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DYNAMIC STABILITY OF SPACE VEHICLES

Volume V - Impedance Testing for Flight Control Parameters

by David R. Lukens

Prepared by
GENERAL DYNAMICS CORPORATION
San Diego, Calif.
for George C. Marshall Space Flight Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1967

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for George C. Marshall Space Flight Center

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FOREWORD

This report is one of a series in the field of structural dynamics prepared under contract NAS 8-11486. The series of reports is intended to illustrate methods used to determine parameters required for the design and analysis of flight control systems of space vehicles. Below is a complete list of the reports of the series.

Volume I	Lateral Vibration Modes
Volume II	Determination of Longitudinal Vibration Modes
Volume III	Torsional Vibration Modes
Volume IV	Full Scale Testing for Flight Control Parameters
Volume V	Impedence Testing for Flight Control Parameters
Volume VI	Full Scale Dynamic Testing for Mode Determination
Volume VII	The Dynamics of Liquids in Fixed and Moving Containers
Volume VIII	Atmospheric Disturbances that Affect Flight Control Analysis
Volume IX	The Effect of Liftoff Dynamics on Launch Vehicle Stability and Control
Volume X	Exit Stability
Volume XI	Entry Disturbance and Control
Volume XII	Re-entry Vehicle Landing Ability and Control
Volume XIII	Aerodynamic Model Tests for Control Parameters Determination
Volume XIV	Testing for Booster Propellant Sloshing Parameters
Volume XV	Shell Dynamics with Special Applications to Control Problems

The work was conducted under the direction of Clyde D. Baker and George F. McDonough, Aero Astro Dynamics Laboratory, George C. Marshall Space Flight Center. The General Dynamics Convair Program was conducted under the direction of David R. Lukens.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1/INTRODUCTION

The stability of a launch vehicle may be affected by the structural compliance of the control system components and their mounting and associated brackets. The effective spring rates of the sensors and control elements can cause large variations in actual phase and gain margins. Structural compliance can also affect the interaction (coupling) between the flight control system and the structure. This interaction is dependent upon the forcing mechanical impedance as well as other factors. For control studies the frequency range below 50 Hz (twice the bandwidth of the servo system) is usually all that is important.

The above problems are dependent on the flexibility of the structure. This monograph presents ways of determining equivalent spring rates that describe the local flexibility of the structures. These spring rates and impedance techniques used to determine them may also be used to evaluate motions for clearance and environmental studies.

2/STATE OF THE ART

In the developmental phase, the spring rates used to perform stability and control analyses are analytically calculated. As the hardware becomes available, spring rates are determined by applying known static loads and measuring the resulting deflections. As the structure becomes more complete, impedance tests may be run to provide data to confirm or update the spring rate values originally obtained for the stability and control analyses. Late in the program, frequency response tests are usually run to experimentally check the autopilot system parameters. The test configuration may include the launch vehicle structure, engine actuators, engines, and the autopilot system (see Figure 1).

Tests of this type provide a very practical way to verify that the mathematical model of the structure is accurate in describing the important structural dynamic properties. A model of the gross vehicle elastic vibration modes may not include enough degrees of freedom; a frequency response test may reveal these higher modes of vibration and any local modes of vibration that were not included in the gross vehicle model. These localized dynamic modes may interact with the autopilot system and cause significant phase and gain variations.

3/CRITERIA

The use of the spring rate, determined from static measurements, can result in significant errors in predicting dynamic response. This comes about from two basic phenomena. First, the motion of the vehicle in flight may be different from that of a restrained test article. This causes the apparent spring rate to differ from that of the test article. Second, the motions may be close to or above the natural frequency of the system in question. Then the true impedance of the structure will be governed by the affected mass. It is necessary, therefore, that static testing be augmented by dynamic testing. This can usually be accomplished with a conventional dynamic (shake) test. Impedance techniques, however, can be used to determine the characteristics required for dynamic analysis.

4/RECOMMENDED PRACTICES

4.1 IMPEDANCE METHODS FOR DETERMINING SPRING RATES

The mechanical impedance (or its inverse- mobility) at a point on a structure can provide a very useful description of structural dynamic properties. The effective mass and the effective spring rate versus frequency can be determined if the mechanical impedance is known. Also, equivalent descriptions of the structure can be determined by synthetical means if point and transfer impedances are known.

The mechanical impedance of a spring, a mass, and a dash pot are presented. The mechanical impedance of several simple types of structures are calculated, and the general rules for calculation of impedance are presented.

4.1.1 IMPEDANCE OF CLASSICAL SPRING, MASS, DASHPOT SYSTEMS. Mechanical impedance (Z) is defined as the ratio of the force (F) through an element to the velocity (V) across the element (Reference 1).

$$Z = \frac{F}{V}$$

For steady-state sinusoidal excitation the complex mechanical impedance of the spring, the mass, and the dashpot are: spring $Z = \frac{K}{j\omega}$, mass $Z = j\omega m$, dashpot $Z = C$. As defined herein

- Z is the complex mechanical impedance (lb-sec/in.)
- K is the spring rate (lb/in.)
- ω is the circular frequency (rad/sec)
- m is the mass (lb)
- C is the coefficient of viscous damping (lb-sec/in.)
- j is the $\sqrt{-1}$

A derivation of the impedances of these elements is given in References 1 and 5.

The complex mechanical impedance of a combined spring, mass, and dashpot system, as sketched in Figure 1, is

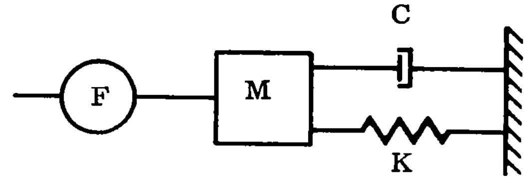


Figure 1. Combined System

$$Z_{11} = C + \frac{K}{j\omega} + j\omega m$$

$$= 2\,\frac{C}{C_c}\sqrt{Km} + j(\omega m - \frac{K}{\omega})$$

where

$$C_c = 2\sqrt{Km}$$

The impedance of the preceding flexibly mounted mass is plotted in Figure 2. If the vibratory force, F_1, is applied at the resonant frequency

$$\omega = \omega_n = \sqrt{\frac{K}{m}}$$

Then Z reduces to C or

$$Z_{11} = \frac{\sqrt{K_m}}{Q} = \frac{K}{Q\omega_n}$$

where

$$Q = \frac{C_c}{2C}$$

This expression has the same value as that commonly used in electrical circuits or the gain of the system at resonance. Values of Q from 20 to 50 are common in mechanical structures, with values in excess of 200 possible.

The remainder of this section will present a discussion on how impedance may be calculated.

One advantage of using impedance methods is the ease of calculation. If the velocity of a point "0" due to an oscillating force is

$$V_0 e^{j\omega t}$$

then

$$V_0 = \frac{F}{Z_0}$$

For a number of impedances in parallel the velocity can be obtained by addition

$$V_0 = \frac{F}{Z_0}$$

where

$$Z_0 = Z_1 + Z_2 \cdot \cdot \cdot \cdot \cdot \cdot \cdot Z_n;$$

or,

$$V_0 = \frac{F}{\Sigma Z_i}$$

When the units are in series the addition is slightly more time consuming.

$$V_0 = \frac{F}{Z_0}$$

where

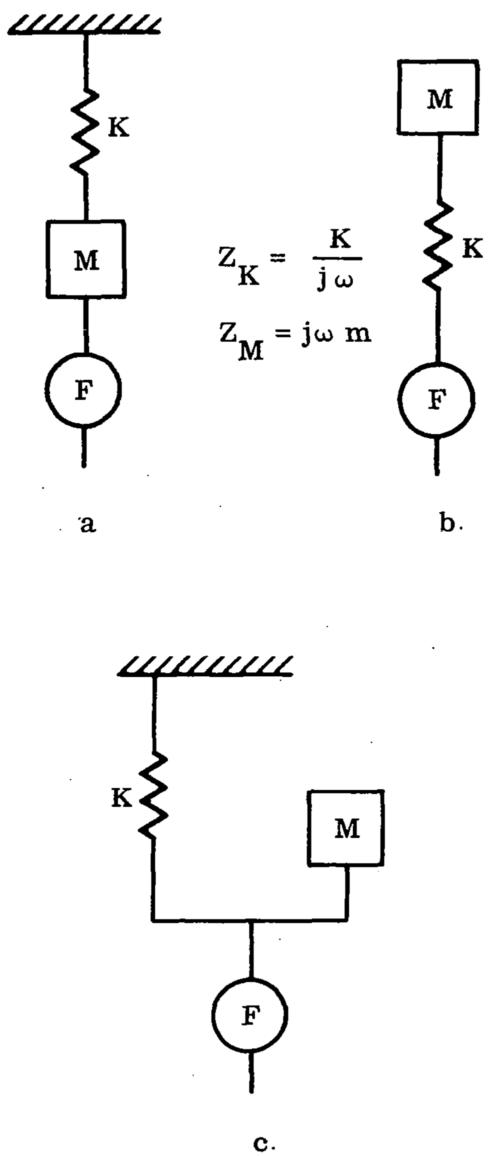
$$\frac{1}{Z_0} = \frac{1}{Z_1} + \frac{1}{Z_2} \cdots\cdots \frac{1}{Z_n}$$

or

$$V_0 = F \Sigma \frac{1}{Z_i}$$

Since almost all nonredundant structures can be broken down into either series or parallel combinations of the basic elements, these two relationships enable one to calculate the impedances of complex systems easily and quickly.

To help in correctly combining impedances (Z) it is worthwhile to note one obvious trap at this time. Consider the following two systems.



$$Z_K = \frac{K}{j\omega}$$

$$Z_M = j\omega\, m$$

Figure 2. System Diagrams

The system sketched in Figure 2a is in parallel, while 2b is in series. To illustrate the parallel relationship the elements of 2a are redrawn as Figure 2c.

To illustrate the correlation between impedance methods and classical vibrations let us solve for the impedance of the system of Figure 2a.

$$Z_0 = \Sigma Z = \frac{K}{j\omega} + j\omega m$$

$$V_0 = \frac{F}{Z_0} = \frac{F}{\frac{K}{j\omega} + j\omega m}$$

$$= \frac{Fj\omega/m}{\frac{K}{m} - \omega^2}$$

Letting natural frequency $= \omega_n$ we obtain the familiar form

$$\frac{V}{F} = \frac{1}{Z} = \frac{j\omega/K}{1 - (\omega/\omega_n)^2}$$

As a final illustration consider the following vibration damper system, Figure 3a.

To clarify the series/parallel relationship 3a is redrawn in the form shown in Figure 3b.

When solving for the impedance of this system to a forcing function, F, it is obvious that

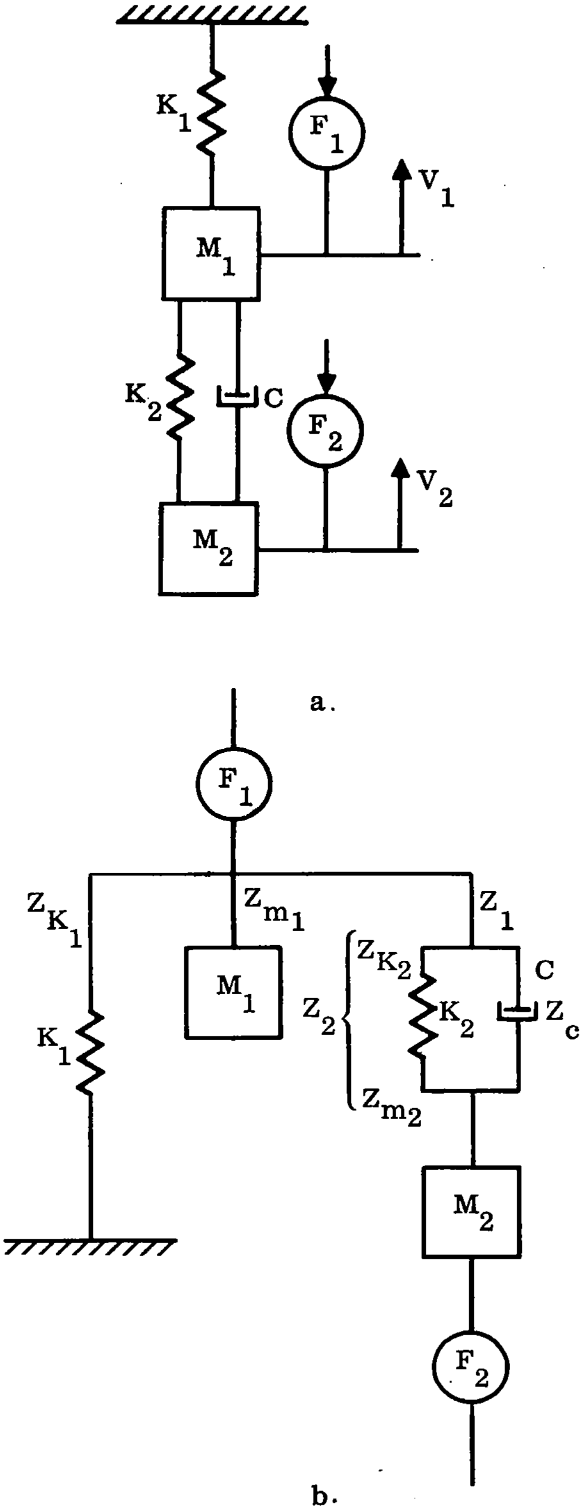


Figure 3. Vibration Damper System

Z_1, Z_m, and Z_{K_1} are in parallel,

Z_{K_2} and Z_c are in parallel, and

Z_2 and Z_{m_2} are in series.

Thus we have

$$Z_2 = Z_{K_2} + Z_c,$$

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_{m_2}}, \text{ and}$$

$$Z_{11} = Z_{K_1} + Z_{m_1} + Z_1$$

which can be used to calculate the system impedance

$$Z_{11} = \frac{F_1}{V_1}$$

If an F_2 is used then to solve for $Z_{22} = \dfrac{F_2}{V_2}$ the following relationship exists.

Z_2 and Z_3 are in series,

Z_{K_2} and Z_c are in parallel,

Z_{m_1} and Z_{K_1} are in parallel, and

Z_{m_2} and Z_1 are in parallel.

Using the preceding relationships we get

$$Z_3 = Z_{K_1} + Z_{m_1}$$

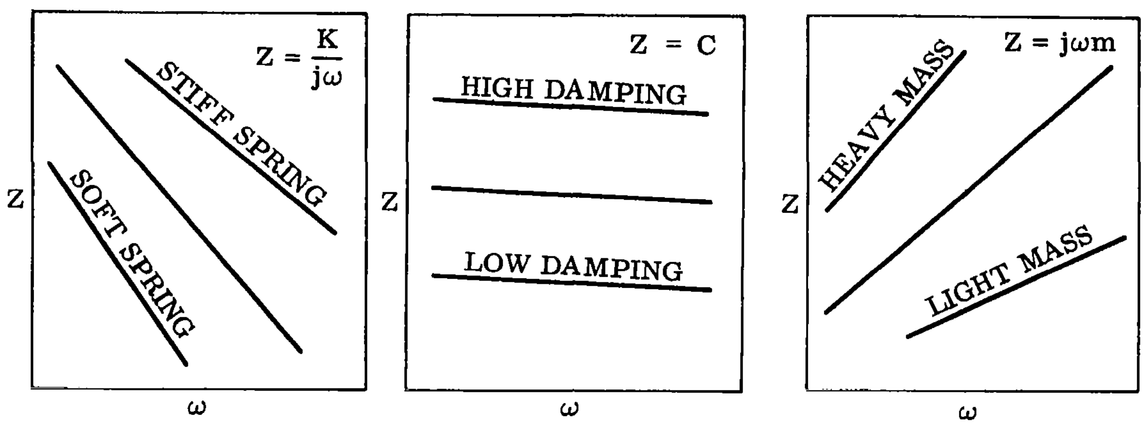
$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$Z_{22} = Z_1 + Z_{m_2}$$

which gives us the effective impedance of the system at M_2 for a forcing function, F_2.

4.1.2 TYPICAL FORMS OF THE IMPEDANCE PLOT. In order to interpret or duplicate analytically the impedance observed from tests it is necessary to have some idea as to the types of responses that various elements and systems yield.

The following is a log-log representation of the responses from the three basic elements: spring, damper, and mass.

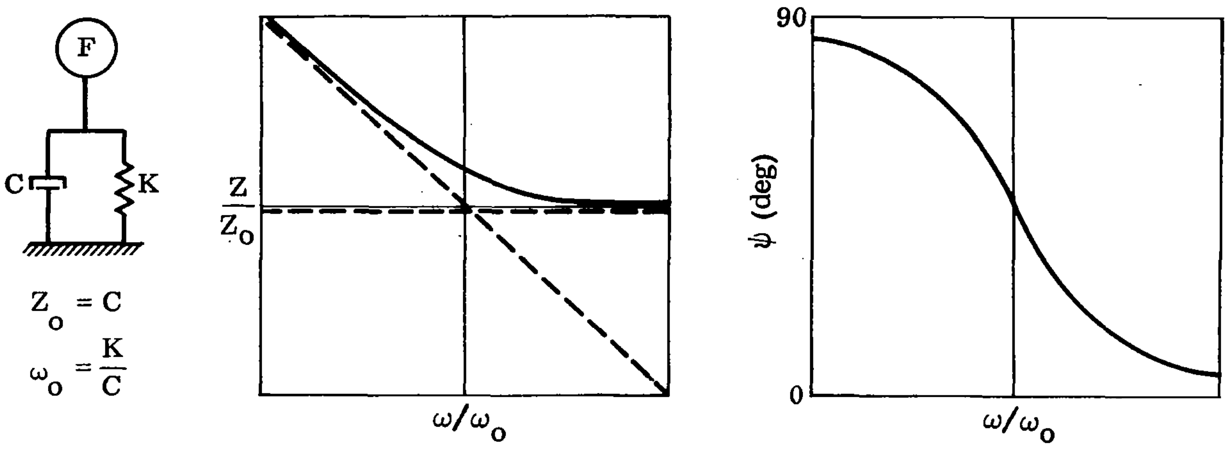


a. Spring b. Damper c. Mass

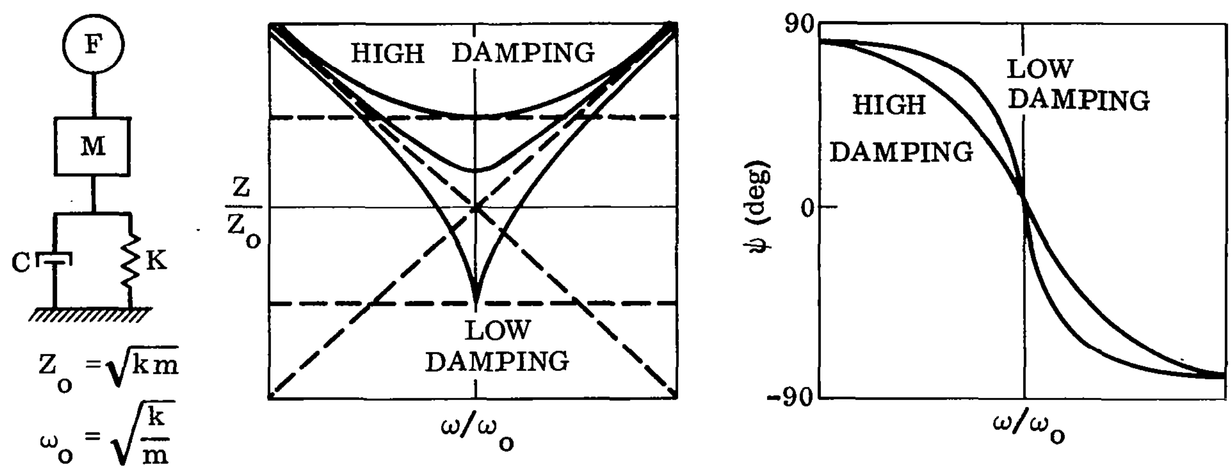
Figure 4. Log-Log Representation of Three Basic Responses

Combining these three plots can be done on an additive process similar to that used for construction of Bode plots in servomechanism synthesis. A representation of two basic combinations is shown in Figure 5.

A plot of the impedance of a typical system of the type shown in Figure 1 is given in Figure 6. In addition, two more plots are presented. First, a plot for a system of spring masses in parallel is shown in Figure 7. This is the plot of a typical system in bending and is described by the typical modal solution. The second, from a system of spring masses in series, is shown in Figure 8. This is representative of the torsional vibration of a shaft; however, it may be encountered in other systems. Figure 6 and 7 are of the so-called terminated type, i.e., there is what appears to be a spring restraint to forces applied to the system. This can be readily seen by observing the system behavior at low frequencies. The spring restraint value is readily read off the curve. Figure 8 is of a free-free vibration. This can be seen by noting that the curve trend is to follow a constant mass line at all frequencies.



a



b

Figure 5. Plot Combinations

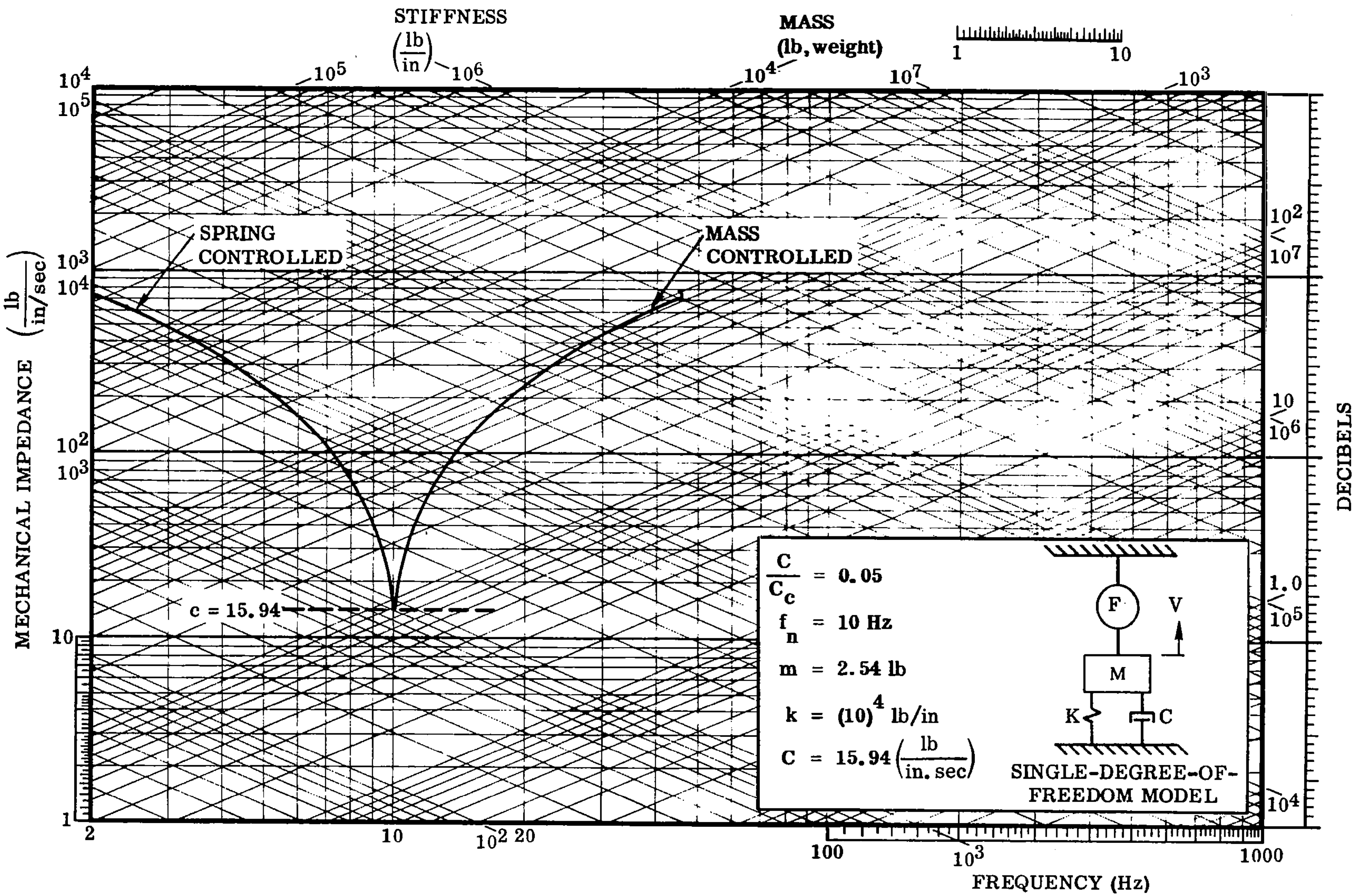
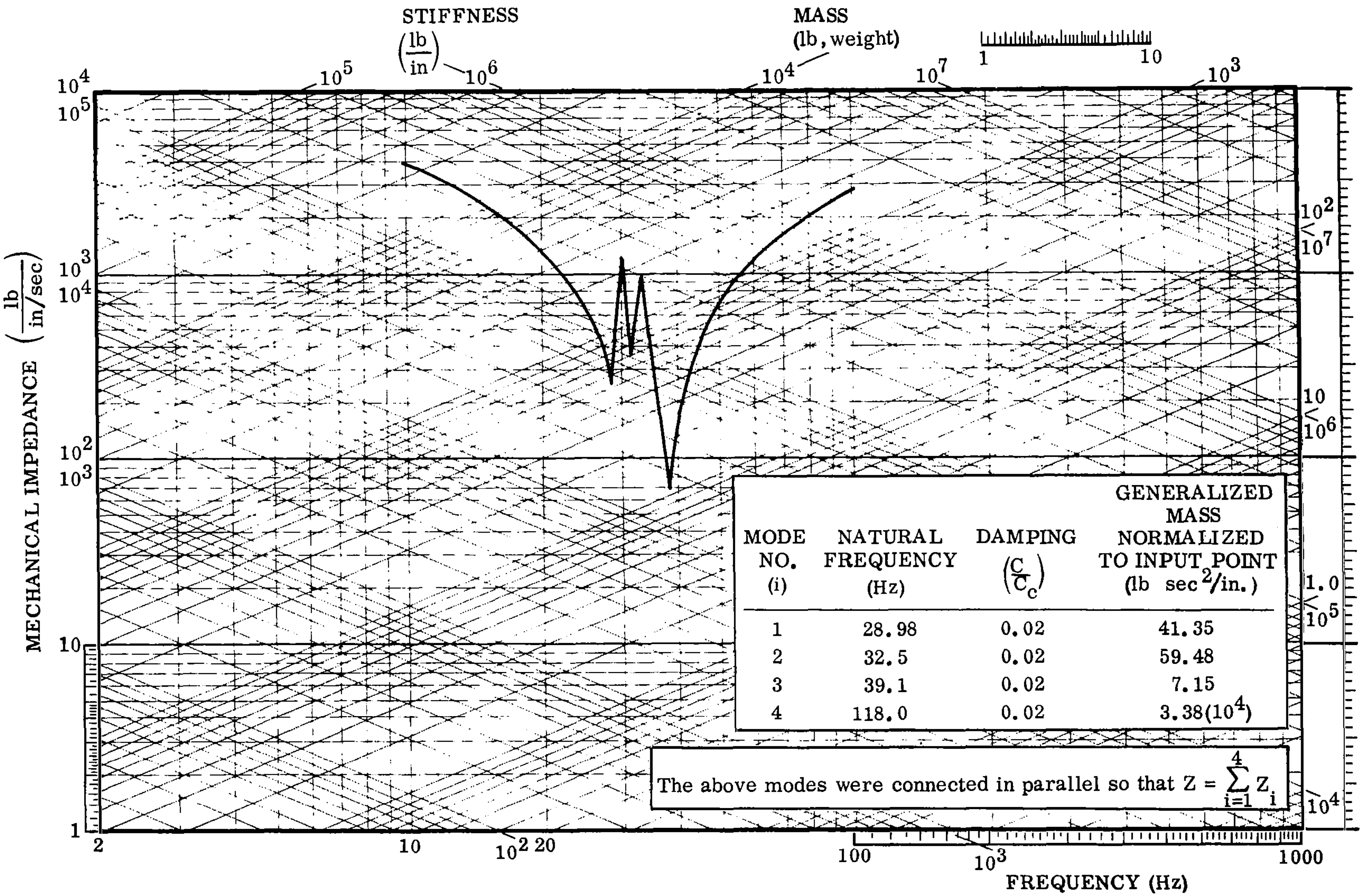


Figure 6. Mechanical Impedance of Single-Degree-of-Freedom System



MODE NO. (i)	NATURAL FREQUENCY (Hz)	DAMPING $\left(\frac{C}{C_c}\right)$	GENERALIZED MASS NORMALIZED TO INPUT POINT (lb sec^2/in.)
1	28.98	0.02	41.35
2	32.5	0.02	59.48
3	39.1	0.02	7.15
4	118.0	0.02	3.38(10^4)

Figure 7. Mechanical Impedance of Multiple-Degree-of-Freedom System

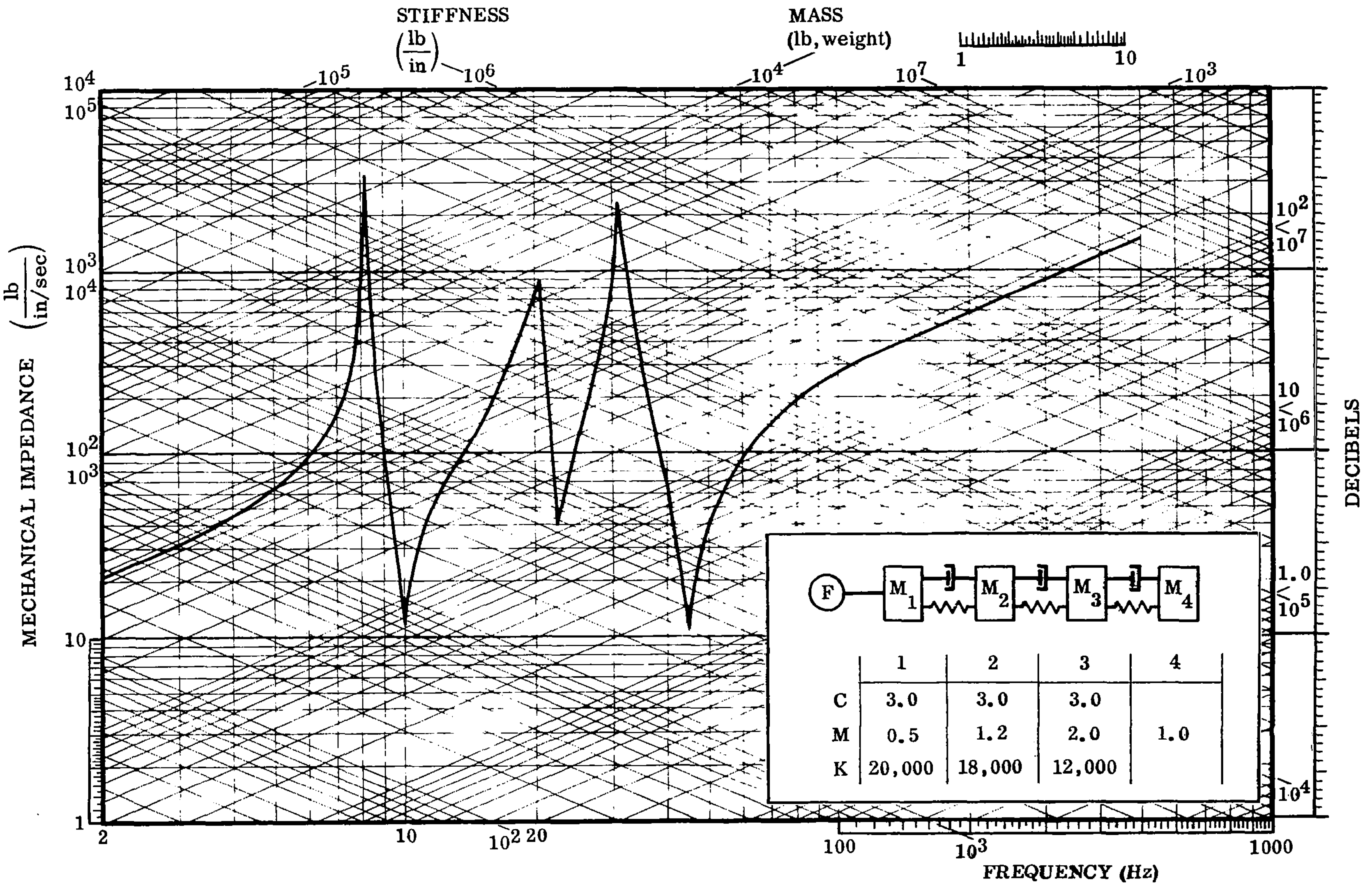


Figure 8. Mechanical Impedance of Series Arrangement of Spring Mass Systems

References 1 and 4 present a number of solutions for simple systems. For more complicated systems, analog or digital computer solution may be required.

4.2 DETERMINATION OF MECHANICAL IMPEDANCE BY TEST

4.2.1 GENERAL TEST CONFIGURATION. The test configuration may be similar to the test setup that is ordinarily used for determining spring rates statically. The main differences are that dynamic forces and motions are applied and measured respectively in place of static forces and displacements. A typical set of measurements for determination of impedance for engine gimbal analysis is shown in Figure 9.

The following paragraphs outline the force and motion requirements, frequency response requirements, and the instrumentation requirements. If applicable, the resonant response of the test stand and the resonant responses of the shaker plus suspension system should be determined. These responses may affect the interpretation of the structural response.

Most of the requirements are usually based on using a steady-state sinusoidal excitation. However, a transient or an impulsive excitation may be used also (Reference 4, Ch. 10). The transient method involves an evaluation of the Fourier frequency spectrum; therefore, very accurate data must be obtained for this method.

4.2.2 FORCE AND MOTION REQUIREMENTS. The dynamic forces may be produced by vibration shakers or by gimbaling the engine and engine actuators. Usually a steady-state sinusoidal force or motion that can be varied from less than 1 to 50 Hz is used. The maximum amplitude of the force is dependent on either the actual flight-control loads, or the test loads may be tolerated. A low force should be used during preliminary frequency sweeps to verify that the test is being controlled properly. When the large forces are applied, several force amplitudes should be used to determine the linearity of the structural response. When the steady-state vibratory force is swept through the frequency range, either the amplitude of the input force or the amplitude of the input motion (usually velocity or acceleration) can be used for controlling the structural response. By measuring the force and the velocity (or acceleration) at a point on the structure the point mechanical impedance can be determined. Transfer impedances can also be determined to aid in describing the vibratory response. Reference 1 presents a thorough discussion of point and transfer impedance.

4.2.3 INSTRUMENTATION REQUIREMENTS. The instrumentation requirements are dependent on the following factors:

a. The magnitude of the forces and motions

b. The dynamic range of the measured quantity

c. Adequate signal-to-noise ratio

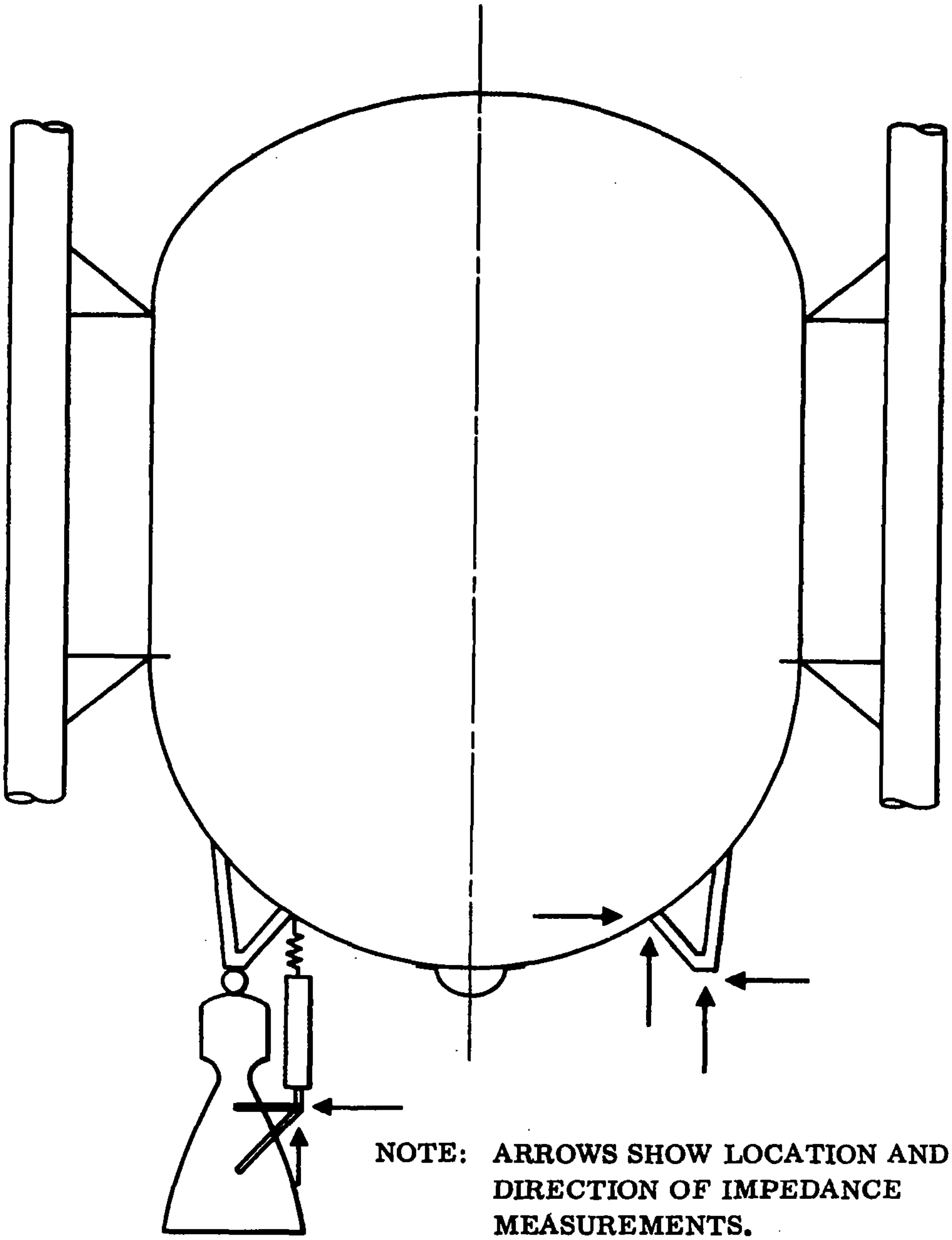


Figure 9. Typical Thrust Section Test Configuration

d. Operating frequency range

e. Required accuracy of phase angle measurements

f. Data reduction requirements (see Section 3.1.c)

The first factor aids in determining the types of transducers that are required. An impedance head that consists of a force transducer and an accelerometer may be used. Reference 2 presents a complete discussion on the development of impedance heads. Factors that should be considered are: resonant frequency of accelerometer, transverse sensitivity of accelerometer, rocking sensitivity, bending strain sensitivity, linearity of accelerometer, and stiffness characteristics of the force gage.

The dynamic range of the measured quantity can be a stringent requirement. The mechanical impedance of a system may vary by 3 orders of magnitude (60 db) depending on the dynamic properties of the system (mainly at resonances and anti-resonances).

The signal-to-noise ratio is closely associated with the dynamic range of the instrumentation system. When the force or motion becomes small due to a resonance or an anti-resonance, the sensitivity of the transducer should assure a large signal-to-noise ratio. Provisions should be made for being able to select the gain on the instrumentation system during the test.

The frequency range is dependent on the frequency response requirements of the space booster. The range is usually from 0 (static) to less than 50 Hz (twice the bandwidth of the servo system).

The accuracy of the phase angle measurements depends upon the intended use of the impedance data. A phase angle error that is less than 3 degrees is usually adequate.

4.2.4 DATA REDUCTION. The data recording and the data reduction can be accomplished either manually or electronically. For frequencies up to 50 Hz the manual method may be the most practical. This would permit very exacting measurements of the mechanical impedance at low frequencies below resonances where the system is probably spring-controlled and at the system resonances or anti-resonances. Appendix 3.7 of Reference 3 presents a method of manually recording and reducing impedance data.

4.3 MEASURED IMPEDANCE FROM FULL SCALE TESTS

4.3.1 SPRING RATES. During developmental tests of the high specific impulse Centaur upper stage vehicle, the mechanical impedance was measured at the nose fairing hinge points. The spring rate was determined from the measured impedance below resonance. Static spring rates were also determined during separate tests.

This particular structure had the characteristics of a softening spring. A comparison of the spring rates was made when the applied force was in the same range. The dynamic spring rate was 20 percent lower than the static spring rate in compression. Several factors may explain the difference.

a. During the impedance test no static preload was applied on the nose fairing hinge. Vibratory tension and compression loads were applied to the hinge. The discontinuous structural joints may have tended to effectively soften the spring rate.

b. The influence of structural vibration modes may not have been completely accounted for.

During another test of the Centaur vehicle the longitudinal mechanical impedances at the engine gimbal blocks were measured. Calculations of the mechanical impedance were made by using a mathematical spring-lumped mass model of the Centaur vehicle. A comparison of the analytical and experimental impedances was made to determine if the mathematical model could provide a basic description of the elastic properties of the vehicle below 50 Hz.

The most apparent difficulties were the following:

a. The damping in some of the analytical modes differed from the experimentally determined damping.

b. Some of the higher frequency modes were not included in the mathematical model.

These difficulties can be expected to recur in all tests of this nature; therefore, exact matches between analytical and experimental impedance cannot usually be obtained.

The experimental equipment and techniques have improved rapidly in the past several years. Today, different impedance heads (force and motion transducer) are available according to different force ranges. Also, automatic data reduction equipment is available for plotting mechanical impedances.

4.3.2 DISCUSSION OF PROBLEM AREAS. The instrumentation and electronic data reduction system is probably the largest problem area. Some of the problems may have originated elsewhere (such as vibration shaker harmonics). These effects add to the complexity of analyzing the data.

Electrical noise (60 Hz) presented some problems. This noise reduced the signal-to-noise ratio. As a result, more electronic filtering of the data was required. In severe cases, data were lost due to excessive moisture on the test specimen and instrumentation. The moisture provided low-resistance electrical paths for stray electrical currents and increased the noise floor. Phase shifts of the reduced data can occur due to incompatible filters within the data reduction system. A complete calibration and checkout of the data reduction system with known electrical signals should be performed before the experimental data are reduced. In order to do this the data should be recorded on magnetic tape so that almost no limitation on data processing is imposed.

4.4 USE OF IMPEDANCE IN STABILITY AND CONTROL ANALYSIS

4.4.1 REQUIREMENT FOR INCLUSION IN STABILITY AND CONTROL ANALYSIS. The need for an accurate model of the mechanical properties of a system is always present. It is not possible to adequately describe equipment attach points or small masses by conventional analytical means (modal simulations). These items must normally be considered as additional degrees of freedom in the system.

Normal practice is to model the elastic properties of a structure by means of normal or elastically uncoupled modes, Reference 7. This method gives an excellent description of the overall vehicle in the range of frequencies within the bandwidth of the usual flight control system. The practice does have one shortcoming which the use of impedance techniques can help to overcome.

This occurs when components having small mass in relation to the mass of the overall vehicle have to be considered. The motion of these components cannot be adequately described by modal analysis. This is caused partly by the effect of damping, not considered in modal solutions, and partly by the inability of present numerical programs to handle the wide disparity of numerical values encountered. Typical examples of this phenomena are gyro mounts, guidance platforms, actuator attach points, etc. A more detailed explanation of this problem can be found in Reference 8.

To handle such items analytically they must be considered as additional degree-of-freedom systems added to the basic modal solution. Even though these systems cannot be considered part of the basic modes it is not always possible to correctly separate them analytically from the rest of the structure. This is where impedance methods can be invaluable in flight control system analysis and synthesis.

The effect of damping within these attached masses, such as a hydraulic actuator, friction, or an active gimbal system for a stabilized platform could result in different values for each frequency and amplitude. These values usually cannot be analytically determined to sufficient accuracy but must be determined by impedance testing. If it is expected that these values will be impedance tested later in the program, then it would be expedient to perform the calculations using impedance methods.

The system simulation for stability and control analysis can be performed in several ways. The three general methods to be considered are:

a. The use of an effective spring/mass

b. The simulation of an equivalent system

c. The direct inclusion of an amplitude/frequency plot into frequency response studies.

These three methods will be described and possible areas of applicability noted.

4.4.2 DETERMINATION OF EQUIVALENT SPRING OR MASS. For a known flight control system frequency (ω) the impedance can be used to generate directly a spring rate for use in stability studies. Thus when the mechanical impedance (Z) is known, then the constant (K) can be determined by multiplying Z by ω (see Section 4.1.1). If the flight control system frequency is not known then the K and frequency will have to be converged on by iterative means. The use of the exact frequency is very important in the high frequency range (above 3 Hz in Figure 2 or 15 Hz in Figure 3). For frequencies well above resonance (20 Hz in Figure 2 or 60 Hz in Figure 3) it is often desirable to use a mass rather than a spring for the impedance. This mass is obtained by dividing Z by ω. Determination of an approximation of a multiple-degree-of-freedom system including damping is a more complicated process. This is discussed in Section 4.4.3.

The application of equivalent mass is basically self explanatory. When the system dynamics equations are written, the equations for the components to be simulated are written either as a single spring or single mass. The choice depends upon the characteristics of the components. Depending upon the expected frequency, it may look like either a mass or a spring (see Figure 6). This scheme is not usually employed where the characteristics change rapidly with frequency. It may, however, be used in an iterative loop where the following sequence is used:

a. The frequency and amplitude are assumed and a mass or spring value taken.

b. The equations are solved for frequency and also mode shape when required.

c. This frequency and amplitude are used to determine a revised mass or spring value.

d. Repetition of b. and so on.

The preceding sequence is repeated until the desired agreement between assumed and actual values is obtained.

Using this procedure the variation in modal characteristics due to amplitude may be obtained.

4.4.3 SIMULATION OF EQUIVALENT SYSTEMS. In this approach an attempt is made to duplicate the measured response with analytic functions. Although this is more complicated than the effective mass technique, it yields transfer functions which are good over a wide range of frequency. This greatly increases the validity of subsequent analytical studies as the results can be applied over a wide frequency range.

For equivalent systems the spring mass or combinations of spring mass systems, either in series or in parallel are used. In this section three simple systems are illustrated to show the types of responses which can be achieved by use of these systems (Figures 6, 7, and 8).

The decision to use a series system, Figure 8, or a parallel (modal) system, Figure 7, is usually a matter of preference. The only exception to this occurs when the physical system can best be described by a series of known rigid masses attached such as to have most of the motion occurring outside the mass. In this case a small amount of correction of the calculated spring rates is usually sufficient to model the measured impedance.

In theory this may be done for all systems. In practice some difficulties are encountered. These fall into two general classes which will be noted briefly.

First, duplication of more than two frequencies by an arbitrary system usually becomes a trial and error procedure. This can be quite time consuming for several frequencies. Procedures for obtaining solutions are outlined in Reference 8.

Second, the duplication of several characteristics may lead to a solution which requires negative masses or spring constants. While not aesthetically pleasing, this is not wrong per se. It is merely an indication that coupling exists within the system that is not duplicated in the model.

As a third alternative, the amplitude and phase may be substituted directly into the system when frequency response techniques are employed.

4.4.4 RECOMMENDATIONS. The use of impedance techniques is normally limited to an equivalent spring or mass. This method yields simulations which may easily be checked for accuracy. The methods of incorporation are quite straightforward. The direct simulation of a complicated system using only impedance data, no a priori knowledge of the model, is normally not attempted. The use of impedance measurements to correct calculated modes should be employed as soon as test specimens become available. The modes should be corrected as more complete or representative models become available. Development of mathematical models for the actual flight conditions must be carried out concurrently with development of the launch vehicle. This effort, coupled with tests to verify models used, will employ techniques similar to those given in References 8 and 9.

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