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by

S. N. Kirpichnikov

Vestnik Leningradskogo Universiteta (Leningrad University Herald), No. 7, 1965, pp. 144-156.

Translated from the Russian

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OPTIMUM FLIGHTS WITH PRESSURE OF LIGHT TAKEN INTO ACCOUNT

by

S. N. Kirpichnikov

This article has for its subject the construction of single-impulse trajectories of interorbital flights with a minimum consumption of mass in the gravitational field of the Sun -- a spherical central body radiating light symmetrically. It disregards the perturbing action on the space vehicle caused by objects in space located on the specified boundaries of the Kepler orbits. It assumes that all three orbits, the initial, intermediate, and final, are located in one plane and the motion along these orbits is proceeding in one direction.

The pressure of light can be, on one hand, regarded as a small correction factor for flights of conventional spaceships. On the other hand, the pressure of light becomes comparable with the force of the Sun's gravity in cases of research flights of probes, which are made of thin hollow shells (cylinders) whose inside is filled with low-pressure gas and whose outside is covered with a good light-reflecting coating. The use of such shells was suggested by Ehricke[1] for investigating the cosmic space of our solar system and also for transporting useful loads. It notes the simplicity of these probes, their light weight, and their ability to carry useful loads. Such objects have an intense brightness, which makes it possible to increase by many times the probability of successful observation of the probes with the aid of telescopes. These shell-probes can be launched from the Earth and also directly from spaceships or from artificial satellites.

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[1] K. Ehricke, "Instrumented Comets-Astronautics of Solar and Planetary Probes," VIII International Astronautical Congress, Barcelona, 1957.

1. The Pressure of Light

Let us find the main vector and the main moment of forces of light pressure acting upon a stationary body whose radiated surface is S. The derivation of expressions in this chapter will be based on the quantum theory of light; the effect due to anisotropism of the reradiation will be neglected.

Let us introduce a stationary system of coordinates x, y, z; the z-axis coincides with the heliocentric radius-vector of the irradiated body; i. e., it is directed along the parallel beam falling on the body. Let us separate an elementary area (dS) on the body's surface, with R as its coefficient of reflection. Let $\bar{\rho} = \{\xi, \eta, \zeta\}$ be the radius-vector of some point on (dS), the origin of this vector is located at point 0 of the irradiated body; α is the angle of incidence equal to the angle of reflection; β is the angle between the projection of the outside normal to the area on plane xy and the x-axis counted in positive direction from the x-axis. In this case, the elementary change in momentum during the time dt, will be

$$d\bar{K} = (M_2 c\bar{e}_1 - M_1 c\bar{e}_2)\, dt \quad , \tag{1}$$

where c is the velocity of light; $\bar{e}_1$ is a unit or basis vector in direction of the incident beam; $\bar{e}_2$ is a unit-vector in direction of the reflected beam of rays; M_1, M_2 are, respectively, the mass of the photons incident to the area (dS) and reflected from it per unit of time.

Based on the principle of equivalence of mass and energy, let us find

$$M_1 = \frac{E}{c^2} \cos\alpha \; dS, \quad M_2 = R\,\frac{E}{c^2} \cos\alpha \; dS \ . \tag{2}$$

Here, E is the solar constant (intensity of solar radiation per unit of area) for the area located at a distance r from the Sun, calculated as:

$$E = \frac{E_{\odot}\, r_{\odot}^2}{r^2} \ , \quad E_{\odot}\, r_{\odot}^2 = 0.302 \cdot 10^{33} \text{ ergs/sec} \ , \tag{3}$$

where $r_{\odot}$ is the average distance between the Earth and the Sun and $E_{\odot}$ is the solar constant for the distance $r_{\odot}$. Using the theorem of momentum, we find the force of the light-pressure $\bar{f}$ on the area (dS)

$$\left.\begin{aligned} f_x &= -R\,\frac{E}{c}\sin 2x\cos\alpha\cos\beta\, dS\ , \\ f_y &= -R\,\frac{E}{c}\sin 2\alpha\cos\alpha\sin\beta\, dS\ , \\ f_z &= \frac{E}{c}\,(1 + R\cos 2\alpha)\cos\alpha\, dS\ . \end{aligned}\right\} \qquad (4)$$

It is then easy to find the projection of the main vector $\overline{F}$ and of the main moment $\overline{L}$ for the point 0 on the axes of the coordinates

$$\left.\begin{aligned} F_x &= -\frac{2E}{c}\iint_{(S)} R\cos^2\alpha\sin\alpha\cos\beta\, dS\ , \\ F_y &= -\frac{2E}{c}\iint_{(S)} R\cos^2\alpha\sin\alpha\sin\beta\, dS\ , \\ F_z &= \frac{E}{c}\iint_{(S)} (1 + R\cos 2\alpha)\cos\alpha\, dS\ , \end{aligned}\right\} \qquad (5)$$

$$\left.\begin{aligned} L_x &= \frac{E}{c}\iint_{(S)} [\eta(1 + R\cos 2\alpha) + \zeta R\sin 2\alpha\sin\beta]\cos\alpha\, dS\ , \\ L_y &= -\frac{E}{c}\iint_{(S)} [\zeta R\sin 2\alpha\cos\beta + \xi(1 + R\cos 2\alpha)]\cos\alpha\, dS\ , \\ L_z &= \frac{E}{c}\iint_{(S)} R(\eta\cos\beta - \xi\sin\beta)\sin 2\alpha\cos\alpha\, dS\ . \end{aligned}\right\} \qquad (6)$$

Let us find the pressure exerted by light on a spherical body with radius R and with an index of reflection R identical for the entire surface. Let us introduce on the body's surface a spherical system of coordinates (longitude φ, latitude ψ) with the origin in the center of the sphere; the longitude φ will be counted from the direction to the Sun; therefore,

$$\cos\alpha = \cos\varphi\cos\psi\ . \qquad (7)$$

Due to the symmetry, the system of forces of light pressure is reduced to a resultant force applied to the geometrical center of the sphere and equal to

$$F_z = \frac{E_{\odot} r_{\odot}^2}{cr^2} \pi R^2 \quad . \tag{8}$$

Consequently, the force acting upon a stationary, spherical, irradiated body is directed along its heliocentric radius-vector and is independent of the reflecting capacity of the latter; this result is fully similar to the result of Radziyevskiy.[1] Therefore, it makes sense when a well-reflecting coating is used only to prevent an excessive overheating.

NOTE 1. If the irradiated body is moving with a velocity U, both the value of the force of light pressure and its direction will change by a value of about U/c (for more details, see Radziyevskiy*). In view of the insignificance of the last ratio, this change will be disregarded.

2. Heliocentric Trajectories with Pressure of Light Taken into Consideration

Let us consider a body with a mass m moving in fields of Newton gravitation and of the Sun's radiated light, whose mass will be designated by M. The components F_x and F_y of the forces of the pressure of light will be disregarded. These components are exactly equal to zero for spherical bodies.

The force of attraction to the Sun is k^2Mm/r^2, where k^2 is the gravitational constant and the force of repulsion by light is B/r^2 which, for a sphere with radius R, is

$$B = \frac{E_{\odot} r_{\odot}^2}{c} \pi R^2 \quad , \tag{9}$$

while B for other bodies can be readily found by using the expressions (3) and (5). Composing the equations for the relative motion of the irradiated body and assuming that $m << M$, we find for its heliocentric radius-vector $\bar{r}$

$$\ddot{\bar{r}} = - \frac{k^2 M}{r^3} \bar{r} + \frac{B}{mr^3} \bar{r} \quad . \tag{10}$$

Let us introduce the "reduced" mass of the Sun

[1] V. V. Radziyevskiy, "Braking by Radiation in the Solar System and the Growth of Rings of Saturn," Astronomicheskiy Zhurnal (Astronomical Journal), <u>29</u>, 3, 1952.

$$M' = M(1 - \delta) \quad , \tag{11}$$

where δ is a parameter characterizing the "decrease" in the mass of the Sun and is equal to

$$\delta = \frac{B}{k^2 m M} \quad . \tag{12}$$

Consequently, with the "reduced" mass of the Sun, all expressions in a problem for two bodies will be valid and, formally, it is only necessary to replace everywhere the value

$$K = k\sqrt{M} \tag{13}$$

with a new value of

$$\tilde{K} = K\sqrt{1 - \delta} \tag{14}$$

We will assume that the parameter is $\delta \in |0, 1)$. At $\delta = 1$, the gravitational attraction will be balanced by the repulsion of light and rectilinear inertial flights are possible in any direction. At $\delta > 1$, the irradiated bodies will fly out the solar system along hyperbolas in which the Sun is located in the external focus.

Let us estimate the value of δ for thin hollow spherical shell-probes having a radius R, a shell-thickness h, and a density γ. Assuming that the shell-probe carries a useful load whose mass is m_0, therefore, the overall mass of the probe, is

$$m = m_0 + 4\pi R^2 \gamma h \tag{15}$$

and the parameter δ will be

$$\delta = \frac{E_{\delta}^{+} r_{\delta}^{+2} \pi R^2}{K^2 c(m_0 + 4\pi R^2 \gamma h)} = \frac{E_{\delta}^{+} r_{\delta}^{+2}}{K^2 c\left(\frac{m_0}{\pi R^2} + 4\gamma h\right)} \quad . \tag{16}$$

If $m_0 = 0$, then δ is independent of the radius of the probe and is dependent only on the product γh. Let us calculate the values of δ for different h and at $\gamma = 1$ gram/cm^3, by assuming that m_0 is much smaller than the mass of the shell:

δ	0.5	0.1	0.05	0.01	0.005	0.001
h, microns	0.38	1.9	3.8	19.0	38.0	190

This estimate is qualitatively in agreement with the estimate of Ehricke.[1]

The constant elements of the orbits with "reduced" mass of the Sun will be called geometrical. We will use for such elements:

$$p = \frac{1}{\sqrt{l}}, \quad q = \frac{e}{\sqrt{l}}, \quad \omega, \; T, \tag{17}$$

where l is a focal parameter, e is the eccentricity, ω is the angular distance of the pericenter, and T is the moment of passing through the pericenter. For the independent variable which determines the position in the orbit we will use the polar angle ϑ, so that the positive direction of the counting coincides with the direction of the motion.

The values pertaining to an osculating orbit will be provided with a subscript "ock." Therefore, we have:

$$U_r = Kq_{ock} \sin\left(\vartheta - \omega_{ock}\right) = K\sqrt{1 - \delta q} \sin(\vartheta - \omega), \tag{18}$$

$$U_\vartheta = K\left[p_{ock} + q_{ock} \cos\left(\vartheta - \omega_{ock}\right)\right] = K\sqrt{1 - \delta}\,[p + q \cos(\vartheta - \omega)], \tag{19}$$

$$\frac{1}{r} = p^2_{ock} + p_{ock}q_{ock} \cos\left(\vartheta - \omega_{ock}\right) = p^2 + pq \cos(\vartheta - \omega), \tag{20}$$

$$\psi = K(t_2 - t_1) = \int_{\vartheta_1 - \omega_{ock}}^{\vartheta_2 - \omega_{ock}} \frac{d\upsilon}{p_{ock}\left(p_{ock} + q_{ock} \cos \upsilon\right)^2}$$

$$= \frac{1}{\sqrt{1 - \delta}} \int_{\vartheta_1 - \omega}^{\vartheta_2 - \omega} \frac{d\upsilon}{p(p + q \cos \upsilon)^2}, \tag{21}$$

where t_1, t_2 are, respectively, the moments of time corresponding to ϑ_1, ϑ_2; U_r, U_ϑ are, respectively, the radial and transversal velocity components. Note

[1] K. Ehricke, op. cit.

that in the first integral it is necessary to take into account only the explicit relationship of the integrand function of υ. Both integrals in (21) are readily taken (for example, for an elliptical motion through an eccentric anomaly: the osculating for the first integral and the geometrical for the second). From the relationships (18) to (21) we can readily find

$$p_{ock} = \frac{p}{\sqrt{1 - \delta}}, \tag{22}$$

$$q_{ock} = \left\{q^2(1 - \delta) - 2pq\delta \cos(\vartheta - \omega) + \frac{p^2\delta^2}{1 - \delta}\right\}^{1/2}, \tag{23}$$

$$\mathrm{tg}\,\omega_{ock} = \frac{q(1 - \delta)\sin\omega - p\delta \sin\vartheta}{q(1 - \delta)\cos\omega - p\delta\cos\vartheta}, \tag{24}$$

$$T_{ock} = T + \frac{1}{K\sqrt{1 - \delta}} \int_0^{\vartheta - \omega} \frac{d\upsilon}{p(p + q\cos\upsilon)^2}$$

$$- \frac{1}{K} \int_0^{\vartheta - \omega_{ock}} \frac{d\upsilon}{p_{ock}\left(p_{ock} + q_{ock}\cos\upsilon\right)^2}, \tag{25}$$

here, the numerator and denominator signs in expression (24) coincide with the signs of $\sin\omega_{ock}$ and $\cos\omega_{ock}$, respectively; in expression (25), the osculating elements should be considered to be constant for calculating the second integral. Note that all osculating elements (during an elliptical motion in geometrical elements) are periodic functions of the angle ϑ with a period 2π.

3. Setting Up the Problem of an Optimum (in Terms of Energy) Flight with the Effect of Pressure of Light Taken into Account. The System of Necessary Conditions.

It is necessary to accomplish a flight of a space vehicle within given boundary orbits with the aid of a single thrust -- an impulse at the initial orbit. The value of the characteristic velocity of this impulse is reduced to a minimum to assure a minimum consumption of fuel. All orbits are coplanary. The pressure of light is taken into account only for the intermediate orbit. Depending on the specific physical problem, the space vehicle may be the shell-probe described above, or any other spaceship for which the pressure of light must be taken into account.

Let the p_1, q_1, ω_1, and T_1 be the elements of the initial orbit and p_2, q_2, ω_2, and T_2 be the elements of the final orbit. It is assumed that the ratio of the effective transverse section to the mass of the objects in space is small at these orbits and the elements of the orbits are practically unaffected by the pressure of light.

During the moment of time t_1, when the polar angle is equal to ϑ_1, there takes place the thrust-impulse which causes the space vehicle to change its flight to the intermediate orbit. During the moment of time t_2 when the polar angle is ϑ_2, the intermediate orbit intersects the final orbit and the space vehicle collides with an object in space located on the final orbit.

With the elements of the boundary orbits specified, it is necessary to find the intermediate orbit of the flight, i. e., to find its geometrical elements p, q, and ω; also the angles ϑ_1, ϑ_2 and the moments of time t_1, t_2 which will reduce to a minimum the characteristic velocities of the initial impulse. After these unknown values are found, we will have for the moment of passage through the pericenter

$$T = t_i - \frac{1}{K\sqrt{1-\delta}} \int_0^{\vartheta_i - \omega} \frac{d\upsilon}{p(p + q\cos\upsilon)^2}, \quad i = 1, 2 \quad , \qquad (26)$$

and the expressions (22) to (25) can be used to calculate the osculating elements.

The following conditions must be satisfied:

$$\varphi_1 = p^2 + pq\cos(\vartheta_1 - \omega) - p_1^2 - p_1 q_1 \cos(\vartheta_1 - \omega_1) = 0 \ , \qquad (27)$$

$$\varphi_2 = p^2 + pq\cos(\vartheta_2 - \omega) - p_2^2 - p_2 q_2 \cos(\vartheta_2 - \omega_2) = 0 \ , \qquad (28)$$

$$\varphi_3 = \psi_1 + \psi + \psi_2 - \alpha = 0 \ , \qquad (29)$$

where the function of ψ is expressed through the geometrical elements as per the expression (21), and

$$\psi_i = K\left(t_i - T_i\right) = \int_0^{\vartheta_i - \omega_i} \frac{d\upsilon}{p_i(p_i + q_i\cos\upsilon)^2}, \quad i = 1, 2 \quad , \qquad (30)$$

$$\alpha = K(T_2 - T_1) \ . \qquad (31)$$

The relationships (27) and (28) indicate a continuity of the radii-vectors at the starting and finishing points, while the relationship (29) represents the condition for the coincidence of the time spent in motion prior to reaching the final point, on one hand, along the initial and intermediate orbits and, on the other hand, along the final orbit.

The characteristic velocity ΔU of the initial thrust can be expressed through the elements of the initial and intermediate orbits[1] for similar computations) as follows:

$$\Delta U = K\Big\{q^2(1 - \delta) - p^2(1 - \delta) + q_1^2 + 3p_1^2 - 2p_1^2\delta - \frac{2p_1^3\sqrt{1 - \delta}}{p}$$

$$-2qq_1\sqrt{1 - \delta}\cos(\omega_1 - \omega)$$

$$- 2q_1\sqrt{1 - \delta}\left[p + \frac{p_1^2}{p} - p_1\frac{2 - \delta}{\sqrt{1 - \delta}}\right]\cos(\vartheta_1 - \omega_1)\Big\}^{1/2}, \tag{32}$$

and for the angle of inclination of thrust Φ (which is counted from the transversal in a direction opposite to the motion), we have

$$\operatorname{tg}\Phi = \frac{q\sqrt{1 - \delta}\sin(\vartheta_1 - \omega) - q_1\sin(\vartheta_1 - \omega_1)}{\left(\frac{p_1\sqrt{1 - \delta}}{p} - 1\right)[p_1 + q_1\cos(\vartheta_1 - \omega_1)]}, \tag{33}$$

where the signs of the numerator and denominator coincide, respectively, with the signs of $\sin\Phi$ and $\cos\Phi$.

Let us find the minimum of the function

$$g = \frac{(\Delta U)^2}{2K^2\sqrt{1 - \delta}} \tag{34}$$

in a class of variables p, q, ω, ϑ_1, ϑ_2 which are dependent and related with the conditions (27) to (29), i. e., we find ourselves in a class of a conditional extremum of a function of a finite number of variables. Introducing the constant factors λ_1, λ_2, λ_3, we compose a Lagrange function

[1] S. N. Kirpichnikov, "Optimum Complanary Flight Between Orbits," Vestnik LGU (Leningrad University Herald), No. 1, 1964.

$$g + \sum_{i-1}^{3} \lambda_i \varphi_i \quad . \tag{35}$$

As we know, the partial derivatives of Lagrange functions must be equal to zero for all variables

$$q_1 p_1 \left[\frac{p}{p_1} + \frac{p_1}{p} - \frac{2 - \delta}{\sqrt{1 - \delta}}\right] \sin(\vartheta_1 - \omega_1)$$

$$+ \lambda_1 [p_1 q_1 \sin(\vartheta_1 - \omega_1) - pq \sin(\vartheta_1 - \omega)] + \lambda_3 \left(\frac{\partial \psi}{\partial \vartheta_1} + \frac{\partial \psi_1}{\partial \vartheta_1}\right) = 0 \ , \tag{36}$$

$$\lambda_2 [p_2 q_2 \sin(\vartheta_2 - \omega_2) - pq \sin(\vartheta_2 - \omega)] + \lambda_3 \left(\frac{\partial \psi}{\partial \vartheta_2} - \frac{\partial \psi_2}{\partial \vartheta_2}\right) = 0 \ , \tag{37}$$

$$-p\sqrt{1 - \delta} + \frac{p_1^3}{p} + q_1 \left(\frac{p_1^2}{p^2} - 1\right) \cos(\vartheta_1 - \omega_1) + \lambda_1 [2p + q \cos(\vartheta_1 - \omega)]$$

$$+ \lambda_2 [2p + q \cos(\vartheta_2 - \omega)] + \lambda_3 \frac{\partial \psi}{\partial p} = 0 \ , \tag{38}$$

$$q\sqrt{1 - \delta} - q_1 \cos(\omega_1 - \omega) + \lambda_1 p \cos(\vartheta_1 - \omega) + \lambda_2 p \cos(\vartheta_2 - \omega) + \lambda_3 \frac{\partial \psi}{\partial q} = 0 \ , \tag{39}$$

$$-qq_1 \sin(\omega_1 - \omega) + \lambda_1 pq \sin(\vartheta_1 - \omega) + \lambda_2 pq \sin(\vartheta_2 - \omega) + \lambda_3 \frac{\partial \psi}{\partial \omega} = 0 \ . \tag{40}$$

The obtained equations (36) to (40) together with the equations (27) to (29) form a system of required conditions made up of eight equations containing eight unknown p, q, ω, ϑ_1, ϑ_2, λ_1, λ_2, λ_3. These equations must be solved jointly.

NOTE 2. If a problem is under consideration in which specific motions are not taken into account, i. e., the initial configuration of objects in space is arbitrary, the condition (29) for the coincidence of the time spent in motion prior to reaching the final point should be omitted, while in the remaining system of seven equations it is necessary to assume that $\lambda_3 = 0$.

NOTE 3. The pressure of light can be readily taken into account at the final orbit. For this is sufficient to consider the elements p_2, q_2, ω_2, T_2 as geometrical and to replace in all equations the function of ψ_2 with $\tilde{\psi}_2 = \psi_2(1 - \delta_2)^{-1/2}$ where δ_2 is the parameter δ computed for an object in space located at the final orbit.

4. Flight Between Circular Orbits

Assume that the initial and final orbits are circular with radii r_1 and r_2, respectively, and their values are $q_1 = 0$ and $q_2 = 0$.

It can be shown that from the system of required conditions will follow that

$$\lambda_3 = 0 \ . \tag{41}$$

Therefore, it is first necessary to solve the problem without taking specific motions into account and the solution will contain only the differences $\vartheta_1 - \omega$, $\vartheta_2 - \omega$ and, consequently, one of the angles ϑ_1, ϑ_2, ω will be arbitrary.

Thereafter, from the condition (29) for the coincidence of the time spent in motion prior to reaching the final point, it is easy to find, for example, the angle ϑ_1

$$\vartheta_1 = [(\alpha - \psi) p_1^3 p_2^3 + \omega_1 p_2^3 - \omega_2 p_1^3 + f p_1^3](p_2^3 - p_1^3)^{-1} \ , \tag{42}$$

where ψ/K is the time spent in motion along the optimum orbit; $f = \vartheta_2 - \vartheta_1$ is the difference between the true anomalies of the starting and finishing points.

Let us consider a problem about a flight without taking specific motions into account. It follows from the equations (36), (37) that

$$\lambda_1 \sin(\vartheta_1 - \omega) = 0 \ , \quad \lambda_2 \sin(\vartheta_2 - \omega) = 0 \ . \tag{43}$$

The last equations can be satisified by using three methods, since the conditions $\lambda_1 = 0$, $\lambda_2 = 0$ contradict the equation (39).

<u>Method I.</u> Assume that

$$\sin(\vartheta_1 - \omega) = 0 \ , \quad \sin(\vartheta_2 - \omega) = 0 \ . \tag{44}$$

The only solution in this case (with an accuracy of up to an arbitrary choice of the angle ϑ_1 of the starting point) is a Homan ellipse in geometrical elements. For it, we have

$$p = \sqrt{\frac{p_1^2 + p_2^2}{2}} \quad , \quad q = \pm\frac{p_1^2 - p_2^2}{\sqrt{2(p_1^2 + p_2^2)}} \quad , \tag{45}$$

$$\lambda_1 = \frac{p_2^4\sqrt{1 - \delta} - p_1^3 p}{4p^4} \quad , \quad \lambda_2 = \frac{p_1^3(p_1\sqrt{1 - \delta} - p)}{4p^4} \quad , \tag{46}$$

$$\Delta U = \frac{Kp_1}{p}|p_1\sqrt{1 - \delta} - p| \quad . \tag{47}$$

The upper symbols are for flights to orbits having a large radius

$$r_2 > r_1 \quad , \quad p_2 < p < p_1 \quad , \quad \vartheta_1 = \omega \quad , \quad \vartheta_2 = \omega + \pi \quad , \quad \Phi = 0 \text{ at } p < p_1\sqrt{1 - \delta} \ ,$$

$$\Phi = \pi \text{ at } p > p_1\sqrt{1 - \delta} \ ; \tag{48}$$

the lower symbols are for flights to orbits of smaller radius

$$r_2 < r_1 \quad , \quad p_2 > p > p_1 \quad , \quad \vartheta_1 = \omega - \pi \quad , \quad \vartheta_2 = \omega \quad , \quad \Phi = \pi \ . \tag{49}$$

Method II. We will satisfy the equations (43), as follows

$$\sin(\vartheta_1 - \omega) = 0 \quad , \quad \lambda_2 = 0 \quad . \tag{50}$$

From the remaining equations of the system of required conditions, we find

$$p = p_1\sqrt{1 - \delta} \quad , \quad q = \frac{p_1\delta}{\sqrt{1 - \delta}} \quad , \tag{51}$$

$$\vartheta_1 = \omega \quad , \quad \cos(\vartheta_2 - \omega) = \frac{p_2^2 - p_1^2(1 - \delta)}{p_1^2\delta} \quad , \tag{52}$$

$$\Delta U = 0 \quad . \tag{53}$$

If $\delta < 0.5$, a solution exists only at

$$\frac{r_1}{1 - 2\delta} > r_2 > r_1 \quad , \quad \sqrt{1 - 2\delta}\, p_1 < p_2 < p_1 \quad , \tag{54}$$

however, if $\delta \geqslant 0.5$, the solution is at $r_2 > r_1$.

The take-off takes place at the pericenter of the flight's orbit, no additional consumption of fuel is required and, therefore, these orbits offer a better advantage in terms of energy than a Homan ellipse. For flying shell-probes to reach the obtained orbits, it is sufficient to inflate the shell. Note that at $r_2 = r_1/1 - 2\delta$, the obtained orbit will coincide with a Homan ellipse; it will be a parabola at $\delta = 0.5$ and an hyperbola at $\delta > 0.5$, at which the Sun is located in the internal focus.

Method III. In this case

$$\lambda_1 = 0 \quad , \quad \sin(\vartheta_2 - \omega) = 0 \ . \tag{55}$$

The remaining equations of the system of required conditions will yield

$$p = \frac{p_2^4\sqrt{1-\delta}}{p_1^3} , \quad q = \frac{p_1^6 - p_2^6(1-\delta)}{p_1^3 p_2^2\sqrt{1-\delta}} , \tag{56}$$

$$\cos(\vartheta_1 - \omega) = \frac{p_1^8 - p_2^8(1-\delta)}{p_2^2[p_1^6 - p_2^6(1-\delta)]} , \quad \vartheta_2 = \omega \ . \tag{57}$$

Such orbits are possible only at

$$\sigma^2 r_1 \leqslant r_2 < r_1 \quad , \quad p_2 > p_1 \geqslant \sigma p_2 , \tag{58}$$

where σ is the only positive root of the equation

$$\sigma^8 + \sigma^6 = 2(1-\delta) \ . \tag{59}$$

The finishing point coincides with the pericenter of the flight's orbit. At $r_2 = \sigma^2 r_1$, the obtained orbit coincides with a Homan ellipse. The characteristic velocity and the tangent of the angle of inclination of the thrust are determined as follows:

$$\Delta U = K\left\{3p_1^2 - 2p_2^2 - \frac{p_1^6}{p_2^4} + 2\delta\,(p_2^2 - p_1^2)\right\}^{1/2} , \tag{60}$$

$$\mathrm{tg}\Phi = \frac{p_2^2[p_1^6 - p_2^6(1-\delta)]\sin(\vartheta_1 - \omega)}{p_1^4(p_1^4 - p_2^4)} \ . \tag{61}$$

During the same interval (58), a Homan-type of a flight is possible (45)-(49). Let us compose the difference between the squares of the characteristic velocities along a Homan ellipse and along the new orbit, which is equal to

$$\frac{K^2 p_2^4}{p_1^2 + p_2^2}\left[\frac{p_1^3\sqrt{p_1^2 + p_2^2}}{p_2^4} - \sqrt{2(1-\delta)}\right]^2 > 0 \ . \tag{62}$$

Consequently, the obtained orbits have a better optimum than a Homan ellipse.

Therefore, the function (34) assumes its least value on orbits described in II and III, provided that the conditions of (54) or of (58) are satisfied, respectively; otherwise, the least value will be on a Homan ellipse (45)-(49).

NOTE 4. The value of the parameter δ in many problems is small and in such cases the approximate magnitude of the value σ is

$$\sigma = 1 - \frac{1}{7}\delta - \frac{43}{686}\delta^2 + \ldots . \tag{63}$$

5. Flight Between Low-Eccentricity Orbits

Assuming that q_1, q_2 are small, let us introduce a small parameter ϵ as per the expressions

$$q_1 = q_1'\epsilon \;, \quad q_2 = q_2'\epsilon \;. \tag{64}$$

The equations (27)-(29) and (36)-(40) will be solved in form of series of powers of ϵ; we will use a vinculum for the coefficients of the sought series with the first powers of ϵ. Similar expansions were obtained by Kirpichnikov[1] for $\delta = 0$.

The solution will be carried out with the assumption that r_1, r_2 are not located in the zones of (54) or (58); i. e., a Homan ellipse will be taken as zero approximation.

Let us first not take into account the specific motions. In zero-order approximation for ϵ, we have only one solution -- a Homan ellipse (45)-(49) (with an accuracy of up to an arbitrarily chosen one of the angles of ϑ_1, ϑ_2, ω). The angular distance of the pericenter ω is determined by the first approximation with the aid of equations (36), (37), and (40), as follows:

$$\mathrm{tg}\omega = \frac{q_1'p_1\left[\lambda_1 + \frac{2p_1}{p} - \frac{2-\delta}{\sqrt{1-\delta}}\right]\sin\omega_1 - q_2'p_2\lambda_2\sin\omega_2}{q_1'p_1\left[\lambda_1 + \frac{2p_1}{p} - \frac{2-\delta}{\sqrt{1-\delta}}\right]\cos\omega_1 - q_2'p_2\lambda_2\cos\omega_2} \,. \tag{65}$$

The following expressions provide an exact solution of the first order:

[1] Ibid.

$$p' = \frac{q_1'p_1 \cos(\vartheta_1 - \omega_1) + q_2'p_2 \cos(\vartheta_2 - \omega_2)}{4p} \;,$$

$$p' = -\frac{q_1'p_1(q \mp 2p) \cos(\vartheta_1 - \omega_1) + q_2'p_2(q \pm 2p) \cos(\vartheta_2 - \omega_2)}{4p^2} \;, \qquad (66)$$

$$\omega' = \frac{-\alpha_* A - \beta_* B + C}{\alpha_* + \beta_* + \gamma_*} \;, \qquad \vartheta_1' = \omega' + A \;, \qquad \vartheta_2' = \omega' + B \;, \qquad (67)$$

$$\lambda_1' = \frac{pR_* - (q \mp 2p) S_*}{4p^2} \;, \qquad \lambda_2' = \frac{pR_* - (q \mp 2p) S_*}{4p^2} \;, \qquad (68)$$

$$\Delta U' = \frac{K^2\sqrt{1-\delta}}{\Delta U}\left\{\frac{p_1^3 - p^3\sqrt{1-\delta}}{p^2} p' + q\sqrt{1-\delta}\, p' \right.$$

$$\left. + \frac{q_1'p_1(2p - 2p_1\sqrt{1-\delta} - p^3) \cos(\vartheta_1 - \omega_1)}{p\sqrt{1-\delta}}\right\} \;, \qquad (69)$$

$$\Phi' = \pm \frac{pq\sqrt{1-\delta}(\vartheta_1' - \omega')}{p_1(p_1\sqrt{1-\delta} - p)} - \frac{q_1'p \sin(\vartheta_1 - \omega_1)}{p_1(p_1\sqrt{1-\delta} - p)} \;, \qquad (70)$$

where

$$A = \pm \frac{q_1'p_1}{\lambda_1 pq}\left[\frac{p}{p_1} + \frac{p_1}{p} + \lambda_1 - \frac{2-\delta}{\sqrt{1-\delta}}\right] \sin(\vartheta_1 - \omega_1) \;,$$

$$B = \mp \frac{q_2'p_2 \sin(\vartheta_2 - \omega_2)}{pq} \;, \qquad (71)$$

$$C = q_1'\left(\frac{p_1^2 - p^2}{p^2} p' - p_1\lambda_1' \mp q'\right) \sin(\vartheta_1 - \omega_1) - q_2'\lambda_2'p_2 \sin(\vartheta_2 - \omega_2) \;, \qquad (72)$$

$$\alpha_* = q_1'p_1\left[\frac{p}{p_1} + \frac{p_1}{p} + \lambda_1 - \frac{2-\delta}{\sqrt{1-\delta}}\right] \cos(\vartheta_1 - \omega_1) \;,$$

$$\beta_* = q_2'p_2\lambda_2 \cos(\vartheta_2 - \omega_2) \;,$$

$$\gamma_* = \pm\, q_1'q \cos(\vartheta_1 - \omega_1) \;, \qquad (73)$$

$$R_* = q_1'\left(1 - \frac{p_1^2}{p^2}\right) \cos(\vartheta_1 - \omega_1)$$

$$+ \left(\sqrt{1-\delta} + \frac{2p_1^3}{p^3} - 2\lambda_1 - 2\lambda_2\right) p' \mp (\lambda_1 - \lambda_2)\, q' \;, \qquad (74)$$

$$S_{*} = \pm q_1' \cos(\vartheta_1 - \omega_1) \mp (\lambda_1 - \lambda_2) p' - \sqrt{1 - \delta} q' \quad . \tag{75}$$

The solution of a problem of "energy-optimum" one-thrust flight without taking into account the pressure of light[1] with an accuracy of up to the terms of the first order of ϵ is obtained with the aid of the expressions (45)-(49), (65)-(75), provided that it is assumed that $\delta = 0$. In case the parameter δ of the same order as the eccentricities of the boundary orbits, is

$$\delta = \epsilon \delta' \quad , \tag{76}$$

for this solution, the corrections of the first order which are made necessary by the presence of light-pressure, will be

$$\left.\begin{aligned}
& p' = 0 \quad , \quad q' = 0 \\
& \omega' = \vartheta_1' = \vartheta_2' \\
& = \frac{2 q_1' q_2' p_1^4 p_2 p^3 (p_1 - p)^2 \sin(\omega_2 - \omega_1) \cos^2 \omega}{\{q_1' p_1 [8p^3(p_1 - p) + p_2^4 - p_1^3 p] \cos \omega_1 - q_2' p_1^3 p_2 (p_1 - p) \cos \omega_2\}^2} \delta' \quad . \\
& \lambda_1' = -\frac{p_2^4}{8p^4} \delta' \quad , \quad \lambda_2' = \frac{p_1^4}{8p^4} \delta' \quad . \\
& \Delta U' = \mp \frac{p_1^2}{2p} \delta' \quad , \quad \Phi' = 0 \quad .
\end{aligned}\right\} \tag{77}$$

Consequently, for a case where the parameter δ and the eccentricities of the boundary orbits are small, the effect of the pressure of light which was taken into account was reduced to a turn of the orbit of the flight by a value of the first order with respect to an orbit corresponding to an optimum problem in which the pressure of light is not taken into account. The size and the shape of the orbit and the true anomalies of its starting and final points change only by values of the second order.

Let us proceed with solving the problem with taking into account the specific motions. With a zero-order approximation, the only solution is a Homan ellipse (45)-(49) which satisfies the equality (41), while for the function ψ, we have

$$\psi = \frac{\pi}{(p^2 - q^2)^{3/2} \sqrt{1 - \delta}} \quad . \tag{78}$$

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[1] Ibid.

The starting angle ϑ_1 is found from the equality (43), as follows:

$$\vartheta_1 = \pi \frac{\left(\frac{p_1^2 + p_2^2}{2}\right)^{3/2} \frac{1}{\sqrt{1-\delta}} - p_1^3}{p_1^3 - p_2^3} + \frac{\omega_2 p_1^3 - \omega_1 p_2^3 - \alpha p_1^3 p_2^3}{p_1^3 - p_2^3} . \tag{79}$$

In the approximation of the first order, from the system of required conditions we find for p', q' the same expressions (66), while for ϑ_1', ϑ_2', and ω, we have:

$$\vartheta_1' = \frac{\tilde{A}(p_1^4 p_2 \sqrt{1-\delta} - pp_2^4) - \tilde{B}(p_1^4 p_2 \sqrt{1-\delta} - pp_1^4) - \tilde{C} p_1^4 p_2^4 \sqrt{1-\delta}}{p_1 p_2 (p_1^3 - p_2^3)\sqrt{1-\delta}} , \tag{80}$$

$$\vartheta_2' = \frac{\tilde{A}(p_2^4 p_1 \sqrt{1-\delta} - pp_2^4) - \tilde{B}(p_2^4 p_1 \sqrt{1-\delta} - pp_1^4) - \tilde{C} p_1^4 p_2^4 \sqrt{1-\delta}}{p_1 p_2 (p_1^3 - p_2^3)\sqrt{1-\delta}} \tag{81}$$

$$\omega' = \frac{\tilde{A}(p_2^4 p_1 \sqrt{1-\delta} - pp_2^4) - \tilde{B}(p_1^4 p_2 \sqrt{1-\delta} - pp_1^4) - \tilde{C} p_1^4 p_2^4 \sqrt{1-\delta}}{p_1 p_2 (p_1^3 - p_2^3)\sqrt{1-\delta}} , \tag{82}$$

where

$$\tilde{A} = A \pm \frac{p_1\sqrt{1-\delta} - p}{\lambda_1 p_1^4 pq \sqrt{1-\delta}} \lambda_3' , \quad \tilde{B} = B \mp \frac{p - p_2\sqrt{1-\delta}}{\lambda_2 p_2^4 pq \sqrt{1-\delta}} \lambda_3' , \tag{83}$$

$$\tilde{C} = \frac{3\pi(pp' - qq')}{(p^2 - q^2)^{5/2}\sqrt{1-\delta}} + \frac{2q_1' \sin(\vartheta_1 - \omega_1)}{p_1^4} - \frac{2q_2' \sin(\vartheta_2 - \omega_2)}{p_2^4} , \tag{84}$$

the function A and B are computed with the aid of expressions (71), and

$$\lambda_3' = \frac{q_1' p_1 \left(\frac{2p_1}{p} + \lambda_1 - \frac{2-\delta}{\sqrt{1-\delta}}\right) \sin(\vartheta_1 - \omega_1) + q_2' p_2 \lambda_2 \sin(\vartheta_2 - \omega_2)}{p_2^{-3} - p_1^{-3}} . \tag{85}$$

6. Condition for Energy-Saving Use of Shell-Probes Carrying Useful Loads

Let us consider in which case it is more profitable (by using less fuel) to deliver to a final orbit a useful load with a mass m_0: with the aid of a hollow shell-probe, or by delivering this load directly to a corresponding orbit of the flight. In the latter case, the pressure of light can be disregarded. We will assume that both cases require a characteristic velocity $\tilde{U}$ to overcome the attraction of the body in space from which, or from the artificial satellite of which the start takes place. We will confine ourselves with a case of circular boundary orbits, where $r_2 > r_1$. At $r_1 > r_2$, it is more profitable to launch a load so, that the resultant of the forces of repulsion by light should be as small as possible.

In a flight in which the pressure of light is not taken into account, a Homan ellipse requires a minimum of fuel consumption. The characteristic velocity ΔU_r of the initial thrust is defined as follows:

$$\Delta U_r = \frac{Kp_1(p_1 - p)}{p} \, . \tag{86}$$

The minimum value of the characteristic velocity of a flight which takes into account the pressure of light ($\delta < 0.5$), is equal to:

$$\Delta U = 0 \text{ at } \frac{r_1}{1 - 2\delta} > r_2 > r_1 \, , \tag{87}$$

$$\Delta U = \frac{Kp_1(p_1\sqrt{1 - \delta} - p)}{p} \text{ at } r_2 > \frac{r_1}{1 - 2\delta} \, . \tag{88}$$

Let c_1 be the velocity of the escaping gases; then, as we know,

$$\Delta U + \tilde{U} = c_1 \ln \frac{m_H}{m_0 + m_1} \, , \quad \Delta U_\Gamma + \tilde{U} = c_1 \ln \frac{m_H - m_1 + \Delta m}{m_0} \, , \tag{89}$$

where m_H is the initial mass of the space vehicle at the launching of the useful load with the shell-probe; Δm is the change in the mass of the fuel during a direct launching of the useful load to the orbit; m_1 is the mass of the shell-probe, for which the following is valid:

$$m_1 = 4\pi R^2 \gamma h \, . \tag{90}$$

The use of a shell-probe offers an advantage at $\Delta m > 0$, while at $\Delta m < 0$, less fuel is required to launch the useful load directly to the flight-orbit.

From the relationships (89), it is easy to find

$$\Delta m = m_0\left(e^{\frac{\Delta U_\Gamma + \tilde{U}}{c_1}} - e^{\frac{\Delta U + \tilde{U}}{c_1}}\right) - m_1\left(e^{\frac{\Delta U + \tilde{U}}{c_1}} - 1\right), \tag{91}$$

where ΔU_Γ, ΔU are determined by the expressions (86)-(88). At specific γ, h, R, m_0, p_2, p_1, $\tilde{U}$, the last relationship makes it possible to estimate which of the methods of launching the load will require less energy.

The condition for a smaller consumption of fuel by using a shell-probe is obtained in the following form:

$$\chi = \frac{m_0}{m_1} > \frac{e^{\frac{\Delta U}{c_1}} - e^{-\frac{\tilde{U}}{c_1}}}{e^{\frac{\Delta U_\Gamma}{c_1}} - e^{\frac{\Delta U}{c_1}}}, \tag{92}$$

however, it should be borne in mind that the right side depends on the parameter δ which, in its turn, depends as follows on χ:

$$\delta = \frac{E_\delta r_\delta^2}{4K^2\gamma h(1 + \chi)c}. \tag{93}$$

The expressions (92) and (93) make it possible to find the required relationship between γ, h, χ, p_1, p_2, $\tilde{U}$.

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13. ABSTRACT

Discussed is the construction of single-impulse trajectories of interorbital flights with a minimum consumption of mass in the gravitational field of the Sun -- a spherical central body radiating light symmetrically. Disregarded is the perturbing action of the space vehicle caused by objects in space located on the specified boundaries of the Kepler orbits. It is assumed that all three orbits, the initial, intermediate, and final, are located in one plane and that the motion along these orbits is proceeding in one direction.

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