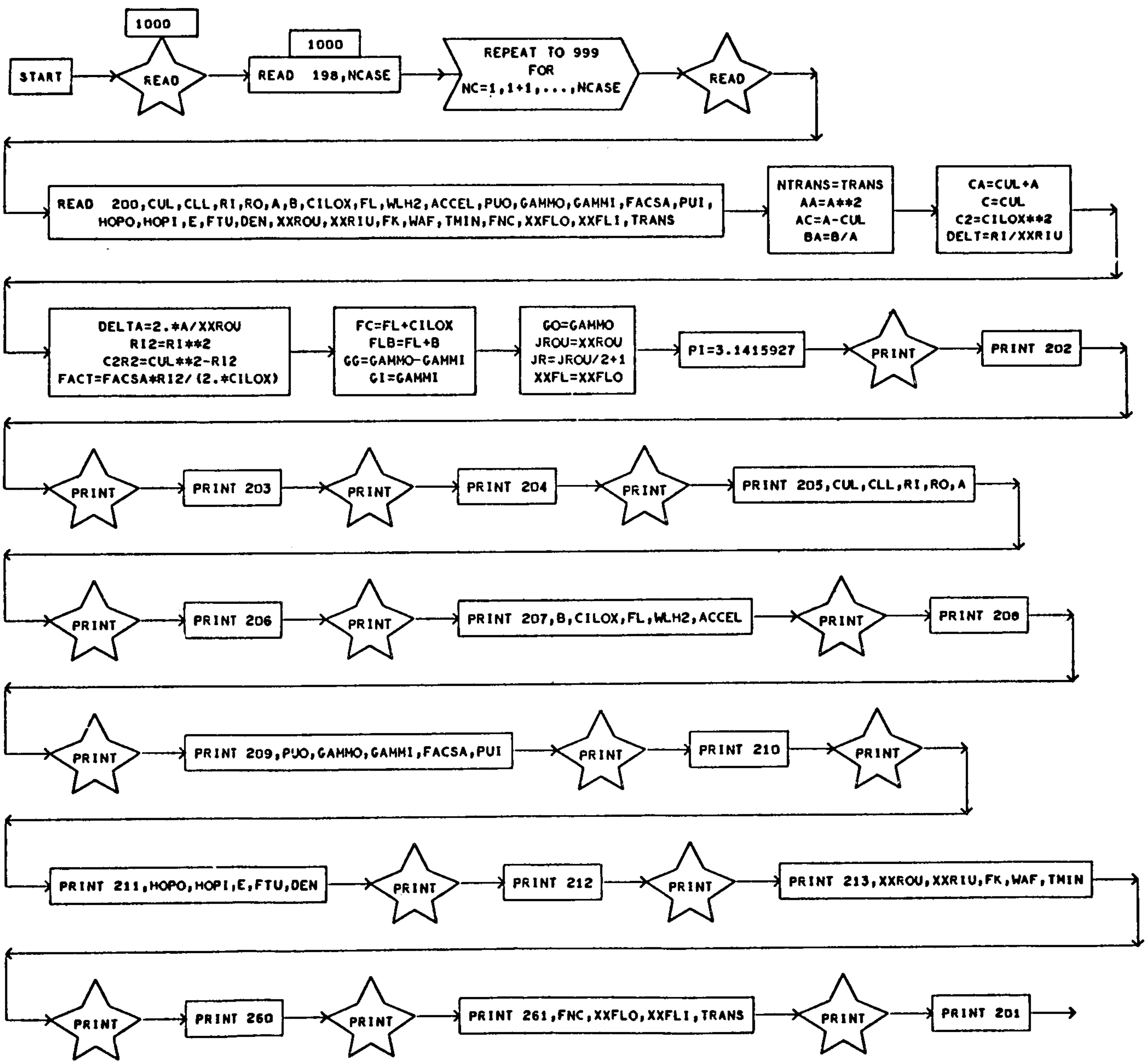
X	Y	FPHI	FTHETA	T	TAVG	WTELE
0.	2.000	307.1	307.1	0.003	0.003	0.004
2.000	1.732	276.8	212.9	0.003		

```
  4.000       0.         153.5      -307.2      0.020       0.011       0.058

MONOCOQUE WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)=          0.1

REDUCE THE MONOCOQUE WEIGHT BY SUBSTITUTING APPROXIMATE WAFFLE PATTERN WEIGHTS
(INPUT WAF) FOR THE HOOP COMPRESSION AREAS OF THE HEAD

APPROX. REDUCED WEIGHT (LOWER OBLATE SPHEROIDAL HEAD)=          0.0



TOTAL CENTER SUPPORT REACTION=    228567.

FPHI (CENTER SUPPORT) =    9099.2

FTHETA (CENTER SUPPORT) =        0.


TOTAL LIQUID REACTION (LOWER SKIRT) =   1037969.

FPHI (LOWER SKIRT) =     834.3

FTHETA (LOWER SKIRT) =        0.

TOTAL WEIGHT LIQUID=   1266655.
```

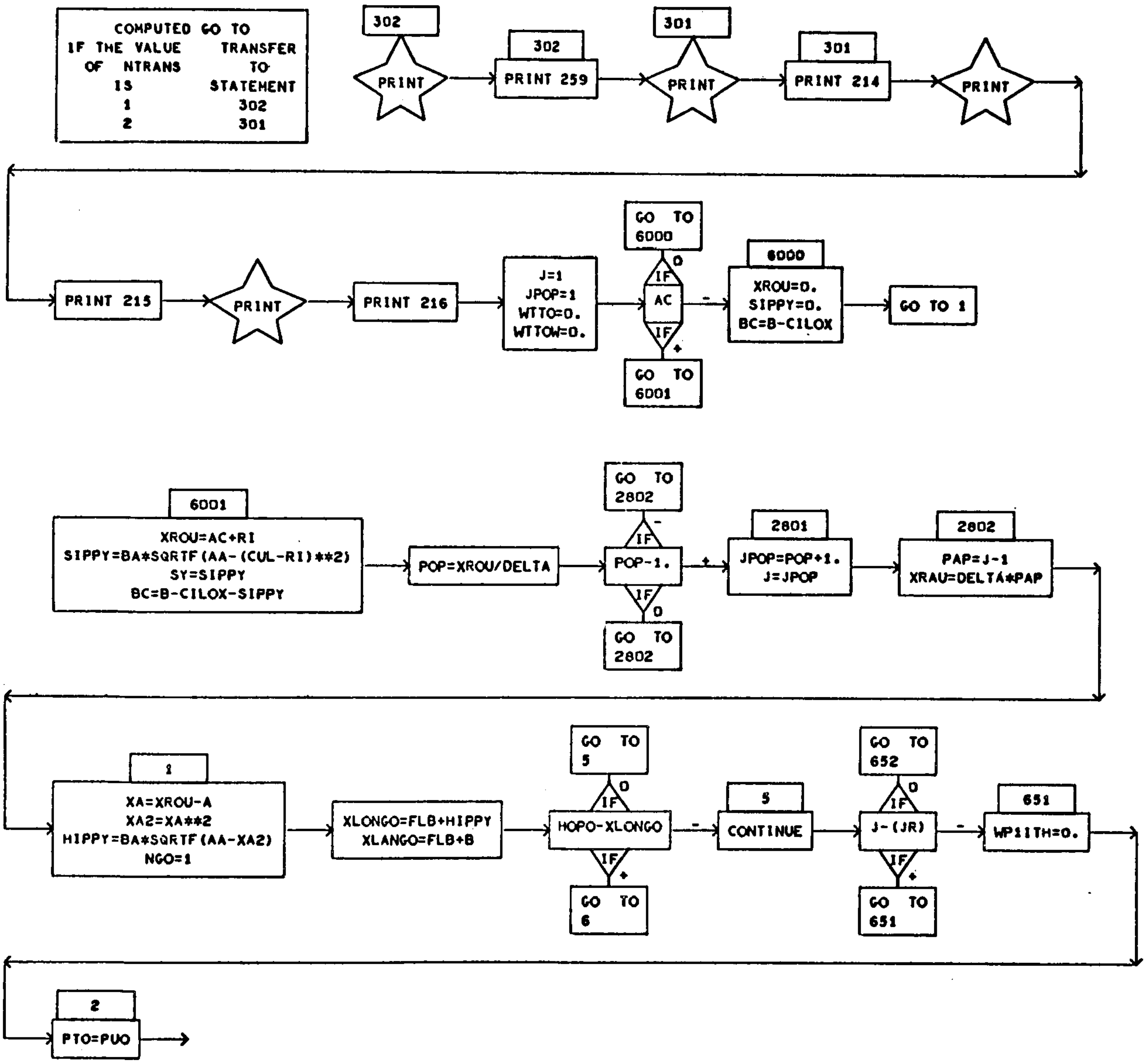
7. Detailed Flow Chart

A detailed flow chart is here included for the convenience of the program user.

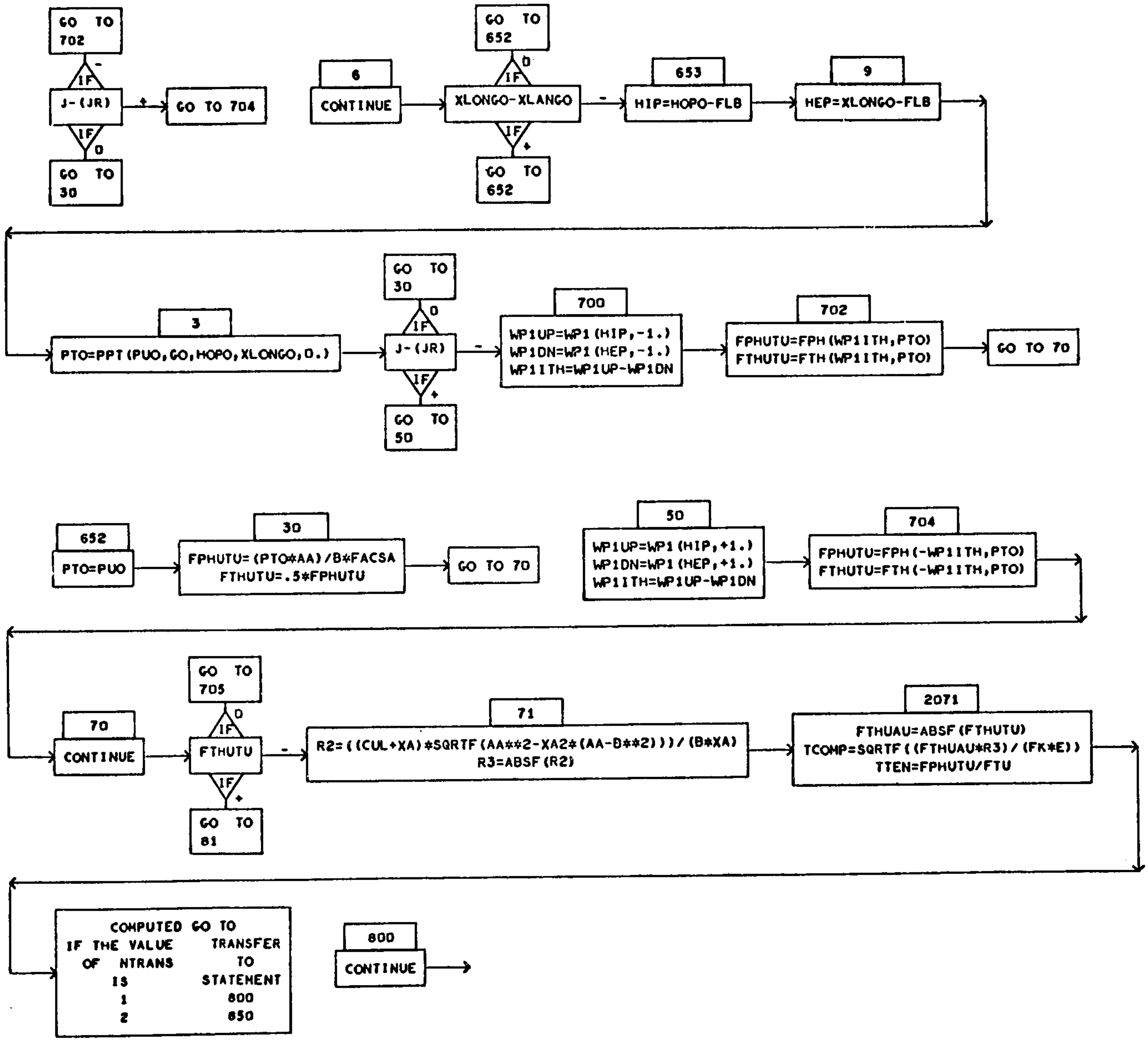
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



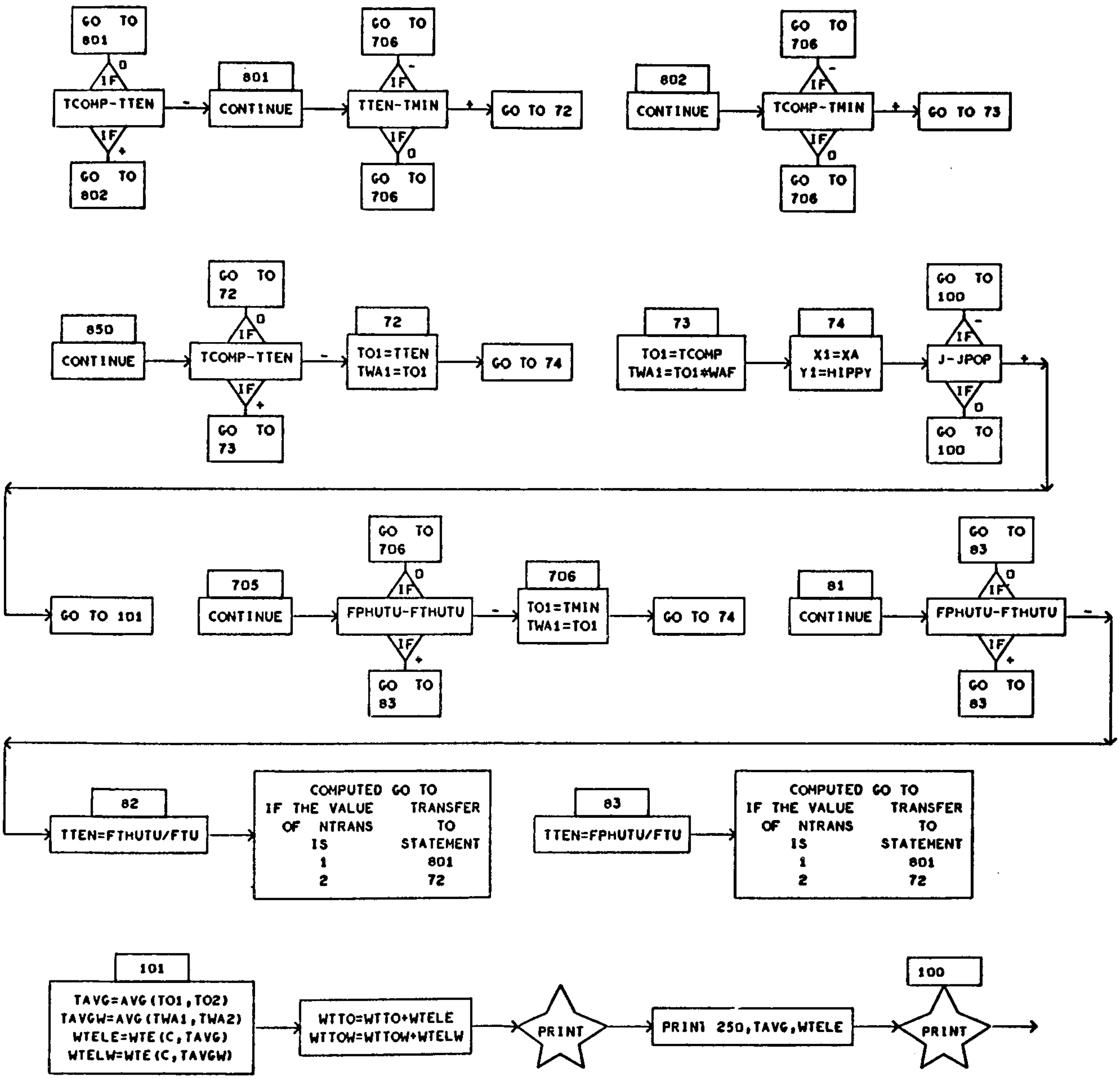
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



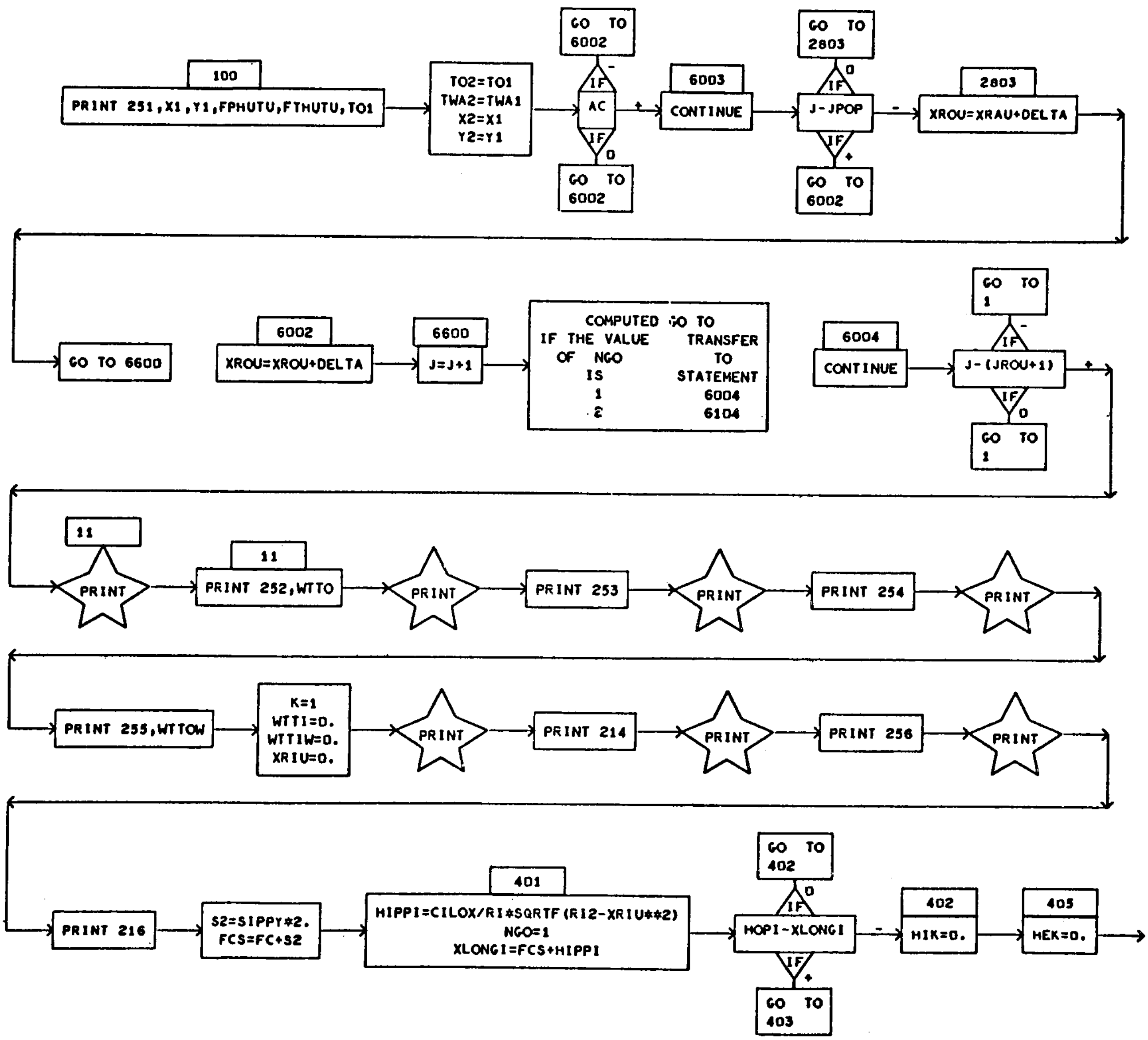
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



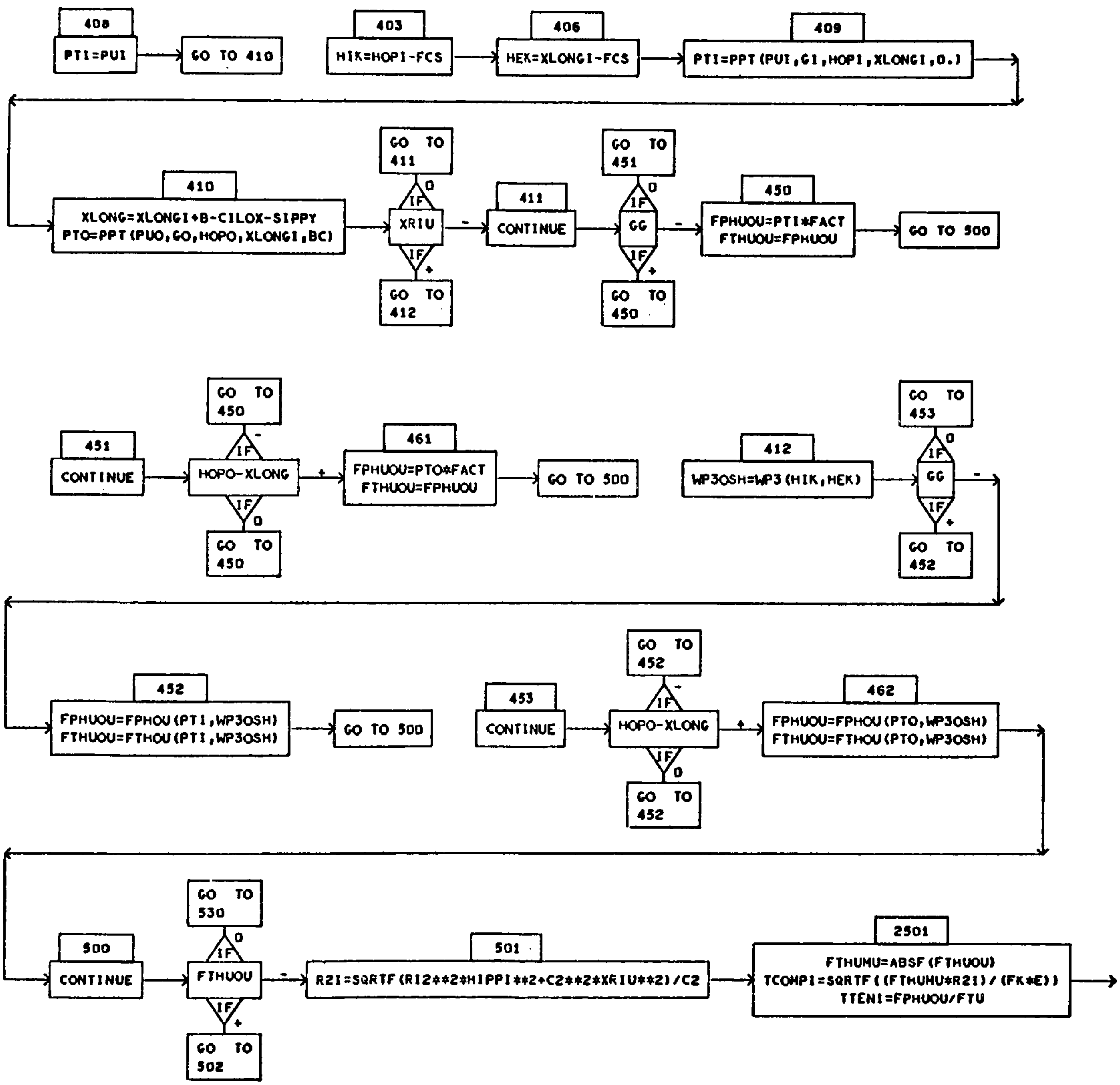
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

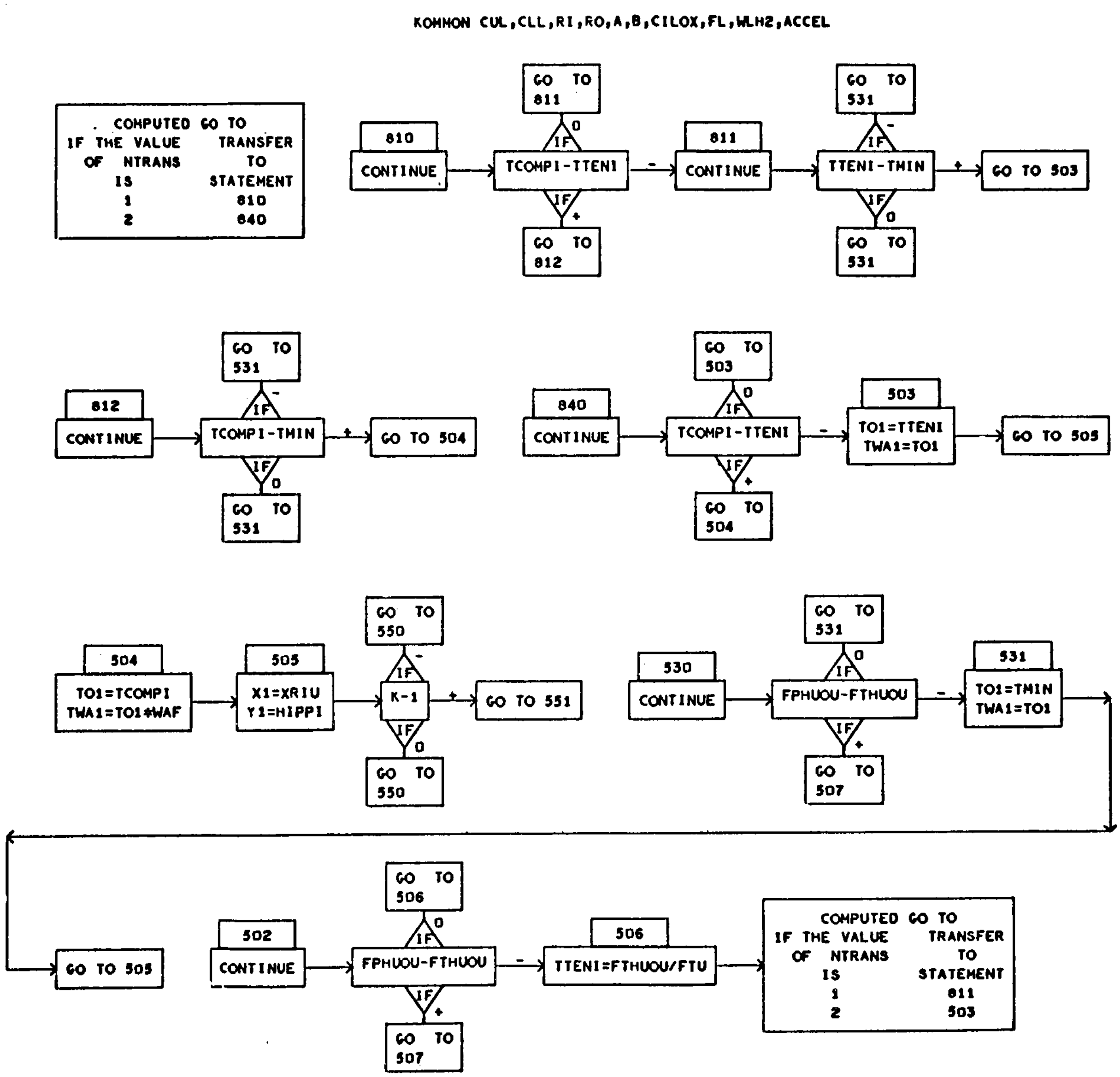


KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

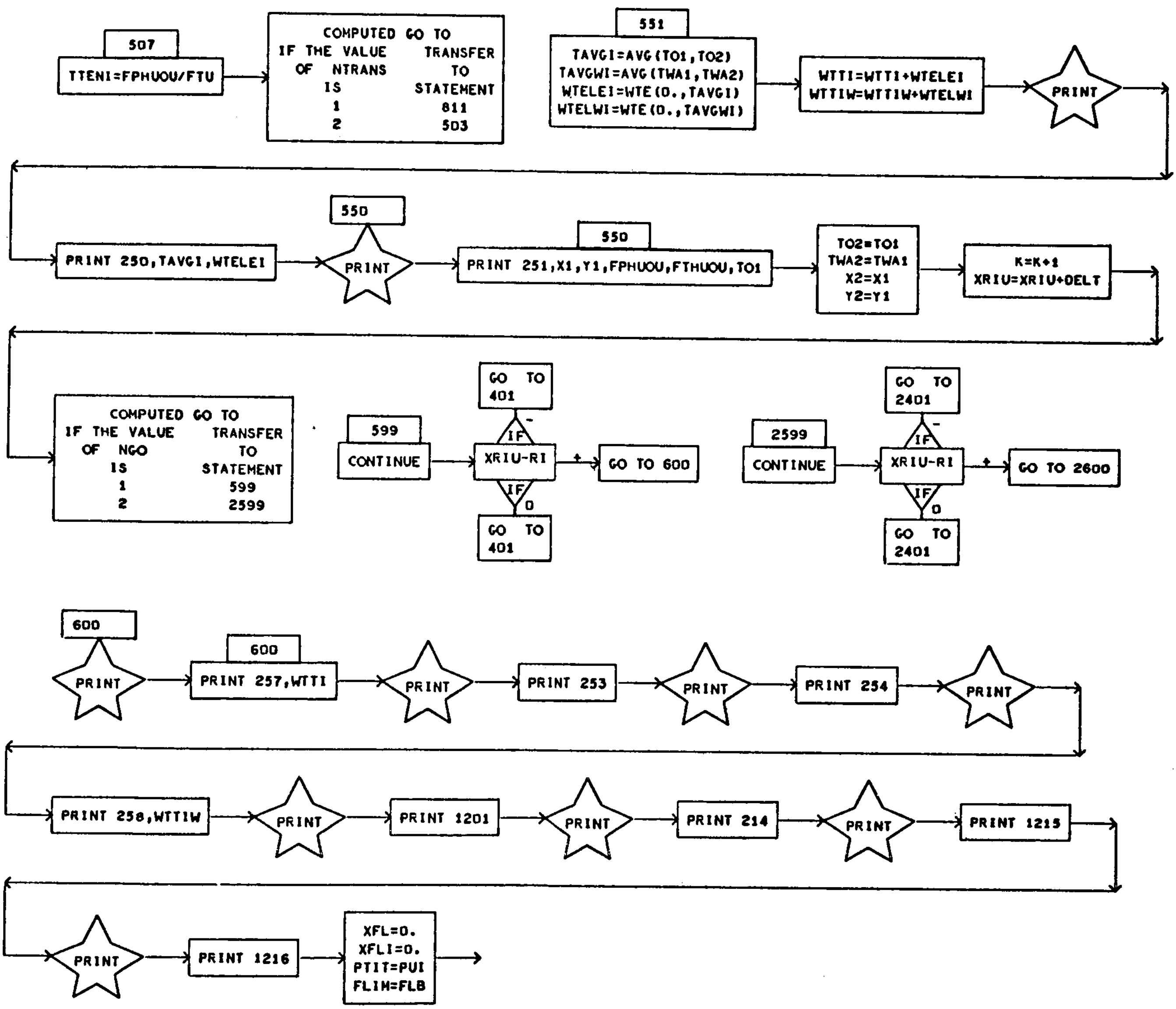


KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

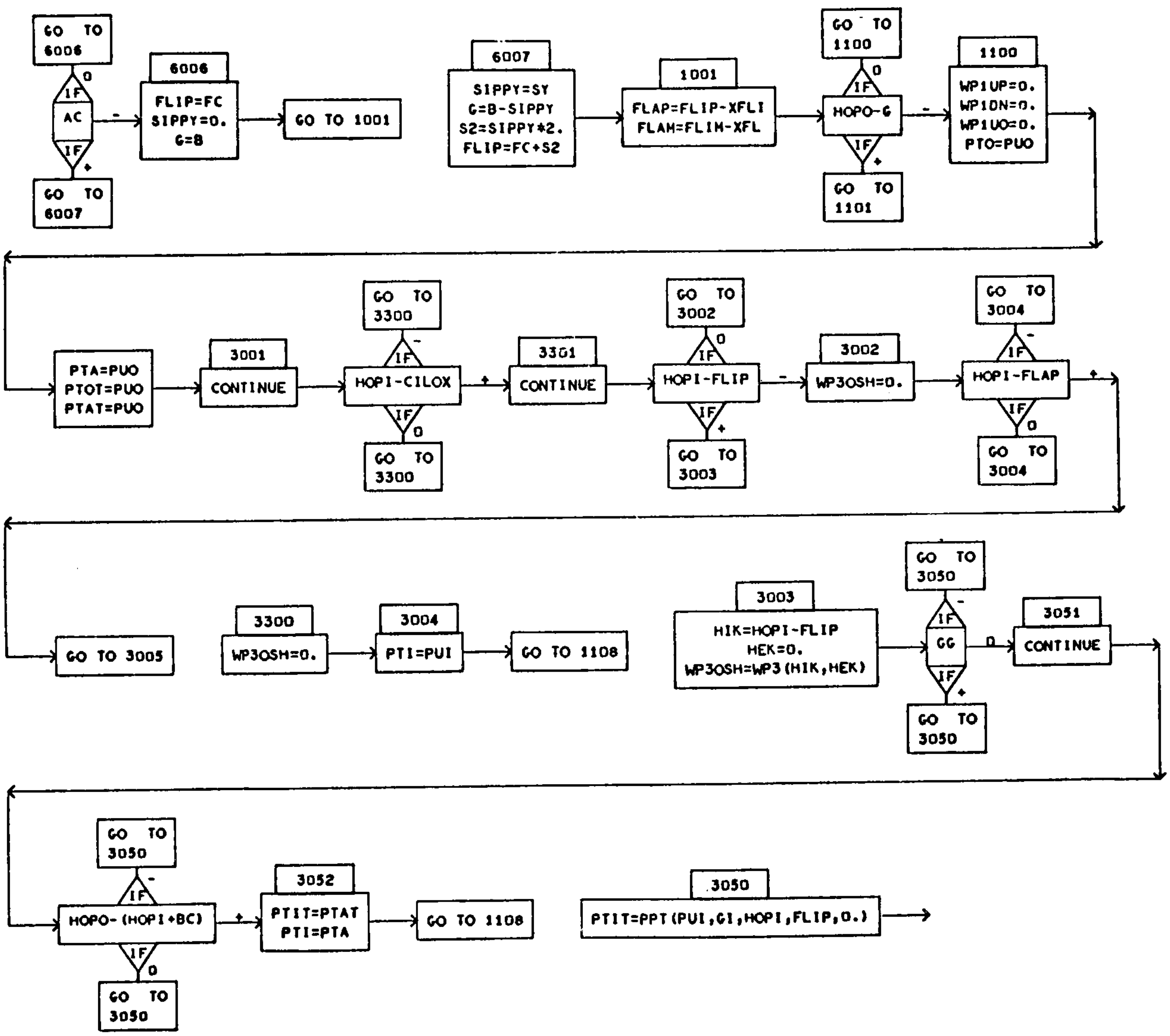



KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
COMPUTED GO TO
IF THE VALUE OF NTRANS IS 1 TRANSFER TO STATEMENT 810
IF THE VALUE OF NTRANS IS 2 TRANSFER TO STATEMENT 840
810
CONTINUE
IF
TCOMPI-TTENI
GO TO 811
GO TO 812
811
CONTINUE
TTENI-TMIN
GO TO 531
GO TO 531
GO TO 503
812
CONTINUE
TCOMPI-TMIN
GO TO 531
GO TO 531
GO TO 504
840
CONTINUE
TCOMPI-TTENI
GO TO 503
GO TO 504
503
TO1=TTENI
TWA1=TO1
GO TO 505
504
TO1=TCOMPI
TWA1=TO1*WAF
505
X1=XRIU
Y1=HIPPI
K-1
GO TO 550
GO TO 550
GO TO 551
530
CONTINUE
FPHUOU-FTHUOU
GO TO 531
GO TO 507
531
TO1=TMIN
TWA1=TO1
GO TO 505
502
CONTINUE
FPHUOU-FTHUOU
GO TO 506
GO TO 507
506
TTENI=FTHUOU/FTU
COMPUTED GO TO
IF THE VALUE OF NTRANS IS 1 TRANSFER TO STATEMENT 811
IF THE VALUE OF NTRANS IS 2 TRANSFER TO STATEMENT 503

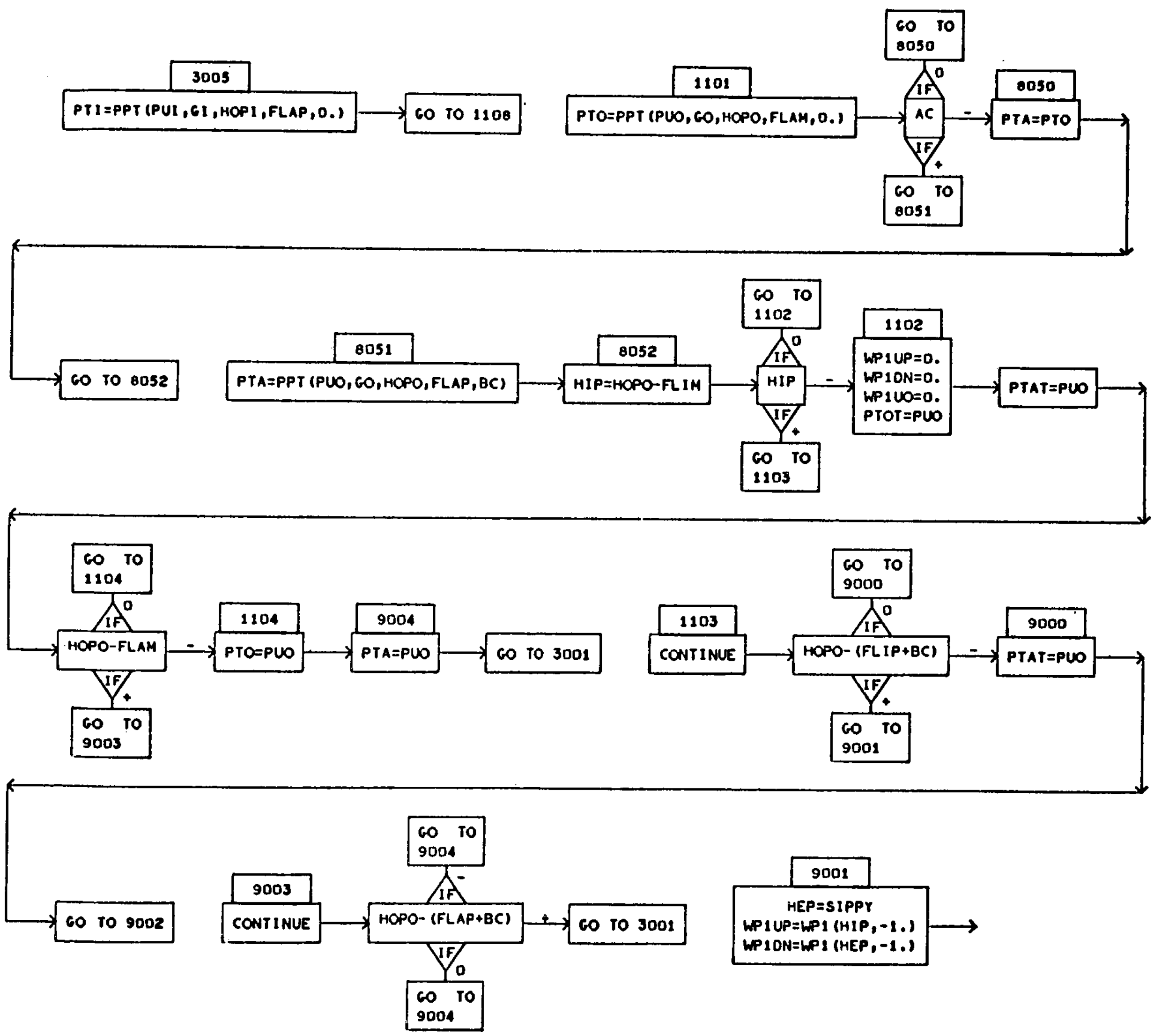
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



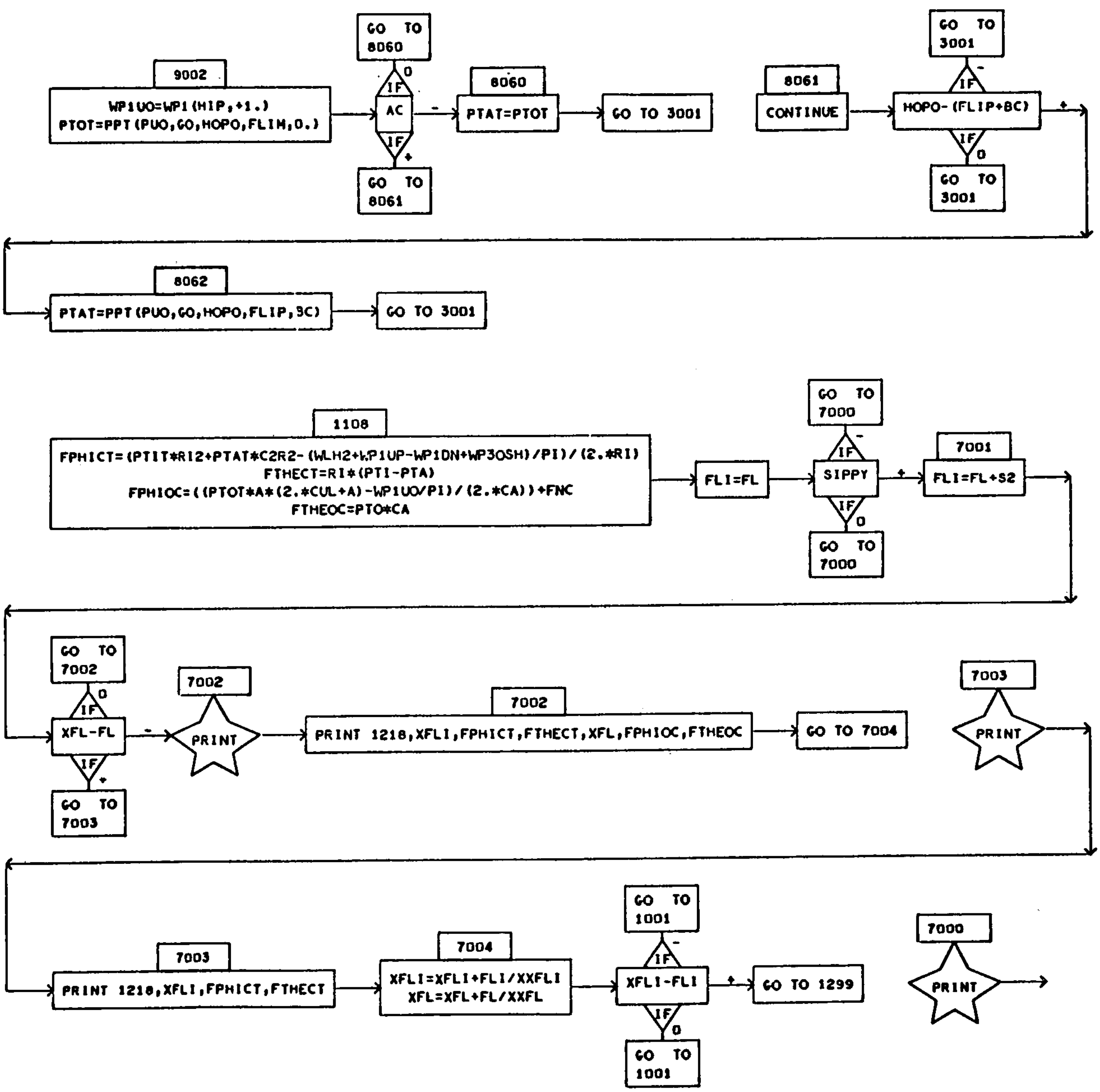
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

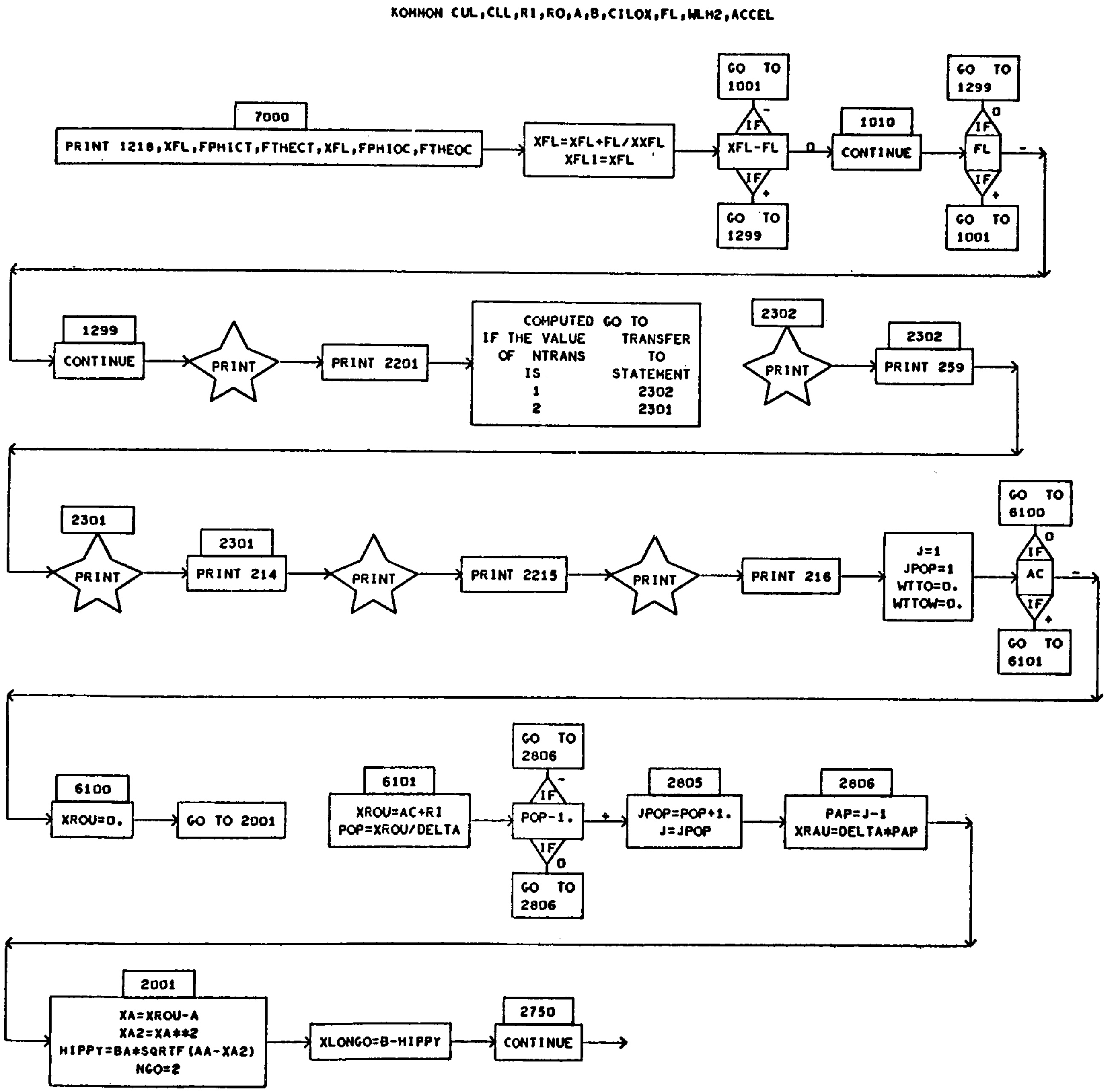


KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



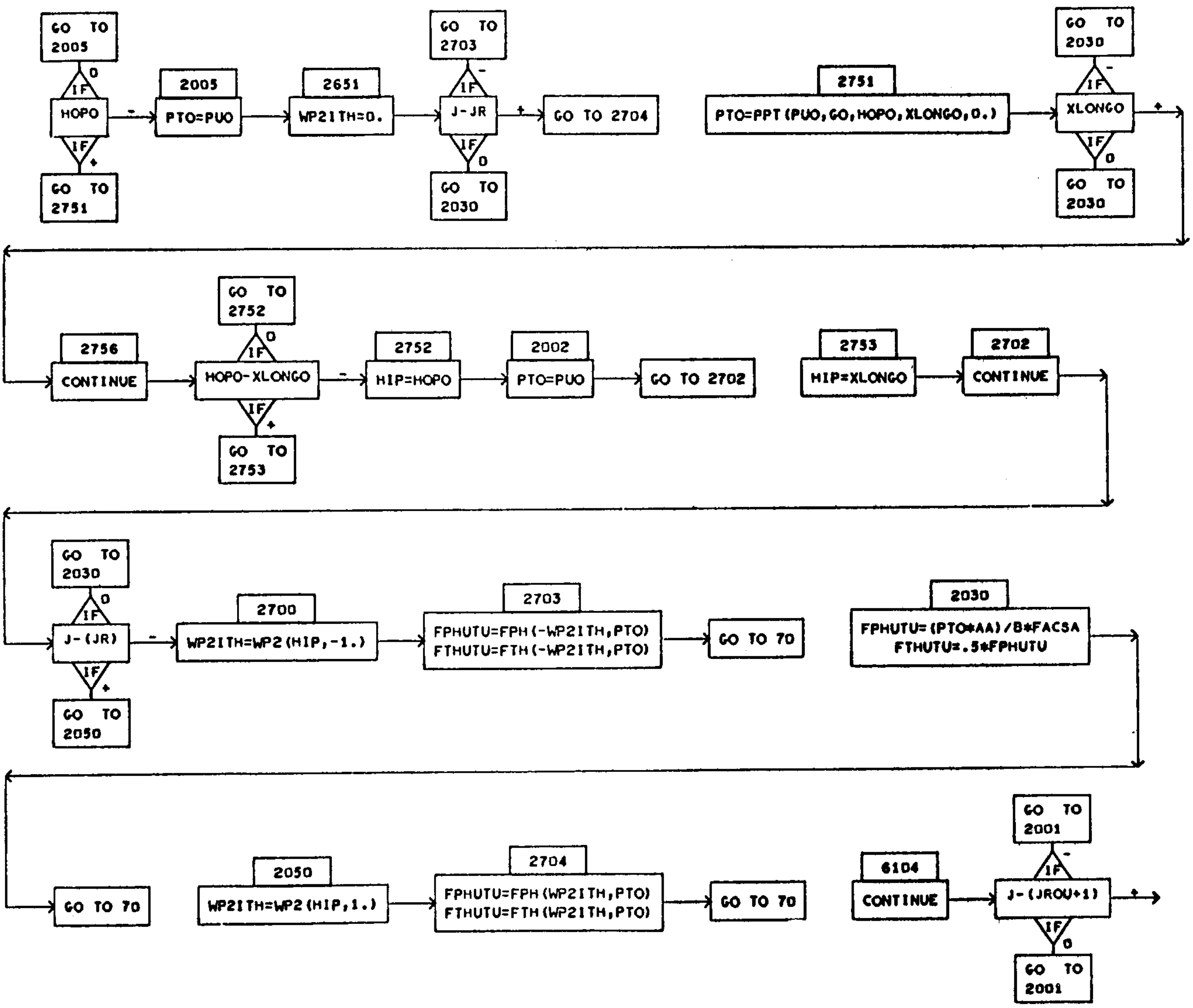
KOHMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



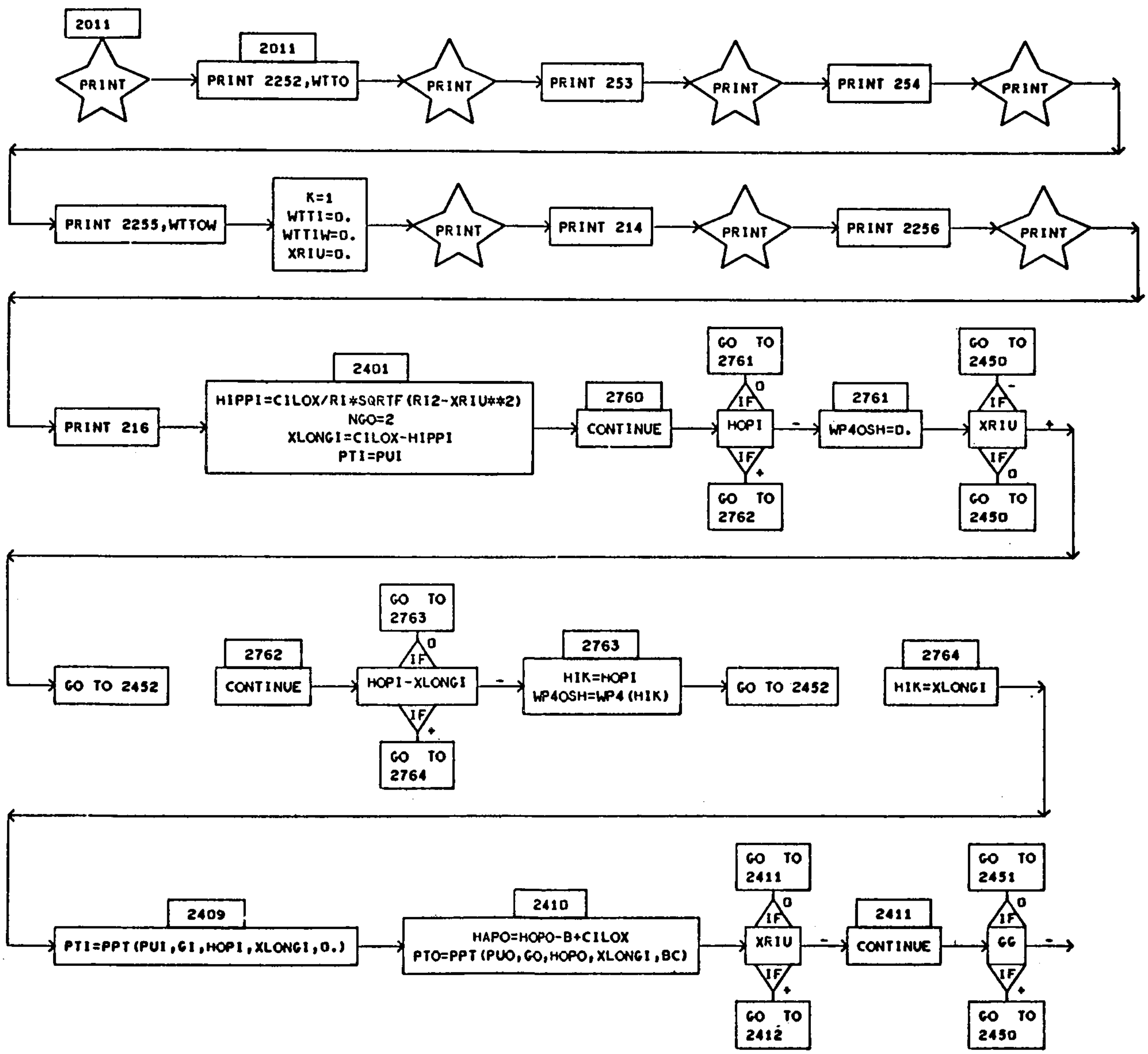


KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL
7000
PRINT 1218,XFL,FPHICT,FTHECT,XFL,FPHIOC,FTHEOC
XFL=XFL+FL/XXFL
XFL1=XFL
GO TO 1001
IF
XFL-FL
IF
GO TO 1299
1010
CONTINUE
GO TO 1299
IF
FL
IF
GO TO 1001
1299
CONTINUE
PRINT
PRINT 2201
COMPUTED GO TO
IF THE VALUE OF NTRANS IS
TRANSFER TO STATEMENT
1 2302
2 2301
2302
PRINT
2302
PRINT 259
2301
PRINT
2301
PRINT 214
PRINT
PRINT 2215
PRINT
PRINT 216
J=1
JPOP=1
WTTO=0.
WTTOW=0.
GO TO 6100
IF
AC
IF
GO TO 6101
6100
XROU=0.
GO TO 2001
6101
XROU=AC+RI
POP=XROU/DELTA
GO TO 2806
IF
POP-1.
IF
GO TO 2806
2805
JPOP=POP+1.
J=JPOP
2806
PAP=J-1
XRAU=DELTA*PAP
2001
XA=XROU-A
XA2=XA**2
HIPPY=BA*SQRTF(AA-XA2)
NGO=2
XLONGO=B-HIPPY
2750
CONTINUE

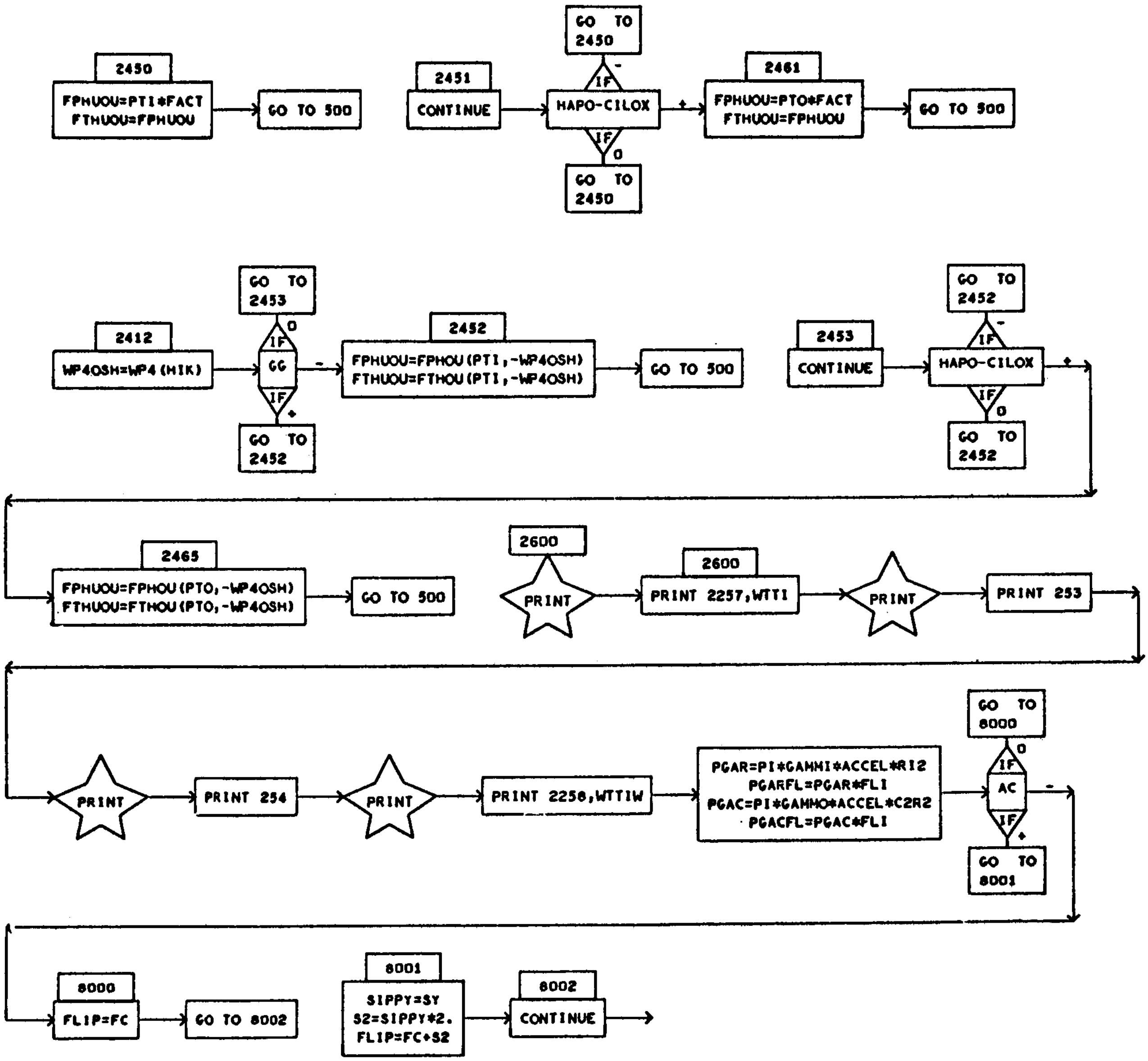
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



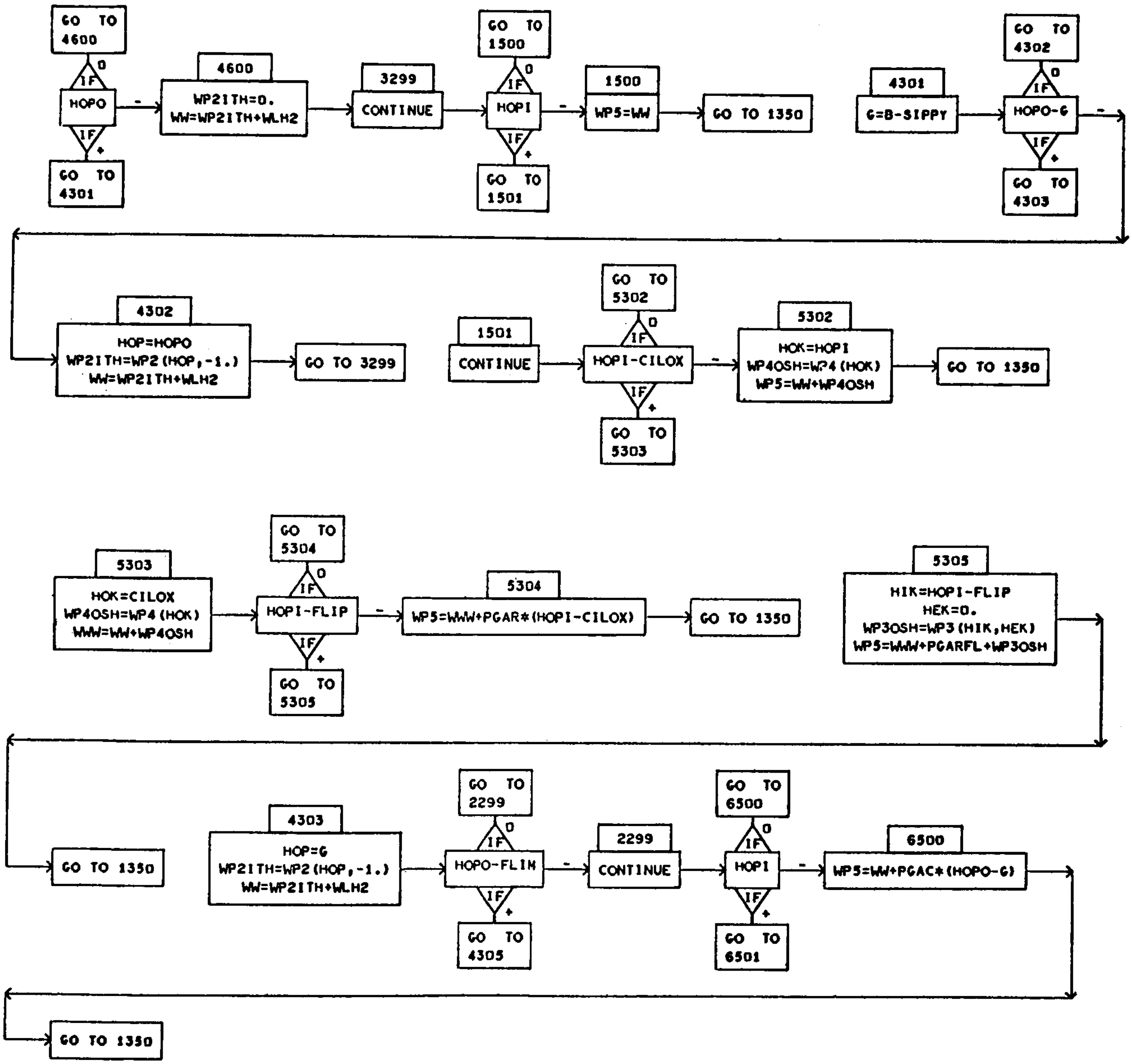
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



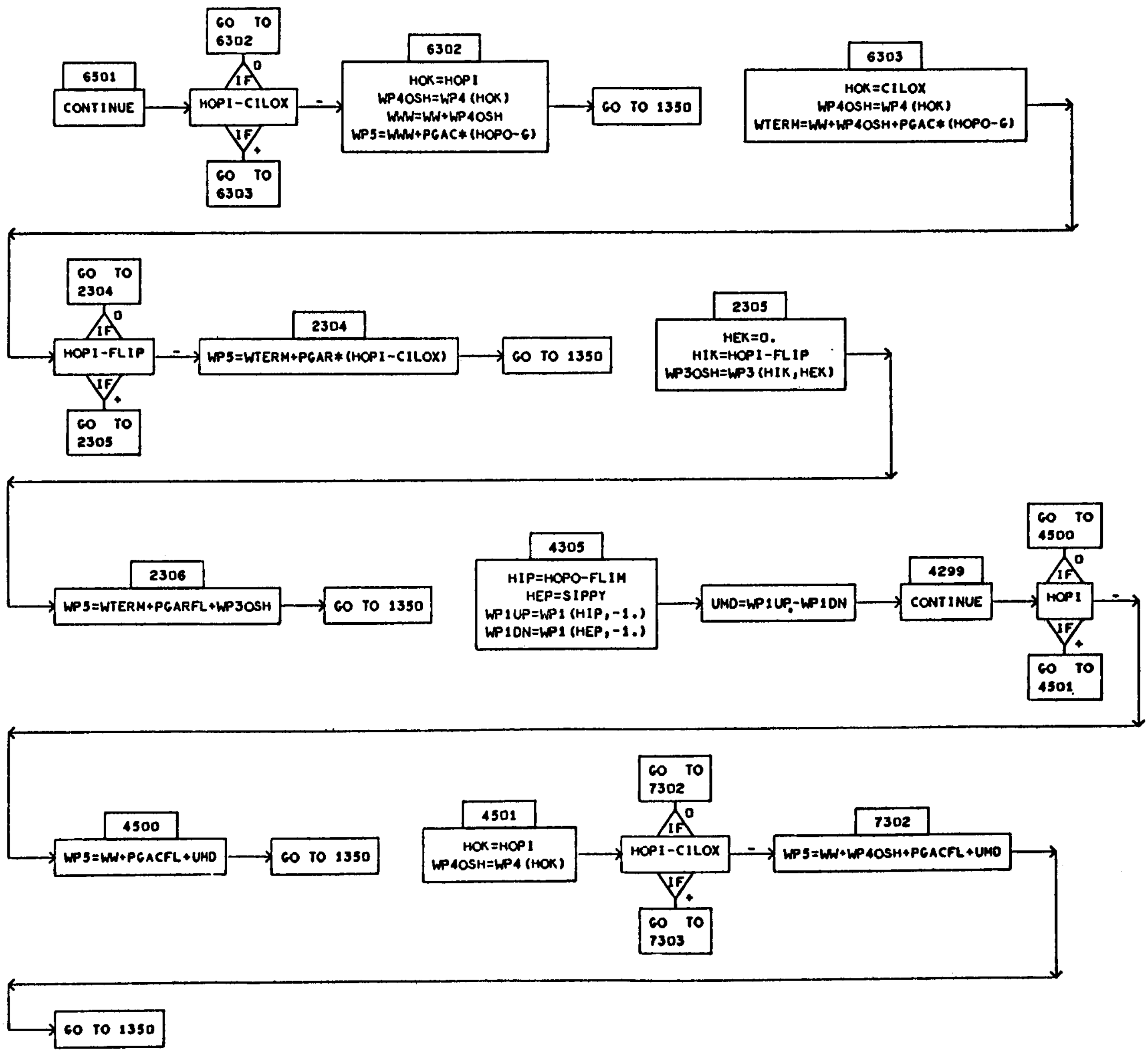
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



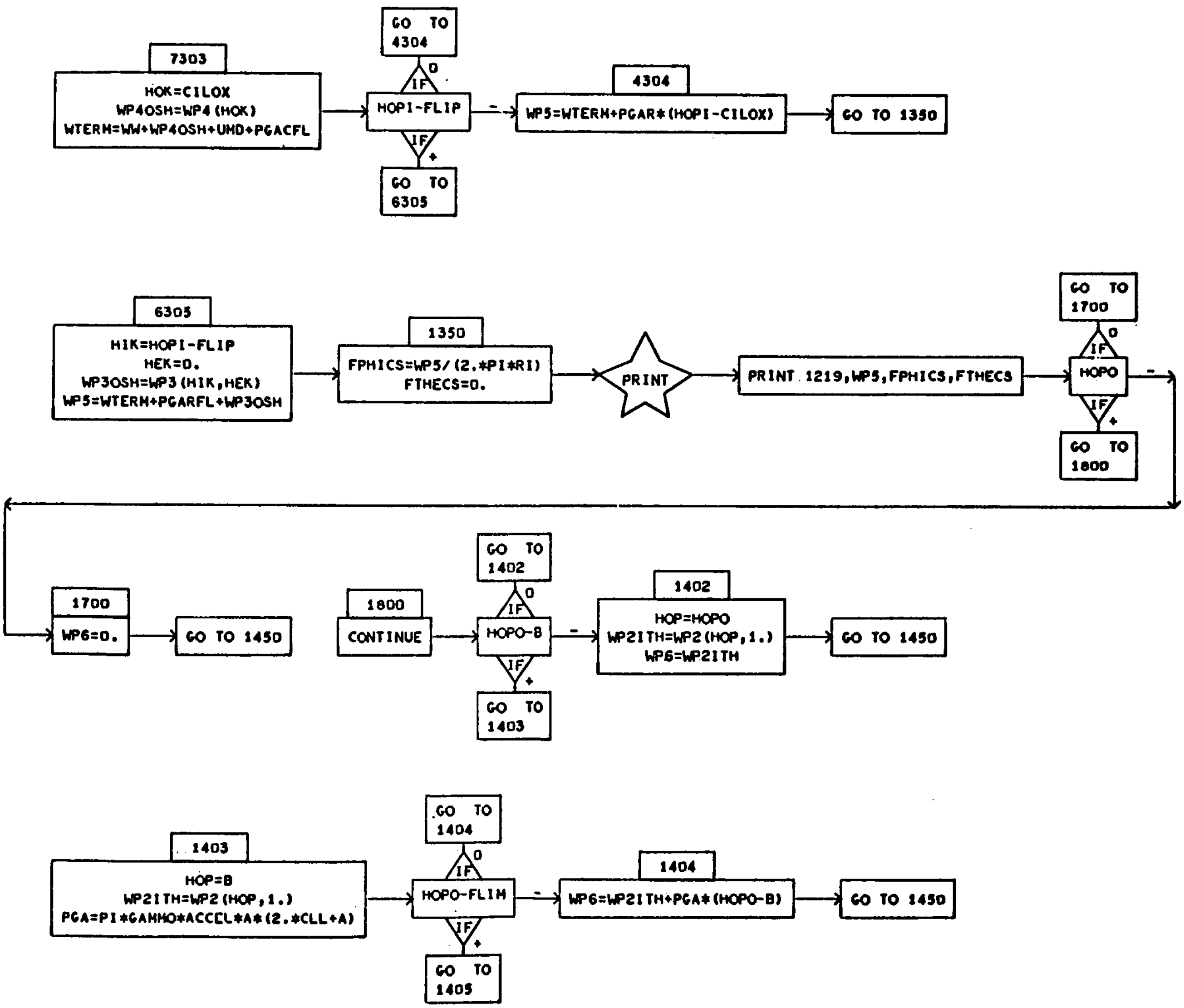
KOHMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL



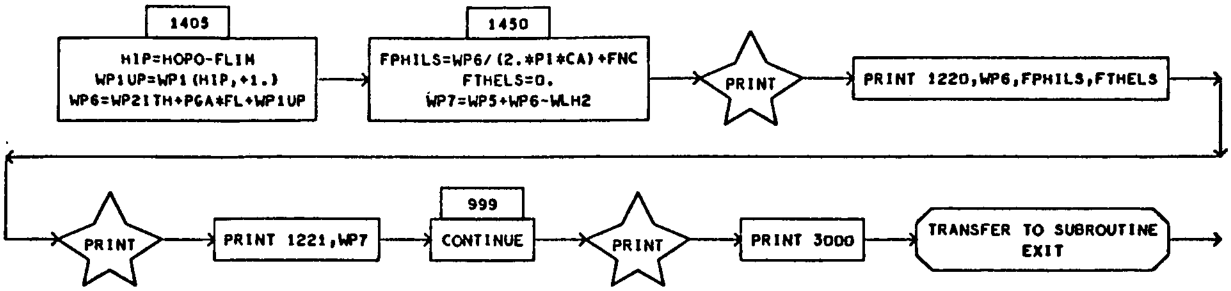
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

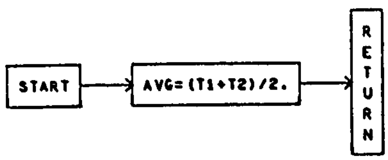


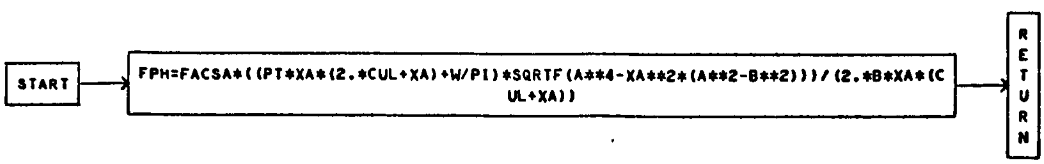
KOHHON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

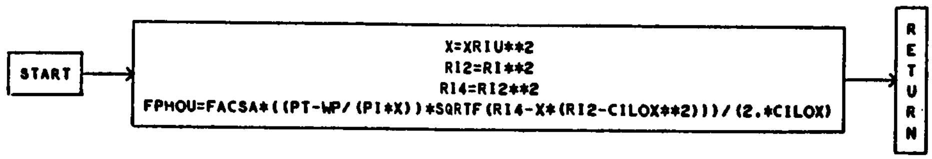


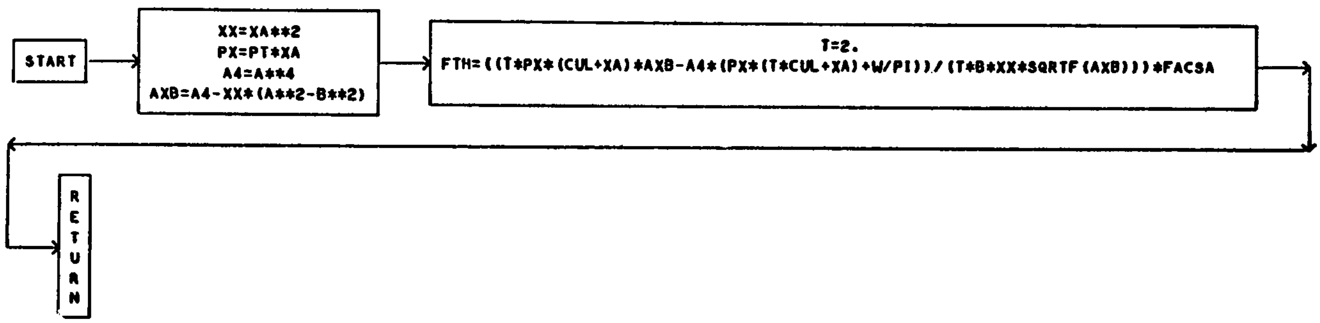
KOMMON CUL,CLL,RI,RO,A,B,CILOX,FL,WLH2,ACCEL

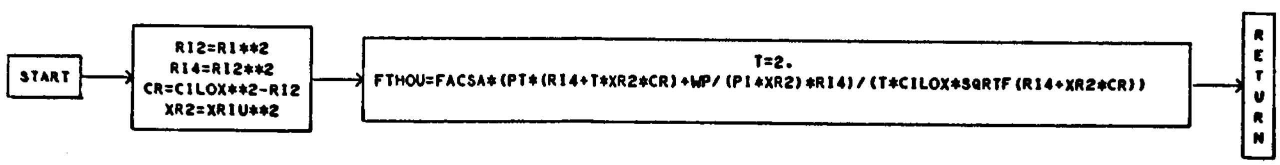


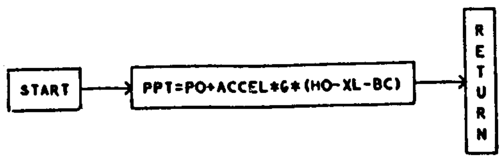


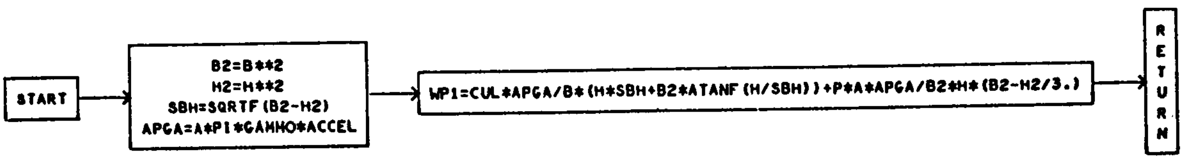


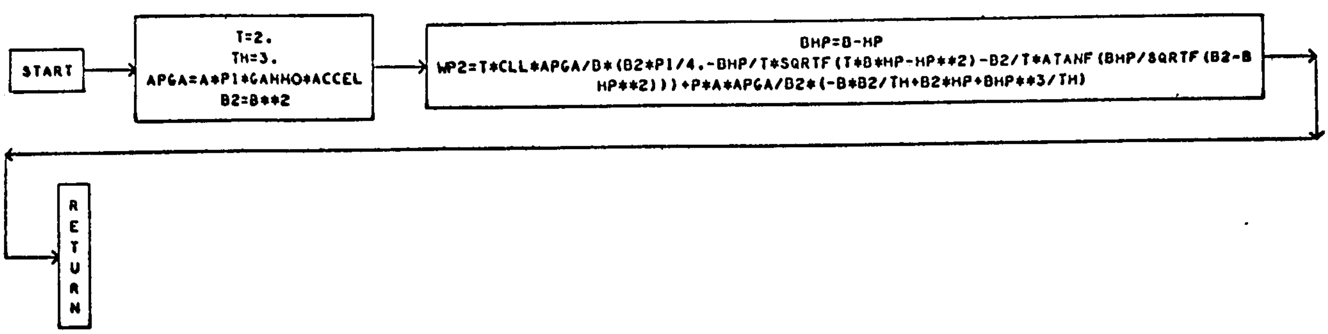

START
X=XRIU**2
RI2=RI**2
RI4=RI2**2
FPHOU=FACSA*((PT-WP/(PI*X))*SQRTF(RI4-X*(RI2-CILOX**2)))/(2.*CILOX)
RETURN

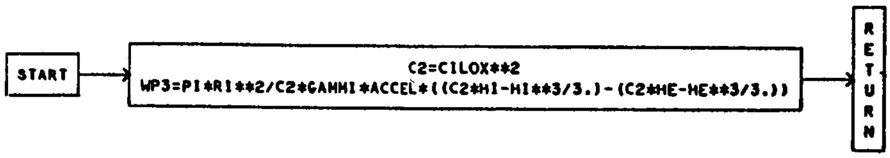

START
XX=XA**2
PX=PT*XA
A4=A**4
AXB=A4-XX*(A**2-B**2)
T=2.
FTH=((T*PX*(CUL+XA)*AXB-A4*(PX*(T*CUL+XA)+W/PI))/(T*B*XX*SQRTF(AXB)))*FACSA
RETURN

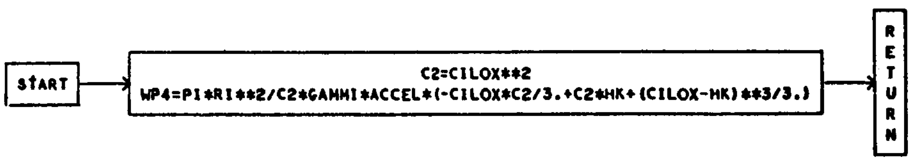

START
RI2=RI**2
RI4=RI2**2
CR=CILOX**2-RI2
XR2=XRIU**2
T=2.
FTHOU=FACSA*(PT*(RI4+T*XR2*CR)+WP/(PI*XR2)*RI4)/(T*CILOX*SQRTF(RI4+XR2*CR))
RETURN

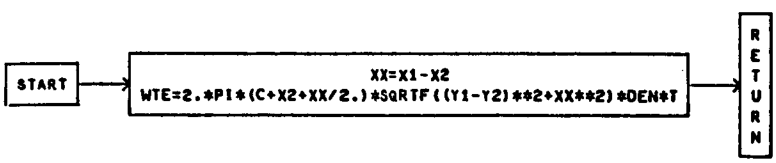

START
PPT=PO+ACCEL*G*(HO-XL-BC)
RETURN


START
B2=B**2
H2=H**2
SBH=SQRTF(B2-H2)
APGA=A*PI*GAMMO*ACCEL
WP1=CUL*APGA/B*(H*SBH+B2*ATANF(H/SBH))+P*A*APGA/B2*H*(B2-H2/3.)
RETURN


START
T=2.
TH=3.
APGA=A*PI*GAMMO*ACCEL
B2=B**2
BHP=B-HP
WP2=T*CLL*APGA/B*(B2*PI/4.-BHP/T*SQRTF(T*B*HP-HP**2)-B2/T*ATANF(BHP/SQRTF(B2-B
HP**2)))+P*A*APGA/B2*(-B*B2/TH+B2*HP+BHP**3/TH)
RETURN


START
C2=CILOX**2
WP3=PI*RI**2/C2*GAMMI*ACCEL*((C2*HI-HI**3/3.)-(C2*HE-HE**3/3.))
RETURN


START
C2=CILOX**2
WP4=PI*RI**2/C2*GAMMI*ACCEL*(-CILOX*C2/3.+C2*HK+(CILOX-HK)**3/3.)
RETURN


START
XX=X1-X2
WTE=2.*PI*(C+X2+XX/2.)*SQRTF((Y1-Y2)**2+XX**2)*DEN*T
RETURN