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A GENERAL ANALYTICAL METHOD FOR ARTIFICIAL-SATELLITE LIFETIME DETERMINATION

by Frank Garcia, Jr.

Manned Spacecraft Center

Houston, Texas

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ABSTRACT

An expression for obtaining the lifetimes of artificial satellites in circular orbits is developed in this paper. A complete derivation of the method is presented to allow the user to evaluate its assumptions according to specific needs. The accuracy of the developed method is verified using Earth and Mars as examples and comparing the results to the results obtained from numerically integrated and approximate analytical trajectories. Characteristic altitude histories are presented for the trajectory methods and the described analytical solution. Expressions for computing lifetimes of elliptical orbits are also included, with the necessary graphical presentations to provide rapid solution of these expressions.

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SUMMARY

Existing analytical techniques for the determination of satellite lifetimes incorporate assumptions which result in simplified expressions at the expense of generality. This paper develops an expression for obtaining lifetimes of circular orbits which carries terms in the order of the cube of the quotient of the altitude over the radius of the planet which is necessary when considering planets smaller than Earth. A complete derivation of the method is presented to allow the user to evaluate its assumptions according to his specific needs. The accuracy of the developed method is verified using Earth and Mars as examples and comparing the results to the results obtained from numerically integrated and approximate analytical trajectories. Characteristic altitude histories are presented for the trajectory methods and the described analytical solution. Expressions for computing lifetimes of elliptical orbits are also included, with the necessary graphical presentations to provide rapid solution of these expressions.

INTRODUCTION

Any artificial satellite which enters the atmosphere of a planet during a portion of its orbit loses a small amount of energy per revolution as a result of atmospheric drag. This loss of energy may be categorized as occurring during two phases: (1) that which takes place when the orbit has appreciable eccentricity and (2) that which takes place when the orbit is nearly circular. If the orbit of a satellite has an appreciable eccentricity, most of the air drag is encountered in the vicinity of periapsis. Thus, the satellite suffers a slight velocity retardation as it passes through periapsis. This retardation leads to a distance decrease at apoapsis, while scarcely altering the periapsis distance. A good physical example of this phenomenon is observed in Explorer I, which has suffered a velocity decrease at periapsis of 14 n. mi. and a distance decrease at apoapsis of 695 n. mi. in a period of 9 years. Thus, air drag tends to circularize an orbit of initial appreciable eccentricity. An orbit of very low or zero initial eccentricity will manifest its energy loss as slight contractions of the orbit. These energy losses may be used to assess satellite lifetimes using analytical techniques enhanced by simplified but realistic assumptions.

A critical review of currently available mathematical tools to estimate artificial-satellite lifetimes is presented in reference 1. The technique presented in this paper has been developed from the same energy considerations and differs principally in the treatment of the mathematics and the inclusion of higher-order terms. It is considered that the resulting expressions now provide a somewhat more accurate expression than similar solutions for Earth alone. In particular, the inclusion of higher-order terms makes the satellite lifetime expression amenable for use with smaller planets such as Mars.

SYMBOLS

A	satellite reference area, ft^2
a	semimajor axis, ft
BN	ballistic number, $m/C_D A$
C_D	drag coefficient
D	aerodynamic drag, lb
E	total energy per unit mass, ft^2/sec^2-slug
e	eccentricity of orbit
exp	exponential function
H	altitude above planet, n. mi.
$I_1(\beta x_o)$	modified Bessel function with argument βx_o
K	time conversion factor, 3153.6×10^4 sec/yr
KE	kinetic energy per unit mass, ft^2/sec^2-slug
m	satellite mass, slugs
N	revolutions
n	variable in integration of N
P	orbital period, sec/revolution
$\overline{P}$	mean orbital period, sec/revolution

PE	potential energy per unit mass, ft^2/sec^2-slug
R	radius of the planet, n. mi.
r	radial distance from center of planet to satellite, n. mi.
T	normalized satellite lifetime, t/BN
t	satellite lifetime, yr or days
V	orbital speed, ft/sec
$V_{C_{()}}$	circular speed at ( ), ft/sec
x	average altitude, $\left(H_a - H_p\right)/2$
β	density scale height factor, 1/n. mi.
μ	universal gravitational constant, ft^3/sec^2
ρ	density of air, $slugs/ft^3$

Subscripts:

a	apoapsis
H	altitude above planet, n. mi.
o	initial or surface value
p	periapsis
( )	quantity evaluated at point in parentheses
⊕	Earth

METHOD OF ANALYSIS

The analysis is made for orbits of initial zero eccentricity and for those with appreciable eccentricity. The lifetime of a circular orbit is defined as the time that it takes an artificial satellite to decay from its initial altitude to an altitude corresponding to a density of about 1×10^{-10} $slugs/ft^3$. However, the lifetime of a satellite in an orbit of appreciable eccentricity includes the time that it takes it to decay to a circular orbit from its initial eccentricity.

It is assumed that the gravitational forces present are those caused by a purely inverse-square potential. The nongravitational forces are assumed to be caused by a nonrotating exponential atmosphere.

$$\left.\begin{aligned} \rho_H &= \rho_o \exp(-\beta H) \\ D &= -\frac{1}{2}\rho_H C_D A V_C^{\,2} \end{aligned}\right\} \qquad (1)$$

These assumptions are standard in the approach to an analytical expression for satellite lifetimes.

Presented in reference 1 is an expression similar to the one presented in this paper for the lifetime in revolutions. Also presented in reference 1 is a similar expression which can be obtained from equation (A14) in appendix A by neglecting terms in the order of $(H/R)^2$ and assuming that $H >> 1/\beta$. A lifetime expression derived by using an arithmetic mean is given in reference 2.

The method presented in this paper carries terms in the order of $(H/R)^3$ and uses an integrated mean which would reduce error introduced by taking large altitude intervals when using the criteria already suggested in the form of an exponential atmosphere. The choice of an integrated mean instead of an arithmetic mean for the period $\overline{P}$ was suggested by the nonlinearity of $\overline{P}$ with respect to r. Terms of higher order were carried to make the lifetime expression amenable for use with smaller planets such as Mars.

Circular and Near-Circular Orbits

At each point on its orbit, a satellite is subject to drag in the opposite direction of the velocity V_C. The circular speed is

$$V_C = \left(\frac{\mu}{r}\right)^{1/2} = V_{C_o}\left(\frac{R}{R + H}\right)^{1/2} \qquad (2)$$

During one revolution, the orbit remains circular with a radius of r if it is assumed that the variation in Δr is small. This assumption can be verified by integrated trajectories which show, if the orbit is initially circular or near circular, that during lifetime of the orbit this near-circular condition is maintained with eccentricities on the order of 0.0001 or less (fig. 1). Then, for one revolution the change in total energy is caused by drag (fig. 2), and this change is commonly expressed as

$$\Delta E = \oint Dr\, d\theta \qquad (3)$$

Using this energy approach, in appendix A it is shown that the satellite lifetime may be expressed as

$$T = \frac{2}{5(H_1 - H_2)}\left(\frac{r_1^2}{V_{C_{r_1}}} - \frac{r_2^2}{V_{C_{r_2}}}\right)\frac{1}{K\rho_o\beta R^2}\left\{\exp(\beta H_1)\left[1 - \left(H_1 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right)\right.\right.$$

$$\left. + \left(\frac{H_1}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_1}{R}\right)^3\right] - \exp(\beta H_2)\left[1 - \left(H_2 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right)\right.$$

$$\left.\left. + \left(\frac{H_2}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_2}{R}\right)^3\right]\right\} \tag{4}$$

where $T = t/BN$ yr-ft^2/slug. In computing lifetimes using equation (4), it should be noticed that β and ρ_o are assumed to remain constant between H_1 and H_2. A semi-log plot of density versus altitude will dictate which H_1 and H_2 to choose for the best results; that is, H_1 and H_2 should be chosen so that the density scale height is essentially constant within this interval. The method may be continued analytically in this way until the final desired altitude is reached. However, for the sake of expediency, equation (4) was set up on a digital computer, with a subroutine which computes ρ_o and β as a function of altitude.

Orbits of Appreciable Eccentricity

A satellite on an orbit of appreciable eccentricity $(\beta x_o > 2)$ suffers its greatest loss of energy at periapsis, and this loss will have more effect on the decrease of velocity at periapsis than on the radius there. The net effect would be similar to an impulse applied at periapsis. Thus, it is assumed that the radial distance at periapsis remains nearly constant (fig. 3), making use of the exponential atmosphere

$$\rho_H = \rho_o \exp\left[\beta\left(r_{\rho_o} - r_H\right)\right] \tag{5}$$

The restriction on βx_o is caused by the expansion of the modified Bessel function $I_1(\beta x_o)$, which appears in the expression for lifetime. In reference 3 it is shown that the lifetime may be expressed as

$$T = \frac{V_{C(a_o)}}{K\mu\beta\rho_{p_o}} f(\beta x_o, e_o) \tag{6}$$

where β is computed at the initial perigee point p_o and remains constant. Here again, $T = t/BN$ yr-ft^2/slug, and it is shown in appendix B that

$$f(\beta x_o, e_o) = \frac{\beta x_o \exp(\beta x_o)}{2I_1(\beta x_o)} \left[\frac{1}{48}\left(48 \exp e_o - 88e_o \exp e_o + 87e_o^2 \exp e_o + \frac{42e_o}{\beta x_o} \exp e_o\right)\right] \tag{7}$$

where only terms in the order of e_o^2 are retained. The assumption that the radial distance at periapsis remains nearly constant is valid in the physical sense as long as BN does not approach zero. Conversely, as BN approaches large values, this assumption agrees well with the actual situation.

The parameters $f(\beta x_o, e_o)$ and $g(\beta x_o)$ (shown in figs. 4 and 5) may be used to compute lifetimes of satellites in elliptical orbits (ref. 3).

Sample Computation of the Lifetime of an Earth Satellite

The lifetime of a satellite in an initial elliptical orbit may be divided into two phases. The first phase is that time which the satellite theoretically spends in producing a nearly circular orbit (fig. 3), and the second phase is the time that the satellite spends in the decay spiral (fig. 1). The sum of the time spent in these two phases becomes the total lifetime. The following is an example to clarify the computation method.

The planetary constants used are

$$\left.\begin{aligned} \mu_{\oplus} &= 1.4020066 \times 10^{16}\ \mathrm{ft}^3/\mathrm{sec}^2 \\ R_{\oplus} &= 0.209020 \times 10^{8}\ \mathrm{ft} \\ V_{C_{o\oplus}} &= 25\,898.33\ \mathrm{ft/sec} \end{aligned}\right\} \tag{8}$$

If

$$\left.\begin{aligned} B &= 10\ \mathrm{slugs/ft}^2 \\ H_a &= 2000\ \mathrm{n.\ mi.} \\ H_p &= 300\ \mathrm{n.\ mi.} \\ \beta &= 0.02218/\mathrm{n.\ mi.} \\ \rho_{p_o} &= 2 \times 10^{-15}\ \mathrm{slug/ft}^3 \end{aligned}\right\} \tag{9}$$

then the necessary constants are

$$\left.\begin{aligned} e_o &= 0.185 \\ \beta x_o &= 18.85 \\ a_o &= 4588.7 \text{ n. mi.} \\ V_{C_{a_o}} &= 20591.3 \text{ ft/sec} \\ K\mu\rho_{(p_o)}\beta &= 3225.7 \text{ lb-sec/yr-ft}^2 \\ V_{C_{a_o}} \Big/ K\mu\rho_{p_o}\beta &= 6.3835 \text{ yr-ft}^2/\text{ slug} \end{aligned}\right\} \tag{10}$$

Entering e_o and βx_o into figure 4

$$f(\beta x_o, e_o) \simeq 110 \tag{11}$$

therefore, the lifetime in this phase is

$$T_1 = 702.185 \text{ yr-ft}^2/\text{ slug} \tag{12}$$

With $H = 300$ n. mi. in figure 6

$$T_2 \simeq 2700 \text{ days-ft}^2/\text{slug} = 7.397 \text{ yr-ft}^2/\text{slug} \tag{13}$$

The total lifetime T is then the sum of T_1 and T_2, or

$$T_1 + T_2 = T = 709.582 \text{ yr-ft}^2/\text{slug} \qquad (14)$$

RESULTS AND DISCUSSION

Comparison of Results

The results obtained from the analytical expression developed in appendix A for computing circular orbit lifetimes are presented for both Earth and Mars. The results for Earth are compared to two trajectory simulations (fig. 6), whereas the Mars results are compared to only one simulation (fig. 7).

The two trajectory simulations are termed as the analytical ephemeris generator (AEG) and the numerically integrated point mass program (GEM). The AEG program makes use of Lagrange's planetary equations with an oblate Earth and drag effects. The 1962 U.S. standard atmosphere (ref. 4) shown on figure 8 is used, and the oblateness is described by the second, third, and fourth harmonics of the Earth. The main use of this program is not for the prediction of orbital decay; but because it is a relatively fast program, it may be used for this purpose. The GEM program (ref. 5) considers the point mass in three-dimensional space and integrates with a fourth-order Runge-Kutta integration routine. This simulation also uses the 1962 U.S. standard atmosphere.

Analysis of Results

As stated previously, the definition of a lifetime as used in this report is the time from the initial orbital-insertion altitude to the arrival at that altitude where the density reaches about 10^{-10} slugs/ft^3. This value corresponds to about 50 n. mi. for Earth and 90 n. mi. for Mars with the models used. It was also noted in the GEM that at about this point the magnitude of the flight-path angle increased greatly, and thus, the time to impact became negligible as compared to the overall lifetime of the satellite.

The lifetime of a satellite in orbit about the Earth, as calculated by the three prediction methods using the U.S. standard atmosphere (fig. 8), is presented in figure 6 for comparison purposes. The maximum deviation of the analytical technique from the numerically integrated trajectory is less than 13 percent. The maximum deviation from the Lagrangian planetary trajectory is less than 25 percent. These deviations occur at the higher altitudes and decrease rapidly thereafter. At 200 n. mi, the deviation of the presented technique from the GEM and AEG is less than 9 and 19 percent, respectively.

The lifetime for a satellite of Mars is shown in figure 7. The results were obtained using the GEM and the analytical technique with the atmospheric model of figure 9, which was obtained from reference 7. Here, the maximum deviation is less than 10 percent, proving that the analytical technique can be used to predict the lifetime with reasonable accuracy.

CONCLUDING REMARKS

Any simulation which requires numerical integration is not only costly but subject to accumulative errors when used continuously for a long time. Of the three methods used in this report for orbital-decay determination, the closed-form method is by far the least expensive, with the numerically integrated point mass program being prohibitive whenever large lifetimes are expected. The analytical ephemeris generator is adequate for lifetimes of 2 years or less.

The deviations of the analytical technique from either the numerically integrated point mass program or the analytical ephemeris generator trajectory are within the present capability to predict either the atmosphere or the drag coefficient of a body in rarefied flow. Therefore, the relatively small deviation from the numerically integrated point mass program implies that this analytical technique is adequate for orbital lifetime prediction.

Manned Spacecraft Center
National Aeronautics and Space Administration
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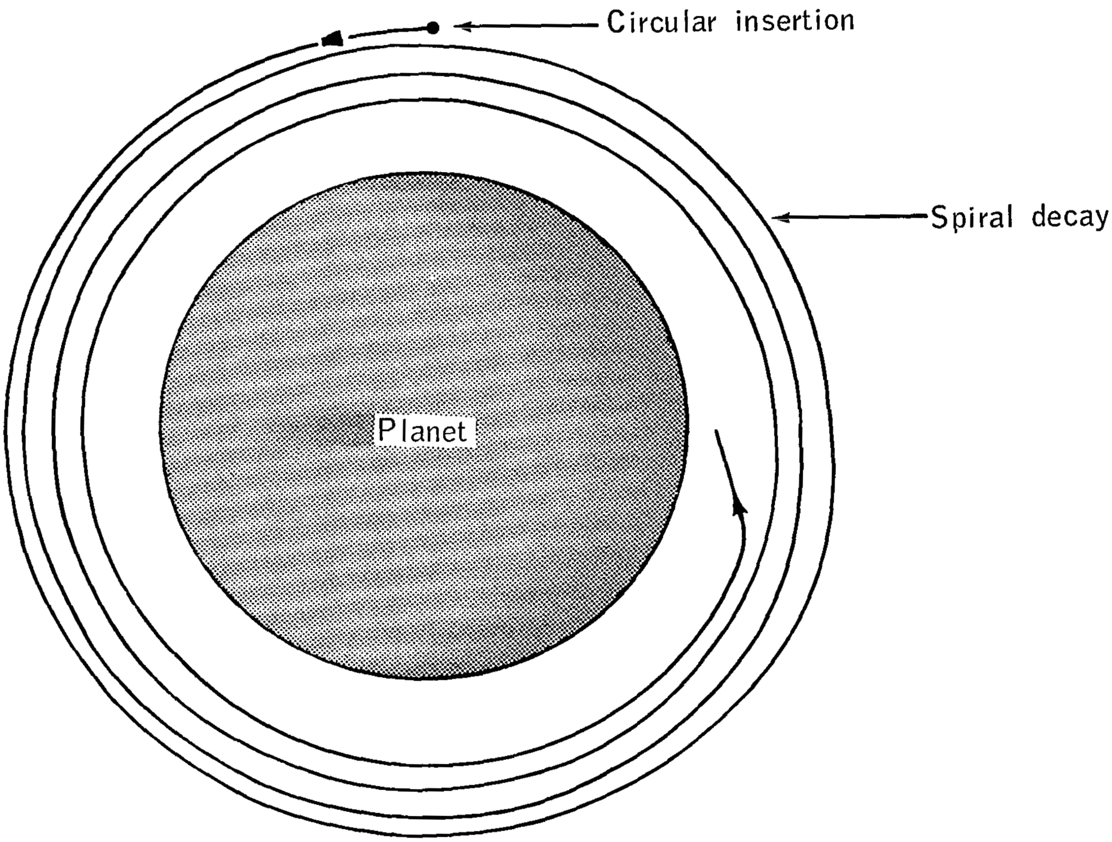

Figure 1. - Theoretical decay of an initially circular orbit perturbed by a planet atmosphere.

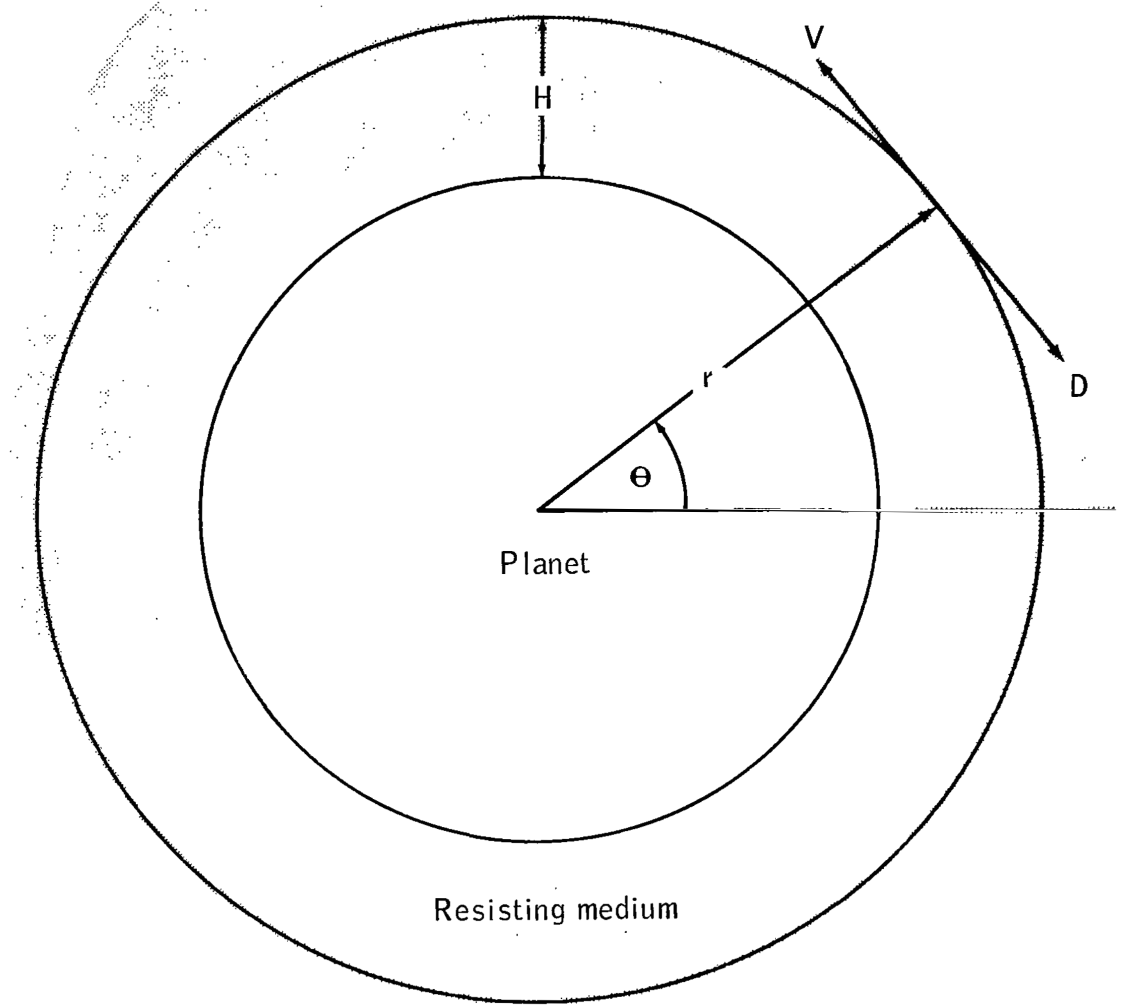

Figure 2. - Definition of quantities used in orbital-lifetime prediction from an initially circular orbit.

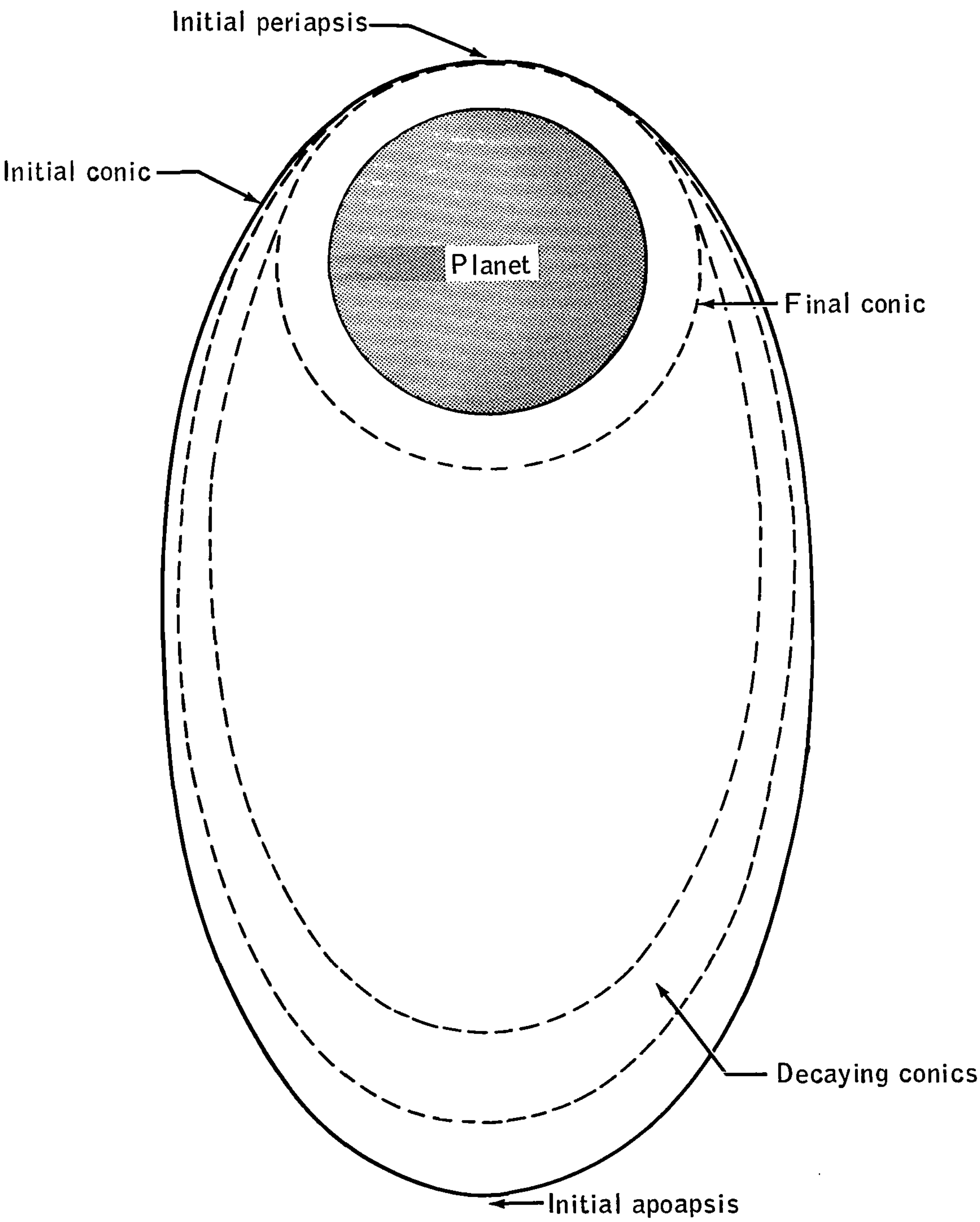

Figure 3. - Definition of conics used in orbital-lifetime prediction from an initially eccentric orbit.

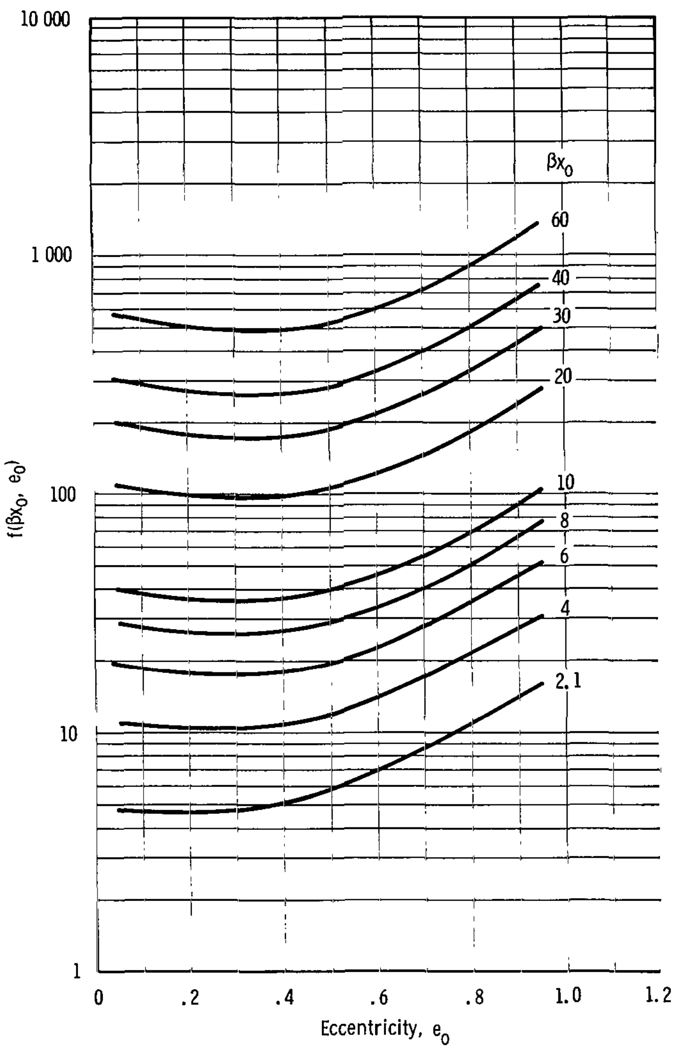

Figure 4. - Elliptical orbit function.

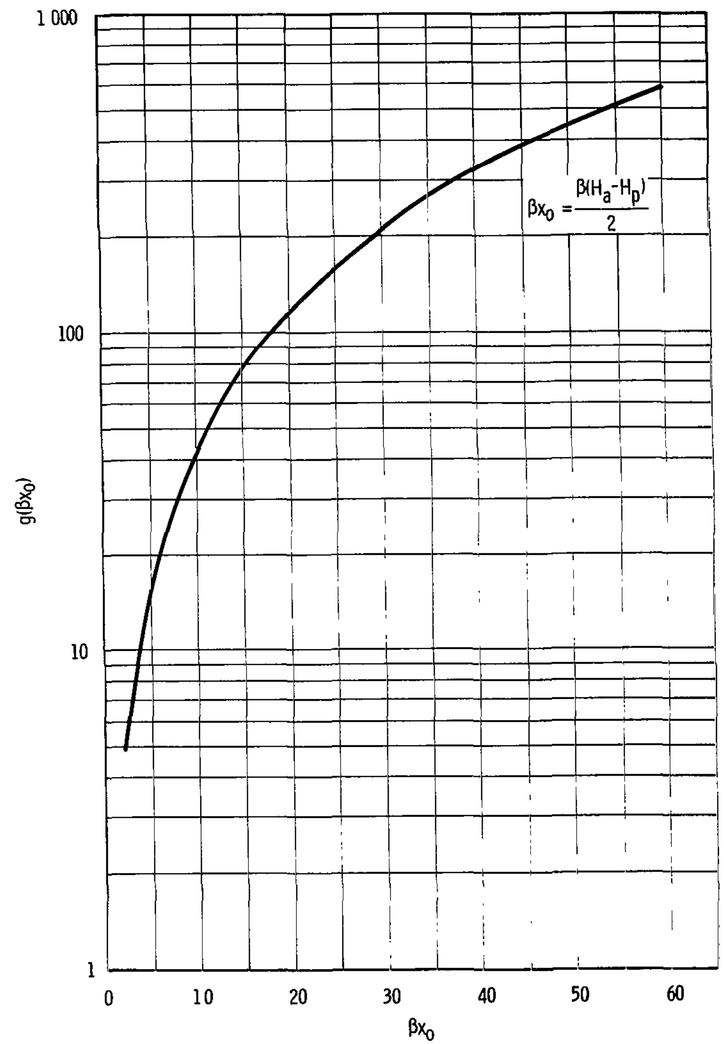

Figure 5. - Modified Bessel function for elliptical orbits.

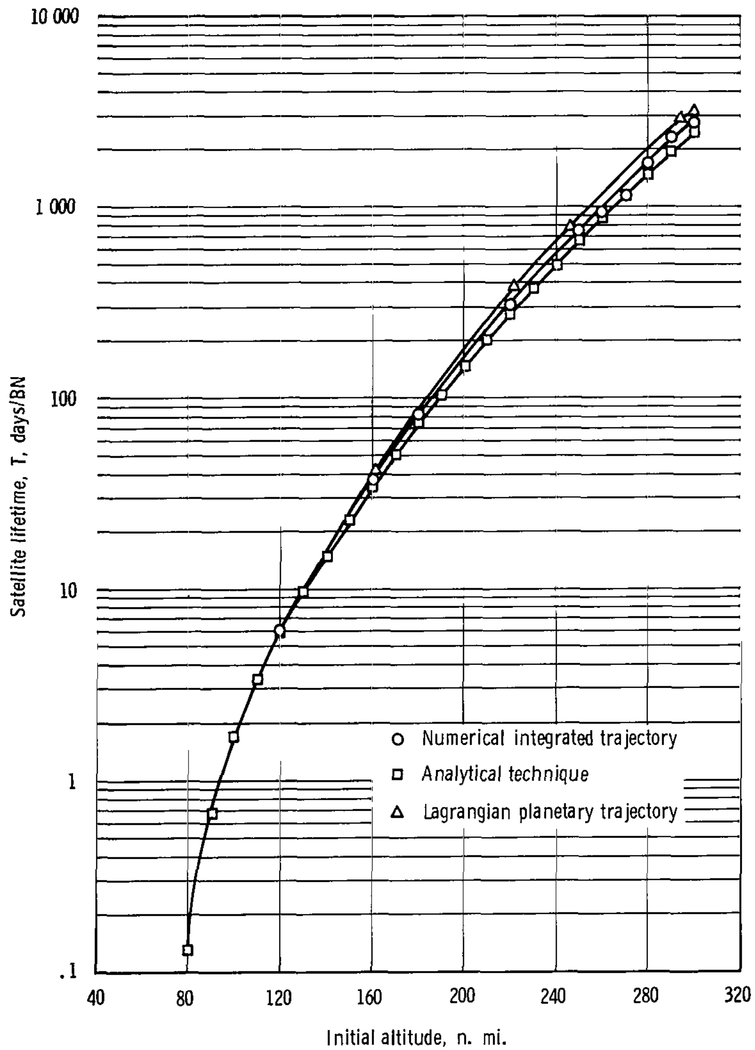

Figure 6. - Lifetime of a satellite in an initially circular orbit about Earth.

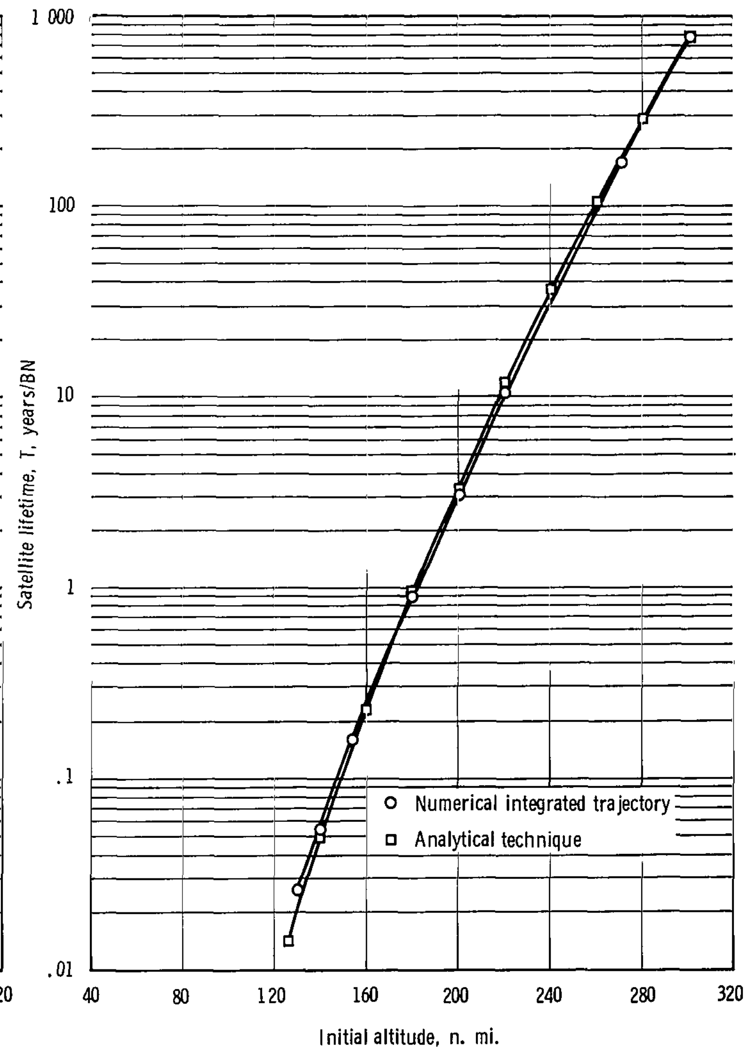

Figure 7. - Lifetime of a satellite in an initially circular orbit about Mars.

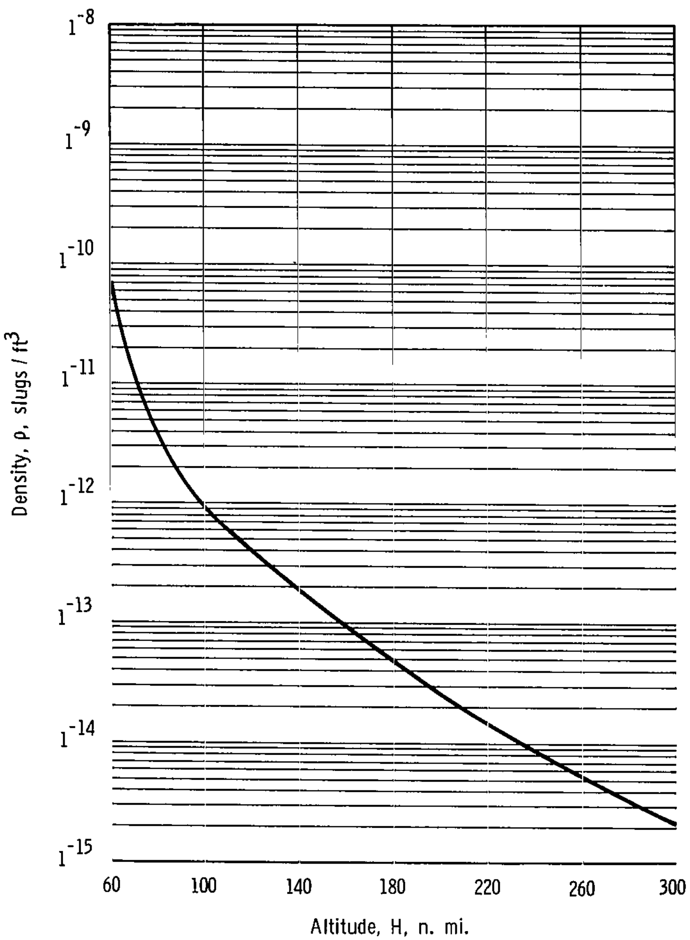

Figure 8. - Earth atmospheric model.

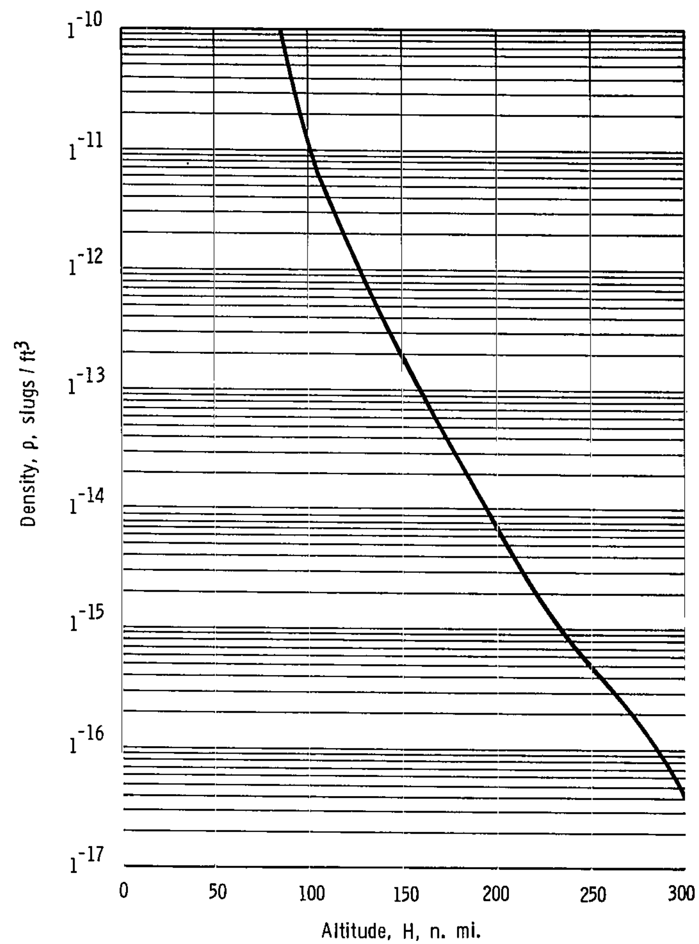

Figure 9. - Mars atmospheric model.

APPENDIX A

CIRCULAR-ORBIT LIFETIME

For one revolution, the change in energy is

$$\Delta E = \oint Dr \, d\theta = -\oint \frac{1}{2} \rho C_D A V_C{}^2 r \, d\theta \tag{A1}$$

or

$$\Delta E = -\frac{1}{2} \mu C_D A \oint \rho \, d\theta \tag{A2}$$

where $D = -\frac{1}{2} \rho C_D A V_C{}^2$ and $V_C{}^2 = \mu/r$. The closed-path integral is taken along a circle of radius r with θ as the argument being swept out (fig. 2). Where an exponential atmosphere is assumed, these equations become

$$\Delta E = -\frac{1}{2} \mu \rho_o C_D A \int_0^{2\pi} \exp(-\beta H) \, d\theta, \tag{A3}$$

and assuming that during one revolution the change in altitude is small, the loss of energy per revolution is

$$\Delta E = -\mu \rho_o C_D A \pi \exp(-\beta H) \tag{A4}$$

The total energy per unit mass is

$$E = KE + PE = \frac{V_C{}^2}{2} - \frac{\mu}{r} \tag{A5}$$

$$E = -\frac{\mu}{2r} \tag{A6}$$

If the mass of the satellite is m, then

$$E = -\frac{m\mu}{2r} \tag{A7}$$

and

$$\Delta E = m\Delta\left(\frac{-\mu}{2r}\right) = -\frac{m\mu}{2r^2}\,\Delta r \tag{A8}$$

assuming that m remains constant.

Since the change in energy is equal to the work done, substituting equation (A4) for ΔE in equation (A8) yields

$$\Delta r = \frac{2\Delta E}{m}\frac{r^2}{\mu} = -\frac{2\pi r^2}{(BN)}\rho_o \exp(-\beta H) \tag{A9}$$

Because Δr is small, the first variation may be taken as the derivative, and recalling that Δr represents the change in radius for one revolution, equation (A9) may be rewritten as

$$(R + H)^{-2}\exp(\beta H)\,dH = -\frac{2\pi}{(BN)}\rho_o\,dn \tag{A10}$$

Expanding $(R + H)^{-2}$ in powers of (H/R)

$$(R + H)^{-2} = R^{-2}\left[1 - 2\left(\frac{H}{R}\right) + 3\left(\frac{H}{R}\right)^2 - 4\left(\frac{H}{R}\right)^3 + \cdots\right] \tag{A11}$$

Retaining terms in the order of $(H/R)^3$ only and substituting equation (A11) into equation (A10)

$$\left.\begin{aligned} (R + H)^{-2} &= R^{-2}\left[1 - 2\left(\frac{H}{R}\right) + 3\left(\frac{H}{R}\right)^2 - 4\left(\frac{H}{R}\right)^3\right] \\ (R + H)^{-2}\exp(\beta H)\,dH &= R^{-2}\left[1 - 2\left(\frac{H}{R}\right) + 3\left(\frac{H}{R}\right)^2 - 4\left(\frac{H}{R}\right)^3\right]\exp(\beta H)\,dH \end{aligned}\right\} \tag{A12}$$

therefore

$$R^{-2}\left[1 - 2\left(\frac{H}{R}\right) + 3\left(\frac{H}{R}\right)^2 - 4\left(\frac{H}{R}\right)^3\right]\exp(\beta H)\,dH = -\frac{2\pi}{(BN)}\rho_o\,dn \tag{A13}$$

Integrating the left-hand side of equation (A13) from H_2 to H_1 and the right-hand side from zero to N

$$\begin{aligned} &\frac{1}{\beta R^2}\exp\left(\beta H_1\right)\left[1 - \left(H_1 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right) + \left(\frac{H_1}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_1}{R}\right)^3\right] \\ &- \exp\left(\beta H_2\right)\left[1 - \left(H_2 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right) + \left(\frac{H_2}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_2}{R}\right)^3\right] = \frac{2\pi}{(BN)}\rho_o N \end{aligned} \tag{A14}$$

where N is the number of revolutions to decay from H_1 to H_2. It is now necessary to obtain an average period to get an expression for lifetime in terms of time.

$$N\overline{P} = t \tag{A15}$$

Making use of the first mean-value theorem for integrals,

$$\overline{P} = \frac{2\pi}{\mu^{1/2}}\int_{r_2}^{r_1} r^{3/2}\,dr = \frac{4}{5}\frac{\pi}{r_1 - r_2}\left(\frac{r_1^{\,2}}{V_{C_{r_1}}} - \frac{r_2^{\,2}}{V_{C_{r_2}}}\right) \tag{A16}$$

The integrated mean of the period from r_1 to r_2 is $\bar{P}$ as expressed in a general manner in reference 6. The lifetime expression may be obtained by solving equation (A15) for N, substituting for $\bar{P}$ from equation (A16), and substituting for N in equation (A14) to obtain the final desired expression

$$T = \frac{2}{5(H_1 - H_2)}\left(\frac{r_1^2}{V_{C_{r_1}}} - \frac{r_2^2}{V_{C_{r_2}}}\right)\frac{1}{\rho_o K\beta R^2}\left\{\exp(\beta H_1)\left[1 - \left(H_1 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right)\right.\right.$$

$$\left. + \left(\frac{H_1}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_1}{R}\right)^3\right] - \exp(\beta H_2)\left[1 - \left(H_2 - \frac{1}{\beta}\right)\left(\frac{2}{R} + \frac{6}{R^2\beta} + \frac{24}{\beta^2 R^3}\right)\right.$$

$$\left.\left. + \left(\frac{H_2}{R}\right)^2\left(3 + \frac{12}{\beta R}\right) - 4\left(\frac{H_2}{R}\right)^3\right]\right\} \qquad \text{(A17)}$$

APPENDIX B

ELLIPTICAL-ORBIT LIFETIME

Reference 3 shows that the satellite lifetime is

$$t = \frac{e_o^2}{2CK}\left(1 - \frac{11e_o}{6} + \frac{29e_o^2}{16} + \frac{7}{8\beta a_o}\right) \tag{B1}$$

$$C = \frac{2\pi}{P_o} \rho_{p_o} x_o I_1(\beta x_o) \exp(-\beta x_o - e_o) \tag{B2}$$

$$t = \frac{(BN)p_o e_o^2 \exp(\beta x_o) \exp e_o}{4\pi\rho_{(p_o)} x_o I_1(\beta x_o) K}\left(1 - \frac{11e_o}{6} + \frac{29e_o^2}{16} + \frac{7}{8\beta a_o}\right) \tag{B3}$$

which may be simplified by using the following relationships.

$$P_o = 2\pi\sqrt{\frac{a_o^3}{\mu}} \tag{B4}$$

$$x_o = a_o e_o \tag{B5}$$

$$V_{C(a_o)} = \sqrt{\frac{\mu}{a_o}} \tag{B6}$$

$$T = \frac{\beta x_o \exp(\beta x_o)}{2I_1(\beta x_o)K} \frac{V_{C(a_o)}}{\mu\rho_{(p_o)} 48\beta}\left(48 \exp e_o - 88e_o \exp e_o + 87e_o^2 \exp e_o + \frac{42e_o \exp e_o}{\beta x_o}\right) \tag{B7}$$

or

$$T = \frac{V_{C_{(a_o)}}}{K\mu P_{(p_o)}{}^{\beta}} f\left(\beta x_o, e_o\right) \tag{B8}$$

where

$$g\left(\beta x_o\right) = \frac{\beta x_o \exp\left(\beta x_o\right)}{2I_1\left(\beta x_o\right)} \tag{B9}$$

$$f\left(\beta x_o, e_o\right) = \frac{g\left(\beta x_o\right)}{48}\left(48 \exp e_o - 88e_o \exp e_o + 87e_o{}^2 \exp e_o + \frac{42e_o \exp e_o}{\beta x_o}\right) \tag{B10}$$

REFERENCES

1. Billik, B: Survey of Current Literature on Satellite Lifetimes. ARS J., vol. 32, no. 11, Nov. 1962, p. 1641.

2. Jensen, Jorgen: Orbital Flight Manual. The Martin-Marietta Co., Rept. ER-11648, July 1961.

3. Deutsch, Ralph: Orbital Dynamics of Space Vehicles. Prentice-Hall, Inc. (Englewood Cliffs, N. J.), 1963.

4. Anon.: U. S. Standard Atmosphere. U. S. Government Printing Office, 1962.

5. Dennison, A. J.; and Butler, J. F.: Missile and Satellite Systems Program for the IBM 7090. General Electric Missile and Space Div., Rept. 61SD170 (Philadelphia, Pa.), Feb. 1962.

6. Fulks, Watson: Advanced Calculus — An Introduction to Analysis. John Wiley and Sons, Inc., 1964.

7. Evans, Dallas; Pitts, D.; Kraus, G. L.: Venus and Mars Nominal Natural Environment for Advanced Manned Planetary Mission Programs. NASA SP-3016, 1965.

NASA-Langley, 1967 —— 30 S-166

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