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A GENERAL METHOD FOR DESIGNING CIRCULAR ARRAY ANTENNAS TO OBTAIN QUASI-OMNIDIRECTIONAL PATTERNS

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SUMMARY

A criterion is established for determining the number of sources needed around the periphery of a spacecraft to obtain small far-field pattern fluctuations; that is, quasi-omnidirectional antenna patterns. The criterion assumes that the source pattern is expressible as a finite Fourier cosine series and that the circumference in wavelengths of the body is greater than or equal to 10. The number of sources obtained by using this criterion will always be adequate; however, if the representation of the source pattern is known, an optimum number of sources can be obtained. The criterion is not valid when the source pattern is isotropic. Two curves are given which relate the number of sources and the circumference of the body in wavelengths for 0.5 dB or less and 2.0 dB or less fluctuation.

INTRODUCTION

In general, spacecraft have a shape which is circularly symmetric about a spin axis. By arraying identical, equally spaced antennas (see fig. 1) around the periphery of this type spacecraft, the pattern of such an array in the plane orthogonal to the spin axis can be controlled; that is, it can be made omnidirectional. The far-field pattern fluctuation, that is, deviation from omnidirectional, in this plane is dependent upon the number of sources S and the circumference of the spacecraft in wavelengths Z. If enough sources are used, a quasi-omnidirectional antenna pattern in the plane of the sources can be obtained.

The equation representing the antenna array pattern has been derived in papers by Knudsen (ref. 1) and Chu (ref. 2). The source (antenna) patterns in these papers are assumed to be expressible as a finite Fourier cosine series. By examining the fluctuation curves, that is, deviation from omnidirectional, in references 2 and 3, one can observe that sources spaced between one-half and one wavelength will produce small fluctuations for the source patterns considered. More than two sources for every wavelength would also give small fluctuations, but utilizing more sources than necessary results in a more complex feed network. Therefore, the choice of the number of sources needed for a particular fluctuation is of great concern in designing circular array antennas.

In the Telstar experiment, the diameter of the satellite was 34.5 inches (87.7 cm) (ref. 4). The two microwave antennas employed in this experiment operated at frequencies around 4 GHz and 6 GHz. The circumference in wavelengths Z for the 4 GHz frequency was 36.8 for which 48 sources were used. For the 6 GHz frequency, the circumference in wavelengths Z was 55.0 for which 72 sources were used. From these numbers the spacing of the sources is approximately 0.764 wavelength.

Another example of choosing the number of sources needed to produce a quasi-omnidirectional antenna pattern is given by Croswell and Knop (ref. 5). In this paper, 54 circumferential slots S on a large cylinder whose circumference in wavelengths Z is 39.5 were needed to obtain a quasi-omnidirectional antenna pattern. Actually, as pointed out in reference 5, 48 to 53 slots could have been used, but for better bandwidth coverage, 54 slots were chosen. Hence, the spacing for each of the 54 slots is approximately 0.733 wavelength to obtain a quasi-omnidirectional antenna pattern.

If the element pattern of the sources to be arrayed is known and expressible as a finite Fourier cosine series, fluctuation curves similar to the ones shown in references 2 and 3 can be plotted; and, from these curves the optimum number of sources S for a particular circumference in wavelengths Z can be obtained for a selected fluctuation. To eliminate the task of computing and plotting these curves, a criterion based on the works of Knudsen (ref. 1) and Chu (ref. 2) is developed herein that relates the number of sources S to the circumference in wavelengths Z for small fluctuations. This criterion is applicable to source patterns expressible as $F(\phi') = \sum_{n=0}^{\infty} A_n \cos n\phi'$ where A_n are the coefficients of the Fourier cosine series and ϕ' is the modified far-field angle. The only restriction on the coefficients is that $A_1 \neq 0$ when the other coefficients are zero. For physically realizable sources, the coefficients A_n must necessarily be restricted.

SYMBOLS

a	radius of array
A_n	coefficients of Fourier cosine series
b	weighting factor

D	fluctuation, dB $\left(20 \log_{10} \frac{\lvert\Phi_{max}\rvert}{\lvert\Phi_{min}\rvert}\right)$
i	a particular source
$J_S(Z)$	ordinary Bessel function of first kind
$\left\lvert\overline{J_0(Z)}\right\rvert$	envelope of the magnitude
j	imaginary number, $\sqrt{-1}$
$J_S^{(n)}(Z)$	nth derivative of Bessel function, $\frac{d^n J_S(Z)}{dZ^n}$
L_λ	linear dimension in wavelengths
N	upper limit of Fourier cosine series
P	far-field point
r	distance from center of array to far-field point
S	number of antennas (sources)
u_i	angular spacing between antennas, $2\pi i/S$
x,y	rectangular coordinates
Z	circumference in wavelengths, $2\pi a/\lambda$
$F(\phi')$	sources pattern
λ	wavelength
ψ_i	distance between center of array and i-antenna in direction of far-field point, radians
Φ	far-field pattern
$\lvert\Phi_{max}\rvert$	magnitude of maximum far-field pattern

$|\Phi_{min}|$ magnitude of minimum far-field pattern

ϕ angle of far-field point, radians

ϕ' modified angle of far-field point, radians

Subscripts:

max maximum

min minimum

DERIVATION OF CRITERION EQUATION

In this section a criterion based on the far-field equation given by Knudsen and Chu is derived. The criterion enables one to determine an always adequate number of sources S for an acceptable fluctuation D and requires only the knowledge of the operating frequency and diameter of the circular array. The total far-field pattern (in the plane of the array) of a circular array of sources as shown in figure 1 may be expressed as (ref. 2)

$$\Phi \approx S \sum_{n=0}^{N} A_n(-j)^n \frac{d^n}{dZ^n}\left[J_0(Z) + 2j^S J_S(Z) \cos S\phi\right] \qquad (S > Z) \tag{1}$$

where the A_n values are the Fourier cosine series coefficients determined by the element pattern, Z is the circumference of the array in wavelengths, and S is the number of sources in the array.

Equation (1) assumes that the single element pattern is expressible as a finite Fourier cosine series. The single element pattern is written as (ref. 2)

$$F(\phi') = \sum_{n=0}^{N} A_n \cos^n \phi' \tag{2}$$

The criterion is originally based on a two-term element pattern series. The one-term-assumed element pattern $\left(F(\phi') = A_0\right)$ must be eliminated since there exist values of Z that cause the fluctuations in the far-field pattern to approach infinity $\left(|J_0(Z)| \rightarrow 0\right)$. Therefore, the minimum number of terms required to express an element pattern from equation (2) may be written as

$$F(\phi') = A_0 + A_1 \cos \phi' \tag{3}$$

Hence, the total far-field pattern expression becomes

$$\Phi \approx S\left\{A_0\left[J_0(Z) + 2j^S J_S(Z) \cos S\phi\right] - jA_1\left[J_0^{(1)}(Z) + 2j^S J_S^{(1)}(Z) \cos S\phi\right]\right\} \tag{4}$$

With the preceding assumptions and limitations, the criterion is established by investigating the first term of equation (4). The form of the first term of equation (4) always appears in the expansion of equation (1) regardless of the number of terms utilized to represent the source pattern from equation (2). Hence, the first term of equation (4) with $A_0 \neq 0$ is sufficient for establishing the criterion with certain limitation on Z and S. Only the second element of this term produces a variation in the far-field pattern; therefore, by restricting the amplitude of this element, the fluctuations in the far-field pattern may be adjusted to a small value. Since small fluctuations are of primary interest, the magnitude of $2J_S(Z)$ is to be less than the magnitude of the envelope of $J_S(Z)$ in equation (4) or

$$\left|2J_S(Z)\right| = b\left|\overline{J_0(Z)}\right| \qquad (0 < b < 1.0) \tag{5}$$

The bar over a function will represent the envelope of that function. By choosing the proper value of b, the fluctuation can be restricted to small values. Equation (5) is now identified as the "criterion equation." At the zeros of $J_0(Z)$, the first term in equation (4) is zero since equation (5) is zero. At these values of Z the criterion is established on the second term of equation (4) since the elements of this term are of the same order of magnitude as the elements of the first term.

The magnitude of $2J_S(Z)$ is plotted as a function of Z for $S > Z$ in figure 2. The function $\left|\overline{J_0(Z)}\right|$, $0.1\left|\overline{J_0(Z)}\right|$, $0.01\left|\overline{J_0(Z)}\right|$, $0.001\left|\overline{J_0(Z)}\right|$, $0.0001\left|\overline{J_0(Z)}\right|$, and $0.00001\left|\overline{J_0(Z)}\right|$ are also plotted in this figure. The reason for plotting the many values of $\left|\overline{J_0(Z)}\right|$ will become apparent later. These plots are known as the "criteria plots." Use of these criteria plots in conjunction with several assumed element patterns is shown in the next section.

CALCULATIONS USING ASSUMED ELEMENT PATTERNS

Computations of the fluctuation D as a function of the number of sources S over a wide range of circumferences in wavelength Z were made for the coefficients A_0 and A_1 in equation (4). The following values were assumed:

$$\left.\begin{array}{ll} A_0 = 1 & A_1 = 1/16 \\ A_0 = 1 & A_1 = 1/8 \\ A_0 = 1 & A_1 = 1/4 \\ A_0 = 1 & A_1 = 1/2 \\ A_0 = 1 & A_1 = 3/4 \\ A_0 = 1 & A_1 = 1.0 \\ A_0 = 1 & A_1 = 2.0 \end{array}\right\} \qquad (6)$$

From these computations, fluctuations of 0.5 dB and 2.0 dB were chosen as typically acceptable amounts of ripple. From the fluctuation curves appearing in references 2 and 3, the 0.5 dB and 2.0 dB levels of fluctuation associated with a particular Z and S can be readily obtained, or, the levels can be obtained from tabulated values of fluctuations. In some cases, these fluctuations may occur more than once; when this happens, the first occurrence is chosen. The 0.5 dB fluctuation for each of the corresponding values of S and Z is plotted on a set of "criteria plots" in figure 2. Similarly, the 2.0 dB fluctuation and its corresponding values of Z and S are plotted in figure 3. Since the curves in figures 2 and 3 are close together, only three of the sets of A_0 and A_1 in equation (6) were plotted.

STATEMENT OF CRITERIA

One can observe by inspecting figure 2 that all the plotted data for 0.5 dB fluctuation fall above the $0.001|\overline{J_0(Z)}|$ curve. Therefore, if a cross plot is made of the intersection of $0.001|\overline{J_0(Z)}|$ and $|2J_S(Z)|$, a relationship between the number of sources S and the circumference in wavelengths Z can be obtained and a value of b in equation (5) can be established. Thus, the criterion equation for 0.5 dB fluctuation becomes

$$|2J_S(Z)| = 0.001|\overline{J_0(Z)}| \qquad (S > Z) \qquad (7)$$

The indicated cross plot is given in figure 4 where Z/S as a function of Z is shown. For a particular circumference in wavelengths Z, this curve will give the required spacing of the sources S to obtain 0.5 dB or less fluctuation. Also, the number of sources can be obtained from this curve.

Similarly, from figure 3, all the plotted data for 2.0 dB fluctuation fall above the $0.01|\overline{J_0(Z)}|$ curve. By using the intersection of this curve and the $|2J_S(Z)|$ curve, a cross plot similar to one in figure 4 can be made. This cross plot is shown in figure 5.

The criterion equation for 2.0 dB fluctuation is

$$\left|2J_S(Z)\right| = 0.01\left|\overline{J_0(Z)}\right| \qquad (S > Z) \qquad (8)$$

In figures 4 and 5, the number of sources S associated with a particular circumference in wavelength Z is a "conservative" number, that is, an always adequate number. By arraying this conservative number of sources around its respective circumference in wavelengths Z, the antenna pattern will be quasi-omnidirectional for the source patterns considered. However, if the source pattern is known, the number of sources needed for small fluctuations will be equal to or less than this conservative number.

Since this criterion is based only on the first term of equation (4), the second term of this equation must be shown to satisfy the criteria. By examining the derivatives of $\left|J_0(Z)\right|$ and $\left|J_S(Z)\right|$, one can observe that the envelope of $\left|J_0^{(1)}(Z)\right|$ $\left(\text{that is, } \left|\overline{J_0^{(1)}(Z)}\right|\right)$ is of the same order of magnitude as $\left|\overline{J_0(Z)}\right|$. Also, $\left|J_S^{(1)}(Z)\right|$ is less than $\left|J_S(Z)\right|$ for large values of Z, provided that the spacing of the sources is greater than 0.5λ but less than 1.0λ. Since the assumption for the criterion is

$$\left|2J_S(Z)\right| < \left|\overline{J_0(Z)}\right| \qquad (S > Z) \qquad (9)$$

it must be shown that

$$\left|2J_S^{(1)}(Z)\right| < \left|\overline{J_0^{(1)}(Z)}\right| \qquad (S > Z) \qquad (10)$$

is true. Plots of $\left|2J_S^{(1)}(Z)\right|$ and $\left|\overline{J_0^{(1)}(Z)}\right|$ are shown in figure 6. By comparing these plots with the plots shown in figure 2 for a specified Z and S, one can observe that the inequality in equation (10) is satisfied for $Z \geqq 10$. Hence, for $Z \geqq 10$, the criterion is satisfied.

APPLICATION TO PHYSICALLY REALIZABLE ANTENNAS

Upon critical inspection of the criteria curves given in figures 4 and 5, a clear restriction on the use of such curves arises when one considers application to physically realizable antennas. First, for $Z < 10$, the criteria requires source spacings $\leqq 0.5\lambda$ to 0.6λ depending upon the choice of ripple D desired. Many antennas commonly used for spacecraft, that is, circumferential $\lambda/2$ slots, $\lambda/2$ dipoles, and so forth, cannot be arranged $\leqq 0.5\lambda$ apart. Hence, for $Z < 10$, the criteria cannot be satisifed for circular arrays using such sources. Secondly, antennas that can be spaced less than $\lambda/2$ apart, that is, axial slots, dipoles, and so forth, exhibit strong mutual coupling effects which are not accounted for in the original derivation of equation (1). Hence, application of the spacing criteria derived in this paper must be restricted to cases where $Z \geqq 10$.

When the source pattern is expressed as a two-term cosine series, the half-power beam width can be no less than 90°. Therefore, the typical source patterns considered in establishing the criterion have a half-power beam width $\geqq 90^\circ$. From Kraus (ref. 6) a uniformly illuminated rectangular aperture has a half-power beam width of $51^\circ/L_\lambda$, where L_λ is the length of the aperture in wavelengths. Thus, the two-term cosine series restricts the linear aperture dimension L_λ to $\leqq 0.567$. (It is assumed that typical sources can be treated as equivalent linear apertures.)

However, many antennas have half-power beam widths less than 90° and hence require more than two terms of a cosine series to represent them. Such is the case for the uniformly illuminated rectangular aperture with $L_\lambda > 0.567$. The criterion established is valid for antennas requiring more than two terms of a Fourier cosine series to represent them.

Many authors have investigated realizable antennas that are appropriate for arraying on circular bodies. From Harrington (ref. 7), a current filament placed 0.25λ away from a conducting cylinder having a circumference in wavelengths of 23.5 ($Z = 23.5$) has a half-power beam width of approximately 130°. Again from Harrington (ref. 7), a circumferential slot 0.65 long in a conducting cylinder having a circumference in wavelengths $\geqq 9.4$ ($Z \geqq 9.4$) has a half-power beam width of approximately 80°. From Wait (ref. 8), a circumferential slot of 0.5λ long on a conducting cylinder for $Z \geqq 3$ produces patterns with half-power beam widths of approximately 80° to 90°. For the circumferential slot of 0.5 inch (1.27 cm) in length considered by Croswell and Knop (ref. 5) on a cylinder of circumference in wavelengths equal to 39.5 ($Z = 39.5$), a half-power beam width of approximately 90° was obtained. The pattern of a slot in this paper is shown to be approximated by a three-term cosine series.

Hence, for many typical physically realizable sources only two or three terms in the Fourier cosine series are needed to obtain an adequate representation of the source pattern. However, for large values of Z, that is, $Z \geqq 10$ and D small, the criterion is satisfied regardless of the number of cosine terms used. See the higher order derivatives of $J_0(Z)$ and $J_S(Z)$ in the appendix. These higher derivatives are related to the number of terms in the source pattern. Plots of $\left|J_0^{(n)}(Z)\right|$ and $\left|2J_S^{(n)}(Z)\right|$ for n up to six are shown in figures 6 to 11 over a limited range of Z. From these curves or from the equations appearing in the appendix, one can observe that the inequalities

$$\left.\begin{aligned}
\left|2J_S^{(1)}(Z)\right| &< \left|\overline{J_0^{(1)}(Z)}\right| \\
\left|2J_S^{(2)}(Z)\right| &< \left|\overline{J_0^{(2)}(Z)}\right|
\end{aligned}\right\}$$

Equations (11) continued on next page

$$\left.\begin{aligned}
\left|2J_S^{(3)}(Z)\right| &< \left|\overline{J_0^{(3)}(Z)}\right| \\
\left|2J_S^{(4)}(Z)\right| &< \left|\overline{J_0^{(4)}(Z)}\right| \\
\left|2J_S^{(5)}(Z)\right| &< \left|\overline{J_0^{(5)}(Z)}\right| \\
\left|2J_S^{(6)}(Z)\right| &< \left|\overline{J_0^{(6)}(Z)}\right|
\end{aligned}\right\} \qquad (S > Z) \qquad (11)$$

are satisfied and, therefore, the criterion in equation (9) is satisfied.

For example, if the criterion is applied to the slots appearing in the paper by Croswell and Knop (ref. 5), the criterion equation would be the same as equation (7); that is,

$$\left|2J_S(Z)\right| = 0.001\left|\overline{J_0(Z)}\right| \qquad (S > Z) \qquad (12)$$

Hence, the curve in figure 4 can be used to determine the number of sources needed to obtain a fluctuation of 0.5 dB or less on a circumference in wavelengths of 39.5. The number from the curve is 52 sources compared with 54 sources used by Croswell and Knop, but, as was stated by these authors, as few as 48 could have been chosen.

CONCLUDING REMARKS

A criterion is established for determining the number of sources needed around the periphery of a spacecraft to obtain small far-field pattern fluctuations, that is, quasi-omnidirectional antenna patterns. The criterion assumes that the source pattern is expressible as a finite Fourier cosine series and that the circumference in wavelengths of the body is $\geqq 10$. The number of sources obtained by using this criterion will always be adequate; however, if the representation of the source pattern is known, an optimum number of sources can be obtained. The criterion is not valid when the source pattern is isotropic. Two curves are given which relate the number of sources and the circumference of the body in wavelengths for 0.5 dB or less and 2.0 dB or less fluctuation.

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Langley Station, Hampton, Va., May 11, 1967,
125-22-02-02-23.

APPENDIX

HIGHER ORDER DERIVATIVES OF BESSEL FUNCTIONS

The higher derivatives of $J_S(Z)$, from which the higher derivatives of $J_0(Z)$ may be obtained, is shown in this section. Since $u = J_S(Z)$ must satisfy Bessel's equation

$$\frac{d^2u}{dZ^2} + \frac{1}{Z}\frac{du}{dZ} + \left(1 - \frac{S^2}{Z^2}\right)u = 0 \tag{A1}$$

the higher derivatives of $J_S(Z)$ can be expressed in terms of $J_S^{(1)}(Z)$ and $J_S(Z)$. By substituting $u = J_S(Z)$ into equation (A1), the second derivative of $J_S(Z)$ is

$$J_S^{(2)}(Z) = \left(-1 + \frac{S^2}{Z^2}\right)J_S(Z) - \frac{1}{Z}J_S^{(1)}(Z) \tag{A2}$$

From equation (A2), the higher order derivatives of $J_S(Z)$ become

$$\left.\begin{aligned}
J_S^{(3)}(Z) &= \left(\frac{1}{Z} - \frac{3S^2}{Z^3}\right)J_S(Z) + \left(-1 + \frac{S^2 + 2}{Z^2}\right)J_S^{(1)}(Z) \\
J_S^{(4)}(Z) &= \left(1 - \frac{2S^2 + 3}{Z^2} + \frac{S^4 + 11S^2}{Z^4}\right)J_S(Z) + \left(\frac{2}{Z} - \frac{6S^2 + 6}{Z^3}\right)J_S^{(1)}(Z) \\
J_S^{(5)}(Z) &= \left(-\frac{2}{Z} + \frac{12S^2 + 12}{Z^3} - \frac{10S^4 + 50S^2}{Z^5}\right)J_S(Z) \\
&\quad + \left(1 - \frac{2S^2 + 7}{Z^2} + \frac{S^4 + 35S^2 + 24}{Z^4}\right)J_S^{(1)}(Z) \\
J_S^{(6)}(Z) &= \left(-1 + \frac{3S^2 + 9}{Z^2} - \frac{3S^4 + 78S^2 + 60}{Z^4} + \frac{S^6 + 85S^4 + 274S^2}{Z^6}\right)J_S(Z) \\
&\quad + \left(-\frac{3}{Z} + \frac{18S^2 + 33}{Z^3} - \frac{15S^4 + 225S^2 + 120}{Z^5}\right)J_S^{(1)}(Z)
\end{aligned}\right\} \tag{A3}$$

The $J_S^{(1)}(Z)$ term in equations (A2) and (A3) can be expressed in terms of $J_{S+1}(Z)$ and $J_{S-1}(Z)$.

APPENDIX

The higher derivatives of $J_0(Z)$ are obtained simply by letting $S = 0$ in these equations; that is,

$$
\left.
\begin{aligned}
J_0^{(1)}(Z) &= -J_1(Z) \\
J_0^{(2)}(Z) &= -J_0(Z) + \frac{J_1(Z)}{Z} \\
J_0^{(3)}(Z) &= \frac{J_0(Z)}{Z} + \left(1 - \frac{2}{Z^2}\right)J_1(Z) \\
J_0^{(4)}(Z) &= \left(1 - \frac{3}{Z^2}\right)J_0(Z) + \left(-\frac{2}{Z} + \frac{6}{Z^3}\right)J_1(Z) \\
J_0^{(5)}(Z) &= \left(-\frac{2}{Z^2} + \frac{12}{Z^3}\right)J_0(Z) + \left(-1 + \frac{7}{Z^2} - \frac{24}{Z^4}\right)J_1(Z) \\
J_0^{(6)}(Z) &= \left(-1 + \frac{9}{Z^2} - \frac{60}{Z^4}\right)J_0(Z) + \left(\frac{3}{Z} - \frac{33}{Z^3} + \frac{120}{Z^5}\right)J_1(Z)
\end{aligned}
\right\}
\tag{A4}
$$

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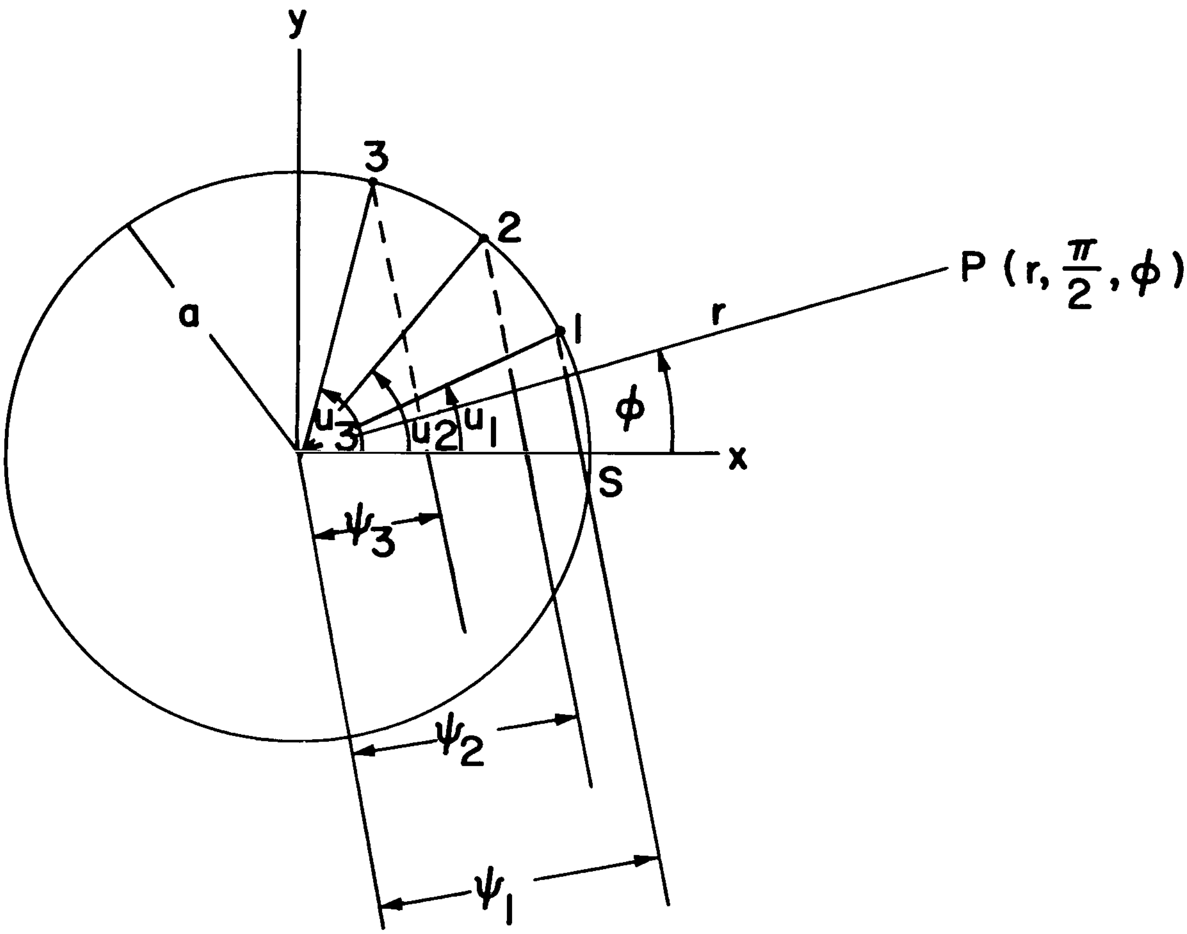
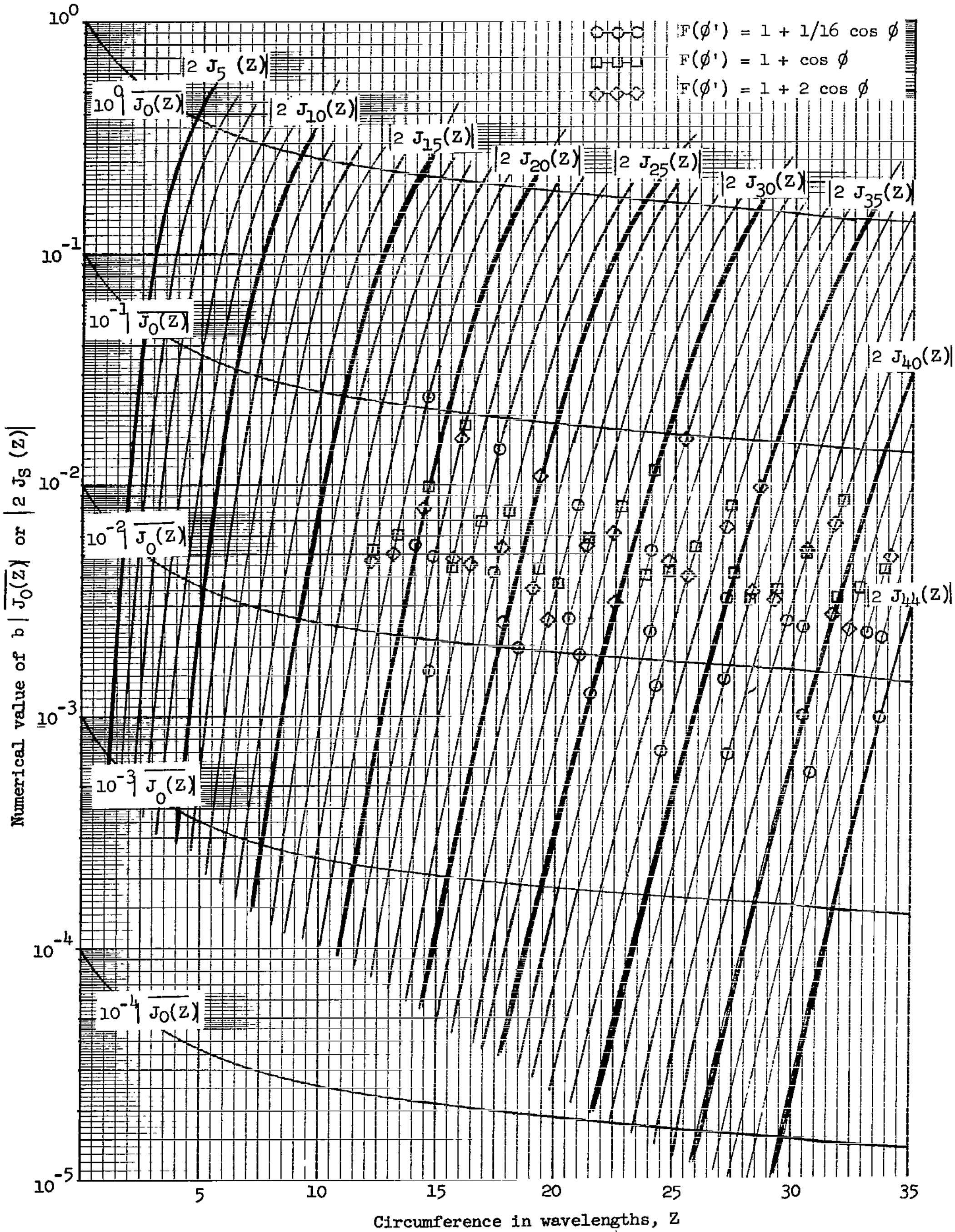
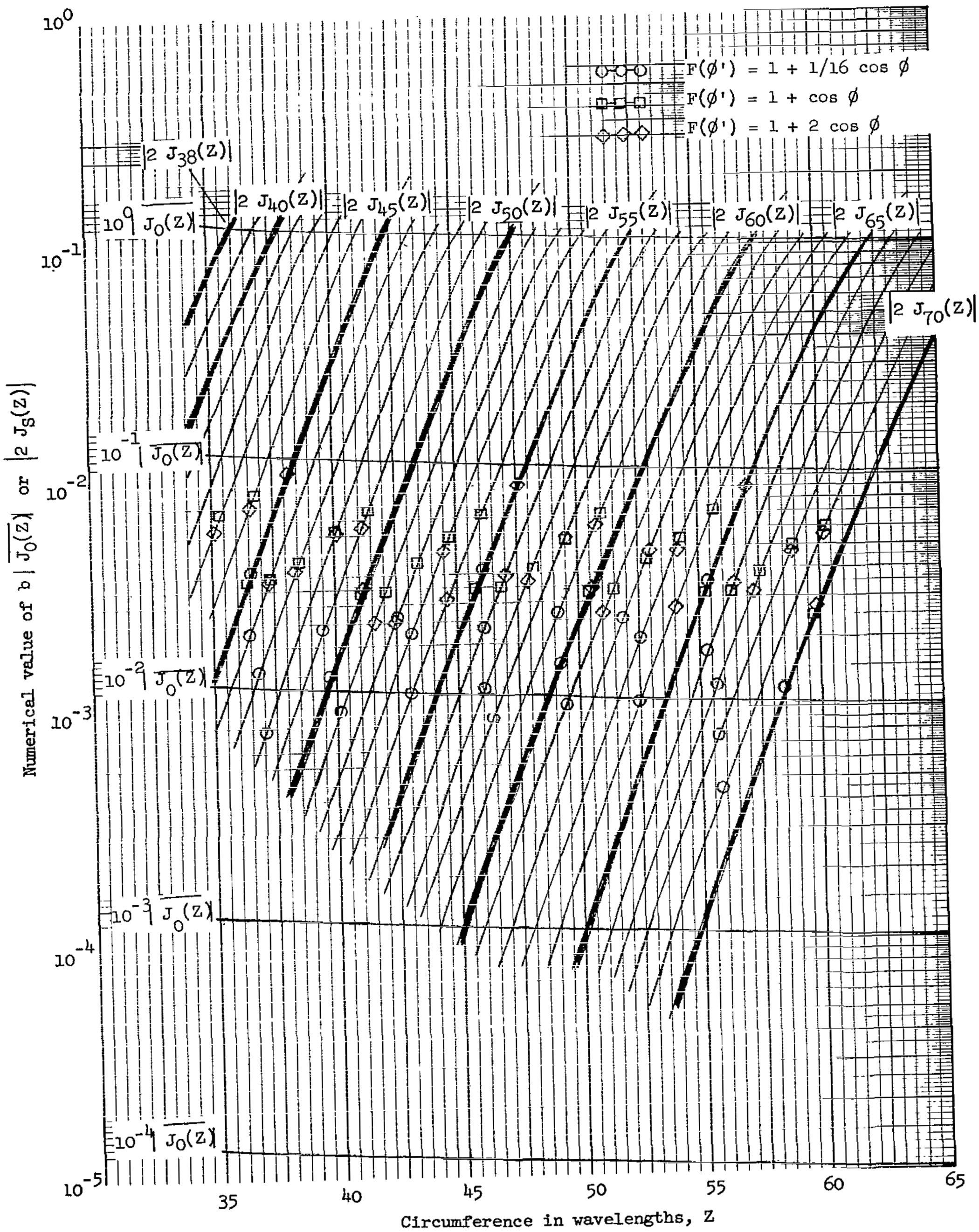


Figure 1.- Uniform circular array and far-field point.



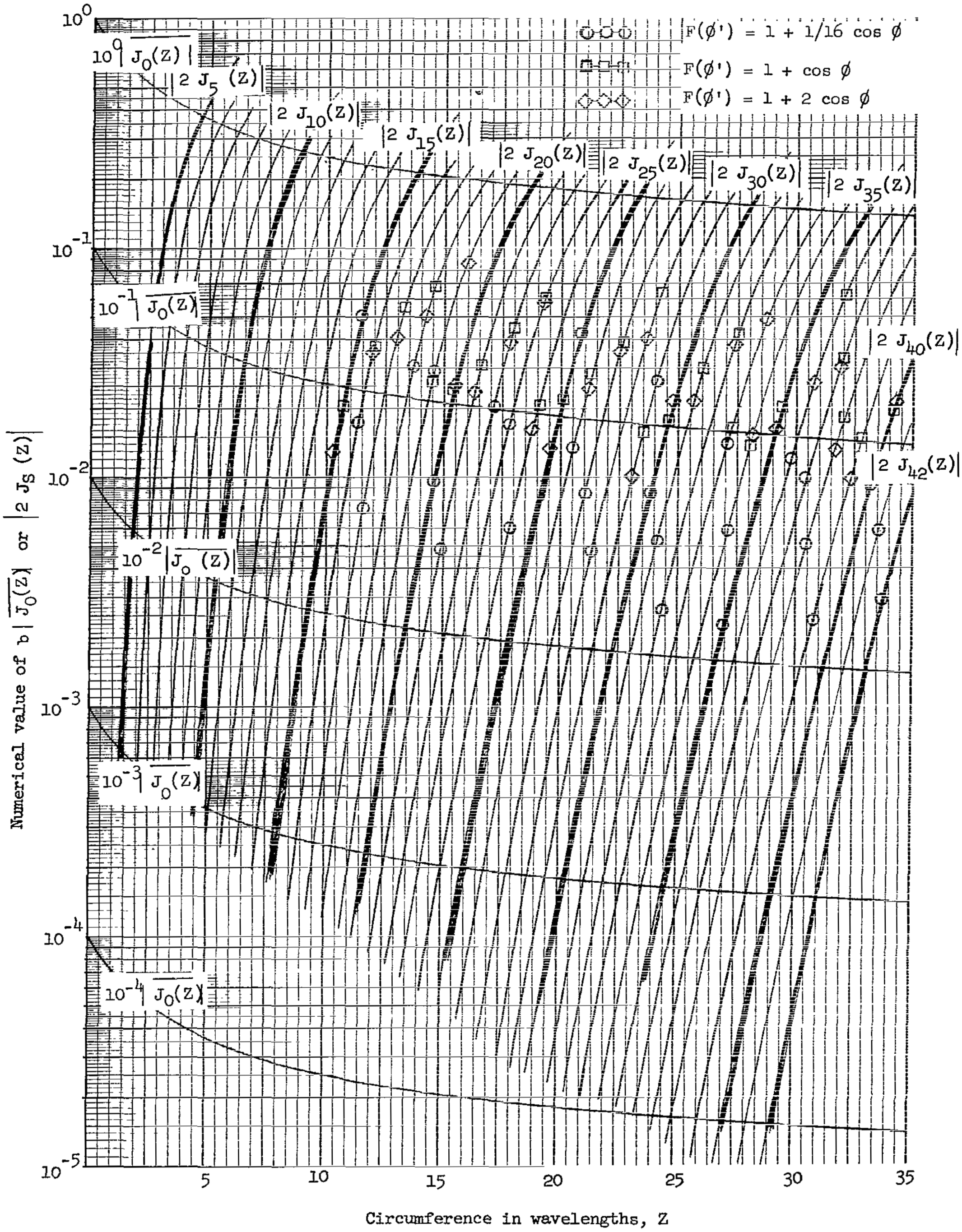
(a) $0 \leqq Z \leqq 35$.

Figure 2.- Comparison of $b|\overline{J_0(Z)}|$ with $|2J_S(Z)|$ where values of S and corresponding Z are plotted for assumed source patterns for a 0.5 dB fluctuation.



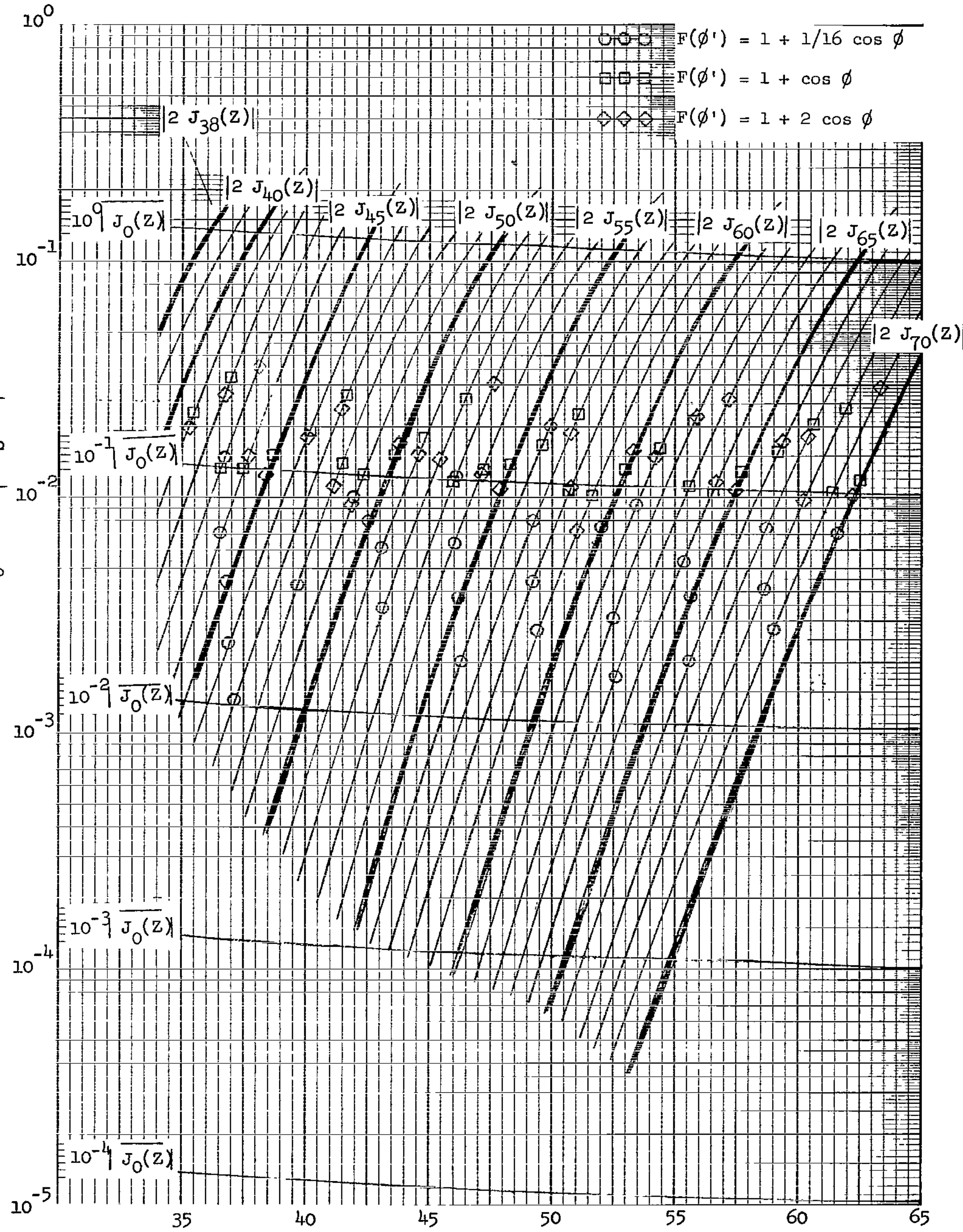
(b) $35 \leqq Z \leqq 65$.

Figure 2.- Concluded.



(a) $0 \leqq Z \leqq 35$.

Figure 3.- Comparison of $b|\overline{J_0(Z)}|$ with $|2J_S(Z)|$ where values of S and corresponding Z are plotted for assumed source patterns for a 2.0 dB fluctuation.



(b) $35 \leqq Z \leqq 65$.

Figure 3.- Concluded.

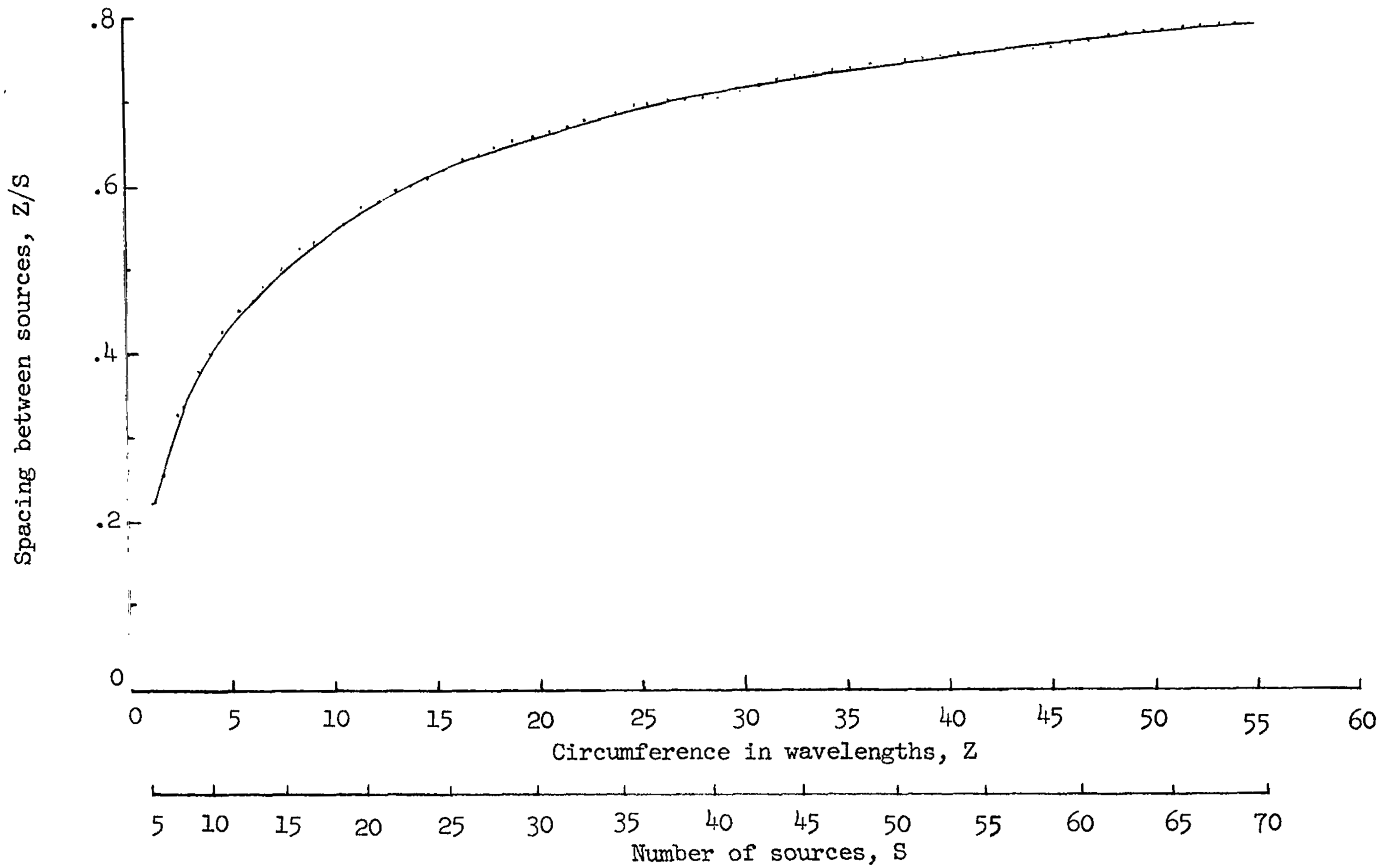


Figure 4.- The criteria curve for 0.5 dB fluctuation.

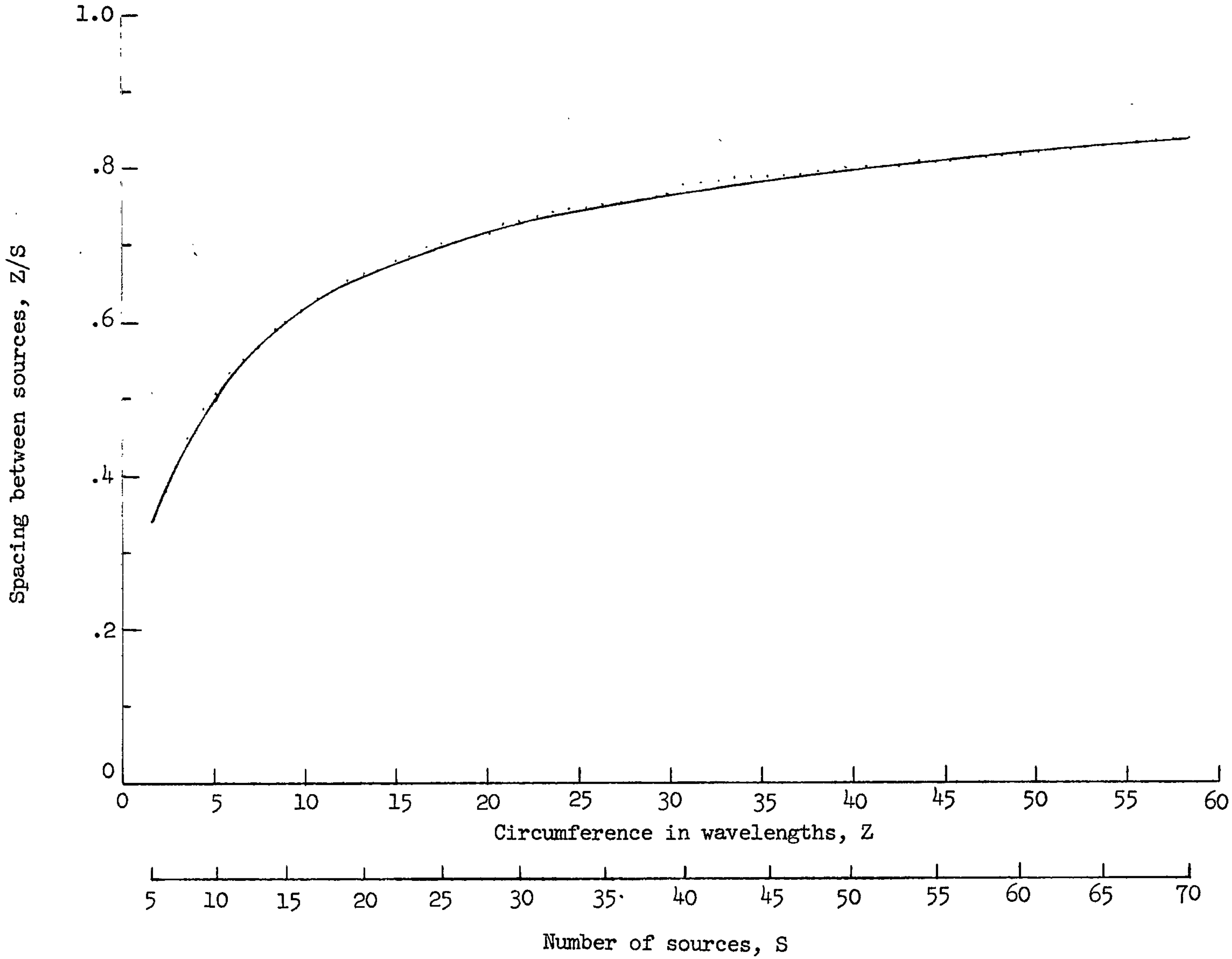


Figure 5.- The criteria curve for 2.0 dB fluctuation.

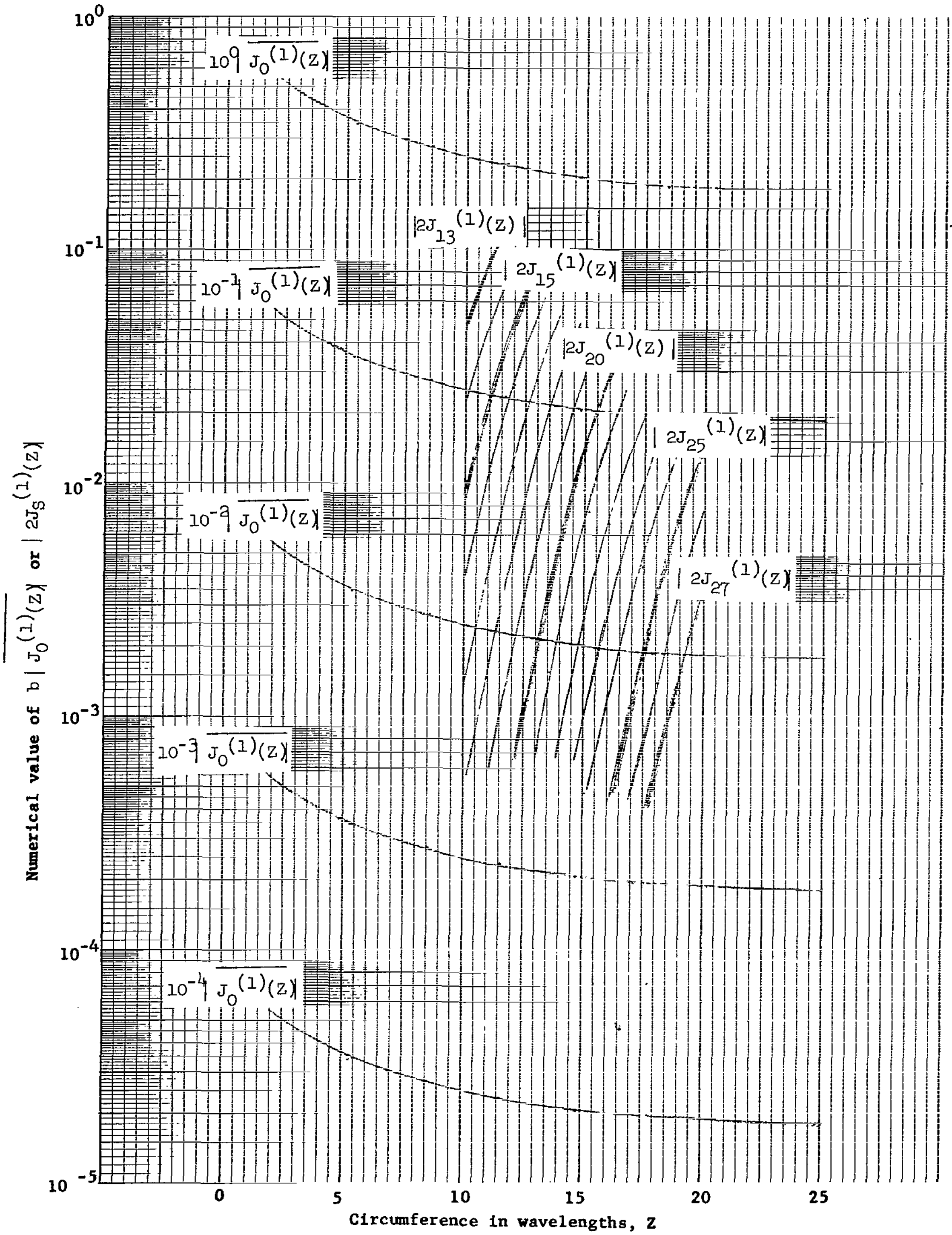


Figure 6.- Comparison of $b\left|\overline{J_0^{(1)}(Z)}\right|$ with $|2J_S^{(1)}(Z)|$.

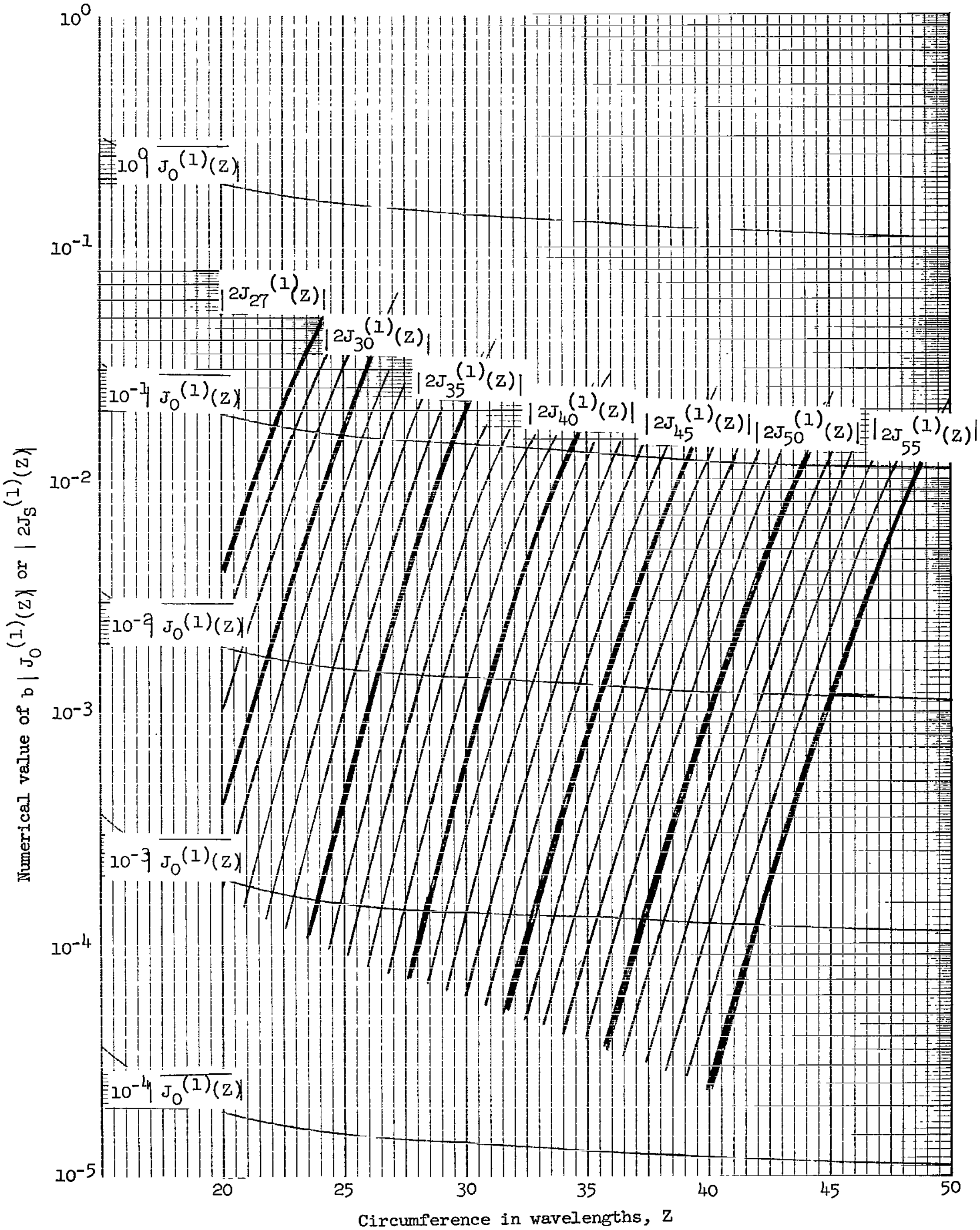


Figure 6.- Concluded.

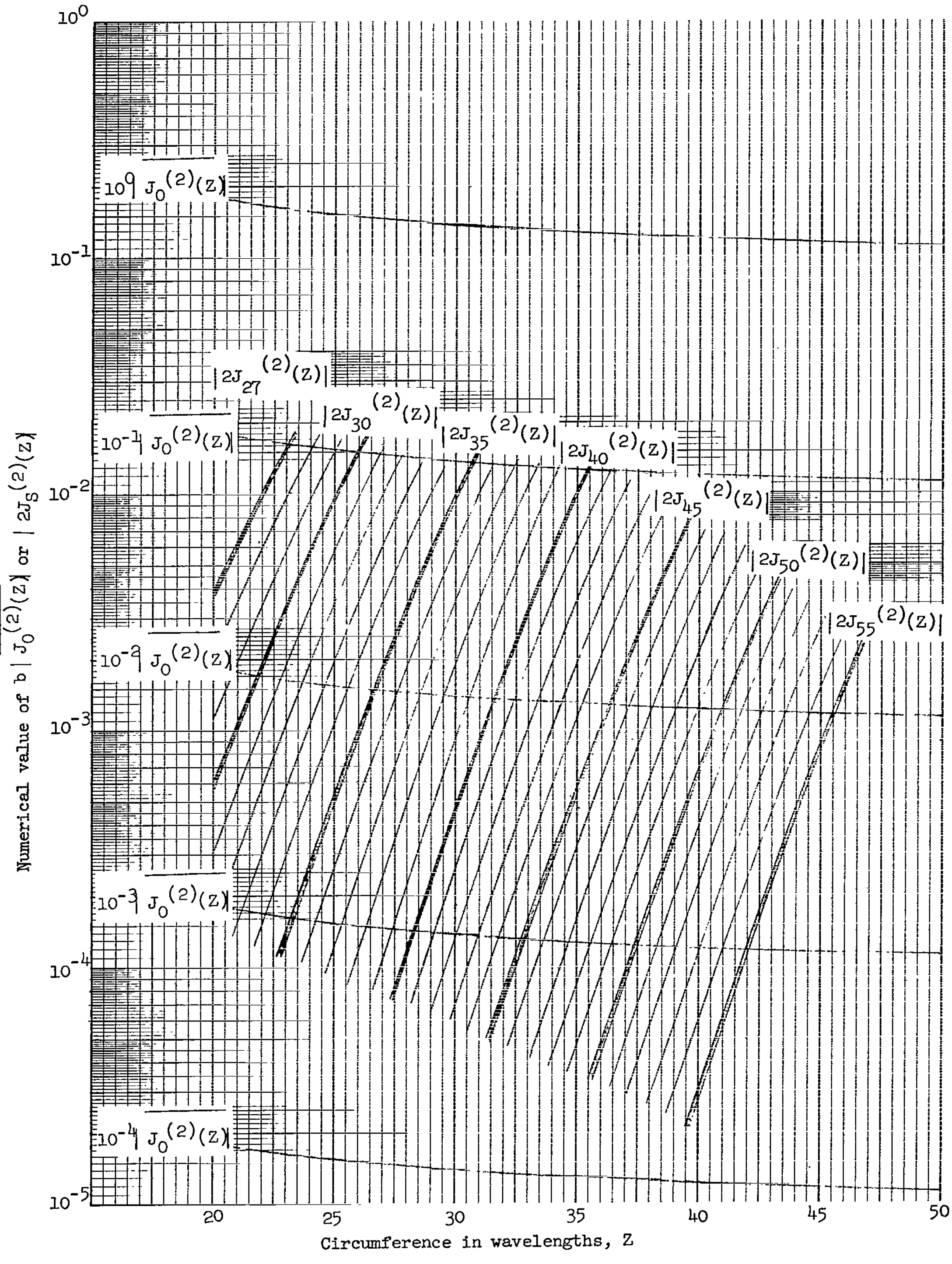


Figure 7.- Comparison of $b\left|\overline{J_0^{(2)}(Z)}\right|$ with $\left|2J_S^{(2)}(Z)\right|$.

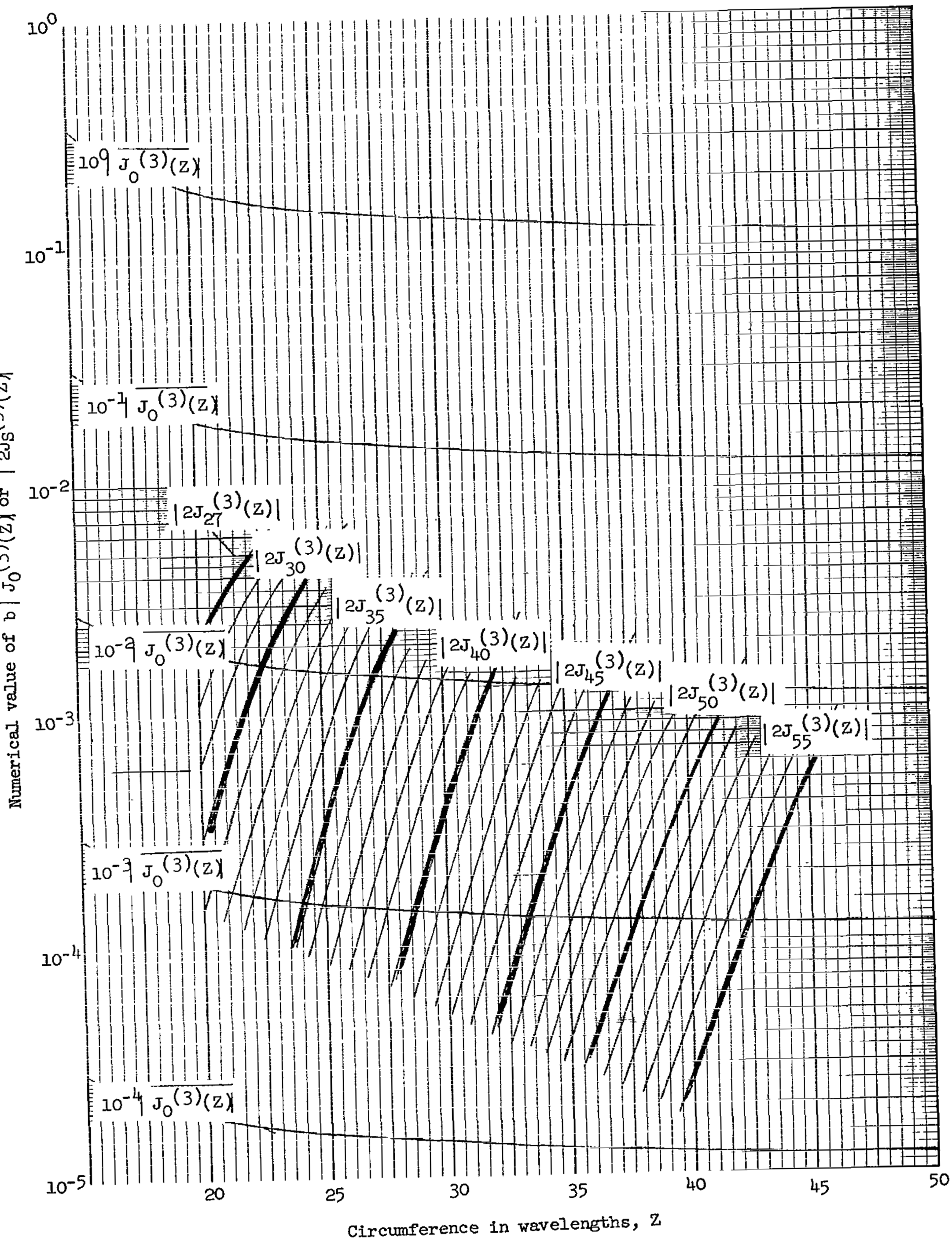


Figure 8.- Comparison of b$\left|\overline{J_0^{(3)}(Z)}\right|$ with $\left|2J_S^{(3)}(Z)\right|$.

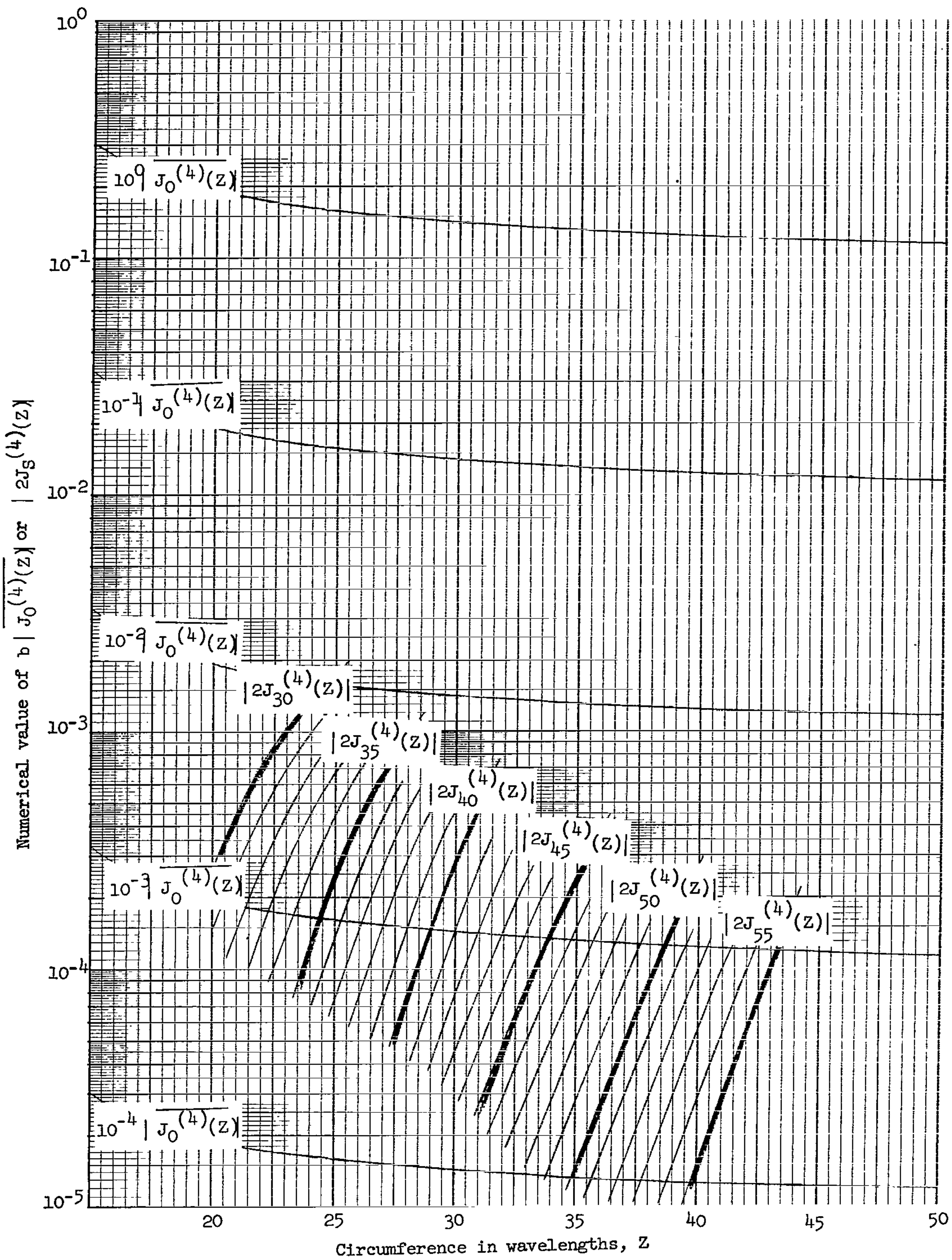


Figure 9.- Comparison of $b\left|\overline{J_0^{(4)}(Z)}\right|$ with $\left|2J_S^{(4)}(Z)\right|$.

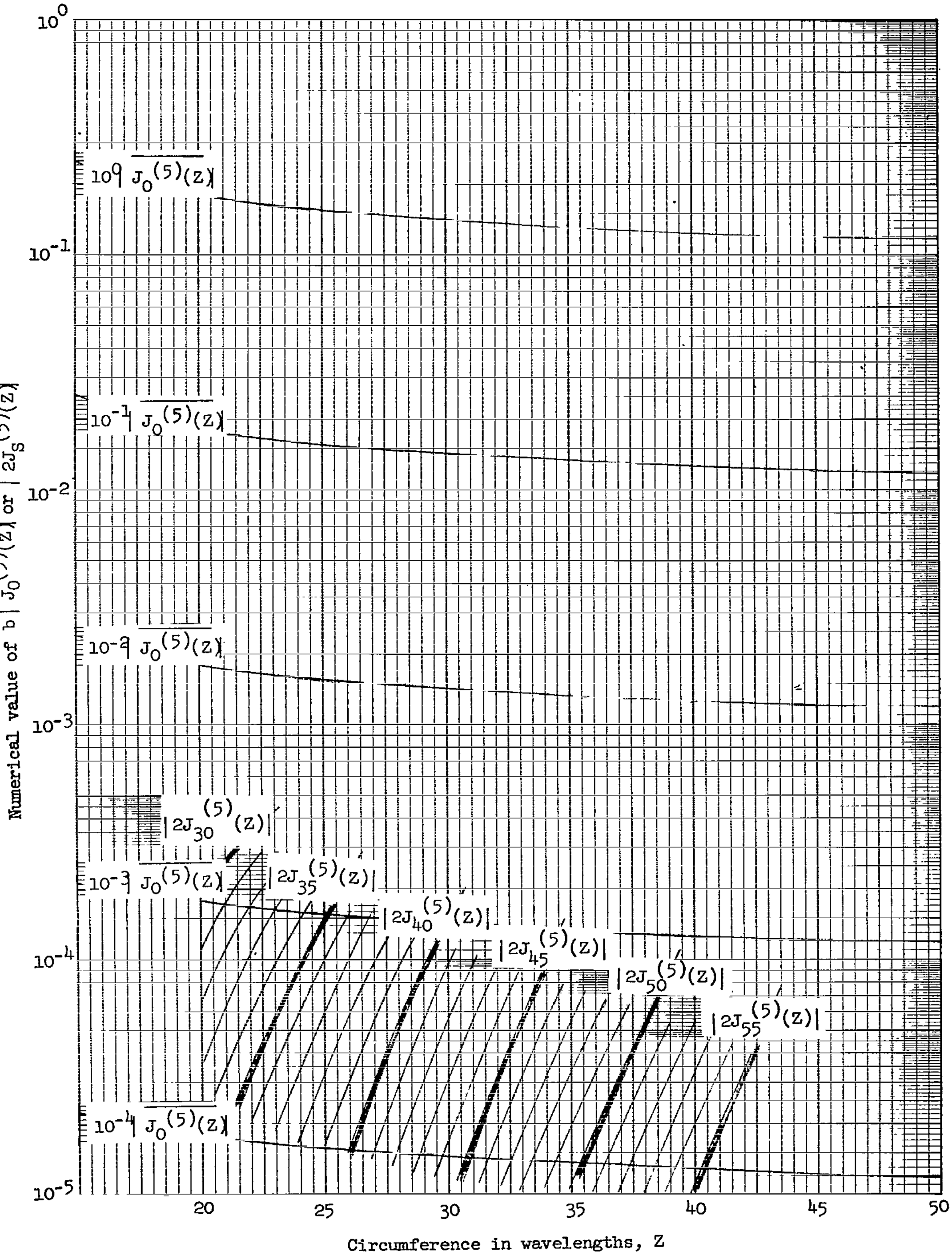


Figure 10.- Comparison of $b\left|\overline{J_0^{(5)}(Z)}\right|$ with $\left|2J_S^{(5)}(Z)\right|$.

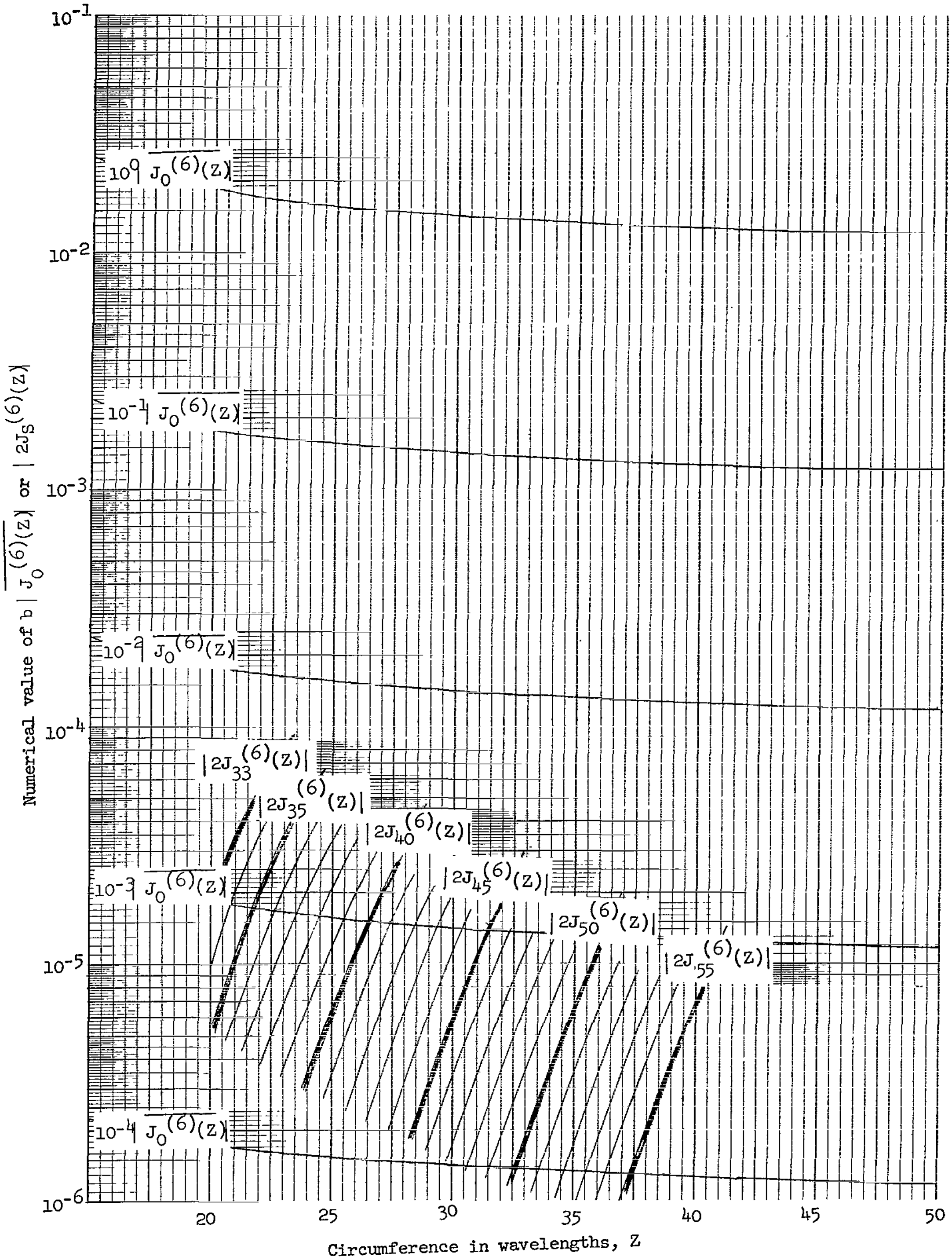


Figure 11.- Comparison of $b\left|\overline{J_0^{(6)}(Z)}\right|$ with $\left|2J_S^{(6)}(Z)\right|$.

NASA-Langley, 1967 —— 7 L-5572

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