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LONG-PERIOD VARIATIONS IN THE ECCENTRICITY AND ARGUMENT OF PERIGEE OF NIMBUS 2

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Summary

Long-period variations have been observed in the mean values of eccentricity and argument of perigee (published by the Goddard Space Flight Center) for Nimbus 2. It is shown that the use of a more recent determination of the earth zonal harmonic coefficients, plus additional first-order earth oblateness perturbations (for small eccentricity satellites) not included in the mathematical model for orbit determination, explains these variations.

LONG-PERIOD VARIATIONS IN THE ECCENTRICITY AND ARGUMENT OF PERIGEE OF NIMBUS 2

INTRODUCTION

In two earlier papers (References 1 and 2), the problem of small divisors in the case of low eccentricity satellites was analyzed at some length. Eccentricity was found to occur as a divisor in analytic expressions (both long and short period) for the perturbations, caused by the non-sphericity of the earth, in eccentricity, mean anomaly, and argument of perigee. Formulas for these perturbations were presented, and contain terms involving up to J_2^3 and $(J_i/J_2)^3$, where J_i is any odd zonal harmonic. In addition, the long-period terms were found to explain satisfactorily the observed variations in eccentricity and argument of perigee for the satellites Alouette 1 and Tiros 8.

Similar long-period variations in eccentricity and argument of perigee were observed in the case of Nimbus 2, another low eccentricity satellite. It will be shown here that the long-period expressions in References 1 and 2 are adequate representations of these perturbations.

LUNAR AND SOLAR EFFECTS

Mean values for the eccentricity and argument of perigee, developed at the Goddard Space Flight Center, were first corrected for lunar and solar gravitational effects and the effects caused by solar radiation pressure and lunar and solar tides; the formulas used to compute these perturbations appear in Reference 3 and 4. For the most part, these effects were fairly small, except for three near-resonant solar terms in the argument of perigee perturbations. However, since these terms all had periods in excess of 10^4 days, their effects over the time interval studied would appear to be secular and therefore wouldn't affect the analysis of the periodic variations.

The corrected values of eccentricity and argument of perigee appear in Tables 1 and 2 as e_c'' and g_c'', respectively.

LONG-PERIOD ZONAL HARMONIC PERTURBATIONS

Analytic formulas for the major long-period zonal harmonic effects on the eccentricity and argument of perigee for a low eccentricity satellite appear in References 1 and 2; they are reproduced here for convenience:

$$\Delta e = -\left[Q - \frac{Q^3}{8e_1^2} - \frac{J_2^2}{64e_1^2 a''^4}\left(28 - 101\cos^2 i'' + 218\cos^4 i''\right) Q\right]\cos\overline{\theta}$$

$$-\left[\frac{Q^2}{4e_1} + \frac{9J_2^2}{64e_1 a''^4}\sin^2 i''\left(1 - 3\cos^2 i''\right)\right]\cos 2\overline{\theta}$$

$$-\left[\frac{Q^3}{8e_1^2} + \frac{27J_2^2}{128e_1^2 a''^4}\sin^2 i''\left(1 - 3\cos^2 i''\right) Q\right]\cos 3\overline{\theta}$$

$$\Delta g = \left(\frac{Q}{e_1} + \frac{Q^3}{4e_1^3}\right)\sin\overline{\theta} + \left[\frac{Q^2}{2e_1^2} + \frac{9J_2^2\sin^2 i''\left(1 - 3\cos^2 i''\right)}{32e_1^2 a''^4}\right]\sin 2\overline{\theta}$$

$$+\left[\frac{Q^3}{3e_1^3} + \frac{9J_2^2\sin^2 i''\left(1 - 3\cos^2 i''\right)}{16e_1^3 a''^4} Q\right]\sin 3\overline{\theta}, \qquad (1)$$

where a'', e_1, and i'' are semi-major axis, eccentricity, and inclination constants, respectively. In addition,

$$\overline{\theta} = \text{constant} + \text{secular term} \simeq g'' + \frac{\pi}{2}$$

$$g'' = \text{argument of perigee}$$

$$Q = \frac{M}{N},$$

where

$$N = -\frac{3J_2}{4a''^3\sqrt{a''}}(1 - 5\cos^2 i'') + \frac{3J_2^2}{64a''^5\sqrt{a''}}(7 - 114\cos^2 i'' + 395\cos^4 i'')$$

$$- \frac{15J_4}{32a''^5\sqrt{a''}}(3 - 36\cos^2 i'' + 49\cos^4 i'') + \cdots$$

$$M = \frac{3J_3 \sin i''}{8a''^4\sqrt{a''}}(1 - 5\cos^2 i'') + \frac{15J_5 \sin i''}{32a''^6\sqrt{a''}}(1 - 14\cos^2 i'' + 21\cos^4 i'')$$

$$+ \frac{105J_7 \sin i''}{1024a''^8\sqrt{a''}}(5 - 135\cos^2 i'' + 495\cos^4 i'' - 429\cos^6 i'')$$

$$+ \frac{315J_9 \sin i''}{4096a''^{10}\sqrt{a''}}(7 - 308\cos^2 i'' + 2002\cos^4 i'' - 4004\cos^6 i'' + 2431\cos^8 i'')$$

$$+ \frac{3465J_{11} \sin i''}{131072a''^{12}\sqrt{a''}}(21 - 1365\cos^2 i'' + 13650\cos^4 i'' - 46410\cos^6 i''$$

$$+ 62985\cos^8 i'' - 29393\cos^{10} i'') + \cdots \qquad (2)$$

However, the terms $-Q\cos\overline{\theta}$ (for Δe) and $(Q/e_1)\sin\overline{\theta}$ (for Δg) have already been used in the mathematical model for orbit determination, at least for the zonal harmonics J_2, J_3, J_4, and J_5. The following values were used for J_3 and J_5:

$$J_3 = -2.285 \times 10^{-6}, \quad J_5 = -0.232 \times 10^{-6}.$$

It is suggested that Kozai's 1964 determination of the harmonic coefficients (see Reference 5) form a better set – they are

$$J_2 = 1.082645 \times 10^{-3} \qquad J_3 = -2.546 \times 10^{-6}$$

$$J_4 = -1.649 \times 10^{-6} \qquad J_5 = -0.210 \times 10^{-6}$$

$$J_7 = -0.333 \times 10^{-6} \qquad J_9 = -0.053 \times 10^{-6}$$

$$J_{11} = 0.302 \times 10^{-6} .$$

It is further suggested that the observed variations in e and g would reflect the differences between the more accurate J_3, J_5 values and the values which were actually used in the $-Q\cos\overline{\theta}$ and $(Q/e_1)\sin\theta$ terms.

Therefore, setting

$$\Delta J_3 = -2.546 \times 10^{-6} - (-2.285 \times 10^{-6}) = -0.261 \times 10^{-6}$$

$$\Delta J_5 = -0.210 \times 10^{-6} - (-0.232 \times 10^{-6}) = 0.22 \times 10^{-7}$$

$$\Delta M = \frac{3\Delta J_3 \sin i''}{8a''^4\sqrt{a''}}(1 - 5\cos^2 i'') + \frac{15\Delta J_5 \sin i''}{32a''^6\sqrt{a''}}(1 - 14\cos^2 i'' + 21\cos^4 i'')$$

$$+ \frac{105J_7 \sin i''}{1024a''^8\sqrt{a''}}(5 - 135\cos^2 i'' + 495\cos^4 i'' - 429\cos^6 i'')$$

$$+ \cdots$$

$$\Delta Q = \frac{\Delta M}{N}, \tag{3}$$

the expressions to be compared with the observed variations are

$$\Delta e = -\left[\Delta Q - \frac{Q^3}{8e_1^2} - \frac{J_2^2}{64e_1^2a''^4}(28 - 101\cos^2 i'' + 218\cos^4 i'')Q\right]\cos\overline{\theta}$$

$$-\left[\frac{Q^2}{4e_1} + \frac{9J_2^2}{64e_1a''^4}\sin^2 i''(1 - 3\cos^2 i'')\right]\cos 2\overline{\theta}$$

$$-\left[\frac{Q^3}{8e_1^2} + \frac{27J_2^2}{128e_1^2a''^4}\sin^2 i''(1 - 3\cos^2 i'')Q\right]\cos 3\overline{\theta}$$

$$\Delta g = \left(\frac{\Delta Q}{e_1} + \frac{Q^3}{4e_1^3}\right) \sin \overline{\theta} + \left[\frac{Q^2}{2e_1^2} + \frac{9J_2^2 \sin^2 i'' (1 - 3\cos^2 i'')}{32 e_1^2 a''^4}\right] \sin 2\overline{\theta}$$

$$+ \left[\frac{Q^3}{3e_1^3} + \frac{9J_2^2 \sin^2 i'' (1 - 3\cos^2 i'')}{16 e_1^3 a''^4} Q\right] \sin 3\overline{\theta} \; . \tag{4}$$

Using

$a'' = 1.1782076$ earth radii

$e_1 = .00560$

$i'' = 100^\circ\!.306$

and Kozai's 1964 determination of the zonal harmonic coefficients, Equations (4) become

$$\Delta e = -(1.0923971 \times 10^{-4}) \cos \overline{\theta} - (0.63515970 \times 10^{-4}) \cos 2\overline{\theta} + (0.85403736 \times 10^{-5}) \cos 3\overline{\theta}$$

$$\Delta g = 1^\circ\!.3454747 \sin \overline{\theta} + 1^\circ\!.2997132 \sin 2\overline{\theta} + 0^\circ\!.23301303 \sin 3\overline{\theta}. \tag{5}$$

OBSERVED VARIATIONS

The values of $e''_{c^\cdot}$ and $g''_{c^\cdot}$ were then fit, by means of least squares, to the following models:

$$e''_{c^\cdot} = e_0 + \sum_{n=1}^{9} A_n \cos n\overline{\theta}$$

$$g''_{c^\cdot} = g_0 + \dot{g}''_{c^\cdot}(t - t_0) + \sum_{n=1}^{9} B_n \sin n\overline{\theta}, \tag{6}$$

where e_0, g_0, A_n, and B_n are constants, $\dot{g}''_{c^\cdot}$ is the secular motion of $g''_{c^\cdot}$, $t - t_0$ is the elapsed time in days, and

$$\overline{\theta} = 51^\circ\!.34033 - (2^\circ\!.3536297/\text{day})\,(t - t_0).$$

The constant $51\overset{\circ}{.}34033$ is merely the first value of g''_c plus $90°$, and the secular motion of $\bar{\theta}$ was computed from the expression for dg''/dt appearing in Reference 6. The results are

$$e''_c = 0.55936191 \times 10^{-2} - (1.200321 \times 10^{-4}) \cos \bar{\theta} - (0.648964 \times 10^{-4}) \cos 2\bar{\theta}$$

$$+ (0.44700 \times 10^{-5}) \cos 3\bar{\theta} + (1.5399 \times 10^{-6}) \cos 4\bar{\theta} + (1.7144 \times 10^{-6}) \cos 5\bar{\theta}$$

$$- (0.9668 \times 10^{-6}) \cos 6\bar{\theta} + (0.34168 \times 10^{-5}) \cos 7\bar{\theta} + (1.9838 \times 10^{-6}) \cos 8\bar{\theta}$$

$$+ (0.8763 \times 10^{-6}) \cos 9\bar{\theta}$$

$$g''_c = 679\overset{\circ}{.}62285 - (2\overset{\circ}{.}3542110/\text{day})\ (t - t_0) + 1\overset{\circ}{.}3007803 \sin \bar{\theta}$$

$$+ 0\overset{\circ}{.}8192370 \sin 2\bar{\theta} - 0\overset{\circ}{.}0202026 \sin 3\bar{\theta} + 0\overset{\circ}{.}0196037 \sin 4\bar{\theta}$$

$$+ 0\overset{\circ}{.}0099007 \sin 5\bar{\theta} + 0\overset{\circ}{.}0441686 \sin 6\bar{\theta} + 0\overset{\circ}{.}0034652 \sin 7\bar{\theta}$$

$$+ 0\overset{\circ}{.}0379263 \sin 8\bar{\theta} + 0\overset{\circ}{.}0279919 \sin 9\bar{\theta}. \tag{7}$$

A comparison between Equations (5) and (7) indicates that the major observable periodic variations in e and g (the $\bar{\theta}$ and $2\bar{\theta}$ terms) are explainable by the theory. There is somewhat of a discrepancy between the amplitudes of the $\sin 2\bar{\theta}$ terms. However, it should be noted (see Equations (4)) that the theoretical amplitude is sensitive to the value of e_1 used; use of a larger value would bring closer agreement.

Figures 1 and 2 show the closeness of the least squares fits.

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REFERENCES

1. Felsentreger, Theodore L., and Victor, Eric L., "On the Long Period Perturbations in the Motion of Small Eccentricity Satellites," GSFC X-547-66-577, December, 1966.

2. Felsentreger, Theodore L., and Steinberg, Ellen L., "On the Perturbations of Small Eccentricity Satellites," GSFC X-547-67-102, March, 1967.

3. Murphy, James P., and Felsentreger, Theodore L., "Analysis of Lunar and Solar Effects on the Motion of Close Earth Satellites," NASA TN D-3559, August, 1966.

4. Fisher, David, and Felsentreger, Theodore L., "Effects of the Solar and Lunar Tides on the Motion of an Artificial Satellite," GSFC X-547-66-560, November, 1966.

5. Kozai, Y., "New Determination of Zonal Harmonics Coefficients of the Earth's Gravitational Potential," SIAO Special Report No. 165, November 2, 1964.

6. Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," Astronomical Journal 64 (9), pp. 378-397, November, 1959.

Table 1. Eccentricity of Nimbus 2

$t - t_0$ (days)	$e_c'' \times 10^2$	$\left(e_0 + \sum_{n=1}^{9} A_n \cos n\bar{\theta}\right) \times 10^2$
0	.55332889	.55309715
7	.54728658	.54689101
14	.54168522	.54263530
21	.54233060	.54216936
28	.54606277	.54230895
35	.54525536	.54553649
42	.54996346	.55172237
49	.55606657	.55742556
55	.55957010	.56257593
62	.56235129	.56728237
69	.56480941	.56868472
76	.56463475	.56837489
83	.56371704	.56742564
90	.56290166	.56565369
97	.56267565	.56413156
104	.56470174	.56492539
111	.56764970	.56686812
118	.57106100	.56810100
125	.57244757	.56866868
132	.57036198	.56814157
139	.56764492	.56465560
146	.56238443	.55874767
153	.55574878	.55306014
160	.54816344	.54685197
167	.54255936	.54262321
174	.54007409	.54216969
181	.54265645	.54231507
188	.54266910	.54557165
195	.55178840	.55176106
202	.55480257	.55746235
209	.56466682	.56344980
216	.56788385	.56768853
223	.56750003	.56869821
230	.56620046	.56827280
237	.56768515	.56721607
244	.56621362	.56534977
251	.56548537	.56408524
258	.56634020	.56520973
264	.56908258	.56687891
271	.57008731	.56810638
278	.57087450	.56867023
285	.56981932	.56813067
292	.56534614	.56462217
299	.55772803	.55870967
306	.55368415	.55302309

e_0 = .0055936191 A_1 = −.0001200321 A_2 = −.0000648964 A_3 = .0000044700

A_4 = .0000015399 A_5 = .0000017144 A_6 = −.0000009668 A_7 = .0000034168

A_8 = .0000019838 A_9 = .0000008763 $\bar{\theta} = 51^\circ\!.34033 - (2^\circ\!.3536297/\text{day})\,(t - t_0)$

$t - t_0$ = days since 5/22/66, 0 hrs. U.T.

Table 2. Argument of Perigee of Nimbus 2

$t - t_0$ (days)	g''_{c} (deg.)	$g''_{c} - [g_0 + \dot{g}''_{c}(t - t_0)]$ (deg.)	$\sum_{n=1}^{9} B_n \sin n\bar{\theta}$ (deg.)
0	681.34033	1.71748	1.81008
7	664.56280	1.41943	1.42397
14	647.43303	0.76913	0.98474
21	630.18123	-0.00319	0.12833
28	612.91141	-0.79353	-0.84045
35	596.17881	-1.04665	-1.32491
42	579.25093	-1.49506	-1.74587
49	562.60975	-1.65676	-1.81638
55	548.70462	-1.43662	-1.60563
62	532.58137	-1.08040	-1.22130
69	516.59107	-0.59122	-0.63771
76	500.47859	-0.22422	-0.25701
83	484.09840	-0.12494	0.00321
90	467.60543	-0.13843	0.15816
97	450.82827	-0.43611	0.03784
104	434.13663	-0.64828	-0.14006
111	417.82345	-0.48198	-0.08924
118	401.75990	-0.06605	0.16244
125	385.67054	0.32406	0.46239
132	369.87252	1.00552	1.01109
139	354.07590	1.68838	1.49166
146	337.83500	1.92696	1.77110
153	321.44596	2.01739	1.80866
160	305.04411	2.09502	1.42115
167	287.97163	1.50202	0.98117
174	270.32661	0.33647	0.12129
181	252.80550	-0.70516	-0.84488
188	236.12142	-0.90976	-1.32760
195	218.76501	-1.78669	-1.74801
202	202.22859	-1.84364	-1.81532
209	186.04263	-1.55012	-1.56182
216	169.99133	-1.12194	-1.13988
223	153.95519	-0.67861	-0.56128
230	137.79547	-0.35885	-0.21829
237	121.45899	-0.21585	0.04051
244	105.04672	-0.14865	0.15577
251	88.51341	-0.20248	0.00728
258	71.94073	-0.29568	-0.15223
264	57.97534	-0.13581	-0.08798
271	41.83007	0.19840	0.16406
278	25.69696	0.54477	0.46502
285	9.76031	1.08759	1.01492
292	- 6.29829	1.50843	1.49380
299	-22.50038	1.78586	1.77253
306	-38.95179	1.81392	1.80722

$g_0 = 679^\circ.62285$ $\quad \dot{g}''_{c} = -2^\circ.3542110$/day $\quad B_1 = 1^\circ.3007803$ $\quad B_2 = 0^\circ.8192370$

$B_3 = -0^\circ.0202026$ $\quad B_4 = 0^\circ.0196037$ $\quad B_5 = 0^\circ.0099007$ $\quad B_6 = 0^\circ.0441686$

$B_7 = 0^\circ.0034652$ $\quad B_8 = 0^\circ.0379263$ $\quad B_9 = 0^\circ.0279919$

$\bar{\theta} = 51^\circ.34033 - (2^\circ.3536297/\text{day})(t - t_0)$

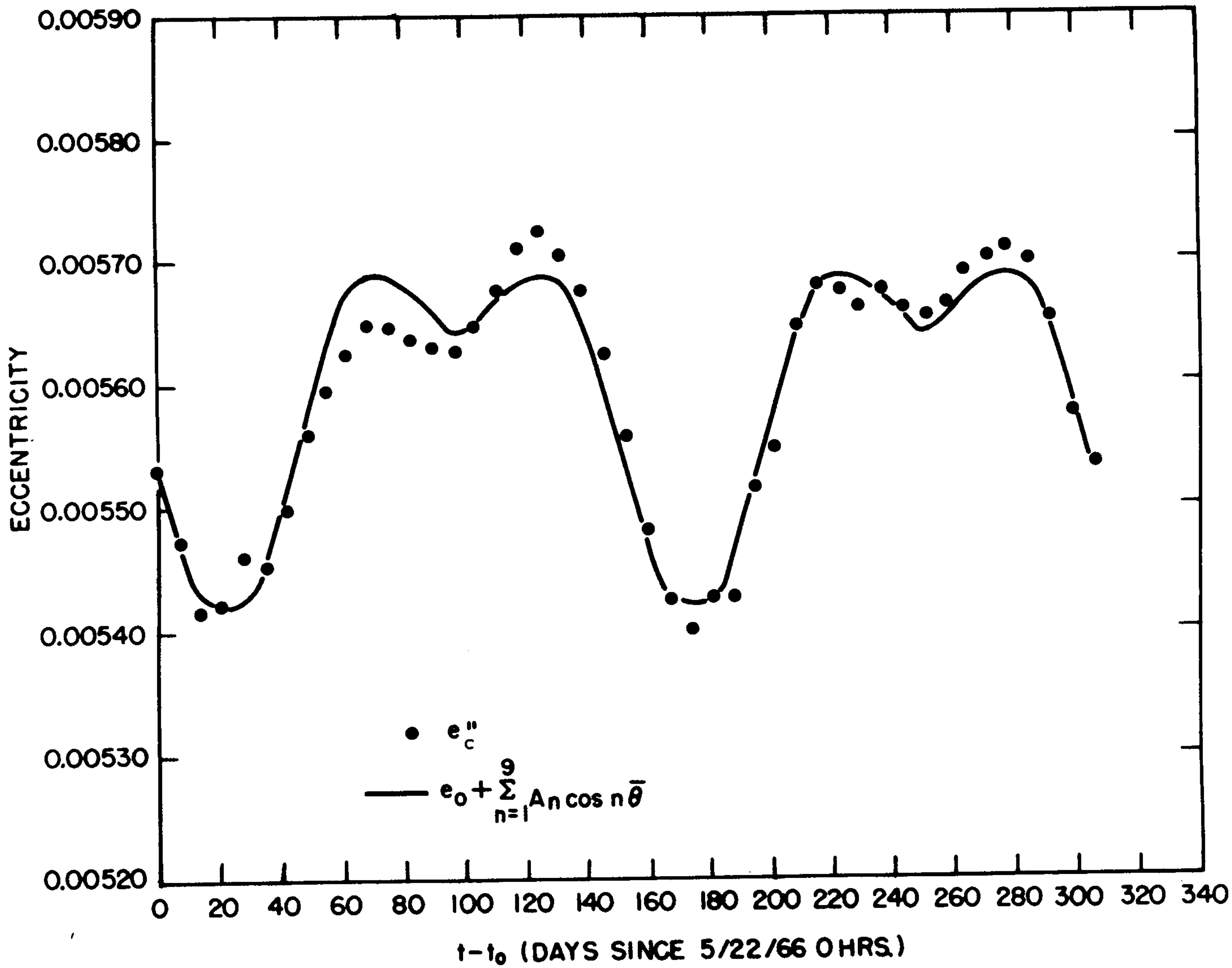

Figure 1–Eccentricity of Nimbus 2.

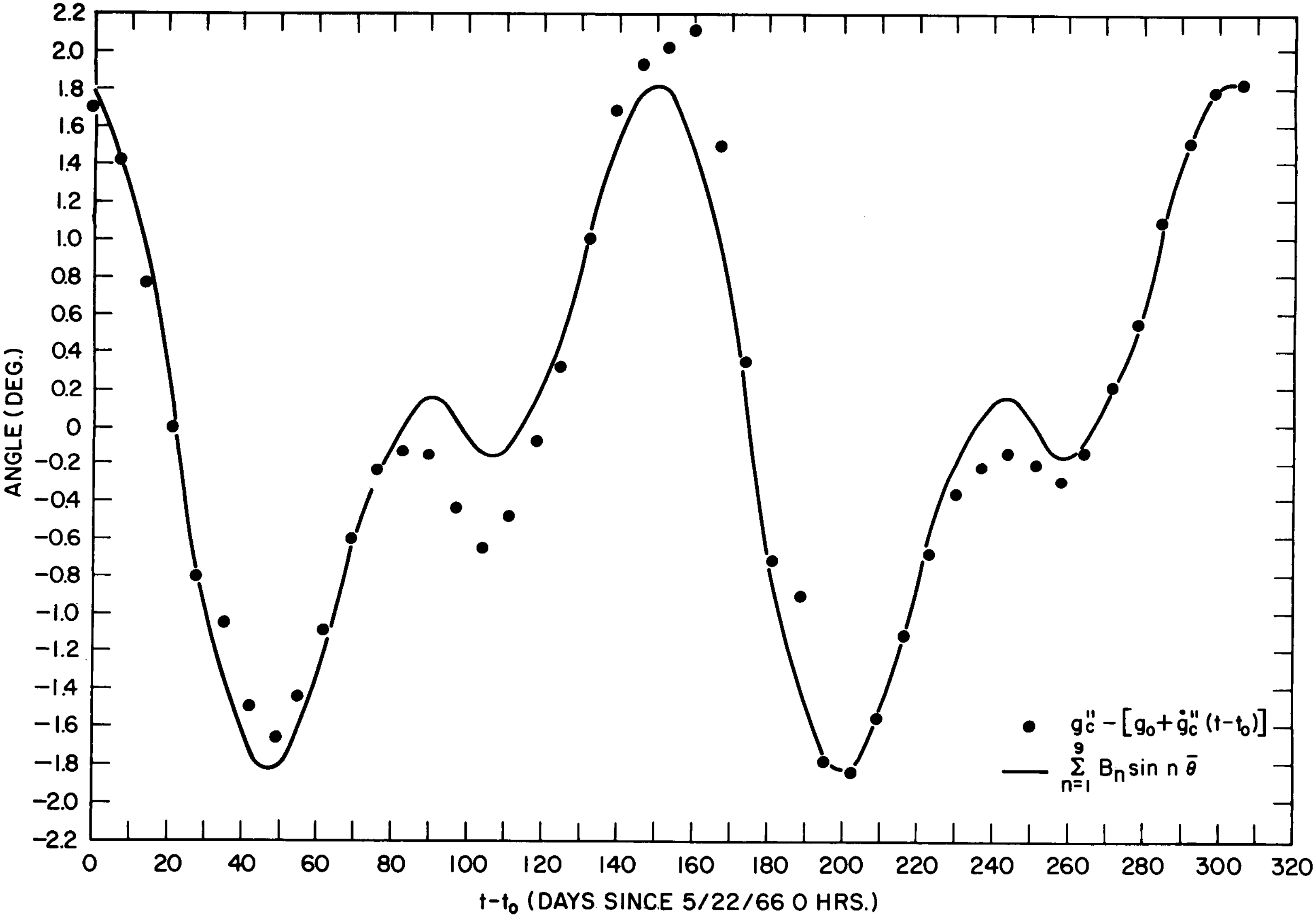

Figure 2–Argument of Perigee of Nimbus 2.