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WEAK LOCALLY HOMOGENEOUS TURBULENCE AND HEAT TRANSFER WITH COMBINED TWO-DIMENSIONAL SHEAR AND NORMAL STRAIN

by Robert G. Deissler

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EXPERIMENTAL HEAT-TRANSFER INVESTIGATION OF NONWETTING, CONDENSING MERCURY FLOW IN HORIZONTAL, SODIUM-POTASSIUM-COOLED TUBES

By Roy A. Lottig, Richard W. Vernon, and William D. Kenney

May 1967

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SUMMARY

An analysis of the effects of shear and normal strain on weak turbulence and turbulent heat transfer is made using two-point correlation equations obtained from the Navier-Stokes and energy equations. The correlation equations are converted to spectral form by taking their Fourier transforms. The resulting first-order partial differential equations are then reduced to an equivalent set of ordinary differential equations. The normal strain is taken as two-dimensional, uniform, and incompressible, with a contraction occuring in the same transverse direction as the uniform shear and temperature gradients. The turbulence is assumed to be initially isotropic but becomes anisotropic in the presence of the shear and normal strain.

INTRODUCTION

The effects of uniform shear and of normal strain on weak turbulence have been analyzed separately in references 1 to 4. There are important cases, however, where shear and normal strain act simultaneously, as in the boundary layer of fluid flowing through a contraction. Effects which are absent when one or the other types of strain acts by itself may be present when they act simultaneously. For instance, an apparent laminarization seems to occur in the boundary layers of certain accelerating flows (refs. 5 to 7).

A complete solution of the turbulent boundary layer in accelerating flows (or of <u>any</u> turbulent boundary layer) from first principles appears to be beyond the capabilities of our present methods of analysis. In this work, a simplified model is analyzed in an attempt to obtain some understanding of the effects of combined shear and normal strain on turbulence and turbulent heat transfer. Uniform shear and uniform normal velocity gradients, as well as a uniform transverse temperature gradient, are assumed to be acting on a field of initially isotropic turbulence. The turbulence quickly becomes anisotropic under the influence of the mean gradients. The turbulent field, although homogeneous in the transverse directions, is assumed to be only locally homogeneous in the longitudinal or flow direction; that is, the effects of changes in the intensity of the turbulence over a correlation or mixing length in the longitudinal direction are negligible. The normal strains in the present model correspond to a two-dimensional contraction with the transverse normal strains occuring in the same direction as the transverse shear and temperature gradients.

A system of correlation equations can be constructed by writing the incompressible Navier-Stokes and energy equations at two points in the turbulent fluid. The system is made determinate by assuming that the turbulence is weak enough for terms containing triple correlations to be negligible compared with the other terms in the equations. Although that assumption may limit the highest Reynolds number for which the analysis is valid, the analysis is of interest in that it gives an asymptotically exact solution for turbulence in the limit of low Reynolds numbers. Moreover, when mean velocity and temperature gradient terms are present in the equations, the tubulence may not have to be as weak for the triple correlation terms to be negligible compared with other terms as it would if the terms were not present.

The correlation and spectral equations used in the analysis will be considered in the next section.

BASIC EQUATIONS

Two-point steady-state correlation equations for weak locally homogeneous turbulence are obtained in references 3 and 4 and are given by

$$\overline{u_k u_j'} \frac{\partial U_i}{\partial x_k} + \overline{u_i u_k'} \frac{\partial U_j}{\partial x_k} + \frac{\partial U_k}{\partial x_l} r_l \frac{\partial}{\partial r_k} \overline{u_i u_j'} + U_k \frac{\partial}{\partial x_k} \overline{u_i u_j'} = -\frac{1}{\rho}\left(\frac{\partial}{\partial r_j} \overline{u_i p'} - \frac{\partial}{\partial r_i} \overline{p u_j'}\right) + 2\nu \frac{\partial^2 \overline{u_i u_j'}}{\partial r_k \, \partial r_k} \tag{1}$$

$$\frac{1}{\rho} \frac{\partial^2 \overline{u_i p'}}{\partial r_j \, \partial r_j} = -2 \frac{\partial U_j}{\partial x_j} \frac{\partial \overline{u_i u_k'}}{\partial r_j} \tag{2}$$

$$\frac{1}{\rho} \frac{\partial^2 \overline{pu_j'}}{\partial r_i \, \partial r_i} = 2 \frac{\partial U_i}{\partial x_k} \frac{\partial \overline{u_k u_j'}}{\partial r_i} \tag{3}$$

$$\overline{\tau u_k'} \frac{\partial U_j}{\partial x_k} + U_k \frac{\partial}{\partial x_k} \overline{\tau u_j'} + r_l \frac{\partial U_k}{\partial x_l} \frac{\partial}{\partial r_k} \overline{\tau u_j'} + \overline{u_k u_j'} \frac{\partial T}{\partial x_k} = -\frac{1}{\rho} \frac{\partial}{\partial r_i} \overline{\tau p'} + (\nu + \alpha) \frac{\partial^2 \overline{\tau u_j'}}{\partial r_k \, \partial r_k} \tag{4}$$

$$\frac{1}{\rho} \frac{\partial^2 \overline{\tau p'}}{\partial r_j \, \partial r_j} = -2 \frac{\partial U_j}{\partial x_k} \frac{\partial \overline{\tau u_k'}}{\partial r_j} \tag{5}$$

where u_i and u_j' are fluctuating velocity components at the arbitrary points P and P', U_i is a mean velocity component, x_i is a space coordinate, r_i is a component of the vector extending from a point P to P', t is the time, ρ is the density, ν is the kinematic viscosity, p is the instantaneous pressure, and τ is the temperature fluctuation. Bars over terms designate correlations or averaged quantities. The subscripts can take on the values 1, 2, or 3, and a repeated subscript in a term indicates a summation.

In obtaining equations (1) to (5), the instantaneous velocities and temperatures in the incompressible Navier-Stokes and energy equations were first broken into mean and fluctuating components. The resulting equations were then written at two points in the turbulent field, multiplied by appropriate temperatures or velocity components and averaged. The equations for correlations involving pressures were obtained by taking the divergence of the Navier-Stokes equation and applying continuity. In order to make the locally-homogeneous approximation, the turbulence was considered homogeneous over a correlation length, or the scale of the inhomogeneity was much greater than the scale of the turbulence. Thus, $\partial/\partial x_i << \partial/\partial r_i$, where the operators operate on two-point correlations. (A calculation for axially decaying turbulence without mean velocity gradients (ref. 2, fig. 3) implies that this is a good approximation except in the region very close to the virtual origin of the turbulence.) Also, for locally homogeneous turbulence, the mean velocity and mean temperature gradients could be considered to vary linearly over distances for which the correlations are appreciable. Finally, in order to make the set of equations determinate, the turbulence was assumed to be weak enough to neglect terms containing triple correlations. The turbulence in a flow with large velocity or temperature gradients may not have to be as weak as that in a flow without mean gradients. The terms containing those gradients may be large compared with triple correlation terms, even if the turbulence is moderately strong.

Equations (1) to (5) can be converted to spectral form by introducing the usual three-dimensional Fourier transforms defined as follows:

$$\overline{u_i u_j'} = \int_{-\infty}^{\infty} \varphi_{ij} e^{i\vec{\kappa}\cdot\vec{r}}\, d\vec{\kappa} \tag{6}$$

$$\overline{p u_j'} = \int_{-\infty}^{\infty} \lambda_j e^{i\vec{\kappa}\cdot\vec{r}}\, d\vec{\kappa} \tag{7}$$

$$\overline{u_i p'} = \int_{-\infty}^{\infty} \lambda_i' e^{i\vec{\kappa}\cdot\vec{r}}\, d\vec{\kappa} \tag{8}$$

$$\overline{\tau u_j'} = \int_{-\infty}^{\infty} \gamma_j e^{i\vec{\kappa}\cdot\vec{r}}\, d\vec{\kappa} \tag{9}$$

$$\overline{\tau p'} = \int_{-\infty}^{\infty} \zeta' e^{i\vec{\kappa}\cdot\vec{r}}\, d\vec{\kappa} \tag{10}$$

where $\vec{\kappa}$ is a wave number vector having the dimension 1/length and $d\vec{\kappa} = d\kappa_1\, d\kappa_2\, d\kappa_3$. Taking the Fourier transforms of equations (1) to (5), eliminating the pressure-velocity and pressure-temperature terms, and using continuity result in (see ref. 3)

$$U_k \frac{\partial}{\partial x_k} \varphi_{ij} = \frac{\partial U_l}{\partial x_k}\left[\left(2\frac{\kappa_l \kappa_j}{\kappa^2} - \delta_{jl}\right)\varphi_{ik} + \left(2\frac{\kappa_l \kappa_i}{\kappa^2} - \delta_{il}\right)\varphi_{kj} + \kappa_l \frac{\partial \varphi_{ij}}{\partial \kappa_k}\right] - 2\nu\kappa^2 \varphi_{ij} \tag{11}$$

$$U_k \frac{\partial \gamma_j}{\partial x_k} = \frac{\partial U_l}{\partial x_k}\left[\left(2\frac{\kappa_l \kappa_j}{\kappa^2} - \delta_{jl}\right)\gamma_k + \kappa_l \frac{\partial \gamma_j}{\partial \kappa_k}\right] - \frac{\partial T}{\partial x_k}\varphi_{kj} - (\alpha + \nu)\kappa^2 \gamma_j \tag{12}$$

where δ_{ij} is the Kronecker delta.

Equations (11) and (12) give contributions of various processes to the rates of change (with x_k) of spectral components of the turbulent energy tensor $\overline{u_i u_j}$ and of the turbulent heat transfer vector $\overline{\tau u_j}$. The terms in the equations which are proportional to $\partial/\partial\kappa_k$ are transfer terms which transfer activity into or out of a spectral component by the stretching or compressing of turbulent vortex filaments by the mean velocity gradient, as discussed in references 2 to 4. The terms with κ^2 in the denominator are spectral components of pressure-velocity or pressure-temperature correlations and transfer activity between directional components (ref. 2). The last terms in the equations are dissipation terms, which dissipate activity by viscous or by conduction effects. The dissipation term in equation (12) contains both viscous and conduction effects because it dissipates spectral components of velocity-temperature correlations. The remaining terms in the equations produce energy or activity by mean velocity or temperature gradient effects.

For the present model, a two-dimensional contraction with the through-flow in the x_1-direction and the contraction in the x_3-direction is considered. The shear and temperature gradients also occur in the x_3-direction. Thus, the mean gradients present in the flow are $\partial U_1/\partial x_1$, $\partial U_3/\partial x_3$, $\partial U_1/\partial x_3$, and $\partial T/\partial x_3$. These gradients are all taken to be independent of position. By continuity of the mean flow,

$$\frac{\partial U_1}{\partial x_1} = -\frac{\partial U_3}{\partial x_3} \equiv s \tag{13}$$

Similarly, set

$$\frac{\partial U_1}{\partial x_3} \equiv a \tag{14}$$

and

$$\frac{\partial T}{\partial x_3} \equiv b \tag{15}$$

In addition, it is assumed that the turbulence is homogeneous in the transverse directions and that it changes only in the longitudinal or x_1 direction, so that

$$U_k \frac{\partial}{\partial x_k} = U_1 \frac{\partial}{\partial x_1} \tag{16}$$

where the operators operate on the correlations or their Fourier transforms. For the model considered, then, equations (11) and (12) can be written as

$$U_1 \frac{\partial \varphi_{ij}}{\partial x_1} = a\left[\left(2\frac{\kappa_1\kappa_j}{\kappa^2} - \delta_{j1}\right)\varphi_{i3} + \left(2\frac{\kappa_1\kappa_i}{\kappa^2} - \delta_{i1}\right)\varphi_{3j} + \kappa_1 \frac{\partial \varphi_{ij}}{\partial \kappa_3}\right]$$

$$+ s\left[\left(2\frac{\kappa_1\kappa_j}{\kappa^2} - \delta_{j1}\right)\varphi_{i1} + \left(2\frac{\kappa_1\kappa_i}{\kappa^2} - \delta_{i1}\right)\varphi_{1j} - \left(2\frac{\kappa_3\kappa_j}{\kappa^2} - \delta_{j3}\right)\varphi_{i3}\right.$$

$$\left. - \left(2\frac{\kappa_3\kappa_i}{\kappa^2} - \delta_{i3}\right)\varphi_{3j} + \kappa_1\frac{\partial \varphi_{ij}}{\partial \kappa_1} - \kappa_3 \frac{\partial \varphi_{ij}}{\partial \kappa_3}\right] - 2\nu\kappa^2\varphi_{ij} \tag{17}$$

and

$$U_1 \frac{\partial \gamma_j}{\partial x_1} = a\left[\left(2\frac{\kappa_1\kappa_j}{\kappa^2} - \delta_{j1}\right)\gamma_3 + \kappa_1\frac{\partial \gamma_j}{\partial \kappa_3}\right] + s\left[\left(2\frac{\kappa_1\kappa_j}{\kappa^2} - \delta_{j1}\right)\gamma_1\right.$$

$$\left. - \left(2\frac{\kappa_3\kappa_j}{\kappa^2} - \delta_{j3}\right)\gamma_3 + \kappa_1\frac{\partial \gamma_j}{\partial \kappa_1} - \kappa_3\frac{\partial \gamma_j}{\partial \kappa_3}\right] - b\varphi_{3j} - (\alpha + \nu)\kappa^2\gamma_j \tag{18}$$

In these equations the shear and normal strain terms are separated and written as the first and second bracketed terms on the right sides of the equations.

For solving equations (17) and (18) it is assumed that the turbulence is isotropic at $x_1 = (x_1)_0$. That condition is satisfied by the relation

$$(\varphi_{ij})_0 = \frac{J_0}{12\pi^2}(\kappa^2\delta_{ij} - \kappa_i\kappa_j) \tag{19}$$

where J_0 is a constant that depends on initial conditions (refs. 2 (eq. (43)), 8, and 9). For the initial condition on γ_i $\left(\text{at } x_1 = (x_1)_0\right)$ it is assumed that

$$(\gamma_i)_0 = 0 \tag{20}$$

Thus, if the initial turbulence is produced by flow through a grid, that grid is unheated, and the temperature fluctuations are produced by the interaction of the mean temperature gradient with the turbulence.

SOLUTION OF SPECTRAL EQUATIONS

Equations (17) and (18) are first-order partial differential equations in the three independent variables x_1, κ_1, and κ_3. In solving the equations, it is convenient to introduce the velocity ratio c which, for a uniform normal strain,' is

$$c \equiv \frac{U_1}{(U_1)_0} = 1 + \frac{x_1 - (x_1)_0}{(U_1)_0} s = 1 + s^* \tag{21}$$

Then,

$$U_1 \frac{\partial}{\partial x_1} = sc \frac{\partial}{\partial c} \tag{22}$$

In order to reduce equations (17) and (18) to ordinary differential equations, the running variables ξ_1, ξ_3, and η are considered, of which κ_1, κ_3, and c are particular values such that $\xi_1 = \kappa_1$ and $\xi_3 = \kappa_3$ when $\eta = c$. If ξ_1, ξ_3, and η are introduced into the set of equations in place of κ_1, κ_3, and c, the resulting equations will, of course, automatically satisfy the original set.

Equation (17) (and eq. (18) with φ_{ij} replaced by γ_j) will then be of the form

$$-\xi_1 \frac{\partial \varphi_{ij}}{\partial \xi_1} + \eta \frac{\partial \varphi_{ij}}{\partial \eta} + \left(\xi_3 - \frac{a}{s}\xi_1\right) \frac{\partial \varphi_{ij}}{\partial \xi_3} = F(\xi_1, \xi_3, \varphi_{ij}, \kappa_2)$$

To determine under what conditions

$$-\xi_1 \frac{\partial \varphi_{ij}}{\partial \xi_1} + \eta \frac{\partial \varphi_{ij}}{\partial \eta} + \left(\xi_3 - \frac{a}{s}\xi_1\right) \frac{\partial \varphi_{ij}}{\partial \xi_3} = -\xi_1 \frac{d\varphi_{ij}}{d\xi_1} \tag{23}$$

note that φ_{ij} is a function of ξ_1, ξ_3, η, and κ_2, so that

$$-\xi_1 \frac{d\varphi_{ij}}{d\xi_1} = -\xi_1 \frac{\partial \varphi_{ij}}{\partial \xi_1} - \xi_1 \frac{\partial \varphi_{ij}}{\partial \xi_3} \frac{d\xi_3}{d\xi_1} - \xi_1 \frac{\partial \varphi_{ij}}{\partial \eta} \frac{d\eta}{d\xi_1}$$

Comparison of this last equation with equation (23) shows that they are equivalent if

$$-\xi_1 \frac{d\xi_3}{d\xi_1} = \xi_3 - \frac{a}{s}\xi_1$$

and

$$-\xi_1 \frac{d\eta}{d\xi_1} = \eta$$

or

$$\xi_1\xi_3 - \frac{1}{2}\frac{a}{s}\xi_1^2 = (\text{constant})_1 = \kappa_1\kappa_3 - \frac{1}{2}\frac{a}{s}\kappa_1^2 \tag{24}$$

and

$$\eta\xi_1 = (\text{constant})_2 = c\kappa_1 \tag{25}$$

Thus, equation (23) will hold if $\xi_1\xi_3 - (1/2)(a/s)\xi_1^2$ and $\eta\xi_1$ are constant during integration. With the introduction of equations (21) to (25), equations (17) and (18) become ordinary differential equations, components of which are

$$\frac{d\varphi_{11}(\xi_1)}{d\xi_1} = -\frac{2}{\xi_1}\left(2\frac{\xi_1^2}{h^2} - 1 - \frac{\nu h^2}{s}\right)\varphi_{11} - \frac{2}{\xi_1}\left[\frac{a}{s}\left(2\frac{\xi_1^2}{h^2} - 1\right) - 2\frac{\xi_1 g}{h^2}\right]\varphi_{13} \tag{26}$$

$$\frac{d\varphi_{13}(\xi_1)}{d\xi_1} = -2\frac{g}{h^2}\varphi_{11} - \frac{2}{\xi_1}\left[\frac{\xi_1 g}{h^2}\left(\xi_1^2 - g^2 + \frac{a}{s}\xi_1 g\right) - \frac{\nu h^2}{s}\right]\varphi_{13} - \frac{1}{\xi_1}\left[-2\frac{\xi_1 g}{h^2} + \frac{a}{s}\left(2\frac{\xi_1^2}{h^2} - 1\right)\right]\varphi_{33} \tag{27}$$

$$\frac{d\varphi_{33}(\xi_1)}{d\xi_1} = -4\frac{g}{h^2}\varphi_{13} - \frac{2}{\xi_1}\left[-\left(2\frac{g^2}{h^2} - 1\right) + 2\frac{a}{s}\frac{\xi_1 g}{h^2} - \frac{\nu h^2}{s}\right]\varphi_{33} \tag{28}$$

$$\frac{d\varphi_{ii}(\xi_1)}{d\xi_1} = -\frac{2}{\xi_1}(\varphi_{33} - \varphi_{11}) + \frac{2}{\xi_1}\frac{a}{s}\varphi_{13} + \frac{2}{\xi_1}\frac{\nu h^2}{s}\varphi_{ii} \tag{29}$$

$$\frac{d\gamma_1(\xi_1)}{d\xi_1} = \frac{b}{\xi_1 s}\varphi_{13} - \frac{1}{\xi_1}\left[2\frac{\xi_1^2}{h^2} - 1 - (\alpha + \nu)\frac{h^2}{s}\right]\gamma_1 - \frac{1}{\xi_1}\left[\frac{a}{s}\left(2\frac{\xi_1^2}{h^2} - 1\right) - 2\frac{\xi_1 g}{h^2}\right]\gamma_3 \tag{30}$$

$$\frac{d\gamma_3(\xi_1)}{d\xi_1} = \frac{b}{\xi_1 s}\varphi_{33} - 2\frac{g}{h^2}\gamma_1 - \frac{1}{\xi_1}\left[1 - 2\frac{g^2}{h^2} + 2\frac{a}{s}\frac{\xi_1 g}{h^2} - (\alpha + \nu)\frac{h^2}{s}\right]\gamma_3 \tag{31}$$

where

$$g \equiv \frac{1}{\xi_1}\left[\frac{1}{2}\frac{a}{s}\left(\xi_1^2 - \kappa_1^2\right) + \kappa_1\kappa_3\right] \tag{32}$$

and

$$h^2 \equiv \xi_1^2 + \kappa_2^2 + \frac{1}{\xi_1^2}\left[\frac{1}{2}\frac{a}{s}\left(\xi_1^2 - \kappa_1^2\right) + \kappa_1\kappa_3\right]^2 \tag{33}$$

In these equations ξ_3 has been eliminated by equation (24). The first three equations are independent of the remaining ones, but the converse is not true.

In order to apply initial conditions to the set of equations (26) to (31), let $\varphi_{ij}(\xi_1) = \left[\varphi_{ij}(\xi_1)\right]_0$ and $\gamma_i(\xi_1) = \left[\gamma_i(\xi_1)\right]_0$ when $\eta = 1$. These conditions will then automatically satisfy the desired initial conditions that $\varphi_{ij}(\kappa_1) = \left[\varphi_{ij}(\kappa_1)\right]_0$ and $\gamma_i(\kappa_1) = \left[\gamma_i(\kappa_1)\right]_0$ when $c = 1\left[(U_1 = (U_1)_0\right]$ since, by definition, $\xi_1 = \kappa_1$ when $\eta = c$. Equation (25) shows that

$$(\xi_1)_0 = c\kappa_1 \tag{34}$$

Equation (34) gives the value of ξ_1 at which to start the integration for given values of κ_1 and c. In order to satisfy the initial conditions (19) and (20), let

$$
\left.
\begin{aligned}
\varphi_{11}(\xi_1) &= \frac{J_0}{12\pi^2}\left(h^2 - \xi_1^2\right) \\
\varphi_{13}(\xi_1) &= -\frac{J_0}{12\pi^2}\,\xi_1 g \\
\varphi_{33}(\xi_1) &= \frac{J_0}{12\pi^2}\left(h^2 - g^2\right) \\
\varphi_{ii}(\xi_1) &= \frac{J_0}{6\pi^2}\,h^2 \\
\gamma_i &= 0
\end{aligned}
\right\} \text{ when } \xi_1 = (\xi_1)_0
$$

where g and h are again given by equations (32) and (33). The integration of equations (26) to (31) then goes from $(\xi_1)_0$ to $\xi_1 = \kappa_1$. We are mainly interested in the final values of φ_{ij} and γ_i, for which $\xi_1 = \kappa_1$ (and $\xi_2 = \kappa_2$ and $\eta = c$). The quantity ξ_1 can be considered as a dummy variable of integration.

In order to solve equations (26) to (31) numerically, it is convenient to convert them to dimensionless form by introducing the following dimensionless quantities:

$$\kappa_i^* = \left[\frac{\nu(x - x_0)}{U_0}\right]^{1/2} \kappa_i \tag{35}$$

$$\xi_1^* = \left[\frac{\nu(x - x_0)}{U_0}\right]^{1/2} \xi_1 \tag{36}$$

$$\varphi_{ij}^* = \left[\frac{(x - x_0)\nu}{J_0 U_0}\right] \varphi_{ij} \tag{37}$$

$$\gamma_i^* = \left(\frac{\nu}{J_0 b}\right) \gamma_i \tag{38}$$

$$a^* = \left[\frac{(x - x_0)}{U_0}\right] a \tag{39}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha} \tag{40}$$

In addition, spherical coordinates were introduced into the equations by using the transformations

$$\left.\begin{aligned} \kappa_1 &= \kappa \cos \varphi \sin \theta \\ \kappa_2 &= \kappa \sin \varphi \sin \theta \\ \kappa_3 &= \kappa \cos \theta \end{aligned}\right\} \tag{41}$$

The integrations were carried out numerically on a high-speed computer for various fixed values of κ^*, θ, φ, a, and c. Directionally integrated spectrum functions can be obtained from (see refs. 2 and 10)

$$\begin{pmatrix} \Psi_{ij} \\ \Gamma_i \\ \Lambda_{ij} \end{pmatrix} = \int_0^{\pi} \int_0^{2\pi} \begin{pmatrix} \varphi_{ij} \\ \gamma_i \\ \Omega_{ij} \end{pmatrix} \kappa^2 \sin \theta \, d\varphi \, d\theta \tag{42}$$

In this equation, Ω_{ij} is the vorticity spectrum tensor given in reference 11 as

$$\Omega_{ij} = (\delta_{ij}\kappa^2 - \kappa_i\kappa_j)\varphi_{kk} - \kappa^2\varphi_{ij} \tag{43}$$

The spectrum functions given by equations (42) can be integrated over all wave numbers to give

$$\begin{pmatrix} \overline{u_i u_j} \\ \overline{\tau u_i} \\ \overline{\omega_i \omega_j} \end{pmatrix} = \int_0^{\infty} \begin{pmatrix} \Psi_{ij} \\ \Gamma_i \\ \Lambda_{ij} \end{pmatrix} d\kappa \tag{44}$$

Thus, ψ_{ij}, Γ_i, and Λ_{ij} show how contributions to $\overline{u_i u_j}$, $\overline{\tau u_i}$, and $\overline{\omega_i \omega_j}$ are distributed among various wave numbers or eddy sizes. Computed spectra and correlations will be considered in the next section. For the quantities which involve temperature gradients, the curves will be given for a gas with a Prandtl number Pr of 0.7.

RESULTS AND DISCUSSION

Calculated dimensionless energy spectra (spectra of $\overline{u_i u_i^*}$) and $\overline{\tau u_3^*}$ spectra are plotted in figures 1 and 2. The spectra are plotted for several values of the shear parameter and the normal strain parameter which are, respectively, proportional to $\partial U_1/\partial x_3$ and $\partial U_1/\partial x_1$. Both parameters are, in addition, proportional to longitudinal distance, so that increasing longitudinal distance has an effect similar to that of increasing the velocity gradients.

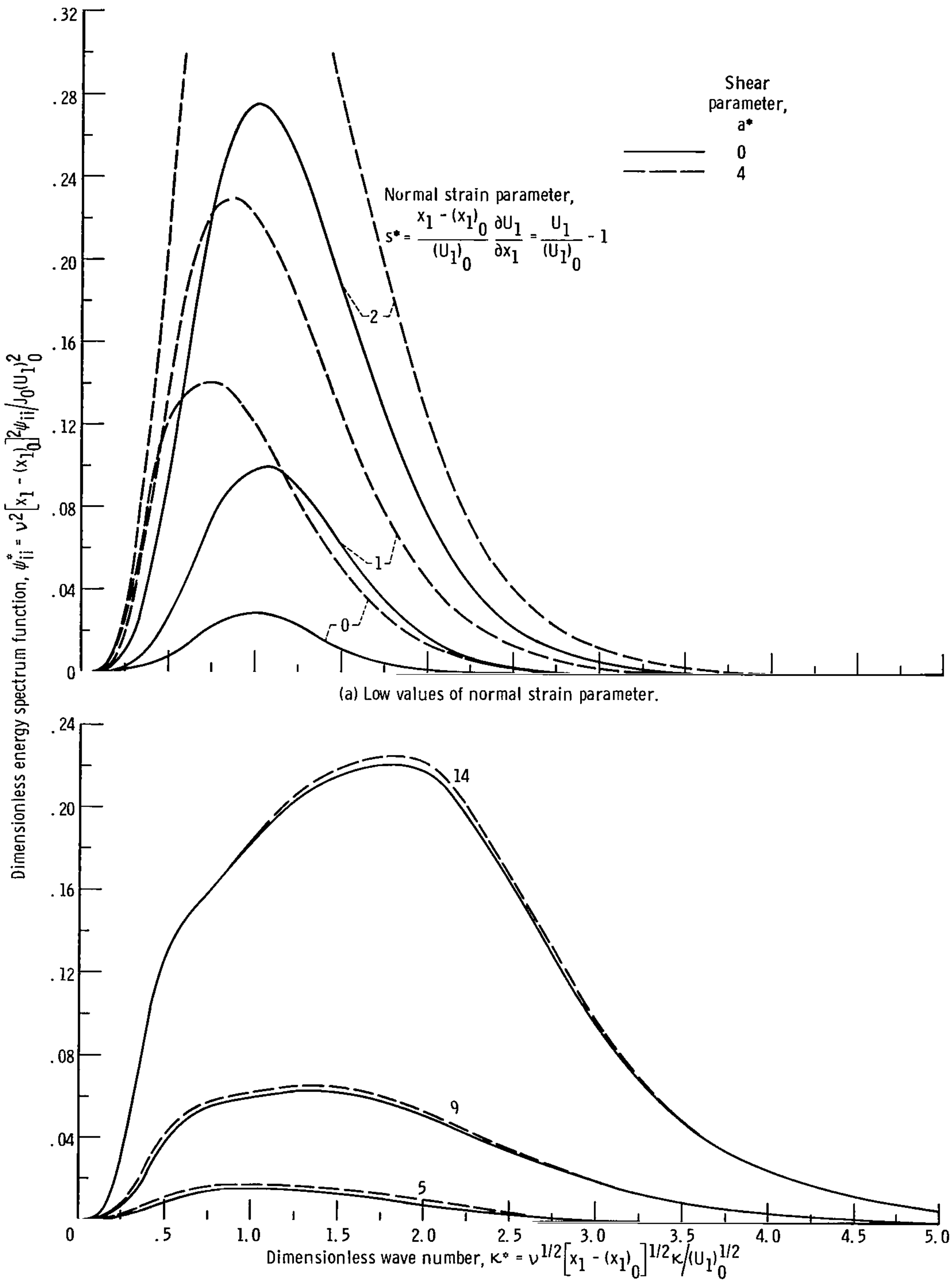
When plotted by using the similarity variables shown in figures 1 and 2, the dimensionless spectra for no shear and normal strain effects ($a^* = s^* = 0$) are the same for all values of x_1, although the turbulence itself decays. Comparison of the various curves indicates how normal strain and shear effects will alter the spectra for a given position and initial mean velocity. If, for instance, a dimensionless spectrum lies above the curve for $a^* = s^* = 0$, the turbulent activity for that case is greater than it would be for no shear or normal strain effects.

The curves in figures 1 and 2 (as well as the succeding ones) are all for positive values of s^* and correspond to an accelerating flow. The curves indicate that, in general, the effects of both shear and normal strain in an accelerating flow are to feed energy or activity into the turbulent field. The effect of shear on the spectra is greater at small values of s^* than at larger ones; that is, it is greater when the ratio a/s is large.

A turbulent velocity-component parameter $(\nu/s)^{5/2}\,\overline{u_i^2}/J_0$, with $i = 1, 2,$ and 3, is plotted against longitudinal velocity ratio in figure 3. This parameter, in contrast to the spectral parameters in figures 1 and 2, does not contain $x_1 - (x_1)_0$, and thus can be used to show how $\overline{u_i^2}$ changes with longitudinal position (or velocity ratio) as well as with shear. Included in the plot is the curve obtained by solving equation (17) with the effects of shear and normal strain absent. This solution gives

$$\left(\frac{\nu}{s}\right)^{5/2}\frac{\overline{u_i^2}}{J_0} = \frac{\ln^{-5/2} c}{48\sqrt{2\pi}} \tag{45}$$

Although the turbulence was taken to be initially isotropic, the results here show the turbulence as already strongly anisotropic from the effects of shear and normal strain. As was the case for the spectra, these results show that $\overline{u_i^2}$ is increased by the shear and



(a) Low values of normal strain parameter.

(b) High values of normal strain parameter.

Figure 1. - Effects of uniform shear and normal strain on spectra of dimensionless turbulent energy.

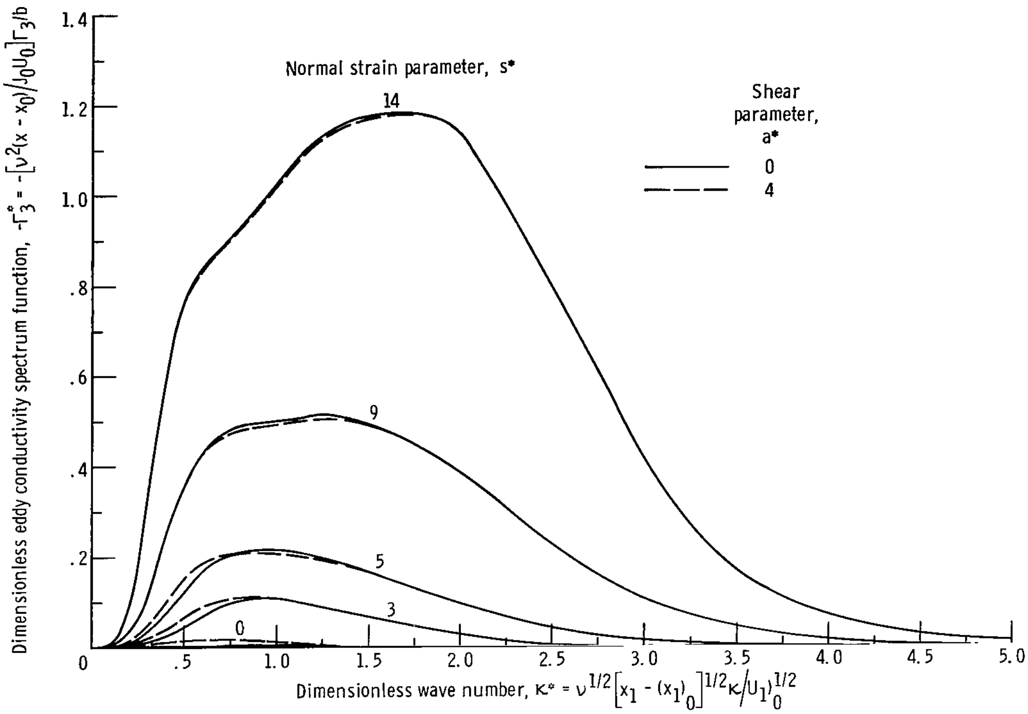


Figure 2. - Effects of uniform shear and normal strain on spectra of dimensionless eddy conductivity; Prandtl number, 0.7.

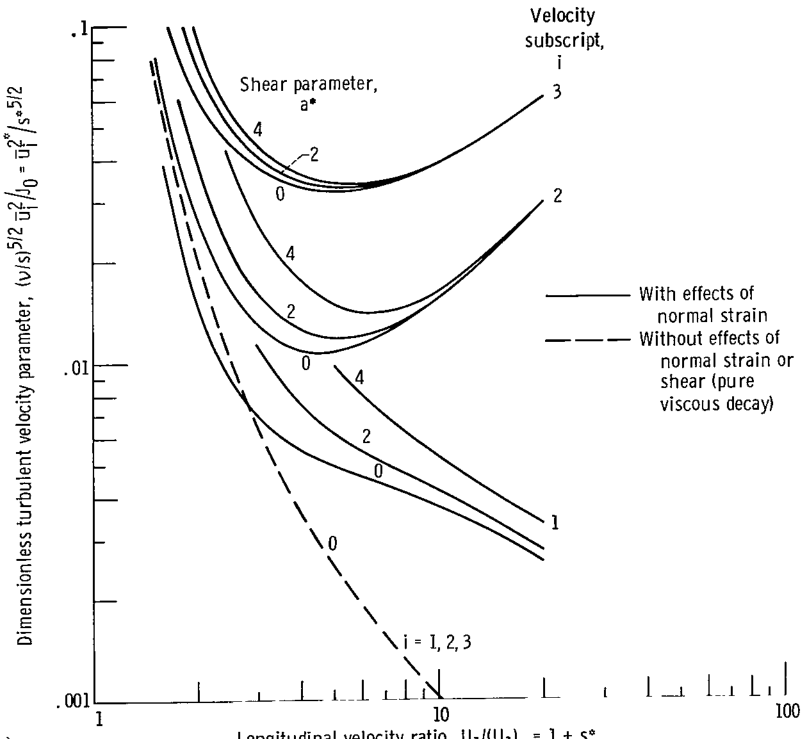


Figure 3. - Effect of uniform shear and normal strain (velocity ratio) on dimensionless variance of turbulent velocity components.

that the effect of shear is greatest at low values of velocity ratio or normal strain parameter. For low values of velocity ratio, all components decay because of the effects of viscosity. In that region the lateral components decrease less rapidly than they would if the effects of normal strain were neglected (compare with dashed curve) and, for $a^* = 0$, the longitudinal component decays more rapidly. At larger velocity ratios, the components in the x_2- and x_3-directions begin to increase as the effects of normal strain offset those of viscosity. The component in the x_1-direction continues to decrease, but at a slower rate than it would if the effects of normal strain were absent. In this way, the curves in figure 3, which are for a two-dimensional contraction, differ from those for the axially symmetric strains in references 1, 3, and 4. For the axially symmetric strains, the longitudinal component decays more rapidly than it would for no effects of strain, whereas, in the present two-dimensional contraction, it decays less rapidly, except at small velocity ratios. Thus, in this case energy is fed into each of the three components of the turbulent energy by normal strain.

In an attempt to understand the trends shown in figure 3, the three components of the dimensionless turbulent vorticity $(\nu/s)^{7/2}\,\overline{\omega_i^2}/J_0$ are plotted against velocity ratio in figure 4 for $a^* = 0$. The dashed curve for no effects of strain was obtained from the equation

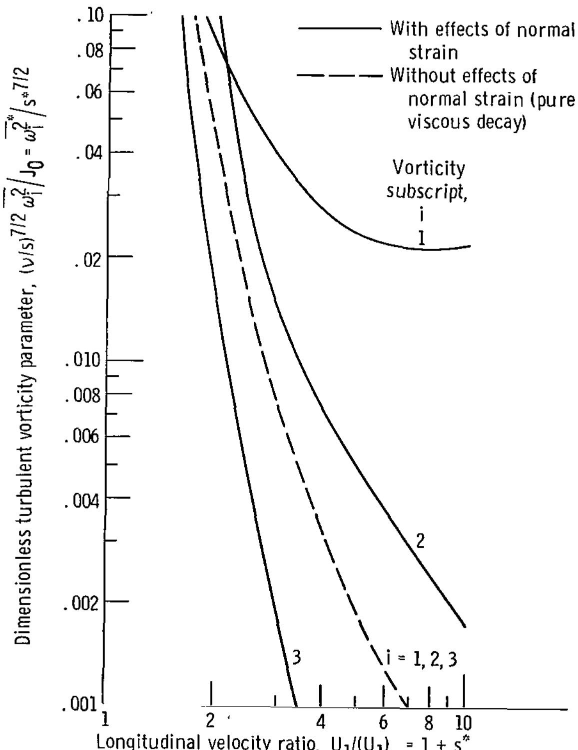


Figure 4. - Effect of uniform normal strain (velocity ratio) on dimensionless variance of turbulent vorticity components; shear parameter, a*, 0.

$$\left(\frac{\nu}{s}\right)^{7/2} \frac{\overline{\omega_i^2}}{J_0} = \frac{5}{192\sqrt{2\pi}} \ln^{-7/2} c$$

The plot shows that the vorticity components in both the x_1- and x_2-directions decay less rapidly than they would if the effects of strain were absent, while the x_3-component decays more rapidly. On the other hand, for the axially symmetric cases considered previously, only the longitudinal component $\overline{\omega_1^2}$ decayed less rapidly (refs. 3 and 4). Thus, although in the axially symmetric case, the turbulent vortex filaments all tended to line up in the longitudinal direction, in the present two-dimensional contraction there is also a tendency (although less pronounced) for an alinement to occur in the x_2-direction (direction of no contraction). These trends are in agreement with the trends for velocity fluctuations shown in figure 3. The velocities associated with a vortex filament will, of course, lie in planes normal to the direction of the filament. Thus, the vortex filaments alined in the longitudinal direction will tend to feed energy into the two lateral velocity components, while those alined in the x_2-direction can give energy to the longitudinal velocity component, as well as to the x_3-component.

It may be of interest to compare the behavior of the components of turbulent energy at large velocity ratios for several types of mean strain. A qualitative comparison is given in the following table:

Type of mean normal strain	Behavior of turbulent energy components at large velocity ratios in accelerating flow
Incompressible axisymmetric strain for flow in a cone (ref. 3)	Lateral components increase with longitudinal distance. Longitudinal component decreases faster than it would without effect of strain.
Uniform incompressible axisymmetric strain (refs. 1 and 4)	Lateral components approach steady state. Longitudinal component decreases faster with distance than it would without effects of strain.
Uniform compressible longitudinal axisymmetric strain (no lateral strain) (ref. 12)	Lateral components decrease less rapidly with distance than they would without effects of strain. Longitudinal component decreases more rapidly.
Uniform incompressible two-dimensional strain (present analysis)	Lateral components increase with longitudinal distance. Longitudinal component decreases, but at a rate slower than it would without effect of strain.

Figure 5 shows the effect of uniform shear and normal strain (velocity ratio) on ratios of the turbulent energy components for accelerating flow. Both $\overline{u_3^2}/\overline{u_1^2}$ and $\overline{u_2^2}/\overline{u_1^2}$ tend to decrease with increasing shear parameter a* and to increase with increasing normal strain parameter s*; that is, the effect of shear is to make $\overline{u_3^2}$ and $\overline{u_2^2}$

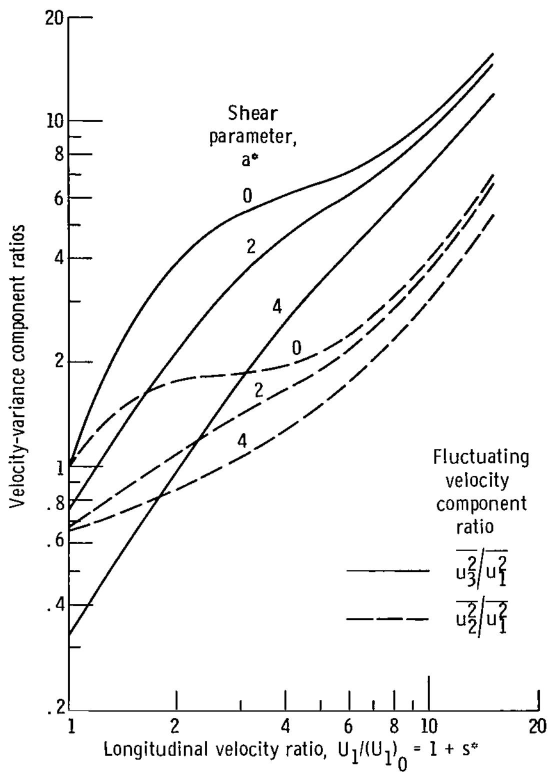


Figure 5. - Effect of uniform shear and normal strain (velocity ratio) on velocity-variance component ratios.

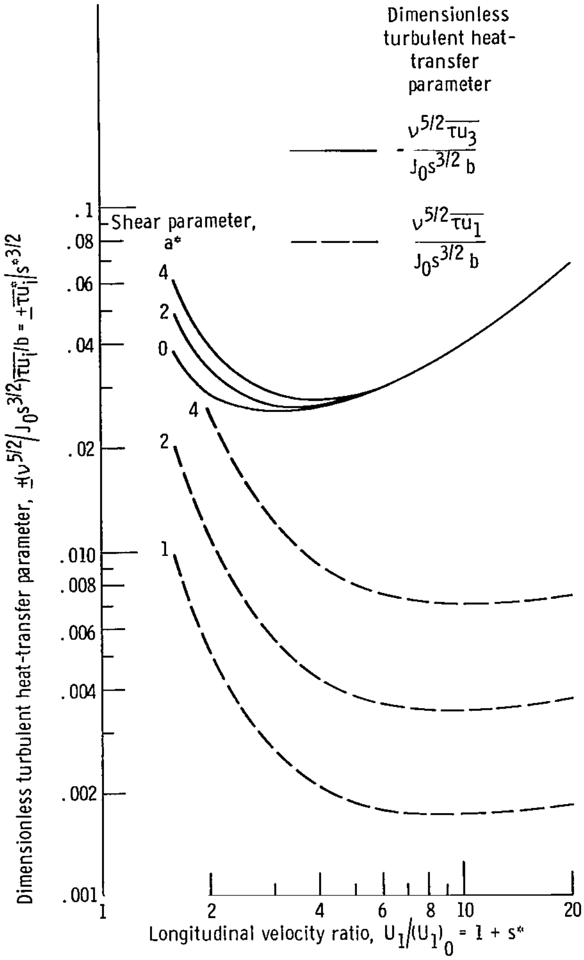


Figure 6. - Effect of uniform shear and normal strain (velocity ratio) on dimensionless temperature-velocity correlations; Prandtl number, 0.7.

less than $\overline{u_1^2}$, and normal strain tends to make those quantities greater than $\overline{u_1^2}$.

Dimensionless turbulent heat-transfer parameters $(\nu^{5/2}/J_0 s^{3/2})\overline{\tau u_i}/b$ with $i = 3$ and 1 are plotted in figure 6 as functions of velocity ratio and shear parameter. The trends shown here are qualitatively similar to those for the dimensionless velocity parameter shown in figure 3. It might seem surprising that there should be turbulent heat transfer in the longitudinal direction x_1, as given by the temperature-velocity correlation $\overline{\tau u_1}$, since there is no temperature gradient in the x_1-direction. However, since there is a correlation between τ and u_3 (because of the temperature gradient dT/dx_3) and a correlation between u_1 and u_3 (because of the velocity gradient dU_1/dx_3), it seems reasonable that there should be a correlation between τ and u_1, and thus a heat transfer in the x_1-direction.

Shear correlation coefficient $-\overline{u_1 u_3}\Big/\left(\overline{u_1^2}\right)^{1/2}\left(\overline{u_3^2}\right)^{1/2}$ is plotted as a function of longitudinal velocity ratio and shear parameter in figure 7. The shear correlation is, of course, zero for zero shear and increases as a^* increases. Except at small velocity ratios and large values of shear parameter, where some increase in correlation with increasing normal strain (velocity ratio) occurs, normal strain tends to destroy the shear correlation.

Figure 8 shows the ratio of eddy conductivity to eddy viscosity plotted as a function of velocity ratio and shear parameter. The eddy conductivity and eddy viscosity are defined by the relations

$$\epsilon_h = -\frac{\overline{\tau u_3}}{\frac{dT}{dx_3}}$$

and

$$\epsilon = -\frac{\overline{u_1 u_3}}{\frac{dU_1}{dx_3}}$$

As the shear parameter a^* increases, the ratio ϵ_h/ϵ increases, reaches a maximum, and then decreases, although the trend is confined to moderately low values of velocity ratio. Results from reference 10 for no normal strain show that ϵ_h/ϵ ultimately approaches 1 as a^* continues to increase. The results in figure 8 indicate that ϵ_h/ϵ reaches a maximum with increasing velocity ratio, as well as with increasing a^*.

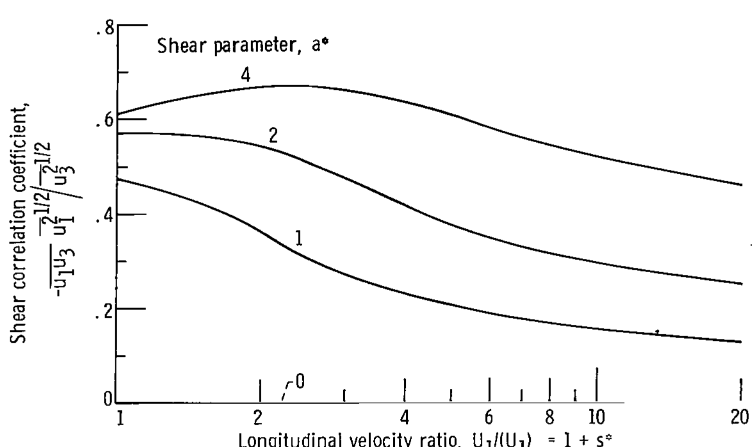


Figure 7. - Effect of uniform shear and normal strain (velocity ratio) on shear correlation coefficient.

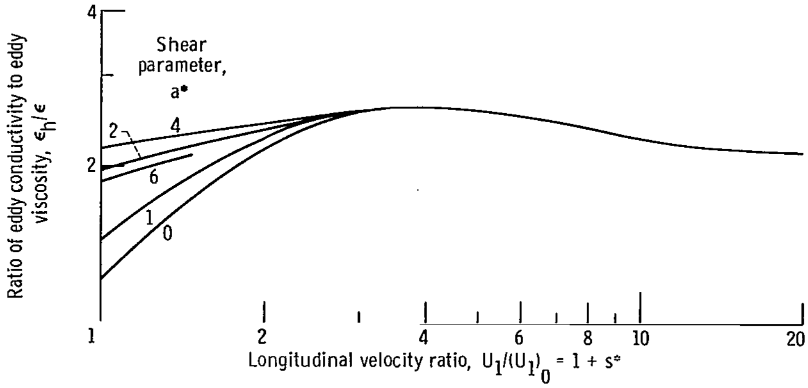


Figure 8. - Effect of uniform shear and normal strain (velocity ratio) on ratio of eddy conductivity to eddy viscosity; Prandtl number, 0.7.

As mentioned in the INTRODUCTION an apparent laminarization sometimes occurs in the turbulent boundary layers of accelerating flows. Some observed low heat-transfer values for flow in nozzles can evidently be explained by the fact that the mean velocity increases with distance in an accelerating flow (ref. 3). Some of the visual observations, however, seem to indicate that the turbulent energy itself decreases (ref. 5). Back, Massier, and Gier (ref. 7) suggested that the effect may be caused by a normal strain term in the energy equation which acts like a sink for turbulent energy. The term $2s\left(\overline{u_3^2} - \overline{u_1^2}\right)$ corresponds to the second term in equation (29), if that term is multiplied by $-\xi_1$ and integrated over all wave numbers. The term can act like a sink only if $\overline{u_1^2}$ is greater than $\overline{u_3^2}$; otherwise, it acts like a normal strain production term. The results in figure 5 for $\overline{u_3^2}/\overline{u_1^2}$ indicate that the normal-strain production term will be negative at low values of velocity ratio and high values of shear parameter. On the other hand, the shear production term $-2a\overline{u_1u_3}$ (which corresponds to the third term in equation (29) multiplied by $-\xi_1$ and integrated over all wave numbers) will always be positive.

The ratio of the two production terms is shown in figure 9 as a function of longitudinal velocity ratio and shear parameter. The curves show that the normal-strain production term can be negative and thus act like a sink term for turbulent energy at low velocity ratios and high shear. However, in order for that term to offset the effect of the shear production term, the ratio of the two terms would, of course, have to be less than -1, and that does not occur for the results in figure 9. It is possible that the ratio could be less than -1 at sufficiently large values of shear parameter. There appears to be a problem in making the normal strain production term sufficiently negative to offset the effect of the shear production term. The shear must be large to make the normal strain production term negative $\left(\text{by making } \overline{u_1^2} > \overline{u_3^2}\right)$. In that case, however, the shear production term will also be large. The curves in figure 9 show that as velocity ratio (or normal strain parameter) increases, the normal strain production term becomes strongly positive, since the effect of normal strain is to make $\overline{u_1^2} < \overline{u_3^2}$.

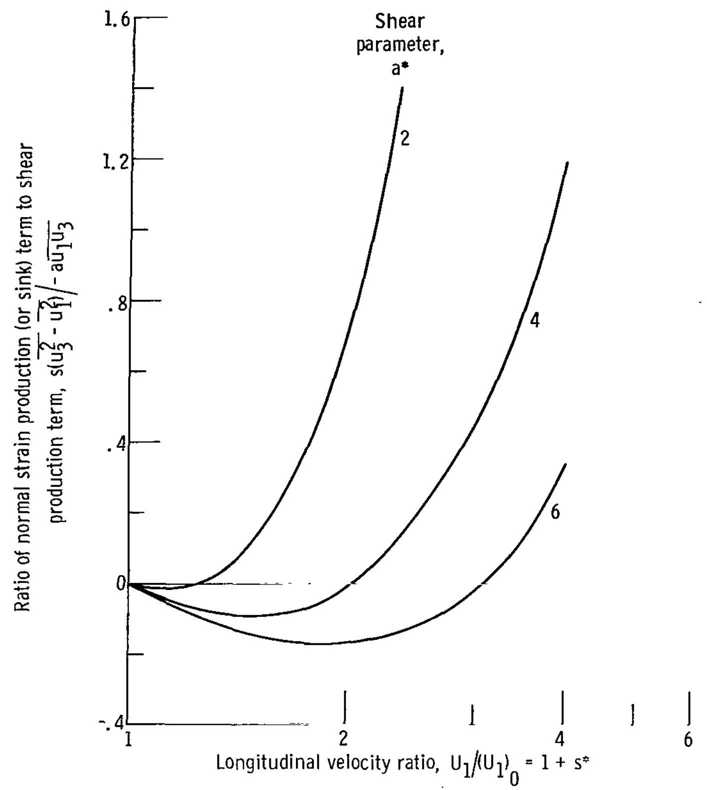


Figure 9. - Ratio of normal strain production (or sink) term to shear production term in turbulent energy equation (second and third terms in eq. (29) multiplied by ξ_1 and integrated over all wave numbers) as function of shear and normal strain (velocity ratio).

SUMMARY OF RESULTS

The results indicate that, in general, both shear and normal strain in an accelerating flow increase the energy in the turbulent field in comparison to that which would be present for no shear or normal strain. This increase occurs in spite of the normal-strain production term in the turbulent energy equation that can, under certain conditions of combined shear and normal strain, be negative and thus act as a turbulent energy sink. For the results computed, the shear production term more than offsets the effect of the sink term, and the net result is that the turbulent energy increases.

The present results for a two-dimensional contraction show that the lateral components of the turbulent energy increase with longitudinal distance at large mean velocity ratios. The longitudinal component decreases, but at a slower rate than it would if the effects of normal strain were absent. Thus, energy is fed into each of the three components of the turbulent energy by normal strain (and shear). This case differs from axially symmetric strain cases of accelerating flows, where the longitudinal turbulence component decays faster with distance than it would if the effects of normal strain were absent. For the two-dimensional contraction, although most of the vortex filaments tend to line up in the longitudinal direction, there is also some tendency for them to aline in the transverse direction of no normal strain.

The normal strain and shear both tend to produce anisotropy in the turbulence, but they work in opposite directions. The normal strain increases the ratios of the lateral components to the longitudinal component of the turbulent energy, while shear decreases the ratio.

In general, the turbulent shear correlation tends to be destroyed by the normal strain. An exception occurs at small velocity ratios and large shear, where some increase in correlation with increasing normal strain (velocity ratio) occurs.

As either the shear or normal strain parameter increases, the ratio of eddy conductivity to eddy viscosity reaches a maximum and then decreases. In the presence of lateral mean-shear velocity gradients and lateral temperature gradients, turbulent heat transfer occurs in the longitudinal as well as in the lateral direction.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 22, 1967,
129-01-11-05-22.

APPENDIX - SYMBOLS

a	shear velocity gradient, dU_1/dx_3
a^*	shear parameter, defined by eq. (39)
b	transverse temperature gradient, dT/dx_3
c	axial mean velocity ratio, $U_1/(U_1)_0$
g	defined by eq. (32)
h	defined by eq. (33)
J_0	constant that depends on initial conditions
Pr	Prandtl number, ν/α
p	pressure
$\vec{r}$	vector between points P and P'
s	longitudinal normal strain, $\partial U_1/\partial x_1$
s^*	normal strain parameter, defined by eq. (21)
T	mean temperature
U_i	mean velocity component
u_i	fluctuating velocity component
$\overline{u_i u_j^*}$	$\dfrac{\nu^{5/2}(x - x_0)^{5/2}}{J_0 U_0^{5/2}} \overline{u_i u_j}$
x_i	space coordinate
α	thermal diffusivity
Γ_i^*	dimensionless spectrum function for eddy conductivity, $\dfrac{\nu^2(x - x_0)}{J_0 U_0} \dfrac{\Gamma_i}{b}$
Γ_j	defined by eq. (42)
γ	defined by eq. (9)
γ_i^*	defined by eq. (38)
δ_{ij}	Kronecker delta
ϵ	eddy viscosity
ϵ_h	eddy conductivity
ζ'	defined by eq. (10)
η	running or dummy variable that equals c when $\xi_1 = \kappa_1$ and $\xi_3 = \kappa_3$
θ, φ	angular coordinates (see eq. (41))
κ_i	wave number component
κ_i^*	defined by eq. (35)
Λ_{ij}	defined by eq. (42)
Λ_{ij}^*	$\Lambda_{ij} \dfrac{\nu^3(x - x_0)^3}{J_0 U_0^3}$
ν	kinematic viscosity
ξ_1, ξ_3	running or dummy variables for which κ_1 and κ_3 are particular values
ρ	density
τ	temperature fluctuation

$\overline{\tau u_i}$ temperature-velocity correlation

$\overline{\tau u_i^*}$ $\dfrac{\nu^{5/2}(x - x_0)^{3/2}}{J_0 U_0^{3/2}} \dfrac{\overline{\tau u_i}}{b}$

φ_{ij} defined by eq. (6)

φ_{ij}^* defined by eq. (37)

ψ_{ij} defined by eq. (42)

ψ_{ij}^* $\psi_{ij} \dfrac{\nu^2 (x - x_0)^2}{J_0 U_0^2}$

Ω_{ij} given by eq. (43)

$\overline{\omega_i \omega_j}$ turbulent vorticity variance

$\overline{\omega_i \omega_j}^*$ $\overline{\omega_i \omega_j} \dfrac{\nu^{7/2}(x - x_0)^{7/2}}{J_0 U_0^{7/2}}$

Subscripts

0 at virial origin of turbulence where turbulent energy would be infinite (It is assumed that turbulence is isotropic at x_0 and that velocity and temperature gradients begin to act there.)

1 in flow direction

3 in direction of transverse strain and of mean shear velocity and temperature gradients

Superscripts

' at point P'

$*$ dimensionless quantities

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