X-643-67-501

NASA TM X-63029

CHARACTERISTICS OF PLANETARY FLY-BY TRAJECTORIES

J. A. BEHUNCIK

OCTOBER 1967

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-643-67-501

CHARACTERISTICS OF PLANETARY FLY-BY TRAJECTORIES

J. A. Behuncik

October 1967

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

CHARACTERISTICS OF PLANETARY FLY-BY TRAJECTORIES

J. A. Behuncik

SUMMARY

This paper presents the results of an initial investigation into the phenomenon of planetary fly-by and the possible free-fall transfer trajectories associated with them. The analysis consists of two parts. The fly-by phase consists of determining for various encounter conditions the variation of maximum changes in velocity, speed, and energy that occur during planetary fly-by. The maximum change in flight-path angle also is presented for each planet. The transfer phase consists of determining the transfers that emanate from earth and terminate at the circle-of-influence with the appropriate encounter conditions that result in a fly-by with a maximum change in one of the above-mentioned parameters. Time of flight and transfer angle are presented for such possible transfers.

It is not possible to obtain with free-fall transfers from earth the theoretically possible maximum velocity change from any of the planets except Venus and Mars. The maximum velocity change of 7.3 km/sec can be obtained from Venus with a free-fall transfer of approximately 50 days duration. The heliocentric injection velocity for such transfers is in the range of from 27 to 33 km/sec. The maximum velocity change of 3.6 km/sec may be obtained from Mars with a free-fall transfer of almost 145 days duration. The necessary heliocentric injection velocity at earth for such transfers is approximately 32 km/sec. Free-fall transfers that derive maximum speed change from Mercury do not exist. The maximum speed change due to fly-by may be obtained from Jupiter, Saturn, Uranus, Neptune, and Pluto. For example, a free-fall transfer of 700 days duration may result in the maximum positive speed change of 24 km/sec from fly-by of Jupiter. The necessary heliocentric injection velocity for such a transfer, however, is close to escape velocity at one astronomical unit, i. e. $\sim$42 km/sec.

PRECEDING PAGE BLANK NOT FILMED.

CONTENTS

TABLES

LIST OF SYMBOLS

a	Semi-major axis of orbit
e	eccentricity of orbit
E_1	eccentric anomaly of transfer orbit evaluated at point of entry into circle-of-influence
E_o	eccentric anomaly of transfer orbit evaluated at intersection of transfer orbital path and earth orbital path
f	true anomaly of orbit
h	angular momentum of fly-by trajectory
P	semi-latus rectum, or parameter, of orbit
R_S	radius of circle-of-influence
r_π	perifocal distance
t_f	time of flight of transfer trajectory
v_E	encounter velocity, i.e., velocity at point of entry into circle-of-influence, but in heliocentric reference
v_{PE}	post-encounter velocity, i.e., velocity at point of exit from circle-of-influence, but in heliocentric reference
v_p	velocity of planet
v_π	velocity at perifocus
v'_∞	velocity at circle-of-influence in fly-by planet reference
$D\beta$	flight-path angle change due to planetary fly-by
ϵ	encounter angle measured clockwise from line of motion of fly-by planet
η	argument of perifocus of fly-by trajectory measured clockwise from positive x axis

θ angle from positive x axis to position vector of point of entry into circle-of-influence

μ_p gravitational parameter of fly-by planet.

CHARACTERISTICS OF PLANETARY FLY-BY TRAJECTORIES

INTRODUCTION

With the proposal of various scientific programs intended to probe the solar system there has been a renewed interest in the principle of planetary fly-by, or swing-by, as a means of shaping a heliocentric transfer trajectory. This is evident from current literature as indicated, for example, in References 1, 2, and 3. Some reports, however, present only maximum positive velocity changes occurring as a result of fly-by, while others use the fly-by of an intermediate body to obtain a desired change in flight-path angle in designing missions to a more distant planet. These reports present little information concerning the process whereby the velocity and/or flight-path angle change is actually obtained. Others seem to imply that the maximum possible change is, indeed, always obtained from a planetary fly-by. Thus, there appeared to be a need for a more detailed investigation into the phenomenon of planetary fly-by to determine more fully the behavior of certain performance parameters and their effect on the heliocentric transfer trajectory.

This paper presents the results of an investigation into the phenomenon of planetary fly-by, or swing-by, trajectories with the intention of determining the maximum values of certain performance parameters and how they vary with different encounter geometries, as well as their associated free-fall transfer trajectories. The intent of the investigation is to give a more definitive description of what changes occur in a trajectory during a planetary fly-by. The parameters considered are the following: maximum velocity change, defined as the magnitude of the maximum velocity change vector; maximum speed change, defined as the maximum value of the difference of the magnitudes of the encounter and post-encounter velocity vectors; and, finally, the energy change in the heliocentric transfer due to planetary fly-by. The parameters presented for the transfer are time of flight and transfer angle.

The procedure was to perform a parametric study with a simplified mathematical model for which certain basic assumptions were made. The results, however, are meaningful and informative since the assumptions are really first order approximations to the problem.

ANALYSIS

Assumptions

For this study certain basic assumptions were made to simplify the required mathematical model. It was assumed that:

- the motion of the planets is coplanar;
- the planets move in circular orbits at their mean distances from the sun;
- the use of two-body equations of motion provided sufficiently accurate representation of the true motion; and
- the coordinate system used when in the vicinity of the fly-by planet is space-fixed.

The concept of spheres-of-influence is used; but since all motion is planar it is understood that the spheres have been reduced to circles-of-influence.

Figure 1 shows the parameters and axis system used for computations inside the circle-of-influence. The parameters are identified in the list of symbols. All physical data used in the study is presented in Table 1. All data in the table are from Reference 4 with the exception of the radius of the planet, Pluto, and the circular velocities of the planets. The radius of Pluto is the value presented in Reference 5. The circular velocities of the planets are computed values, obtained using the mean distances of the planets and the gravitational parameter of the sun.

Procedure

The procedure consisted of performing a parametric study with the mathematical model established in accordance with the assumptions stated in the previous section and obtaining the maximum changes in velocity, speed, flight-path angle, and energy. It was decided to present only the maxima of the aforementioned parameters to avoid presentation of voluminous data. The results that are presented prove sufficiently informative since they show that energy can be gained or lost during encounter with a planetary fly-by (or swing-by).

At a distance equal to the radius of the circle-of-influence escape velocity is calculated. Using the fact that energy of the fly-by trajectory is conserved inside the circle-of-influence, one computes the velocity at perifocus. With this information the elements of the hyperbola are computed using standard two-body equations as presented in Reference 6. With the elements of the hyperbolic fly-by trajectory the state is computed at the entry and exit points of the circle-of-influence. The appropriate transformation is performed to obtain the state in heliocentric space for both of these points. Having the orbital data before and after encounter of the fly-by planet, one obtains the effects of the fly-by. A more detailed description of the procedure is presented in the Appendix.

The independent parameters that are varied follow. The hyperbolic velocity with respect to the planet under consideration is varied from parabolic velocity to an arbitrarily large value, approximately 50 km/sec, for fixed values of perifocal distance, r_π, and argument of perifocus, η. (See Figure 1 and list of symbols.) Perifocal distance is then changed and, again holding argument of perifocus fixed, hyperbolic velocity is changed through its full range of variation. This is repeated until perifocal distance is varied from one to five planetary radii. Finally, with perifocal distance again set at one planetary radius, the argument of perifocus is varied by ten degree increments from zero to 360 degrees, allowing hyperbolic velocity to run through its full range of variation for each value of argument of perifocus. In this manner full families of hyperbolae are generated for all combinations of encounter geometry, allowing one to see how the dependent parameters change under all possible conditions. Perifocal distance of one planetary radius only was used while varying argument of perifocus because it presents the theoretical maximum effect that can be obtained from a planet.

The dependent parameters whose maxima are presented in this paper are as follows:

- the velocity change, DV, defined as

$$DV = |\overline{\Delta V}| = |\overline{V}_{P.E.} - \overline{V}_E| ,$$

  where $\overline{V}_{P.E.}$ is the post-encounter velocity vector in heliocentric space, and $\overline{V}_E$ is the encounter velocity vector in heliocentric space;

- the speed change, DS, defined as

$$DS = |\overline{V}_{P.E.}| - |\overline{V}_E| ; \tag{1}$$

- the change in flight-path angle, $D\beta$, defined as

$$D\beta = \beta_{P.E.} - \beta_E , \tag{2}$$

  where $\beta_{P.E.}$ and β_E are the flight-path angles of the post-encounter and encounter velocity vectors, respectively; and

- the energy change, DE, defined as

$$DE = \left[\frac{1}{2} \cdot |\overline{V}_{P.E.}|^2 - |\overline{V}_E|^2\right]. \tag{3}$$

It should be emphasized that the parameters, DV and DS, are both scalars but in a different sense. Velocity change, DV, is the magnitude of a vector and, hence, is scalar, while speed change, DS, is simply the difference between two scalar quantities, i.e., speed change is not the magnitude of the velocity change vector.

After the various maxima are obtained for each configuration for all planets one then attempts to generate free-fall heliocentric transfers that match the state at the respective entry points of the circles-of-influence. With the state at the circle-of-influence transformed to heliocentric coordinates the semi-major axis, eccentricity, perihelion distance, and aphelion distance in sun reference are computed and used to determine whether a free-fall transfer from earth is possible. In the context of this study a transfer is considered possible if the heliocentric transfer trajectory merely intersects the orbital path of the earth. No attempt is made to determine the geometry-time constraint, i.e., whether the earth is indeed at the point on its orbital path intersected by the transfer trajectory. Nor is effort made to match the heliocentric injection conditions at earth's orbit with a launch trajectory emanating from an existing launch site. Such considerations are beyond the scope of the present study.

In this study there is no possible free-fall transfer to an inner planet if:

- aphelion is less than one astronomical unit; and/or

- the transfer trajectory is hyperbolic.

If the aphelion distance of the free-fall transfer trajectory — an ellipse with the sun at one focus — is less than one astronomical unit, the ellipse crosses the orbital path of the proposed fly-by planet but does not intersect the orbit of earth. If the transfer trajectory is hyperbolic the probe escapes the solar system. It is assumed that such a case is of no interest and, therefore, no consideration is given to such transfers.

There is no possible transfer to an outer planet if:

- perihelion distance is greater than one astronomical unit; and/or

- the transfer trajectory is hyperbolic.

If the perihelion distance of the transfer trajectory is greater than one astronomical unit, the ellipse crosses the orbital path of the proposed fly-by planet but does not intersect the orbital path of the earth.

This precédure is followed for each of the planets except earth which is excluded from the present study since this planet is the originator planet.

RESULTS AND DISCUSSION

This section is divided into two parts. The first part is a discussion of the planetary fly-by phase. The second part contains a discussion of the transfer trajectories that reach the circles-of-influence of the fly-by planets.

Fly-by Phase

Presented in Figure 2 is the variation of velocity change, DV, with true anomaly, f, for the point of entry into the circle-of-influence for Mercury. The change in velocity is presented for perifocal distances of one planetary radius as well as one radius plus 1000, 3000, and 6000 kilometers. The argument of périfocus, η, the angle between the positive x-axis and the perifocal vector, counterclockwise being the positive direction, is zero. The velocity change increases, reaches a maximum, and decreases, approaching zero. The increase is due to the increase in hyperbolic velocity. It is interesting to note that the velocity change does not increase indefinitely. That it does not continue to increase is due to the fact that as the hyperbolic velocity is made larger the energy of the hyperbola becomes so high that a probe flying such a path is moving so rapidly that the effect of fly-by becomes negligible. The hyperbola becomes more flattened and approaches a rectilinear path running through the circle-of-influence. From the figure it is seen that the maximum velocity change is 3.0 km/sec for a hyperbola that enters the circle-of-influence at a true anomaly of -120 degrees. These values are in excellent agreement with the analytical work Niehoff has done on this problem (Ref. 4).

Variation of argument of perifocus did not alter the behavior of velocity change with true anomaly of entry point into the circle-of-influence, nor did it change the maximum value of velocity change. Thus, 3.0 km/sec is the maximum velocity change that can be derived from a fly-by of the planet, Mercury.

It is evident also from Figure 2 that increasing the magnitude of perifocal distance, r_π, decreases the velocity change, as expected.

Figure 3 presents velocity change versus true anomaly, f, of the entry point into the circle-of-influence for all the planets. The argument of perifocus,

η, is zero and the perifocal distance, r_π, is one planetary radius. The maximum velocity change, DV_{MAX}, varies from 3.0 km/sec for Mercury to 42.5 km/sec for Jupiter. These maxima are presented in Table 2. Table 2 also includes maximum values of speed change, change in flight-path angle, and energy change due to fly-by of the planets. The entries of the table are ordered by the amount of maximum velocity change imparted by the planets.

Figure 4 shows the variation of maximum velocity change, DV_{MAX}, with perifocal distance, r_π, for values up to, and including, five planetary radii for the planets. It is obvious from the definition that DV will decrease to zero as perifocal distance approaches the radius of the circle-of-influence for any given planet.

Up to this point the discussion has been concerned with velocity changes only. From its definition in Equation 1 it is clear that the minimum velocity change, is zero. This is not to imply, however, that only a gain might be realized from a planetary fly-by. There is need, therefore, of a parameter that can show both loss and gain in performance due to a fly-by. Speed change, DS, defined in Equation 2, is such a parameter. A positive speed change indicates a gain in performance, a negative speed change a loss. This parameter can also show no effect due to fly-by by being zero. While not presented in this paper, results of the calculations of speed change show that, as was the case with velocity change, a maximum occurs. Further, the maximum values of speed change no longer occur for a hyperbola that originates at the same point on the circle-of-influence for all the planets. Rather, the maximum value of speed change, DS_{MAX}, varies with argument of perifocus, η, for each planet. Also, the maximum speed change, DS_{MAX}, ranges from positive to negative values. Thus, there can be a maximum speed gain or a maximum speed loss from a fly-by of a particular planet. Since, however, the main point of interest is to see how the planetary fly-by trajectories relate to possible transfer trajectories emanating from the earth, it is more convenient to refer the maximum speed change to another parameter. That parameter is the encounter angle, ϵ, an angle measured positive in a counterclockwise sense from the line of motion of the planet to the position vector of the entry point on the circle-of-influence (see Figure 1)

Figure 5 presents maximum speed change, DS_{MAX}, versus encounter angle, ϵ, for all the planets. For the data presented in this figure the perifocal distance is one planetary radius for each planet. Jupiter again has the most pronounced effect. It produces a maximum speed change of approximately ± 24 km/sec, positive for a trajectory that originates on the circle-of-influence at $\epsilon = 0°$ and negative for a trajectory with an encounter angle of 230 degrees. For each of the planets it is possible to gain or lose performance from a fly-by depending upon encounter. Note also that for each of the planets there are two values of encounter angle, ϵ, for which no speed change is derived. In the case of Jupiter, for example, no speed change is derived for trajectories with encounter angles of 120 and 317 degrees. The fact that the maximum speed change for all the planets do not all become zero for the same point of entry into the circle-of-influence is due in part to the fact that this parameter does not take into account the change in flight-path angle that also takes place during the fly-by. This figure shows more dramatically the possible results of planetary fly-by trajectories.

The maximum speed changes obtained from fly-by are also presented in Table 2. The values presented may be either additive or subtractive depending upon the encounter geometry. To be emphasized is the fact that these values of speed change are maximum. Indeed, if entry of the circle-of-influence is not made with the appropriate velocity, a speed change less than the maximum presented in Figure 5 for a particular value of encounter angle will occur.

Another important result of a fly-by is change in flight-path angle. Presented in Table 2 are the maximum changes in this quantity. It is seen that all the planets from Jupiter out to Pluto are capable of reversing the original line of flight of a vehicle that might fly by them. Of course, the amount of deflection depends upon the encounter conditions.

The final quantity to be mentioned is the change in energy imparted to a spacecraft during fly-by (see Equation 4). Presented in Figure 6 is the variation of maximum energy change, DE_{MAX}, versus encounter angle, ϵ, for all the planets. The perifocal distance is one planetary radius. The absolute maxima of energy gain for the planets occurs at an encounter angle of 60 degrees, a result which is in excellent agreement with Niehoff (Ref. 4). As expected, Jupiter is again the most prominent in its effect on fly-by trajectories, producing an absolute maximum energy change of 550 $(km/sec)^2$. A positive energy change is possible through the range of encounter angles from 330 to 150 degrees — a 180 degree arc not symmetric about the line of motion of the planets. This is due to the fact that the abscissa is encounter angle, a quantity that is 120 degrees out of phase with the argument of perifocus, η. This can be seen if 120 degrees is added to encounter angle. The 180 degree arc is then symmetric to the line of motion, namely from 90 to 270 degrees. It is then seen that energy is gained from trajectories with points of closest approach behind the planet, and energy is lost for those trajectories with points of closest approach in front of the planet. All data presented in Figure 6 are maximum values. Therefore, for any given encounter angle it is possible to have a fly-by trajectory that results in an energy change less than the value presented in this figure. The absolute maximum energy change for each planet is presented in Table 2.

In concluding the discussion of the fly-by phase it is reasonable to rate the planets in some way. Consideration of Table 2 clearly indicates that Jupiter is the most effective planet for fly-by. It produces the largest maximum changes in velocity, speed, and energy. It is also the nearest planet to earth capable of imparting a flight-path angle change of 180 degrees. The next most effective planet is Saturn. It is interesting to note, however, that from a consideration of energy change Venus is second in capability. Venus is also able to impart the second largest change in flight-path angle.

Transfer Trajectory Phase

Inner Planets — There are no possible free-fall transfer trajectories to Mercury for either maximum velocity or maximum speed change conditions. This is because the elliptical transfers associated with the encounter conditions at the circle-of-influence transformed into heliocentric space have aphelion distances of less than one astronomical unit. If it were desired to fly by Mercury energy would have to be added to the transfer trajectory in order to have it intersect the orbit of earth. This would result in causing the encounter velocity to be so large as to preclude any beneficial effect of the resultant fly-by. Considering this fact with the relatively slight effects to be expected from a fly-by of this planet, Mercury hardly seems a likely candidate for a planetary fly-by. Of course, Mercury's proximity to the sun offers many other disadvantages which also might exclude it from consideration.

Figure 7-a presents maximum speed change, time of flight and transfer angle versus encounter angle for possible free-fall transfer trajectories that derive maximum speed change from Venus. There are possible transfers but it is interesting to note that the largest maximum gain in speed is approximately 4 km/sec, a reduction of almost 50 percent from the theoretically possible maximum speed change shown by the positive peak of DS_{max} at $\epsilon = 50$ degrees. Also, a large segment of the window of possible transfers results in speed reductions. Indeed, the maximum speed reduction of 7.4 km/sec may be obtained for a fly-by trajectory with an encounter angle of 250 degrees. All possible free-fall transfers that enter the circle-of-influence with encounter angles between 110 and 180 degrees are transfers on which the probe passes through perihelion before encountering Venus thus explaining the longer times of flight.

Figure 7-b presents time of flight and transfer angle versus encounter angle for transfer trajectories that result in maximum velocity change for Venus. All possible free-fall transfers that enter the circle-of-influence with encounter angles between 104 and 180 degrees are transfers on which the probe passes through perihelion before encountering Venus. A transfer with a flight time of approximately 50 days and a transfer angle of about 70 degrees can gain

the maximum velocity change of 7.3 km/sec with an encounter angle of 230 degrees.

Outer Planets — Figure 8 presents for Mars the same parameters as were presented in Figure 7 for Venus. The window of possible free-fall transfers is almost symmetric about the line of motion of Mars for both maximum speed and velocity changes. From Figure 8-a it is seen that it is possible to obtain almost the largest of maximum speed changes (~ 3.5 km/sec). Such a transfer would have an encounter angle of approximately 47 degrees. It would have transferred through an angle of almost 140 degrees in approximately 190 days. For Mars there are possible transfers that result in no change or in possible speed reductions of up to 2 km/sec. Transfer trajectories that arrive at Mars with an encounter angle between approximately 306 and 360 degrees are trajectories on which a probe passes through aphelion before encountering Mars, thus explaining the larger flight times in this region. From Figure 8-b it is seen that the maximum velocity change of 3.6 km/sec may be obtained with a transfer trajectory that encounters Mars with an encounter angle of 20 degrees. Such a transfer would have traversed a transfer angle of 105 degrees in almost 170 days. In this figure transfers with encounter angles between about 318 and 360 degrees pass through aphelion before arriving at Mars.

It is not possible to obtain by free-fall elliptical transfer trajectories the theoretical maximum velocity change from Jupiter, Saturn, Uranus, Neptune and Pluto. For these planets the velocities required at the entry points of their respective circles-of-influence can be matched only with trajectories that are hyperbolic with respect to the sun. The remaining figures, therefore, present information concerning transfers that result only in maximum speed changes due to fly-by of these planets.

Figure 9 presents maximum speed change, time of flight and transfer angle versus encounter angle for Jupiter. Transfers with encounter angles between approximately 305 and 360 degrees are trajectories on which the probe passes through aphelion before encountering Jupiter. Flight times are seen to increase in an almost asymptotic fashion. Transfers whose times of flight are so large are those with aphelion distances of hundreds of astronomical units (~ 500-600 a.u.). These transfers may be neglected since they are fast approaching escape ($e \to 1.0$). Indeed, transfers with times of flight in excess of 10,000 days (~ 30 years) may be ignored as being impractical. Further, since such transfers result in no appreciable positive speed change from a fly-by of this planet they may be undesirable. But transfer times of up to 40,000 days (~ 100 years) have been presented to give some idea of the long duration of possible free-fall transfer trajectories to Jupiter.

Figure 10 presents the same parameters for Saturn as were discussed above for Jupiter. Transfers that encounter this planet after aphelion passage are those with encounter angles between approximately 296 and 360 degrees. Only a small part of the encounter window produces trajectories that result in negative or no speed changes, and that is the region where times of flight are very large. The largest of possible maximum speed gains may be obtained through a fly-by of Saturn. Times of flight for such transfers are of the order of 1600 days (or ~ 4 years).

Parameters associated with transfer trajectories to Uranus are presented in Figure 11. Transfers that encounter Uranus after aphelion passage are those that have encounter angles in the range from about 300 to 360 degrees. A transfer time of about 4900 days (or > 10 years) is required to achieve the largest maximum speed change from a fly-by of Uranus. But this free-fall trajectory is nearly parabolic with respect to the sun, as indicated by the magnitude of the transfer angle (~ 0.5 degrees). This same transfer requires an heliocentric injection velocity at earth of 41 km/sec. Recall that heliocentric escape velocity at one astronomical unit is approximately 42 km/sec.

Neptune and Pluto vary somewhat from the other outer planets in that the encounter window splits into two parts for both of these planets (see Figures 12 and 13). The gap in the encounter window is due to the following fact: at these distances from the sun and with the encounter velocities required at the circles-of-influence the free-fall heliocentric transfer trajectories are such that perihelion distances are greater than one astronomical unit. Therefore, there are no free-fall transfers associated with encounter angles in these gaps.

In Figure 12 it is seen that it is possible to derive the largest of maximum speed changes from a fly-by of Neptune with a free-fall transfer that enters the circle-of-influence with an encounter angle of approximately five degrees. The transfer time is more than 9000 days (or almost 25 years), and the transfer angle is approximately 10 degrees. Note that even the shortest transfer time is of the order of 5700 days (or > 15 years). This is due to the fact that the free-fall transfer has an aphelion distance of greater than 55 astronomical units. The second of the two encounter windows allows the largest of possible maximum speed gains from fly-by of Neptune.

Figure 13 presents transfer data for Pluto. The encounter window is in two parts and each is quite narrow. As opposed to Neptune the first window is the one that produces the largest of maximum speed changes from fly-by. However, this window is so narrow that the guidance system or injection accuracy for such transfers might be such as to preclude using Pluto for a fly-by. Of course, launch propulsion capability might also exclude using this planet. The second window, even more restrictive in its size, is associated with transfers of the shorter transfer times, e.g., almost 20 years. But such transfers may produce maximum speed changes that are less than half the largest possible change of almost 8 km/sec.

Table 3 contains a summary of the maximum changes that can be obtained from fly-by of the planets by free-fall transfer trajectories. The maxima do not occur simultaneously for any one planet. Thus, each column is a listing of an independent quantity. A dash appearing in a column indicates that a maximum change in that quantity can not be obtained from that particular planet through the use of a free-fall transfer from earth.

CONCLUSIONS

It has been shown that it is not possible to obtain with free-fall transfer trajectories from earth the theoretically possible maximum velocity change from any of the planets except Venus and Mars. Indeed, it is not possible with such transfers to obtain even a maximum speed change from Mercury due to the required encounter conditions at the circle-of-influence of that planet. It is possible to obtain from all the planets, except Mercury, the largest — or near largest — maximum positive speed changes due to fly-by (see Figures 7 through 13).

It is convenient that the nearest neighbors of earth, Venus and Mars, and the giant planet, Jupiter, can be reached with free-fall transfers that result in such large changes due to fly-by of these planets, since they undoubtedly will be the first choice for such use from a technological point of view. It is possible to obtain the maximum velocity change of 7.3 km/sec from Venus with a free-fall transfer of approximately 50 days duration. The heliocentric injection velocity for such transfers is in the range of from 27 to 33 km/sec. The maximum velocity change of 3.6 km/sec may be obtained from fly-by of Mars with a free-fall transfer of almost 145 days duration. The necessary heliocentric injection velocity at earth for such transfers is approximately 32 km/sec. It is possible to obtain from Jupiter with a free-fall transfer of almost 700 days duration the maximum positive speed change of 24 km/sec. The necessary heliocentric injection velocity at earth is almost 42 km/sec. (At one astronomical unit escape velocity is approximately 42 km/sec.)

Due to the very large flight times associated with such transfers it appears unlikely that direct fly-by of the planets beyond Jupiter with free-fall transfers from earth will be used to shape trajectories of deep space probes. However, fly-by of these planets may well be not only likely but also reasonable for transfers emanating from some other celestial body, for example in multiple fly-by missions.

Further study should be made of the phenomenon of planetary fly-by to determine how the effects presented in this paper will be affected by such things as the introduction of: the third dimension; a closer approximation of the true motion of the planets; and, perhaps, spatial-time constraints such as launch possibilities. In consideration of spatial-time constraints it should be mentioned that it is possible that some of the transfers presented in this paper may intersect the circle-of-influence in two places. The solution to this problem was postponed to the more detailed study that takes into account the motion of the planets. A sensitivity study to determine the effects of errors in injection conditions on the encounter conditions would be more indicative of realistic results of planetary fly-by.

ACKNOWLEDGMENTS

I am indebted to Mr. Robert Blanchard whose idea it was to investigate this problem, and with whom I had many enlightening and fruitful discussions. Mr. James Moore prepared the figures and performed many tasks during the course of this investigation.

REFERENCES

1. Minovitch, M. A., A Method for Determining Interplanetary Free-Fall Reconnaissance Trajectories, J.P.L. Technical Memo No. 312-130, 1961.

2. Minovitch, M. A., The Determination and Potentialities of Advanced Free-Fall Interplanetary Trajectories, Proceedings of the First Graduate Academy of the University of California, Spring Recess, 1963.

3. Niehoff, J., An Analysis of Gravity Assisted Trajectories in the Ecliptic Plane, Report No. T-12 of the Astro Sciences Center of the IIT Research Institute, Chicago, Illinois, 1965.

4. Ehricke, Kraft A., Space Flight, I. Environment and Celestial Mechanics, D. Van Nostrand Co., Inc., 1960.

5. Blanco, V. M. and McCuskey, S. W., Basic Physics of the Solar System, Addison-Wesley Publishing Co., Inc., 1961.

6. Battin, Richard H., Astronautical Guidance, McGraw-Hill Book Company, 1964.

APPENDIX

Presented herein are the equations which were used to obtain the numerical results of the planetary swing-by study.

1. The equations which were used for calculations inside the circle-of-influence are the following.

   The velocity at the circle-of-influence was calculated by:

$$v'_\infty = (2\mu_p/R_s)^{1/2} + \epsilon$$

where ϵ is a small positive number ($\sim 1 \times 10^{-6}$).

The velocity was increased by the small amount, ϵ, because theoretically the velocity at infinite distances is zero. Because the distance at which the velocity was calculated was finite, R_s, the velocity was in fact non-zero. To be consistent, however, the velocity was increased by ϵ.

The velocity at perifocus was next calculated by:

$$v_\pi = \left(v'^2_\infty + 2\mu_p \left[1/r_\pi - 1/R_s\right]\right)^{1/2}.$$

The angular momentum was then evaluated by:

$$h = r_\pi v_\pi .$$

The semi-latus rectum was then obtained by:

$$p = h^2/\mu_p .$$

The semi-major axis is found from the vis-viva equation:

$$\frac{1}{a} = \frac{v_\pi^2}{\mu_p} - \frac{2}{r_\pi}$$

The eccentricity is found by:

$$e = p/r_\pi - 1 .$$

The tangential and radial velocity components are obtained by the following:

$$v_f = h/R_s$$

$$v_r = -(v_\infty'^2 - v_f^2)^{1/2} .$$

The true anomaly was calculated from the following:

$$f = \tan^{-1}\left[e \sin f / e \cos f\right]$$

where

$$e \sin f = v_r p/h$$

$$e \cos f = p/R_s - 1$$

The transformation angle, θ, was obtained from the identity:

$$\theta = f + \eta .$$

The velocity components referred to the cartesian coordinate system were then obtained from the transformation:

$$\begin{pmatrix} v_x \\ \\ v_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \\ \sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} v_r \\ \\ v_f \end{pmatrix}$$

The magnitude of the velocity is given by:

$$v = \left[(v_x)^2 + (v_y)^2\right]^{1/2}$$

At this point the encounter velocity was computed as follows:

$$(v_x)_E = v_x - v_p$$

$$(v_y)_E = v_y$$

$$v_E = \left[(v_x)_E^2 + (v_y)_E^2\right]^{1/2}$$

From this point the equations are essentially the equivalent of a geometrical reflection of the orbit about the perifocal vector.

The new transformation angle was obtained as follows:

$$\theta' = -f + \eta .$$

The new radial and transverse velocity components were:

$$v_r' = -v_r$$

$$v_f' = v_f .$$

The velocity components referred to the cartesian coordinates for the point of exit of the circle-of-influence were obtained from the transformation:

$$\begin{pmatrix} v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix} \cdot \begin{pmatrix} v'_r \\ v'_f \end{pmatrix}$$

$$v' = \left[(v'_x)^2 + (v'_y)^2 \right]^{1/2}$$

At this point the post-encounter velocity was computed:

$$(v_x)_{PE} = (v'_x) - v_p$$

$$(v_y)_{PE} = (v'_y)$$

$$v_{PE} = \left[(v_x)^2_{PE} + (v_y)^2_{PE} \right]^{1/2}$$

The vector velocity change was then obtained.

$$DV_x = (v_x)_{PE} - (v_x)_E$$

$$DV_y = (v_y)_{PE} - (v_y)_E$$

$$DV = \left[(DV_x)^2 + (DV_y)^2 \right]^{1/2}$$

The change in flight-path angle was obtained by computing the flight-path angle for the encounter velocity and post-encounter velocitv and differencing the two values.

$$D\beta = \beta_{PE} - \beta_E$$

where

$$\beta_{PE} = \tan^{-1} \left[(v_y)_{PE} / (v_x)_{PE} \right]$$

$$\beta_E = \tan^{-1} \left[(v_y)_E / (v_x)_E \right] .$$

The encounter angle, ϵ, was obtained from:

$$\epsilon = 180. + \theta$$

2. The equations which were used to generate the heliocentric transfer orbits are the following.

The velocity components have already been obtained. The appropriate transformation was performed to obtain heliocentric position components.

The semi-major axis was obtained from the vis-viva equation:

$$\frac{1}{a} = \frac{2}{r_i} - \frac{v_i^2}{\mu}$$

where μ is the gravitational parameter of the sun.

The eccentric anomaly was computed from:

$$E_i = \tan^{-1} \left[(e \sin E_i) / (e \cos E_i) \right]$$

where:

$$(e \sin E_i) = \frac{\bar{r}_i \cdot \bar{v}_i}{\sqrt{\mu a}} \text{, and}$$

$$(e \cos E_i) = 1 - r_i / a .$$

The eccentricity was then obtained from:

$$e = \left[(e \sin E_i)^2 + (e \cos E_i)^2\right]^{1/2}$$

The eccentric anomaly for injection into the transfer orbit — originating at the earth — was then obtained from:

$$E_o = \tan^{-1}\left[\sin E_o / \cos E_o\right]$$

where:

$$\cos E_o = (1 - r_o/a)/e,$$

and

$$\sin E_o = \left[1 - \left(|\cos E_o|\right)^2\right]^{1/2}$$

The time of transfer was then computed by:

$$\Delta T = \frac{a^3}{\sqrt{\mu}}\left[\left(\beta + \frac{\overline{r}_i \cdot \overline{v}_i}{\sqrt{\mu a}}\right)(1 - \cos\beta) - \left(1 - \frac{r_i}{a}\sin\beta\right)\right]$$

where

$$\beta = \Delta E.$$

Finally the position and velocity at injection — the earth — were obtained from the following equations:

$$\bar{r}_o = \left[\left\{ \left(1 - \frac{a}{r_i}\right) \left(1 - \cos \beta\right) \right\} \bar{r}_i + \left\{ \Delta t - \frac{(\beta - \sin \beta)}{\sqrt{\mu / a^3}} \right\} \bar{v}_i \right]$$

$$\bar{v}_o = \left[\left\{ - \frac{\sqrt{\mu a}}{r_i \, r_o} \sin \beta \right\} \bar{r}_i + \left\{ 1 - \left(\frac{a}{r_i}\right) \left(1 - \cos \beta\right) \right\} \bar{v}_i \right]$$

TABLE 1

Planetary Physical and Orbital Data

Planet	Gravitational Parameter, μ, $\left(\frac{km^3}{sec^2}\right)$	Mean Distance a, (a.u.)	Radius of Circle of Influence $\times 10^{-3}$, R_s, (km)	Radius of Planet, r_p, (km)	Velocity of Planet, v_p, (km/sec)
Mercury	2.16494×10^4	.387099	111.9	2500.	47.769
Venus	3.2423×10^5	.723332	618.0	6200.	34.945
Mars	4.2906×10^4	1.523691	567.0	3310.	24.112
Jupiter	1.26498×10^8	5.202803	48240.0	69880.	13.030
Saturn	3.78811×10^7	9.538843	48690.0	57550.	9.623
Uranus	5.79364×10^6	19.181973	51900.0	25500.	6.786
Neptune	6.86004×10^6	30.057707	87075.0	25000.	5.421
Pluto	3.31237×10^5	39.51774	35490.0	3000.	4.728

gravitational parameter, μ, for Sun = $1.324948 \times 10^{11} \frac{km^3}{sec^2}$

TABLE 2

Theoretical Maximum Changes Due to Planetary Fly-by

Planet	Velocity km/sec	Speed km/sec	Flight-Path Angle degrees	Energy $(km/sec)^2$
Jupiter	42.5	24.0	180.0	555.0
Saturn	25.5	17.0	180.0	247.0
Neptune	16.6	9.8	180.0	90.0
Uranus	15.1	11.3	180.0	102.0
Pluto	10.5	7.8	180.0	50.0
Venus	7.3	7.4	17.0	255.0
Mars	3.6	3.6	10.0	87.0
Mercury	3.0	3.0	3.0	145.0

TABLE 3

Maximum Changes Due to Planetary Fly-by
Possible with Free-Fall Transfer Trajectories

Planet	Velocity km/sec	Speed km/sec	Flight-Path Angle degrees
Jupiter	—	24.0	180.0
Saturn	—	17.0	180.0
Neptune	—	9.8	180.0
Uranus	—	11.3	180.0
Pluto	—	7.8	180.0
Venus	7.3	4.2	11.0
Mars	3.6	3.5	10.0
Mercury	—	—	—

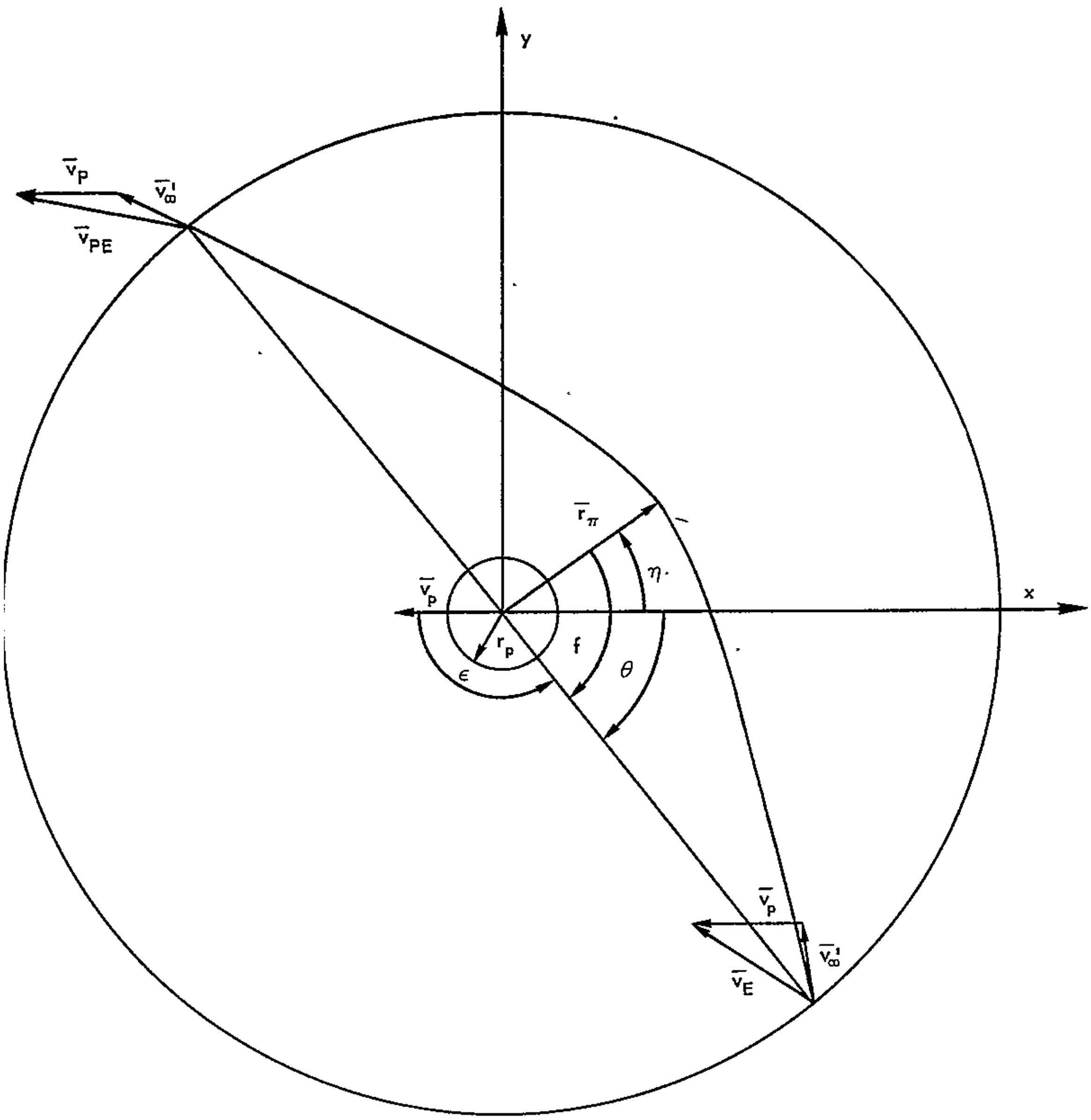

Figure 1. Fly-by Geometry, Parameters and Axis System

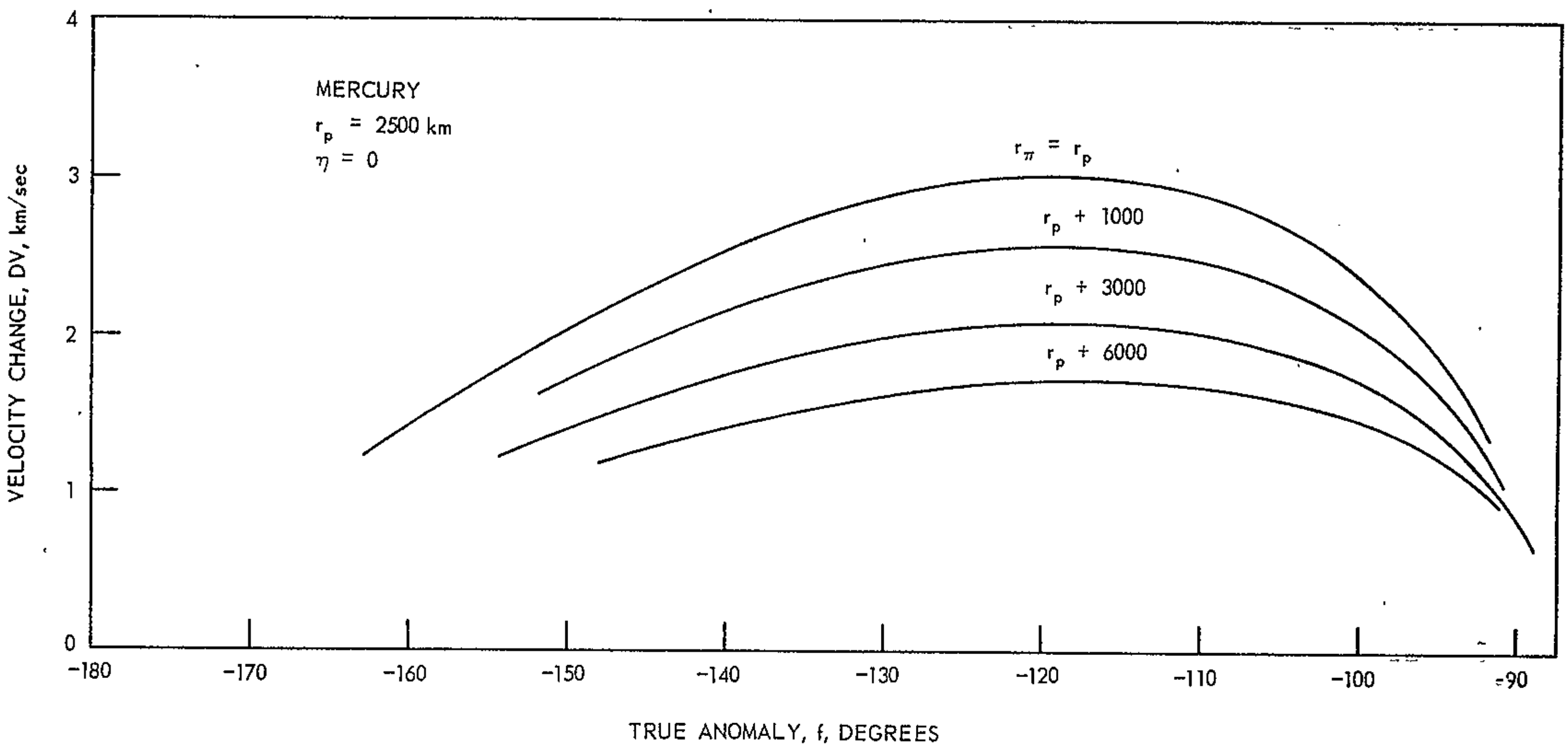

Figure 2. Variation of Velocity Change with True Anomaly of Entry Point and Perifocal Distance for Mercury

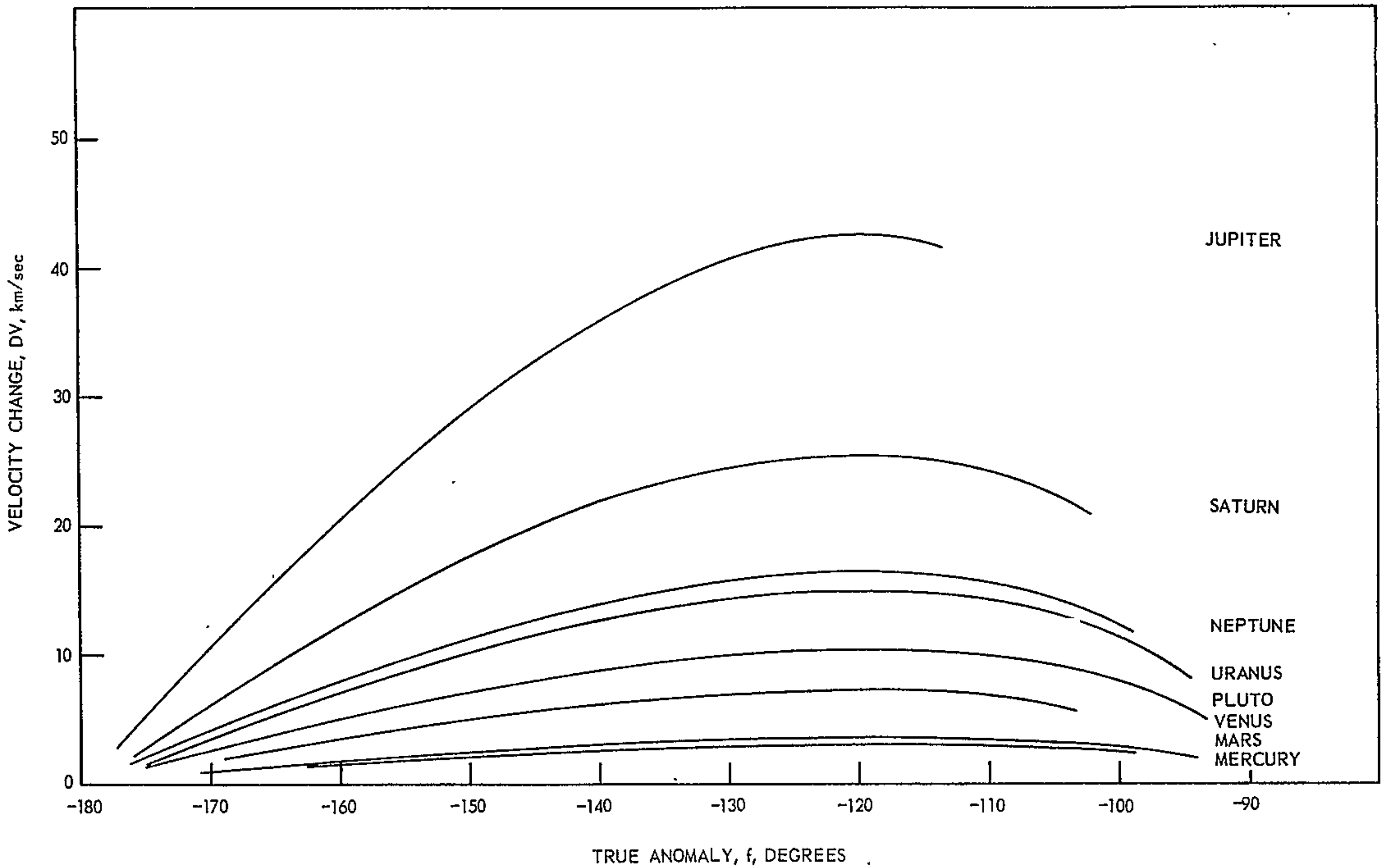

Figure 3. Variation of Velocity Change with True Anomaly of Entry Point for the Planets.

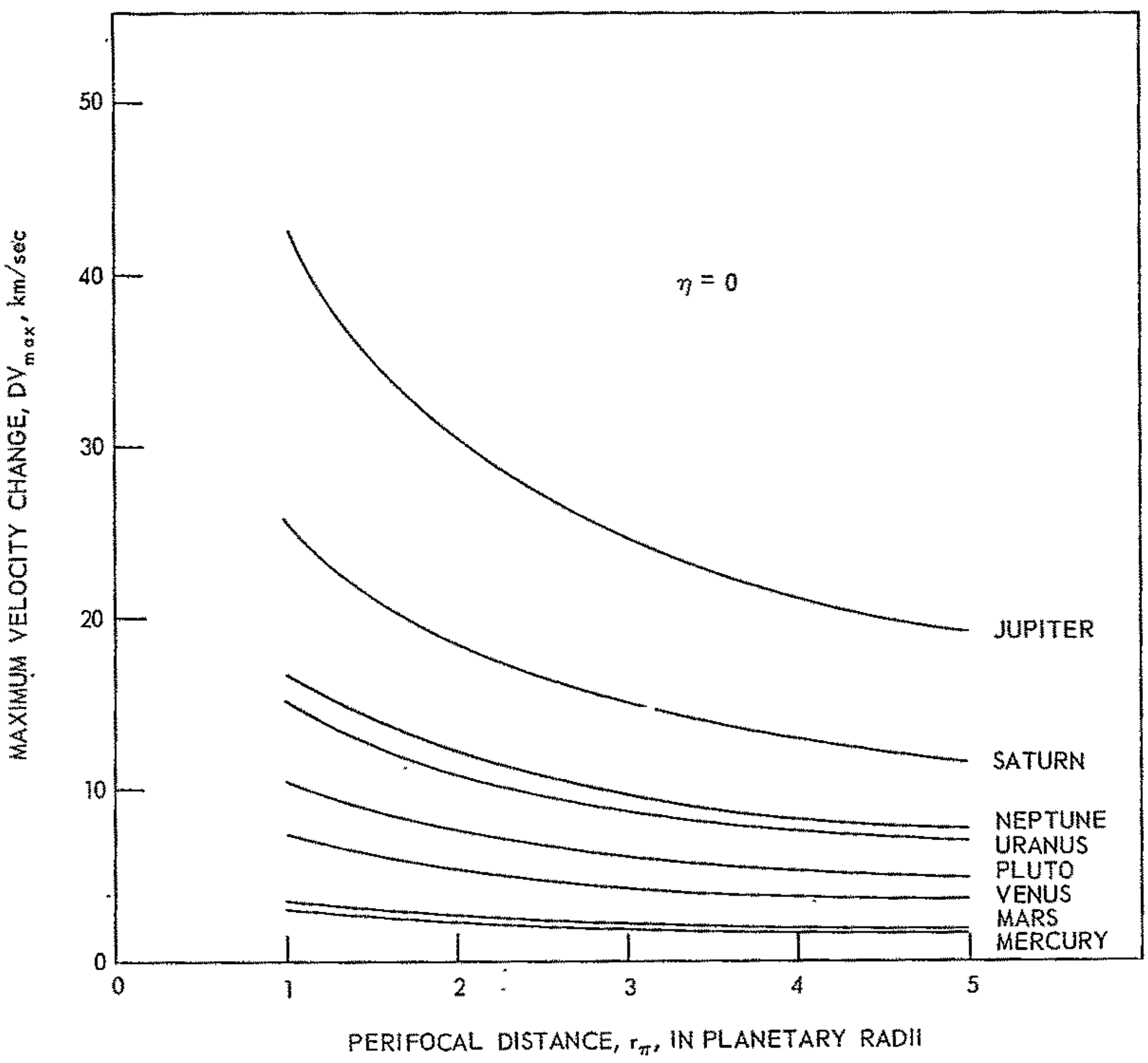

Figure 4. Variation of Maximum Velocity Change with Perifocal Distance for the Planets

26

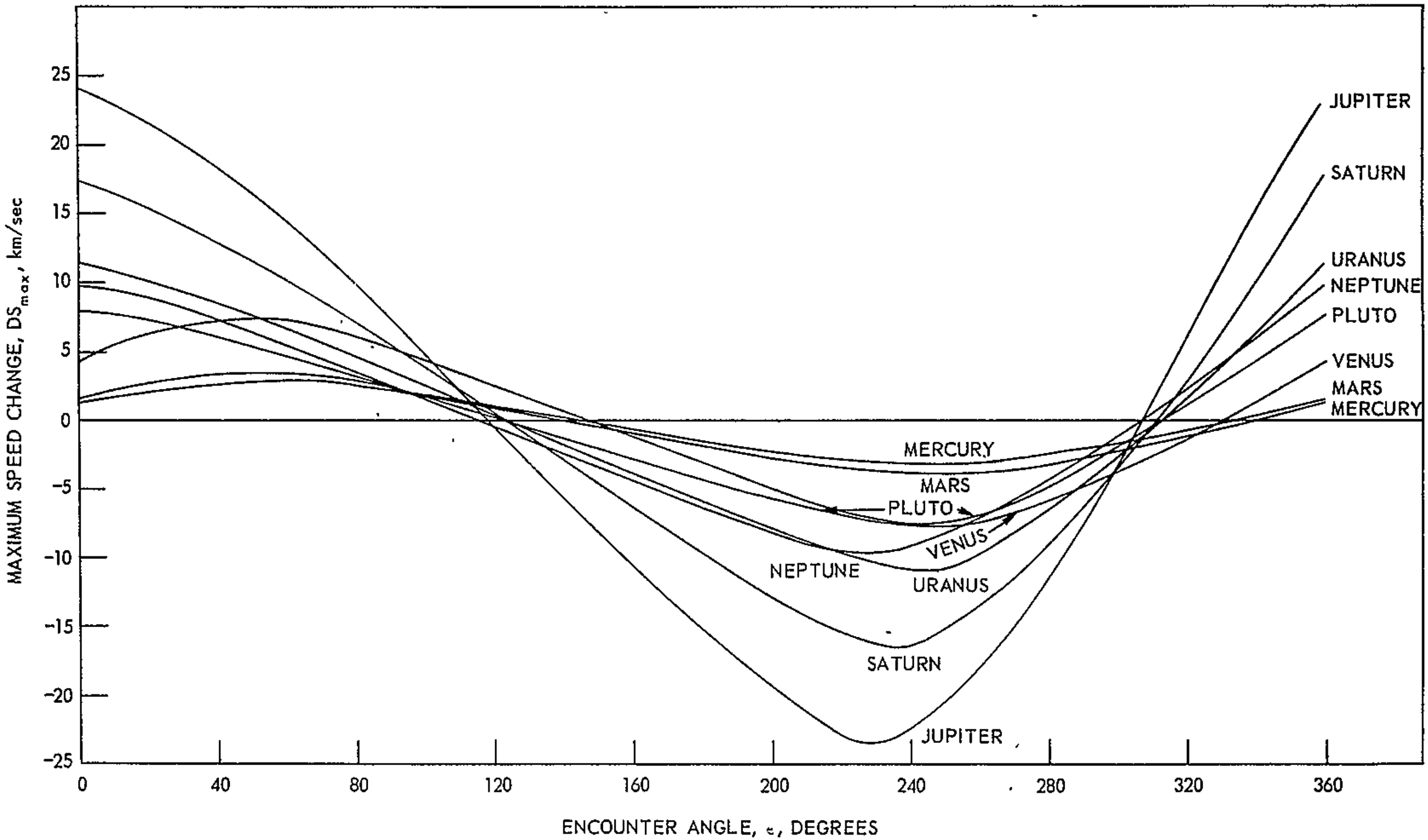

Figure 5. Variation of Maximum Speed Change with Encounter Angle for the Planets

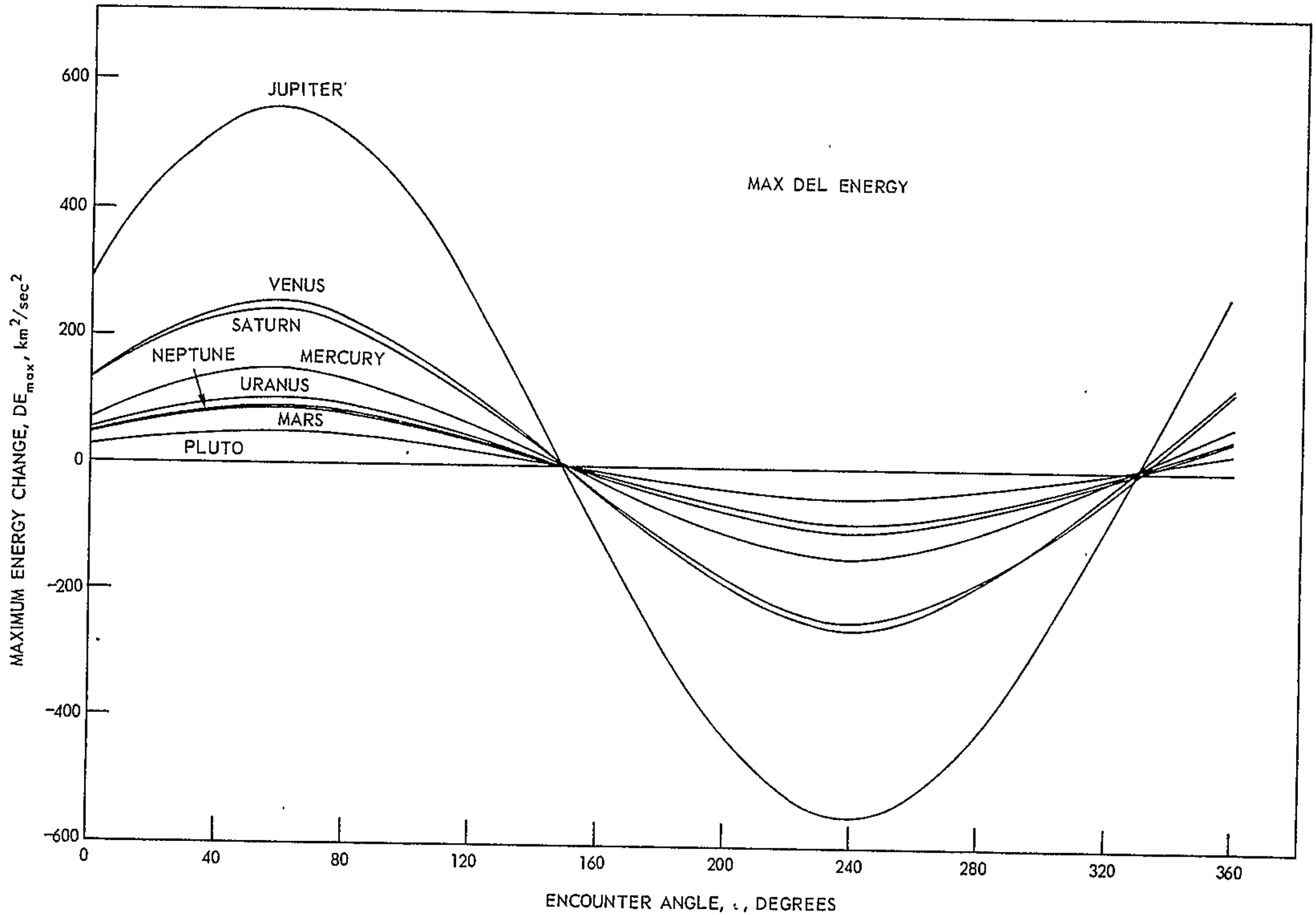

Figure 6. Variation of Maximum Energy Change with Encounter Angle for the Planets

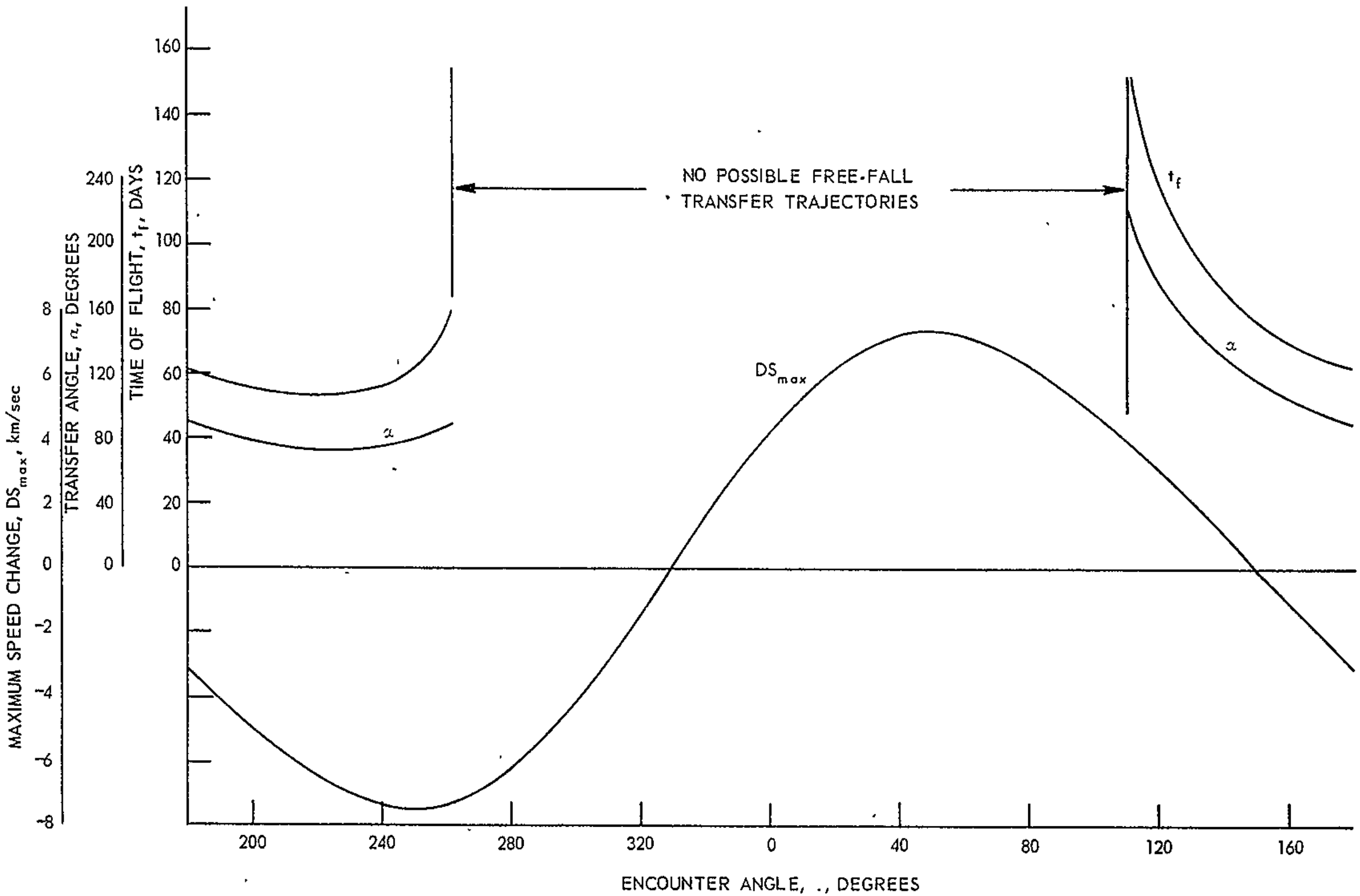

Figure 7-a. Characteristics of Transfer Trajectories that Result in Maximum Speed Changes for Fly-by of Venus

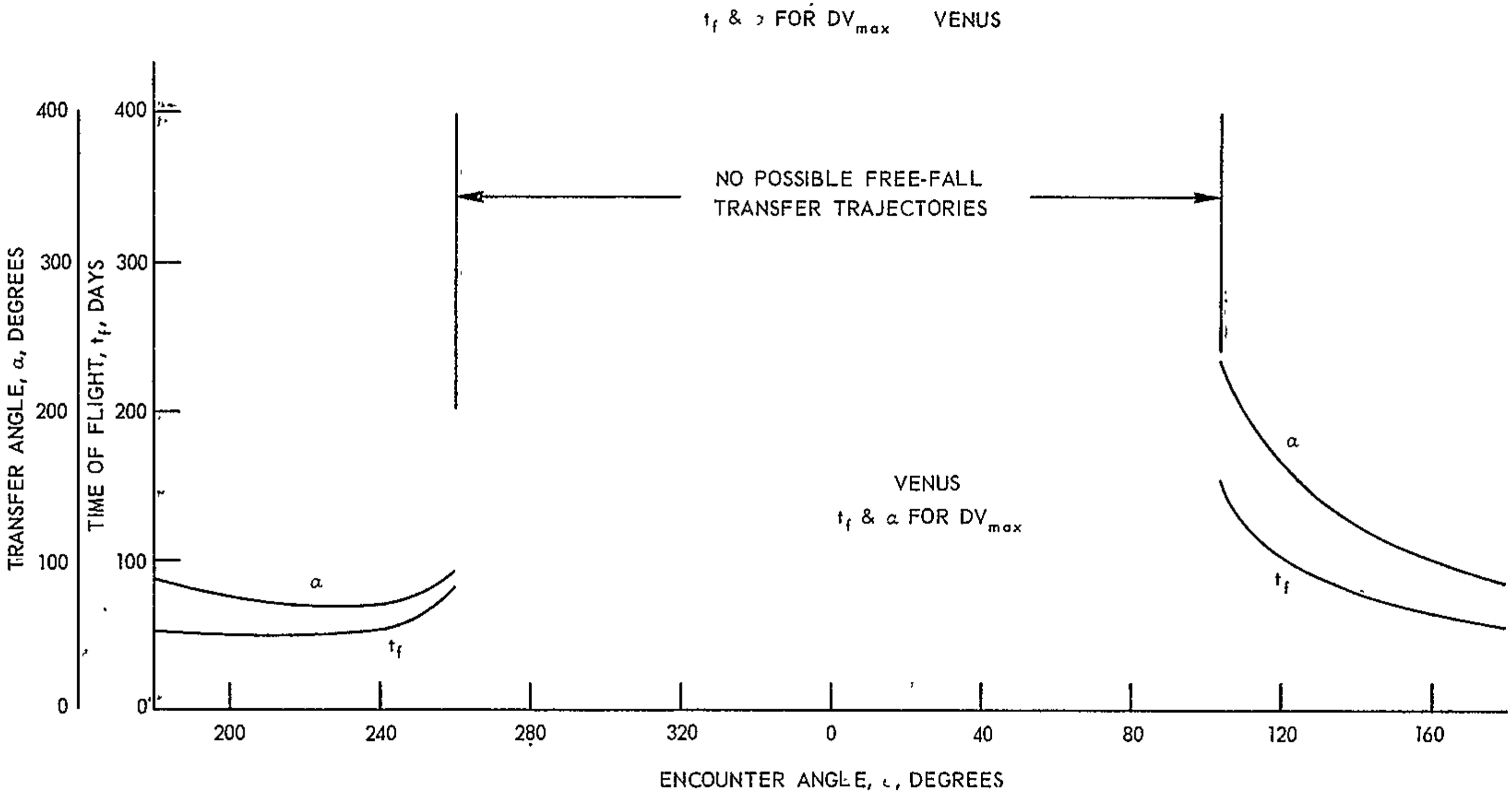

Figure 7-b. Characteristics of Transfer Trajectories that Result in Maximum Velocity Changes for Fly-by of Venus

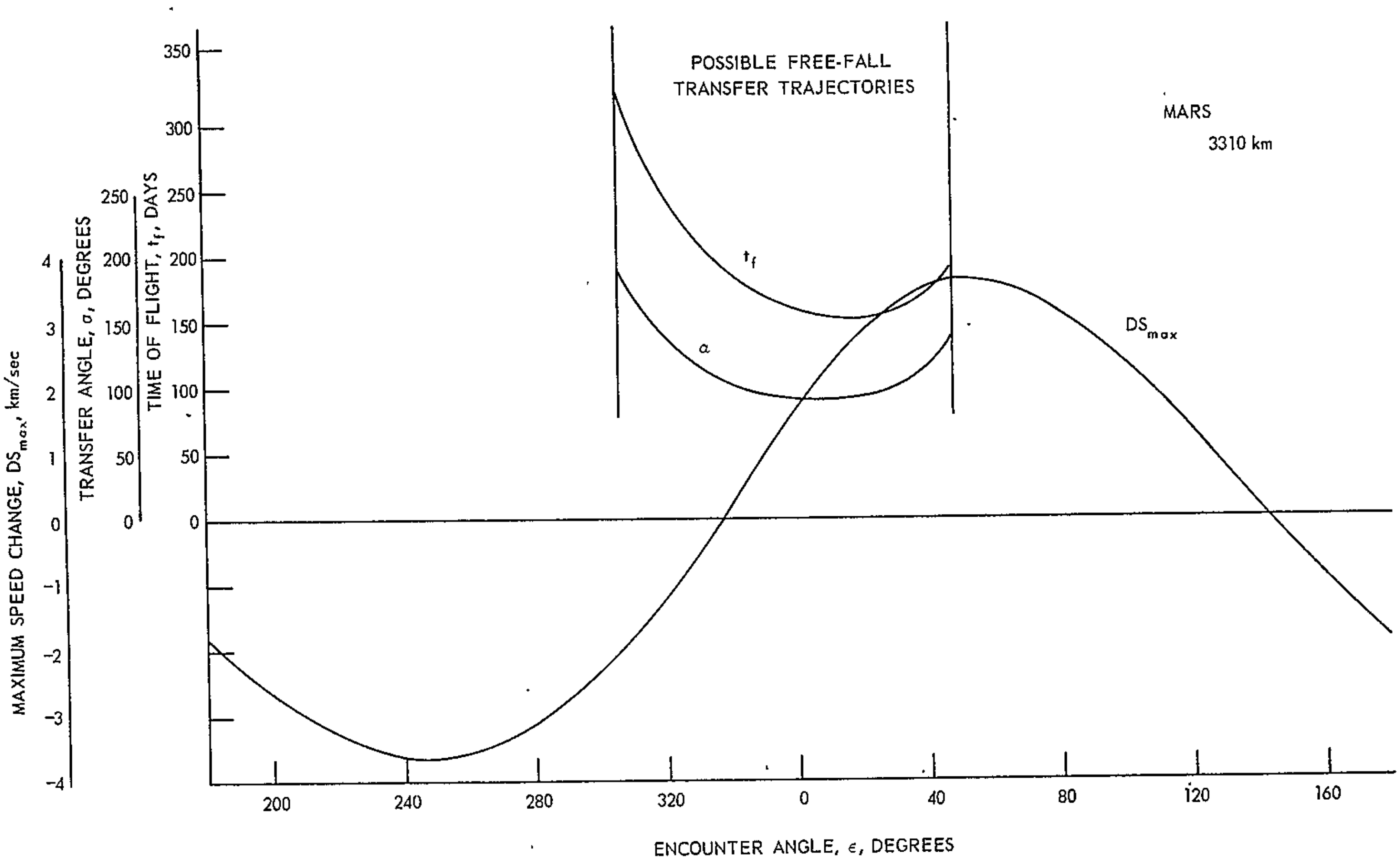

Figure 8-a. Characteristics of Transfer Trajectories that Result in Maximum Speed Changes for Fly-by of Mars

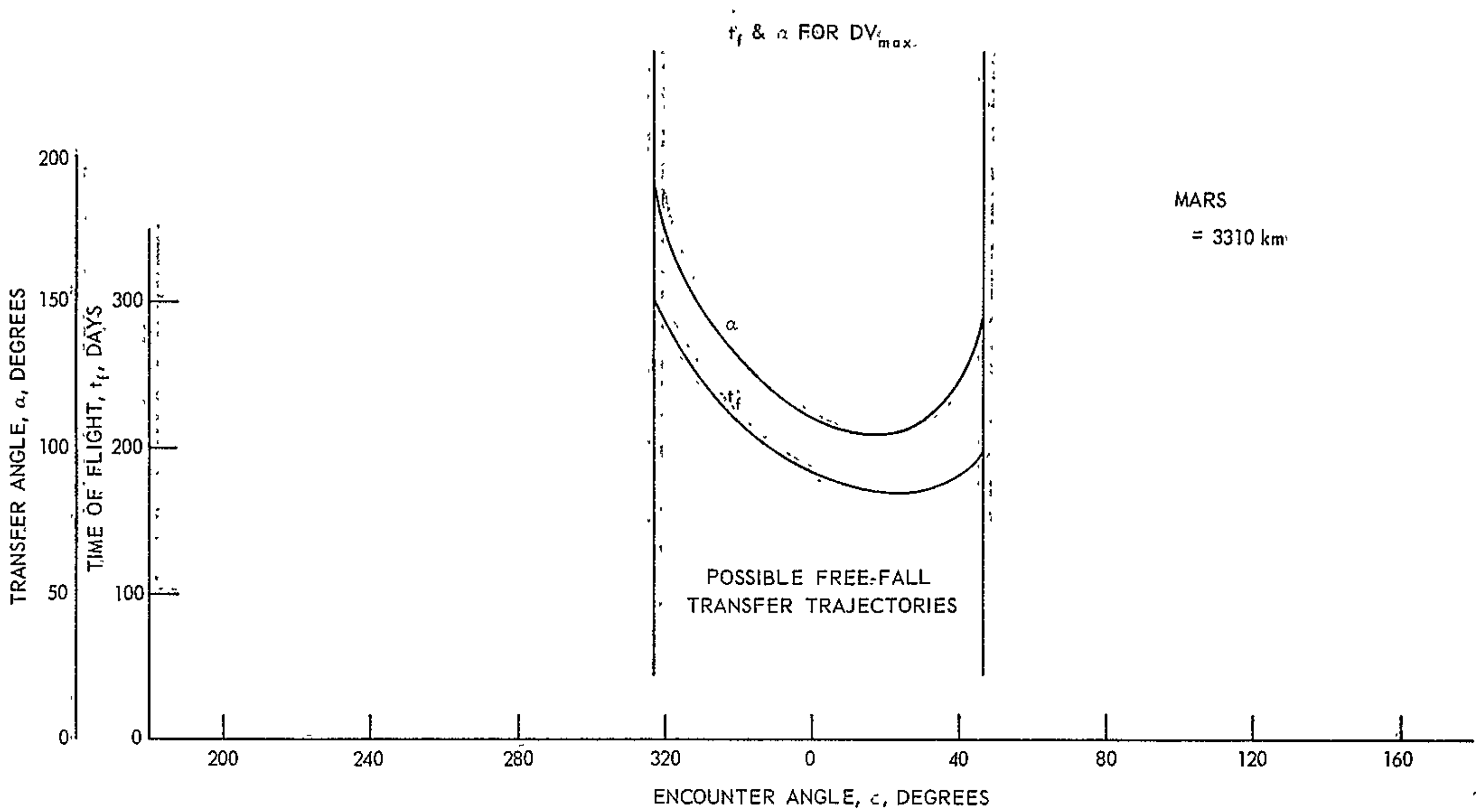

Figure 8-b. Characteristics of Transfer Trajectories that Result in Maximum Velocity Changes for Fly-by of Mars

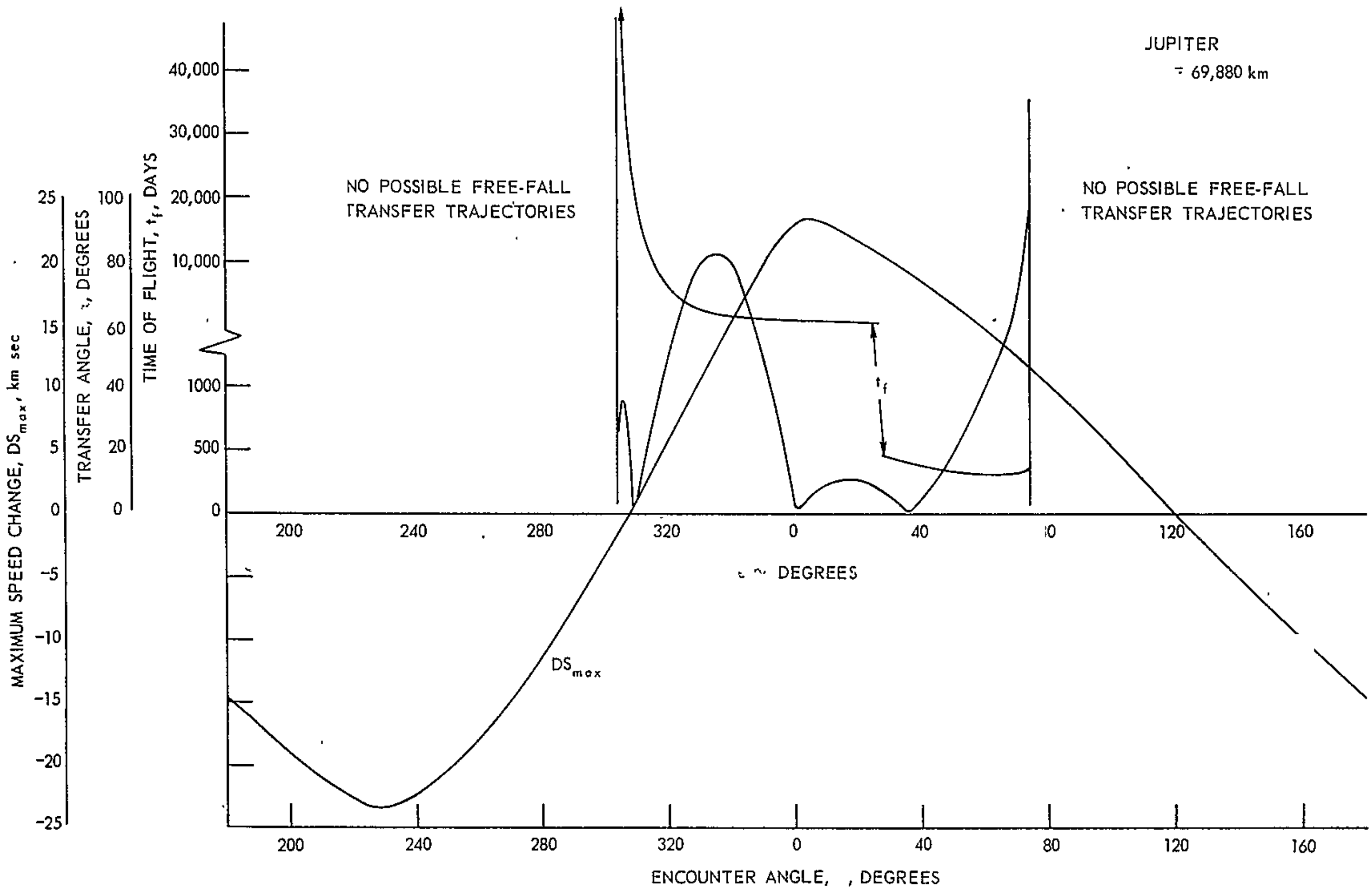

Figure 9. Characteristics of Transfer Trajectories that Result in Maximum Speed Change for Fly-by of Jupiter

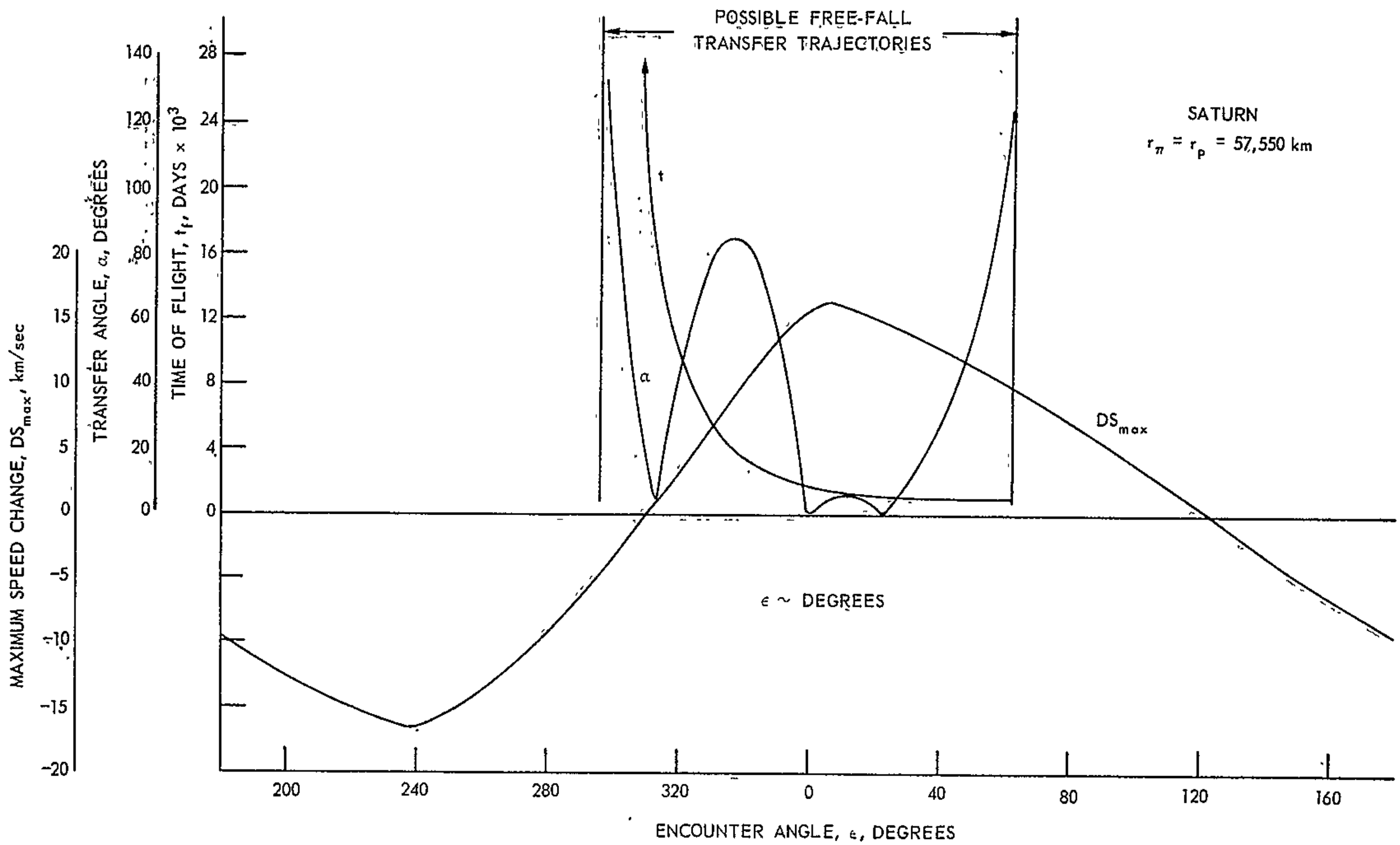

Figure 10. Characteristics of Transfer Trajectories that Result in Maximum Speed Changes for Fly-by of Saturn

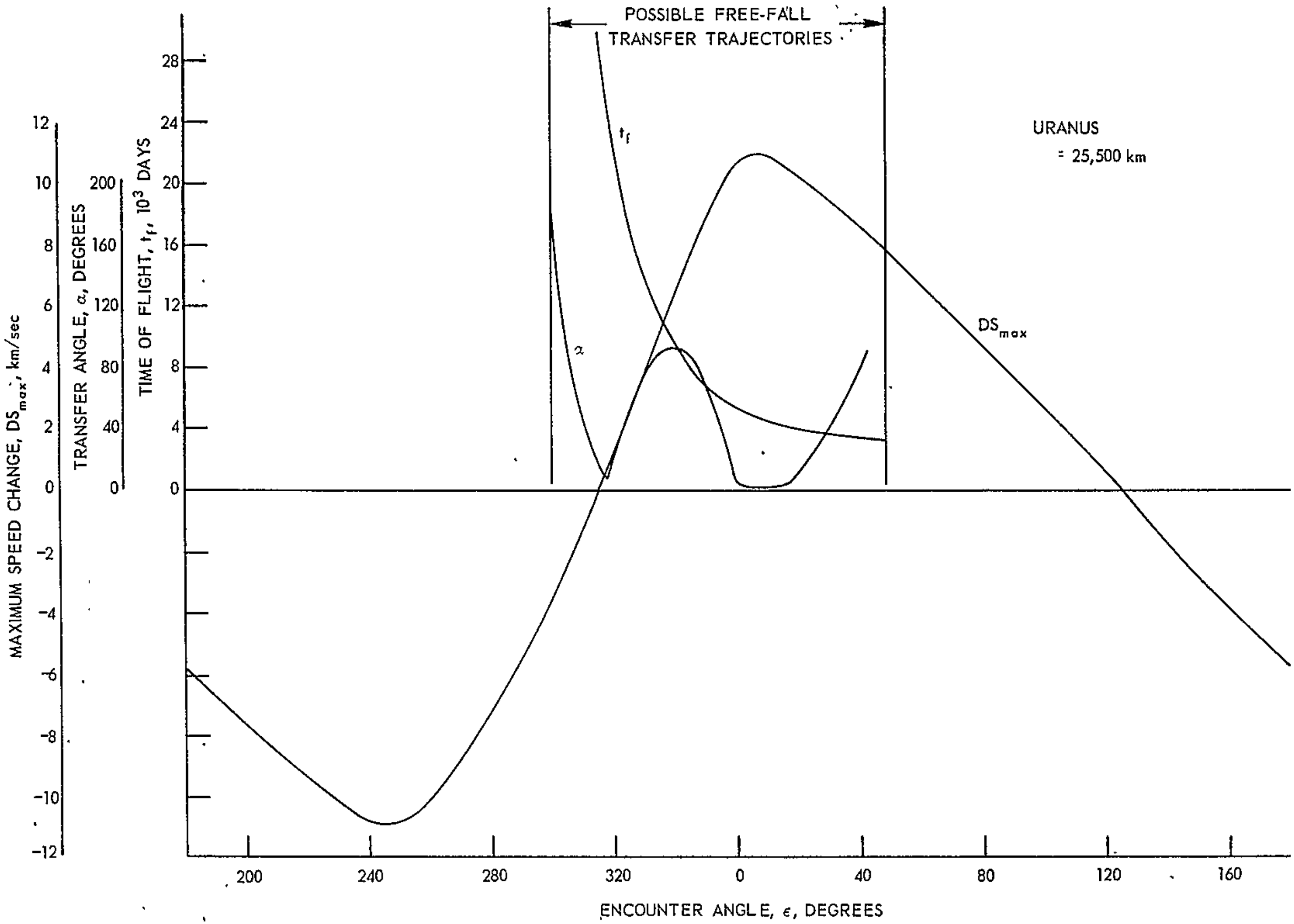

Figure 11. Characteristics of Transfer Trajectories that Result in Maximum Speed Changes for Fly-by of Uranus

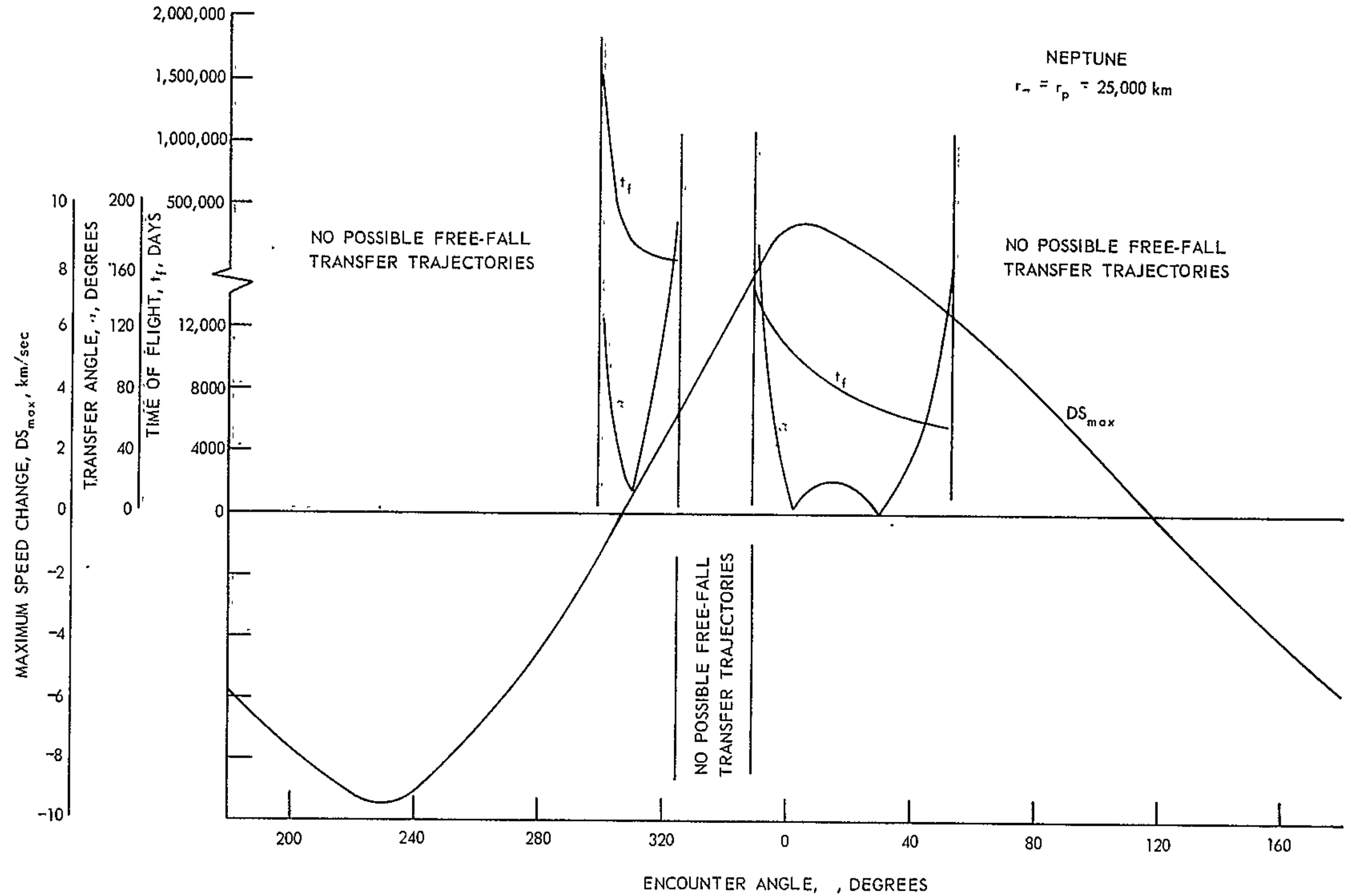

Figure 12. Characteristics of Transfer Trajectories that Result in Maximum Speed Changes for Fly-by of Neptune

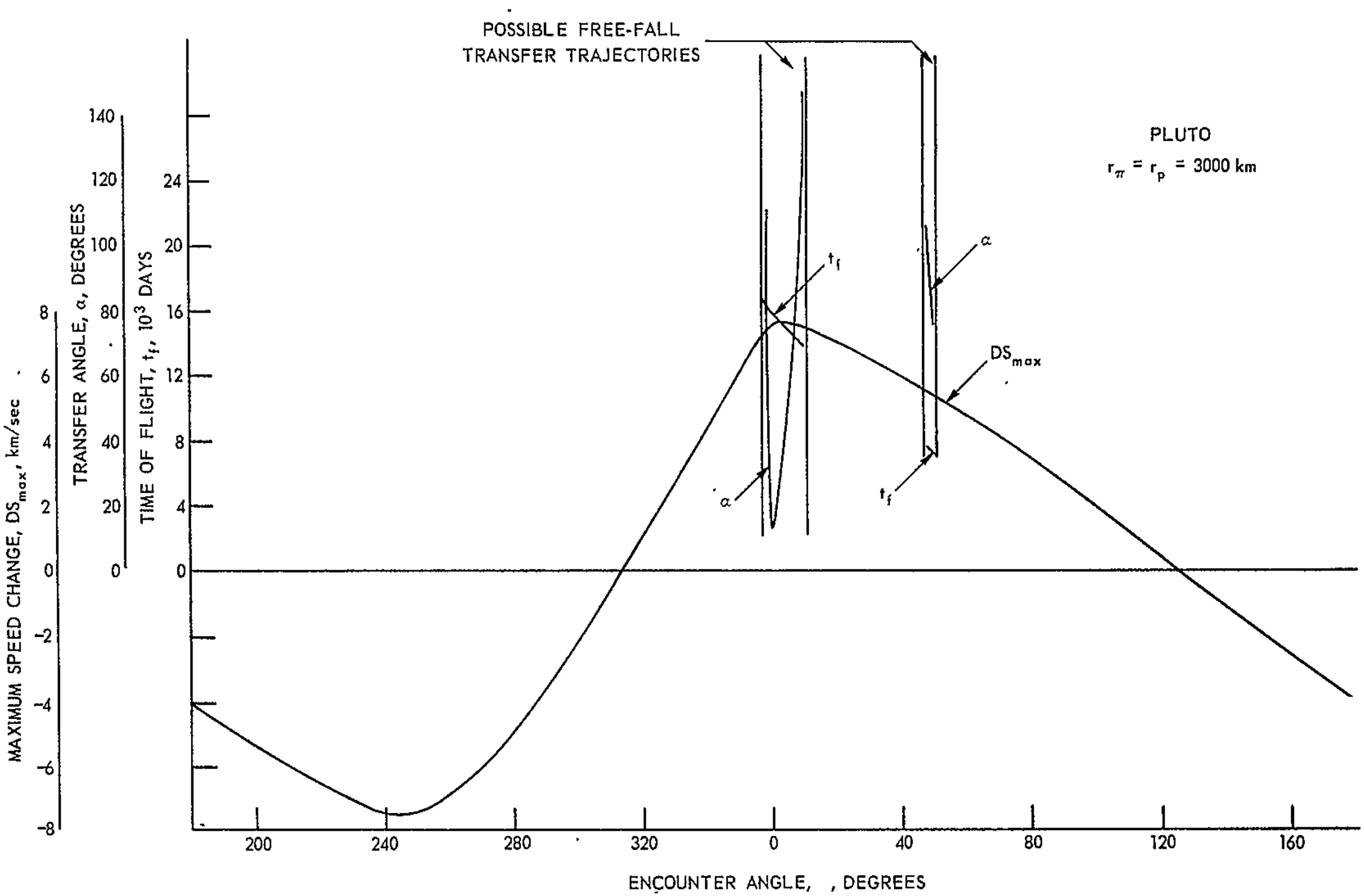

Figure 13. Characteristics of Transfer Trajectories that Result in Maximum Speed Change for Fly-by of Pluto