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Newtonian Planetary Ephemerides 1800–2000: Development Ephemeris Number 28

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Abstract

Dynamically consistent Newtonian ephemerides of the nine principal planets have been generated from 1800–2000 with epoch 1899 December 12, 0^h ET (Julian Ephemeris Date 241 5000.5). The fifty-fourth order set of differential equations determining the motion of the nine planets is solved by numerical methods and the integrated ephemerides are adjusted to make a best fit in the least squares sense to suitably modified classical source ephemerides. Secular deficiencies in the Newtonian modified source ephemerides are determined and analyzed.

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I. Introduction

The classical planetary ephemerides used by astronomers are generated from general theories which usually are only first-order approximations to the solution of the differential equations determining the motion of a planet. Second and higher order perturbation terms have largely been ignored and rather rough approximations have been made for secular terms, while mixed secular terms of the form $at \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} bt$ have not been considered at all.

With the advance of space exploration and more precise techniques of observation, the need for more accurate planetary ephemerides has become evident. With present large high-speed digital computers, it has become relatively easy to generate more precise ephemerides than was possible only a few decades ago.

Since numerical integrations can be made over a longer time span than those for which actual observations are readily available, it has become necessary and conventional to fit the numerically integrated positions to one of the classical theories, rather than to actual observations.

The widely distributed Development Ephemeris (DE) series, DE 3 (Ref. 1) and DE 19 (Ref. 2), of the Jet Propulsion Laboratory are two examples. These ephemerides have been generated by assuming that the orbits of (N-1) bodies are known and by numerically integrating the differential equations for a single body (Ref. 3).

One problem which arises when fitting a source theory rather than actual observations is the manner in which to treat known deficiencies of the source theory. A perfect example appears in the usual Newcomb (Ref. 4) source theories of the three inner planets.

The general theory was derived, of necessity, in a fixed coordinate system, but for the sake of convenience, the final results were referred to the mean equator and equinox of date. This implies that some definite value of the speed of general precession in longitude has been employed—in the case of Newcomb, the value is 5024″.82 per tropical century at 1900.0. The fact that the internationally adopted value is 5025″.64 per tropical century introduces the following dilemma: (1) one may either adopt Newcomb referred to the mean equinox of date (as given by S. Newcomb) as defining the position of the planet and use the *new* Newcomb general precession in longitude, 5025″.64, to determine the position of the planet referred to a fixed equinox; or (2) one may adopt Newcomb referred to the mean equinox of date (as given by S. Newcomb), adjust for the difference in precession, and then use the new Newcomb precession to determine the position referred to a fixed equator and equinox. The two alternatives differ by +0″.82 T in the perihelia and nodes, where T is measured in centuries from 1900.

From the mathematical point of view, the second alternative is preferred, although there is no guarantee that alternative (2) will be closer to the actual position of the planet than alternative (1). The American Ephemeris adopts the first alternative, and P. Herget adopts the second one (although his ephemeris is strictly consistent with yet a third value of the general precession in longitude) in his work on the sun (Ref. 5).

Another deficiency in the usual Newcomb source theories for the inner planets deals with the so-called empirical terms applied by S. Newcomb. He assumed that the force between two bodies is proportional to $r^{-(2+\delta)}$, $\delta = 0.000\,000\,1612$, rather than the Newtonian r^{-2}. This has the effect that the longitude of perihelion of a planet is advanced by an amount proportional to $a^{-3/2}\,T$, rather than $a^{-5/2}\,T$ predicted by general relativity.

Of course, these dilemmas may be circumvented by using the same epoch as Newcomb and fitting Newcomb over equal time spans on either side of the epoch, since in the least squares process, any secular error in the source theory will remain in the final residuals and will not be absorbed by the integration. If, however, one adopts a significantly different epoch, it becomes important to distinguish between the alternatives.

A third problem occurs when one applies, for example, the differential corrections as found by G. M. Clemence (Ref. 6) to the Newcomb theory of Mercury, or those given by R. Duncombe for Venus and the earth (Ref. 7). The corrections usually include secular terms which are primarily due to a different system of planetary masses or another value of the general precession in longitude; hence, these corrections should not be indiscriminately applied. The fact that the observations were analyzed before the introduction of ephemeris time also contributes theoretical reasons for not using the corrections without considerable thought.

In most of the published numerically integrated fits to source theories, the preceding problems have either been ignored or specific details of what has been done are lacking. While there is little justification in stating that such fitted ephemerides do not represent the real world as well as a set in which the preceding source deficiencies are corrected, there is considerable difficulty in comparing results with other sources.

B. G. Marsden (Ref. 8) has listed the theoretical corrections which should be applied to the classical source theories to render them mathematically consistent. His recommendations have generally been followed in this investigation.

J. Schubart and P. Stumpff (Ref. 9) of the Astronomisches Rechen-Institut in Heidelberg have recently developed an N-body computer program in which the equations of motion of N-bodies are numerically integrated simultaneously. A modification of their program has been employed in this investigation to develop a dynamically consistent set of planetary ephemerides from 1800–2000; this set is ~~referred to as~~ Development Ephemeris (DE) Number 28. A by-product of the fit to the source theories as modified by the theoretical corrections is an ability to determine secular errors in the Newtonian source theories.

II. Source Ephemerides

A. System of Masses

The planetary masses adopted by the International Astronomical Union (IAU) (Ref. 10) and used in the following integration are listed in Table 1. The mass ratio of moon/earth is 1/81.30.

These masses are sufficiently close to those employed by S. Newcomb. They are identical with the masses used by G. Clemence in his theory of Mars and by Eckert, Brouwer, and Clemence in the SSEC (selective sequence electronic calculator) integration of the outer planets.

Table 1. Planetary masses

Planet	Mass of sun/mass of planet
Mercury	6,000,000
Venus	408,000
Earth-moon	329,390
Mars	3,093,500
Jupiter	1,047.355
Saturn	3,501.6
Uranus	22,869
Neptune	19,314
Pluto	360,000

B. Theoretical Secular Corrections

To avoid the philosophical problem of which relativity metric to use in the integration, it was decided to perform a Newtonian integration and, after the integration, to advance the planetary perihelia by an amount

$$\frac{12\pi^2 a^2}{c^2 P^2 (1-e^2)}$$

per revolution period P, where a, e, and c are the semimajor axis, eccentricity, and speed of light, respectively.

To render the basic source ephemerides Newtonian and consistent with the Newcomb general precession in longitude of $5025''.64$ per tropical century at 1900.0, it is necessary to add the secular corrections listed in Table 2 to compensate for the non-gravitational nature of the source theories.

In Table 2, T is measured in Julian centuries from 1900. With the exception of the outer planets, the values are identical with those proposed by Marsden in Ref. 8.

The source ephemeris for Mercury is an evaluation of the Newcomb theory of Mercury (Ref. 4) by Duncombe, Tufekcioglu, and Larson (Ref. 11). The ephemeris by Duncombe is a strict evaluation of the Newcomb theory referred to the mean equinox of date and precessed to 1950.0 with the IAU value of the Newcomb general precession in longitude (personal communication). The corrections listed in Table 2 represent removal of the empirical term and adjustment for precession.

The source ephemeris for Venus is P. Herget's evaluation (Ref. 12) of the Newcomb theory of Venus (Ref. 4). The corrections represent removal of the empirical term and adjustment for precession.

Table 2. Secular corrections to basic source ephemerides

Planet	Source	Corrections to be applied
Mercury	Newcomb	$\Delta\tilde{\omega} = -42''.55T$, $\Delta\Omega = +0''.82\ T$
Venus	Newcomb	$\Delta\tilde{\omega} = -16''.16T$, $\Delta\Omega = +0''.82\ T$
Earth-moon	Newcomb	$\Delta\tilde{\omega} = -17''.31\ T$
Mars	Clemence	$\Delta\tilde{\omega} = -1''.35\ T$
Jupiter	SSEC	Clemence corrections $+\ e\Delta\tilde{\omega} = -0''.00301\ T$
Saturn	SSEC	Clemence corrections $+\ e\Delta\tilde{\omega} = -0''.00077\ T$
Uranus	SSEC	Clemence corrections $+\ e\Delta\tilde{\omega} = -0''.000112\ T$
Neptune	SSEC	Clemence corrections
Pluto	SSEC	Clemence corrections

The source ephemeris for the earth–moon barycenter is P. Herget's evaluation (Ref. 5) of the Newcomb theory of the sun (Ref. 4). Herget has already applied a correction $\Delta\tilde{\omega} = -4''.78\ T$ to the Newcomb longitude of the earth perihelion; hence, the correction $\Delta\tilde{\omega} = -12''.53\ T$ is applied to Herget's evaluation. The Newcomb ephemeris for the earth–moon barycenter includes a secular term $\Delta\tilde{\omega} = (10''.45 + 7''.68)\ T$, where the first value is the empirical (relativity) term and the second value represents the secular acceleration of the barycenter perihelion due to the moon.

The source ephemeris for Mars is the preliminary evaluation of the Clemence theory of Mars (Refs. 13 and 14) by Clemence and Duncombe (Refs. 15 and 16). The source ephemeris includes $\Delta\tilde{\omega} = +1''.35\ T$ due to relativity, and the Table 2 correction removes this term.

The source ephemerides for the five outer planets are those calculated by Eckert, Brouwer, and Clemence (Ref. 17) on the SSEC. G. M. Clemence (Ref. 18) has published tables of corrections necessary to render the SSEC coordinates heliocentric. These corrections have been applied to the SSEC coordinates (with the exception of one term in the Jupiter y-coordinate in which the typographical error $-140 \times 10^{-9}\, T \sin g_5$ was changed to $-104 \times 10^{-9}\, T \sin g_5$) for the five outer planets. Since the Clemence corrections contain relativistic effects in the coordinates of Jupiter, Saturn, and Uranus, the relativistic contributions were removed in the same manner as they were included in the tables.

C. Constant Corrections

In addition to the secular corrections to the basic source ephemerides listed in Table 2, the constant corrections of Table 3 were applied.

The constant corrections for Mercury are those determined by Clemence (Ref. 6) reduced to the Newcomb theory of Mercury and consistent with ephemeris time

Table 3. Constant corrections to source ephemerides

Planet	Corrections to be applied
Mercury	$\Delta\lambda = +0''.23$, $\Delta e = -0''.40$, $\Delta\tilde{\omega} = +0''.92$, $\Delta I = +0''.06$
Venus	$\Delta e = -0''.12$, $e\Delta\tilde{\omega} = +0''.01$, $\sin I\ \Delta\Omega = +0''.21$, $\Delta I = +0''.08$
Earth-moon	$\Delta e = -0''.10$, $e\Delta\tilde{\omega} = -0''.07$

and the IAU general precession in longitude. Clemence lists a correction

$$\Delta\lambda = -0''.58 \pm 0''.08 - (0''.67 \pm 0''.14)\,T$$

to the mean longitude of Mercury, but the approximation which he employed to remove the effects of the non-uniform nature of universal time (UT) exceeds those due to the adopted definition of ephemeris time (ET) by $\Delta\lambda = +0''.81 + 0''.75\ T$. Thus the correction found by Clemence to the Newcomb mean longitude of Mercury measured in ET is

$$\Delta\lambda = +0''.23 \pm 0''.08 + (0''.08 \pm 0''.14)\,T$$

The adopted correction employed in this investigation is $\Delta\lambda = +0''.23$.

The constant corrections for Venus are those determined by R. Duncombe (Ref. 7). The correction to the mean longitude of Venus was not employed, since the Duncombe corrections to the mean longitudes of Venus and the earth are in the ratio of their mean motions and, hence, are ascribed to deficiencies in the approximation used to remove effects of the non-uniform nature of UT.

The constant corrections for the earth–moon barycenter are taken from Duncombe's work on Venus (Ref. 7). The correction to the mean longitude was not employed, as noted in the preceding paragraph, to avoid tampering with the definition of ephemeris time. The value of obliquity employed is that of Newcomb for 1950.0, $\epsilon = 23^\circ 26' 44''.836$.

The net corrections applied to the basic source ephemerides are the sum of terms listed in Tables 2 and 3. The source ephemerides corrected in this manner will be referred to as modified source ephemerides (MSE).

After the numerical integration was fitted to the modified source ephemerides, the relativistic advance of the longitude of perihelion of each planet was computed and is listed as $\Delta\mathbf{r}$ in ephemeris tapes DE 28. The relativistic contributions were calculated from the values in Table 4.

In Table 4, T is measured in Julian centuries from 241 5000.5. The correction listed for the earth–moon barycenter is a combination of the effects of relativity ($3''.839\ T$) and the acceleration of the moon on the barycenter ($+7''.700\ T$).

Table 4. Relativistic corrections to perihelia

Planet	$\Delta\tilde{\omega}$
Mercury	$+42''.982\ T$
Venus	$-8''.6250$
Earth-moon	$+11''.539\ T$
Mars	$+1''.3509\ T$
Jupiter	$+6.2304 \times 10^{-2}\ T$
Saturn	$+1.3547 \times 10^{-2}\ T$
Uranus	$+2.3468 \times 10^{-3}\ T$
Neptune	$+7.8187 \times 10^{-4}\ T$
Pluto	$+4.2304 \times 10^{-4}\ T$

The epoch of osculation selected for the integration is Julian Date 241 5000.5 (1899 December 12 0^h ET). The integrations cover the period 237 8520.5 to 245 1480.5 (1800 January 25 0^h ET to 1999 October 29 0^h ET).

In this investigation, it was assumed that the real positions of the planets at 1900 were those given by the modified source epemerides. Thus, any Newtonian secular error in the modified source ephemerides is not absorbed in a least squares fit from 1800–2000 with epoch 1900, but will appear in the final residuals after the best fit has been made. The final residuals were then examined for such secular errors in the modified source theories.

III. Numerical Integration

A. Method

The Adams-Størmer method of numerical integration using backward differences is employed in the Heidelberg N-body integration program which forms a basis for this investigation. Only a brief outline will be given here and further details may be found in Ref. 9.

The differential equations integrated in a center of mass of the solar system reference frame are given by:

$$\ddot{\mathbf{R}}_i = -k^2 \sum_{\substack{j=1\\ j\neq i}}^{10} m_j \frac{\mathbf{R}_i - \mathbf{R}_j}{R_{ij}^3} \qquad (i = 1, 10) \qquad (1)$$

where

$\ddot{\mathbf{R}}_i$ = vector from center of mass to ith body
k^2 = square of Gaussian gravitational constant

$k =$ 0.017202 09895

$m_j =$ ratio of mass of jth body to mass of sun

and

$$\mathbf{R}_{ij} = \mathbf{R}_j - \mathbf{R}_i$$

$$R_{ij} = |\mathbf{R}_{ij}|$$

Bodies are numbered in ascending order proceeding outward from the sun, with the sun being body 1 and Pluto body 10.

The integral of the center of mass

$$\sum_{i=1}^{10} m_i \ddot{\mathbf{R}}_i = 0$$

is used to eliminate the differential equations determining the motion of the sun about the center of mass of the solar system, and only nine second-order differential equations in equatorial rectangular coordinates x, y, and z are integrated.

Heliocentric equatorial rectangular coordinates are then evaluated from

$$\mathbf{r}_i = \mathbf{R}_i - \mathbf{R}_1 \qquad (i = 2, 10) \tag{2}$$

where $\mathbf{r}_i$ is the heliocentric vector to the ith planet.

B. Integration Accuracy Tests

To monitor the integration procedure, the integrals of energy

$$E = \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\mathbf{R}}_i \cdot \dot{\mathbf{R}}_i - k^2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{m_i m_j}{R_{ij}} \tag{3}$$

and angular momentum about the center of mass

$$\mathbf{L} = \sum_{i=1}^{N} m_i (\mathbf{R}_i \times \dot{\mathbf{R}}_i) \tag{4}$$

were calculated at each step. The highest order differences retained were also monitored.

An option of using either the predictor method alone or the predictor-corrector method was built into the program; and tests were performed to determine the best step size and method. An integration performed at half-day steps, retaining eleventh differences in the accelerations and using the predictor formula only, proved superior to using the predictor with one corrector cycle at one-day steps retaining the same order of differences.

The integrations were thus performed with epoch 241 5000.5 at half-day steps retaining eleventh differences and using only the predictor formula. The starting table was iterated until the accelerations retained a relative accuracy of 10^{-15}. The integration was then performed from 1900–2000 and 1900–1800. After the final fit to the modified source ephemerides, the positions and velocities at 1800 and 2000 were used to integrate 1800–1900 and 2000–1900. The coordinates generated in the normal integration (1900–2000 and 1900–1800) were then compared with those of the reverse integration (2000–1900 and 1800–1900).

The Mercury through Pluto portions of Table 5 summarize the results of the reversibility test in the sense of normal minus reverse integration for each planet. The integral portion of the table summarizes the relative deviations of the energy and angular momentum integrals from their epoch values. The integral values at 241 5000.5 are:

$$E = -3.32 \times 10^{-8}$$

$$L = +6.11 \times 10^{-5}$$

$$L_x = +1.68 \times 10^{-6}$$

$$L_y = -2.38 \times 10^{-5}$$

$$L_z = +5.62 \times 10^{-5}$$

where L_x, L_y, and L_z are the total angular momentum components of the solar system in an equatorial frame referred to 1950.0. Graphs of the relative changes in energy and angular momentum integrals from their epoch values are given in Figs. C-1 and C-2.

The number of steps required to integrate the solar system from 241 5000.5 to 245 1480.5 or from 241 5000.5 to 237 8520.5 in half-day intervals is 72,960. Examination of the reversibility statistics for Mercury indicates that the accumulation of errors as a function of the number of steps is about a factor of three less than that given by D. Brouwer (Ref. 19). The reversibility statistics for Mercury could be improved by including one corrector cycle in the integration for Mercury.

The integrations were performed on the IBM 7094/7040 DCOS (direct couple operating system) at Yale University. The nine-planet integrations were performed at the rate of 10.5 steps/s, and the 200-yr integration required a total of 232 min of computer time. The five outer planets could be integrated at 40-day steps for 400 yr in about 4 min.

Table 5. Integration statistics

Residuals in position	Standard deviation	Maximum difference	Minimum difference
Mercury			
dx, AU	1.46 E −08	5.68 E −08	−5.66 E −08
dy, AU	1.31 E −08	4.34 E −08	−4.37 E −08
dz, AU	7.08 E −09	2.44 E −08	−2.46 E −08
cosB dL, ″	1.16 E −02	3.80 E −02	−3.82 E −02
dB, ″	1.00 E −03	4.50 E −03	−4.53 E −03
dr/r, ″	1.67 E −03	5.56 E −03	−5.53 E −03
Venus			
dx, AU	5.94 E −12	2.70 E −11	−2.70 E −11
dy, AU	5.42 E −12	2.43 E −11	−2.48 E −11
dz, AU	2.46 E −12	1.10 E −11	−1.12 E −11
cosB dL, ″	1.96 E −06	7.81 E −06	−5.14 E −09
dB, ″	1.01 E −07	4.58 E −07	−4.56 E −07
dr/r, ″	1.27 E −08	5.06 E −08	−8.35 E −08
Earth-moon barycenter			
dx, AU	2.29 E −12	8.65 E −12	−9.00 E −12
dy, AU	2.10 E −12	7.96 E −12	−8.16 E −12
dz, AU	9.11 E −13	3.45 E −12	−3.54 E −12
cosB dL, ″	6.68 E −07	1.89 E −06	−1.51 E −06
dB, ″	3.87 E −10	1.08 E −09	−1.08 E −09
dr/r, ″	7.56 E −09	3.62 E −08	−2.80 E −08
Mars			
dx, AU	3.32 E −12	1.26 E −11	−1.17 E −11
dy, AU	3.02 E −12	1.20 E −11	−1.00 E −11
dz, AU	1.39 E −12	5.51 E −12	−4.62 E −12
cosB dL, ″	5.44 E −07	1.99 E −06	−5.49 E −07
dB, ″	1.49 E −08	5.76 E −08	−5.19 E −08
dr/r, ″	4.20 E −08	1.59 E −07	−1.49 E −07
Jupiter			
dx, AU	1.69 E −12	4.83 E −12	−5.20 E −12
dy, AU	1.57 E −12	4.97 E −12	−4.69 E −12
dz, AU	6.74 E −13	2.15 E −12	−2.03 E −12
cosB dL, ″	5.89 E −08	2.26 E −07	−4.52 E −11
dB, ″	1.59 E −09	5.05 E −09	−4.56 E −09
dr/r, ″	4.86 E −09	9.91 E −09	−1.68 E −08
Saturn			
dx, AU	9.96 E −14	3.30 E −13	−4.06 E −13
dy, AU	9.20 E −14	2.01 E −13	−4.15 E −13
dz, AU	3.82 E −14	8.06 E −14	−1.69 E −13
cosB dL, ″	2.91 E −09	5.47 E −09	−9.70 E −09
dB, ″	1.74 E −10	6.04 E −10	−6.92 E −10
dr/r, ″	3.43 E −10	5.87 E −10	−1.50 E −09
Uranus			
dx, AU	2.27 E −13	3.67 E −13	−1.01 E −12
dy, AU	1.56 E −13	4.81 E −13	−2.22 E −13
dz, AU	7.68 E −14	2.36 E −13	−9.95 E −14
cosB dL, ″	3.07 E −09	4.50 E −09	−1.07 E −08
dB, ″	1.75 E −10	4.95 E −10	−3.83 E −10
dr/r, ″	6.68 E −10	2.93 E −09	−5.80 E −11
Neptune			
dx, AU	1.25 E −13	4.46 E −13	−1.47 E −13
dy, AU	3.93 E −14	1.78 E −13	−9.06 E −14
dz, AU	1.35 E −14	5.15 E −14	−3.38 E −14
cosB dL, ″	7.97 E −10	1.06 E −09	−2.99 E −09
dB, ″	6.15 E −11	1.27 E −10	−1.42 E −10
dr/r, ″	4.08 E −10	1.46 E −09	−3.85 E −10
Pluto			
dx, AU	4.34 E −14	8.08 E −14	−1.02 E −13
dy, AU	2.31 E −14	1.03 E −13	−2.49 E −14
dz, AU	2.57 E −14	7.73 E −14	−2.31 E −14
cosB dL, ″	1.99 E −10	5.26 E −10	−3.63 E −10
dB, ″	8.09 E −11	2.24 E −10	−6.68 E −11
dr/r, ″	1.13 E −10	4.27 E −10	−5.02 E −11
Integrals			
dE/E	3.40 E −14	1.48 E −13	−5.71 E −14
dL/L	2.42 E −14	9.27 E −14	−2.84 E −14
dL_x/L_x	2.60 E −13	4.21 E −13	−5.22 E −13
dL_y/L_y	2.43 E −14	8.33 E −14	−5.07 E −14
dL_z/L_z	2.44 E −14	9.52 E −14	−2.79 E −14

IV. Ephemeris Fitting Procedure

The parameters adopted for correction are identical with the Eckert-Brouwer Set III elements (Ref. 20), except that their parameter $\Delta M_0 + \Delta r$ is replaced by a correction to the mean longitude at epoch

$$\Delta\lambda_0 = \Delta M_0 + \Delta\varpi_0$$

where ΔM_0 and $\Delta\varpi_0$ are the corrections to the mean anomaly and longitude of perihelion at epoch, respectively. The partial derivative matrices were modified accordingly. In evaluating the partial derivative matrices, osculating elements and **PQR** matrices at epoch were used with the integrated positions and velocities.

Initial positions and numerically differentiated velocities were taken from the modified source ephemerides at Julian Date 241 5000.5 to commence the integration. An integration ±2000 days from the epoch was made, and the differential corrections were applied to the initial conditions. The integration was successively made ±4000 days, ±25 yr, ±50 yr, and ±100 yr from epoch and corrected until no significant corrections remained.

Integrated coordinates of the four inner planets were compared in 4-day intervals with the modified source ephemerides, while the five outer planets were compared at 40-day intervals (to avoid introducing interpolation errors into the SSEC coordinates).

Difficulties were experienced in fitting the integrated positions of Neptune to the modified source ephemerides. (This difficulty also occurred in JPL DE 3 ephemeris of Neptune.) The difficulty can be traced to the fact that Clemence's corrections to the SSEC coordinates of Neptune and Pluto do not completely include the effect of a change in scale from the SSEC coordinates to heliocentric positions. The effect is that Clemence disregarded a term in the coordinates of Neptune and Pluto with the period of the planet (see Figs. C-17 through C-20). After the integration interval became larger than the linear range of this neglected term, the differential corrections converged nicely.

V. Results

The ephemerides integrated in this investigation contain Newtonian heliocentric equatorial rectangular coordinates referred to the mean equator and equinox of 1950.0. Differences in position, relativistic minus Newtonian, are also given in the ephemeris tape.

A. The Fit to Modified Source Ephemerides

Final residuals and statistics of the fit, in the sense of modified source ephemeris minus DE28, along with initial conditions are given in Tables B-1 through B-9. Rectangular coordinates in astronomical units (AU) are heliocentric equatorial and are referred to the mean equator and equinox of 1950.0; velocities are in AU/day. Angular residuals, in seconds of arc, for $\cos B\ dL$, dB, and dr/r, where L, B, and r are celestial longitude, latitude, and heliocentric distance, respectively, are also referred to 1950.0. Epoch values of the osculating equatorial positions and velocities along with ecliptic elements are also given. The osculating **PQR** matrix is listed for reduction from the plane of the orbit to heliocentric equatorial coordinates referred to 1950.0. The value of obliquity used is $23°26'44''.836$ at 1950.0.

Plots of the residuals, MSE minus DE28, in heliocentric equatorial coordinates and celestial longitude, latitude, and radius are given in Figs. C-3 through C-20.

The mixed secular residual with a period of one year, zero at 1850, and growing with time, apparent in the five outer planets, is due to the earth. G. M. Clemence used the coefficient of time in the earth mean anomaly to compute the corrections of the outer planets due to the earth. For his purposes, this correction was sufficient, but the residuals presented in the plots of Figs. C-11 through C-20 suggest that it would have been better to use the sidereal mean motion of the earth (determined from its mean longitude), which differs from that determined from the mean anomaly by $1100''.0$ per century. The long-period residuals in the Neptune and Pluto plots are attributed to effects of the change in scale from SSEC coordinates to heliocentric coordinates which were below the significant level in Clemence's corrections. Using the data Clemence gives for Δr in his corrections (Ref. 18), these defects in the modified source ephemerides can be removed.

The secular trend of residuals in the Jupiter x-coordinate, shown in Fig. C-11, can be attributed to a secular change in the Jupiter eccentricity, $-0''.00117\ T$, due to the inner planets—an effect which Clemence did not include in his corrections (Ref. 18).

B. Secular Deficiencies in Newtonian Source Ephemerides

After making the best fit to the modified source ephemerides, the final residuals were examined in an attempt to detect secular deficiencies of the Newtonian modified source ephemerides in the elements e, ϖ, Ω, and I. A significant reduction of residuals was found in the ephemerides of Venus, earth-moon, and Jupiter.

The modified source ephemerides corrected by the deduced secular variations will be called improved modified source ephemerides (IMSE). The corrections applied and the new statistics are listed in Tables D-1 through D-3. Plots of the IMSE minus DE28 are given in Figs. D-1 through D-6.

The modified source ephemeris of the earth-moon barycenter was analyzed in an attempt to detect a secular error in the Newcomb obliquity. Observations suggest that a correction to the Newcomb obliquity of approximately $-0''.3\ T$ is needed (Ref. 7). Clemence and Duncombe (Ref. 21) report that P. Herget has determined a correction of $-0''.47\ T$ to the Newcomb obliquity by fitting a numerical integration of the earth-moon barycenter to Newcomb. Such an error would appear in the latitude residuals as a mixed secular term with a period of one year and amplitude of $-0''.47\ T$. This term is not evident in the 200-yr integration of this investigation (see Fig. D-4), and the deduced correction to $\Delta(\sin I \cos \Omega)$ of the earth-moon barycenter is $-(0''.0057 \pm 0''.0009)\ T$.

It is known that $d\epsilon/dt \approx \sin \pi_1 \cos \Pi_1$ (Ref. 22) where π_1 is the angle at the node between a fixed ecliptic and the mean ecliptic of date, and Π_1 is the longitude of ascending node of the mean ecliptic of date on a fixed ecliptic. Since $\Omega_{\oplus☾} = \Omega_{\odot} + 180°$, with $\Omega_{\odot} = \Pi_1$, it is seen that

$$\Delta \frac{d\epsilon}{dt} \approx -\Delta(\sin I \cos \Omega)_{\oplus☾}$$

Thus, it appears from this investigation that the secular change in obliquity given by Newcomb is approximately correct, at least as far as Newtonian interactions are concerned.

The correction $\Delta\tilde{\omega} = +1''.19 \pm 0''.03$ to the longitude of perihelion for the modified source ephemeris of the earth-moon barycenter indicates that Herget's evaluation of Newcomb plus $\Delta\tilde{\omega} = -4''.78\ T$ is better than the one adopted for the modified source ephemeris, Newcomb plus $\Delta\tilde{\omega} = -5''.77\ T$ ($-17''.31$ Newtonian + $11''.539$ for relativity and secular effect of the moon). The difference between Herget's correction to Newcomb and the one adopted is due to a value of the general precession in longitude, $5026''.65$ per tropical century, recommended by G. M. Clemence (Ref. 23) in 1948. The present investigation suggests that Herget's correction is better, not for the quoted reason of precession, but because of a Newtonian deficiency in the Newcomb theory.

As noted previously, the modified source ephemeris of Jupiter lacks a term $\Delta e = -0''.00117\ T$ (Ref. 18) due to the inner planets. This term would appear in the x-coordinate residuals of Jupiter as $\Delta x = -43 \times 10^{-9}\ T$. It is seen that the correction found, $\Delta e = -0''.00183 \pm 0''.00001$, is in good agreement with the known deficiency.

Nomenclature

a	semimajor axis, AU
B	celestial latitude referred to mean ecliptic and equinox 1950.0
dB	residual in ecliptic latitude,″
dL	residual in ecliptic longitude,″
dx, dy, dz	residuals in heliocentric equatorial rectangular coordinates, AU
e	eccentricity, rad
E	energy integral
I	inclination to mean ecliptic of 1950.0, rad
k^2	square of Gaussian constant, $k = 0.017202\ 09895$
$\mathbf{L}$	angular momentum integral about center of mass
L	celestial longitude referred to mean ecliptic and equinox of 1950.0
L_x, L_y, L_z	angular momentum components referred to mean equator and equinox of 1950.0
M_0	mean anomaly, osculating at epoch, rad
m_j	mass of jth body relative to sun
n	mean motion, osculating at epoch, rad/day
$\mathbf{P}$	unit vector directed toward perihelion, osculating at epoch
$\mathbf{Q}$	$\mathbf{R} \times \mathbf{P}$
$\mathbf{R}$	unit vector normal to the orbital plane, osculating at epoch
$\mathbf{R}_i$	vector from center of mass to ith body
$\mathbf{R}_{ij}$	$\mathbf{R}_j - \mathbf{R}_i$
$\mathbf{r}_j$	vector from sun to jth body
r	heliocentric distance
T	time in Julian centuries from Julian Date 241 5000.5
x, y, z	heliocentric equatorial rectangular coordinates referred to mean equator and equinox of 1950.0, AU
$\dot{x}, \dot{y}, \dot{z}$	heliocentric equatorial rectangular velocities referred to 1950.0, AU/day
Ω	longitude of ascending node, osculating at epoch, on ecliptic of 1950.0, rad
$\tilde{\omega}$	longitude of perihelion, osculating at epoch, rad
ω	argument of perihelion, osculating at epoch, rad

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Appendix A

Tape Format

The set of ephemerides DE28 resulting from the Newtonian fit to the modified source ephemerides is available on tape over the interval of Julian Dates 237 8520.5 to 245 1480.5 (1800 January 25 to 1999 October 29) at four-day intervals in the standard DCOS, 460 words per physical record, format. The first four logical records contain the following reference data:

(1) Record 1 contains 24 BCD words written in binary and identifying the tape as DE28.

(2) Record 2 contains the masses used in generating DE28. The masses are in double precision with the sun mass first and Pluto mass last.

(3) Record 3 contains the osculating elements at 241 5000.5 of the nine planets in double precision. The order of elements is a (AU), n (rad/day), e (rad), I (rad), Ω (rad), ω (rad), M_0 (rad). Mercury elements are listed first and Pluto elements are last.

(4) Record 4 contains the osculating **PQR** matrices at epoch in double precision for reduction to the equator of 1950.0. Mercury is listed first and Pluto is last.

The data records contain the following information: the Julian Date in single precision; the body number, its equatorial Newtonian position and velocity in double precision referred to 1950.0; and the single-precision difference in position between the relativistic and Newtonian coordinates (Table 4). The format is Julian Date (body number I, position in AU, velocity in AU/day, $\Delta\mathbf{r}$ in AU) for $I = 2,10$; where $I = 2$ indicates Mercury and $I = 10$ indicates Pluto.

The record immediately following JD 245 1480.5 contains a $-8.$ in the Julian Date position.

Appendix B

Initial Conditions and Statistics for DE28

Table B-1. The fitted ephemeris of Mercury

<table>
<tr><th colspan="6">Statistical summary (modified source minus DE28)</th></tr>
<tr><th>Residuals in position</th><th>Mean square residuals</th><th>Mean residuals</th><th>Standard deviations</th><th>Maximum residuals</th><th>Minimum residuals</th></tr>
<tr><td>dx, AU</td><td>6.50E −14</td><td>−7.05E −09</td><td>2.55E −07</td><td>8.47E −07</td><td>−8.69E −07</td></tr>
<tr><td>dy, AU</td><td>7.03E −14</td><td>−1.41E −08</td><td>2.65E −07</td><td>9.88E −07</td><td>−8.59E −07</td></tr>
<tr><td>dz, AU</td><td>8.58E −14</td><td>−2.45E −08</td><td>2.92E −07</td><td>8.88E −07</td><td>−9.87E −07</td></tr>
<tr><td>$\cos B\, dL$, ″</td><td>1.59E −02</td><td>1.92E −04</td><td>1.26E −01</td><td>4.88E −01</td><td>−4.80E −01</td></tr>
<tr><td>dB, ″</td><td>2.45E −02</td><td>−8.02E −03</td><td>1.56E −01</td><td>4.78E −01</td><td>−4.93E −01</td></tr>
<tr><td>dr/r, ″</td><td>2.19E −02</td><td>2.40E −02</td><td>1.46E −01</td><td>3.50E −01</td><td>−3.87E −01</td></tr>
<tr><th colspan="6">Osculating values at JD 241 5000.5</th></tr>
<tr><td colspan="3">x = −1.10729 98321 32043D −01 AU
y = 2.56801 39813 66735D −01 AU
z = 1.48892 38800 73276D −01 AU
$\dot{x}$ = −3.19700 94618 65669D −02 AU/day
$\dot{y}$ = −8.90445 12293 25886D −03 AU/day
$\dot{z}$ = −1.47683 53555 47968D −03 AU/day</td><td colspan="3">a = 3.87097 75793 17067D −01 AU
n = 7.14250 40257 46578D −02 rad/day
e = 2.05625 54122 00547D −01
I = 1.22292 71650 06895D −01 rad
Ω = 8.34298 97004 37712D −01 rad
ω = 5.02591 83313 10970D −01 rad
M_0 = 3.92282 29482 57636D −01 rad</td></tr>
<tr><th colspan="6">Equatorial PQR matrix</th></tr>
<tr><td colspan="2">P = 2.34443 59973 10055D −01
Q = −9.67919 99131 51598D −01
R = 9.03719 47846 82575D −02</td><td colspan="2">8.66856 82375 16508D −01
1.66074 33231 72862D −01
−4.70083 57050 69548D −01</td><td colspan="2">4.39994 82458 36168D −01
1.88547 62411 08036D −01
8.77983 11360 85723D −01</td></tr>
</table>

Table B-2. The fitted ephemeris of Venus

<table>
<tr><th colspan="6">Statistical summary (modified source minus DE28)</th></tr>
<tr><th>Residuals in position</th><th>Mean square residuals</th><th>Mean residuals</th><th>Standard deviations</th><th>Maximum residuals</th><th>Minimum residuals</th></tr>
<tr><td>dx, AU</td><td>1.06E −13</td><td>7.81E −09</td><td>3.26E −07</td><td>1.10E −06</td><td>−1.10E −06</td></tr>
<tr><td>dy, AU</td><td>3.79E −14</td><td>5.89E −09</td><td>1.95E −07</td><td>7.17E −07</td><td>−8.08E −07</td></tr>
<tr><td>dz, AU</td><td>1.57E −14</td><td>3.67E −09</td><td>1.25E −07</td><td>4.99E −07</td><td>−4.14E −07</td></tr>
<tr><td>$\cos B\, dL$, ″</td><td>1.03E −02</td><td>5.71E −05</td><td>1.01E −01</td><td>3.19E −01</td><td>−3.16E −01</td></tr>
<tr><td>dB, ″</td><td>8.08E −04</td><td>4.56E −04</td><td>2.84E −02</td><td>1.09E −01</td><td>−1.07E −01</td></tr>
<tr><td>dr/r, ″</td><td>1.92E −03</td><td>−6.93E −03</td><td>4.33E −02</td><td>1.47E −01</td><td>−1.46E −01</td></tr>
<tr><th colspan="6">Osculating values at JD 241 5000.5</th></tr>
<tr><td colspan="3">x = 4.88159 75692 62222D −01 AU
y = −4.80515 78943 34433D −01 AU
z = −2.47283 13064 12261D −01 AU
$\dot{x}$ = 1.48600 98031 27145D −02 AU/day
$\dot{y}$ = 1.26529 27926 43026D −02 AU/day
$\dot{z}$ = 4.75939 13961 13112D −03 AU/day</td><td colspan="3">a = 7.23325 75593 39194D −01 AU
n = 2.79628 27226 92774D −02 rad/day
e = 6.80681 87625 15753D −03
I = 5.92485 03797 83429D −02 rad
Ω = 1.33291 69380 24584D 00 rad
ω = 9.47022 97082 49603D −01 rad
M_0 = −3.11381 67358 98620D 00 rad</td></tr>
<tr><th colspan="6">Equatorial PQR matrix</th></tr>
<tr><td colspan="2">P = −6.49799 92568 24021D −01
Q = −7.57923 78984 42415D −01
R = 5.75463 75831 03330D −02</td><td colspan="2">6.76829 95693 76232D −01
−6.11403 37850 69234D −01
−4.09984 29011 62613D −01</td><td colspan="2">3.45920 89554 54408D −01
−2.27458 65017 28645D −01
9.10275 39595 80465D −01</td></tr>
</table>

Table B-3. The fitted ephemeris of earth-moon barycenter

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	7.89E −14	1.20E −08	2.81E −07	1.23E −06	−1.07E −06
dy, AU	7.40E −14	−2.71E −08	2.71E −07	1.33E −06	−8.80E −07
dz, AU	3.28E −14	2.08E −10	1.81E −07	5.92E −07	−6.46E −07
cosB dL, ″	5.25E −03	4.57E −05	7.24E −02	2.74E −01	−2.93E −01
dB, ″	1.08E −03	2.26E −03	3.27E −02	1.15E −01	−9.79E −02
dr/r, ″	1.60E −03	−1.05E −02	3.86E −02	1.62E −01	−1.56E −01

Osculating values at JD 241 5000.5

x = 1.62776 19435 35683D −01 AU	a = 1.00000 25634 13442D 00 AU
y = 8.90604 17544 57508D −01 AU	n = 1.72020 58918 00823D −02 rad/day
z = 3.86366 38283 98065D −01 AU	e = 1.67690 64295 52822D −02
$\dot{x}$ = −1.72499 81320 43152D −02 AU/day	I = 1.14128 47135 22253D −04 rad
$\dot{y}$ = 2.55556 00747 39084D −03 AU/day	Ω = −1.07891 26039 10112D −01 rad
$\dot{z}$ = 1.10842 85834 56602D −03 AU/day	ω = 1.88611 09222 58526D 00 rad
	M_0 = −3.61453 12861 02470D −01 rad

Equatorial PQR matrix

P = −2.05939 15334 21594D −01	8.97728 33075 07242D −01	3.89452 06288 86597D −01
Q = −9.78564 79855 43320D −01	−1.88922 10466 14020D −01	−8.19717 84173 73714D −02
R = −1.22895 89297 41080D −05	−3.97985 27929 78960D −01	9.17391 80141 91873D −01

Table B-4. The fitted ephemeris of Mars

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	2.28E −13	7.01E −10	4.77E −07	1.85E −06	−1.50E −06
dy, AU	1.71E −13	3.38E −09	4.14E −07	1.57E −06	−1.28E −06
dz, AU	3.83E −14	9.99E −10	1.96E −07	7.67E −07	−6.38E −07
cosB dL, ″	7.91E −03	−7.99E −05	8.89E −02	2.64E −01	−1.54E −01
dB, ″	4.75E −05	−7.56E −05	6.89E −03	2.75E −02	−3.31E −02
dr/r, ″	2.61E −04	3.27E −04	1.62E −02	5.29E −02	−5.42E −02

Osculating values at JD 241 5000.5

x = 1.33757 33773 62721D −01 AU	a = 1.52367 13186 55129D 00 AU
y = −1.30387 78434 87142D 00 AU	n = 9.14628 16919 77629D −03 rad/day
z = −6.02078 22252 04242D −01 AU	e = 9.32101 80192 46872D −02
$\dot{x}$ = 1.44852 25377 11578D −02 AU/day	I = 3.23610 16727 22402D −02 rad
$\dot{y}$ = 2.40221 52642 67937D −03 AU/day	Ω = 8.60806 75094 40483D −01 rad
$\dot{z}$ = 7.11393 61848 99518D −04 AU/day	ω = −1.29832 25395 30241D 00 rad
	M_0 = −8.85026 81027 62645D −01 rad

Equatorial PQR matrix

P = 9.05424 58177 23178D −01	−3.76009 16384 41312D −01	−1.97036 63473 49160D −01
Q = 4.23797 41354 74490D −01	8.27472 07447 49755D −01	3.68355 42378 87272D −01
R = 2.45372 97995 41911D −02	−4.17021 67170 22240D −01	9.08565 26806 70588D −01

Table B-5. The fitted ephemeris of Jupiter

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	2.12E −15	−3.71E −09	4.58E −08	1.31E −07	−1.23E −07
dy, AU	2.64E −16	−9.82E −10	1.62E −08	4.63E −08	−4.71E −08
dz, AU	5.20E −17	−1.11E −09	7.12E −09	1.99E −08	−2.21E −08
$\cos B\, dL$, ″	2.60E −06	1.15E −06	1.61E −03	4.56E −03	−5.14E −03
dB, ″	6.46E −09	−3.20E −05	7.37E −05	2.27E −04	−2.99E −04
dr/r, ″	1.22E −06	−9.41E −05	1.10E −03	3.46E −03	−4.10E −03

Osculating values at JD 241 5000.5

x = −3.19142 51765 06697D 00 AU
y = −4.01920 45093 95551D 00 AU
z = −1.64634 06237 49956D 00 AU
$\dot{x}$ = 6.00193 49645 17663D −03 AU/day
$\dot{y}$ = −3.72897 89854 49904D −03 AU/day
$\dot{z}$ = −1.74686 19631 84758D −03 AU/day

a = 5.20279 66644 36973D 00 AU
n = 1.45021 83365 04639D −03 rad/day
e = 4.87182 94535 83041D −02
I = 2.28284 56352 71518D −02 rad
Ω = 1.74267 77821 93899D 00 rad
ω = −1.51468 05539 87502D 00 rad
M_0 = −2.36749 59377 15825D 00 rad

Equatorial PQR matrix

P =	9.73864 71451 63290D −01	2.16392 66235 07493D −01	6.90053 15018 36209D −02
Q =	−2.26012 63762 94756D −01	8.93187 53307 10321D −01	3.88759 97530 38516D −01
R =	2.24901 18981 37172D −02	−3.94195 69562 24135D −01	9.18751 29828 53226D −01

Table B-6. The fitted ephemeris of Saturn

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	2.21E −15	−4.57E −10	4.70E −08	1.42E −07	−1.16E −07
dy, AU	1.76E −15	−2.42E −09	4.18E −08	1.16E −07	−1.15E −07
dz, AU	3.14E −16	−2.26E −09	1.76E −08	4.57E −08	−5.05E −08
$\cos B\, dL$, ″	1.78E −06	−2.10E −06	1.33E −03	2.77E −03	−3.28E −03
dB, ″	3.53E −09	−2.71E −05	5.28E −05	1.48E −04	−2.09E −04
dr/r, ″	2.73E −07	2.74E −05	5.22E −04	1.51E −03	−2.04E −03

Osculating values at JD 241 5000.5

x = −5.94956 02928 19203D −01 AU
y = −9.29550 93103 32007D 00 AU
z = −3.81681 73382 18680D 00 AU
$\dot{x}$ = 5.26756 59689 84819D −03 AU/day
$\dot{y}$ = −2.40510 33329 67816D −04 AU/day
$\dot{z}$ = −3.26617 76771 81013D −04 AU/day

a = 9.57974 17954 12653D 00 AU
n = 5.80246 78247 81564D −04 rad/day
e = 5.12004 11642 01615D −02
I = 4.34027 64243 69105D −02 rad
Ω = 1.97873 81929 09210D 00 rad
ω = −3.45741 34833 17933D −01 rad
M_0 = 3.00693 80341 14843D 00 rad

Equatorial PQR matrix

P =	−6.24533 81900 41049D −02	9.21397 20167 31201D −01	3.83571 07794 79403D −01
Q =	−9.97252 85568 65767D −01	−7.29479 13285 67131D −02	1.28586 06932 06811D −02
R =	3.98285 94177 65996D −02	−3.81714 28935 29262D −01	9.23421 83447 74078D −01

Table B-7. The fitted ephemeris of Uranus

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	9.49E −17	1.27E −09	9.66E −09	2.94E −08	−3.12E −08
dy, AU	8.75E −17	−1.02E −10	9.35E −09	3.11E −08	−2.97E −08
dz, AU	1.76E −17	−6.74E −11	4.20E −09	1.28E −08	−1.17E −08
$\cos B\, dL$, ″	1.10E −08	8.42E −08	1.05E −04	3.07E −04	−3.49E −04
dB, ″	1.52E −10	1.27E −07	1.23E −05	4.78E −05	−3.90E −05
dr/r, ″	1.22E −08	1.12E −06	1.11E −04	3.07E −04	−3.57E −04

Osculating values at JD 241 5000.5

x = −6.76950 07781 13280D 00 AU
y = −1.62850 11314 68881D 01 AU
z = −7.03982 80081 07494D 00 AU
$\dot{x}$ = 3.64948 04881 61621D −03 AU/day
$\dot{y}$ = −1.43631 39888 26346D −03 AU/day
$\dot{z}$ = −6.80173 51596 69874D −04 AU/day

a = 1.93133 80668 68614D 01 AU
n = 2.02676 54190 87620D −04 rad/day
e = 4.89964 59177 76881D −02
I = 1.35150 36183 18392D −02 rad
Ω = 1.28781 16045 11192D 00 rad
ω = 1.79256 81297 13942D 00 rad
M_0 = 1.17501 96098 39210D 00 rad

Equatorial PQR matrix

P = −9.98041 52757 27898D −01	5.08555 84225 01821D −02	3.64255 23926 29153D −02
Q = −6.11939 87937 02395D −02	−9.14530 75215 55767D −01	−3.99860 97484 26547D −01
R = 1.29770 98309 56381D −02	−4.01306 88122 04531D −01	9.15851 72490 12400D −01

Table B-8. The fitted ephemeris of Neptune

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	9.12E −16	1.40E −10	3.02E −08	6.78E −08	−7.61E −08
dy, AU	8.16E −16	−3.99E −09	2.83E −08	5.70E −08	−5.74E −08
dz, AU	1.64E −16	−3.96E −09	1.22E −08	2.79E −08	−2.59E −08
$\cos B\, dL$, ″	2.02E −08	3.97E −05	1.36E −04	4.19E −04	−4.22E −04
dB, ″	9.32E −10	−1.40E −05	2.71E −05	7.70E −05	−6.13E −05
dr/r, ″	6.78E −08	−2.45E −04	8.77E −05	−1.73E −05	−4.71E −04

Osculating values at JD 241 5000.5

x = 1.94125 69860 33593D 00 AU
y = 2.76001 39691 69463D 01 AU
z = 1.12571 31503 15774D 01 AU
$\dot{x}$ = −3.14313 20212 10028D −03 AU/day
$\dot{y}$ = 1.75391 80187 76925D −04 AU/day
$\dot{z}$ = 1.50866 52286 66874D −04 AU/day

a = 2.99488 67026 67926D 01 AU
n = 1.04959 58088 43447D −04 rad/day
e = 5.33020 97343 25422D −03
I = 3.09525 01741 01840D −02 rad
Ω = 2.29121 13328 21336D 00 rad
ω = −1.84950 72404 15644D 00 rad
M_0 = 1.05450 12555 94580D 00 rad

Equatorial PQR matrix

P = 9.03678 42422 59001D −01	4.03746 09728 35746D −01	1.42668 82811 90965D −01
Q = −4.27579 66183 77455D −01	8.32693 36959 65117D −01	3.51848 52566 51315D −01
R = 2.32580 81849 40044D −02	−3.78960 21052 12230D −01	9.25120 65184 51511D −01

Table B-9. The fitted ephemeris of Pluto

Statistical summary (modified source minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	3.01E −16	−1.49E −09	1.73E −08	4.03E −08	−7.45E −08
dy, AU	3.65E −16	−5.82E −09	1.82E −08	4.54E −08	−5.42E −08
dz, AU	3.24E −16	−1.92E −09	1.79E −08	7.45E −08	−3.54E −08
cos*B* *dL*, ″	7.47E −09	−5.75E −06	8.62E −05	1.85E −04	−4.97E −04
dB, ″	1.01E −08	−4.05E −06	1.00E −04	4.71E −04	−2.04E −04
dr/*r*, ″	1.39E −08	−7.57E −05	9.02E −05	4.01E −04	−2.73E −04

Osculating values at JD 241 5000.5

x =	1.08898 33456 98867D 01 AU	a =	3.93162 65690 03125D 01 AU
y =	4.44014 79876 26816D 01 AU	n =	6.97788 25104 84292D −05 rad/day
z =	1.07217 00090 29059D 01 AU	e =	2.53032 43405 15995D −01
$\dot{x}$ =	−2.15764 99130 21905D −03 AU/day	I =	2.99767 12637 47195D −01 rad
$\dot{y}$ =	−2.10708 08193 74032D −05 AU/day	Ω =	1.91232 78510 53723D 00 rad
$\dot{z}$ =	6.48833 61690 99614D −04 AU/day	ω =	1.99627 89789 48910D 00 rad
		M_0 =	−2.28468 75731 26375D 00 rad

Equatorial PQR matrix

P =	−6.81713 38551 66848D −01	−7.31227 22445 70322D −01	−2.39500 77666 77484D −02
Q =	6.76644 78112 95817D −01	−6.17701 17309 89745D −01	−4.00745 68110 24311D −01
R =	2.78242 16103 50953D −01	−2.89399 39005 63995D −01	9.15876 24319 96500D −01

Appendix C

Residual Plots, Modified Source Minus DE28

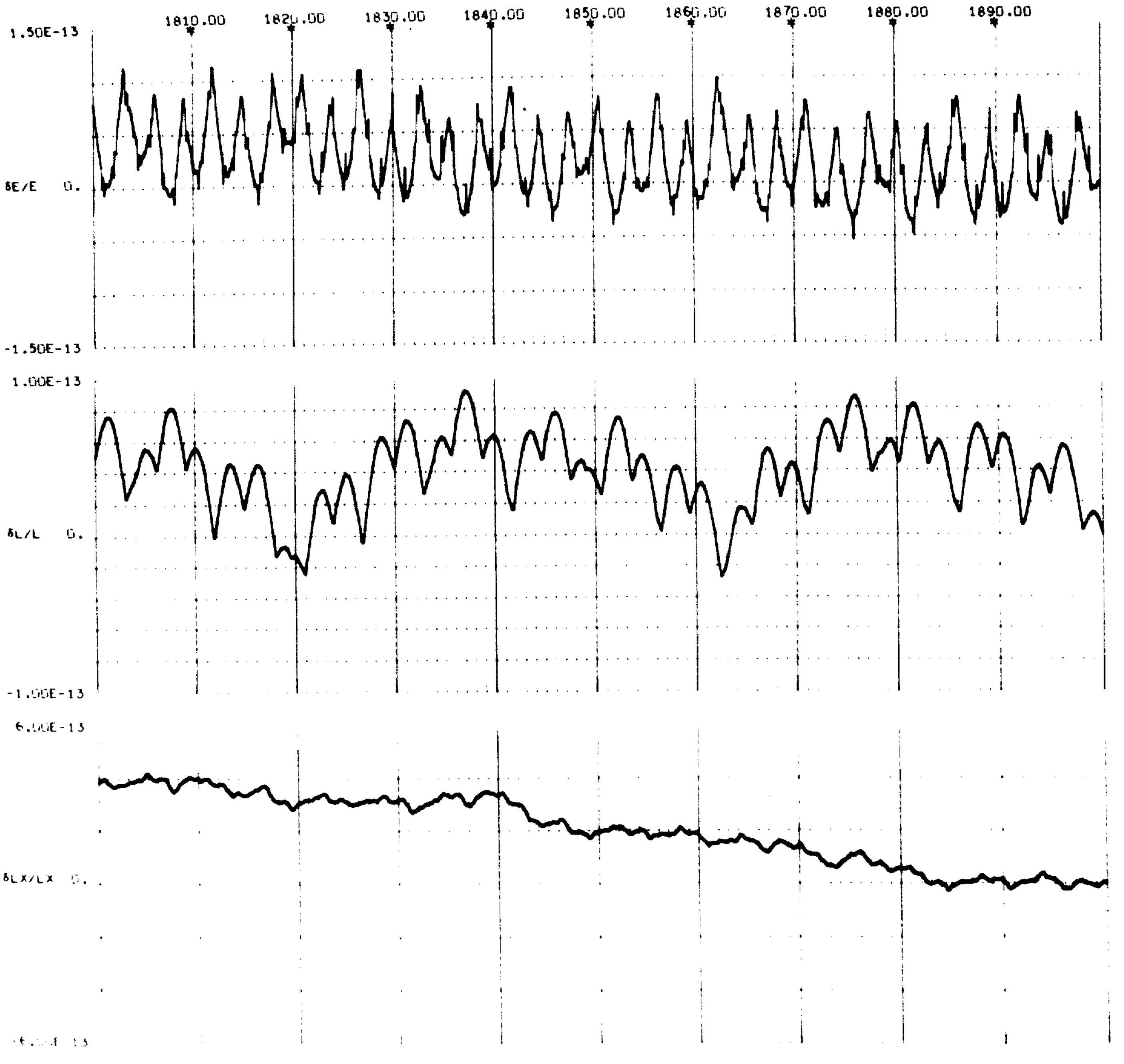


Fig. C-1. Relative changes in energy and angular momentum integrals for 1800–2000

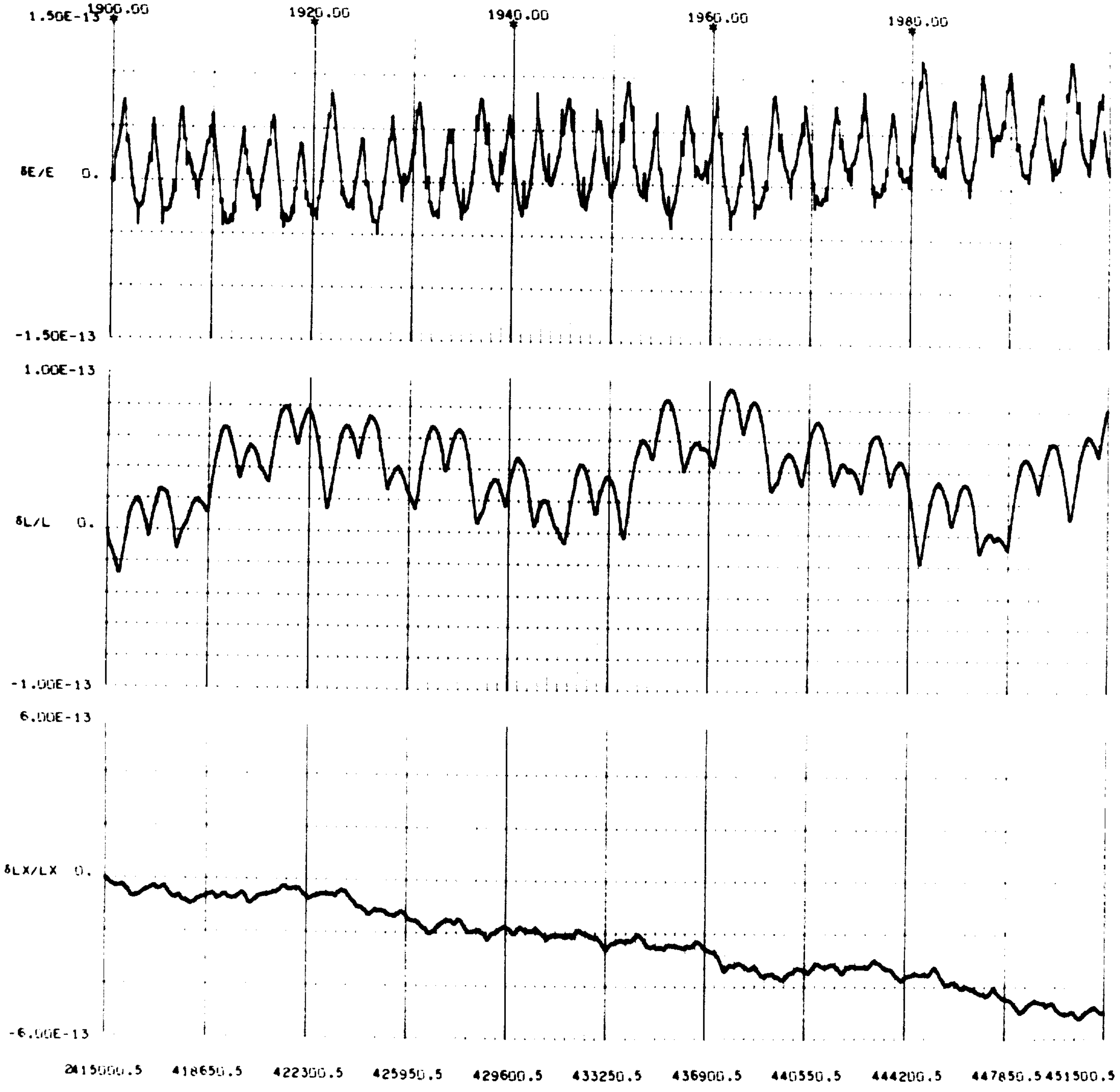

1.50E-13
1900.00
1920.00
1940.00
1960.00
1980.00
δE/E 0.
-1.50E-13
1.00E-13
δL/L 0.
-1.00E-13
6.00E-13
δLX/LX 0.
-6.00E-13
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-1 (contd)

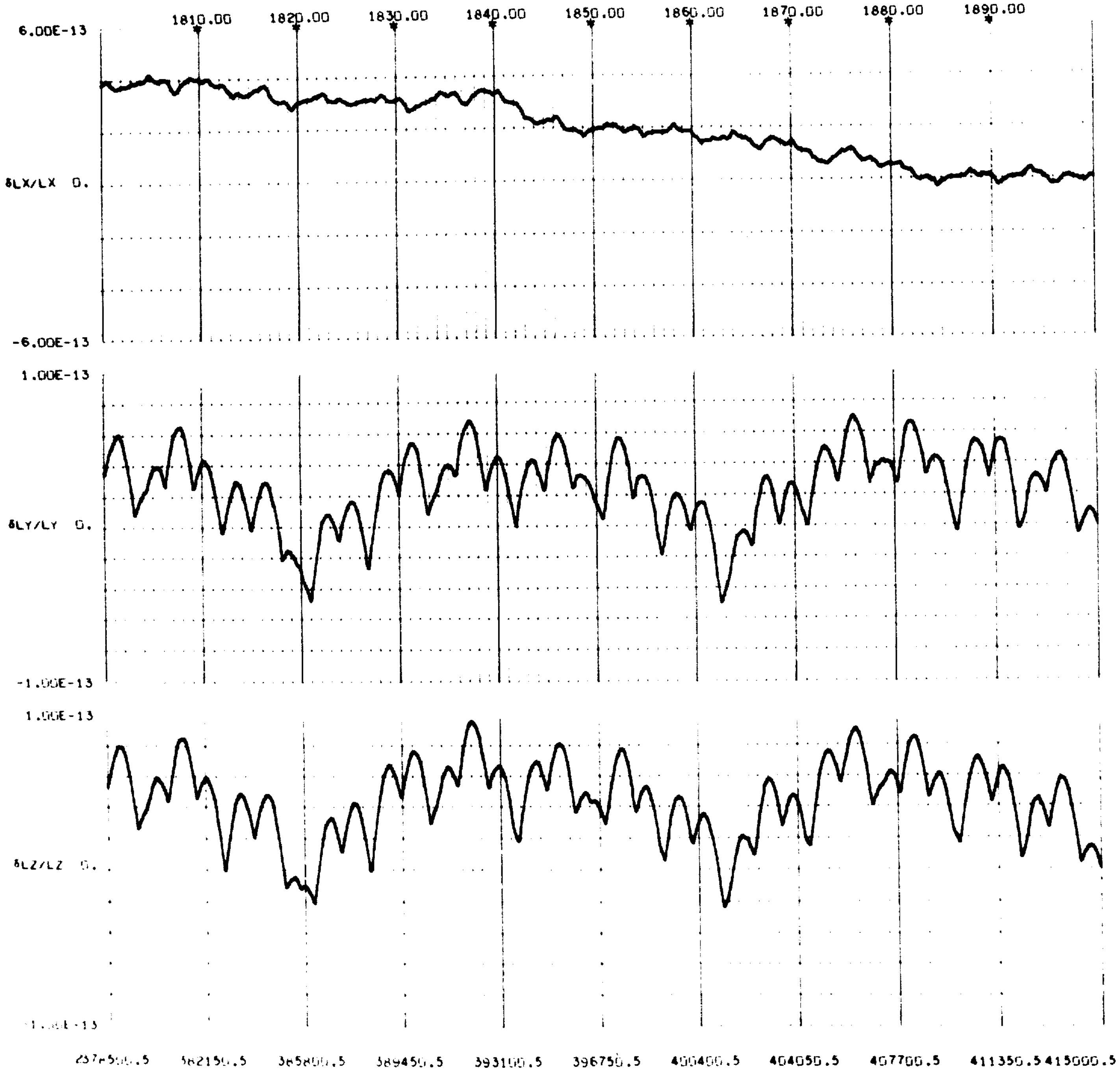


Fig. C-2. Relative changes in angular momentum components for 1800–2000

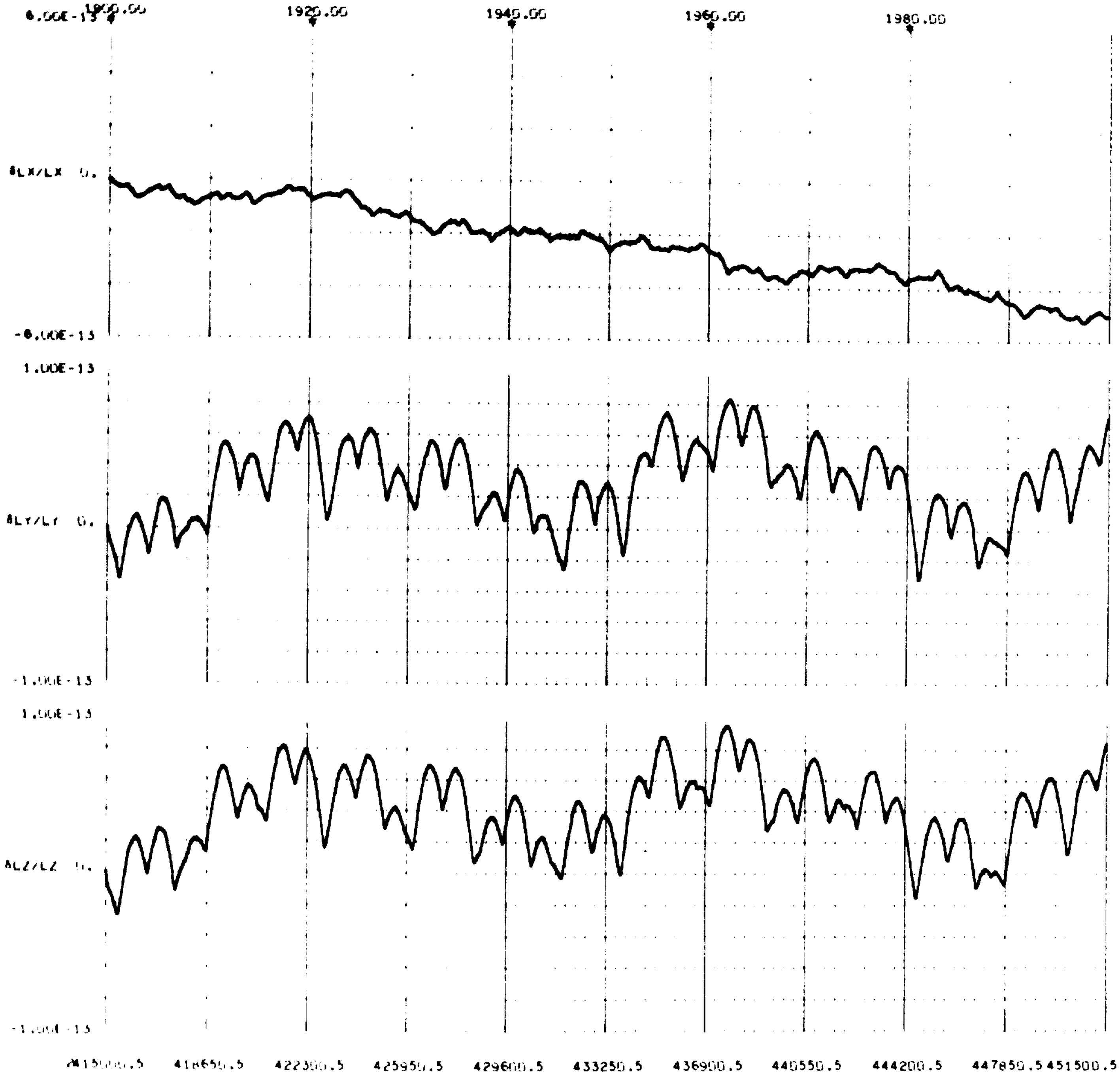

1900.00
1920.00
1940.00
1960.00
1980.00
8.00E-13
δLX/LX 0.
-8.00E-13
1.00E-13
δLY/LY 0.
-1.00E-13
1.00E-13
δLZ/LZ 0.
-1.00E-13
415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-2 (contd)

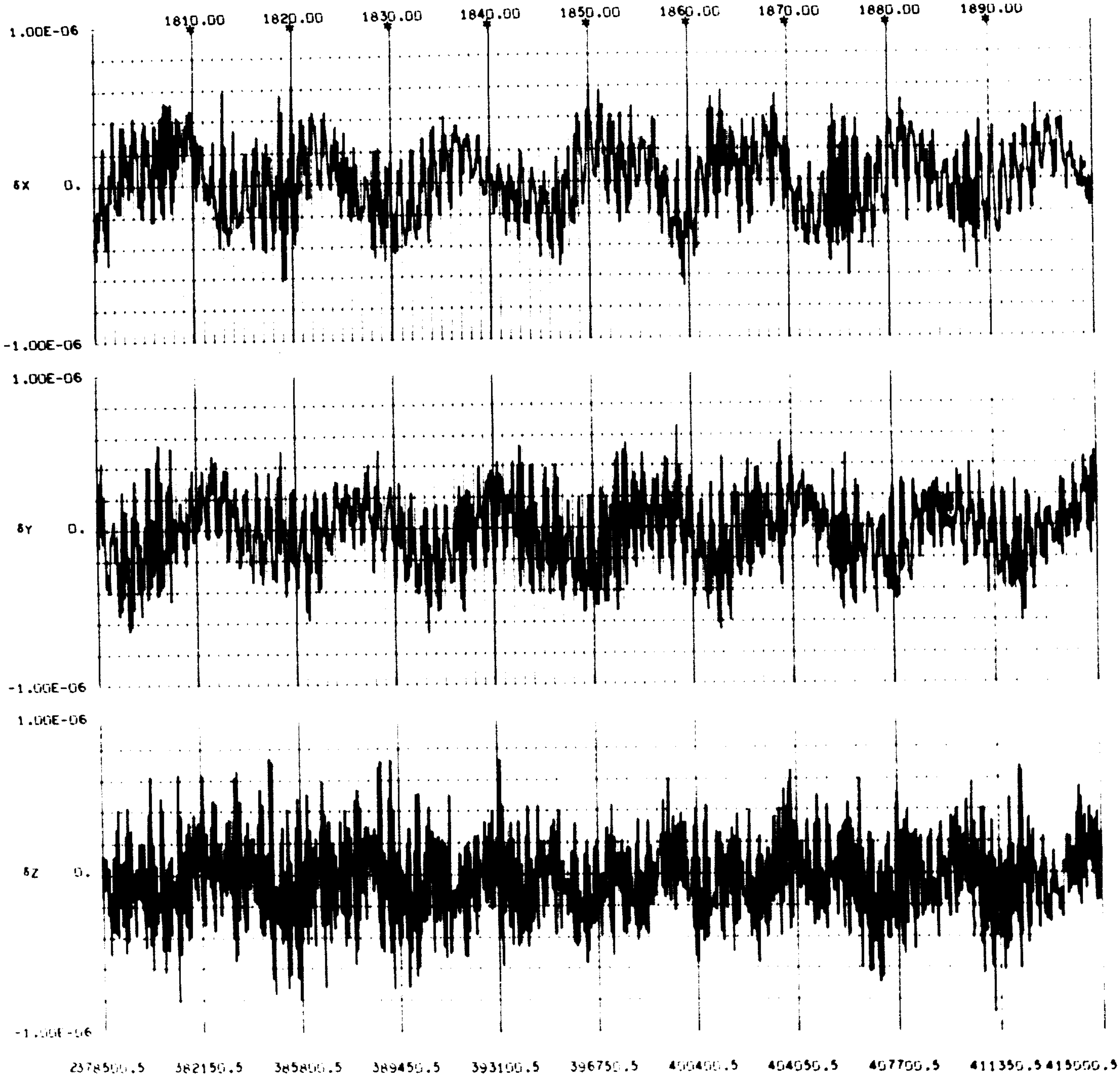


Fig. C-3. Mercury rectangular residuals (modified source minus DE28) for 1800–2000

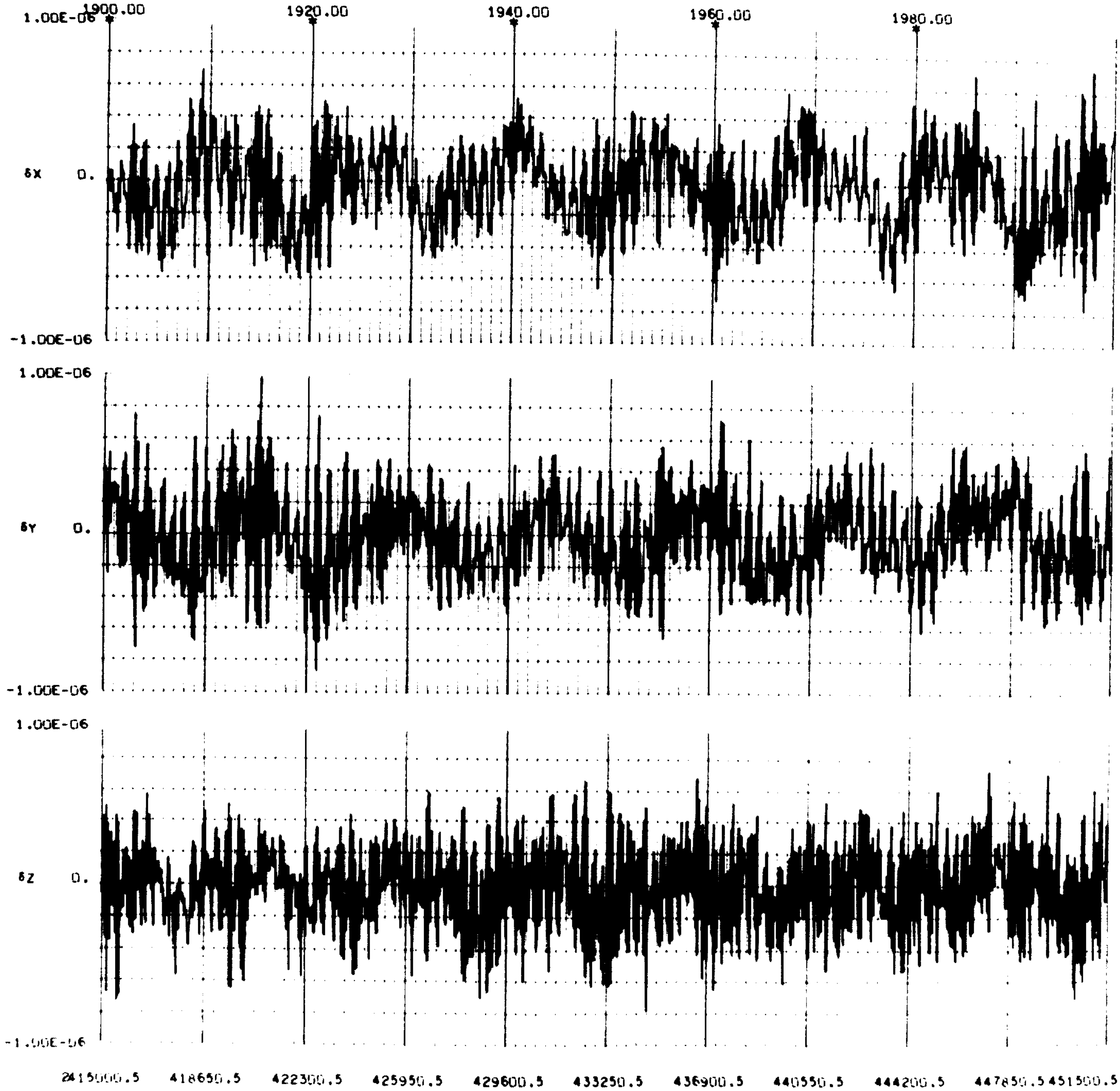

1900.00
1920.00
1940.00
1960.00
1980.00
1.00E-06
δX 0.
-1.00E-06
1.00E-06
δY 0.
-1.00E-06
1.00E-06
δZ 0.
-1.00E-06
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-3 (contd)

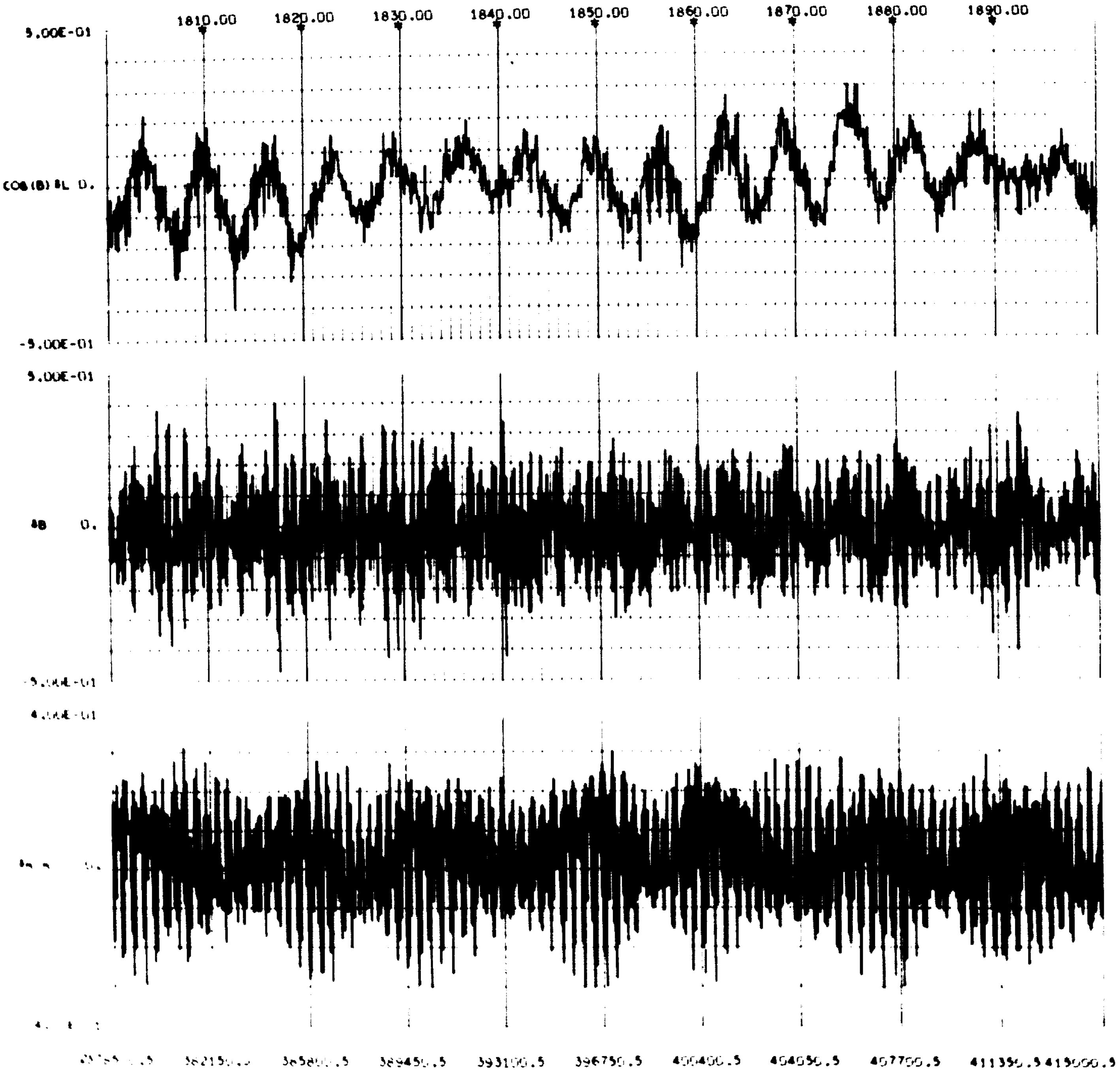


Fig. C-4. Mercury angular residuals (modified source minus DE28) for 1800–2000

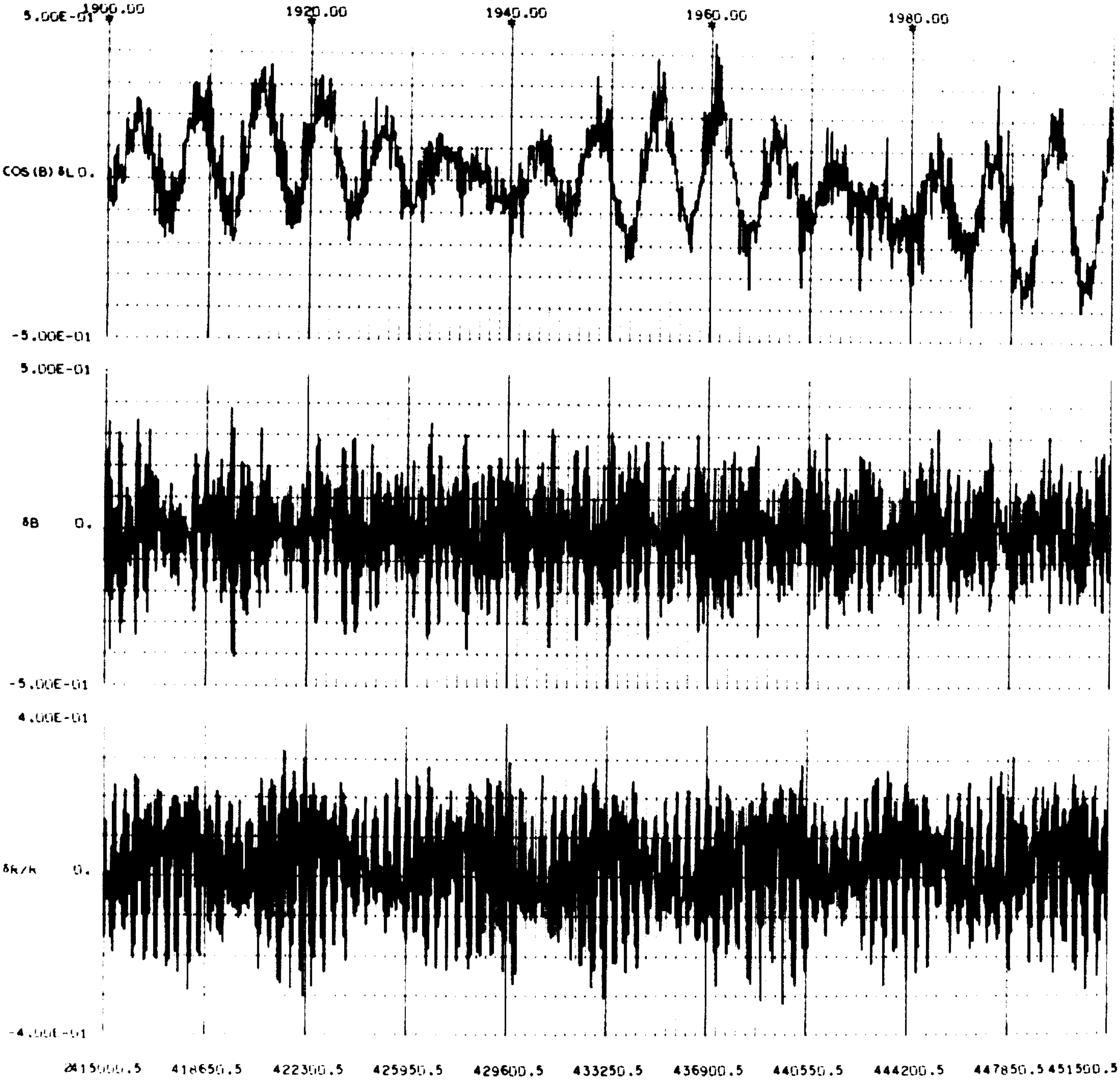

1900.00
1920.00
1940.00
1960.00
1980.00
5.00E-01
COS(B) δL 0.
-5.00E-01
5.00E-01
δB 0.
-5.00E-01
4.00E-01
δR/R 0.
-4.00E-01
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-4 (contd)

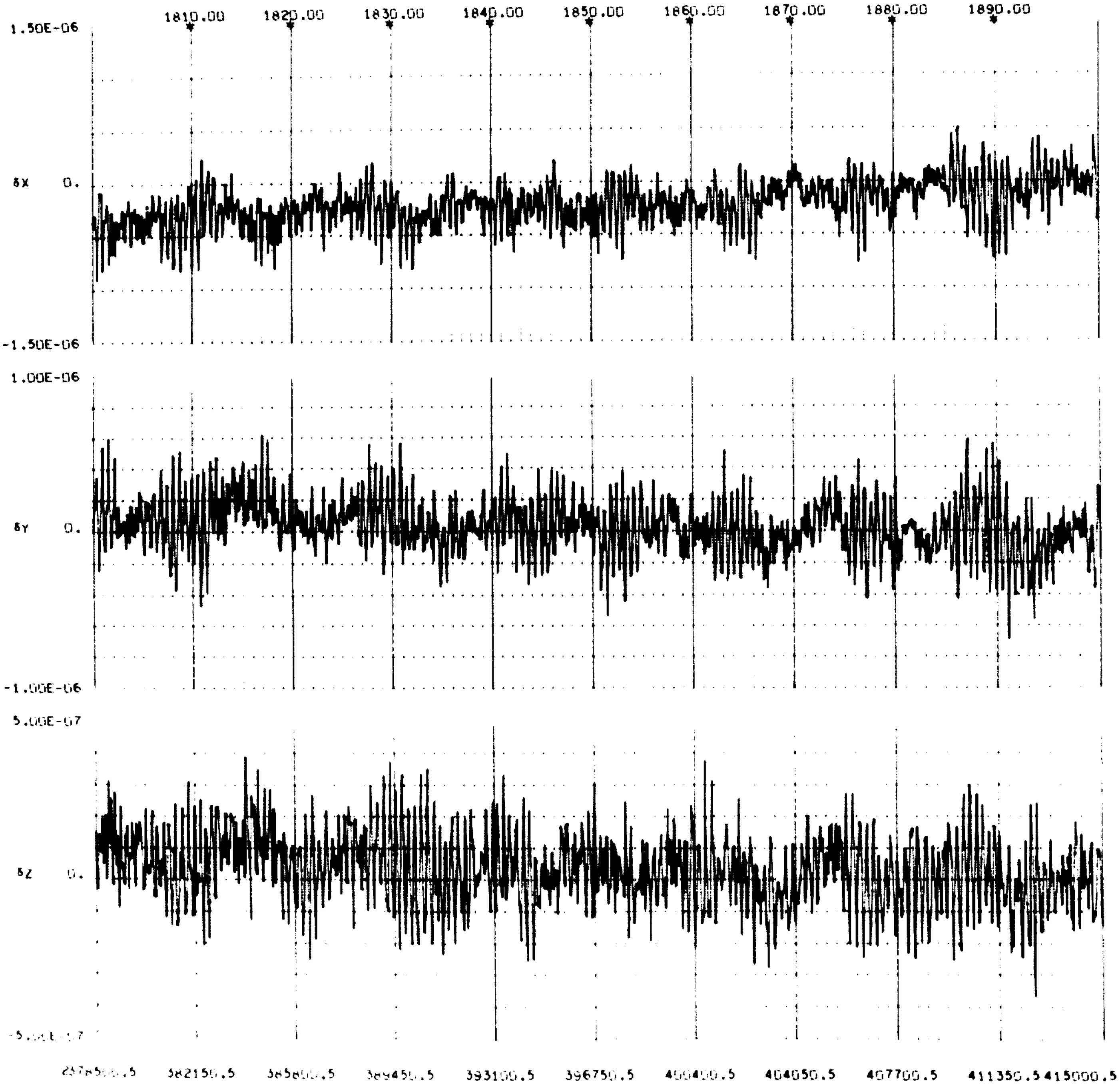


Fig. C-5. Venus rectangular residuals (modified source minus DE28) for 1800–2000

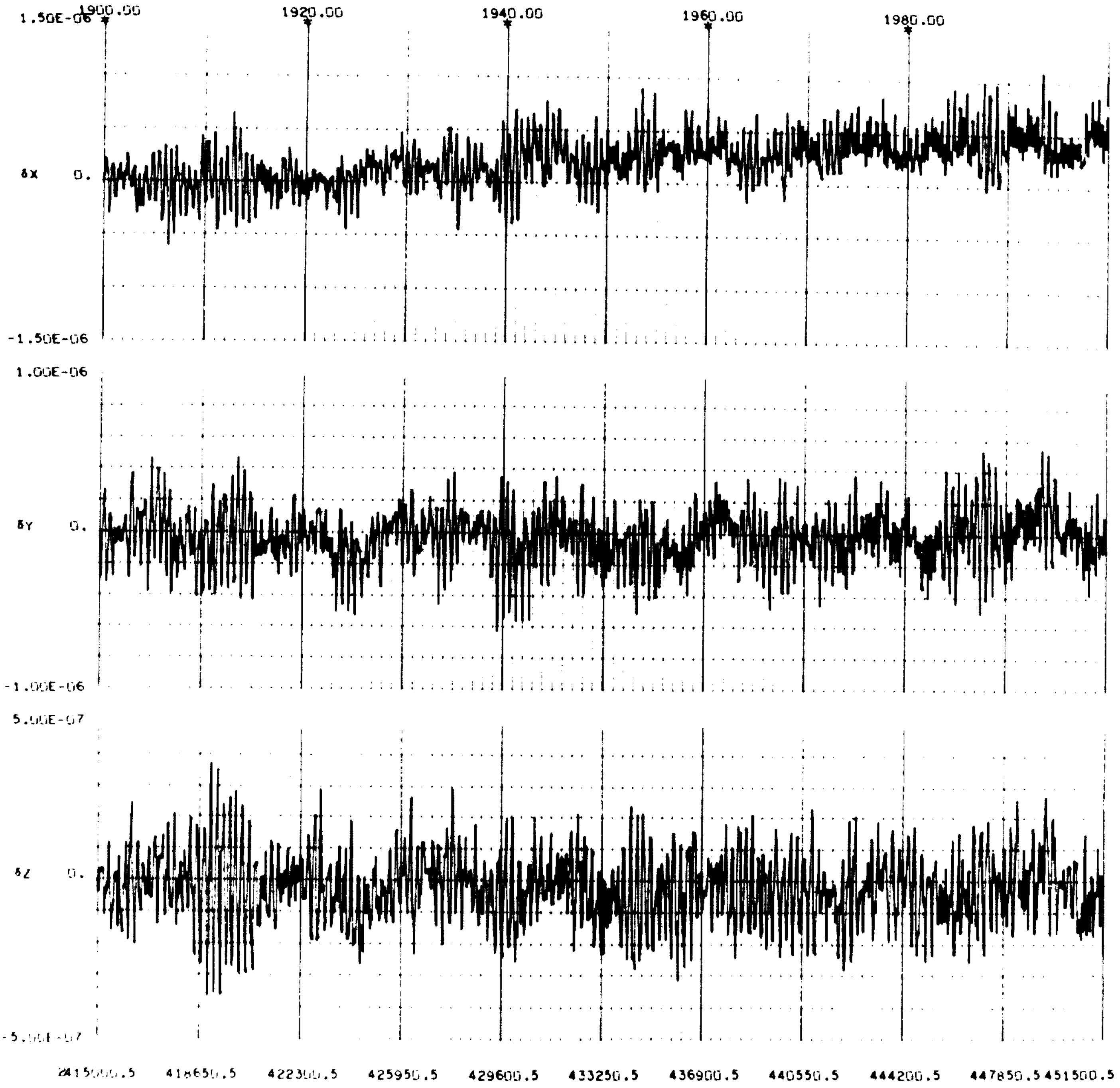

1.50E-06
1900.00
1920.00
1940.00
1960.00
1980.00
δX
0.
-1.50E-06
1.00E-06
δY
0.
-1.00E-06
5.00E-07
δZ
0.
-5.00E-07
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-5 (contd)

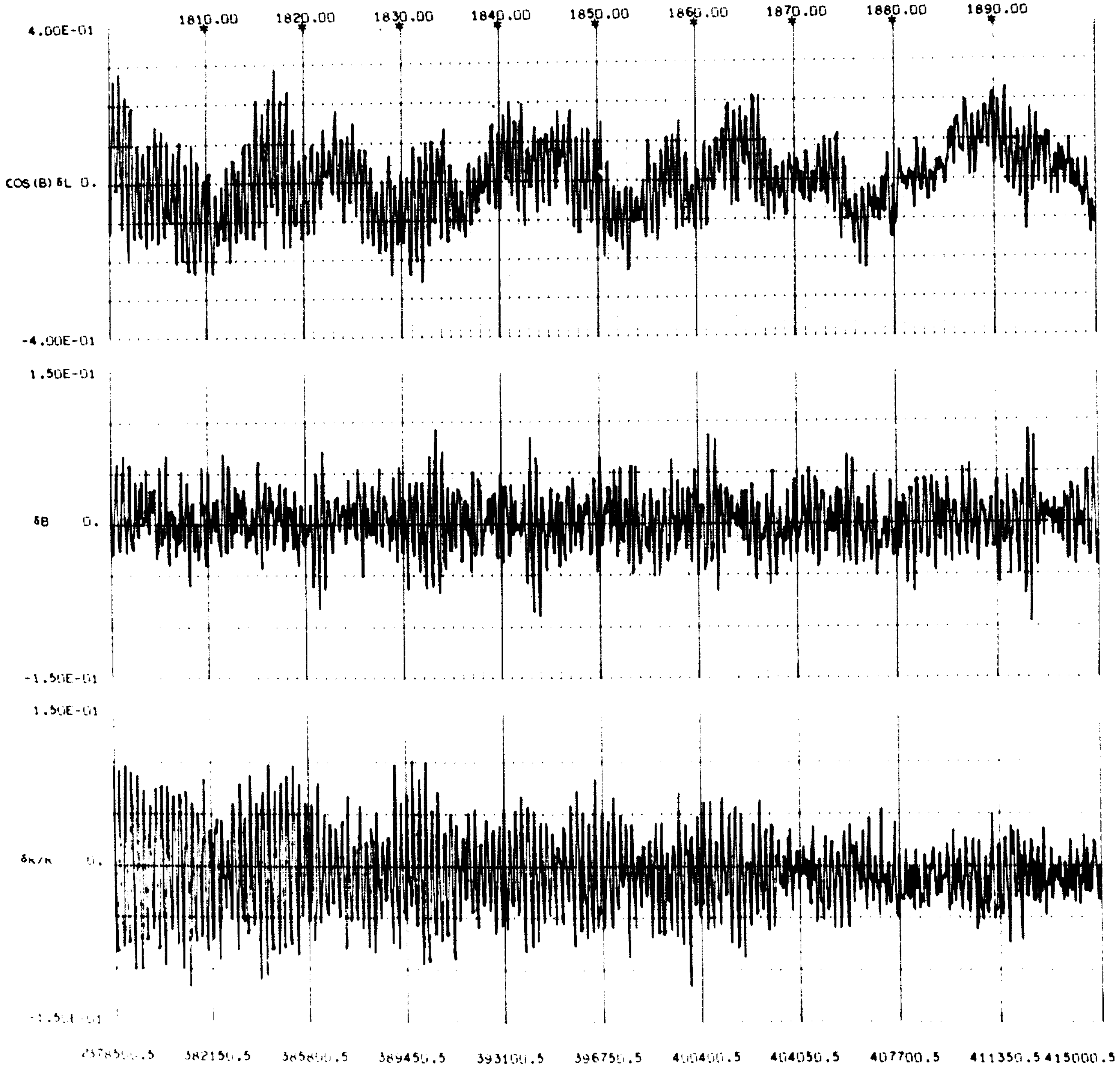


Fig. C-6. Venus angular residuals (modified source minus DE28) for 1800–2000

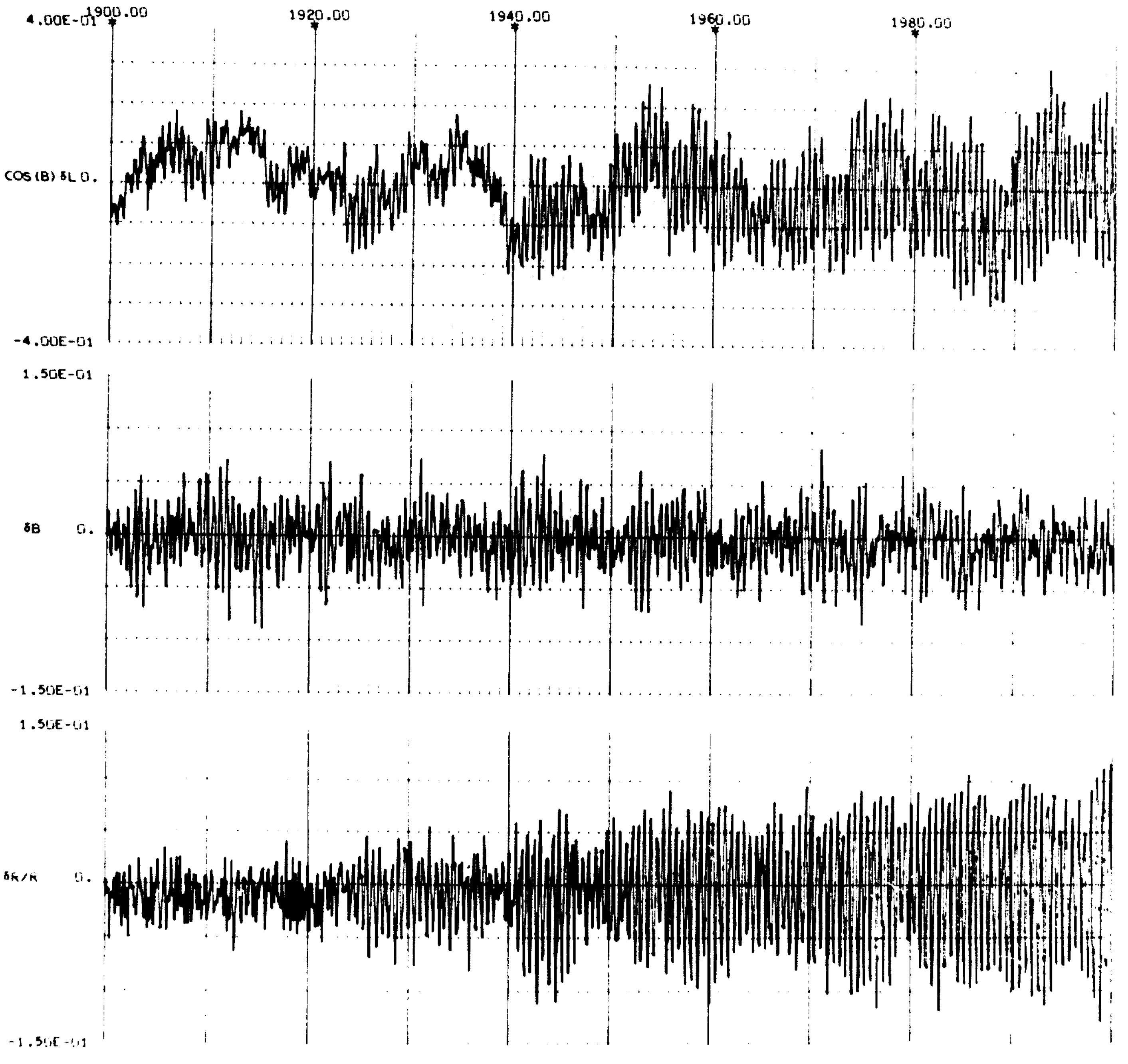

1900.00
1920.00
1940.00
1960.00
1980.00
4.00E-01
COS(B) δL 0.
-4.00E-01
1.50E-01
δB 0.
-1.50E-01
1.50E-01
δR/R 0.
-1.50E-01
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-6 (contd)

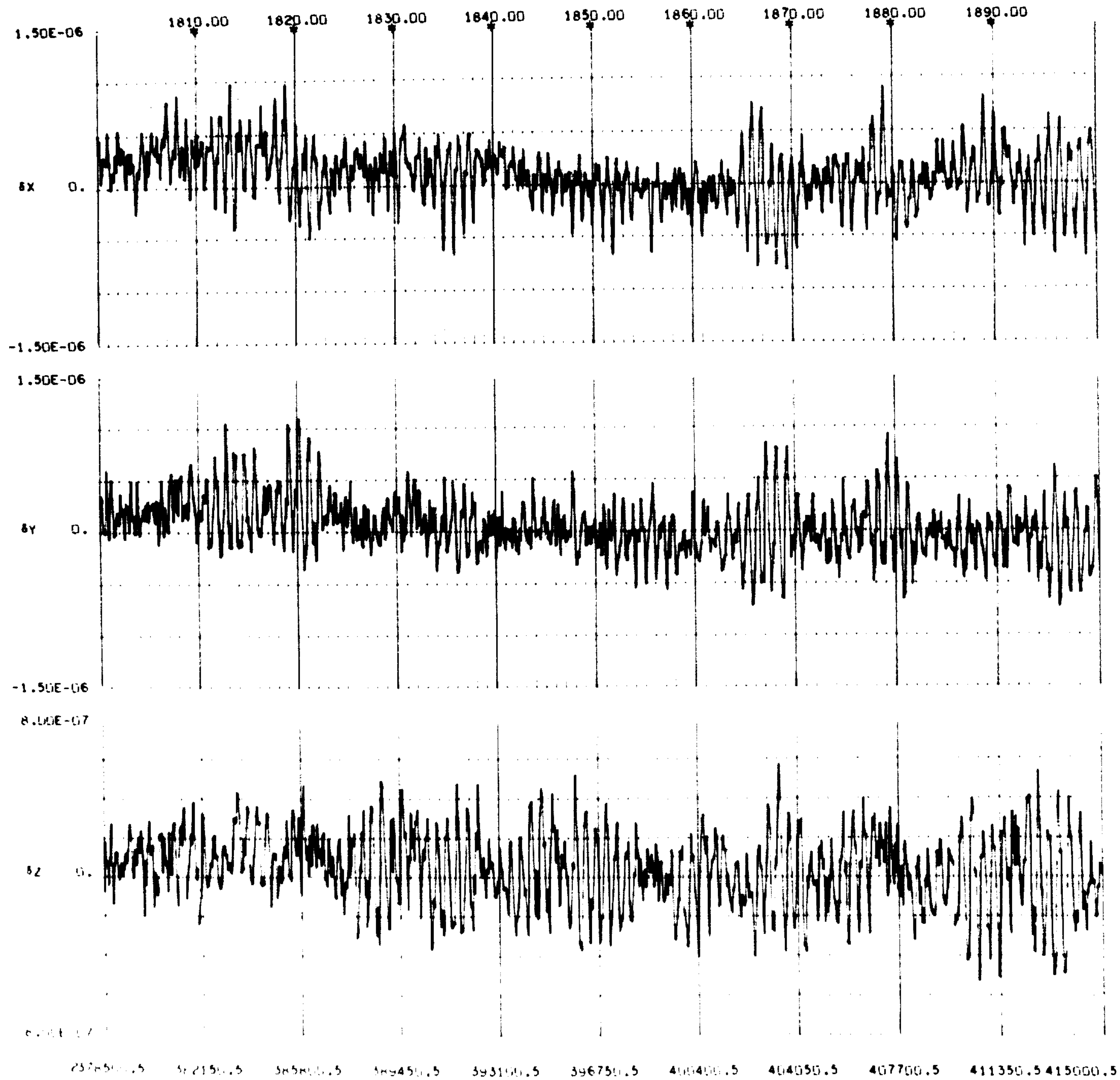


Fig. C-7. Earth-moon barycenter rectangular residuals (modified source minus DE28) for 1800–2000

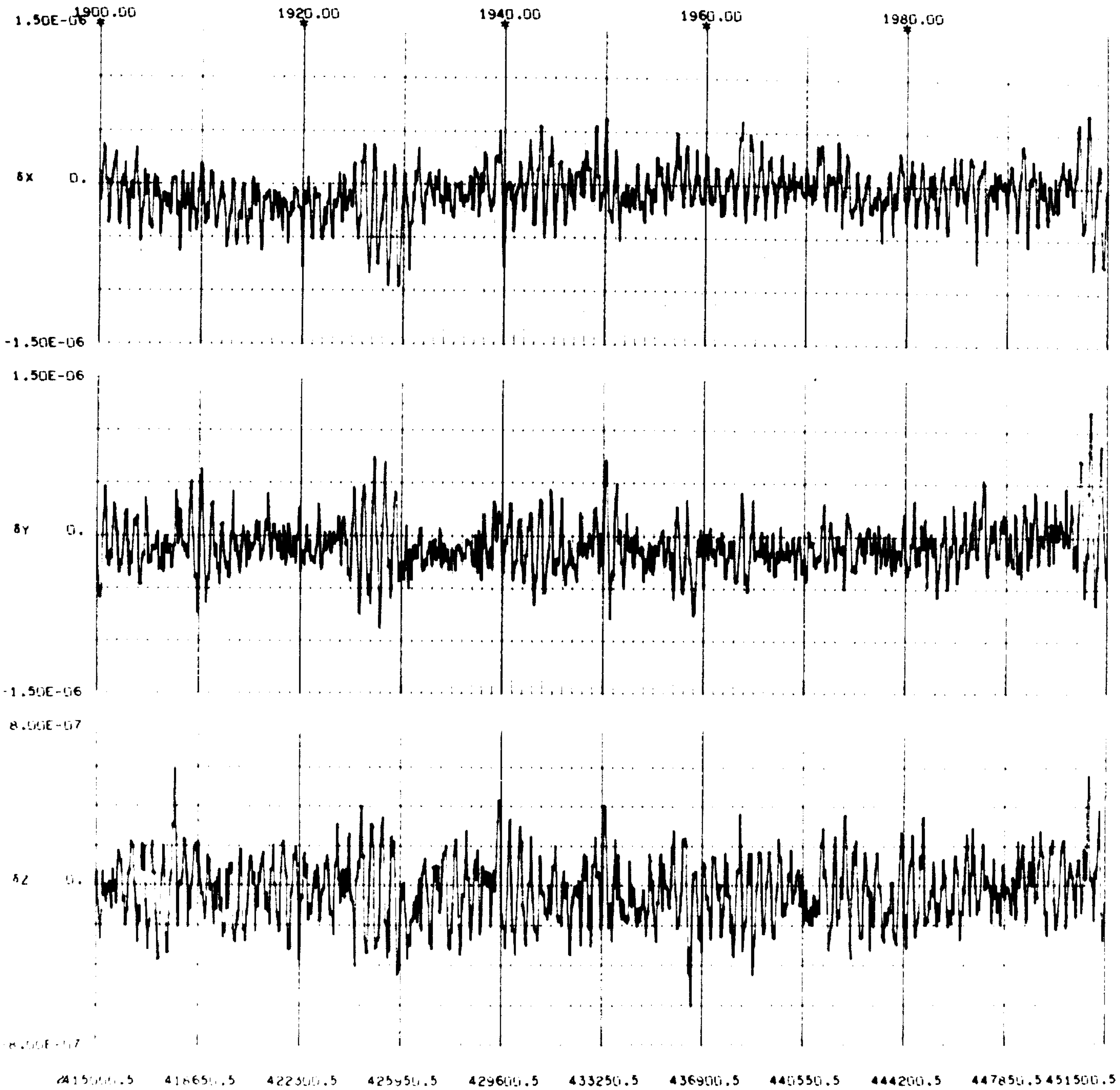
1900.00
1920.00
1940.00
1960.00
1980.00
1.50E-06
δX
0.
-1.50E-06
1.50E-06
δY
0.
-1.50E-06
8.00E-07
δZ
0.
-8.00E-07
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-7 (contd)

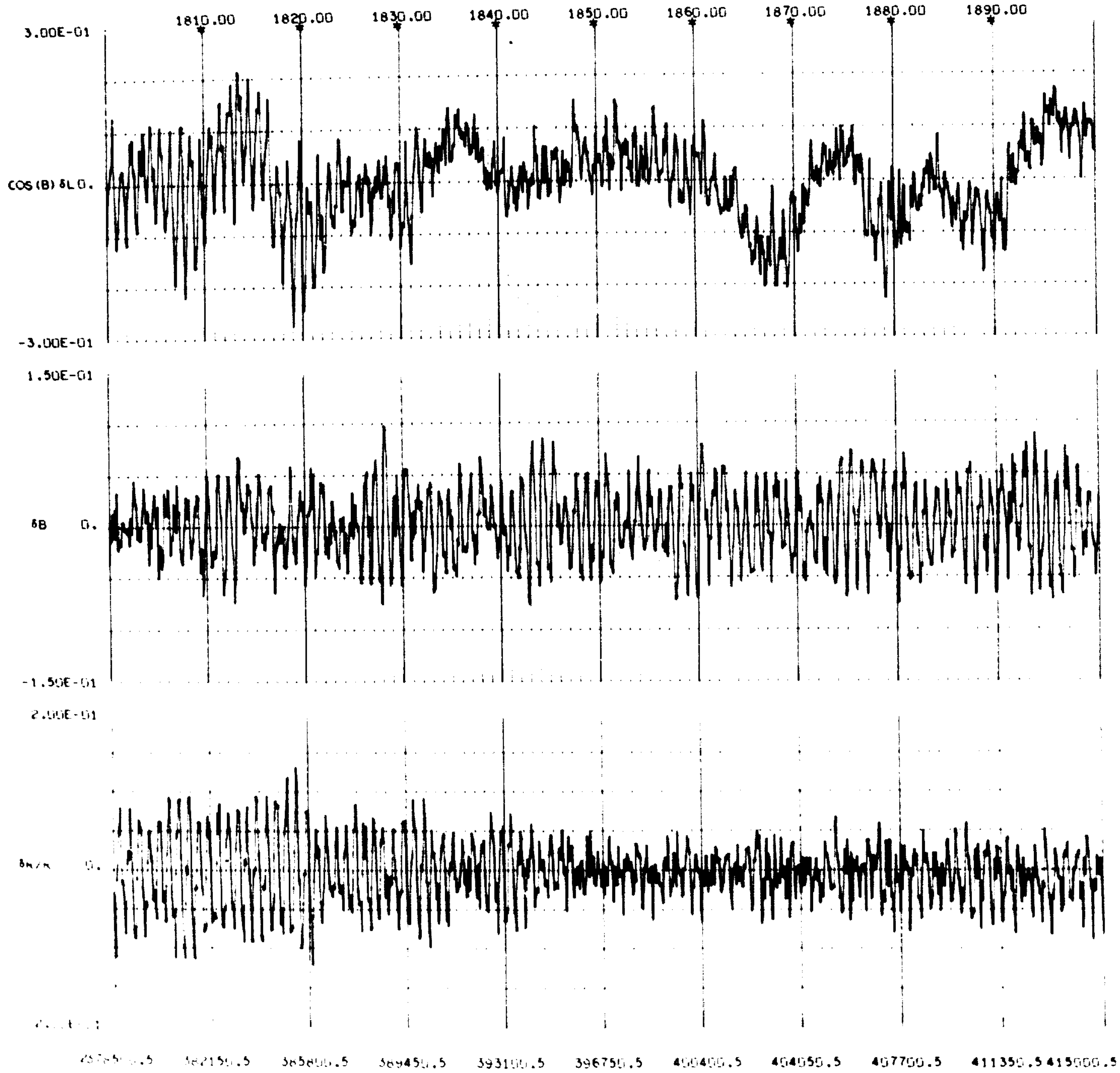


Fig. C-8. Earth-moon barycenter angular residuals (modified source minus DE28) for 1800–2000

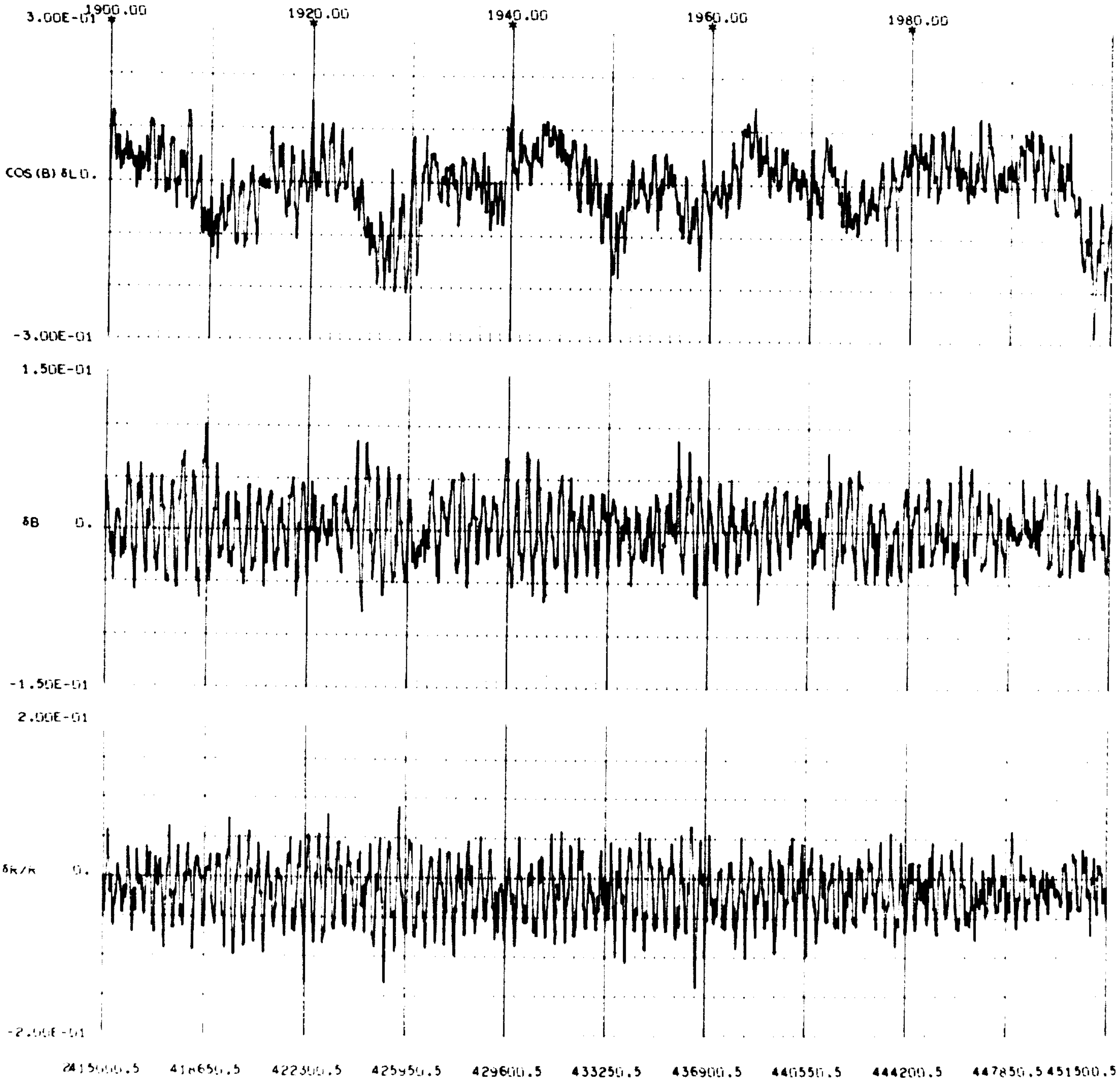

1900.00
1920.00
1940.00
1960.00
1980.00
3.00E-01
COS(B) δL0.
-3.00E-01
1.50E-01
δB 0.
-1.50E-01
2.00E-01
δR/R 0.
-2.00E-01
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-8 (contd)

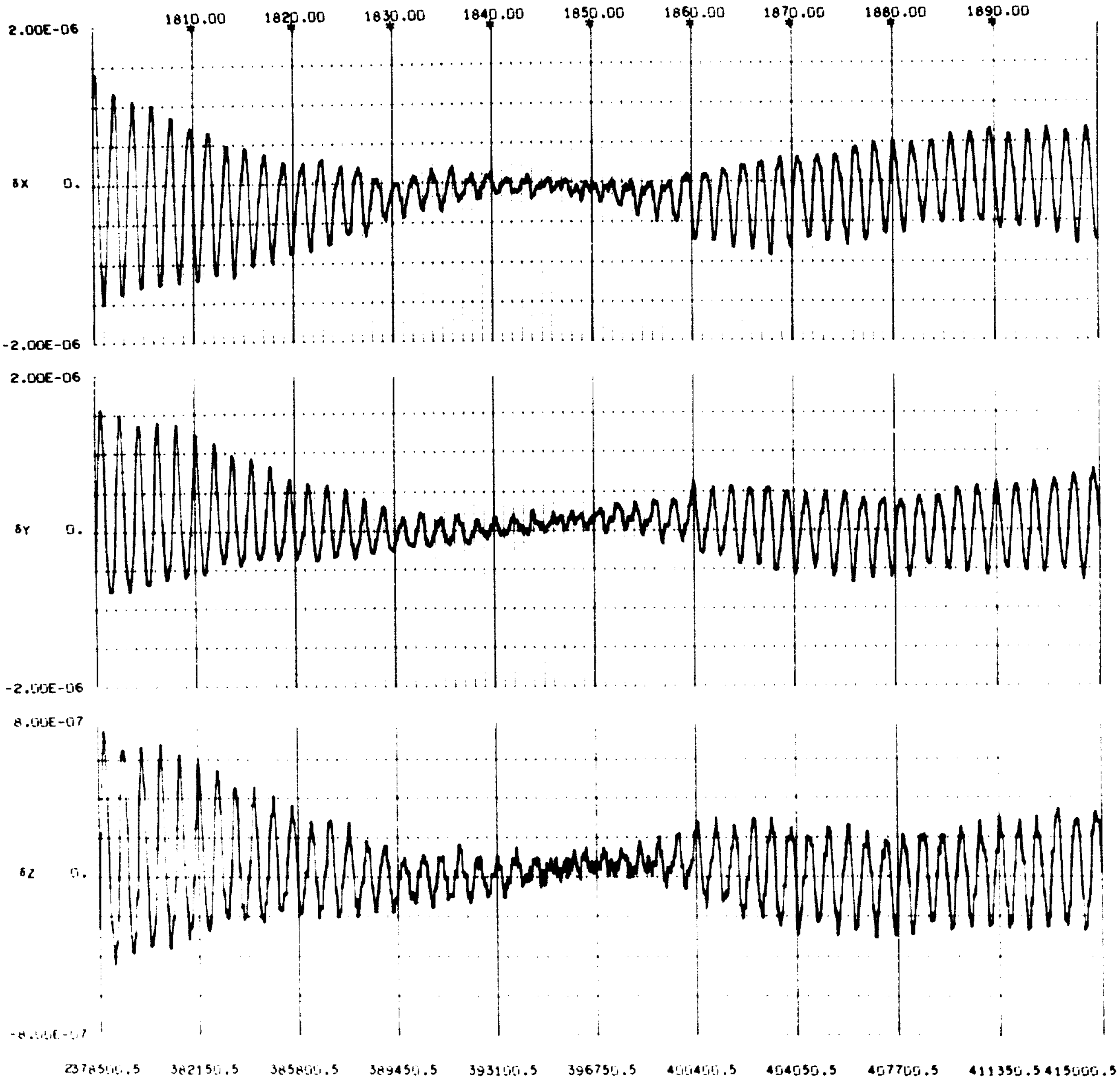


Fig. C-9. Mars rectangular residuals (modified source minus DE28) for 1800–2000

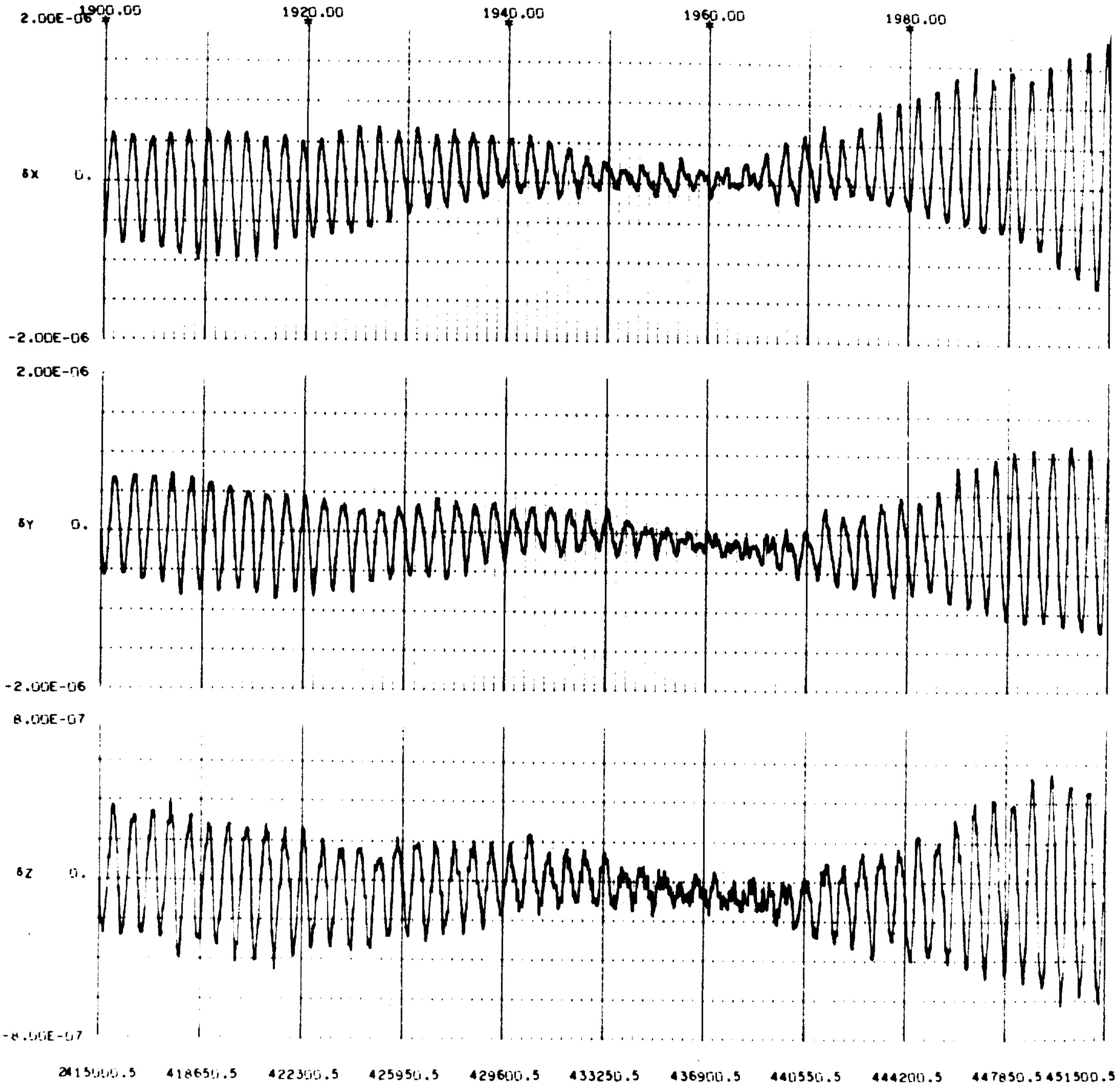

1900.00
1920.00
1940.00
1960.00
1980.00
2.00E-06
δX
0.
-2.00E-06
2.00E-06
δY
0.
-2.00E-06
8.00E-07
δZ
0.
-8.00E-07
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. C-9 (contd)

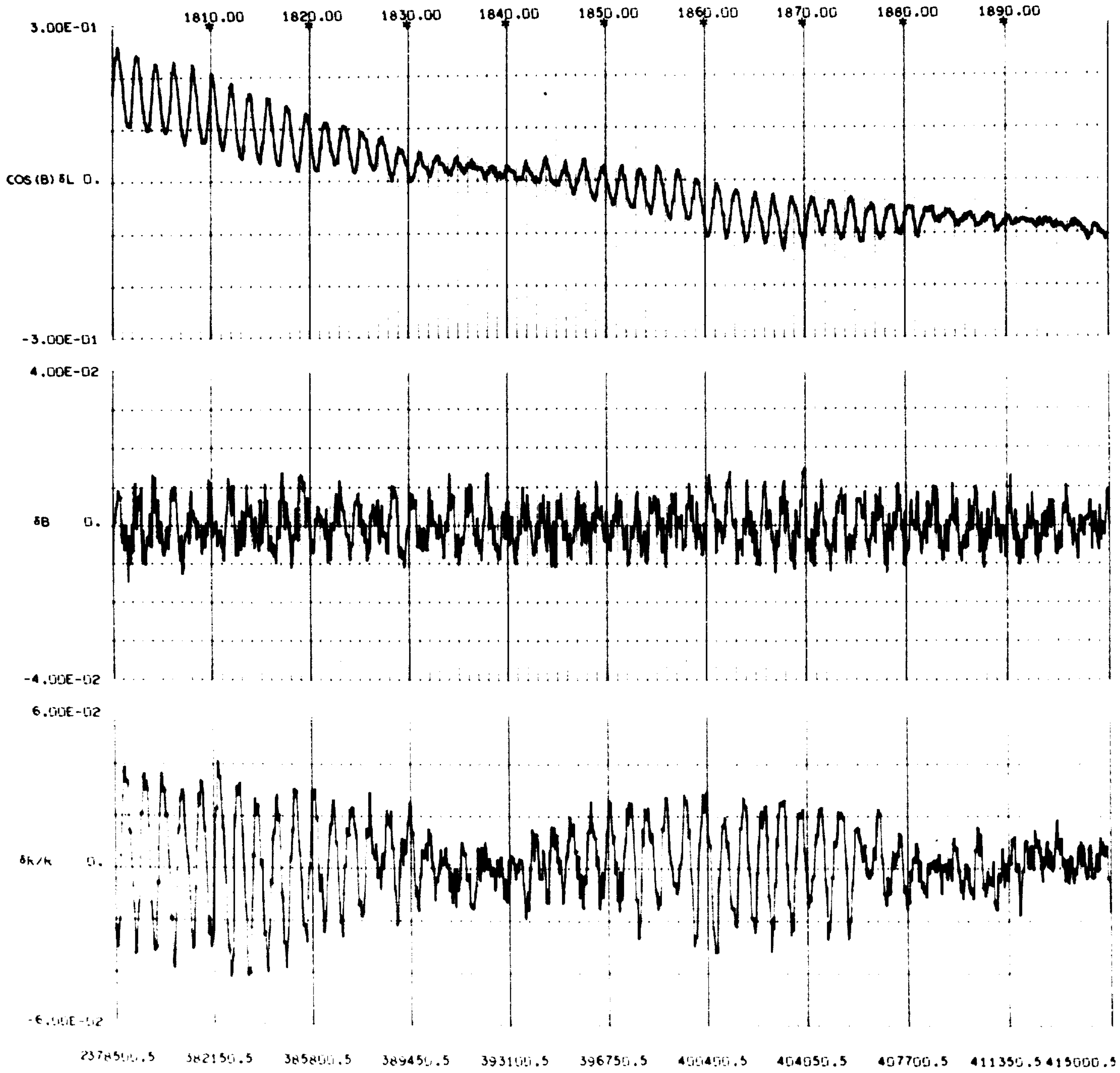


Fig. C-10. Mars angular residuals (modified source minus DE28) for 1800–2000

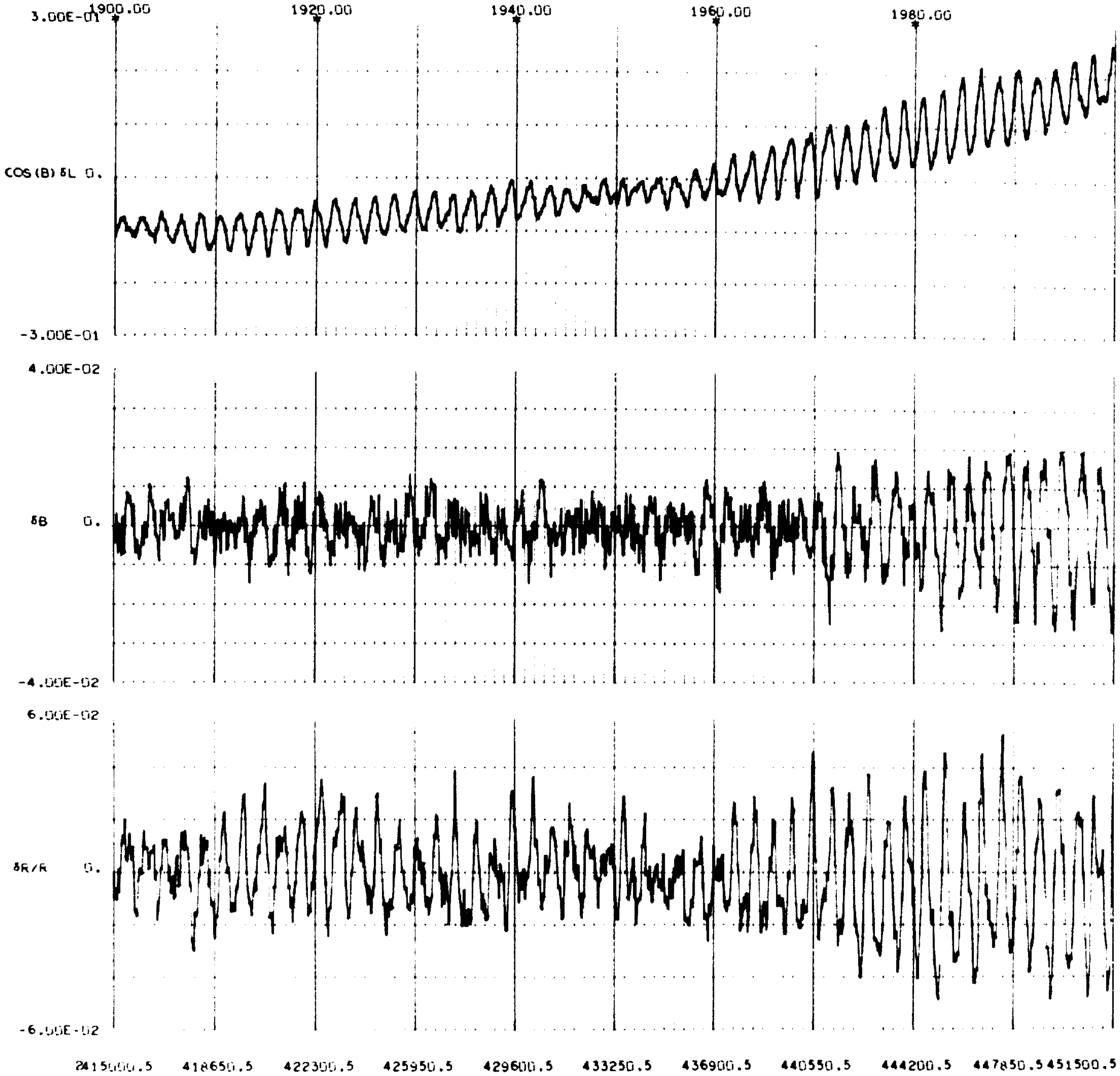


Fig. C-10 (contd)

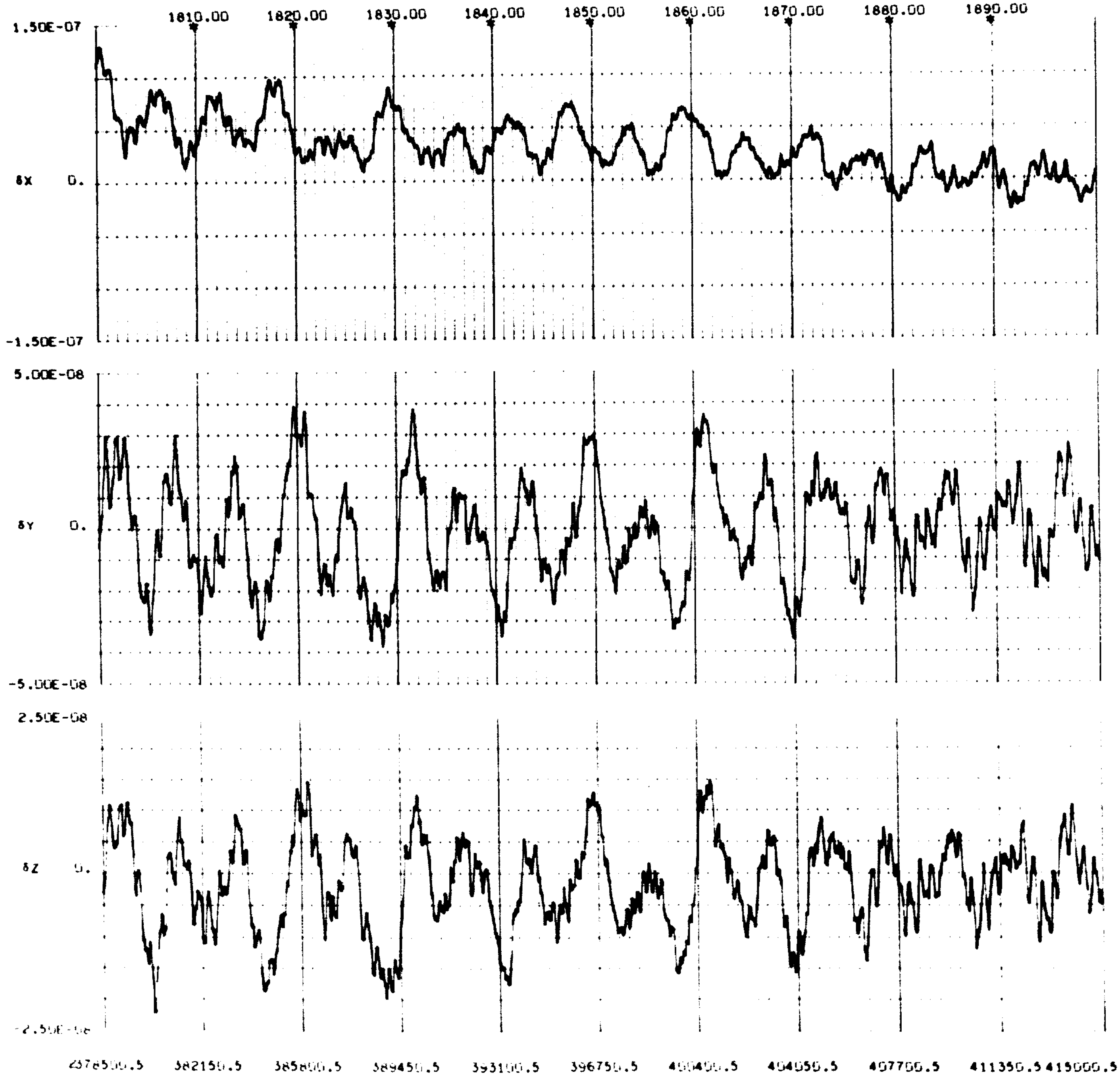


Fig. C-11. Jupiter rectangular residuals (modified source minus DE28) for 1800–2000

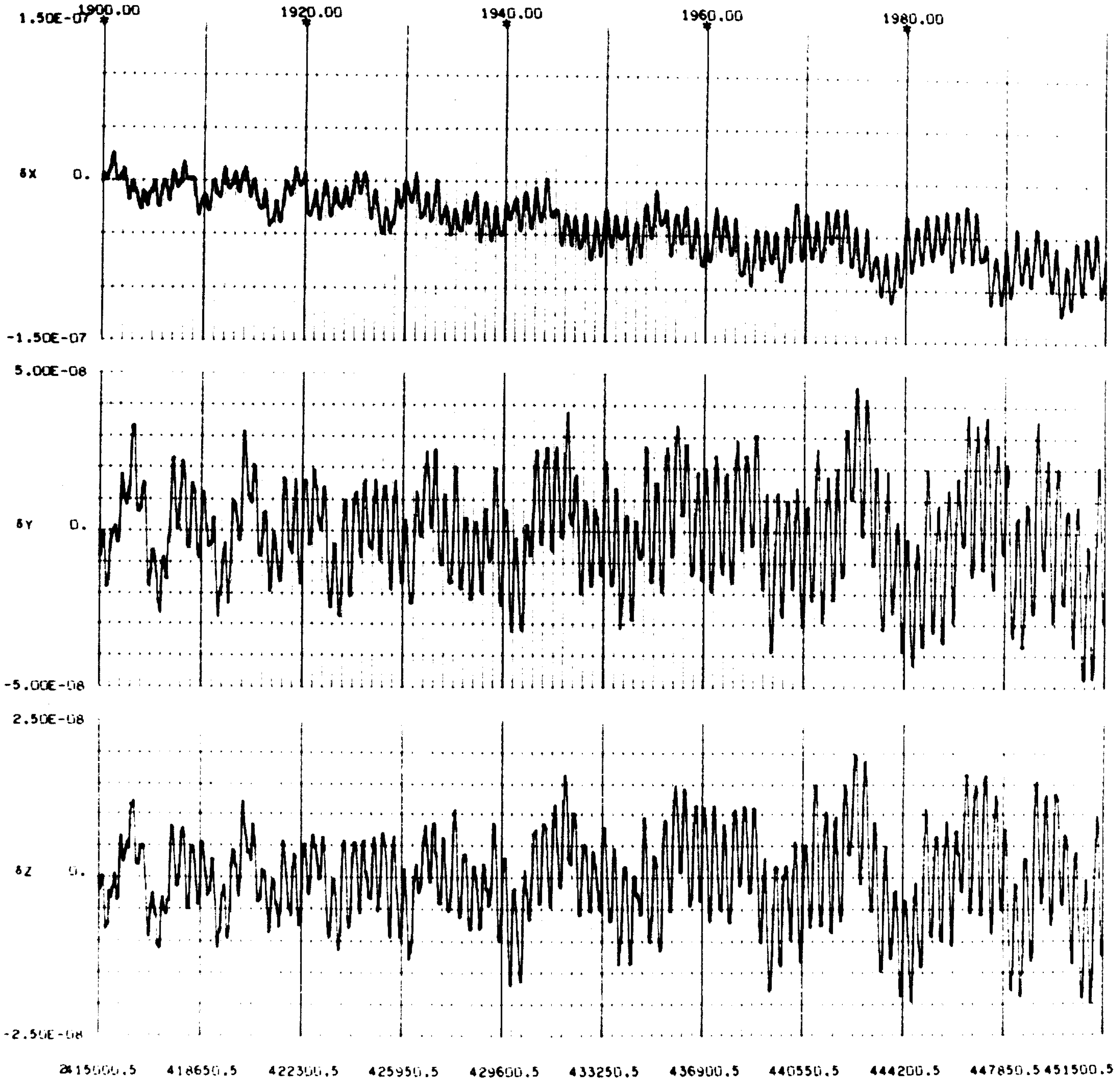


Fig. C-11 (contd)

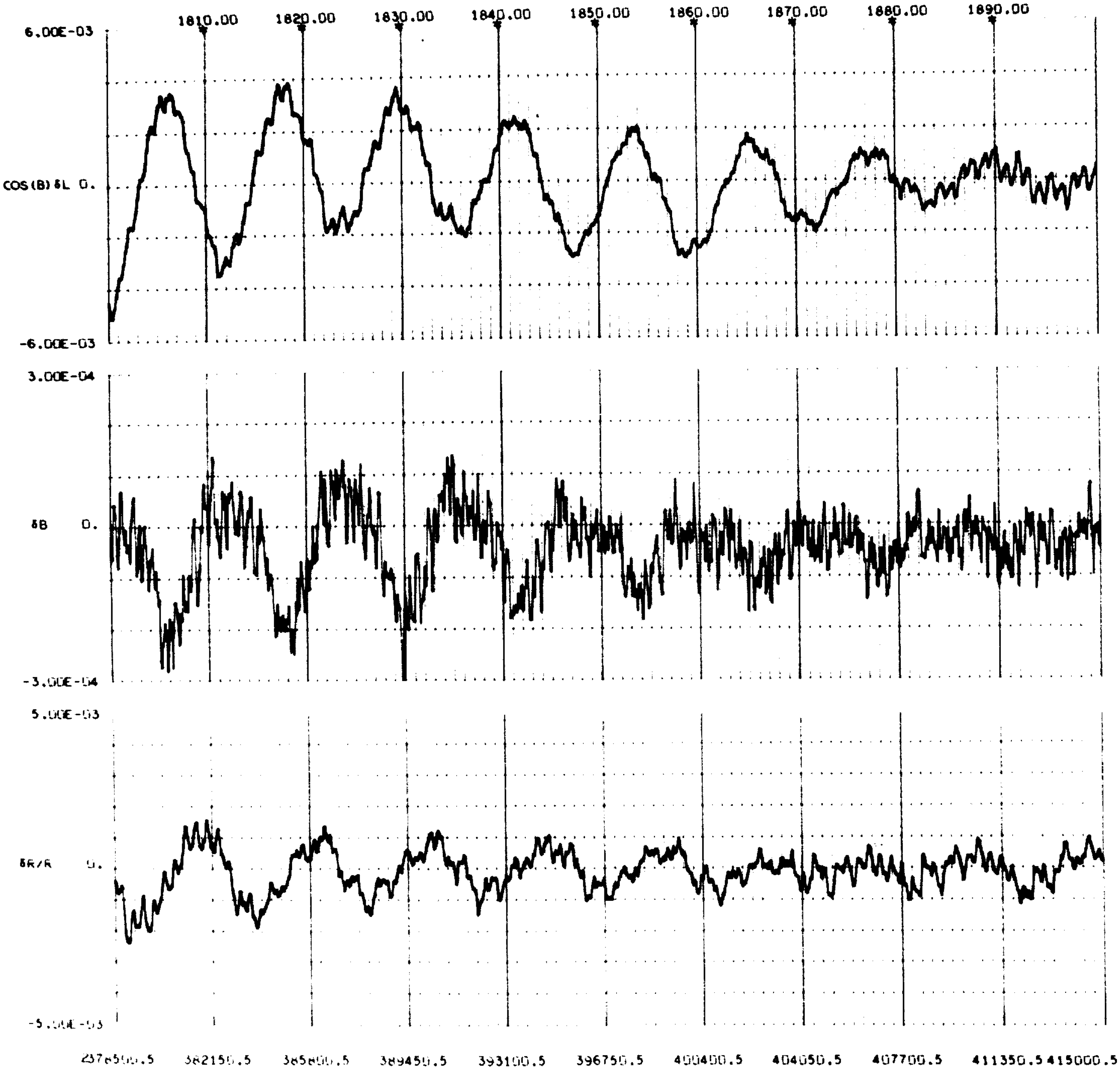


Fig. C-12. Jupiter angular residuals (modified source minus DE28) for 1800–2000

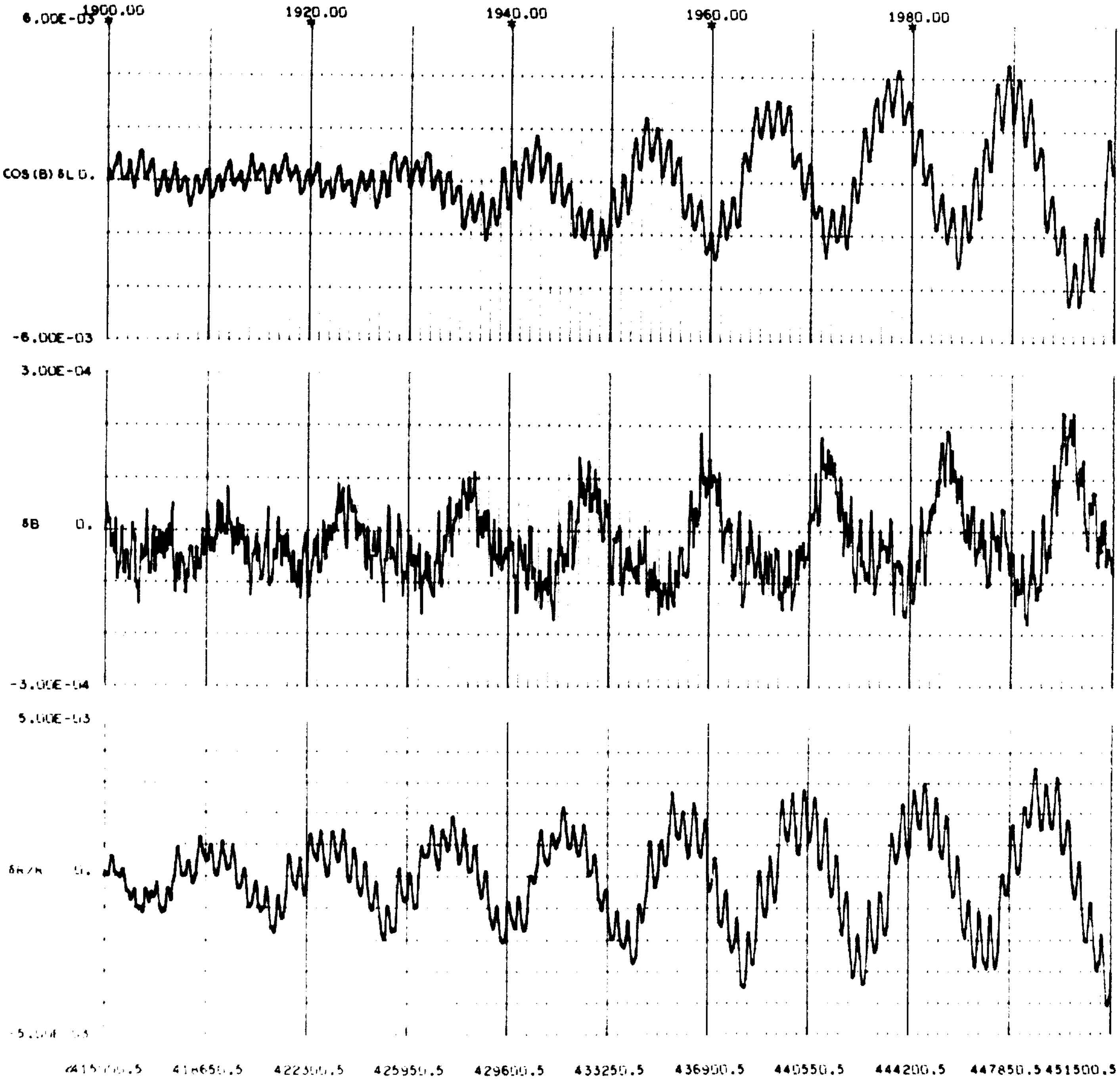


Fig. C-12 (contd)

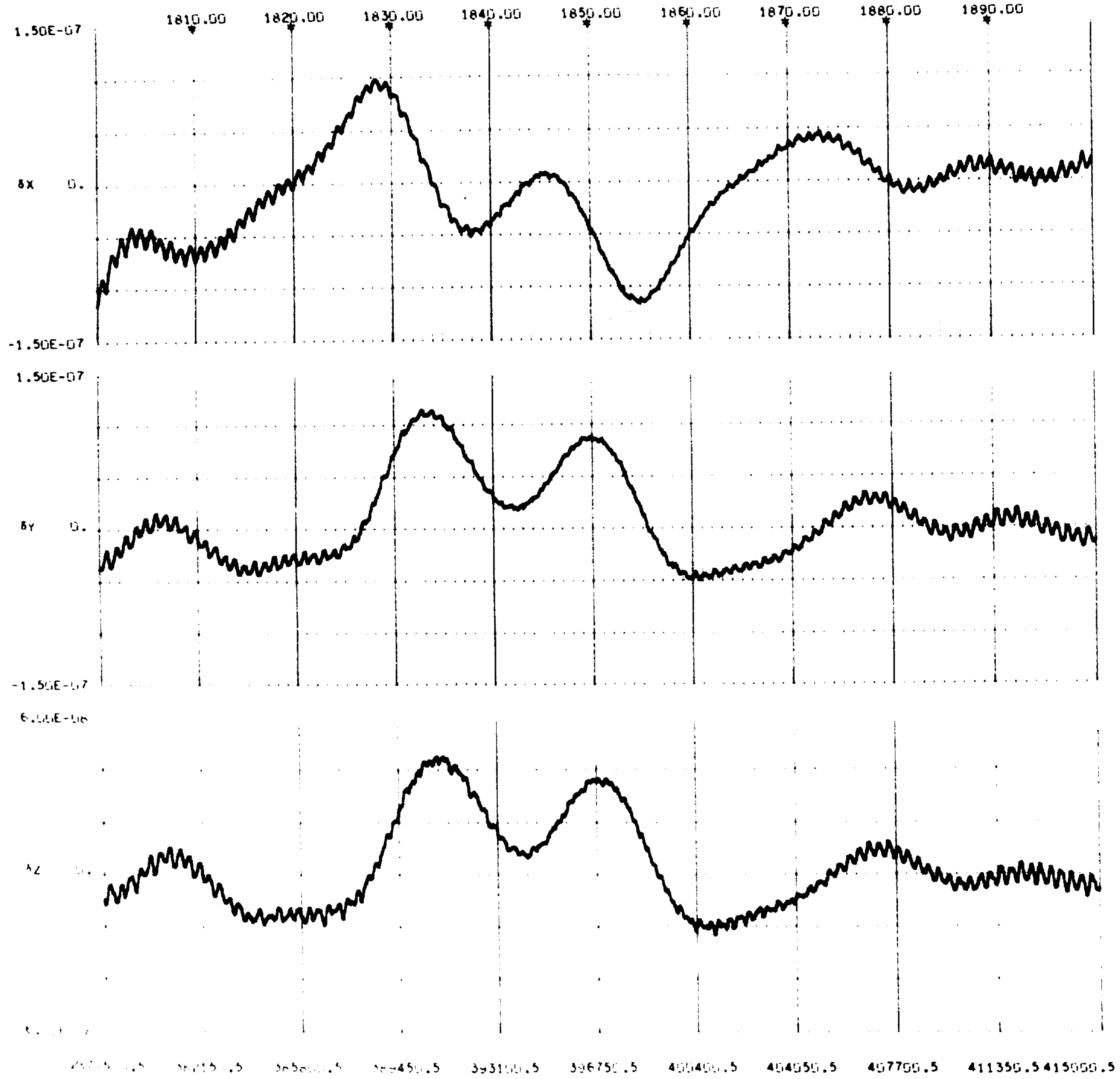


Fig. C-13. Saturn rectangular residuals (modified source minus DE28) for 1800–2000

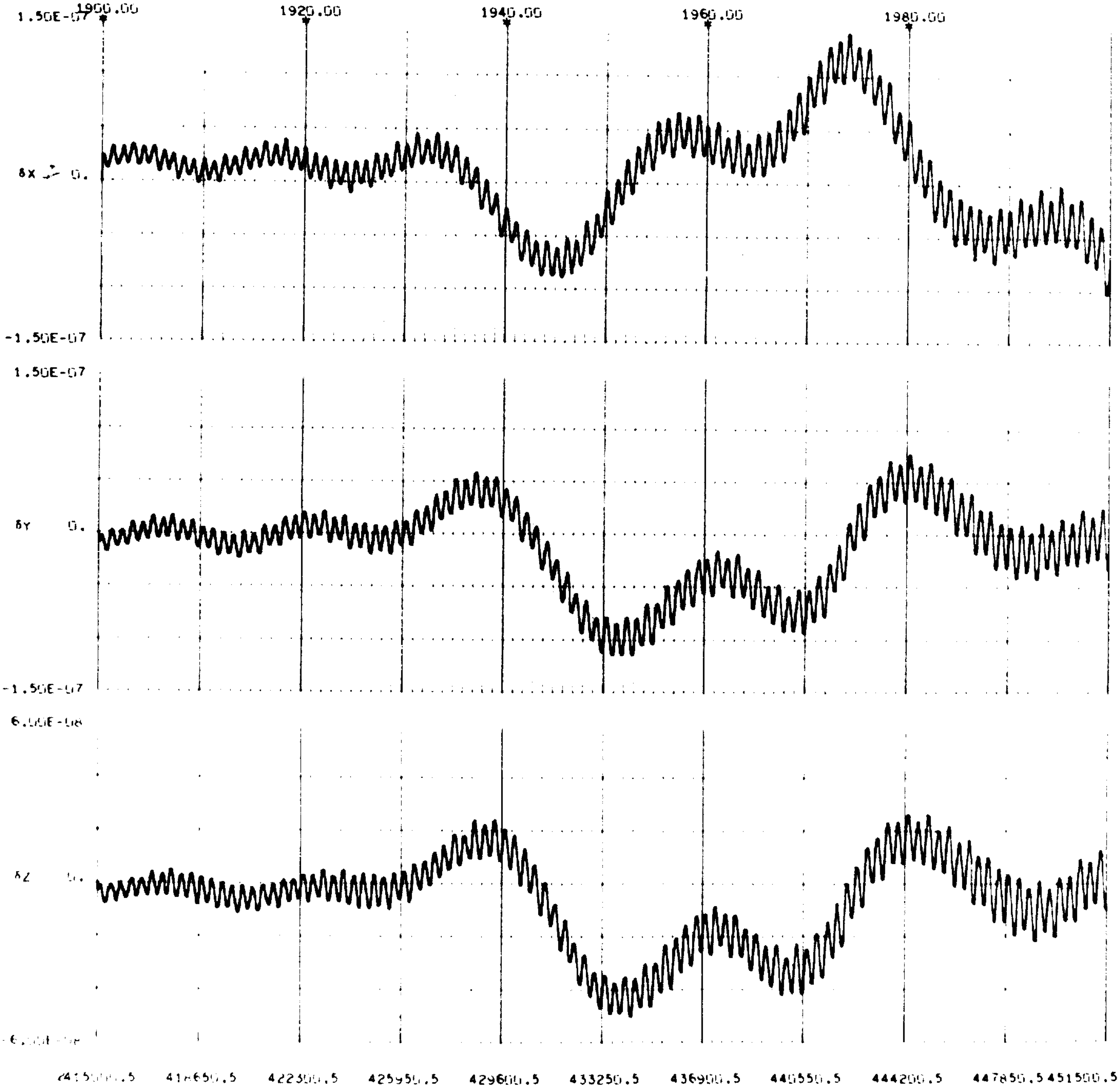


Fig. C-13 (contd)

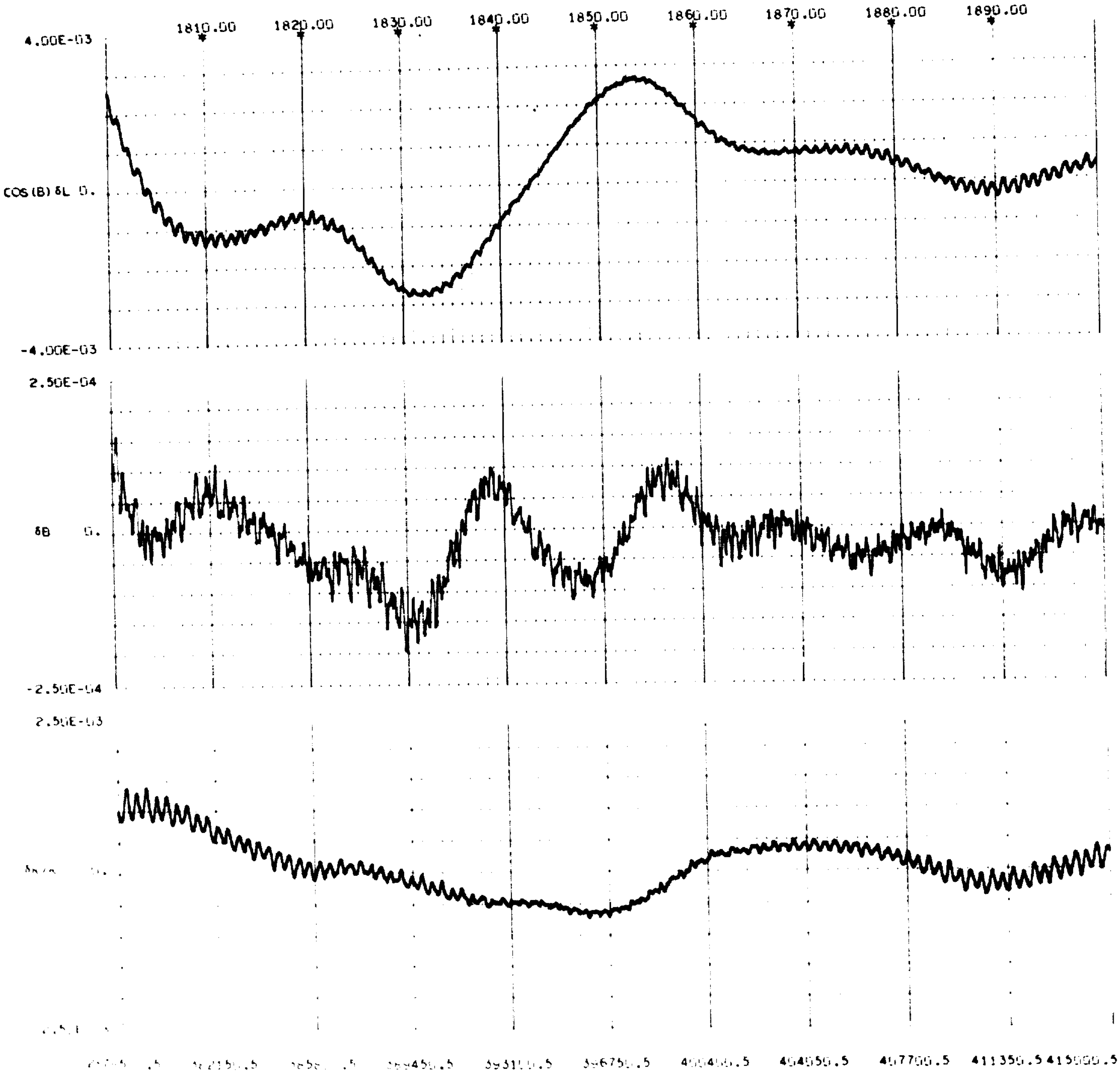


Fig. C-14. Saturn angular residuals (modified source minus DE28) for 1800–2000

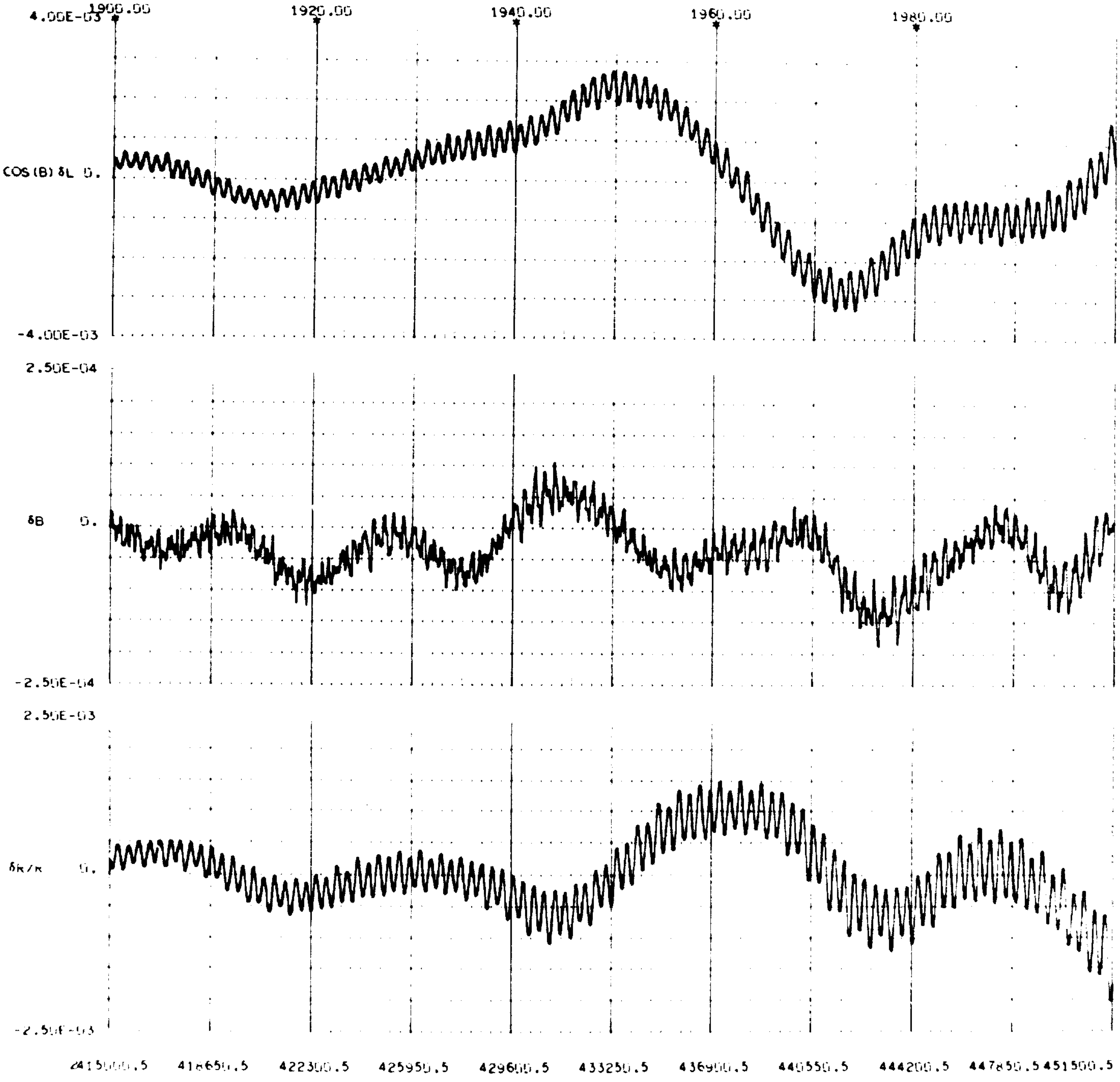


Fig. C-14 (contd)

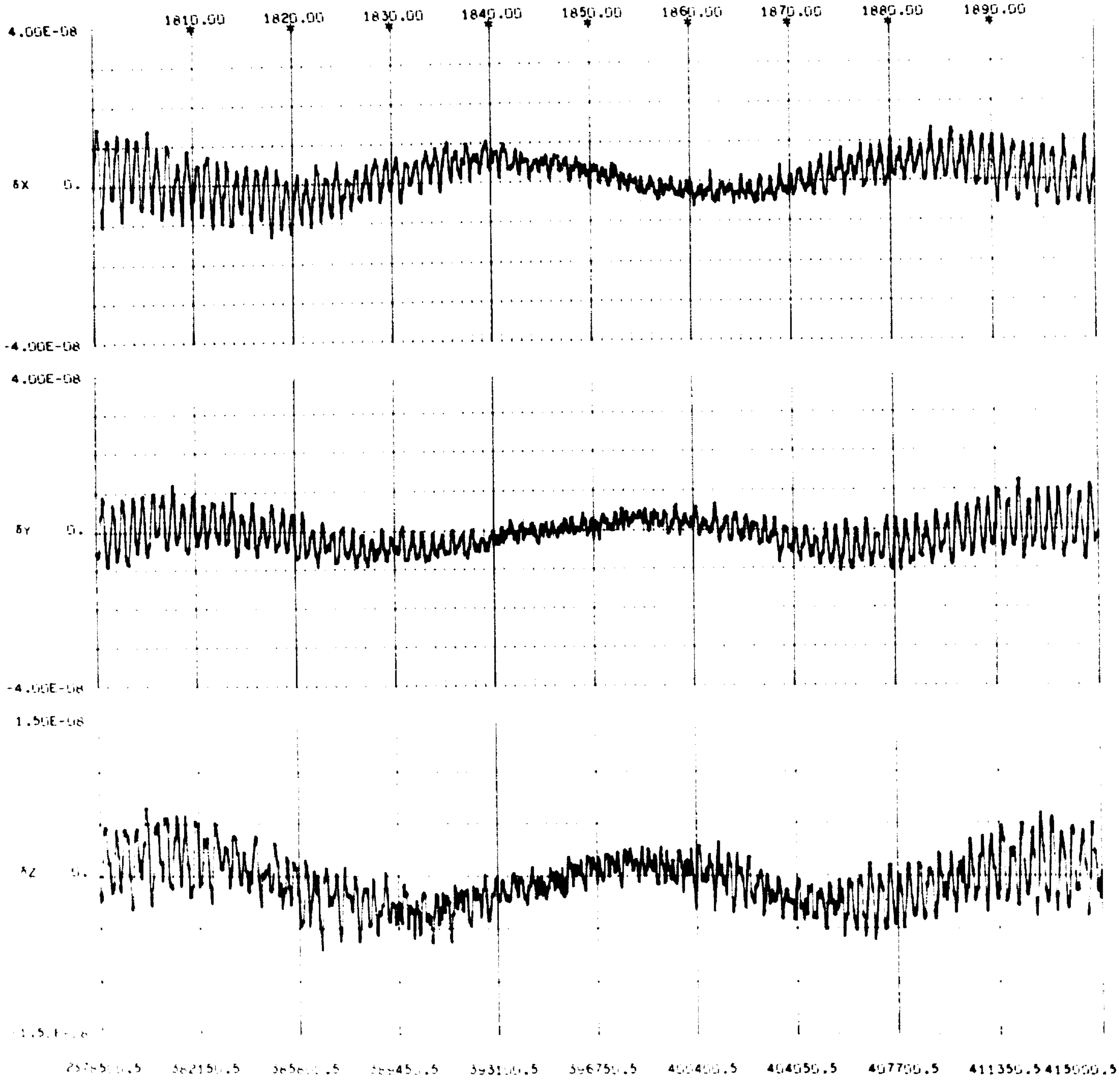


Fig. C-15. Uranus rectangular residuals (modified source minus DE28) for 1800–2000

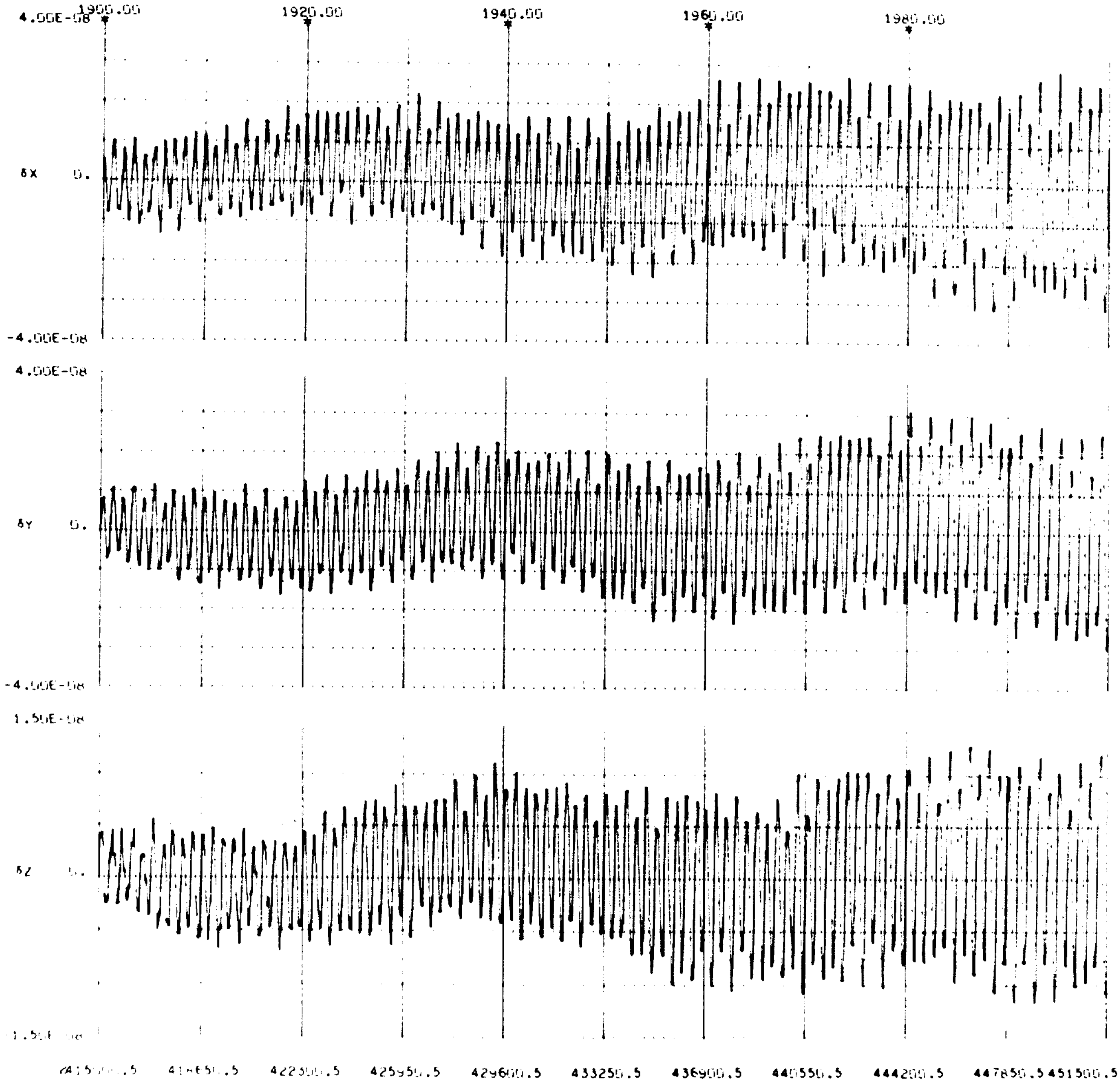


Fig. C-15 (contd)

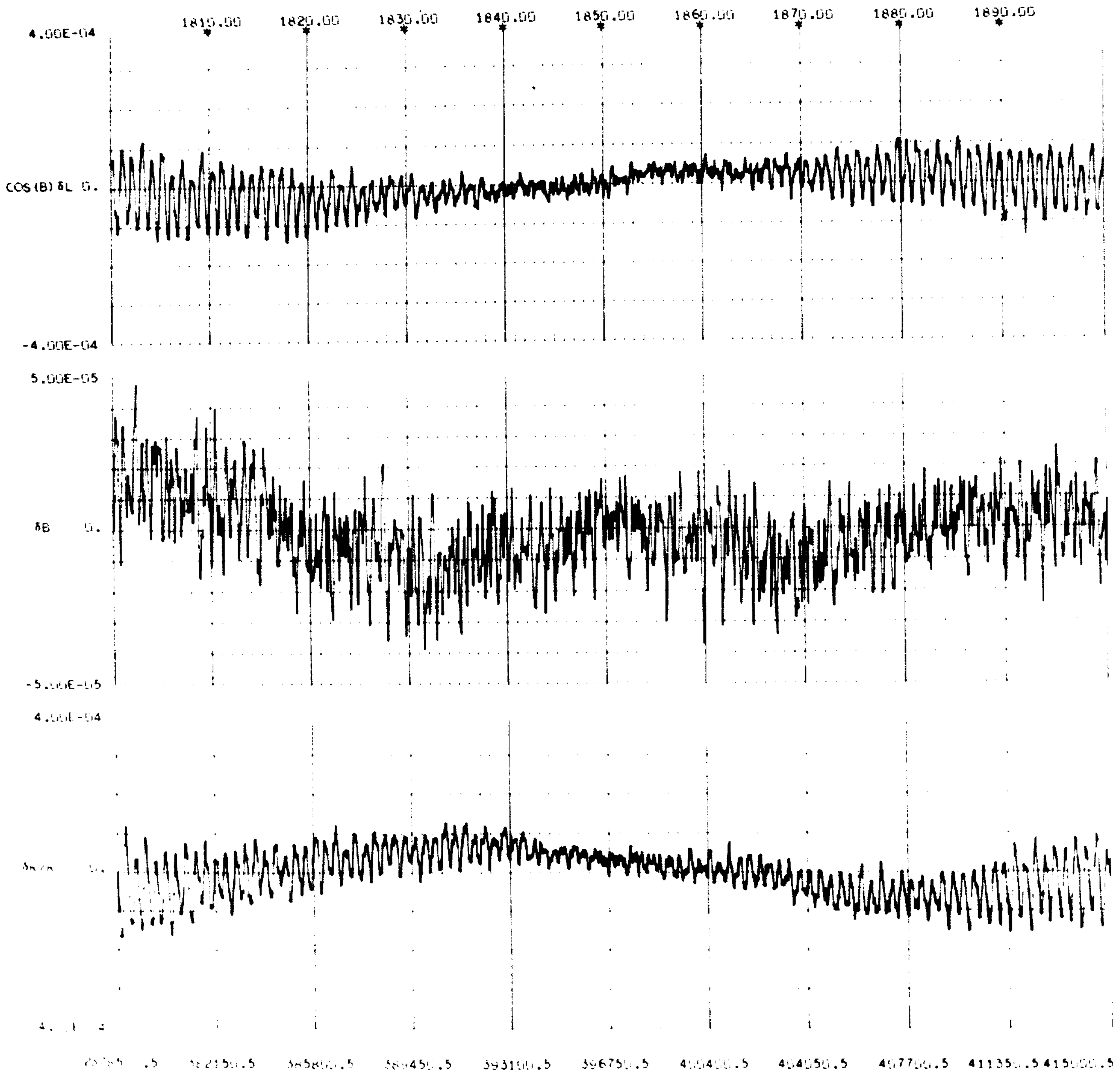


Fig. C-16. Uranus angular residuals (modified source minus DE28) for 1800–2000

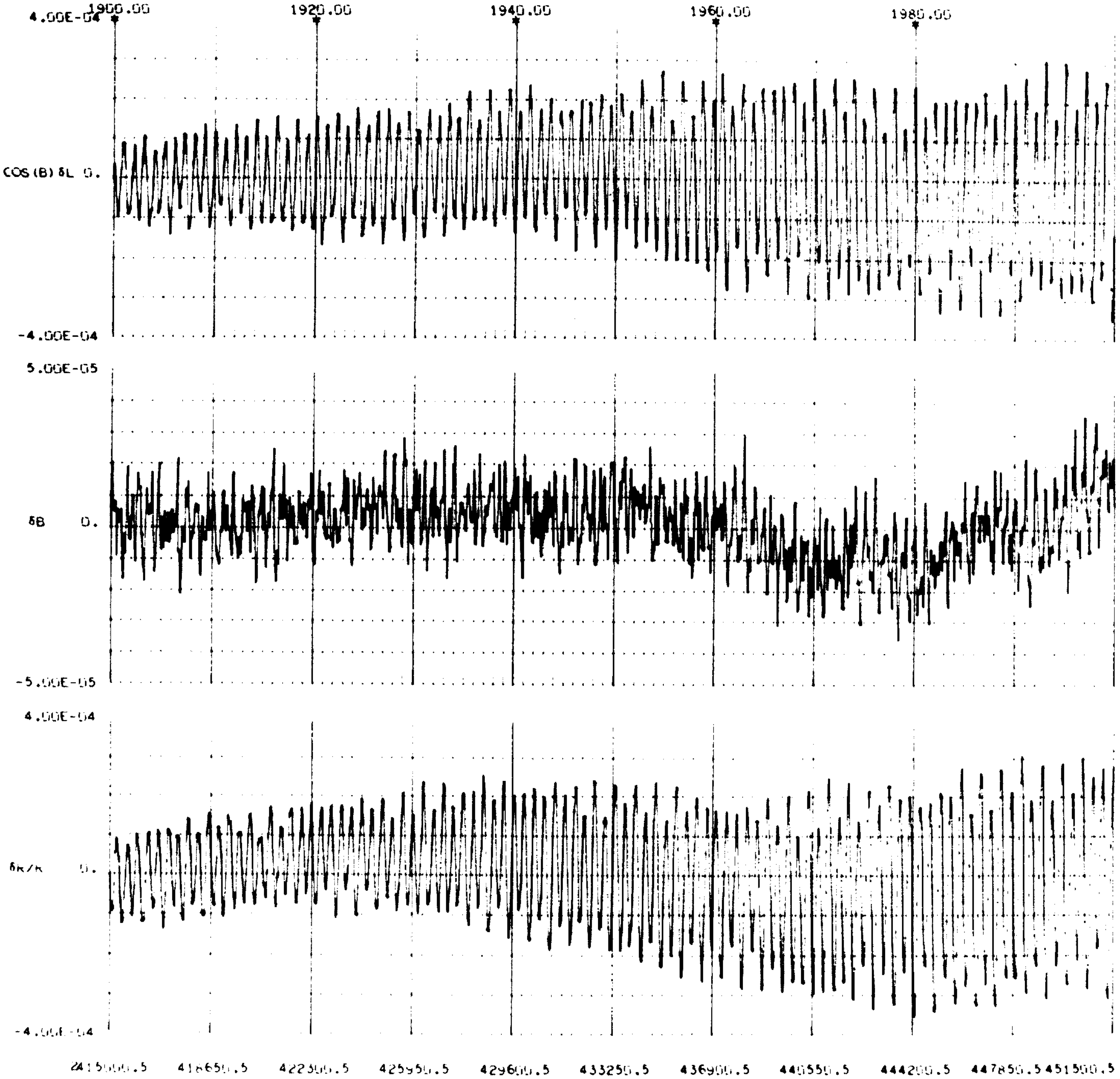


Fig. C-16 (contd)

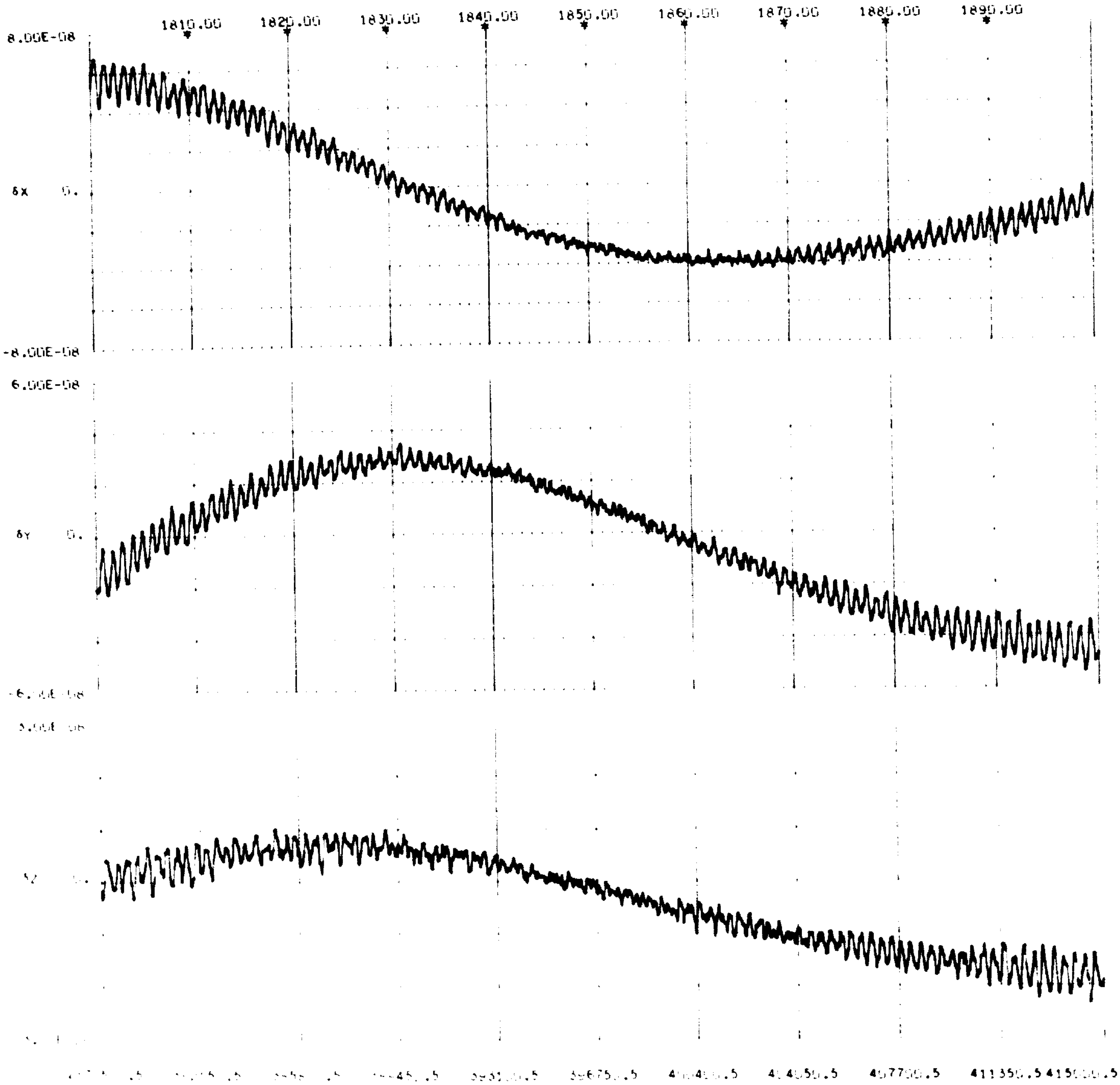


Fig. C-17. Neptune rectangular residuals (modified source minus DE28) for 1800–2000

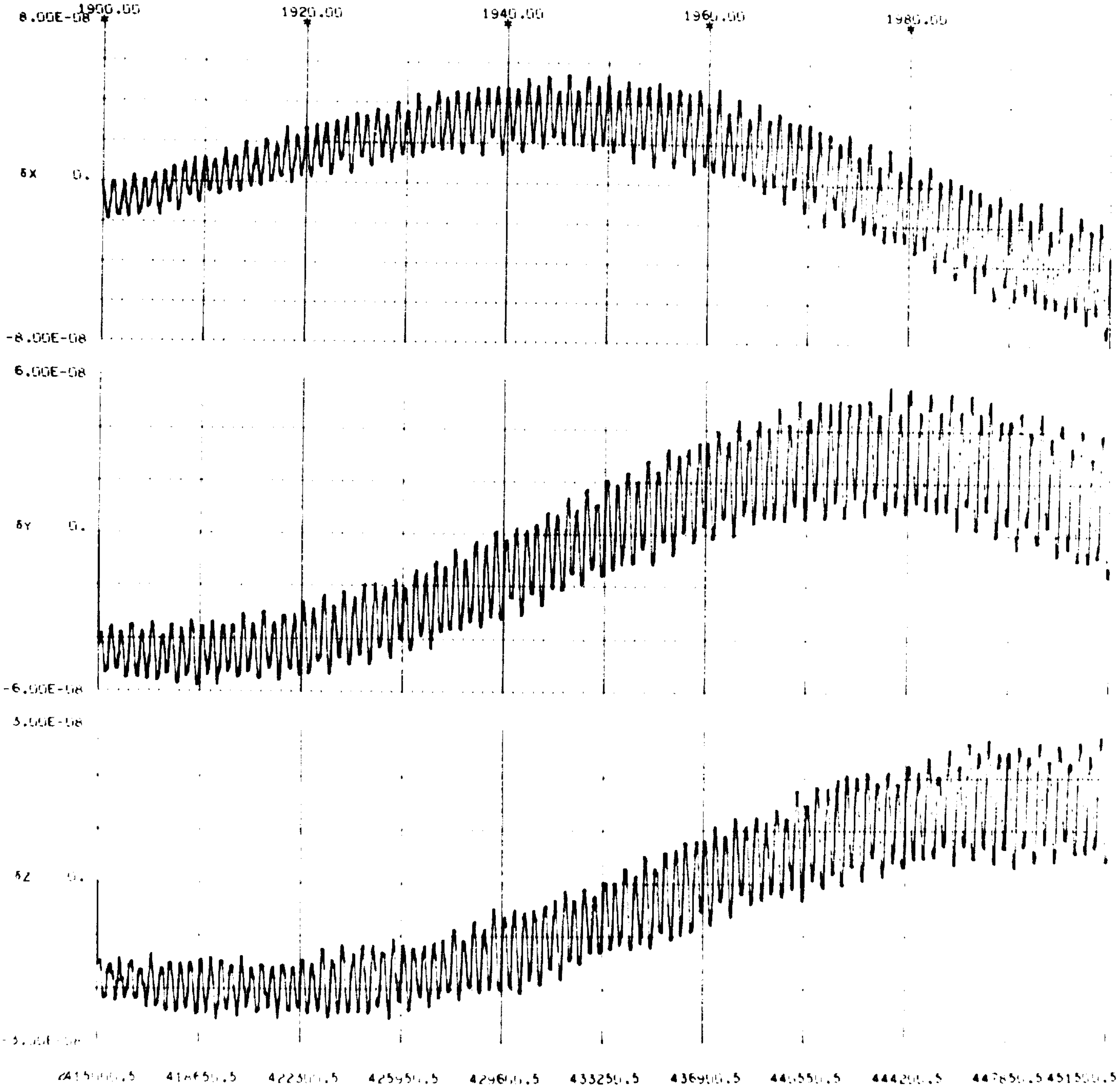


Fig. C-17 (contd)

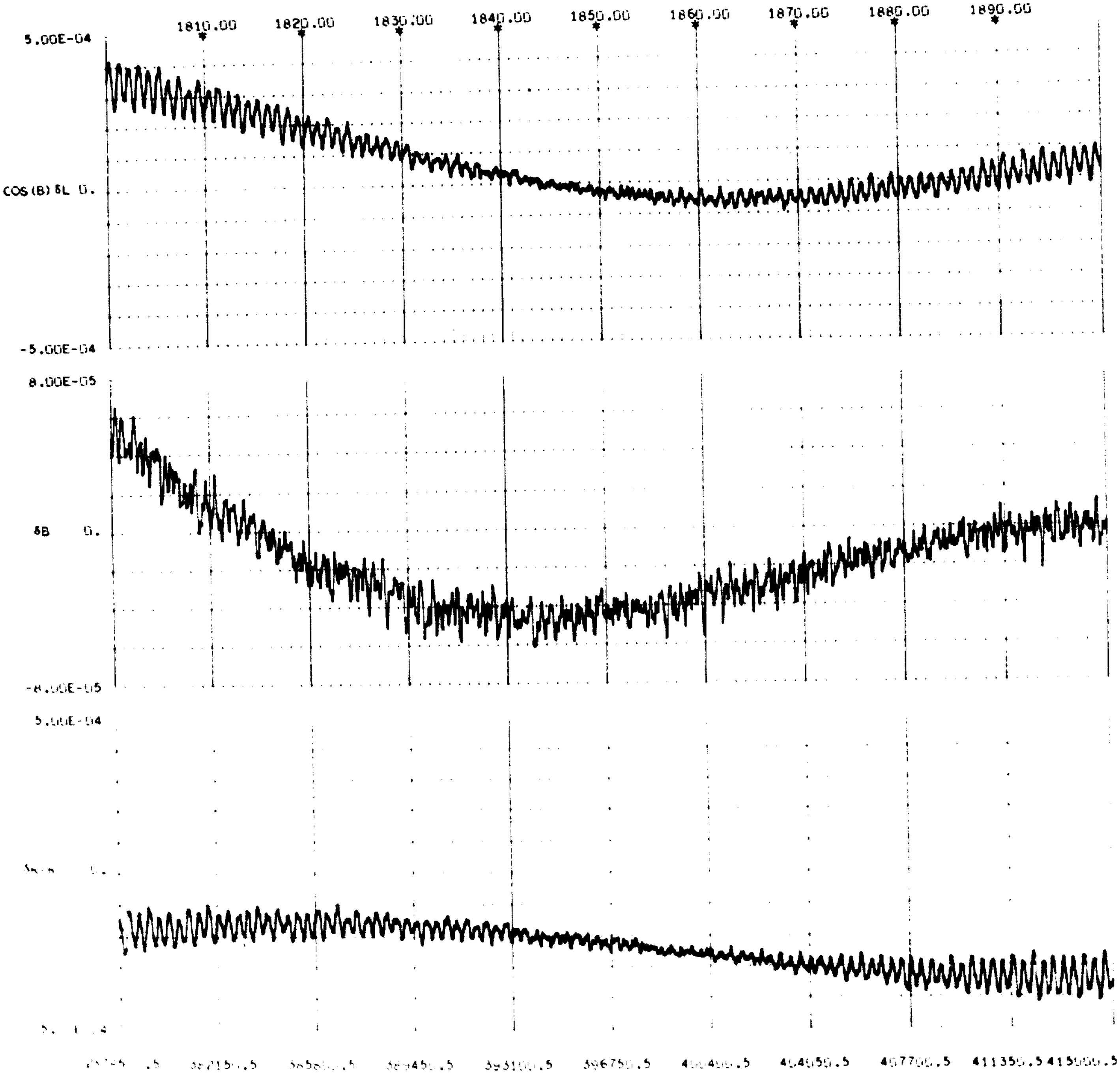


Fig. C-18. Neptune angular residuals (modified source minus DE28) for 1800–2000

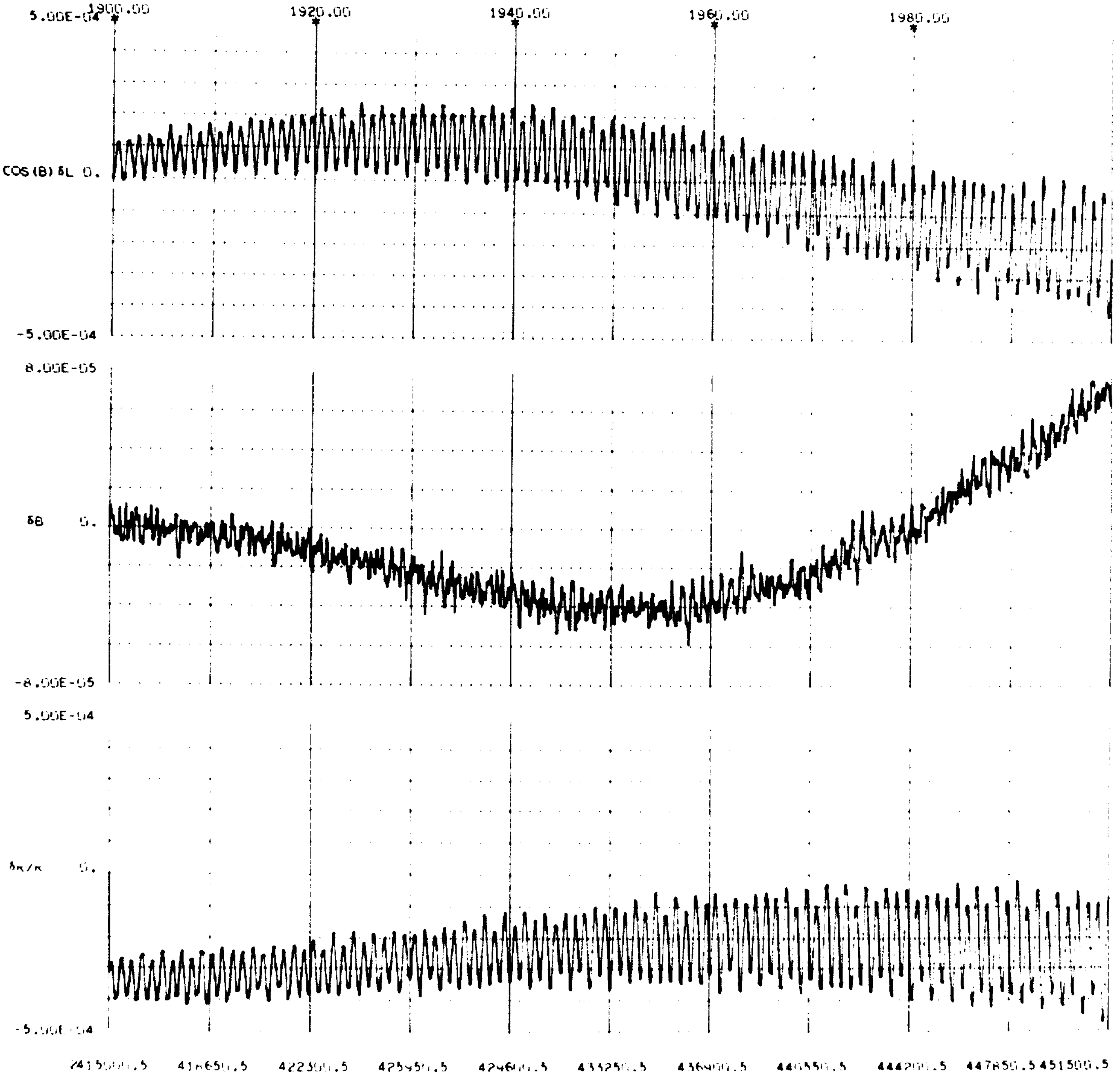


Fig. C-18 (contd)

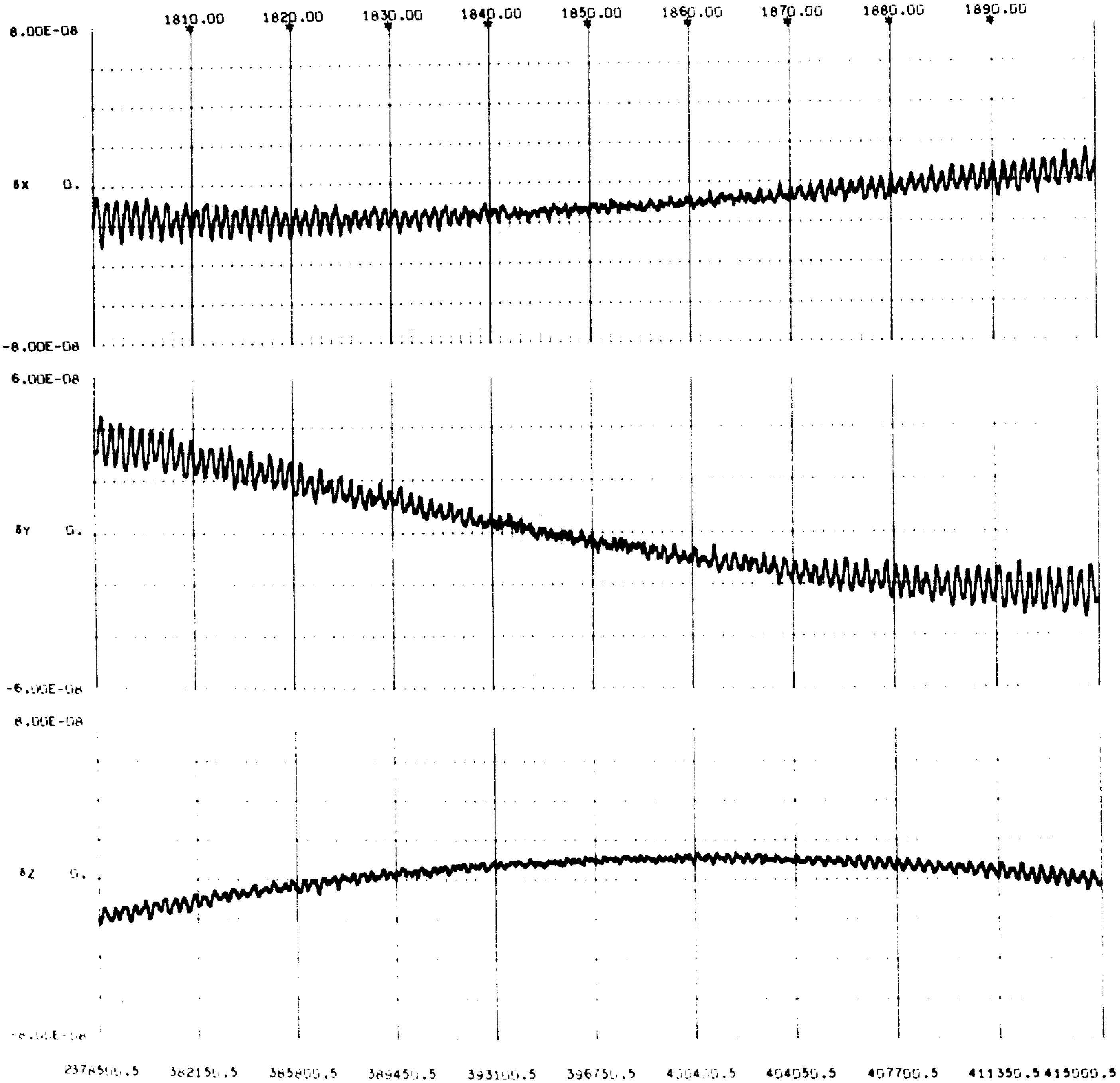


Fig. C-19. Pluto rectangular residuals (modified source minus DE28) for 1800–2000

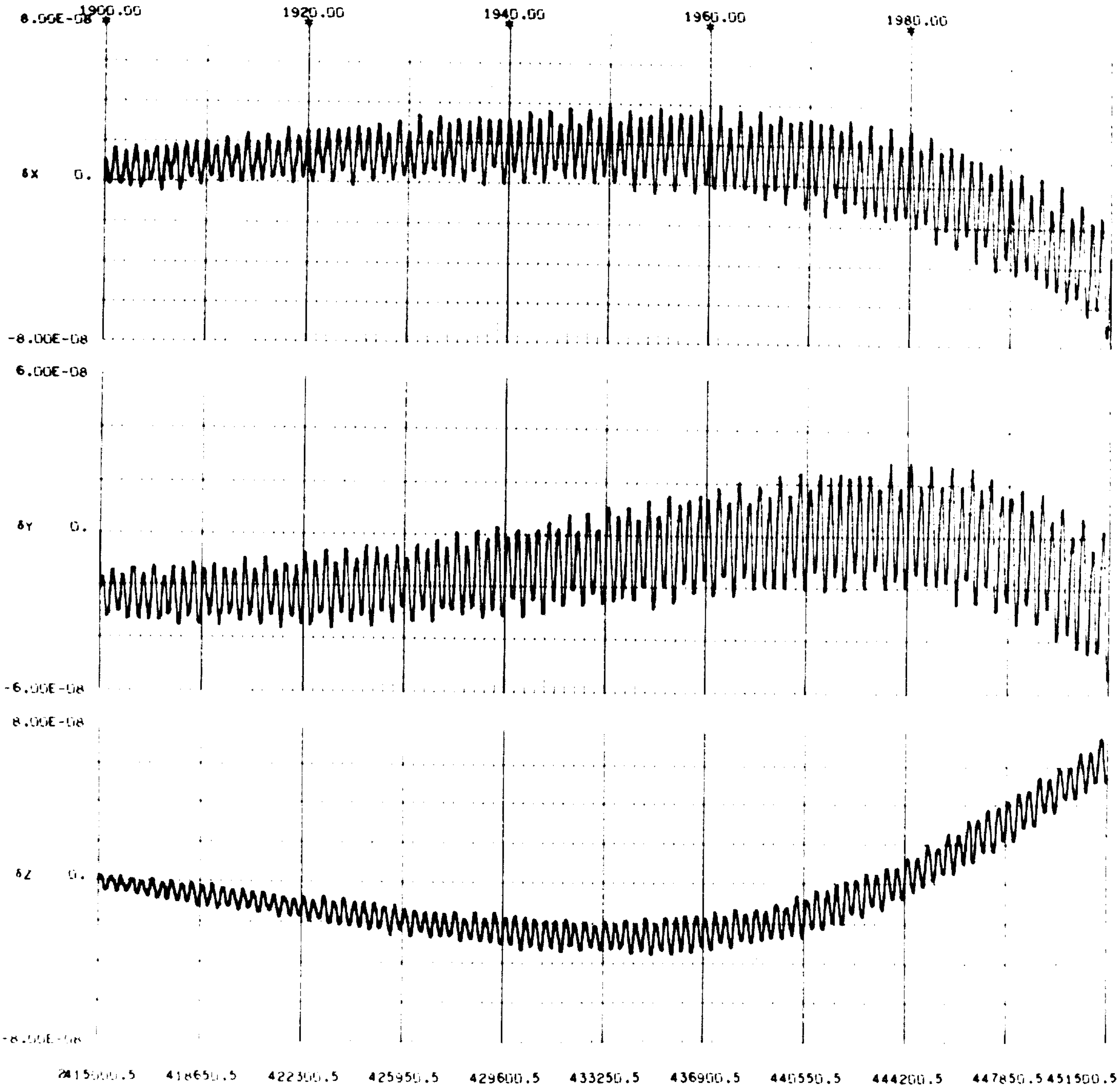


Fig. C-19 (contd)

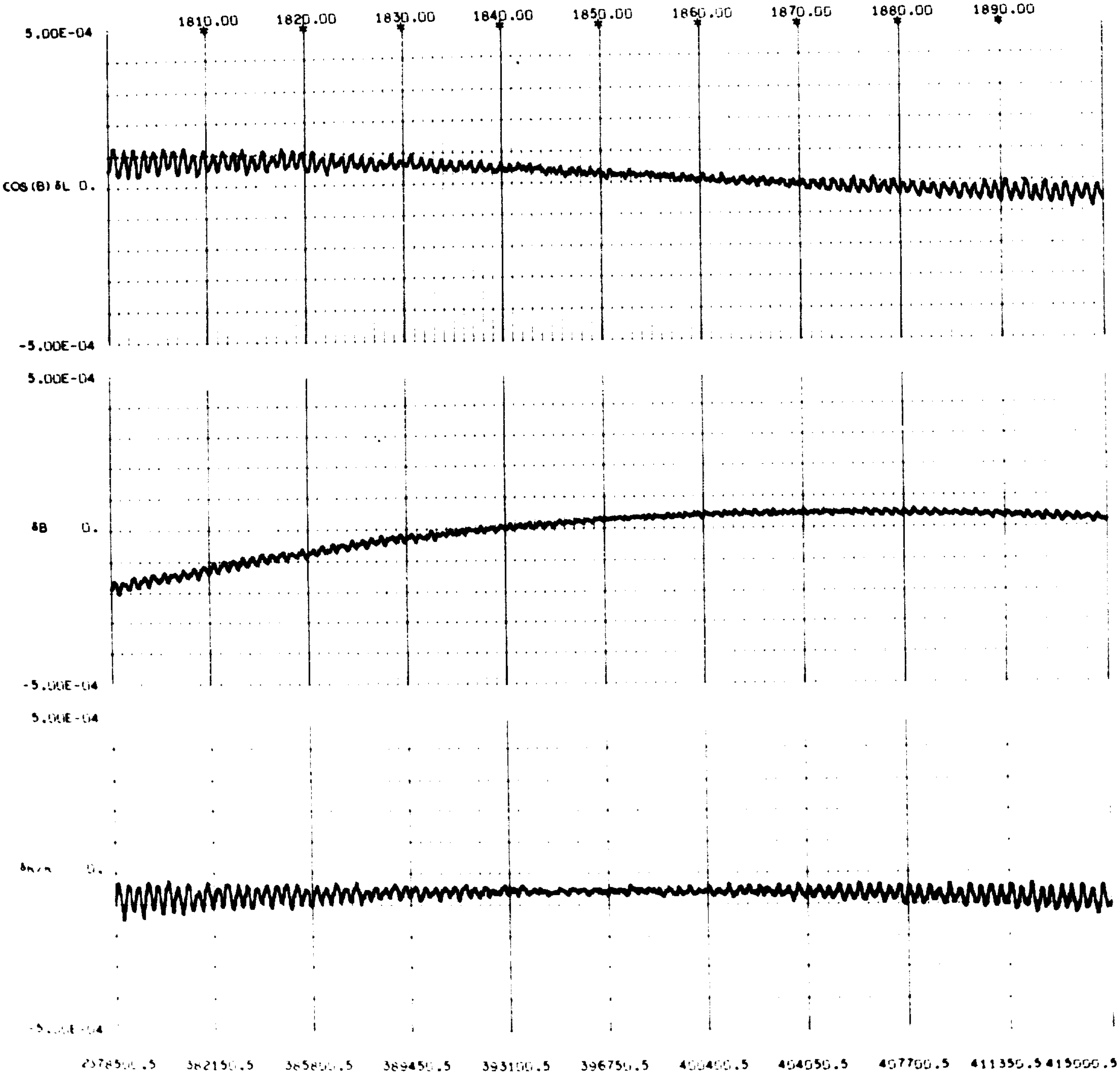


Fig. C-20. Pluto angular residuals (modified source minus DE28) for 1800–2000

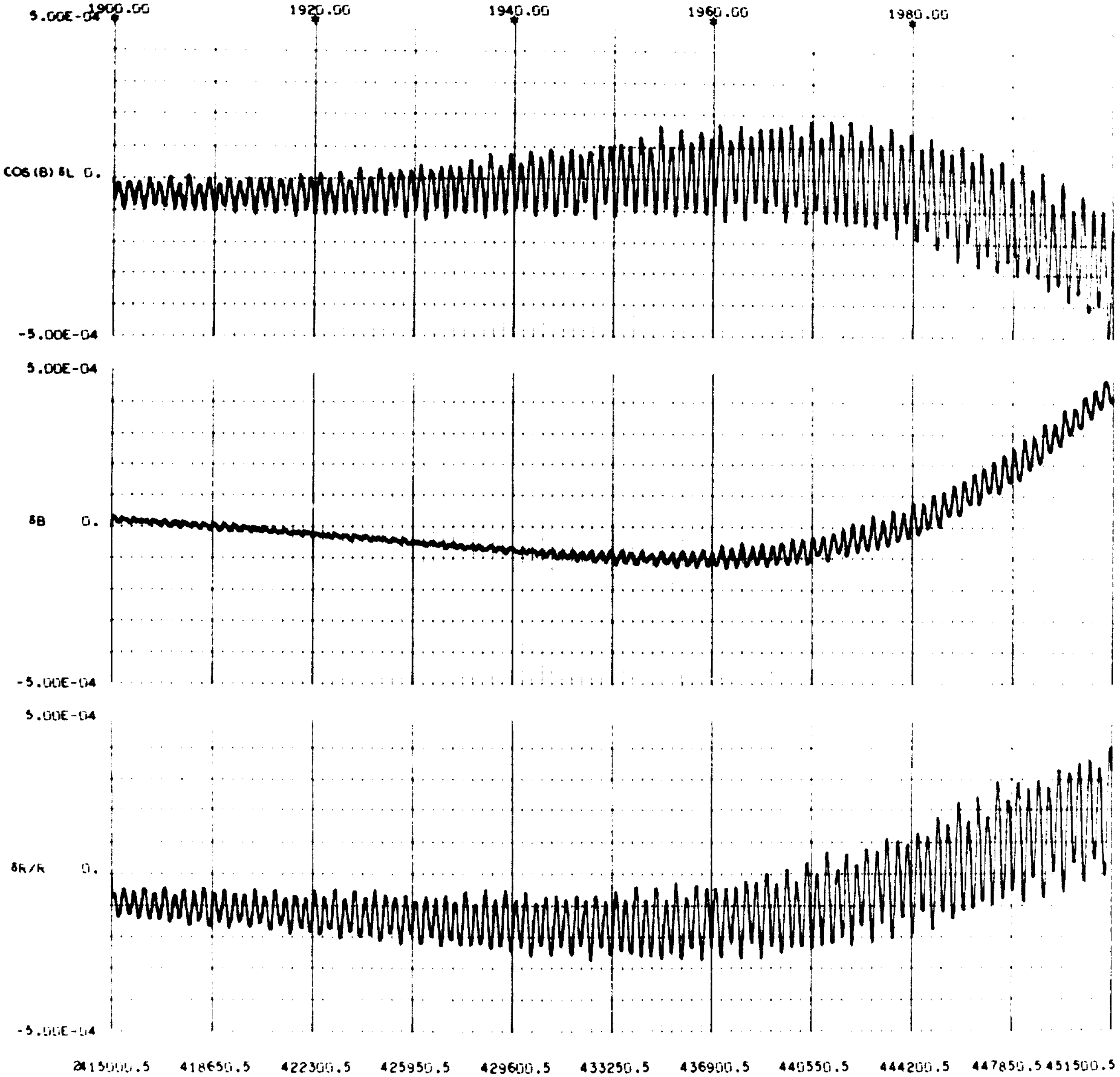


Fig. C-20 (contd)

Appendix D

Secular Deficiencies of Modified Source Ephemerides

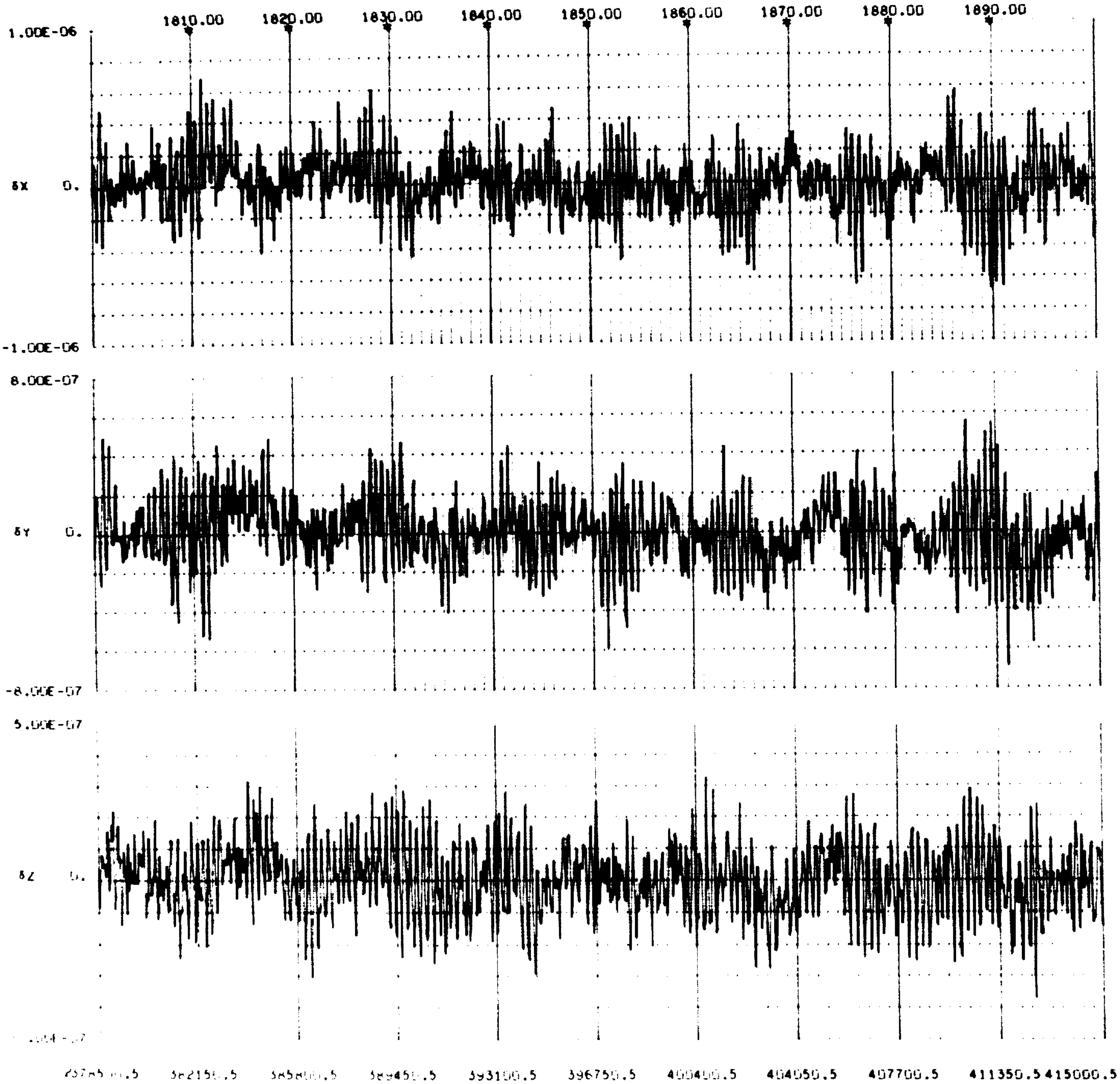


Fig. D-1. Venus rectangular residuals (improved modified source minus DE28) for 1800–2000

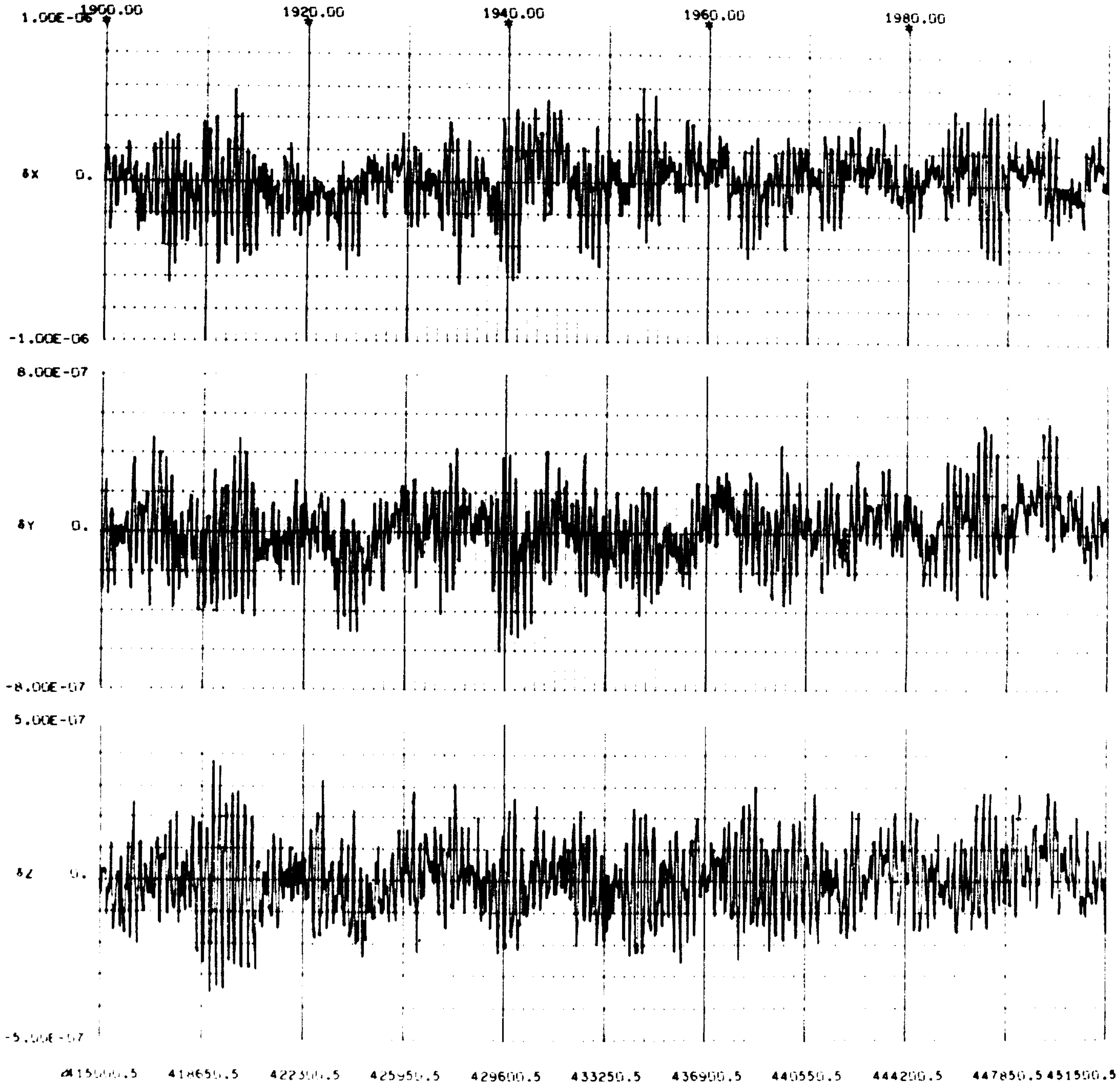

1900.00
1920.00
1940.00
1960.00
1980.00
1.00E-06
δX 0.
-1.00E-06
8.00E-07
δY 0.
-8.00E-07
5.00E-07
δZ 0.
-5.00E-07
415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-1 (contd)

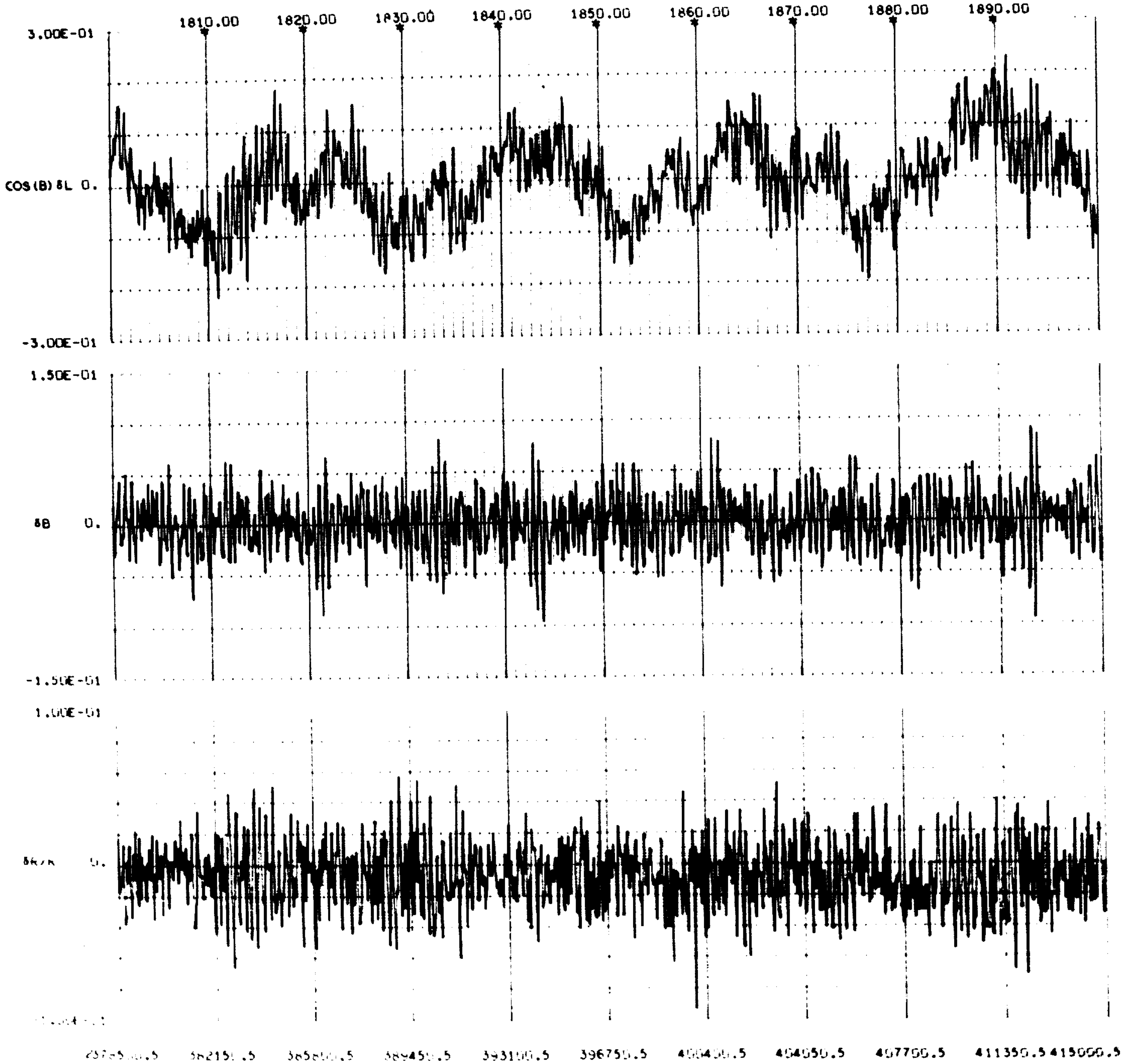


Fig. D-2. Venus angular residuals (improved modified source minus DE28) for 1800–2000

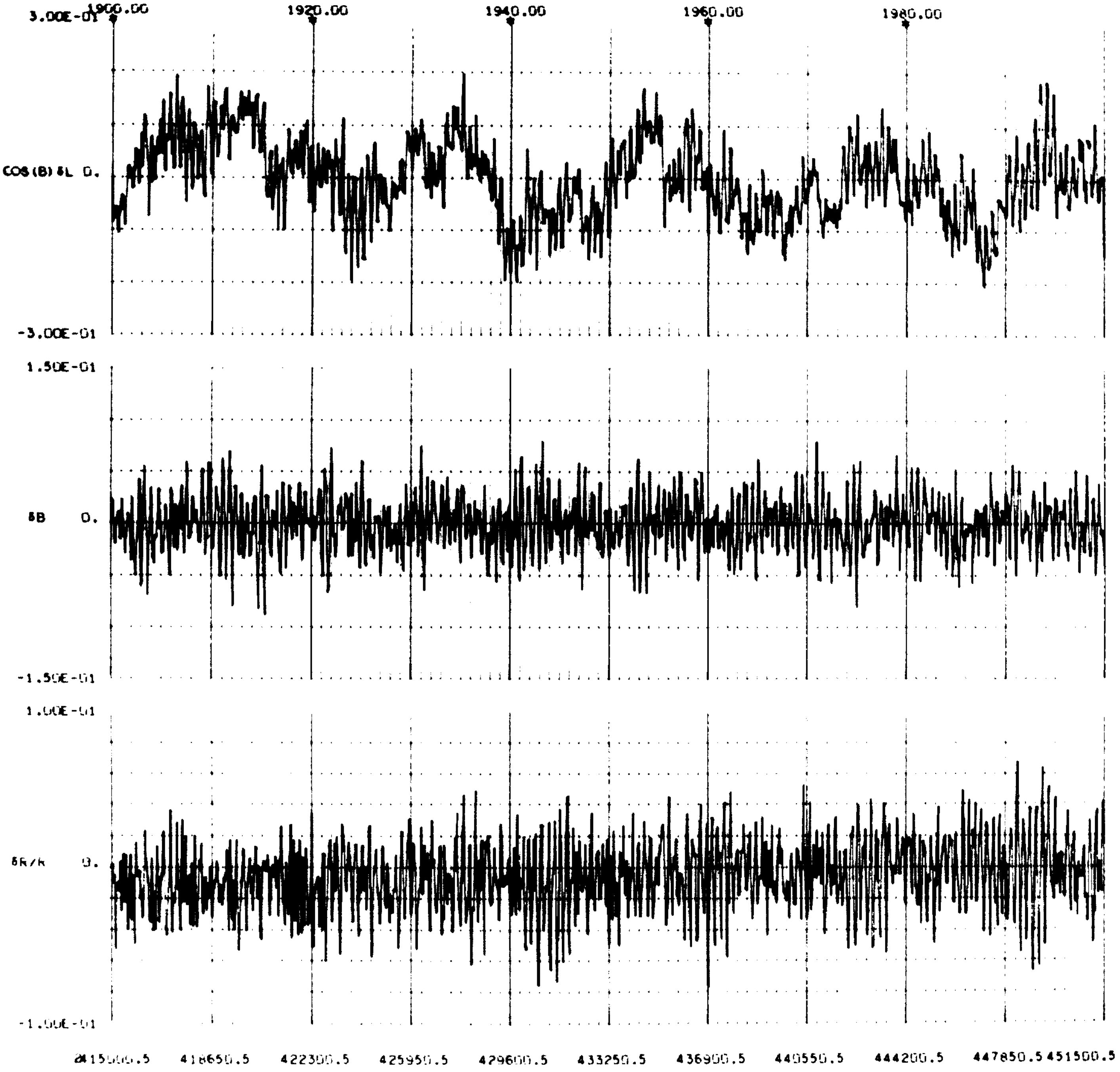

1900.00
1920.00
1940.00
1960.00
1980.00
3.00E-01
COS(B) δL 0.
-3.00E-01
1.50E-01
δB 0.
-1.50E-01
1.00E-01
δR/R 0.
-1.00E-01
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-2 (contd)

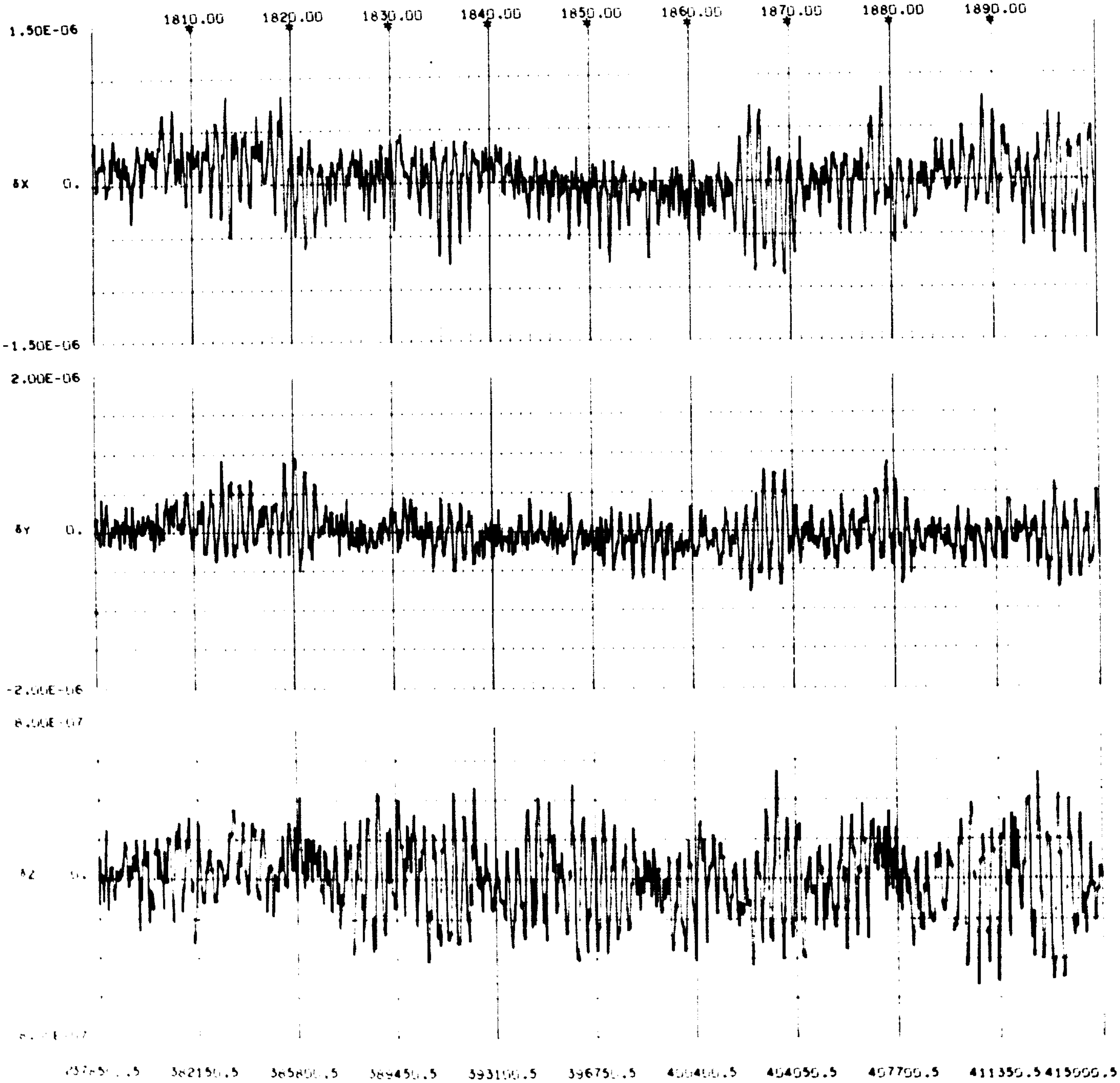


Fig. D-3. Earth-moon barycenter rectangular residuals (improved modified source minus DE28) for 1800–2000

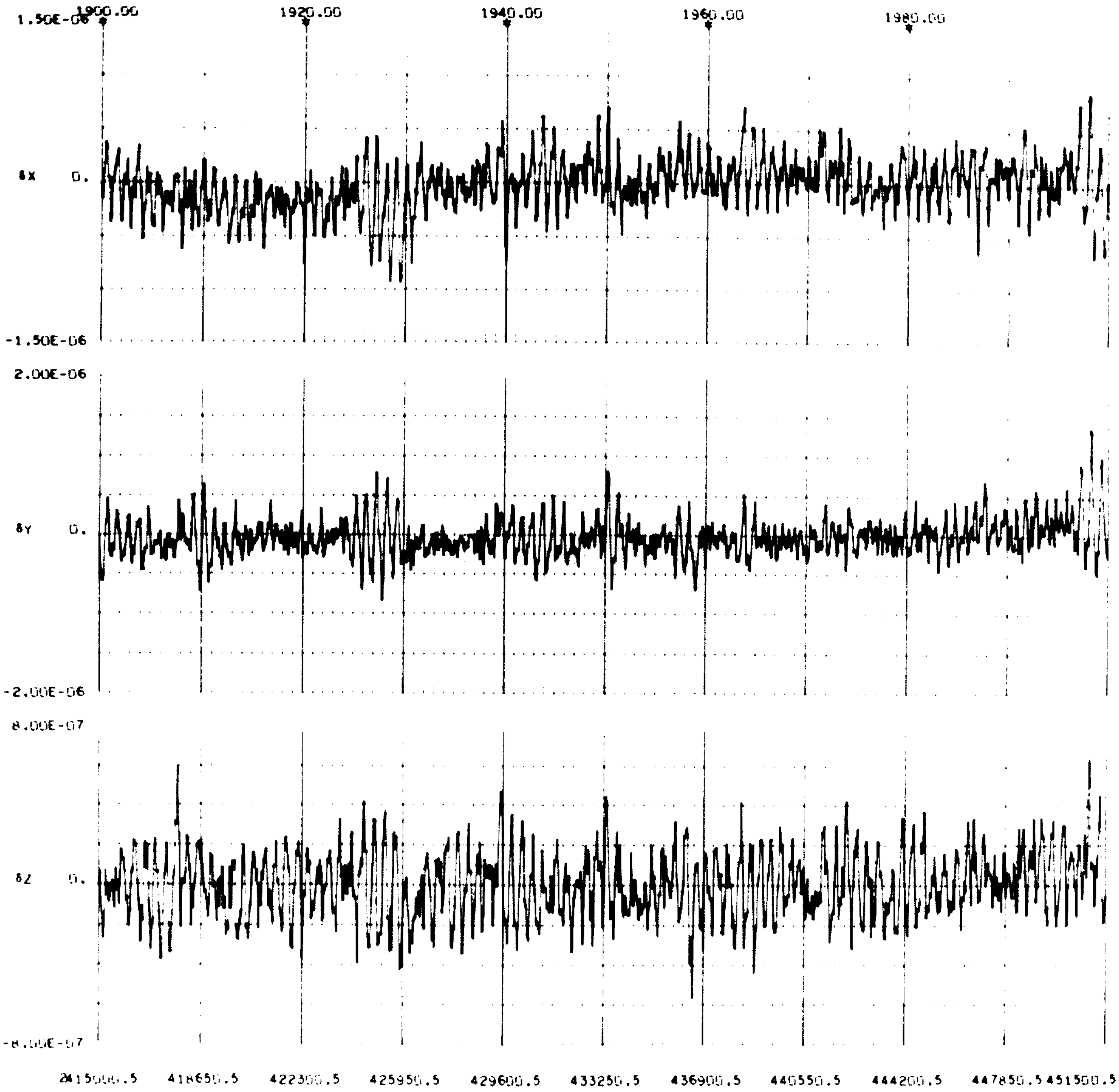

1900.00
1920.00
1940.00
1960.00
1980.00
1.50E-06
δX
0.
-1.50E-06
2.00E-06
δY
0.
-2.00E-06
8.00E-07
δZ
0.
-8.00E-07
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-3 (contd)

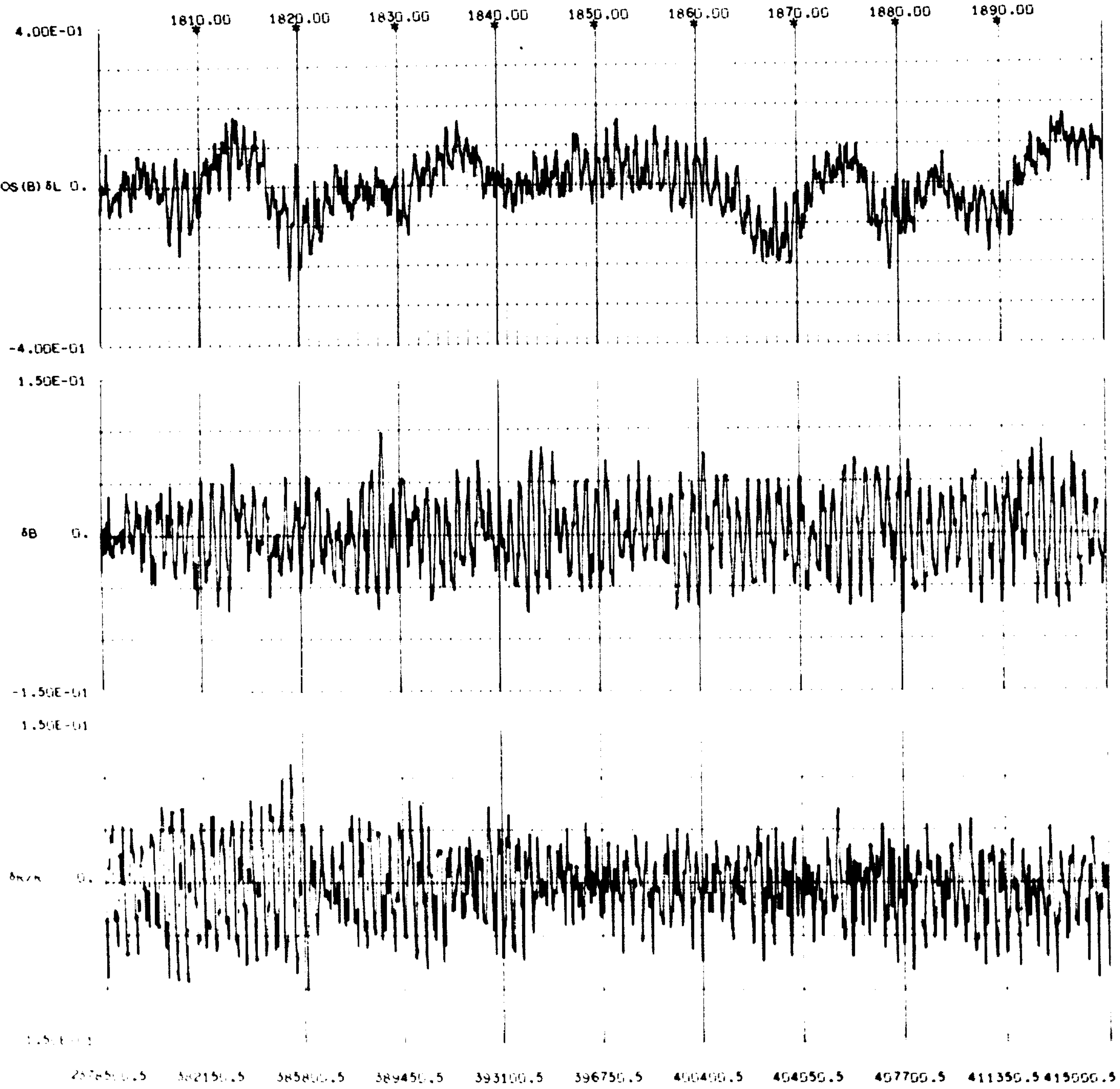


Fig. D-4. Earth-moon barycenter angular residuals (improved modified source minus DE28) for 1800–2000

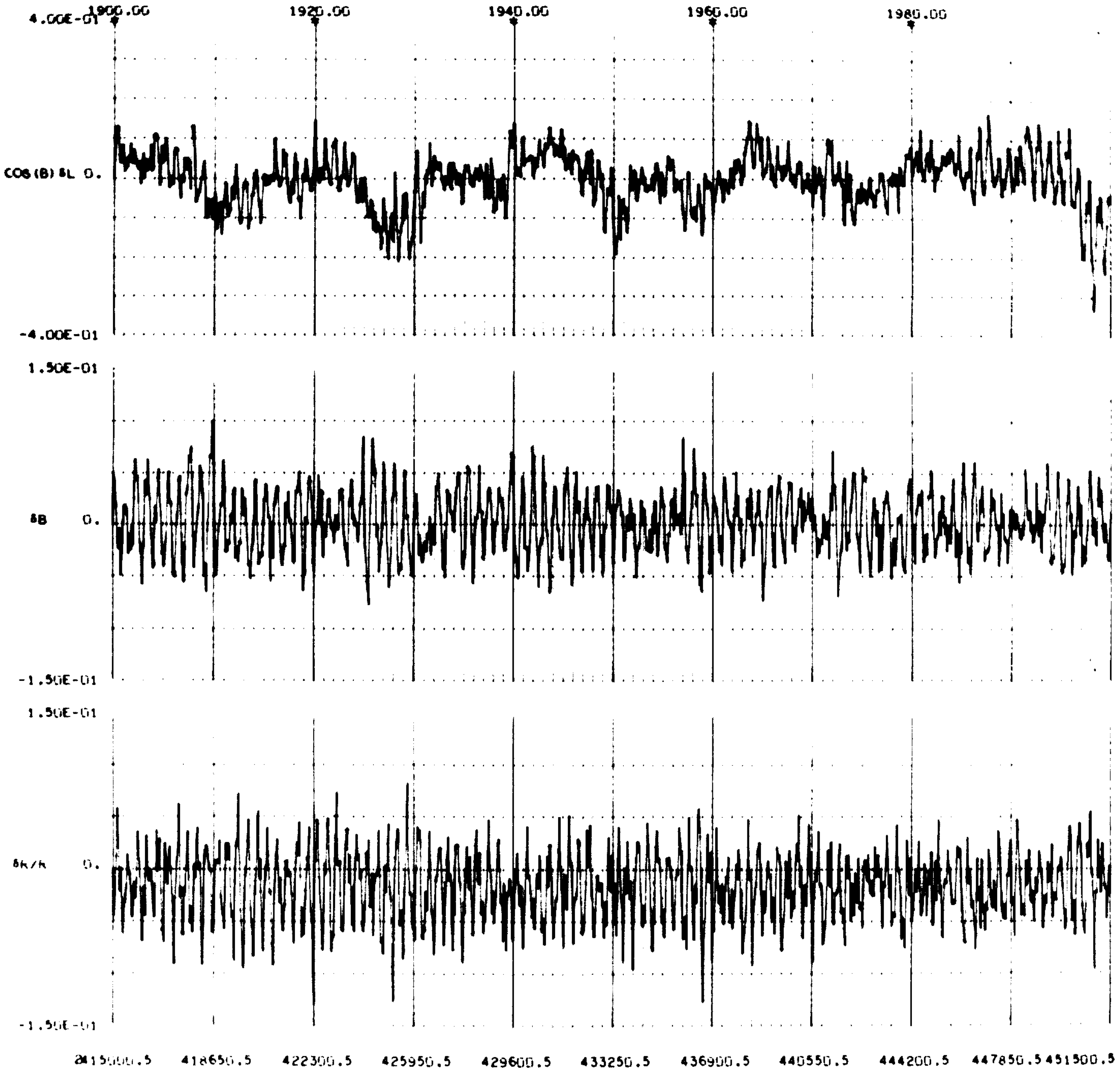

1900.00
1920.00
1940.00
1960.00
1980.00
4.00E-01
COS(B) δL 0.
-4.00E-01
1.50E-01
δB 0.
-1.50E-01
1.50E-01
δR/R 0.
-1.50E-01
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-4 (contd)

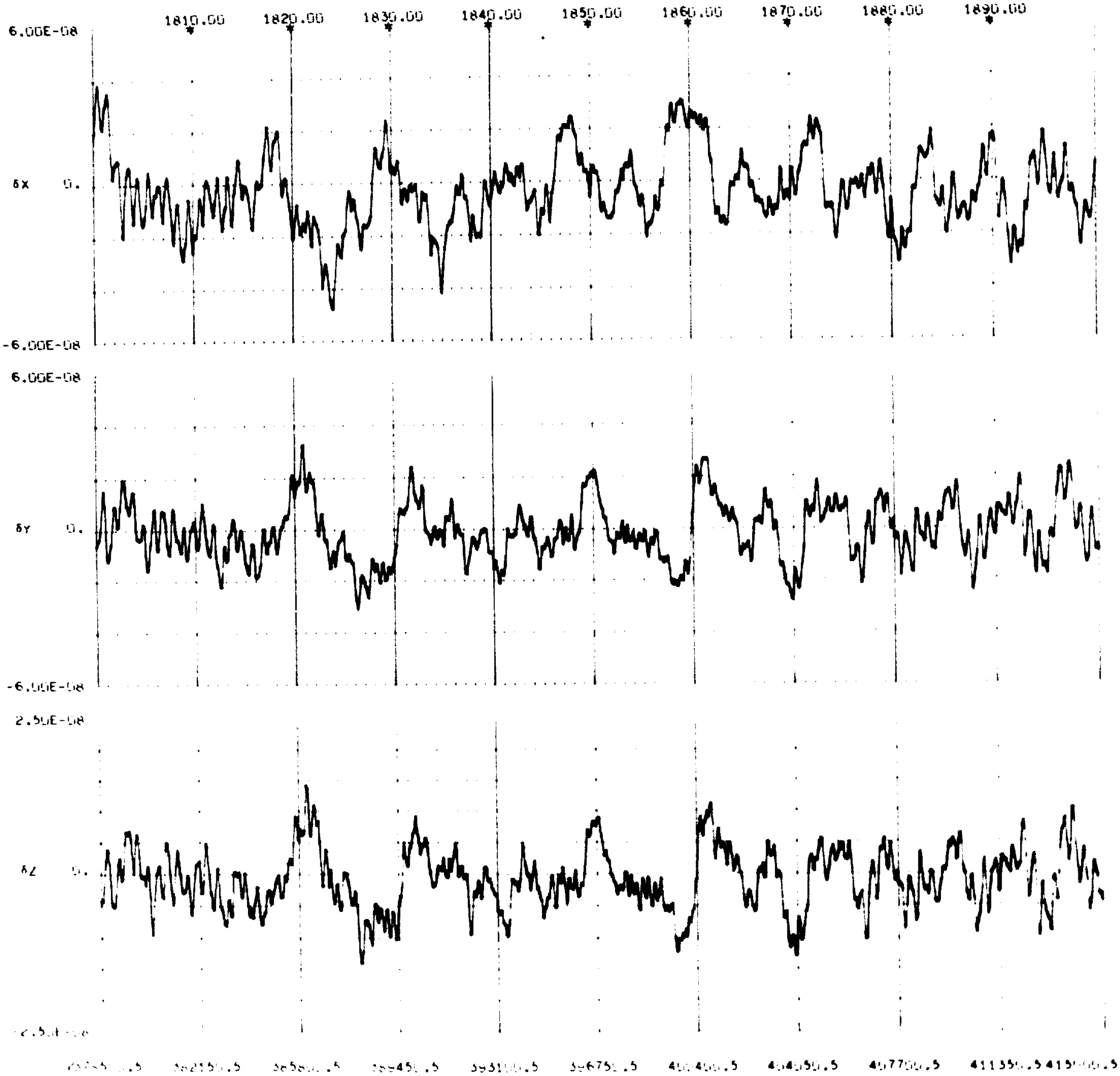


Fig. D-5. Jupiter rectangular residuals (improved modified source minus DE28) for 1800–2000

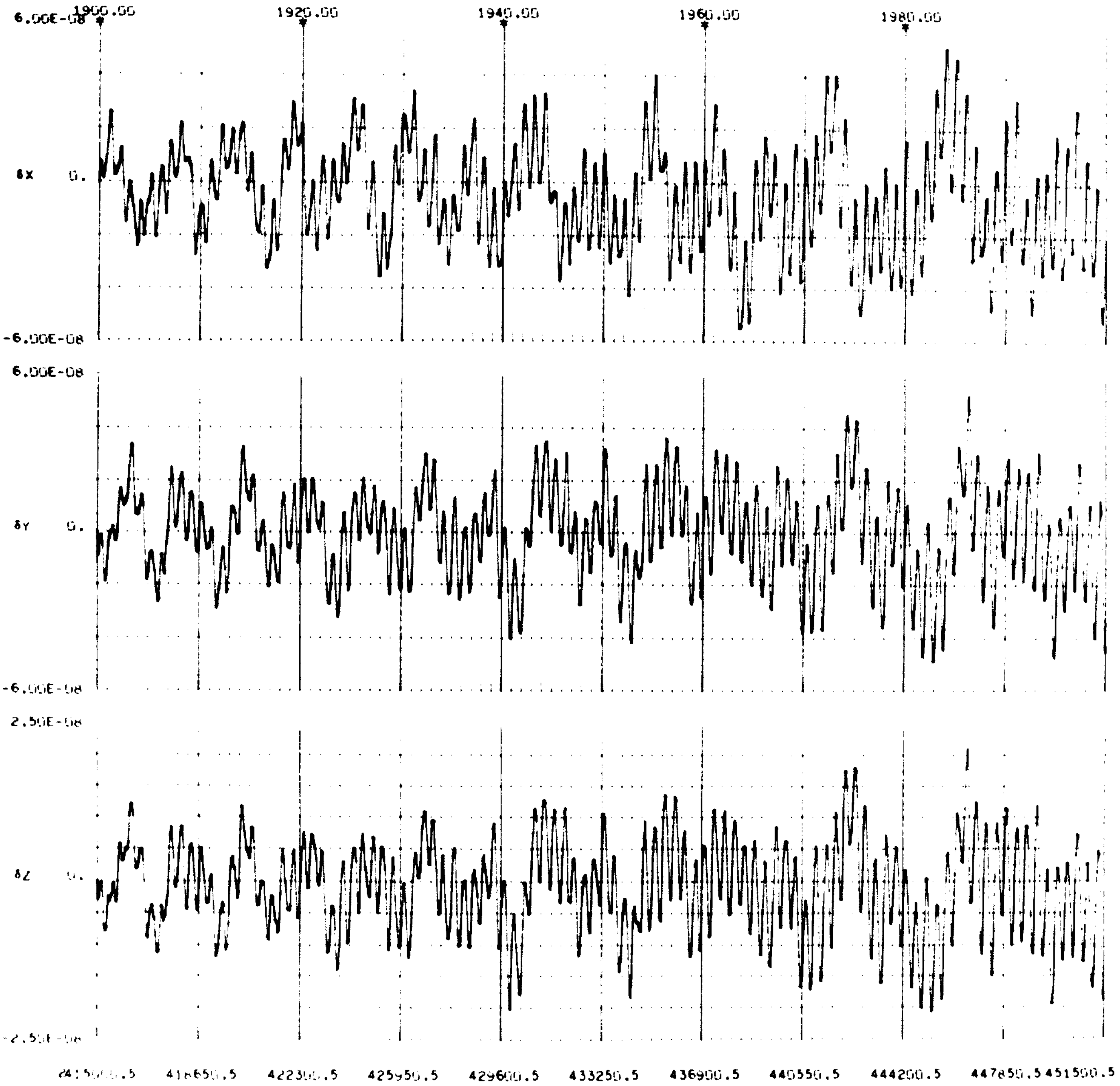

1900.00
1920.00
1940.00
1960.00
1980.00
6.00E-08
δX
0.
-6.00E-08
6.00E-08
δY
0.
-6.00E-08
2.50E-08
δZ
0.
-2.50E-08
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-5 (contd)

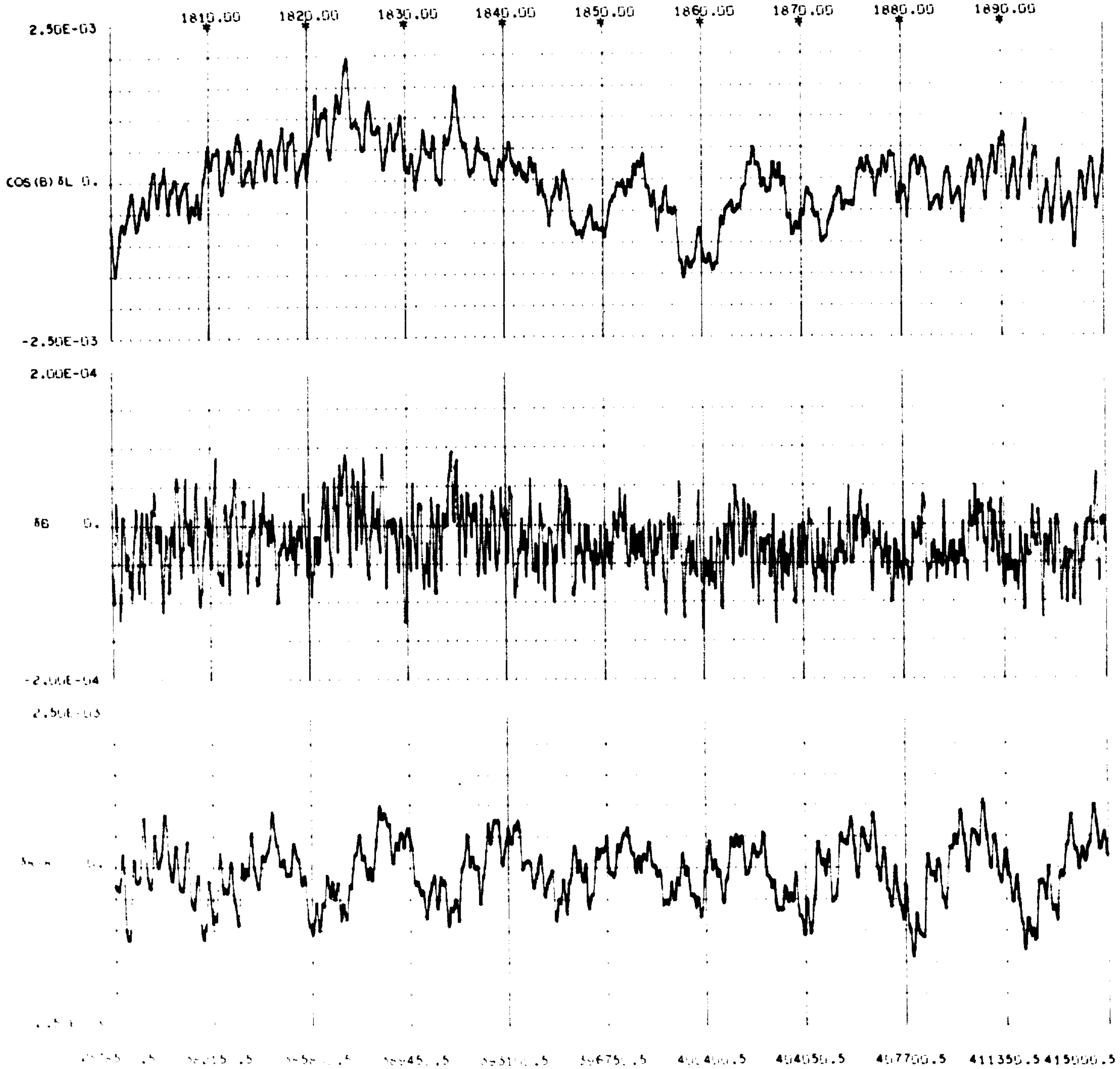
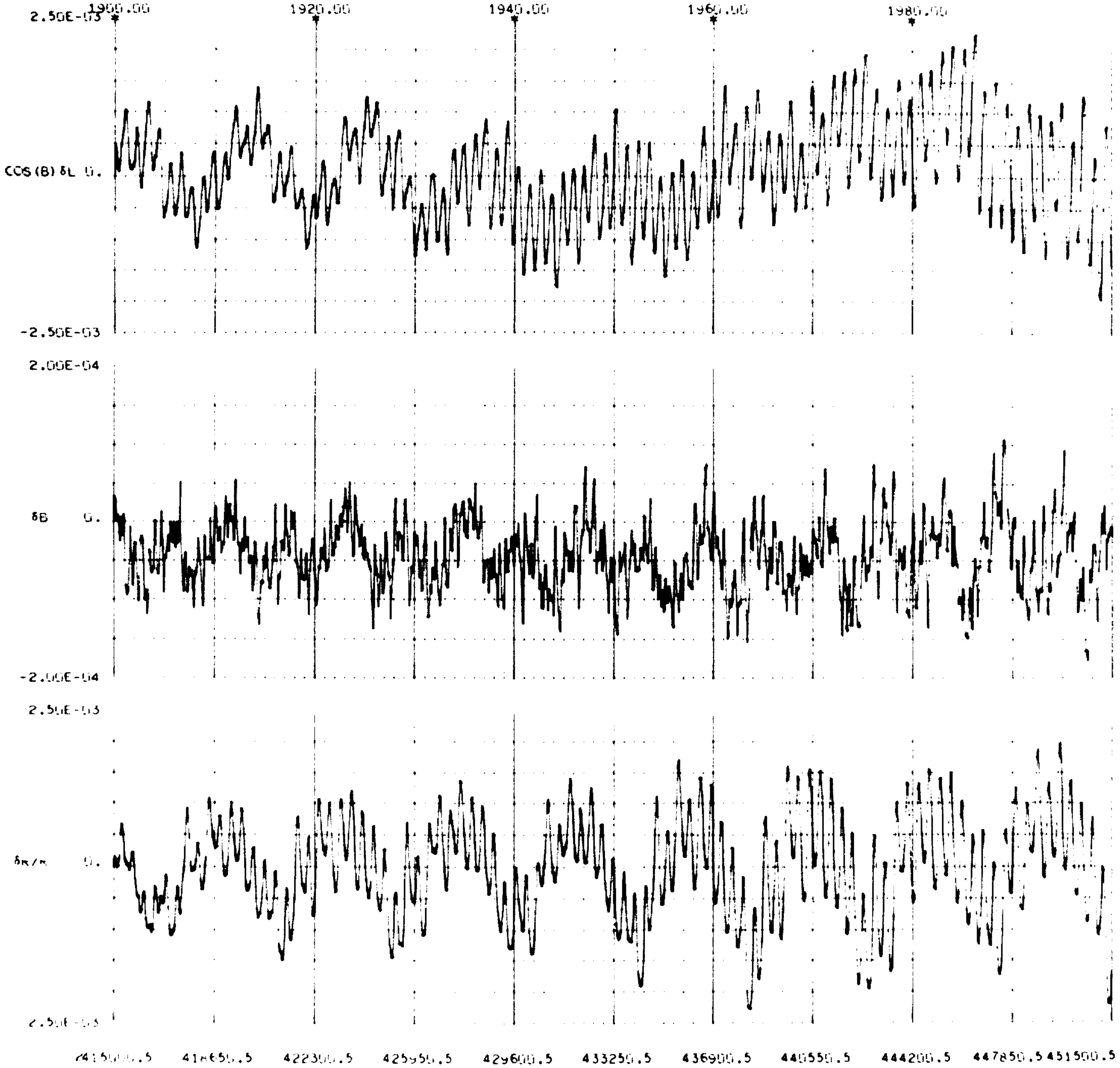


Fig. D-6. Jupiter angular residuals (improved modified source minus DE28) for 1800–2000



1900.00
1920.00
1940.00
1960.00
1980.00
2.50E-03
COS(B)δL
0.
-2.50E-03
2.00E-04
δB
0.
-2.00E-04
2.50E-03
δR/R
0.
-2.50E-03
2415000.5
418650.5
422300.5
425950.5
429600.5
433250.5
436900.5
440550.5
444200.5
447850.5
451500.5

Fig. D-6 (contd)

Table D-1. Improved modified source statistics for Venus

Corrections applied to MS ephemeris

$$\Delta e = -(0''.0668 \pm 0''.0004)\,T$$
$$\Delta\tilde{\omega} = -(7''.43 \pm 0''.06)\,T$$
$$\Delta(\sin I \sin \Omega) = +(0''.010 \pm 0''.001)\,T$$
$$\Delta(\sin I \cos \Omega) = -(0''.0002 \pm 0''.0009)\,T$$

Statistical summary (IMS minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	3.94E −14	8.82E −09	1.98E −07	7.05E −07	−8.17E −07
dy, AU	3.42E −14	5.71E −09	1.85E −07	7.18E −07	−7.78E −07
dz, AU	1.38E −14	3.54E −09	1.18E −07	4.18E −07	−4.03E −07
cos*B dL*, ″	5.71E −03	1.41E −04	7.56E −02	2.66E −01	−2.36E −01
dB, ″	7.77E −04	4.44E −04	2.79E −02	1.01E −01	−1.05E −01
dr/r, ″	6.19E −04	−6.96E −03	2.39E −02	8.73E −02	−9.40E −02

Table D-2. Improved modified source statistics for earth-moon barycenter

Corrections applied to MS ephemeris

$$\Delta e = -(0''.0164 \pm 0''.0004)\,T$$
$$\Delta\tilde{\omega} = +(1''.19 \pm 0''.03)\,T$$
$$\Delta(\sin I \sin \Omega) = +(0''.0002 \pm 0''.0009)\,T$$
$$\Delta(\sin I \cos \Omega) = -(0''.0057 \pm 0''.0009)\,T$$

Statistical summary (IMS minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	7.33E −14	1.16E −08	2.71E −07	1.17E −06	−1.04E −06
dy, AU	6.75E −14	−2.70E −08	2.58E −07	1.50E −06	−8.42E −07
dz, AU	3.17E −14	2.42E −10	1.78E −07	6.39E −07	−6.50E −07
cos*B dL*, ″	4.96E −03	9.76E −05	7.05E −02	2.31E −01	−3.34E −01
dB, ″	1.07E −03	2.26E −03	3.27E −02	1.13E −01	−9.81E −02
dr/r, ″	1.32E −03	−1.04E −02	3.48E −02	1.41E −01	−1.46E −01

Table D-3. Improved modified source statistics for Jupiter

Corrections applied to MS ephemeris

$$\Delta e = -(0\overset{''}{.}00183 \pm 0\overset{''}{.}00001)\,T$$
$$\Delta\tilde{\omega} = +(0\overset{''}{.}0099 \pm 0\overset{''}{.}0003)\,T$$
$$\Delta(\sin I \sin \Omega) = -(5\overset{''}{.}0 \pm 31\overset{''}{.}0) \times 10^{-6}\,\mathrm{T}$$
$$\Delta(\sin I \cos \Omega) = +(1\overset{''}{.}3 \pm 0\overset{''}{.}3) \times 10^{-4}\,T$$

Statistical summary (IMS minus DE28)

Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
dx, AU	2.98E −16	−3.88E −09	1.68E −08	5.28E −08	−5.48E −08
dy, AU	2.25E −16	−9.20E −10	1.50E −08	5.27E −08	−4.91E −08
dz, AU	3.98E −17	−1.11E −09	6.21E −09	2.09E −08	−2.06E −08
cos*B dL*, "	4.59E −07	3.45E −05	6.77E −04	2.28E −03	−2.00E −03
dB, "	3.43E −09	−3.29E −05	4.85E −05	1.06E −04	−1.85E −04
dr/r, "	4.31E −07	−6.66E −05	6.53E −04	1.99E −03	−2.28E −03