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ETC, - A FORTRAN IV Program for Computing the Multigroup P(0) and P(1) Elastic Transfer Coefficients

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ETC, - A FORTRAN IV Program for Computing the Multigroup P(0) and P(1) Elastic Transfer Coefficients

I. INTRODUCTION

A program (ETC) has been written in FORTRAN IV to calculate flux averaged elements of the P(0) and P(1) transfer matrices. The flux is assumed to vary as 1/E. The same quantities are calculated with ETC as are calculated with the DOODLES program[1], and also with other[2] programs. However in deriving the equations for ETC rather than carrying out a double integration numerically one of the integrations has been carried out analytically and the numerical integration need only be carried out over one variable. This results in a more satisfactory numerical treatment, and also a faster calculation.

This work was initiated when it was found that numerical difficulties are encountered with DOODLES which makes it inadequate when the elastic scattering cross section depends strongly on the neutron energy, i.e. where resonances occur. For example, the transfer matrices as calculated by ETC and DOODLES agree quite well for Be, however for Ni there are serious discrepancies. Furthermore there are certain relationships which the results must satisfy (see Section VI), and they are not satisfied by the results of DOODLES.

In the case of Ni, for identical input ETC takes about a third the time that DOODLES takes to calculate the P(0) and P(1) matrices, using either an IBM-7094 or a CDC-6600 computer. The calculation required about 4.3 times as much computer time using the IBM-7094 as compared to the CDC-6600 computer,

viz. for ETC a calculation for Ni required 26 seconds with the 6600 as compared to 1.86 minutes with the 7094.

By taking advantage of the feature described in the next paragraph ETC requires about a sixth the time that DOODLES takes to calculate the transfer matrices for Be.

When carrying out the numerical integration in the DOODLES program, either 100 or 200 mesh points must be used per GAM-group*. An external program is available, such that when a number (enough to "describe" the functions) of values of σ_S (the elastic scattering cross section), and of the f_v's (the Legendre coefficients of the differential scattering cross section in the center of mass coordinate system) are input, a linear interpolation is carried out to provide the 100 or 200 values required for the integration. This calculation is carried out in ETC; also any number of mesh points less than 200 can be used in performing the integration. Furthermore the number of mesh points need not be the same for all GAM groups; this feature of course can be utilized to save computer time.

II. EQUATIONS FOR ELASTIC TRANSFERENCE COEFFICIENTS

The quantities+ required for the GAM library are the flux averaged elastic transfer coefficients, viz.

$$\sigma_{i\to j}^{l} = \int_{E_{j-1}}^{E_{j}} \int_{E_i}^{E_{i-1}} S^{l}(E'\to E)\,\varphi(E')\,dE'\,dE \Big/ \int_{E_i}^{E_{i-1}} \varphi(\xi)\,d\xi \,, \qquad (1)$$

*$\frac{1}{4}$ lethargy intervals, 68 groups from 0.414 eV to 10 Mev.

+Subsequently we use the expressions P(0) and P(1) matrices synonomously with the σ^0 and σ^1 matrices.

where the transference function (see Appendix I) is expressed as

$$\sigma(E' \to E, \mu) = \frac{1}{4\pi} \sum_{l} S^{l}(E' \to E)\, P_{l}(\mu) \tag{2}$$

and

$$S^{l}(E' \to E) = \frac{\sigma(E')(2l+1)}{E'(1-\alpha)}\, P_{l}(\mu) \sum_{v} (2v+1)\, f_{v}(E')\, P_{v}(\mu_{0}) \tag{3}$$

$$\text{if} \qquad \alpha E' \leq E \leq E'$$

otherwise $S^{l}(E' \to E) = 0$.

Also

$$f_{v}(E') = \frac{2\pi}{\sigma(E')} \int_{-1}^{1} \sigma(E', \mu_{0})\, P_{v}(\mu_{0})\, d\mu_{0} \qquad \text{(see Appendix II).}$$

The transference function is the cross section for changing the energy E' and the direction $\vec{\Omega}'$ of a neutron into unit energy interval at E and unit solid angle around $\vec{\Omega}$; the cosine of the scattering angle $\mu = \vec{\Omega} \cdot \vec{\Omega}'$. The transference function contains a delta function since the energy E is uniquely determined by the deflection μ .

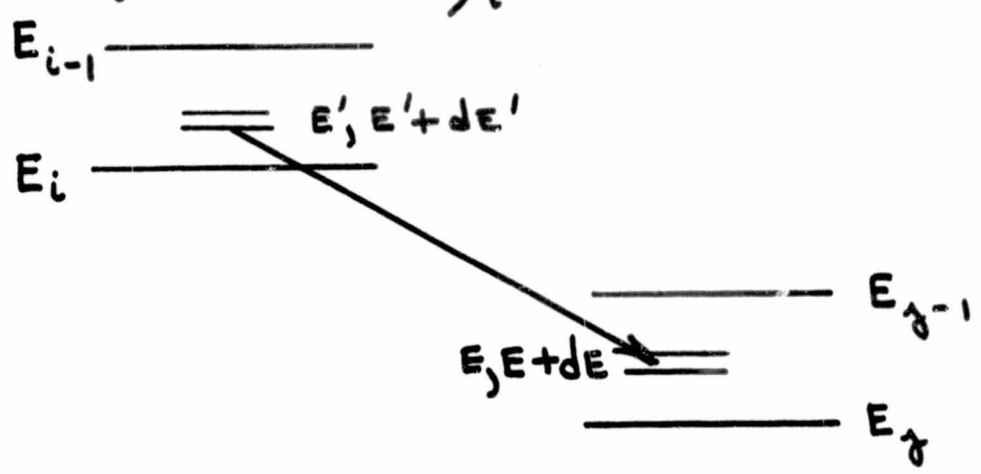


Figure 1 Fine (GAM) Energy Groups

i,j signify two GAM groups;

μ is the cosine of the scattering angle in the lab system of coordinates (target nucleus at rest);

$$\mu = \frac{A+1}{2}\left(\frac{E}{E'}\right)^{\frac{1}{2}} - \frac{A-1}{2}\left(\frac{E'}{E}\right)^{\frac{1}{2}}$$

μ_o is the cosine of the scattering angle in the center of mass system of coordinates;

$$\mu_o = 1 - \frac{(A+1)^2}{2A}\left(1 - \frac{E}{E'}\right)$$

$\alpha = \left(\frac{A-1}{A+1}\right)^2$; $\alpha E'$ is the minimum value of the energy to which a neutron can be reduced in an elastic scattering collision;

E' is the neutron energy before collision;

A is the mass number of the target nucleus;

$P_n(\mu)$ is the n^{th} Legendre polynomial with argument μ;

$\sigma(E')$ is the elastic scattering cross section at energy E'.

The flux ϕ is assumed to be proportional to 1/E, $\sigma(E')$ is an input quantity for ETC, also $f_v(E')$ is an input quantity for ETC that is obtained from the f-code for $v \leq 9$ (see Appendix II). Substituting Equations 3 into Equation 1,

$$\sigma_{ij}^{l} = \frac{2l+1}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \left\{ \frac{\sigma(E')}{(E')^2} \int_{E=C}^{E=d} P_l(\mu) \sum_{v=0}^{9} (2v+1) f_v(E') P_v(\mu_o)\, dE \right\} dE' \qquad (4)$$

For out-of-group scattering (i.e. $i \neq j$) there are six possible situations requiring different sets of limits of integration (a, b, c and d). These cases are given in Table I and are illustrated in Figure 2. For in-group scattering (i.e. $i = j$) there are two cases, see Table II and Figure 3.

Table I Limits of Integration; Out-of-Group Scattering

Case	$\frac{E_j}{\alpha}$	$\frac{E_{j-1}}{\alpha}$	a	b	c	d	a	b	c	d
0	$< E_i$ $< E_{i-1}$	$\leq E_i$ $< E_{i-1}$	Integral is 0							
1	$\leq E_i$ $< E_{i-1}$	$> E_i$ $< E_{i-1}$	E_i	$\frac{E_{j-1}}{\alpha}$	$\alpha E'$	E_{j-1}				
2	$> E_i$ $< E_{i-1}$	$> E_i$ $\leq E_{i-1}$	E_j/α	$\frac{E_{j-1}}{\alpha}$	$\alpha E'$	E_{j-1}	E_i	$\frac{E_j}{\alpha}$	E_j	E_{j-1}
3	$\leq E_i$ $< E_{i-1}$	$> E_i$ $\geq E_{i-1}$	E_i	E_{i-1}	$\alpha E'$	E_{j-1}				
4	$> E_i$ $\leq E_{i-1}$	$> E_i$ $> E_{i-1}$	$\frac{E_j}{\alpha}$	E_{i-1}	$\alpha E'$	E_{j-1}	E_i	$\frac{E_j}{\alpha}$	E_j	E_{j-1}
5	$> E_i$ $\geq E_{i-1}$	$> E_i$ $> E_{i-1}$	E_i	E_{i-1}	E_j	E_{j-1}				

Table II Limits of Integration; In-Group Scattering

Case			a	b	c	d	a	b	c	d
6	$\alpha E_{j-1} \leq E_j$		E_j	E_{j-1}	E_j	E'				
7	$\alpha E_{j-1} > E_j$		$\frac{E_j}{\alpha}$	E_{j-1}	$\alpha E'$	E'	E_j	$\frac{E_j}{\alpha}$	E_j	E'

Figure 2 Limits of Integration; Out-of-Group Scattering

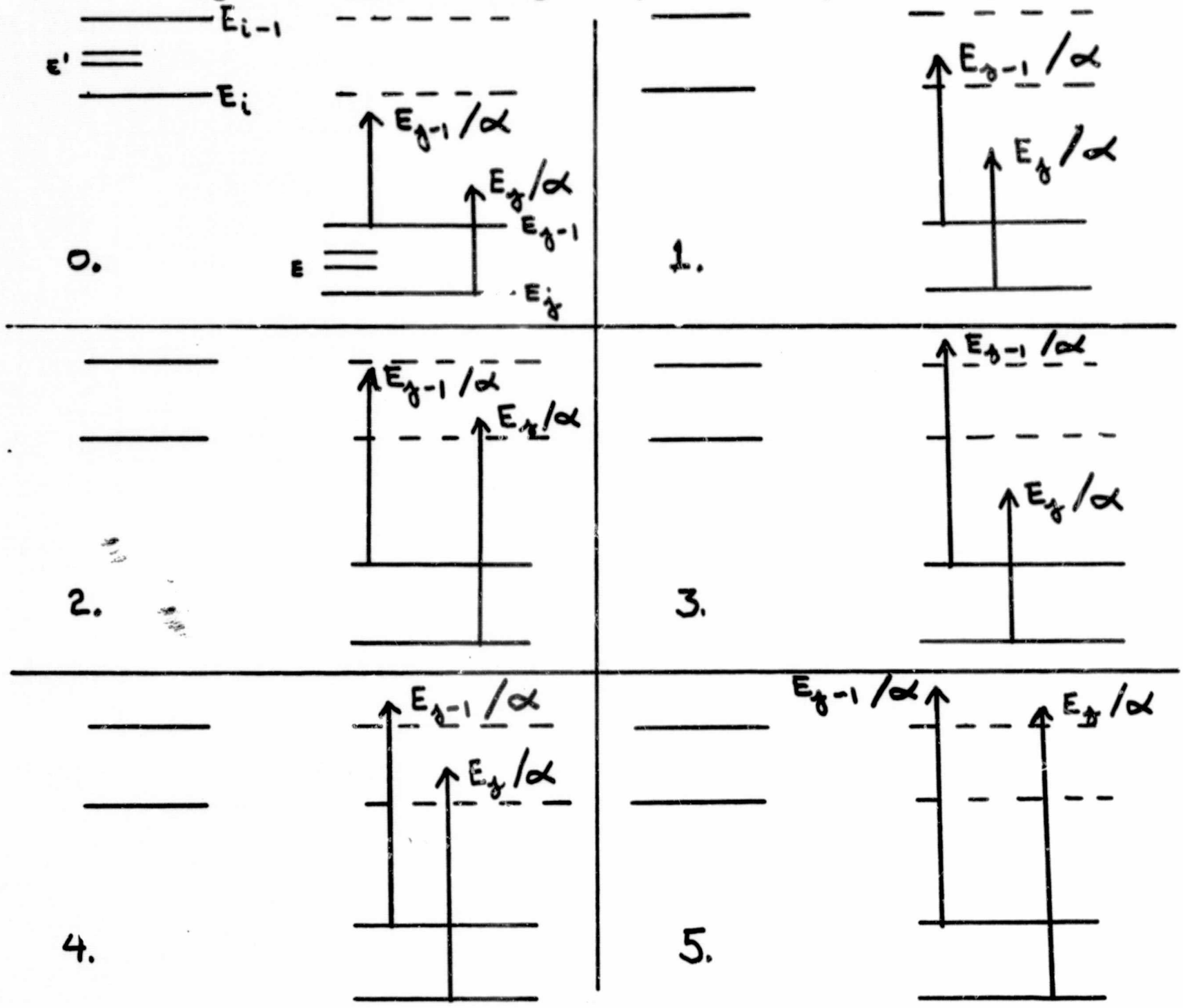
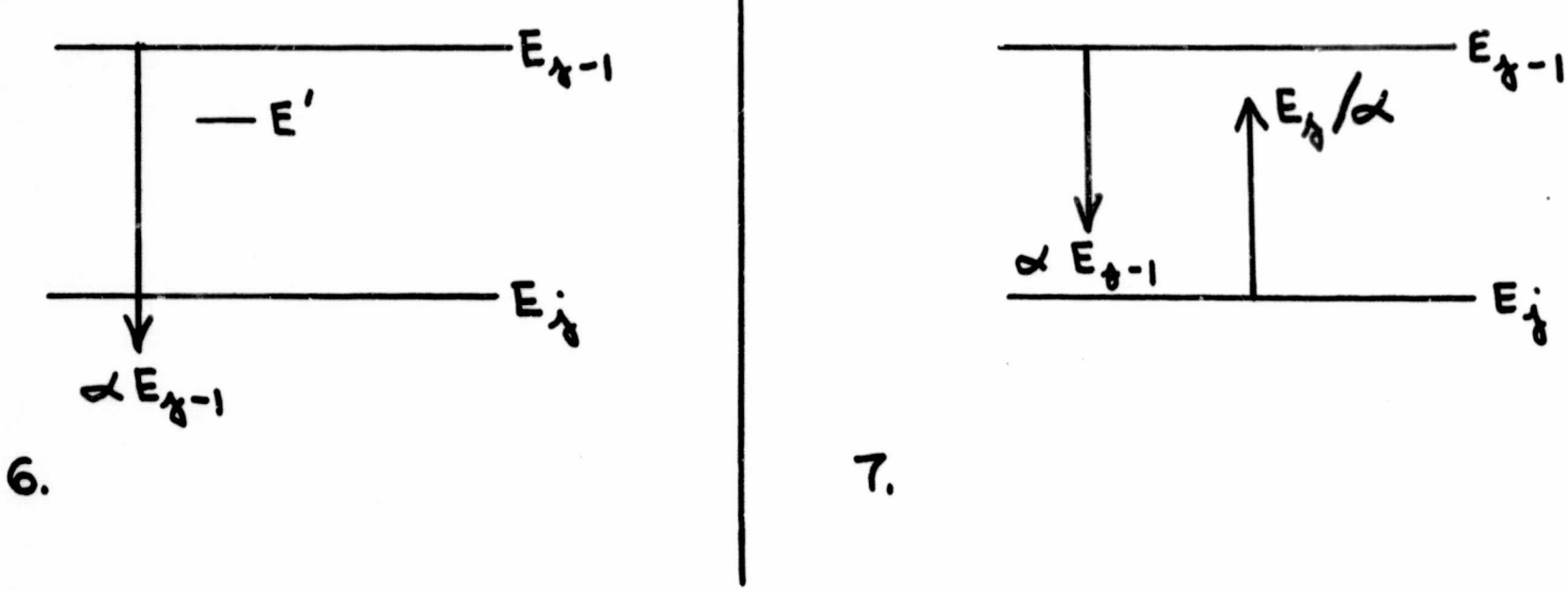


Figure 3 Limits of Integration; In-Group Scattering



For cases 2 and 4 of the out-of-group scattering and case 7 for the in-group scattering the total integral is the sum of two integrals.

From Equation 4 we have:

$$\sigma^0_{ij} \equiv [P\text{-}0]_{ij} = \frac{1}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \left\{ \frac{\sigma(E')}{(E')^2} \sum_{v=0}^{g} (2v+1) f_v(E') \int_{E=C}^{E=d} P_v(\mu_0)\, dE \right\} dE' \qquad (5)$$

and

$$\sigma^1_{ij} \equiv [P\text{-}1]_{ij} = \frac{3}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \left\{ \frac{\sigma(E')}{(E')^2} \sum_{v=0}^{g} (2v+1) f_v(E') \int_{E=C}^{E=d} \mu\, P_v(\mu_0)\, dE \right\} dE' \qquad (6)$$

where P-0 and P-1 are the symbols used in GAM.

III. THE $[P\text{-}0]$ MATRIX, $P(0)_{ij} \equiv \sigma^0_{ij}$

Rewriting Equation 5

$$\sigma^0_{ij} = \frac{1}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \left\{ \frac{\sigma(E')}{(E')^2} \left[(d-c) + \sum_{v=1}^{g} (2v+1) f_v(E') \int_{E=C}^{E=d} P_v(\mu_0)\, dE \right] \right\} dE' \qquad (7)$$

since $f_o = P_o = 1$.

Now $$\mu_0 = 1 - \frac{2}{1-\alpha}\left(1 - \frac{E}{E'}\right) \quad \text{or} \quad -\left(\frac{1+\alpha}{1-\alpha}\right) + \frac{2}{1-\alpha}\left(\frac{E}{E'}\right), \qquad (8)$$

so if we wish to integrate over E, we may hold E' constant and

$$d\mu_0 = \frac{2}{1-\alpha}\frac{dE}{E'} \quad . \qquad (9)$$

Equation 7 becomes

$$\sigma_{ij}^{o} = \frac{1}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \left\{ \frac{\sigma(E')}{(E')^2} \left[(d-c) + \frac{(1-\alpha)E'}{2} \sum_{v=1}^{g} (2v+1) f_v(E') \int_{M_0(E=c)}^{M_0(E=d)} P_v(M_0)\, dM_0 \right] \right\} dE' \qquad (10)$$

where

$$M_0(E=d) = 1 - \frac{2}{1-\alpha}\left(1 - \frac{d}{E'}\right) = 1 - \frac{2}{1-\alpha} + \frac{2d}{(1-\alpha)E'} \quad , \text{ and}$$

$$M_0(E=c) = 1 - \frac{2}{1-\alpha}\left(1 - \frac{c}{E'}\right) = 1 - \frac{2}{1-\alpha} + \frac{2c}{(1-\alpha)E'} \, .$$

Now $(2v+1)P_v = P'_{v+1} - P'_{v-1}$ (see reference 3)

therefore, $\int_{E=C}^{E=d} (2v+1) P_v(M_0)\, dM_0 = \left[P_{v+1}(M_0) - P_{v-1}(M_0) \right]_{M_0(E=c)}^{M_0(E=d)}$

Equation 10 becomes

$$\sigma_{ij}^{o} = \frac{1}{(1-\alpha)\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \frac{\sigma(E')}{(E')^2} \left\{ (d-c) + \frac{(1-\alpha)E'}{2} \sum_{v=1}^{g} f_v(E') \left[P_{v+1}(M_0) - P_{v-1}(M_0) \right]_{M_0(E=c)}^{M_0(E=d)} \right\} dE'$$

or

$$\sigma_{ij}^{o} = \frac{1}{2\ln(E_{i-1}/E_i)} \int_{E'=a}^{E'=b} \frac{\sigma(E')}{E'} \left\{ \frac{2(d-c)}{(1-\alpha)E'} + \sum_{v=1}^{g} f_v(E') \left[P_{v+1}(M_0) - P_{v-1}(M_0) \right]_{M_0(E=c)}^{M_0(E=d)} \right\} dE' \qquad (11)$$

Table III gives the expressions for the first eleven Legendre polynomials. Table IV is the set of terms, $\left[P_{v+1}(M_0) - P_{v-1}(M_0) \right]_{M_0(E=c)}^{M_0(E=d)}$ required for evaluating the integral in Equation 11.

The Program ETC carries out the integration of E' in Equation 11 numerically to obtain σ_{ij}^{o} .

Table III The Legendre Polynomials

$$P_0(M_o) = 1$$

$$P_1(M_o) = M_o$$

$$P_2(M_o) = (1/2)(3M_o^2 - 1)$$

$$P_3(M_o) = (1/2)(5M_o^3 - 3M_o)$$

$$P_4(M_o) = (1/8)(35M_o^4 - 30M_o^2 + 3)$$

$$P_5(M_o) = (1/8)(63M_o^5 - 70M_o^3 + 15M_o)$$

$$P_6(M_o) = (1/16)(231M_o^6 - 315M_o^4 + 105M_o^2 - 5)$$

$$P_7(M_o) = (1/16)(429M_o^7 - 693M_o^5 + 315M_o^3 - 35M_o)$$

$$P_8(M_o) = (1/128)(6435M_o^8 - 12012M_o^6 + 6930M_o^4 - 1260M_o^2 + 35)$$

$$P_9(M_o) = (1/128)(12155M_o^9 - 25740M_o^7 + 18018M_o^5 - 4620M_o^3 + 315M_o)$$

$$P_{10}(M_o) = (1/256)(46189M_o^{10} - 109395M_o^8 + 90090M_o^6 - 30030M_o^4 + 3465M_o^2 - 63)$$

Table IV $\left[P_{\nu+1}(\mu_0) - P_{\nu-1}(\mu_0)\right]_{\mu_0(E=c)}^{\mu_0(E=d)}$

For Evaluating the Integral in Equation 11

$$\Delta P_{2,0} = 1.5\,\Delta^2$$

$$\Delta P_{3,1} = 2.5\,\Delta^3 - 2.5\Delta$$

$$\Delta P_{4,2} = 4.375\,\Delta^4 - 5.25\,\Delta^2$$

$$\Delta P_{5,3} = 7.875\,\Delta^5 - 11.25\,\Delta^3 + 3.375\Delta$$

$$\Delta P_{6,4} = 14.4375\,\Delta^6 - 24.0625\,\Delta^4 + 10.3125\,\Delta^2$$

$$\Delta P_{7,5} = 26.8125\,\Delta^7 - 51.1875\,\Delta^5 + 28.4375\,\Delta^3 - 4.0625\Delta$$

$$\Delta P_{8,6} = 50.2734375\,\Delta^8 - 108.28125\,\Delta^6 + 73.828125\,\Delta^4 - 16.40625\,\Delta^2$$

$$\Delta P_{9,7} = 94.9609375\,\Delta^9 - 227.90625\,\Delta^7 + 184.078125\,\Delta^5 - 55.78125\,\Delta^3 + 4.6484375\Delta$$

$$\Delta P_{10,8} = 180.42578125\,\Delta^{10} - 477.59765625\,\Delta^8 + 445.7578125\,\Delta^6 - 171.4453125\,\Delta^4 + 23.37890625\,\Delta^2$$

where

$$\Delta P_{ij} = \left\{P_i\left[\mu_0(E=d)\right] - P_j\left[\mu_0(E=d)\right]\right\} - \left\{P_i\left[\mu_0(E=c)\right] - P_j\left[\mu_0(E=c)\right]\right\}$$

and

$$\Delta^n = \left[\mu_0(E=d)\right]^n - \left[\mu_0(E=c)\right]^n$$

Note! $\Delta^2 \neq \Delta \times \Delta$ etc.

IV. THE $[P-1]$ MATRIX, $P(1)_{ij} \equiv \sigma^{1}_{ij}$

To solve Equation 6 we need:

$$F(E') = \sum_{\nu=0}^{9} (2\nu+1)\, f_\nu(E') \int_{E=C}^{E=d} \mu\, P_\nu(\mu_0)\, dE. \tag{12}$$

Since

$$\mu = \frac{A+1}{2}\sqrt{\frac{E}{E'}} - \frac{A-1}{2}\sqrt{\frac{E'}{E}} = \frac{1+A\mu_0}{(1+2A\mu_0+A^2)^{1/2}},$$

Equation 12 may be rewritten as

$$F(E') = \sum_{\nu=0}^{9} \frac{E'(1-\alpha)}{2} (2\nu+1) f_\nu(E') \int_{\mu_0(E=C)}^{\mu_0(E=d)} \frac{(1+A\mu_0)\, P_\nu(\mu_0)}{(1+2A\mu_0+A^2)^{1/2}}\, d\mu_0. \tag{13}$$

Expanding Equation 13:

$$\begin{aligned}
F(E') = \frac{(1-\alpha)E'}{2} \int_{\mu_0(E=C)}^{\mu_0(E=d)} \Big\{ & f_0(E')(1+A\mu_0) + 3f_1(E')(\mu_0+A\mu_0^2) + 5f_2(E')\tfrac{1}{2}(3A\mu_0^3 \\
& + 3\mu_0^2 - A\mu_0 - 1) + 7f_3(E')\tfrac{1}{2}(5A\mu_0^4 + 5\mu_0^3 - 3A\mu_0^2 - 3\mu_0) \\
& + \tfrac{9}{8} f_4(E')(35A\mu_0^5 + 35\mu_0^4 - 30A\mu_0^3 - 30\mu_0^2 + 3A\mu_0 + 3) \\
& + \tfrac{11}{8} f_5(E')(63A\mu_0^6 + 63\mu_0^5 - 70A\mu_0^4 - 70\mu_0^3 + 15A\mu_0^2 + 15\mu_0) \\
& + \tfrac{13}{16} f_6(E')(231A\mu_0^7 + 231\mu_0^6 - 315A\mu_0^5 - 315\mu_0^4 + 105A\mu_0^3 + 105\mu_0^2 - 5A\mu_0 - 5) \\
& + \tfrac{15}{16} f_7(E')(429A\mu_0^8 + 429\mu_0^7 - 693A\mu_0^6 - 693\mu_0^5 + 315A\mu_0^4 + 315\mu_0^3 - 35A\mu_0^2 - 35\mu_0) \\
& + \tfrac{17}{128} f_8(E')(6435A\mu_0^9 + 6435\mu_0^8 - 12012A\mu_0^7 - 12012\mu_0^6 + 6930A\mu_0^5 + 6930\mu_0^4 - 1260A\mu_0^3 - 1260\mu_0^2 + 35A\mu_0 + 35) \\
& + \tfrac{19}{128} f_9(E')(12155A\mu_0^{10} + 12155\mu_0^9 - 25740A\mu_0^8 - 25740\mu_0^7 + 18018A\mu_0^6 + 18018\mu_0^5 - 4620A\mu_0^4 - 4620\mu_0^3 + 315A\mu_0^2 + 315\mu_0) \Big\} \\
& \times \left\{ \frac{d\mu_0}{(1+2A\mu_0+A^2)^{1/2}} \right\}.
\end{aligned} \tag{14}$$

Each term in the integrand is of the general form $\int \frac{x^m dx}{(a+bx)^{1/2}}$

By using

$$\int \frac{x^m dx}{(a+bx)^{1/2}} = \frac{2x^m \sqrt{a+bx}}{(2m+1)\,b} - \frac{2ma}{(2m+1)b} \int \frac{x^{m-1}\, dx}{(a+bx)^{1/2}} \quad , \qquad (15)$$

one can obtain the integrations required in Equation 14. These are given in Table V.

Table V Results of the Integrations: $I_n \equiv \int \frac{M_0^n \, dM_0}{\sqrt{(1+A^2)+2AM_0}}$

In the solutions k is defined as: $k \equiv \frac{A^2+1}{A}$.

$$I_{10} = \frac{M_0^{10}\sqrt{(1+A^2)+2AM_0}}{21A} - \frac{10(1+A^2)}{21A} I_9 = \frac{M_0^{10}}{21} I_0 - \frac{10}{21} k I_9$$

$$I_9 = \frac{M_0^9\sqrt{(1+A^2)+2AM_0}}{19A} - \frac{9(1+A^2)}{19A} I_8 = \frac{M_0^9}{19} I_0 - \frac{9}{19} k I_8$$

$$I_8 = \frac{M_0^8\sqrt{(1+A^2)+2AM_0}}{17A} - \frac{8(1+A^2)}{17A} I_7 = \frac{M_0^8}{17} I_0 - \frac{8}{17} k I_7$$

$$I_7 = \frac{M_0^7\sqrt{(1+A^2)+2AM_0}}{15A} - \frac{7(1+A^2)}{15A} I_6 = \frac{M_0^7}{15} I_0 - \frac{7}{15} k I_6$$

$$I_6 = \frac{M_0^6\sqrt{(1+A^2)+2AM_0}}{13A} - \frac{6(1+A^2)}{13A} I_5 = \frac{M_0^6}{13} I_0 - \frac{6}{13} k I_5$$

$$I_5 = \frac{M_0^5\sqrt{(1+A^2)+2AM_0}}{11A} - \frac{5(1+A^2)}{11A} I_4 = \frac{M_0^5}{11} I_0 - \frac{5}{11} k I_4$$

$$I_4 = \frac{M_0^4\sqrt{(1+A^2)+2AM_0}}{9A} - \frac{4(1+A^2)}{9A} I_3 = \frac{M_0^4}{9} I_0 - \frac{4}{9} k I_3$$

$$I_3 = \frac{M_0^3\sqrt{(1+A^2)+2AM_0}}{7A} - \frac{3(1+A^2)}{7A} I_2 = \frac{M_0^3}{7} I_0 - \frac{3}{7} k I_2$$

$$I_2 = \frac{M_0^2\sqrt{(1+A^2)+2AM_0}}{5A} - \frac{2(1+A^2)}{5A} I_1 = \frac{M_0^2}{5} I_0 - \frac{2}{5} k I_1$$

$$I_1 = \frac{M_0\sqrt{(1+A^2)+2AM_0}}{3A} - \frac{(1+A^2)}{3A} I_0 = \frac{M_0}{3} I_0 - \frac{1}{3} k I_0$$

$$I_0 = \frac{\sqrt{(1+A^2)+2AM_0}}{A} = \sqrt{1+2\frac{M_0}{A}+\frac{1}{A^2}} \, .$$

It is obvious that I_n can be written in terms of I_o. In fact it is shown in Appendix III that for $n \geq 1$

$$I_n = \frac{1}{2n+1}\left\{\sum_{i=0}^{n-1}(-1)^{i+1}\frac{n(n-1)(n-2)\cdots(n-i)}{(2n-1)(2n-3)\cdots(2n-2i-1)}k^{i+1}\mu_0^{n-i-1}+\mu_0^{n}\right\}I_o \qquad (16)$$

Using the results given in Table V Equation 14 becomes

$$F(E') = \frac{E'(1-\alpha)}{2}G(E') = \frac{E'(1-\alpha)}{2}\left\{f_0(E')(I_o+AI_1)+3f_1(E')(AI_2+I_1)\right.$$

$$+\frac{5}{2}f_2(E')[3AI_3+3I_2-AI_1-I_o]+\frac{7}{2}f_3(E')[5AI_4+5I_3-3AI_2-3I_1]$$

$$+\frac{9}{8}f_4(E')[35AI_5+35I_4-30AI_3-30I_2+3AI_1+3I_o]$$

$$+\frac{11}{8}f_5(E')[63AI_6+63I_5-70AI_4-70I_3+15AI_2+15I_1]$$

$$+\frac{13}{16}f_6(E')[231AI_7+231I_6-315AI_5-315I_4+105AI_3+105I_2-5AI_1-5I_o]$$

$$+\frac{15}{16}f_7(E')[429AI_8+429I_7-693AI_6-693I_5+315AI_4+315I_3-35AI_2-35I_1]$$

$$+\frac{17}{128}f_8(E')[6435AI_9+6435I_8-12012AI_7-12012I_6+6930AI_5+6930I_4-1260AI_3-1260I_2+35AI_1+35I_o]$$

$$\left.+\frac{19}{128}f_9(E')[12155AI_{10}+12155I_9-25740AI_8-25740I_7+18018AI_6+18018I_5-4620AI_4-4620I_3+315AI_2+315I_1]\right\}_{M_o(E=c)}^{M_o(E=d)} \qquad (17)$$

and

$$\sigma_{ij}^{1} = \frac{3}{2\ln(E_{i-1}/E_i)}\int_{E'=a}^{E'=b}\frac{\sigma(E')}{E'}G(E')\,dE'. \qquad (18)$$

The program ETC carries out the integration over E' numerically in Equation 17 to obtain σ_{ij}^{1}.

In order to carry out the integration of Equation 18 it is useful (because of the numerical computation) to expand (Binomial series) the functions I_n in terms of $(\frac{\mathcal{M}_o}{k})$. This is done in Appendix IV. In order for the series to converge: $\frac{2A\mathcal{M}_o}{1+A^2} << 1$. Since $\mathcal{M}_o$ may have the maximum value of $\mathcal{M}_o = 1$, the expansion is no good for A = 1, i.e. monatomic hydrogen. What is needed to obtain the function G(E') is the difference of the function I_n, with the upper and lower limits (i.e. $\mathcal{M}_o(E = d)$ and $\mathcal{M}_o(E = c)$) substituted. With the following definitions,

and

$$\Delta I_n \equiv I_n[\mathcal{M}_o(E=d)] - I_n[\mathcal{M}_o(E=c)]$$

$$\Delta^n \equiv [\mathcal{M}_o(E=d)]^n - [\mathcal{M}_o(E=c)]^n$$

(note $\Delta^2 \neq \Delta x \Delta$, $\Delta^3 \neq \Delta x \Delta x \Delta$, etc!),

it is proven by induction in Appendix IV that

$$\Delta I_n = \frac{1}{\sqrt{1+A^2}} \sum_{i=0} (-1)^i C_i \frac{\Delta^{n+i+1}}{(n+i+1)} \frac{1}{k^i} \qquad n = 0, 1, 2 \cdots \quad (19)$$

where $c_o = 1$ and $c_i = \frac{(2i-1)(2i-3)\cdots 1}{i!}$ $\qquad i = 1, 2, 3 \ldots$

For the numerical work any number of terms out to fourteen, i.e. $i \leq 13$ may be used in the expansion for each value of n.

V. SOME RELATIONSHIPS THAT THE RESULTS OF ETC MUST SATISFY

Integrating Equation 2 over solid angle yields

$$\sigma(E'\to E)=\int_{-1}^{1}\sigma(E'\to E,\mu)2\pi d\mu=\frac{1}{2}\int_{-1}^{1}\sum_{\ell}S^{\ell}(E'\to E)\,P_{\ell}(\mu)d\mu=S^{0}(E'\to E)$$

Substituting this into Equation 1, it is seen that the quantity $\sum_{j}\sigma_{ij}^{0}$ is the flux averaged(1/E flux) elastic scattering cross section for group i. This quantity is printed out and may be compared with the scattering cross section.

It is shown in Appendix I that for the special case of isotropic scattering in the center-of-mass coordinate system that

$$S^{0}(E'\to E)=\frac{\sigma_{s}(E')}{(1-\alpha)E'}\qquad \text{when } \alpha E'\le E\le E', \tag{20}$$

and $$S^{0}(E'\to E)=0\qquad \text{otherwise,}$$

$$\text{also } S^{1}(E'\to E)=3\mu S^{0}(E'\to E)=3\left[\frac{A+1}{2}\left(\frac{E}{E'}\right)^{1/2}-\frac{A-1}{2}\left(\frac{E'}{E}\right)^{\frac{1}{2}}\right]S^{0}(E'\to E) \tag{21}$$

From Equation 1 and 20:

$$\sum_{j}\sigma_{ij}^{0}=\frac{1}{N}\int_{\alpha E'}^{E'}\frac{dE}{E}\int_{E_i}^{E_{i-1}}dE'\left[\frac{1}{1-\alpha}\frac{\sigma_{s}(E')}{E'^{2}}\right]\quad\text{where}\quad N\equiv\int_{E_i}^{E_{i-1}}\frac{dE}{E}=\ln\frac{E_{i-1}}{E_i}$$

Integrating over E:

$$\sum_{j}\sigma_{ij}^{0}=\frac{1}{N}\int_{E_i}^{E_{i-1}}\left[E\right]_{\alpha E'}^{E'}\frac{1}{1-\alpha}\frac{\sigma_{s}(E')}{(E')^{2}}dE'=\frac{1}{N}\int_{E_i}^{E_{i-1}}\sigma_{s}(E')\frac{dE'}{E'} \tag{22}$$

which equals the flux average cross section for the energy group i, as we have shown holds in general. From Equations 1 and 21:

$$\sum_j \sigma_{ij}^1 = \frac{1}{N} \int_{\alpha E'}^{E'} dE \int_{E_i}^{E_{i-1}} 3 \frac{\sigma_s(E')}{(1-\alpha)(E')^2} \left[\frac{A+1}{2} \left(\frac{E}{E'} \right)^{1/2} - \frac{A-1}{2} \left(\frac{E'}{E} \right)^{1/2} \right] dE'$$

Integrating over E:

$$\sum_j \sigma_{ij}^1 = \frac{1}{N} \int_{E_i}^{E_{i-1}} \frac{3\sigma_s(E')}{2(E')^2} \cdot \frac{A+1}{1-\alpha} \left[\frac{2}{3} \frac{E^{3/2}}{(E')^{1/2}} - \frac{A-1}{A+1} 2(E'E)^{1/2} \right]_{\alpha E'}^{E'} dE'$$

or

$$\sum_j \sigma_{ij}^1 = \frac{2}{A} \frac{1}{N} \int_{E_i}^{E_{i-1}} \sigma_s(E') \frac{dE'}{E'} \qquad (23)$$

Therefore $\sum_j \sigma_{ij}^1 \Big/ \sum_j \sigma_{ij}^0 = \frac{2}{A}$ for the (24)

special case of isotropic scattering in the center-of-mass coordinate system.

The quantities $\sum_j \sigma_{ij}^0$, $\sum_j \sigma_{ij}^1$ and $\frac{1}{3}\sum_j \sigma_{ij}^1 \Big/ \sum_j \sigma_{ij}^0$ are printed out. In the sample calculation for Ni that is given in this report it can be seen that the relationship given in Equation 24 is satisfied over the energy range where $f_v = 0$ for $v \geq 1$; i.e. for GAM groups 20-68 inclusive.

VI. DISCUSSION OF RESULTS

A. Validity of Truncated Binomial Expansion Approximation for ΔI_n

Let us examine the relative size of the 1^{st}, 9^{th}, and 14^{th} terms in the series given by Equation 19. Consider the "worst" case, i.e. $\Delta^n = 2$, $A = 4$.

$$n = 0,\ i = 0: \quad \frac{1}{\sqrt{5}}\, 2 \approx 1$$

$$n = 0,\ i = 8: \quad \frac{1}{\sqrt{5}} \frac{(15 \cdot 13 \cdot 11 \cdots 1)}{8!}\, 2\, \frac{1}{9}\, \frac{1}{(4.25)^8} \approx 5 \times 10^{-5}$$

$$n = 0,\ i = 13: \quad \frac{1}{\sqrt{5}} \frac{(25 \cdot 23 \cdot 21 \cdots 1)}{13!}\, \frac{2}{14}\, \frac{1}{(4.25)^{13}} \approx 2 \times 10^{-7}$$

i.e. for n = 0 the ratio of the 14^{th} to the 1^{st} term is $\approx 2 \times 10^{-7}$.

$$n = 9,\ i = 0: \quad \frac{1}{\sqrt{5}}\, \frac{2}{10} \approx 0.1$$

$$n = 9,\ i = 8: \quad 5 \times 10^{-5}\, \frac{9}{18} \approx 2.5 \times 10^{-5}$$

$$n = 9,\ i = 13: \quad 2 \times 10^{-7}\, \frac{14}{23} \approx 10^{-7}$$

i.e. for n = 9 the ratio of the 14^{th} to the 1^{st} term is $\approx 10^{-7}$.

Since the terms go as $\frac{1}{k^I} \sim \frac{1}{A^I}$, as A increases the relative size of subsequent terms is less, i.e. fewer terms in the series should be required in the calculation for elements of larger mass. For Ni it has been found that if 5 or 14 terms are included in the series the results for P(1) are identical, whereas if only two terms are included the results differ in the 4^{th} significant figure. For Be it has been found that the results differ in the 4^{th} significant figure when retaining 5 or 14 terms in the series.

For Ni the times (IBM-7094) for the calculations as a function of the number of terms retained in Equation 19 were: 14 terms - 1.86 minutes, 5 terms - 1.62 minutes, 2 terms - 1.5 minutes; For Be: 14 terms - 1.44 minutes, 5 terms - 1.08 minutes.

The number of terms to be retained in the series (Equation 19) is specified as an input quantity, I_n. Although in general it would appear that 5 terms ($I_n = 4$) would yield sufficient accuracy, the difference in computational time is sufficiently small that it is recommended 14 terms ($I_n = 13$) be retained.

The results of computations (Be and Ni) carried out on the IBM-7094 (calculations carried out with 9 significant figures) and the CDC-6600 where the calculations are carried out with nearly twice as many significant figures differed, due to round-off, only in the last significant figure (6^{th}) that was printed out.

B. Results* for Ni and Be; Comparison with DOODLES Results

First consider Ni. The out-of-group elements of the P(0) and P(1) matrices as calculated by ETC and DOODLES agree to with $\approx 2\%$ and in most cases differ only in the 4^{th} significant figure except for groups 1 through 4 where the P(1) elements differ by as much as 15%. However, the magnitudes of these four elements are small so this corresponds to disagreements of less than 0.007 barns.

For the in-group elements there is agreement only where the elastic cross section does not depend strongly on energy. For groups 68-35, i.e. from 0.414 eV to 2.04 kev, the in-group elements agree to within $\approx 1\%$. For

*For GAM library, 68 groups, $\frac{1}{2}$ lethargy intervals from 0.414 eV to 10 Mev.

groups 12-1, i.e. from 0.498 Mev to 10 Mev, the P(1) in-group elements agree to within a few percent, and the P(0) in-group elements differ at most by 1/4 barn which for any element corresponds to agreement to within ≈10%. For the energy groups 13-34 where resonances occur, the P(0) in group elements differ by as much as 1.5 barns which corresponds to differences of 30 or 40%. Over this energy range there are great differences in the P(1) elements, e.g. an extreme case is for group 26 which encompasses the large resonance at 15.5 kev. DOODLES gives $P(1)_{26\to 26} = -5$ barns whereas ETC gives $P(1)_{26\to 26} = 11$ barns. (For the out-of-group elements DOODLES gives $P(1)_{26\to 27} = -8.4$ barns and ETC gives $P(1)_{26\to 27} = -8.4$ barns.) Now for group 26, in fact for groups 20-68 inclusive, the scattering is isotropic in the center of mass coordinate system (all f's are zero except $f_o = 1$), and as shown in Section V, $\sum_j P(1)_{ij}/3 \sum_j P(0)_{ij} = \frac{2}{3A} = .0113552$. ETC satisfies this relationship (see printout of sample problem) however DOODLES gives a value of -.07. ETC satisfies this relationship for groups 20-68 inclusive as it should whereas DOODLES does not in the energy range where there are resonances. The quantity $\sum_j P(0)_{ij}$ affords another check of the results. As shown in Section V it represents the flux weighted elastic scattering cross section. This quantity when obtained from the ETC elements gives better agreement with the scattering cross section than the results from the DOODLES program.

For Be the scattering cross section as a function of energy is relatively "smooth" and the DOODLES and ETC results are in better agreement than for Ni. The elements of the P(0) matrix both in-group and out-of-group agree quite well, to within a few percent. The P(1) elements agree out to 4 significant figures

for groups 1-10; for groups 11, 12, 13, and 14 the P(1) elements differ at most by 0.12 barns or 10%; for the remaining groups 15-68 the elements agree to .05 barns. Both sets of results satisfy the conditions described in Section V.

VII. ETC INPUT QUANTITIES, PROGRAM LISTING, SAMPLE PRINTOUT

A. ETC Input

Card	Format	Column	Variable	Comment
1	12A6	1-72	TITLE	Alphanumeric description
2	E12.5	1-12	ATM	Atomic weight (cannot use A = 1)
3	I6	5-6	NOG	Number of groups from which scattering is occuring
	I6	11-12	NFIR	First GAM group from which scattering occurs
	I6	17-18	NLAS	Last GAM group from which scattering occurs
	I6	23-24	NGTO	Number of groups scattered to (not including in-group)*
	I6	29-30	NOFMAX	Number of maximum f-terms (≤ 9) (does not include f_o)
	I6	31-36	NOE	Number of input points (no max.)
	I6	42	IN	Number of terms in series for ΔI_n (n = 0 to 13)

*This quantity may be determined by considering scattering from group 1. Take $(\frac{A-1}{A+1})^2$ 7.79 as the lowest energy the neutron can be scattered to from group 1. Since the GAM gropps are of equal lethargy the number of groups to which a neutron may scatter is independent of the group from which the neutron scatters.

Card	Format	Column	Variable	Comment
3	I6	48	NPRINT+	1, print intermediate steps 0, final print only
4	12I6		NOI	Mesh size of each GAM group from which scattering occurs (max. 200)
5	12I6		NOF	Number of f-terms per GAM group from which scattering occurs (max. 9)
6	E12.5	1-12	EN	Energy (ev) from high to low value (1st > than top boundary energy of first group scattered from)
	E12.5	13-24	SIG	Sigma (barns)
	4E12.5		FV	f-term, max. of 4 on card No. 6.
7	6E12.5		FV	f-terms

Omit card No. 7 if NOFMAX < 4.

+The intermediate steps should be printed out only when necessary. The extra information which is provided is: the case number (see Tables I and II); the limits a, b, c and d, also μ_o (E = c) and μ_o (E = d) (see Equations 11, 17 and 18); and for each mesh point within the limits a and b the P(0) and P(1) contributions for each f-term.

SAMPLE INPUT

NICKEL

```
58.71
    68     1    68     1     8   465     1     0
   100   100   100   100   100   100   100   100   100   100   100   100
   100   100   100   100   100   100   100   100   100   100   100   100
   100   100   100   100   100   100   100   100   100   100   100   100
   100   100   100   100   100   100   100   100   100   100   100   100
   100   100   100   100   100   100   100   100   100   100   100   100
   100   100   100   100   100   100   100   100
     8     8     8     8     8     8     8     8     6     6     5     5
     4     2     2     2     1     1     1     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
 0.13000E 08 0.13500E 01 0.88500E 00 0.77000E 00 0.62800E 00 0.47000E-00
 0.32200E-00 0.17500E-00 0.35000E-01-0.21600E-01
 0.10000E 08 0.17300E 01 0.85800E 00 0.72100E 00 0.56100E 00 0.36100E-00
 0.24500E-00 0.13000E-00 0.35000E-01-0.20000E-01
 0.95000E 07 0.17900E 01 0.85090E 00 0.70960E 00 0.54550E 00 0.33790E-00
 0.22640E-00 0.12240E-00 0.34240E-01-0.19490E-01
 0.90000E 07 0.18600E 01 0.84410E 00 0.69860E 00 0.53060E 00 0.31570E-00
 0.20870E-00 0.11510E-00 0.33510E-01-0.19000E-01
 0.85000E 07 0.19500E 01 0.83690E 00 0.68720E 00 0.51510E 00 0.29260E-00
 0.19010E-00 0.10740E-00 0.32740E-01-0.18500E-01
 0.80000E 07 0.20300E 01 0.83010E 00 0.67620E 00 0.50030E 00 0.27040E-00
 0.17230E-00 0.10010E-00 0.32010E-01-0.18010E-01
 0.75000E 07 0.20900E 01 0.81740E 00 0.65610E 00 0.47800E-00 0.24730E-00
 0.15260E-00 0.88640E-01 0.29730E-01-0.17240E-01
 0.70000E 07 0.21400E 01 0.80490E 00 0.63630E 00 0.45630E-00 0.22480E-00
 0.13330E-00 0.77400E-01 0.27480E-01-0.16490E-01
 0.65000E 07 0.21700E 01 0.79240E 00 0.61660E 00 0.43460E-00 0.20230E-00
 0.11410E-00 0.66160E-01 0.25230E-01-0.15740E-01
 0.60000E 07 0.21900E 01 0.77990E 00 0.59690E 00 0.41290E-00 0.18000E-00
 0.94940E-01 0.54950E-01 0.22980E-01-0.15000E-01
 0.55000E 07 0.21400E 01 0.74760E 00 0.56110E 00 0.38410E-00 0.17750E-00
 0.79060E-01 0.40050E-01 0.18520E-01-0.14500E-01
 0.50000E 07 0.20800E 01 0.71540E 00 0.52540E 00 0.35530E-00 0.17500E-00
 0.63170E-01 0.25160E-01 0.14050E-01-0.14010E-01
 0.45000E 07 0.19800E 01 0.62480E 00 0.46550E-00 0.30790E-00 0.15270E-00
 0.40690E-01 0.12110E-01 0.21020E-02-0.12510E-01
 0.40000E 07 0.19000E 01 0.53350E 00 0.40530E-00 0.26030E-00 0.13010E-00
 0.18120E-00-0.92970E-03-0.99350E-02-0.11010E-01
 0.35000E 07 0.18500E 01 0.44000E-00 0.30000E-00 0.18500E-00 0.74970E-01
```

```
 0.19970E-01 0.45960E-05-0.99910E-02-1.00000E-02
 0.30000E 07 0.17664E 01 0.35060E-00 0.20070E-00 0.11050E-00 0.20370E-01
-0.54490E-01 0.99320E-02 0.98650E-02-1.00000E-02
 0.28000E 07 0.18046E 01 0.32620E-00 0.18530E-00 0.98100E-01 0.13260E-01
-0.53020E-01 1.00000E-02 0.11190E-01-0.92070E-02
 0.26000E 07 0.18534E 01 0.30190E-00 0.17030E-00 0.85930E-01 0.63620E-02
-0.50990E-01 1.00000E-02 0.12410E-01-0.83950E-02
 0.24000E 07 0.19133E 01 0.27930E-00 0.16300E-00 0.78050E-01 0.33900E-02
-0.47660E-01 0.90240E-02 0.10660E-01-0.74140E-02
 0.22000E 07 0.20039E 01 0.25720E-00 0.16300E-00 0.74030E-01 0.41930E-02
-0.42840E-01 0.70170E-02 0.58400E-02-0.62100E-02
 0.20000E 07 0.21561E 01 0.23490E-00 0.16300E-00 0.69980E-01 0.50200E-02
-0.37960E-01 0.49950E-02 0.99870E-02-0.49930E-02
 0.19000E 07 0.22521E 01 0.22410E-00 0.16300E-00 0.67610E-01 0.79820E-02
-0.32430E-01 0.42050E-02 0.80120E-03-0.40060E-02
 0.18000E 07 0.23481E 01 0.21320E-00 0.16300E-00 0.65250E-01 0.10940E-01
-0.26910E-01 0.34150E-02 0.60380E-03-0.30190E-02
 0.17000E 07 0.24540E 01 0.20190E-00 0.16300E-00 0.62780E-01 0.14020E-01
-0.21160E-01 0.25940E-02-0.          -0.
 0.16000E 07 0.25699E 01 0.19090E-00 0.16300E-00 0.60380E-01 0.17020E-01
-0.15560E-01 0.17950E-02-0.          -0.
 0.15000E 07 0.27257E 01 0.17990E-00 0.16300E-00 0.57980E-01 0.20020E-01
-0.99770E-02 0.99880E-03-0.          -0.
 0.14000E 07 0.31318E 01 0.16790E-00 0.16240E-00 0.54380E-01 0.23020E-01
-0.59790E-02 0.79900E-03-0.          -0.
 0.13000E 07 0.32126E 01 0.15590E-00 0.16180E-00 0.50780E-01 0.26020E-01
-0.19790E-02 0.59890E-03-0.          -0.
 0.12000E 07 0.32435E 01 0.14410E-00 0.16120E-00 0.47220E-01 0.28980E-01
 0.19730E-02 0.40140E-03-0.          -0.
 0.11000E 07 0.32791E 01 0.13190E-00 0.16060E-00 0.43560E-01 0.32030E-01
 0.60440E-02 0.19780E-03-0.          -0.
 0.10000E 07 0.33137E 01 0.12000E-00 0.16000E-00 0.40010E-01 0.34990E-01
 0.99850E-02-0.          0.          0.
 0.90000E 06 0.33573E 01 0.11000E-00 0.14750E-00 0.37500E-01 0.34500E-01
 0.90010E-02-0.          0.          0.
 0.80000E 06 0.33999E 01 0.10010E-00 0.13510E-00 0.35020E-01 0.34000E-01
 0.80090E-02-0.          0.          0.
 0.70000E 06 0.34632E 01 0.87400E-01 0.11740E-00 0.24920E-01 0.26940E-01
 0.39680E-02-0.          0.          0.
 0.60000E 06 0.35861E 01 0.74950E-01 0.99910E-01 0.14970E-01 0.19970E-01
-0.          -0.          0.          0.
 0.55000E 06 0.36672E 01 0.67530E-01 0.86550E-01 0.10020E-01 0.15020E-01
-0.          -0.          0.          0.
 0.50000E 06 0.38182E 01 0.60100E-01 0.73190E-01 0.50700E-02 0.10070E-01
-0.          0.          0.          0.
 0.48000E 06 0.39386E 01 0.57180E-01 0.68770E-01 0.38710E-02 0.85890E-02
-0.          0.          0.          0.
```

0.47000E 06	0.40888E 01	0.55760E-01	0.66640E-01	0.33050E-02	0.78810E-02
-0.	0.	0.	0.		
0.45000E 06	0.46392E 01	0.52930E-01	0.62400E-01	0.21720E-02	0.64650E-02
-0.	0.	0.	0.		
0.43000E 06	0.57395E 01	0.49960E-01	0.57880E-01	0.95910E-03	0.49180E-02
-0.	0.	0.	0.		
0.42000E 06	0.26697E 01	0.48940E-01	0.54920E-01	0.	0.29520E-02
-0.	0.	0.	0.		
0.40000E 06	0.25399E 01	0.44980E-01	0.49960E-01	-0.	-0.
0.	0.	0.	0.		
0.38000E 06	0.35900E 01	0.42340E-01	0.44010E-01	-0.	-0.
0.	0.	0.	0.		
0.37000E 06	0.41900E 01	0.40990E-01	0.40920E-01	-0.	-0.
0.	0.	0.	0.		
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0.12000E 06	0.40105E 01	0.99920E-03	0.	0.	0.
0.	0.	0.	0.		
0.11900E 06	0.42608E 01	0.98980E-03	0.	0.	0.
0.	0.	0.	0.		
0.11800E 06	0.45639E 01	0.98030E-03	0.	0.	0.
0.	0.	0.	0.		
0.11700E 06	0.48339E 01	0.97080E-03	0.	0.	0.
0.	0.	0.	0.		
0.11600E 06	0.51908E 01	0.96140E-03	0.	0.	0.
0.	0.	0.	0.		
0.11500E 06	0.55837E 01	0.94880E-03	0.	0.	0.
0.	0.	0.	0.		
0.11400E 06	0.60204E 01	0.93930E-03	0.	0.	0.
0.	0.	0.	0.		
0.11300E 06	0.66718E 01	0.92980E-03	0.	0.	0.
0.	0.	0.	0.		
0.11250E 06	0.70833E 01	0.92350E-03	0.	0.	0.
0.	0.	0.	0.		
0.11200E 06	0.73116E 01	0.92040E-03	0.	0.	0.
0.	0.	0.	0.		

0.11150E 06	0.83905E 01	0.91410E-03	0.	0.	0.
0.	0.	0.	0.		
0.11100E 06	0.10615E 02	0.91090E-03	0.	0.	0.
0.	0.	0.	0.		
0.11070E 06	0.11557E 02	0.90700E-03	0.	0.	0.
0.	0.	0.	0.		
0.11050E 06	0.11695E 02	0.90500E-03	0.	0.	0.
0.	0.	0.	0.		
0.11030E 06	0.11891E 02	0.90300E-03	0.	0.	0.
0.	0.	0.	0.		
0.11020E 06	0.11981E 02	0.90200E-03	0.	0.	0.
0.	0.	0.	0.		
0.11010E 06	0.12050E 02	0.90110E-03	0.	0.	0.
0.	0.	0.	0.		
0.11000E 06	0.12113E 02	0.89000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10990E 06	0.12134E 02	0.87500E-03	0.	0.	0.
0.	0.	0.	0.		
0.10970E 06	0.12086E 02	0.85000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10950E 06	0.11930E 02	0.80000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10930E 06	0.11704E 02	0.76000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10910E 06	0.11568E 02	0.74230E-03	0.	0.	0.
0.	0.	0.	0.		
0.10900E 06	0.11342E 02	0.72000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10890E 06	0.10837E 02	0.69810E-03	0.	0.	0.
0.	0.	0.	0.		
0.10870E 06	0.95750E 01	0.67000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10850E 06	0.84669E 01	0.63000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10830E 06	0.74637E 01	0.58000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10800E 06	0.57606E 01	0.54000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10780E 06	0.46531E 01	0.50000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10750E 06	0.36244E 01	0.45000E-03	0.	0.	0.
0.	0.	0.	0.		
0.10700E 06	0.27752E 01	0.34420E-03	0.	0.	0.
0.	0.	0.	0.		
0.10650E 06	0.27529E 01	0.25580E-03	0.	0.	0.
0.	0.	0.	0.		
0.10600E 06	0.27802E 01	0.16730E-03	0.	0.	0.

0.	0.	0.	0.		
0.10550E 06	0.28285E 01	0.78860E-04	0.	0.	0.
0.	0.	0.	0.		
0.10500E 06	0.31768E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10450E 06	0.35036E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10400E 06	0.38061E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10350E 06	0.41264E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10300E 06	0.44955E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10250E 06	0.50925E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10200E 06	0.57243E 01	-0.	0.	0.	0.
0.	0.	0.	0.		
0.10150E 06	0.63888E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10100E 06	0.70648E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10080E 06	0.73672E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10060E 06	0.75372E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10040E 06	0.76810E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10020E 06	0.78041E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10010E 06	0.78279E 01	0.	0.	0.	0.
0.	0.	0.	0.		
1.00000E 05	0.78494E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99900E 05	0.78565E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99800E 05	0.78592E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99600E 05	0.77886E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99400E 05	0.76879E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99200E 05	0.75202E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.99000E 05	0.72800E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.98800E 05	0.70770E 01	0.	0.	0.	0.
0.	0.	0.	0.		

0.98500E 05	0.67690E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.98000E 05	0.62764E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.97500E 05	0.58008E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.97000E 05	0.53471E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.96500E 05	0.49003E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.96000E 05	0.46869E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.95500E 05	0.46473E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.95000E 05	0.49226E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.94000E 05	0.52009E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.93000E 05	0.54051E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.92000E 05	0.56281E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.91000E 05	0.59070E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.90000E 05	0.63287E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.89500E 05	0.66053E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.89000E 05	0.69925E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.88800E 05	0.71168E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.88600E 05	0.71638E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.88400E 05	0.71881E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.88200E 05	0.72028E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.88000E 05	0.72061E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.87800E 05	0.71758E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.87600E 05	0.70012E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.87500E 05	0.69009E 01	0.	0.	0.	0.	
0.	0.	0.	0.			
0.87400E 05	0.68908E 01	0.	0.	0.	0.	

0.		0.		0.	0.	
0.87200E 05	0.68456E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.87000E 05	0.67390E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.86800E 05	0.66037E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.86500E 05	0.63940E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.86000E 05	0.61714E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.85500E 05	0.61413E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.85000E 05	0.64377E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.84500E 05	0.66439E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.84000E 05	0.68310E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.83500E 05	0.69910E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.83000E 05	0.71411E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.82500E 05	0.72831E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.82000E 05	0.74191E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.81500E 05	0.75501E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.81000E 05	0.76951E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.80000E 05	0.79850E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.79000E 05	0.82499E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.78000E 05	0.85378E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.77000E 05	0.88557E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.76000E 05	0.92505E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.75000E 05	0.97283E 01	0.	0.	0.	0.	
0.		0.		0.	0.	
0.74000E 05	0.10288E 02	0.	0.	0.	0.	
0.		0.		0.	0.	
0.73000E 05	0.11100E 02	0.	0.	0.	0.	
0.		0.		0.	0.	

0.72000E 05	0.12014E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.71000E 05	0.13175E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.70000E 05	0.14536E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.69500E 05	0.15408E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.69000E 05	0.16327E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.68500E 05	0.17488E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.68000E 05	0.18996E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.67500E 05	0.22648E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.67000E 05	0.26394E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.66500E 05	0.28482E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.66200E 05	0.29775E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.66000E 05	0.30414E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.65800E 05	0.31019E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.65500E 05	0.30223E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.65200E 05	0.29376E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.65000E 05	0.28539E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.64500E 05	0.27306E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.64000E 05	0.23013E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.63500E 05	0.18697E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.63000E 05	0.14035E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.62500E 05	0.10105E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.62000E 05	0.59517E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.61500E 05	0.32476E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.61000E 05	0.21421E 01	0.	0.	0.	0.

0.	0.	0.	0.		
0.60500E 05	0.18749E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.60000E 05	0.17783E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.65900E 05	0.18087E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.58000E 05	0.20432E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.57000E 05	0.24377E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.56000E 05	0.30438E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.54000E 05	0.39869E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.52000E 05	0.46005E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.50000E 05	0.51938E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.48000E 05	0.57020E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.46000E 05	0.62420E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.44000E 05	0.69410E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.42000E 05	0.77008E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.40000E 05	0.85616E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.39000E 05	0.90214E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.38000E 05	0.95221E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.37000E 05	0.10055E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.36000E 05	0.10678E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.35000E 05	0.11342E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.34500E 05	0.11712E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.34000E 05	0.12108E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.33500E 05	0.12609E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.33000E 05	0.13502E 02	0.	0.	0.	0.
0.	0.	0.	0.		

0.32500E 05	0.15990E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.32000E 05	0.18162E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.31500E 05	0.20905E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.31200E 05	0.22287E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.31000E 05	0.22854E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30800E 05	0.23000E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30600E 05	0.23007E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30400E 05	0.22900E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30200E 05	0.22447E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30000E 05	0.21514E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.29800E 05	0.19654E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.29500E 05	0.18108E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.29000E 05	0.15298E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.28500E 05	0.12072E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.28000E 05	0.10843E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.27000E 05	0.11015E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.26000E 05	0.11726E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.25000E 05	0.12767E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.24000E 05	0.14144E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.23000E 05	0.15684E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.22000E 05	0.17625E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.21000E 05	0.20265E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.20500E 05	0.22306E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.20000E 05	0.27123E 02	0.	0.	0.	0.

0.	0.	0.	0.		
0.19500E 05	0.33761E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.19000E 05	0.41849E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.18500E 05	0.54797E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.18000E 05	0.68691E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.17500E 05	0.81944E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.17200E 05	0.84329E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.17000E 05	0.86011E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.16800E 05	0.87368E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.16500E 05	0.90476E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.16200E 05	0.93839E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.16000E 05	0.83375E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.15800E 05	0.71287E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.15500E 05	0.59505E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.15000E 05	0.52545E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.14800E 05	0.50412E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.14600E 05	0.48318E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.14400E 05	0.46175E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.14200E 05	0.43808E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.14000E 05	0.39437E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.13500E 05	0.32280E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.13000E 05	0.23648E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.12500E 05	0.17767E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.12000E 05	0.97683E 01	0.	0.	0.	0.
0.	0.	0.	0.		

0.11500E 05	0.61568E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.11000E 05	0.51586E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10500E 05	0.44162E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.10200E 05	0.41464E 01	0.	0.	0.	0.
0.	0.	0.	0.		
1.00000E 04	0.45182E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.95000E 04	0.75314E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.90000E 04	0.93414E 01	0.	0.	0.	0.
0.	0.	0.	0.		
0.85000E 04	0.10500E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.80000E 04	0.11415E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.75000E 04	0.12367E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.70000E 04	0.13741E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.68000E 04	0.14469E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.66000E 04	0.15289E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.64000E 04	0.16231E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.62000E 04	0.17395E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.60000E 04	0.19372E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.59000E 04	0.20803E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.58000E 04	0.22806E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.57000E 04	0.27132E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.56000E 04	0.33419E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.55000E 04	0.42897E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.54000E 04	0.59183E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.53000E 04	0.88552E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.52000E 04	0.14803E 03	0.	0.	0.	0.

0.	0.	0.	0.		
0.51000E 04	0.26851E 03	0.	0.	0.	0.
0.	0.	0.	0.		
0.50000E 04	0.37840E 03	0.	0.	0.	0.
0.	0.	0.	0.		
0.49000E 04	0.26368E 03	0.	0.	0.	0.
0.	0.	0.	0.		
0.48000E 04	0.14207E 03	0.	0.	0.	0.
0.	0.	0.	0.		
0.47000E 04	0.85818E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.46000E 04	0.60664E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.45000E 04	0.48733E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.44000E 04	0.42775E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.43000E 04	0.39492E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.42000E 04	0.37126E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.41000E 04	0.34729E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.40000E 04	0.31935E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.38000E 04	0.26872E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.36000E 04	0.24686E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.34000E 04	0.23107E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.32000E 04	0.21625E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.30000E 04	0.20268E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.27500E 04	0.18758E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.25000E 04	0.17472E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.22500E 04	0.16377E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.20000E 04	0.15635E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.17500E 04	0.15097E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.15000E 04	0.14747E 02	0.	0.	0.	0.
0.	0.	0.	0.		

0.12500E 04	0.14649E 02	0.	0.	0.	0.
0.	0.	0.	0.		
1.00000E 03	0.14749E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.90000E 03	0.14867E 02	0.	0.	0.	0.
0.	0.	0.	0.		
0.40000E-00	0.17234E 02	0.	0.	0.	0.
0.	0.	0.	0.		

B) ETC LISTING EFN SOURCE STATEMENT - IFN(S) -

```
C   ETC   ELASTIC TRANSFER COEFFICIENTS
      COMMON AL, AL1, E(70), NOF1, ALPHA, INTGC, ITEXT, EP, S, Y, X,
     1       AK, AKK(14), ATM, APA, AP, A2, FF(9), UOC, UOD, I, J
      COMMON INN, NPRINT
      DIMENSION NOI(70), NOF(70), ENM(201), FV(9), FV1(9), SIGMA(201),
     1 FINT(201,9), PNOT(68,69), TITLE(12)
      DIMENSION P1(68,69)
      DIMENSION PUNO(2414), PUN1(2414)
      DIMENSION BBB(7)
      DIMENSION SPO(68), SP1(68)
C SECOND DOODLES FOR FINDING PO AND P1 MATRICES
      READ(5,1000) TITLE, ATM                                          1
 1000 FORMAT (12A6/ E12.5)
      READ (5,1001) NOG, NFIR, NLAS, NGTO, NOFMAX, NOE, IN, NPRINT     3
 1001 FORMAT (12I6)
      READ (5,1001) (NOI(I), I=1,NOG)                                  12
      READ (5,1001) (NOF(I), I=1,NOG)
C
C
C ATM - ATOMIC WEIGHT OF TARGET ATOM
C NOG - NUMBER OF GROUPS SCATTERING FROM
C NFIR - GAM NUMBER OF FIRST GROUP FROM WHICH SCATTERING OCCURS
C NLAS - GAM NUMBER OF LAST GROUP FROM WHICH SCATTERING OCCURS
C NGTO - NUMBER OF GROUPS TO WHICH SCATTERING OCCURS
C NOFMAX- NUMBER OF MAXIMUM F-TERMS (MAX. OF 9)
C NOE - NUMBER OF INPUT POINTS
C NOI - MESH SIZE PER GAM GROUP (MAX. SIZE IS 200)
C NOF - NUMBER OF F-TERMS PER GAM GROUP
C IN - NO. OF TERMS IN SERIES FOR DELTA I SUB N
C        (N = 0 TO 13) A MAX. OF 14 TERMS
C NPRINT - 1, PRINT INTERMEDIATE VALUES
C          0, PRINT SUMMARY ONLY
C
C                                                                      19
      ALPHA = ((ATM-1.0)/(ATM+1.0))**2.0                               26
      AL = 1.0-ALPHA
      AL1 = 2.0/AL
      A2 = 2.0/ATM
      AK = ATM+(1.0/ATM)
      AP = AK/ATM
      INN = IN+1
      GO TO (80,81,82,83,84,85,86,87,88,89,90,91,92,93), INN
```

```
      ETC        - EFN   SOURCE STATEMENT  -  IFN(S)  -
   93 AKK(14) = -1269.6044921/AK**13.0                                    29
   92 AKK(13) = 660.19433593/AK**12.0                                     31
   91 AKK(12) = -344.44921875/AK**11.0                                    33
   90 AKK(11) = 180.42578125/AK**10.0                                     35
   89 AKK(10) = -94.9609375/AK**9.0                                       37
   88 AKK(9) = 50.2734375/AK**8.0                                         39
   87 AKK(8) = -26.8125/AK**7.0                                           41
   86 AKK(7) = 14.4375/AK**6.0                                            43
   85 AKK(6) = -7.875/AK**5.0                                             45
   84 AKK(5) = 4.375/AK**4.0                                              47
   83 AKK(4) = -2.5/AK**3.0                                               49
   82 AKK(3) = 1.5/AK**2.0                                                51
   81 AKK(2) = -1.0/AK
   80 AKK(1) = 1.0
      APA = 1.0/SQRT(1.0+ATM*ATM)                                         54
      WRITE (6,2052) TITLE, ATM, NOG, NFIR, NLAS, NGTO, NOFMAX, NOE, IN   55
 2052 FORMAT (1H1, 19X, 12A6 /// 10X, 39HTHE ATOMIC WEIGHT OF THE TARGET
     1 ATOM IS, F9.3 //10X, 56HTHE NUMBER OF GAM GROUPS FROM WHICH SCATT
     2ERING OCCURS IS, I3 //10X, 47HTHE FIRST GROUP FROM WHICH SCATTERIN
     3G OCCURS IS, I3, 22H AND THE LAST GROUP IS, I3//10X, 78HTHE NUMBER
     4 OF GROUPS TO WHICH SCATTERING OCCURS FROM A PARTICULAR GAM GROUP
     5IS, I3 //10X, 32HTHE NUMBER OF MAXIMUM F-TERMS IS, I2 //10X, 47HTH
     6E NUMBER OF ENERGY POINTS READ IN BY CARDS IS, I7 //
     7 10X, 43HNO. OF TERMS IN SERIES FOR DELTA I SUB N IS, I3 //
     8 1H1, 19X, 36HGAM GROUP  MESH SIZE  NO. OF F-TERMS)
      NNN = 1
      DO 101 L=NFIR,NLAS
      WRITE (6,2053) L, NOI(NNN), NOF(NNN)                               60
 2053 FORMAT (23X, I2, 9X, I3, 10X, I1)
  101 NNN = NNN+1
C
C
C THE GROUP STRUCTURE ENERGY COMPUTED IS THE UPPER ENERGY OF EACH
C GAM GROUP
C
C
      YOU = 0.25
      DELYOU = 0.25
      E(1) = 1.0E7
      DO 10 M=2,69
      E(M) = E(1)*EXP(-YOU)                                              70
   10 YOU = YOU+DELYOU
      E(70) = 0.0
```

```
C
C
C INTERMEDIATE STEPS FOR INITIALIZATION
C
C
      NEVER = 0
      N = 1
      MYS = 0
C
C
C DO-LOOP FOR GROUPS SCATTERED FROM
C
C
      DO 16 I=NFIR,NLAS
      SPO(I) = 0.0
      SP1(I) = 0.0
      NOF1 = NOF(N)
      NOI1 = NOI(N)
      NOII = NOI1+1
      ANOI1 = NOI1
      N = N+1
      FIRL = 1.0/(2.0*ALOG(E(I)/E(I+1)))
C
C
C THE FOLLOWING DELTA ENERGY IS USED TO DETERMINE THE ENERGY INTERVAL
C PER GAM GROUP AND FOR USE IN THE TRAPEZOIDAL RULE EQUATION
C
C
      DELYOU =(E(I)-E(I+1))/ANOI1
C
C
C CALCULATION OF ENERGY MESH PER GAM GROUP, LOW TO HIGH ENERGY
C ENM - MESH ENERGY
C
C
      ENM(1)= E(I+1)
      DO 12 M=2,NOI1
   12 ENM(M) = ENM(M-1)+DELYOU
      ENM(NOII) = E(I)
C
C
C CALCULATION OF SIGMA AND F-TERMS FOR EACH ENERGY MESH POINT FROM
C INPUT (CARD) INFORMATION
```

```
C
C
      WRITE (6,303)                                                      103
  303 FORMAT (1H1, 40X, 15HINPUT  OF  FORM // 4X, 2HNO, 3X, 10HENERGY, E
     1V, 4X, 12HSIGMA, BARNS, 9X, 3HF-1, 13X, 3HF-2, 13X, 3HF-3, 13X,
     2 3HF-4 / 12X, 3HF-5, 13X, 3HF-6, 13X, 3HF-7, 13X, 3HF-8, 13X,
     3  3HF-9 //)
      DO 13 KK=1,NOII
      M = NOII+1-KK
      IF (NEVER.GT.0) GO TO 51
   50 NEVER = NEVER+1
      IF (NEVER.GT.NOE) GO TO 53
      IF (NOFMAX.GT.0) GO TO 54
      READ (5,1002) EN,SIG                                               117
      WRITE (6,305) NEVER, EN, SIG                                       118
      GO TO 253
   54 READ (5,1002) EN, SIG, (FV(L), L=1,NOFMAX)                         120
 1002 FORMAT (6E12.5)
      WRITE (6,305) NEVER, EN, SIG, (FV(L), L=1,NOFMAX)                  125
  305 FORMAT (I6, E13.5, 5E16.5 / E19.5, 4E16.5)
C
C
C INPUT QUANTITIES-
C    EN, ENERGY IN EV, RANGING FROM HIGH TO LOW ENERGY
C    SIG, ELASTIC SCATTERING CROSS SECTION IN BARNS
C    FV(L), L=1,NOFMAX, THE NUMBER OF MAXIMUM F-TERMS ONCE SET MUST
C                       CONTINUE TO HAVE A VALUE EVEN IF VALUE IS ZERO.
C                       IF NOFMAX EQUALS 4 OR IS LESS THAN 4 ONLY ONE
C                       CARD PER ENERGY,SIGMA VALUE IS NESSESARY
C                       IF NOFMAX IS GREATER THAN 4 TWO CARDS MUST BE
C                       SUPPLIED EACH TIME.
C
C
  253 IF (NEVER.GT.1) GO TO 51
   52 EN1 = EN
      SIG1 = SIG
      IF (NOFMAX.EQ.0) GO TO 50
      DO 14 L=1,NOFMAX
   14 FV1(L) = FV(L)
      GO TO 50
   51 IF (ENM(M).LE.EN1.AND.ENM(M).GE.EN) GO TO 55
      GO TO 52
C
```

```
C
C INTERPOLATE FOR SIGMA AND F-VALUES
C
C
   55 SIGMA(M) = (ENM(M)-EN1)*((SIG-SIG1)/(EN-EN1))+SIG1
      IF (NOF1-0) 13,13,56
   56 DO 15 K=1,NOF1
   15 FINT(M,K) = (ENM(M)-EN1)*((FV(K)-FV1(K))/(EN-EN1))+FV1(K)
   13 CONTINUE
      WRITE (6,2054) I                                                    165
 2054 FORMAT (1H1, 49X, 9HGAM GROUP, I4)
      IF (NOF1.GT.4) GO TO 111
      IF (NOF1.GT.0) GO TO 254
      WRITE (6,2044)                                                      171
 2044 FORMAT (35H0 NO.  MESH ENERGY,EV  ELAS. SCATT.)
      DO 255 M=1,NOII
  255 WRITE (6,2043) M, ENM(M), SIGMA(M)                                  174
 2043 FORMAT (I4, 1PE16.5, 1PE16.5)
      GO TO 107
  254 NPR = NOF1
      GO TO (108,109,110,111), NOF1
  108 WRITE (6,2055)                                                      181
 2055 FORMAT (35H0 NO.  MESH ENERGY,EV  ELAS. SCATT., 8X, 3HF-1)
      GO TO 122
  109 WRITE (6,2056)                                                      183
 2056 FORMAT (35H0 NO.  MESH ENERGY,EV  ELAS. SCATT., 8X, 3HF-1, 13X,
     1 3HF-2)
      GO TO 122
  110 WRITE (6,2057)                                                      185
 2057 FORMAT (35H0 NO.  MESH ENERGY,EV  ELAS. SCATT., 8X, 3HF-1, 13X,
     1 3HF-2, 13X, 3HF-3)
      GO TO 122
  111 NPR = 4
      WRITE (6,2058)                                                      188
 2058 FORMAT (35H0 NO.  MESH ENERGY,EV  ELAS. SCATT., 8X, 3HF-1, 13X,
     1 3HF-2, 13X, 3HF-3, 13X, 3HF-4)
  122 DO 112 M=1,NOII
  112 WRITE (6,2059) M, ENM(M), SIGMA(M), (FINT(M,K), K=1,NPR)            192
 2059 FORMAT (I4. 6(1PE16.5))
      IF (NOF1.LE.4) GO TO 107
      NPR = NOF1-4
      GO TO (113,114,115,116,117),NPR
  113 WRITE (6,2060)                                                      204
```

```
2060 FORMAT (21H0 NO.  MESH ENERGY,EV, 6X, 3HF-5)
     GO TO 188
 114 WRITE (6,2061)                                                    206
2061 FORMAT (21H0 NO.  MESH ENERGY,EV, 6X, 3HF-5, 13X, 3HF-6)
     GO TO 188
 115 WRITE (6,2062)                                                    208
2062 FORMAT (21H0 NO.  MESH ENERGY,EV, 6X, 3HF-5, 13X, 3HF-6, 13X,
    1 3HF-7)
     GO TO 188
 116 WRITE (6,2063)                                                    210
2063 FORMAT (21H0 NO.  MESH ENERGY,EV, 6X, 3HF-5, 13X, 3HF-6, 13X,
    1 3HF-7, 13X, 3HF-8)
     GO TO 188
 117 WRITE (6,2064)                                                    212
2064 FORMAT (21H0 NO.  MESH ENERGY,EV, 6X, 3HF-5, 13X, 3HF-6, 13X,
    1 3HF-7, 13X, 3HF-8, 13X, 3HF-9)
 188 DO 118 M=1,NOII
 118 WRITE (6,2059) M, ENM(M), (FINT(M,K), K=5,NOF1)                   216
 107 CONTINUE
C
C
C CALCULATION OF FINAL GAM GROUP SCATTERED TO FOR  A PARTICULAR GAM
C GROUP FROM WHICH SCATTERING IS OCCURRING
C
C
      NSEC = I+NGTO
      IF (NSEC-69) 58,58,57
   57 NSEC = 69
C
C
C DO-LOOP FOR GROUPS SCATTERED TO
C
C
   58 DO 16 J=I,NSEC
      EAJ = E(J+1)/ALPHA
      EAJ1 = E(J)/ALPHA
      IF (J.EQ.I) GO TO 72
C
C
C OUT-OF-GROUP SCATTERING
C
C
   59 IF (EAJ.LT.E(I+1).AND.EAJ.LT.E(I).AND.EAJ1.LE.E(I+1).AND.EAJ1
```

```
     1 .LT.E(I)) GO TO 60
    41 IF (EAJ.LE.E(I+1).AND.EAJ.LT.E(I).AND.EAJ1.GT.E(I+1).AND.EAJ1
     1 .LT.E(I)) GO TO 42
    61 IF (EAJ.GT.E(I+1).AND.EAJ.LT.E(I).AND.EAJ1.GT.E(I+1).AND.EAJ1
     1 .LE.E(I)) GO TO 62
    63 IF (EAJ.LE.E(I+1).AND.EAJ.LT.E(I).AND.EAJ1.GT.E(I+1).AND.
     1 EAJ1.GE.E(I)) GO TO 64
    66 IF (EAJ.GT.E(I+1).AND.EAJ.LE.E(I).AND.EAJ1.GT.E(I+1).AND.
     1 EAJ1.GT.E(I)) GO TO 67
    69 IF (EAJ.GT.E(I+1).AND.EAJ.GE.E(I).AND.EAJ1.GT.E(I+1).AND.
     1 EAJ1.GT.E(I)) GO TO 70
    71 WRITE (6,2500) I, E(I+1), E(I), J, E(J+1), EAJ, E(J), EAJ1                277
  2500 FORMAT (1H1, 17H    ERROR MESSAGE // 74H       INTEGRATION CONSTAN
     1T COULD NOT BE FOUND FOR OUT-OF-GROUP SCATTERING // 38H   SCATTERI
     2NG OCCURRING FROM GROUP , I2 / 48H   WHICH LIES WITHIN THE ENERGY
     3RANGE OF E(I+1)=, 1PE12.5, 11H AND E(I) =, 1PE12.5 // 49H   THE GR
     4OUP TO WHICH SCATTERING IS OCCURRING IS , I2 / 48H   WHICH LIES WI
     5THIN THE ENERGY RANGE OF E(J+1)=, 1PE12.5, 19HWHOSE E(J+1)/ALPHA=,
     6 1PE12.5 / 13H   AND E(J) =, 1PE12.5, 17HWHOSE E(J)/ALPHA=,
     7 1PE12.5)
      GO TO 2222
    60 INTGC = 0
       PNOT(I,J) = 0.0
       P1(I,J) = 0.0
       IF (NPRINT.EQ.1) WRITE (6,2065) I, J, INTGC                              287
  2065 FORMAT (6H0   I=, I3, 5H   J=, I3, 8H  INTGC=, I2)
       GO TO 16
    42 INTGC = 1
       A = E(I+1)
       B = EAJ1
       GO TO 75
    62 INTGC = 2
       BP = EAJ1
       GO TO 43
    64 INTGC = 3
    65 A = E(I+1)
       B = E(I)
       GO TO 75
    67 INTGC = 4
    68 BP = E(I)
    43 A = E(I+1)
       B = EAJ
       GO TO 75
```

```
   70 INTGC = 5
      GO TO 65
C
C
C IN-GROUP SCATTERING
C
C
   72 ALPE = ALPHA*E(J)
      IF (ALPE.LE.E(J+1)) GO TO 73
   74 INTGC = 7
      GO TO 68
   73 INTGC = 6
      GO TO 65
C
C
C INITIALIZATION
C
C
   75 YY = 0.0
      XX = 0.0
      ITEXT = 0
      TAU = 0.0
      TAUX = 0.0
      IT2 = 0
      IF (NPRINT.EQ.1) WRITE (6,2066) I, J, INTGC, A, B                    323
 2066 FORMAT (1H1, 18X, 22HSCATTERING FROM GROUP , I2, 10H TO GROUP ,
     1 I2, 15X, 9HCASE NO. , I1 / 8X, 2HA=, 1PE13.5, 4H, B=, 1PE13.5)
C
C USE OF MESH VALUES PER GAM GROUP SCATTERED FROM
C
C
      DO 18 M=1,NOII
      IF (NPRINT.EQ.1)  WRITE (6,3067)  M                                  327
 3067 FORMAT (4H  M=, I4)
      IF (M.EQ.1.OR.A.LE.ENM(M).AND.ENM(M).LE.B) GO TO 151
  145 IF (ENM(M-1).EQ.B) GO TO 144
C
C
C THE LOWER LIMIT B FALL BETWEEN TWO MESH POINTS, INTERPOLATION OF SIGMA
C AND F-TERMS IS NECESSARY.
C
C
  140 YY = YY-Y
```

```
      XX = XX-X
      TAU = TAU+0.5*YY*DELYOU
      TAUX = TAUX+0.5*XX*DELYOU
      IF (NPRINT.EQ.1) WRITE (6,2069) TAU, TAUX                          339
 2069 FORMAT (5X, 4HAREA, 1PE16.5, 1PE14.5)
   45 DEL = B-ENM(M-1)
   46 EP = B
      S = (B-ENM(M-1))*((SIGMA(M)-SIGMA(M-1))/(ENM(M)-ENM(M-1)))+
     1 SIGMA(M-1)
      Z = Y
      ZZ = X
      IF (NOF1.EQ.0) GO TO 139
      DO 20 K=1,NOF1
   20 FF(K) = (B-ENM(M-1))*((FINT(M,K)-FINT(M-1,K))/(ENM(M)-ENM(M-1)))
     1 +FINT(M-1,K)
  139 CALL FVAL                                                          365
      TAU = TAU+0.5*(Z+Y)*DEL
      TAUX = TAUX+0.5*(ZZ+X)*DEL
      IF (NPRINT.EQ.1) WRITE (6,2069) TAU, TAUX
C
C CASE 1 WILL PROCEED TO STATEMENT NUMBER 152, OTHER CASES GO ON
C CHANGING LIMITS A,B AND C,D
C                                                                        367
      IF (INTGC.EQ.1) GO TO 152
      IF (INTGC.EQ.2.AND.ITEXT.EQ.1) GO TO 152
  138 ITEXT = 1
      DEL = ENM(M)-EP
      A = B
      B = BP
      YY = 0.0
      XX = 0.0
      IF (NPRINT.EQ.1) WRITE (6,2067) A, B                               378
 2067 FORMAT (8X, 2HA=, 1PE13.5, 4H, B=, 1PE13.5)
      CALL FVAL                                                          380
      Z=Y
      ZZ = X
      IT2 = 1
      IF (INTGC.GT.2.OR.B.GT.ENM(M)) GO TO 151
      IF (B.LT.ENM(M)) GO TO 47
      IT2 = 2
      GO TO 151
   47 DEL = B-A
```

ETC - EFN SOURCE STATEMENT - IFN(S) -

```
      GO TO 46
C
C
C THE PREVIOUS MESH POINT WAS EQUAL TO LIMIT, B
C
C
  144 IF (INTGC.EQ.1) GO TO 150
C
C
C CASE1 PROCEEDS TO STATEMENT NUMBER 150
C OTHER CASES GO ON CHANGING LIMITS A,B AND C,D
C
C
  143 TAU = TAU+0.5*YY*DELYOU
      TAUX = TAUX+0.5*XX*DELYOU
      IF (NPRINT.EQ.1) WRITE (6,2069) TAU, TAUX                          398
      IF (INTGC.EQ.2.AND.ITEXT.EQ.1) GO TO 152
      A = B
      B = BP
      ITEXT = 1
      IF (NPRINT.EQ.1) WRITE (6,2067)  A, B                              404
      EP = ENM(M-1)
      S = SIGMA(M-1)
      IF (NOF1.EQ.0) GO TO 141
  142 DO 19 K=1,NOF1
   19 FF(K) = FINT(M-1,K)
  141 CALL FVAL                                                          419
      YY = Y
      XX = X
      IF (INTGC.GT.2.OR.B.GT.ENM(M)) GO TO 151
      IF (B.LT.ENM(M)) GO TO 45
      IT2 = 2
      DEL = DELYOU
      Z = YY
      ZZ = XX
  151 EP = ENM(M)
      S = SIGMA(M)
      IF (NOF1.EQ.0) GO TO 148
  149 DO 17 K=1,NOF1
   17 FF(K) = FINT(M,K)
  148 CALL FVAL                                                          444
      IF (IT2.LT.1) GO TO 154
      TAU = TAU+0.5*(Z+Y)*DEL
```

```
      TAUX = TAUX+0.5*(ZZ+X)*DEL
      IF (NPRINT.EQ.1) WRITE (6,2069) TAU, TAUX                          449
      IF (IT2.EQ.2) GO TO 44
      IT2 = 0
      IF (M.EQ.NOII) GO TO 152
      GO TO 147
   44 IT2 = 0
      GO TO 152
  154 IF (M.EQ.1.OR.M.EQ.NOII.OR.EP.EQ.A.OR.EP.EQ.B) GO TO 147
  146 YY = YY+2.0*Y
      XX = XX+2.0*X
      GO TO 18
  147 YY = YY+Y
      XX = XX+X
   18 CONTINUE
C
C
C TRAPEZOIDAL RULE
C
C
  150 TAU = TAU+0.5*YY*DELYOU
      TAUX = TAUX+0.5*XX*DELYOU
      IF (NPRINT.EQ.1) WRITE (6,2069) TAU, TAUX                          470
  152 PNOT(I,J) = FIRL*TAU
      P1(I,J) = 3.0*FIRL*TAUX
      MYS = MYS+1
      PUNO(MYS) = PNOT(I,J)
      PUN1(MYS) = P1(I,J)
      SPO(I) = SPO(I)+PNOT(I,J)
      SP1(I) = SP1(I)+P1(I,J)
      IF (NPRINT.EQ.1) WRITE (6,2070) PNOT(I,J), P1(I,J)                 486
 2070 FORMAT (1H+, 42X, 3HETC, 1PE17.5, 1PE14.5)
   16 CONTINUE
      LOL = NOG+((NGTO+(2*NOG+1-NGTO))/2)
      II = 1
      WRITE (6,3002) TITLE                                               495
 3002 FORMAT (1H1, 10X, 12A6 /// 5X, 21HP-0 SCATTERING MATRIX)
      GO TO 2033
 2032 WRITE (6,3003) TITLE                                               498
 3003 FORMAT (1H1, 10X, 12A6 /// 5X, 21HP-1 SCATTERING MATRIX)
 2033 KK = 0
      WRITE (6,3001) LOL, NOG, NGTO                                      501
 3001 FORMAT (1H0, 43X, 17HSCATTERING MATRIX // 17X,
```

```
      ETC        -  EFN   SOURCE STATEMENT  -  IFN(S)  -
     1 17HLENGTH OF LIST =  ,I4,25H,GROUPS SCATTERED FROM = , I2,
     2 23H,GROUPS SCATTERED TO = , I2)
      JJFIR = NFIR
      IF (NOG.GT.7) GO TO 2020
 2030 JJLAS = NLAS
      GO TO 2021
 2020 NUM = NOG/7
      NUMN = NOG-(NUM*7)
 2029 KK = KK+1
      JJLAS = JJFIR+6
 2021 J2LAS = JJLAS+NGTO
      IF (J2LAS.LE.69) GO TO 2022
      J2LAS = 69
 2022 WRITE (6,3004) (I, I=JJFIR,JJLAS)                                  517
 3004 FORMAT (1H0//7H   FROM, I6, 6I13)
      WRITE (6,3005)                                                     523
 3005 FORMAT (4H  TO)
      DO 2023 J=JJFIR,J2LAS
      K = J-NGTO
      IF (K.GE.JJFIR) GO TO 2024
      K = JJFIR
 2024 JK = J
      IF (J.LE.JJLAS) GO TO 2025
      JK = JJLAS
 2025 L = 0
      DO 1016 I=K,JK
      L = L+1
      IF (II.EQ.1) BBB(L)=PNOT(I,J)
      IF (II.EQ.2) BBB(L)=P1(I,J)
 1016 CONTINUE
      IF (JJFIR.EQ.K) WRITE (6,1007)  J,(BBB(LL), LL=1,L)                551
 1007 FORMAT (I4, 1PE14.5, 1P6E13.5)
      IF (JJFIR.EQ.K-1) WRITE (6,1008) J, (BBB(LL), LL=1,L)              557
 1008 FORMAT (I4, 14X, 1P6E13.5)
      IF (JJFIR.EQ.K-2) WRITE (6,1009) J, (BBB(LL), LL=1,L)              563
 1009 FORMAT (I4, 27X, 1P5E13.5)
      IF (JJFIR.EQ.K-3) WRITE (6,1010) J, (BBB(LL), LL=1,L)              569
 1010 FORMAT (I4, 40X, 1P4E13.5)
      IF (JJFIR.EQ.K-4) WRITE (6,1011) J, (BBB(LL), LL=1,L)              575
 1011 FORMAT (I4, 53X, 1P3E13.5)
      IF (JJFIR.EQ.K-5) WRITE (6,1012) J, (BBB(LL), LL=1,L)              581
 1012 FORMAT (I4, 66X, 1P2E13.5)
      IF (JJFIR.EQ.K-6) WRITE (6,1013) J, (BBB(LL), LL=1,L)              587
```

ETC - EFN SOURCE STATEMENT - IFN(S) -

```
1013 FORMAT (I4, 79X, 1PE13.5)
2023 CONTINUE
     IF (KK.EQ.0) GO TO 2027
     IF (KK.EQ.NUM) GO TO 2028
     JJFIR = JJLAS+1
     GO TO 2029
2028 IF (NUMN.EQ.0) GO TO 2027
     NUMN = 0
     JJFIR = JJLAS+1
     GO TO 2030
2027 IF (II.GT.1) GO TO 2031
     II = 2
     GO TO 2032
2031 WRITE (7,3007) (PUN0(M), M=1,MYS)                                    612
     WRITE (7,3007) (PUN1(M), M=1,MYS)                                    619
3007 FORMAT (6E12.5)
     WRITE (6,97)                                                         626
  97 FORMAT (1H1, 2X, 5HGROUP, 5X, 5HP-NOT, 10X, 3HP-1, 8X,
    1          10HP-1/3P-NOT)
     DO 98 I=NFIR,NLAS
     CHECK = SP1(I)/(3.0*SP0(I))
  98 WRITE (6,99) I, SP0(I), SP1(I), CHECK                                632
  99 FORMAT (4X, I2, 1PE15.5, 1P2E14.5)
     GO TO 2222
  53 WRITE (6,2501)                                                       638
2501 FORMAT (1H1, 10X, 13HERROR MESSAGE // 73H     ENERGY GREATER THAN
    1THAT OF THE LAST GROUP SCATTERED FROM IS MISSING)
2222 STOP
     END
```

02/08/67

```
$IBFTC FVAL     FULIST,REF,M94/2,XR7
```

FVAL - EFN SOURCE STATEMENT - IFN(S) -

```
      SUBROUTINE FVAL
      COMMON AL, AL1, E(70), NOF1, ALPHA, INTGC, ITEXT, EP, S, Y, X,
     1       AK, AKK(14), ATM, APA, AP, A2, FF(9), UOC, UOD, I, J
      COMMON INN, NPRINT
      DIMENSION DELUO(18), BAR(11), ABAR(11), R(9), HOL(9), DELP(9),
     1          GOL(9)
C
CDETERMINES THE C AND D VALUES FOR THE SPECIFIC E-PRIM VALUE
C Y IS DETERMINED
C
C
      IF (INTGC.EQ.1.OR.INTGC.EQ.2.OR.INTGC.EQ.4.OR.INTGC.EQ.7)
     1 GO TO 113
  114 IF (INTGC.EQ.3) GO TO 119
  116 IF (INTGC.EQ.5) GO TO 117
  118 C = E(J+1)
      D = EP
      GO TO 127
  119 C = ALPHA*EP
      D = E(J)
      GO TO 127
  117 C = E(J+1)
      D = E(J)
      GO TO 127
  113 IF (INTGC.EQ.1) GO TO 119
  115 IF (INTGC.EQ.2) GO TO 112
  120 IF (INTGC.EQ.4) GO TO 121
  123 IF (ITEXT.EQ.1) GO TO 125
      GO TO 118
  112 IF (ITEXT.EQ.1) GO TO 119
      GO TO 117
  121 IF (ITEXT.EQ.1) GO TO 119
      GO TO 117
  125 C = ALPHA*EP
      D = EP
  127 DIFP = (2.0*(D-C))/(AL*EP)
      UOC = 1.0-(AL1*(1.0-C/EP))
      UOD = 1.0-(AL1*(1.0-D/EP))
      IF (NPRINT.EQ.1) WRITE (6,3067) C, D, UOC, UOD
 3067 FORMAT(1H+,9X,2HC=,1PE13.5,4H, D=, 1PE13.5,7H, MUOC=, 1PE13.5,
     1    7H, MUOD=, 1PE13.5/ 5X, 6HF-TERM, 6X, 3HP-0, 11X, 3HP-1, 8X,
     2       6HF-TERM, 6X, 3HP-0, 11X, 3HP-1)
```

FVAL - EFN SOURCE STATEMENT - IFN(S) -

```
      NOW = NOF1+15
      DO 200 K=1,NOW
  200 DELUO(K) = UOD**K-UOC**K                                          51    52
      NOW = NOF1+2
      DO 206 K=1,NOW
      BAR(K) = 0.0
      NON = K-1
      DO 207 KK=1,INN
      NN = NON+KK
      XNN = NN
  207 BAR(K) = BAR(K)+(DELUO(NN)/XNN)*AKK(KK)
      BAR(K) = APA*BAR(K)
  206 ABAR(K) = ATM*BAR(K)
      RR = ABAR(2)+BAR(1)
      SUN = RR
      IF (NPRINT.EQ.1) WRITE (6,3068) DIFP, SUN                         78
 3068 FORMAT (5X, 3HF-0, 1PE17.5, 1PE14.5)
      IF (NOF1.EQ.0) GO TO 128
      DO 208 KK =1,NOF1
  208 R(KK) = ABAR(KK+2)+BAR(KK+1)
      GO TO (1,2,3,4,5,6,7,8,9), NOF1
    9 HOL(9) = 0.1484375*FF(9)*(12155.*R(9)-25740.*R(7)+18018.*R(5)
     1 -4620.*R(3)+315.*R(1))
    8 HOL(8) = 0.1328125*FF(8)*(6435.*R(8)-12012.*R(6)+6930.*R(4)
     1 -1260.*R(2)+35.*RR)
    7 HOL(7) = 0.9375*FF(7)*(429.*R(7)-693.*R(5)+315.*R(3)-35.*R(1))
    6 HOL(6) = 0.8125*FF(6)*(231.*R(6)-315.*R(4)+105.*R(2)-5.*RR)
    5 HOL(5) = 1.375*FF(5)*(63.*R(5)-70.*R(3)+15.*R(1))
    4 HOL(4) = 1.125*FF(4)*(35.*R(4)-30.*R(2)+3.*RR)
    3 HOL(3) = 3.5*FF(3)*(5.*R(3)-3.*R(1))
    2 HOL(2) = 2.5*FF(2)*(3.*R(2)-RR)
    1 HOL(1) = 3.0*FF(1)*R(1)
      GO TO (31,32,33,34,35,36,37,38,39), NOF1
   39 DELP(9)=180.42578125*DELUO(10)-477.59765625*DELUO(8)+445.7578125*
     1        DELUO(6)-171.4453125*DELUO(4)+23.37890625*DELUO(2)
   38 DELP(8)=94.9609375*DELUO(9)-227.90625*DELUO(7)+184.078125*DELUO(5)
     1        -55.78125*DELUO(3)+4.6484375*DELUO(1)
   37 DELP(7)=50.2734375*DELUO(8)-108.28125*DELUO(6)+73.828125*DELUO(4)-
     1        16.40625*DELUO(2)
   36 DELP(6)=26.8125*DELUO(7)-51.1875*DELUO(5)+28.4375*DELUO(3)-
     1        4.0625*DELUO(1)
   35 DELP(5)=14.4375*DELUO(6)-24.0625*DELUO(4)+10.3125*DELUO(2)
   34 DELP(4)=7.875*DELUO(5)-11.25*DELUO(3)+3.375*DELUO(1)
```

```
      FVAL      -  EFN   SOURCE STATEMENT  -  IFN(S)  -
   33 DELP(3)=4.375*DELUO(4)-5.25*DELUO(2)
   32 DELP(2)=2.5*(DELUO(3)-DELUO(1))
   31 DELP(1)=1.5*DELUO(2)
      SUMV=0.0
      DO 10 KK=1,NOF1
      GOL(KK) = FF(KK)*DELP(KK)
      SUN = SUN+HOL(KK)
   10 SUMV = SUMV+GOL(KK)
      IF (NPRINT.EQ.1) WRITE (6,3069) (K,GOL(K),HOL(K), K=1,NOF1)                  123
 3069 FORMAT(1H+,42X,2HF-,I1, 1PE17.5,1PE14.5,/ 5X, 2HF-,I1, 1PE17.5,
     1         1PE14.5)
      GO TO 131
  128 SUMV = 0.0
  131 SEP = S/EP
      SUMV = SUMV+DIFP
      Y = SEP*SUMV
      X = SEP*SUN
      IF (NPRINT.EQ.1) WRITE (6,1004) SUMV, SUN, Y, X                              135
 1004 FORMAT (5X, 5HTOTAL, 1PE15.5, 1PE14.5, 3X, 7HFUN(EP), 1PE13.5,
     1        1PE14.5)
      RETURN
      END
```

C) SAMPLE PRINTOUT

NICKEL

THE ATOMIC WEIGHT OF THE TARGET ATOM IS 58.710

THE NUMBER OF GAM GROUPS FROM WHICH SCATTERING OCCURS IS 68

THE FIRST GROUP FROM WHICH SCATTERING OCCURS IS 1 AND THE LAST GROUP IS 68

THE NUMBER OF GROUPS TO WHICH SCATTERING OCCURS FROM A PARTICULAR GAM GROUP IS 1

THE NUMBER OF MAXIMUM F-TERMS IS 8

THE NUMBER OF ENERGY POINTS READ IN BY CARDS IS 465

NO. OF TERMS IN SERIES FOR DELTA I SUB N IS 13

GAM GROUP	MESH SIZE	NO. OF F-TERMS
1	100	8
2	100	8
3	100	8
4	100	8
5	100	8
6	100	8
7	100	8
8	100	8
9	100	6
10	100	6
11	100	5
12	100	5
13	100	4
14	100	2
15	100	2
16	100	2
17	100	1
18	100	1
19	100	1
20	100	0
21	100	0
22	100	0
23	100	0
24	100	0
25	100	0
26	100	0
27	100	0
28	100	0
29	100	0
30	100	0
31	100	0
32	100	0
33	100	0
34	100	0
35	100	0
36	100	0
37	100	0
38	100	0
39	100	0
40	100	0
41	100	0
42	100	0
43	100	0
44	100	0
45	100	0
46	100	0
47	100	0

48	100	0
49	100	0
50	100	0
51	100	0
52	100	0
53	100	0
54	100	0
55	100	0
56	100	0
57	100	0
58	100	0
59	100	0
60	100	0
61	100	0
62	100	0
63	100	0
64	100	0
65	100	0
66	100	0
67	100	0
68	100	0

INPUT OF FORM

NO	ENERGY, EV F-5	SIGMA, BARNS F-6	F-1 F-7	F-2 F-8	F-3 F-9	F-4
1	0.13000E 08	0.13500E 01	0.88500E 00	0.77000E 00	0.62800E 00	0.47000E 00
	0.32200E 00	0.17500E 00	0.35000E-01	-0.21600E-01		
2	0.10000E 08	0.17300E 01	0.85800E 00	0.72100E 00	0.56100E 00	0.36100E 00
	0.24500E 00	0.13000E 00	0.35000E-01	-0.20000E-01		
3	0.95000E 07	0.17900E 01	0.85090E 00	0.70960E 00	0.54550E 00	0.33790E 00
	0.22640E 00	0.12240E 00	0.34240E-01	-0.19490E-01		
4	0.90000E 07	0.18600E 01	0.84410E 00	0.69860E 00	0.53060E 00	0.31570E 00
	0.20870E 00	0.11510E 00	0.33510E-01	-0.19000E-01		
5	0.85000E 07	0.19500E 01	0.83690E 00	0.68720E 00	0.51510E 00	0.29260E 00
	0.19010E 00	0.10740E 00	0.32740E-01	-0.18500E-01		
6	0.80000E 07	0.20300E 01	0.83010E 00	0.67620E 00	0.50030E 00	0.27040E 00
	0.17230E 00	0.10010E 00	0.32010E-01	-0.18010E-01		
7	0.75000E 07	0.20900E 01	0.81740E 00	0.65610E 00	0.47800E 00	0.24730E 00
	0.15260E 00	0.88640E-01	0.29730E-01	-0.17240E-01		

CAM GROUP 1

GAM GROUP 1

NO.	MESH ENERGY, EV	ELAS. SCATT.	F-1	F-2	F-3	F-4
1	7.78801E 06	2.05544E 00	8.24715E-01	6.67678E-01	4.90845E-01	2.60606E-01
2	7.81013E 06	2.05278E 00	8.25277E-01	6.68567E-01	4.91832E-01	2.61628E-01
3	7.83225E 06	2.05013E 00	8.25839E-01	6.69456E-01	4.92818E-01	2.62650E-01
4	7.85437E 06	2.04748E 00	8.26401E-01	6.70346E-01	4.93805E-01	2.63672E-01
5	7.87649E 06	2.04482E 00	8.26963E-01	6.71235E-01	4.94791E-01	2.64694E-01
6	7.89861E 06	2.04217E 00	8.27525E-01	6.72124E-01	4.95778E-01	2.65716E-01
7	7.92073E 06	2.03951E 00	8.28086E-01	6.73013E-01	4.96764E-01	2.66738E-01
8	7.94285E 06	2.03686E 00	8.28648E-01	6.73902E-01	4.97751E-01	2.67760E-01
9	7.96497E 06	2.03420E 00	8.29210E-01	6.74792E-01	4.98738E-01	2.68781E-01
10	7.98709E 06	2.03155E 00	8.29772E-01	6.75681E-01	4.99724E-01	2.69803E-01
11	8.00921E 06	2.02853E 00	8.30225E-01	6.76403E-01	5.00573E-01	2.70809E-01
12	8.03133E 06	2.02499E 00	8.30526E-01	6.76889E-01	5.01227E-01	2.71791E-01
13	8.05345E 06	2.02145E 00	8.30827E-01	6.77376E-01	5.01882E-01	2.72773E-01
14	8.07557E 06	2.01791E 00	8.31128E-01	6.77862E-01	5.02537E-01	2.73755E-01
15	8.09769E 06	2.01437E 00	8.31429E-01	6.78349E-01	5.03192E-01	2.74737E-01
16	8.11981E 06	2.01083E 00	8.31729E-01	6.78836E-01	5.03846E-01	2.75719E-01
17	8.14193E 06	2.00729E 00	8.32030E-01	6.79322E-01	5.04501E-01	2.76702E-01
18	8.16405E 06	2.00375E 00	8.32331E-01	6.79809E-01	5.05156E-01	2.77684E-01
19	8.18617E 06	2.00021E 00	8.32632E-01	6.80296E-01	5.05810E-01	2.78666E-01
20	8.20829E 06	1.99667E 00	8.32933E-01	6.80782E-01	5.06465E-01	2.79648E-01
21	8.23041E 06	1.99314E 00	8.33234E-01	6.81269E-01	5.07120E-01	2.80630E-01
22	8.25253E 06	1.98960E 00	8.33534E-01	6.81756E-01	5.07775E-01	2.81612E-01
23	8.27464E 06	1.98606E 00	8.33835E-01	6.82242E-01	5.08429E-01	2.82594E-01
24	8.29676E 06	1.98252E 00	8.34136E-01	6.82729E-01	5.09084E-01	2.83576E-01
25	8.31888E 06	1.97898E 00	8.34437E-01	6.83215E-01	5.09739E-01	2.84558E-01
26	8.34100E 06	1.97544E 00	8.34738E-01	6.83702E-01	5.10394E-01	2.85541E-01
27	8.36312E 06	1.97190E 00	8.35038E-01	6.84189E-01	5.11048E-01	2.86523E-01
28	8.38524E 06	1.96836E 00	8.35339E-01	6.84675E-01	5.11703E-01	2.87505E-01
29	8.40736E 06	1.96482E 00	8.35640E-01	6.85162E-01	5.12358E-01	2.88487E-01
30	8.42948E 06	1.96128E 00	8.35941E-01	6.85649E-01	5.13013E-01	2.89469E-01
31	8.45160E 06	1.95774E 00	8.36242E-01	6.86135E-01	5.13667E-01	2.90451E-01
32	8.47372E 06	1.95420E 00	8.36543E-01	6.86622E-01	5.14322E-01	2.91433E-01
33	8.49584E 06	1.95066E 00	8.36843E-01	6.87109E-01	5.14977E-01	2.92415E-01
34	8.51796E 06	1.94677E 00	8.37159E-01	6.87610E-01	5.15657E-01	2.93430E-01
35	8.54008E 06	1.94278E 00	8.37477E-01	6.88114E-01	5.16343E-01	2.94452E-01
36	8.56220E 06	1.93880E 00	8.37796E-01	6.88618E-01	5.17028E-01	2.95474E-01
37	8.58432E 06	1.93482E 00	8.38114E-01	6.89123E-01	5.17714E-01	2.96496E-01
38	8.60644E 06	1.93084E 00	8.38433E-01	6.89627E-01	5.18400E-01	2.97518E-01
39	8.62856E 06	1.92686E 00	8.38751E-01	6.90131E-01	5.19085E-01	2.98540E-01
40	8.65068E 06	1.92288E 00	8.39070E-01	6.90636E-01	5.19771E-01	2.99562E-01
41	8.67280E 06	1.91890E 00	8.39388E-01	6.91140E-01	5.20457E-01	3.00583E-01
42	8.69492E 06	1.91491E 00	8.39707E-01	6.91644E-01	5.21143E-01	3.01605E-01
43	8.71704E 06	1.91093E 00	8.40025E-01	6.92149E-01	5.21828E-01	3.02627E-01
44	8.73916E 06	1.90695E 00	8.40344E-01	6.92653E-01	5.22514E-01	3.03649E-01

45	8.76128E 06	1.90297E.00	8.40662E-01	6.93157E-01	5.23200E-01	3.04671E-01
46	8.78340E 06	1.89899E 00	8.40981E-01	6.93662E-01	5.23885E-01	3.05693E-01
47	8.80552E 06	1.89501E 00	8.41300E-01	6.94166E-01	5.24571E-01	3.06715E-01
48	8.82764E 06	1.89102E 00	8.41618E-01	6.94670E-01	5.25257E-01	3.07737E-01
49	8.84976E 06	1.88704E 00	8.41937E-01	6.95175E-01	5.25943E-01	3.08759E-01
50	8.87188E 06	1.88306E 00	8.42255E-01	6.95679E-01	5.26628E-01	3.09781E-01
51	8.89400E 06	1.87908E 00	8.42574E-01	6.96183E-01	5.27314E-01	3.10803E-01
52	8.91612E 06	1.87510E 00	8.42892E-01	6.96688E-01	5.28000E-01	3.11825E-01
53	8.93824E 06	1.87112E 00	8.43211E-01	6.97192E-01	5.28685E-01	3.12847E-01
54	8.96036E 06	1.86713E 00	8.43529E-01	6.97696E-01	5.29371E-01	3.13869E-01
55	8.98248E 06	1.86315E 00	8.43848E-01	6.98201E-01	5.30057E-01	3.14891E-01
56	9.00460E 06	1.85936E 00	8.44163E-01	6.98701E-01	5.30737E-01	3.15904E-01
57	9.02672E 06	1.85626E 00	8.44463E-01	6.99188E-01	5.31396E-01	3.16886E-01
58	9.04884E 06	1.85316E 00	8.44764E-01	6.99674E-01	5.32055E-01	3.17869E-01
59	9.07096E 06	1.85007E 00	8.45065E-01	7.00161E-01	5.32715E-01	3.18651E-01
60	9.09308E 06	1.84697E 00	8.45366E-01	7.00648E-01	5.33374E-01	3.19833E-01
61	9.11520E 06	1.84387E 00	8.45667E-01	7.01134E-01	5.34033E-01	3.20815E-01
62	9.13732E 06	1.84078E 00	8.45968E-01	7.01621E-01	5.34692E-01	3.21797E-01
63	9.15944E 06	1.83768E 00	8.46268E-01	7.02108E-01	5.35351E-01	3.22779E-01
64	9.18156E 06	1.83458E 00	8.46569E-01	7.02594E-01	5.36010E-01	3.23761E-01
65	9.20368E 06	1.83148E 00	8.46870E-01	7.03081E-01	5.36670E-01	3.24743E-01
66	9.22580E 06	1.82839E 00	8.47171E-01	7.03568E-01	5.37329E-01	3.25726E-01
67	9.24792E 06	1.82529E 00	8.47472E-01	7.04054E-01	5.37988E-01	3.26708E-01
68	9.27004E 06	1.82219E 00	8.47773E-01	7.04541E-01	5.38647E-01	3.27690E-01
69	9.29216E 06	1.81910E 00	8.48073E-01	7.05027E-01	5.39306E-01	3.28672E-01
70	9.31428E 06	1.81600E 00	8.48374E-01	7.05514E-01	5.39966E-01	3.29654E-01
71	9.33640E 06	1.81290E 00	8.48675E-01	7.06001E-01	5.40625E-01	3.30636E-01
72	9.35852E 06	1.80981E 00	8.48976E-01	7.06487E-01	5.41284E-01	3.31618E-01
73	9.38064E 06	1.80671E 00	8.49277E-01	7.06974E-01	5.41943E-01	3.32600E-01
74	9.40276E 06	1.80361E 00	8.49578E-01	7.07461E-01	5.42602E-01	3.33582E-01
75	9.42488E 06	1.80052E 00	8.49878E-01	7.07947E-01	5.43261E-01	3.34565E-01
76	9.44700E 06	1.79742E 00	8.50179E-01	7.08434E-01	5.43921E-01	3.35547E-01
77	9.46912E 06	1.79432E 00	8.50480E-01	7.08921E-01	5.44580E-01	3.36529E-01
78	9.49124E 06	1.79123E 00	8.50781E-01	7.09407E-01	5.45239E-01	3.37511E-01
79	9.51336E 06	1.78840E 00	8.51090E-01	7.09905E-01	5.45914E-01	3.38517E-01
80	9.53548E 06	1.78574E 00	8.51404E-01	7.10409E-01	5.46600E-01	3.39539E-01
81	9.55760E 06	1.78309E 00	8.51718E-01	7.10913E-01	5.47286E-01	3.40561E-01
82	9.57972E 06	1.78043E 00	8.52032E-01	7.11418E-01	5.47971E-01	3.41583E-01
83	9.60184E 06	1.77778E 00	8.52346E-01	7.11922E-01	5.48657E-01	3.42605E-01
84	9.62396E 06	1.77513E 00	8.52660E-01	7.12426E-01	5.49343E-01	3.43627E-01
85	9.64608E 06	1.77247E 00	8.52974E-01	7.12931E-01	5.50028E-01	3.44649E-01
86	9.66820E 06	1.76982E 00	8.53288E-01	7.13435E-01	5.50714E-01	3.45671E-01
87	9.69032E 06	1.76716E 00	8.53602E-01	7.13939E-01	5.51400E-01	3.46693E-01
88	9.71244E 06	1.76451E 00	8.53917E-01	7.14444E-01	5.52086E-01	3.47715E-01
89	9.73456E 06	1.76185E 00	8.54231E-01	7.14948E-01	5.52771E-01	3.48737E-01
90	9.75668E 06	1.75920E 00	8.54545E-01	7.15452E-01	5.53457E-01	3.49758E-01
91	9.77880E 06	1.75654E 00	8.54859E-01	7.15957E-01	5.54143E-01	3.50780E-01
92	9.80092E 06	1.75389E 00	8.55173E-01	7.16461E-01	5.54828E-01	3.51802E-01

93	9.82304E 06	1.75124E 00	8.55487E-01	7.16965E-01	5.55514E-01	3.52824E-01
94	9.84516E 06	1.74858E 00	8.55801E-01	7.17470E-01	5.56200E-01	3.53846E-01
95	9.86728E 06	1.74593E 00	8.56115E-01	7.17974E-01	5.56886E-01	3.54868E-01
96	9.88940E 06	1.74327E 00	8.56429E-01	7.18478E-01	5.57571E-01	3.55890E-01
97	9.91152E 06	1.74062E 00	8.56744E-01	7.18983E-01	5.58257E-01	3.56912E-01
98	9.93364E 06	1.73796E 00	8.57058E-01	7.19487E-01	5.58943E-01	3.57934E-01
99	9.95576E 06	1.73531E 00	8.57372E-01	7.19991E-01	5.59628E-01	3.58956E-01
100	9.97788E 06	1.73265E 00	8.57686E-01	7.20496E-01	5.60314E-01	3.59978E-01
101	1.00000E 07	1.73000E 00	8.58000E-01	7.21000E-01	5.61000E-01	3.61000E-01

NO.	MESH ENERGY,EV	F-5	F-6	F-7	F-8
1	7.78801E 06	1.63948E-01	9.52411E-02	3.10433E-02	-1.76835E-02
2	7.81013E 06	1.64819E-01	9.57481E-02	3.11442E-02	-1.77176E-02
3	7.83225E 06	1.65691E-01	9.62551E-02	3.12450E-02	-1.77517E-02
4	7.85437E 06	1.66562E-01	9.67621E-02	3.13459E-02	-1.77857E-02
5	7.87649E 06	1.67434E-01	9.72691E-02	3.14468E-02	-1.78198E-02
6	7.89861E 06	1.68305E-01	9.77761E-02	3.15476E-02	-1.78539E-02
7	7.92073E 06	1.69177E-01	9.82831E-02	3.16485E-02	-1.78879E-02
8	7.94285E 06	1.70048E-01	9.87901E-02	3.17494E-02	-1.79220E-02
9	7.96497E 06	1.70920E-01	9.92970E-02	3.18502E-02	-1.79560E-02
10	7.98709E 06	1.71791E-01	9.98040E-02	3.19511E-02	-1.79901E-02
11	8.00921E 06	1.72628E-01	1.00234E-01	3.20234E-02	-1.80190E-02
12	8.03133E 06	1.73415E-01	1.00557E-01	3.20557E-02	-1.80407E-02
13	8.05345E 06	1.74203E-01	1.00880E-01	3.20880E-02	-1.80624E-02
14	8.07557E 06	1.74990E-01	1.01203E-01	3.21203E-02	-1.80841E-02
15	8.09769E 06	1.75778E-01	1.01526E-01	3.21526E-02	-1.81057E-02
16	8.11981E 06	1.76565E-01	1.01849E-01	3.21849E-02	-1.81274E-02
17	8.14193E 06	1.77353E-01	1.02172E-01	3.22172E-02	-1.81491E-02
18	8.16405E 06	1.78140E-01	1.02495E-01	3.22495E-02	-1.81708E-02
19	8.18617E 06	1.78927E-01	1.02818E-01	3.22818E-02	-1.81924E-02
20	8.20829E 06	1.79715E-01	1.03141E-01	3.23141E-02	-1.82141E-02
21	8.23041E 06	1.80502E-01	1.03464E-01	3.23464E-02	-1.82358E-02
22	8.25253E 06	1.81290E-01	1.03787E-01	3.23787E-02	-1.82575E-02
23	8.27464E 06	1.82077E-01	1.04110E-01	3.24110E-02	-1.82792E-02
24	8.29676E 06	1.82865E-01	1.04433E-01	3.24433E-02	-1.83008E-02
25	8.31888E 06	1.83652E-01	1.04756E-01	3.24756E-02	-1.83225E-02
26	8.34100E 06	1.84440E-01	1.05079E-01	3.25079E-02	-1.83442E-02
27	8.36312E 06	1.85227E-01	1.05402E-01	3.25402E-02	-1.83659E-02
28	8.38524E 06	1.86015E-01	1.05725E-01	3.25725E-02	-1.83875E-02
29	8.40736E 06	1.86802E-01	1.06048E-01	3.26048E-02	-1.84092E-02
30	8.42948E 06	1.87590E-01	1.06370E-01	3.26370E-02	-1.84309E-02
31	8.45160E 06	1.88377E-01	1.06693E-01	3.26693E-02	-1.84526E-02
32	8.47372E 06	1.89165E-01	1.07016E-01	3.27016E-02	-1.84742E-02
33	8.49584E 06	1.89952E-01	1.07339E-01	3.27339E-02	-1.84959E-02
34	8.51796E 06	1.90768E-01	1.07677E-01	3.27677E-02	-1.85180E-02
35	8.54008E 06	1.91591E-01	1.08017E-01	3.28017E-02	-1.85401E-02
36	8.56220E 06	1.92414E-01	1.08358E-01	3.28358E-02	-1.85622E-02
37	8.58432E 06	1.93237E-01	1.08699E-01	3.28699E-02	-1.85843E-02
38	8.60644E 06	1.94060E-01	1.09039E-01	3.29039E-02	-1.86064E-02
39	8.62856E 06	1.94883E-01	1.09380E-01	3.29380E-02	-1.86286E-02

40	8.65068E 06	1.95705E-01	1.09721E-01	3.29721E-02	-1.86507E-02
41	8.67280E 06	1.96528E-01	1.10061E-01	3.30061E-02	-1.86728E-02
42	8.69492E 06	1.97351E-01	1.10402E-01	3.30402E-02	-1.86949E-02
43	8.71704E 06	1.98174E-01	1.10742E-01	3.30742E-02	-1.87170E-02
44	8.73916E 06	1.98997E-01	1.11083E-01	3.31083E-02	-1.87392E-02
45	8.76128E 06	1.99820E-01	1.11424E-01	3.31424E-02	-1.87613E-02
46	8.78340E 06	2.00643E-01	1.11764E-01	3.31764E-02	-1.87834E-02
47	8.80552E 06	2.01465E-01	1.12105E-01	3.32105E-02	-1.88055E-02
48	8.82764E 06	2.02288E-01	1.12446E-01	3.32446E-02	-1.88276E-02
49	8.84976E 06	2.03111E-01	1.12786E-01	3.32786E-02	-1.88498E-02
50	8.87188E 06	2.03934E-01	1.13127E-01	3.33127E-02	-1.88719E-02
51	8.89400E 06	2.04757E-01	1.13468E-01	3.33468E-02	-1.88940E-02
52	8.91612E 06	2.05580E-01	1.13808E-01	3.33808E-02	-1.89161E-02
53	8.93824E 06	2.06403E-01	1.14149E-01	3.34149E-02	-1.89382E-02
54	8.96036E 06	2.07225E-01	1.14490E-01	3.34490E-02	-1.89604E-02
55	8.98248E 06	2.08048E-01	1.14830E-01	3.34830E-02	-1.89825E-02
56	9.00460E 06	2.08863E-01	1.15167E-01	3.35167E-02	-1.90045E-02
57	9.02672E 06	2.09646E-01	1.15490E-01	3.35490E-02	-1.90262E-02
58	9.04884E 06	2.10429E-01	1.15813E-01	3.35813E-02	-1.90479E-02
59	9.07096E 06	2.11212E-01	1.16136E-01	3.36136E-02	-1.90695E-02
60	9.09308E 06	2.11995E-01	1.16459E-01	3.36459E-02	-1.90912E-02
61	9.11520E 06	2.12778E-01	1.16782E-01	3.36782E-02	-1.91129E-02
62	9.13732E 06	2.13561E-01	1.17105E-01	3.37105E-02	-1.91346E-02
63	9.15944E 06	2.14344E-01	1.17428E-01	3.37428E-02	-1.91563E-02
64	9.18156E 06	2.15127E-01	1.17751E-01	3.37751E-02	-1.91779E-02
65	9.20368E 06	2.15910E-01	1.18074E-01	3.38074E-02	-1.91996E-02
66	9.22580E 06	2.16693E-01	1.18397E-01	3.38397E-02	-1.92213E-02
67	9.24792E 06	2.17476E-01	1.18720E-01	3.38720E-02	-1.92430E-02
68	9.27004E 06	2.18259E-01	1.19043E-01	3.39043E-02	-1.92646E-02
69	9.29216E 06	2.19042E-01	1.19366E-01	3.39366E-02	-1.92863E-02
70	9.31428E 06	2.19825E-01	1.19688E-01	3.39688E-02	-1.93080E-02
71	9.33640E 06	2.20609E-01	1.20011E-01	3.40011E-02	-1.93297E-02
72	9.35852E 06	2.21392E-01	1.20334E-01	3.40334E-02	-1.93513E-02
73	9.38064E 06	2.22175E-01	1.20657E-01	3.40657E-02	-1.93730E-02
74	9.40276E 06	2.22958E-01	1.20980E-01	3.40980E-02	-1.93947E-02
75	9.42488E 06	2.23741E-01	1.21303E-01	3.41303E-02	-1.94164E-02
76	9.44700E 06	2.24524E-01	1.21626E-01	3.41626E-02	-1.94381E-02
77	9.46912E 06	2.25307E-01	1.21949E-01	3.41949E-02	-1.94597E-02
78	9.49124E 06	2.26090E-01	1.22272E-01	3.42272E-02	-1.94814E-02
79	9.51336E 06	2.26897E-01	1.22603E-01	3.42603E-02	-1.95036E-02
80	9.53548E 06	2.27720E-01	1.22939E-01	3.42939E-02	-1.95262E-02
81	9.55760E 06	2.28543E-01	1.23275E-01	3.43275E-02	-1.95487E-02
82	9.57972E 06	2.29365E-01	1.23612E-01	3.43612E-02	-1.95713E-02
83	9.60184E 06	2.30188E-01	1.23948E-01	3.43948E-02	-1.95939E-02
84	9.62396E 06	2.31011E-01	1.24284E-01	3.44284E-02	-1.96164E-02
85	9.64608E 06	2.31834E-01	1.24620E-01	3.44620E-02	-1.96390E-02
86	9.66820E 06	2.32657E-01	1.24957E-01	3.44957E-02	-1.96616E-02
87	9.69032E 06	2.33480E-01	1.25293E-01	3.45293E-02	-1.96841E-02

88	9.71244E 06	2.34303E-01	1.25629E-01	3.45629E-02	-1.97067E-02
89	9.73456E 06	2.35126E-01	1.25965E-01	3.45965E-02	-1.97292E-02
90	9.75668E 06	2.35948E-01	1.26301E-01	3.46301E-02	-1.97518E-02
91	9.77880E 06	2.36771E-01	1.26638E-01	3.46638E-02	-1.97744E-02
92	9.80092E 06	2.37594E-01	1.26974E-01	3.46974E-02	-1.97969E-02
93	9.82304E 06	2.38417E-01	1.27310E-01	3.47310E-02	-1.98195E-02
94	9.84516E 06	2.39240E-01	1.27646E-01	3.47646E-02	-1.98421E-02
95	9.86728E 06	2.40063E-01	1.27983E-01	3.47983E-02	-1.98646E-02
96	9.88940E 06	2.40886E-01	1.28319E-01	3.48319E-02	-1.98872E-02
97	9.91152E 06	2.41708E-01	1.28655E-01	3.48655E-02	-1.99097E-02
98	9.93364E 06	2.42531E-01	1.28991E-01	3.48991E-02	-1.99323E-02
99	9.95576E 06	2.43354E-01	1.29327E-01	3.49327E-02	-1.99549E-02
100	9.97788E 06	2.44177E-01	1.29664E-01	3.49664E-02	-1.99774E-02
101	1.00000E 07	2.45000E-01	1.30000E-01	3.50000E-02	-2.00000E-02

NICKEL

P-0 SCATTERING MATRIX

SCATTERING MATRIX

LENGTH OF LIST = 136,GROUPS SCATTERED FROM = 68,GROUPS SCATTERED TO = 1

FROM / TO	1	2	3	4	5	6	7
1	1.84541E 00						
2	4.82684E-02	2.07416E 00					
3		6.41416E-02	2.03314E 00				
4			8.59358E-02	1.80334E 00			
5				1.29511E-01	1.65492E 00		
6					1.57411E-01	1.68600E 00	
7						1.93666E-01	1.93969E 00
8							2.51740E-01

FROM / TO	8	9	10	11	12	13	14
8	2.39158E 00						
9	3.51229E-01	2.85862E 00					
10		3.85323E-01	2.93457E 00				
11			4.08890E-01	3.01750E 00			
12				4.35959E-01	3.17877E 00		
13					4.79833E-01	3.50866E 00	
14						3.61194E-01	3.83745E 00
15							6.35718E-01

FROM / TO	15	16	17	18	19	20	21
15	4.67324E 00						
16	8.47414E-01	5.47669E 00					
17		5.37769E-01	6.19317E 00				
18			1.09323E 00	4.33165E 00			
19				8.93144E-01	5.21288E 00		
20					9.05674E-01	8.07556E 00	
21						2.33369E 00	9.00814E 00
22							5.45201E-01

FROM / TO	22	23	24	25	26	27	28
22	5.19380E 00						
23	1.05030E 00	9.09740E 00					
24		2.13593E 00	1.34306E 01				
25			1.67215E 00	1.56475E 01			
26				4.14281E 00	6.20904E 01		
27					8.01172E 00	2.78165E 01	
28						1.37812E 00	4.72444E 00
29							1.09431E 00

FROM / TO	29	30	31	32	33	34	35
29	9.54365E 00						
30	1.75362E 00	1.50979E 01					
31		4.13667E 00	1.26041E 02				
32			5.94543E 00	2.57815E 01			
33				3.14754E 00	1.76816E 01		
34					2.47384E 00	1.45774E 01	
35						2.13918E 00	1.32215E 01
36							2.00981E 00

FROM / TO	36	37	38	39	40	41	42
36	1.27435E 01						
37	1.97476E 00	1.27246E 01					
38		1.99012E 00	1.29583E 01				
39			2.05079E 00	1.33817E 01			
40				2.11083E 00	1.37204E 01		
41					2.15759E 00	1.39841E 01	
42						2.19401E 00	1.41896E 01
43							2.22237E 00

FROM / TO	43	44	45	46	47	48	49
43	1.43495E 01						
44	2.24447E 00	1.44741E 01					
45		2.26167E 00	1.45712E 01				
46			2.27506E 00	1.46468E 01			
47				2.28550E 00	1.47056E 01		
48					2.29363E 00	1.47515E 01	
49						2.29995E 00	1.47871E 01

50							2.30489E 00

FROM	50	51	52	53	54	55	56
TO							
50	1.48150E 01						
51	2.30872E 00	1.48366E 01					
52		2.31171E 00	1.48535E 01				
53			2.31404E 00	1.48666E 01			
54				2.31585E 00	1.48768E 01		
55					2.31726E 00	1.48848E 01	
56						2.31837E 00	1.48910E 01
57							2.31922E 00

FROM	57	58	59	60	61	62	63
TO							
57	1.48958E 01						
58	2.31988E 00	1.48996E 01					
59		2.32041E 00	1.49025E 01				
60			2.32081E 00	1.49048E 01			
61				2.32113E 00	1.49066E 01		
62					2.32137E 00	1.49080E 01	
63						2.32156E 00	1.49090E 01
64							2.32171E 00

FROM	64	65	66	67	68
TO					
64	1.49099E 01				
65	2.32182E 00	1.49105E 01			
66		2.32191E 00	1.49110E 01		
67			2.32199E 00	1.49114E 01	
68				2.32204E 00	1.49117E 01
69					2.32208E 00

NICKEL

P-1 SCATTERING MATRIX

SCATTERING MATRIX

LENGTH OF LIST = 136,GROUPS SCATTERED FROM = 68,GROUPS SCATTERED TO = 1

FROM / TO	1	2	3	4	5	6	7
1	4.75587E 00						
2	4.28289E-02	5.12404E 00					
3		4.69025E-02	4.69359E 00				
4			1.70422E-02	3.38465E 00			
5				-5.74382E-02	2.29028E 00		
6					-8.18555E-02	1.84548E 00	
7						-1.32350E-01	1.79552E 00
8							-2.12224E-01

FROM / TO	8	9	10	11	12	13	14
8	1.91653E 00						
9	-3.35750E-01	1.89305E 00					
10		-4.03397E-01	1.66552E 00				
11			-4.32285E-01	1.49352E 00			
12				-4.51084E-01	1.35830E 00		
13					-4.81107E-01	1.06446E 00	
14						-3.37031E-01	1.24739E 00
15							-6.00439E-01

FROM / TO	15	16	17	18	19	20	21
15	1.43112E 00						
16	-8.38142E-01	1.05245E 00					
17		-5.52002E-01	1.23175E 00				
18			-8.60482E-01	9.57417E-01			
19				-7.50463E-01	1.07119E 00		
20					-8.59580E-01	2.36509E 00	
21						-2.01050E 00	8.25593E-01
22							-5.00154E-01

FROM	22	23	24	25	26	27	28
TO							
22	1.21082E 00						
23	-9.98113E-01	2.26915E 00					
24		-1.88648E 00	2.10458E 00				
25			-1.59010E 00	4.33102E 00			
26				-3.65685E 00	1.08071E 01		
27					-8.41908E 00	2.56228E 00	
28						-1.56775E 00	1.20695E 00
29							-1.00873E 00

FROM	29	30	31	32	33	34	35
TO							
29	2.06466E 00						
30	-1.67981E 00	4.18123E 00					
31		-3.52600E 00	1.06913E 01				
32			-6.19512E 00	4.09742E 00			
33				-3.11194E 00	3.12127E 00		
34					-2.43467E 00	2.66573E 00	
35						-2.09628E 00	2.48287E 00
36							-1.96401E 00

FROM	36	37	38	39	40	41	42
TO							
36	2.42818E 00						
37	-1.92680E 00	2.44097E 00					
38		-1.93971E 00	2.50861E 00				
39			-1.99732E 00	2.58433E 00			
40				-2.05658E 00	2.64362E 00		
41					-2.10274E 00	2.68979E 00	
42						-2.13868E 00	2.72575E 00
43							-2.16667E 00

FROM	43	44	45	46	47	48	49
TO							
43	2.75376E 00						
44	-2.18848E 00	2.77557E 00					
45		-2.20546E 00	2.79255E 00				
46			-2.21868E 00	2.80578E 00			
47				-2.22898E 00	2.81609E 00		

48					-2.23700E 00	2.82411E 00	
49						-2.24324E 00	2.83036E 00
50							-2.24811E 00

FROM	50	51	52	53	54	55	56
TO							
50	2.83522E 00						
51	-2.25190E 00	2.83901E 00					
52		-2.25485E 00	2.84196E 00				
53			-2.25715E 00	2.84426E 00			
54				-2.25893E 00	2.84605E 00		
55					-2.26033E 00	2.84745E 00	
56						-2.26142E 00	2.84853E 00
57							-2.26226E 00

FROM	57	58	59	60	61	62	63
TO							
57	2.84938E 00						
58	-2.26292E 00	2.85004E 00					
59		-2.26343E 00	2.85055E 00				
60			-2.26384E 00	2.85095E 00			
61				-2.26414E 00	2.85126E 00		
62					-2.26438E 00	2.85150E 00	
63						-2.26457E 00	2.85169E 00
64							-2.26472E 00

FROM	64	65	66	67	68
TO					
64	2.85183E 00				
65	-2.26483E 00	2.85195E 00			
66		-2.26492E 00	2.85204E 00		
67			-2.26499E 00	2.85211E 00	
68				-2.26505E 00	2.85216E 00
69					-2.26509E 00

GROUP	P-NOT	P-1	P-1/3P-NOT
1	1.89368E 00	4.79870E 00	8.44685E-01
2	2.13830E 00	5.17095E 00	8.06084E-01
3	2.11908E 00	4.71063E 00	7.40989E-01
4	1.93285E 00	3.32722E 00	5.73801E-01
5	1.81234E 00	2.20843E 00	4.06184E-01
6	1.87967E 00	1.71313E 00	3.03801E-01
7	2.19143E 00	1.58329E 00	2.40831E-01
8	2.74280E 00	1.58078E 00	1.92113E-01
9	3.24395E 00	1.48965E 00	1.53070E-01
10	3.34346E 00	1.23323E 00	1.22950E-01
11	3.45346E 00	1.04243E 00	1.00618E-01
12	3.65860E 00	8.77192E-01	7.99205E-02
13	3.86986E 00	7.27430E-01	6.26578E-02
14	4.47317E 00	6.46949E-01	4.82096E-02
15	5.52065E 00	5.92983E-01	3.58039E-02
16	6.01446E 00	5.00451E-01	2.77360E-02
17	7.28639E 00	3.71267E-01	1.69845E-02
18	5.22480E 00	2.06954E-01	1.32033E-02
19	6.11856E 00	2.11614E-01	1.15285E-02
20	1.04093E 01	3.54595E-01	1.13551E-02
21	9.55334E 00	3.25438E-01	1.13551E-02
22	6.24411E 00	2.12707E-01	1.13551E-02
23	1.12333E 01	3.82667E-01	1.13551E-02
24	1.51027E 01	5.14481E-01	1.13551E-02
25	1.97903E 01	6.74163E-01	1.13551E-02
26	7.01021E 01	2.38805E 00	1.13551E-02
27	2.91947E 01	9.94527E-01	1.13551E-02
28	5.81875E 00	1.98217E-01	1.13551E-02
29	1.12973E 01	3.84846E-01	1.13551E-02
30	1.92346E 01	6.55232E-01	1.13551E-02
31	1.31987E 02	4.49617E 00	1.13551E-02
32	2.89291E 01	9.85481E-01	1.13551E-02
33	2.01554E 01	6.86604E-01	1.13551E-02
34	1.67166E 01	5.69455E-01	1.13551E-02
35	1.52313E 01	5.18860E-01	1.13551E-02
36	1.47182E 01	5.01381E-01	1.13551E-02
37	1.47147E 01	5.01260E-01	1.13551E-02
38	1.50091E 01	5.11290E-01	1.13551E-02
39	1.54925E 01	5.27756E-01	1.13551E-02
40	1.58779E 01	5.40888E-01	1.13551E-02
41	1.61781E 01	5.51114E-01	1.13551E-02
42	1.64119E 01	5.59077E-01	1.13551E-02
43	1.65940E 01	5.65282E-01	1.13551E-02
44	1.67358E 01	5.70111E-01	1.13551E-02
45	1.68462E 01	5.73872E-01	1.13551E-02
46	1.69323E 01	5.76804E-01	1.13551E-02
47	1.69992E 01	5.79085E-01	1.13551E-02

48	1.70514E 01	5.80862E-01	1.13551E-02
49	1.70920E 01	5.82246E-01	1.13551E-02
50	1.71237E 01	5.83323E-01	1.13551E-02
51	1.71483E 01	5.84164E-01	1.13551E-02
52	1.71675E 01	5.84817E-01	1.13551E-02
53	1.71824E 01	5.85325E-01	1.13551E-02
54	1.71941E 01	5.85723E-01	1.13551E-02
55	1.72032E 01	5.86031E-01	1.13551E-02
56	1.72102E 01	5.86271E-01	1.13551E-02
57	1.72157E 01	5.86460E-01	1.13551E-02
58	1.72200E 01	5.86605E-01	1.13551E-02
59	1.72233E 01	5.86718E-01	1.13551E-02
60	1.72259E 01	5.86808E-01	1.13551E-02
61	1.72279E 01	5.86875E-01	1.13551E-02
62	1.72295E 01	5.86930E-01	1.13551E-02
63	1.72308E 01	5.86972E-01	1.13551E-02
64	1.72317E 01	5.87003E-01	1.13551E-02
65	1.72324E 01	5.87030E-01	1.13551E-02
66	1.72330E 01	5.87048E-01	1.13551E-02
67	1.72335E 01	5.87064E-01	1.13551E-02
68	1.72338E 01	5.87078E-01	1.13551E-02

APPENDIX I - THE LEGENDRE POLYNOMIAL EXPANSION OF THE TRANSFERENCE FUNCTION

The transference function may be expanded in Legendre Polynomials, i.e.

$$\sigma_s(E' \to E, \mu) = \frac{1}{4\pi} \sum_{l=0}^{\infty} S_l(E' \to E)\, P_l(\mu), \qquad (1)$$

where quantities are expressed in the Laboratory system of coordinates (see text for definition of terms). Multiplying Equation 1 by $P_k(\mu)$ and integrating over the solid angle yields:

$$\int P_k(\mu)\sigma_s(E' \to E, \mu)\, d\Omega = 2\pi \int_{-1}^{1} P_k(\mu)\sigma_s(E' \to E, \mu)\, d\mu = \frac{1}{2}\sum_l S_l(E' \to E) \int_{-1}^{1} P_l(\mu) P_k(\mu)\, d\mu$$

$$= \frac{1}{2k+1} S_k(E' \to E),$$

since $\int_{-1}^{1} P_l(\mu)P_k(\mu)\,d\mu = \frac{2}{2k+1}\,\delta_{lk}$ where $\delta_{lk} \equiv \begin{cases} 1 & l = k \\ 0 & l \neq k \end{cases}$

Also

$$S_k(E' \to E) = (2k+1)\, 2\pi \int_{-1}^{1} P_k(\mu)\sigma_s(E' \to E, \mu)\, d\mu \qquad (2)$$

The transference function can be expressed[4] as

$$\sigma_s(E' \to E, \mu) = \sigma_s(E')\, G(\mu)\, \delta\left(E - E' + \frac{2AE'}{(A+1)^2}(1-\mu_0)\right) \qquad (3)$$

where $\sigma_s(E')$ is the total elastic scattering cross section for a neutron of energy E',

$G(\mu)$ is the probability density of a deflection μ in the lab frame,

δ is a delta function which takes into account the fact that the energy loss and angle of scatter are not independent,

and μ_0 is the scattering angle in the center of mass coordinate system.

Now $G(\mu)d\mu = G(\mu_0)d\mu_0$, so by (4)

substituting Equation 4 into Equation 3 one obtains

$$\sigma_s(E'\to E,\mu) = \sigma_s(E')G(\mu_0)\left(\frac{d\mu_0}{d\mu}\right)\delta\left(E-E'+\frac{2AE'}{(A+1)^2}(1-\mu_0)\right) \quad (5)$$

or (see Equation 10, Appendix II)

$$\sigma_s(E'\to E,\mu)=\sigma_s(E')\left[\frac{1}{4\pi}\sum_{\nu=0}^{\infty}(2\nu+1)f_\nu(E')P_\nu(\mu_0)\right]\frac{d\mu_0}{d\mu}\delta\left(E-E'+\frac{2AE'}{(A+1)^2}(1-\mu_0)\right) \quad (6)$$

For the special case of isotropic scattering in the center of mass

$G(\mu_0)=\frac{1}{4\pi}$, i.e. $f_\nu = 0$ for $\nu \geq 1$, and $f_0 = 1$.

Substituting Equation 6 into Equation 2 yields

$$S_k(E'\to E)=\frac{2k+1}{2}\int_{-1}^{1}\sigma_s(E')P_k(\mu)\sum_{v=0}^{\infty}(2v+1)f_v(E')P_v(\mu_0)\,\delta\left(E-E'+\frac{2AE'}{(A+1)^2}(1-\mu_0)\right)d\mu_0 \qquad (7)$$

Let $\frac{2AE'}{(A+1)^2}\mu_0=\xi$ then (8)

$$S_k(E'\to E)=\frac{2k+1}{2}\frac{(A+1)^2}{2A}\int_{-\frac{2AE'}{(A+1)^2}}^{\frac{2AE'}{(A+1)^2}}\frac{\sigma_s(E')}{E'}P_k(\mu)\sum_{v=0}^{\infty}(2v+1)f_v(E')P_v(\xi)\,\delta\left(E-E'+\frac{2AE'}{(A+1)^2}-\xi\right)d\xi \qquad (9)$$

$$\text{or } S_k(E'\to E)=\frac{(2k+1)(A+1)^2}{4A}\frac{\sigma_s(E')}{E'}P_k(\mu)\sum_{v=0}^{\infty}(2v+1)f_v(E')P_v(\mu_0) \qquad (10)$$

where[4] $\mu=\frac{1}{2}(A+1)\left(\frac{E}{E'}\right)^{1/2}-\frac{1}{2}(A-1)\left(\frac{E'}{E}\right)^{1/2}$ and $\mu_0=1-\frac{(A+1)^2}{2A}\left(1-\frac{E}{E'}\right)$ (11)

Now $\frac{(A+1)^2}{4A}=\frac{1}{1-\alpha}$ since $\alpha=\left(\frac{A-1}{A+1}\right)^2$ (12)

$$\text{so } S_k(E'\to E)=\frac{(2k+1)}{1-\alpha}\frac{\sigma_s(E')}{E'}P_k(\mu)\sum_{v=0}^{\infty}(2v+1)f_v(E')P_v(\mu_0). \qquad (13)$$

All $S_k(E' \to E)$ vanish unless[4] $\alpha E' \leq E \leq E'$. As an example we may consider the special case when the scattering is isotropic in the center of mass coordinate system:

$$S_0(E' \to E) = \frac{1}{1-\alpha} \frac{\sigma_s(E')}{E'} \quad \text{When} \quad \alpha E' \leq E \leq E', \tag{14}$$

and = 0 otherwise.

Also

$$S_1(E' \to E) = \frac{3}{1-\alpha} \frac{\sigma_s(E')}{E'} \left[\frac{A+1}{2} \left(\frac{E}{E'} \right)^{1/2} - \frac{A-1}{2} \left(\frac{E'}{E} \right)^{1/2} \right] \quad \text{when} \alpha E' \leq E \leq E' \tag{15}$$

or $S_1(E' \to E) = \frac{3}{1-\alpha} \cdot \frac{\sigma_s(E')}{E'} \mu,$

and = 0 otherwise.

For this special case $S_1(E' \to E) = 3\mu S_0(E' \to E)$ (16)

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APPENDIX II - LEGENDRE POLYNOMIAL EXPANSION OF THE DIFFERENTIAL SCATTERING CROSS SECTION IN THE CENTER OF MASS COORDINATE SYSTEM

Ordinarily differential scattering cross section, $\frac{d\sigma}{d\Omega}$ (barns per steradian), data[5] which are obtained for some incident neutron energy E' are plotted as a function of the cosine of the scattering angle μ_0 in the center of mass coordinate system. We shall use the symbol $\sigma(E', \mu_0)$ for this differential cross section. This cross section may be represented by a Legendre polynomial expansion, i.e.

$$\sigma_s(E', \mu_0) = \frac{1}{4\pi} \sum_{l=0}^{\infty} a_l(E') P_l(\mu_0) \tag{1}$$

Multiplying by $P_k(\mu_0)$ and integrating over solid angle:

$$\int \sigma_s(E', \mu_0) P_k(\mu_0) d\hat{\Omega} = 2\pi \int_{-1}^{1} P_k(\mu_0) \sigma_s(E', \mu_0) d\mu_0 = \frac{1}{2} \sum_{l=0}^{\infty} a_l(E') \int_{-1}^{1} P_l(\mu_0) P_k(\mu_0) d\mu_0 \tag{2}$$

where $d\hat{\Omega} = \sin\theta_0 d\theta_0 d\varphi_0 = -d\mu_0 d\varphi_0$.

Now $\int_{-1}^{1} P_l(\mu_0) P_k(\mu_0) d\mu_0 = \frac{2}{2l+1} \delta_{lk}$ where $\delta_{lk} = \begin{cases} 1 & l = k \\ 0 & l \neq k \end{cases}$ (3)

therefore $2\pi \int_{-1}^{1} P_k(\mu_0) \sigma_s(E', \mu_0) d\mu_0 = \frac{1}{2} a_k \frac{2}{2k+1}$, (4)

or $a_k = 2\pi (2k+1) \int_{-1}^{1} P_k(\mu_0) \sigma_s(E', \mu_0) d\mu_0$. (5)

Also

$$\sigma_s(E';\mu_0)=\sum_{l=0}^{\infty}\frac{2l+1}{2}P_l(\mu_0)\int_{-1}^{1}P_l(\xi)\sigma_s(E';\xi)\,d\xi\,. \qquad (6)$$

Now

$$\sigma_s(E')\equiv\int\sigma_s(E';\mu_0)\,d\hat{\Omega}=2\pi\int_{-1}^{1}\sigma_s(E';\mu_0)\,d\mu_0\;; \qquad (7)$$

Substituting Equation 6 into Equation 7 yields

$$\sigma_s(E')=a_0 \qquad \text{since} \qquad P_0(\mu_0)=1 \qquad (8)$$

Hence given the differential elastic scattering cross section, $\sigma_s(E';\mu_0)$ we may find the a_l's including a_0 which is the total elastic scattering cross section.

Let us define $$f_l(E')=\frac{1}{2l+1}\frac{a_l(E')}{a_0(E')}=\frac{1}{2l+1}\frac{a_l(E')}{\sigma_s(E')}\;; \qquad (9)$$

Equation 1 becomes:

$$\sigma_s(E';\mu_0)=\frac{\sigma_s(E')}{4\pi}\sum_{\nu=0}^{\infty}(2\nu+1)f_\nu(E')\,P_\nu(\mu_0) \qquad (10)$$

where

$$f_\nu(E')=\frac{2\pi}{\sigma_s(E')}\int_{-1}^{1}P_\nu(\mu_0)\sigma_s(E';\mu_0)\,d\mu_0 \qquad (11)$$

For the special case $\nu=0$ $$f_0=\frac{2\pi}{\sigma_s(E')}\int_{-1}^{1}\sigma_s(E';\mu_0)\,d\mu_0=1 \qquad (12)$$

Also $f_1(E')=\bar{\mu}_0$ by definition.

Although the f_ν's and $\sigma_s(E')$ can be obtained from the experimental differential cross section data ($\sigma_s(E', \mu_0)$), experimental data for $\sigma_s(E')$ is normally available. The experimental $\sigma_s(E')$, and the f_ν's calculated from the differential cross section data are used directly to compute the elastic matrices.

A program (f-code) is available[1] for calculating the f_ν's when given $\sigma_s(E', \mu_0)$. This program takes the resulting f_ν values and calculates $\sigma_s(E', \mu_0)$ as a check. The coefficients for the first nine terms ($\nu \leq 9$) in the expansion are obtained with the f-code.

If the experimental differential cross sections are presented in the laboratory coordinate system, a program is available[1] to transform the data to the center-of-mass coordinate system before using it in the f-code program.

APPENDIX III - EXPRESSION FOR I_n IN TERMS OF I_o

Since

$$I_n = \frac{\mu_o^{\,n}}{2n+1} I_o - \frac{n}{2n+1} k I_{n-1} \tag{1}$$

I_n can be written in terms of I_o.

Assertion: for $n \geq 1$

$$I_n = \frac{1}{2n+1}\left\{\sum_{i=0}^{n-1} (-1)^{i+1} \frac{n(n-1)(n-2)\cdots(n-i)}{(2n-1)(2n-3)\cdots(2n-2i-1)} k^{i+1} \mu_o^{\,n-i-1} + \mu_o^{\,n}\right\} I_o \tag{2}$$

For n = 1

$$I_1 = \frac{1}{3}\left\{-k + \mu_o\right\} I_o \quad , \qquad \text{which checks.} \tag{3}$$

Therefore to prove (by induction) that the assertion, Equation 2, is correct we will assume that it is correct for n = p and then prove it holds for n = p + 1.

$$I_{p+1} = \frac{\mu_o^{\,p+1}}{2p+3} I_o - \frac{p+1}{2p+3} k I_p \stackrel{?}{=} \frac{1}{2p+3}\left\{\sum_{i=0}^{p} \frac{(-1)^{i+1}(p+1)p(p-1)\cdots(p+1-i)k^{i+1}}{(2p+1)(2p-1)(2p-3)\cdots(2p-2i+1)} \mu_o^{\,p-i} + \mu_o^{\,p+1}\right\} I_o \tag{4}$$

or

$$\frac{\mu_o^{\,p+1}}{2p+3} I_o - \frac{p+1}{2p+3} k \frac{1}{2p+1}\left\{\sum_{i=0}^{p-1} \frac{(-1)^{i+1} p(p-1)\cdots(p-i)k^{i+1}}{(2p-1)(2p-3)\cdots(2p-2i-1)} \mu_o^{\,p-i-1} + \mu_o^{\,p}\right\} I_o \stackrel{?}{=} \frac{1}{2p+3}\{\quad\} I_o \tag{5}$$

Let us work with the left-hand side of the equation, and let i = j - 1

$$\frac{1}{2p+3}\left\{\mu_o^{\,p+1} - \sum_{j=1}^{p} \frac{(-1)^{j}(p+1)p(p-1)\cdots(p-j+1)k^{j+1}}{(2p+1)(2p-1)\cdots(2p-2j+1)} \mu_o^{\,p-j} - \frac{(p+1)k\mu_o^{\,p}}{(2p+1)}\right\} I_o$$

or

$$\frac{1}{2p+3}\left\{\mu_0^{p+1}+\sum_{\gamma=0}^{p}\frac{(-1)^{\gamma+1}(p+1)\,p(p-1)\cdots(p-\gamma+1)}{(2p+1)(2p-1)\cdots(2p-2\gamma+1)}\,k^{\gamma+1}\,\mu_0^{p-\gamma}\right\}I_0 \qquad (6)$$

which agrees with the right-hand side of Equation 5, hence Equation 2 is proven to be correct.

APPENDIX IV - DERIVATION OF EQUATION 19 IN THE TEXT

In Part I of this Appendix I_o is expressed in terms of a binomial series, and a general expression is asserted to be true for I_n. This expression is proven to be true by induction. Equation 19 of the text is then easily obtainedd from this expression.

In Part IIa the results of the integrations I_n as given in Table V are approximated by truncated binomial series. Expressions are then obtained for the definite integrals, ΔI_n, i.e. $I_n\left[\mathcal{M}_o\ (E = d)\right] - I_n\left[\mathcal{M}_o\ (E = c)\right]$. For ΔI_o twelve terms are included in the series, for ΔI_1 there are eleven terms, etc. till only two terms remain for ΔI_{10}. By examining the coefficients one can verify they can be obtained from Equation 19 of the text.

In Part IIb rather than keeping only a finite number of terms in the binomial expansion, the entire series is retained and it is shown that Equation 19 of the text for ΔI_n is correct for n = 0, 1, 2, 3, 4.

Part II (Sections a and b) are not required for the proof of Equation 19 of the text. But it was in this manner that the expression for ΔI_n, which was subsequently proven to be true by induction as shown in Part I of this appendix, was determined. Also it is instructive to see the first few terms in the expansions.

Part I

Rewriting the expression for I_o gives

$$I_o = \sqrt{1 + 2\frac{\mu_o}{A} + \frac{1}{A^2}} = \sqrt{\frac{k}{A}}\sqrt{1 + 2\frac{\mu_o}{k}} \qquad \text{where } k = \frac{A^2+1}{A} \quad (1)$$

The Binomial expansion can be used if $2\frac{\mu_o}{k} < 1$, so

$$I_o = \sqrt{\frac{k}{A}} \sum_{i=0}^{\infty} b_i \left(2\frac{\mu_o}{k}\right)^i, \qquad (2)$$

where

$$b_i = \frac{1}{i!}\left[\tfrac{1}{2}\left(\tfrac{1}{2}-1\right)\left(\tfrac{1}{2}-2\right)\cdots\left(\tfrac{1}{2}-i+1\right)\right], \qquad i = 1,2,3\ldots \quad (3)$$

and $b_0 = 1$

or $$b_i = \frac{(-1)^{i+1}}{i!\,2^i}(2i-3)(2i-5)(2i-7)\cdots 1 \qquad i = 2,3,4\ldots \quad (4)$$

and $b_0 = 1 \;,\; b_1 = \frac{1}{2}$.

We may rewrite Equation 2 as

$$I_o = \sqrt{\frac{k}{A}} \sum_{i=0}^{\infty} a_i \left(\frac{\mu_o}{k}\right)^i \qquad (5)$$

where

$$a_i = (-1)^{i+1}\frac{1}{i!}(2i-3)(2i-5)\cdots 1 \qquad i = 2,3,4\ldots \quad (6)$$

and

$$a_0 = a_1 = 1$$

Now

$$I_n = \frac{\mu_0^n}{(2n+1)} I_0 - \frac{n}{2n+1} k I_{n-1} \tag{7}$$

and the assertion is made that

$$I_n = k^n \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i}{i+n} a_i \left(\frac{\mu_0}{k}\right)^{i+n} + g_n(k) \quad \text{for } n \geq 1 \tag{8}$$

where a_i is defined by Equation 6 and $g_n(k)$ is a constant which depends only on the parameter k.

$$I_1 = k \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i a_i}{i+1} \left(\frac{\mu_0}{k}\right)^{i+1} + g_1(k) \tag{9}$$

It is shown in Part IIb of this Appendix that this is the correct expression for I_1. Therefore to complete the proof that the assertion (Equation 8) is correct, we assume that it holds for n = p and show that it holds for n = p + 1.

$$I_{p+1} = \frac{\mu_0^{p+1}}{2p+3} I_0 - \frac{p+1}{2p+3} k I_p \stackrel{?}{=} k^{p+1} \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i a_i}{i+p+1} \left(\frac{\mu_0}{k}\right)^{i+p+1} + g_{p+1}(k)$$

or

$$\frac{\mu_0^{p+1}}{2p+3} \sqrt{\frac{k}{A}} \sum_{i=0} a_i \left(\frac{\mu_0}{k}\right)^i - \frac{p+1}{2p+3} k^{p+1} \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i a_i}{i+p} \left(\frac{\mu_0}{k}\right)^{i+p} + \frac{p+1}{2p+3} k g_p(k) \stackrel{?}{=} k^{p+1} \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i a_i}{i+p+1} \left(\frac{\mu_0}{k}\right)^{i+p+1} \tag{10}$$

$$+ g_{p+1}(k)$$

We will show that the left-hand side of the equation can be made to equal the right-hand side.

$$\frac{\mu_0^{p+1}}{2p+3}\sqrt{\frac{k}{A}}\,a_0+\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{i=1}\left\{a_i\frac{\mu_0^{i+p+1}}{k^i}-(p+1)k^{p+1}\frac{i}{i+p}a_i\left(\frac{\mu_0}{k}\right)^{i+p}\right\}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

$$\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{i=1}a_i\frac{\mu_0^{i+p+1}}{k^i}-\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{i=2}(p+1)k^{p+1}\frac{i}{i+p}a_i\left(\frac{\mu_0}{k}\right)^{i+p}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

In the second term let i = j + 1

$$\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{i=1}a_i\frac{\mu_0^{i+p+1}}{k^i}-\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{j=1}(p+1)k^{p+1}\frac{j+1}{p+j+1}a_{j+1}\left(\frac{\mu_0}{k}\right)^{p+j+1}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

$$\text{or}\sqrt{\frac{k}{A}}\frac{1}{2p+3}\sum_{i=1}\left\{a_i\frac{\mu_0^{i+p+1}}{k^i}-(p+1)k^{p+1}\frac{(i+1)}{p+i+1}\cdot\frac{(-1)(2i-1)}{(i+1)}a_i\left(\frac{\mu_0}{k}\right)^{p+i+1}\right\}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

$$\sqrt{\frac{k}{A}}\frac{k^{p+1}}{2p+3}\sum_{i=1}\left[1+\frac{(2i-1)(p+1)}{p+i+1}\right]a_i\left(\frac{\mu_0}{k}\right)^{p+i+1}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

$$\sqrt{\frac{k}{A}}\frac{k^{p+1}}{2p+3}\sum_{i=1}\left[\frac{p+i+1+2ip+2i-p-1}{p+i+1}\right]a_i\left(\frac{\mu_0}{k}\right)^{p+i+1}+\frac{p+1}{2p+3}k\,g_p(k)\stackrel{?}{=}$$

$$\sqrt{\frac{k}{A}}\sum_{i=1}\frac{i}{p+i+1}a_i\frac{(\mu_0)^{p+i+1}}{k^i}+\frac{p+1}{2p+3}k\,g_p(k)=$$

which agrees with the right-hand side of Equation 10, and completes the proof.

From Equation 8 we have:

$$\Delta I_n = \sqrt{\frac{k}{A}} \sum_{i=1} \frac{i}{i+n} a_i \frac{\Delta^{i+n}}{k^i} , \tag{11}$$

where a_i is given by Equation 6 of this appendix and ΔI_n and Δ are defined in Equation 19 of the text. Let i = j + 1 then

$$\Delta I_n = \sqrt{\frac{k}{A}} \left\{ \frac{1}{1+n} a_1 \frac{\Delta^{n+1}}{k} + \sum_{j=1} \frac{(j+1)}{j+1+n} \cdot \frac{(-1)^j (2j-1)(2j-3)\cdots 1}{(j+1)\, j!} \frac{\Delta^{n+j+1}}{k^{j+1}} \right\}$$

or

$$\Delta I_n = \frac{1}{k} \sqrt{\frac{k}{A}} \sum_{j=0} (-1)^j C_j \frac{\Delta^{(n+j+1)}}{(n+j+1)} \frac{1}{k^j} = \frac{1}{\sqrt{1+A^2}} \sum_{j=0} (-1)^j C_j \frac{\Delta^{(n+j+1)}}{(n+j+1)} \frac{1}{k^j}$$

where $C_0 = 1$ and $C_j = \dfrac{(2j-1)(2j-3)(2j-5)\cdots 1}{j!}$ for $j \geq 1$

which is Equation 19 of the text.

Part IIa

The equations on the following pages are the binomial expansions (Equation 5) of the integrals given in Table V.

$$I_0=\sqrt{\frac{k}{A}}\left[1+\frac{\mu_0}{2k}-\frac{1}{2}\left(\frac{\mu_0}{2k}\right)^2+\frac{1}{2}\left(\frac{\mu_0}{2k}\right)^3-\frac{5}{8}\left(\frac{\mu_0}{2k}\right)^4+\frac{7}{8}\left(\frac{\mu_0}{2k}\right)^5-\frac{21}{16}\left(\frac{\mu_0}{2k}\right)^6+\frac{33}{16}\left(\frac{\mu_0}{2k}\right)^7-\frac{429}{128}\left(\frac{\mu_0}{2k}\right)^8+\frac{2145}{384}\left(\frac{\mu_0}{2k}\right)^9-\frac{7293}{768}\left(\frac{\mu_0}{2k}\right)^{10}+\frac{12597}{768}\left(\frac{\mu_0}{2k}\right)^{11}-\frac{88179}{3072}\left(\frac{\mu_0}{2k}\right)^{12}+\cdots\right]$$

$$I_1=\sqrt{\frac{k}{A}}\left[\frac{1}{3}\right]\Big[\cancel{\mu_0}+\frac{\mu_0^2}{2k}-\frac{1}{2}\frac{\mu_0^3}{2k^2}+\frac{1}{2}\frac{\mu_0^4}{2k^3}-\frac{5}{8}\frac{\mu_0^5}{2k^4}+\frac{7}{8}\frac{\mu_0^6}{2k^5}-\frac{21}{16}\frac{\mu_0^7}{2k^6}+\frac{33}{16}\frac{\mu_0^8}{2k^7}-\frac{429}{128}\frac{\mu_0^9}{2k^8}+\frac{2145}{384}\frac{\mu_0^{10}}{2k^9}-\frac{7293}{768}\frac{\mu_0^{11}}{2k^{10}}+\frac{12597}{768}\frac{\mu_0^{12}}{2k^{11}}-\frac{88179}{3072}\frac{\mu_0^{13}}{2k^{12}}$$
$$-k-\cancel{\mu_0}+\frac{1}{2}\frac{\mu_0^2}{2k}-\frac{1}{2}\frac{\mu_0^3}{2k^2}+\frac{5}{8}\frac{\mu_0^4}{2k^3}-\frac{7}{8}\frac{\mu_0^5}{2k^4}+\frac{21}{16}\frac{\mu_0^6}{2k^5}-\frac{33}{16}\frac{\mu_0^7}{2k^6}+\frac{429}{128}\frac{\mu_0^8}{2k^7}-\frac{2145}{384}\frac{\mu_0^9}{2k^8}+\frac{7293}{768}\frac{\mu_0^{10}}{2k^9}-\frac{12597}{768}\frac{\mu_0^{11}}{2k^{10}}+\frac{88179}{3072}\frac{\mu_0^{12}}{2k^{11}}-\cdots\Big]$$

$$I_1=\frac{k}{3}\sqrt{\frac{k}{A}}\left[-1+\frac{3}{2}\left(\frac{\mu_0}{2k}\right)^2-\left(\frac{\mu_0}{2k}\right)^3+\frac{9}{8}\left(\frac{\mu_0}{2k}\right)^4-\frac{3}{2}\left(\frac{\mu_0}{2k}\right)^5+\frac{35}{16}\left(\frac{\mu_0}{2k}\right)^6-\frac{27}{8}\left(\frac{\mu_0}{2k}\right)^7+\frac{693}{128}\left(\frac{\mu_0}{2k}\right)^8-\frac{143}{16}\left(\frac{\mu_0}{2k}\right)^9+\frac{3861}{256}\left(\frac{\mu_0}{2k}\right)^{10}-\frac{3315}{128}\left(\frac{\mu_0}{2k}\right)^{11}+\frac{46189}{1024}\left(\frac{\mu_0}{2k}\right)^{12}+\cdots\right]$$

$$I_2=\frac{1}{5}\sqrt{\frac{k}{A}}\Big[\cancel{\mu_0^3}+\frac{\mu_0^3}{2k}-\frac{1}{2}\frac{\mu_0^4}{2k^2}+\frac{1}{2}\frac{\mu_0^5}{2k^3}-\frac{5}{8}\frac{\mu_0^6}{2k^4}+\frac{7}{8}\frac{\mu_0^7}{2k^5}-\frac{21}{16}\frac{\mu_0^8}{2k^6}+\frac{33}{16}\frac{\mu_0^9}{2k^7}-\frac{429}{128}\frac{\mu_0^{10}}{2k^8}+\frac{2145}{384}\frac{\mu_0^{11}}{2k^9}-\frac{7293}{768}\frac{\mu_0^{12}}{2k^{10}}+\frac{12597}{768}\frac{\mu_0^{13}}{2k^{11}}-\cdots$$
$$+\frac{2}{3}k^2-\cancel{\mu_0^2}+\frac{2}{3}\frac{\mu_0^3}{2k}-\frac{3}{4}\frac{\mu_0^4}{2k^2}+\frac{\mu_0^5}{2k^3}-\frac{35}{24}\frac{\mu_0^6}{2k^4}+\frac{9}{4}\frac{\mu_0^7}{2k^5}-\frac{231}{64}\frac{\mu_0^8}{2k^6}+\frac{286}{48}\frac{\mu_0^9}{2k^7}-\frac{1287}{128}\frac{\mu_0^{10}}{2k^8}+\frac{1105}{64}\frac{\mu_0^{11}}{2k^9}-\frac{92378}{3072}\frac{\mu_0^{12}}{2k^{10}}+\cdots\Big]$$

$$I_2=\frac{k^2}{5}\sqrt{\frac{k}{A}}\left[\frac{2}{3}+\frac{5}{3}\left(\frac{\mu_0}{2k}\right)^3-\frac{5}{4}\left(\frac{\mu_0}{2k}\right)^4+\frac{3}{2}\left(\frac{\mu_0}{2k}\right)^5-\frac{50}{24}\left(\frac{\mu_0}{2k}\right)^6+\frac{25}{8}\left(\frac{\mu_0}{2k}\right)^7-\frac{315}{64}\left(\frac{\mu_0}{2k}\right)^8+\frac{385}{48}\left(\frac{\mu_0}{2k}\right)^9-\frac{429}{32}\left(\frac{\mu_0}{2k}\right)^{10}+\frac{2925}{128}\left(\frac{\mu_0}{2k}\right)^{11}-\frac{60775}{1536}\left(\frac{\mu_0}{2k}\right)^{12}+\cdots\right]$$

$$I_3=\frac{1}{7}\sqrt{\frac{k}{A}}\Big[\cancel{\mu_0^3}+\frac{\mu_0^4}{2k}-\frac{1}{2}\frac{\mu_0^5}{2k^2}+\frac{1}{2}\frac{\mu_0^6}{2k^3}-\frac{5}{8}\frac{\mu_0^7}{2k^4}+\frac{7}{8}\frac{\mu_0^8}{2k^5}-\frac{21}{16}\frac{\mu_0^9}{2k^6}+\frac{33}{16}\frac{\mu_0^{10}}{2k^7}-\frac{429}{128}\frac{\mu_0^{11}}{2k^8}+\frac{2145}{384}\frac{\mu_0^{12}}{2k^9}$$
$$-\frac{3}{5}k^3-\cancel{\mu_0^3}+\frac{3}{4}\frac{\mu_0^4}{2k}-\frac{9}{10}\frac{\mu_0^5}{2k^2}+\frac{5}{4}\frac{\mu_0^6}{2k^3}-\frac{15}{8}\frac{\mu_0^7}{2k^4}+\frac{189}{64}\frac{\mu_0^8}{2k^5}-\frac{77}{16}\frac{\mu_0^9}{2k^6}+\frac{1287}{160}\frac{\mu_0^{10}}{2k^7}-\frac{1755}{128}\frac{\mu_0^{11}}{2k^8}+\frac{12155}{512}\frac{\mu_0^{12}}{2k^9}+\cdots\Big]$$

$$I_3=\frac{k^3}{7}\sqrt{\frac{k}{A}}\left[-\frac{2}{5}+\frac{7}{4}\left(\frac{\mu_0}{2k}\right)^4-\frac{7}{5}\left(\frac{\mu_0}{2k}\right)^5+\frac{7}{4}\left(\frac{\mu_0}{2k}\right)^6-\frac{5}{2}\left(\frac{\mu_0}{2k}\right)^7+\frac{245}{64}\left(\frac{\mu_0}{2k}\right)^8-\frac{49}{8}\left(\frac{\mu_0}{2k}\right)^9+\frac{1617}{160}\left(\frac{\mu_0}{2k}\right)^{10}-\frac{273}{16}\left(\frac{\mu_0}{2k}\right)^{11}+\frac{15015}{512}\left(\frac{\mu_0}{2k}\right)^{12}-\cdots\right]$$

$$I_4=\frac{1}{9}\sqrt{\frac{k}{A}}\Big[\cancel{\mu_0^4}+\frac{\mu_0^5}{2k}-\frac{1}{2}\frac{\mu_0^6}{2k^2}+\frac{1}{2}\frac{\mu_0^7}{2k^3}-\frac{5}{8}\frac{\mu_0^8}{2k^4}+\frac{7}{8}\frac{\mu_0^9}{2k^5}-\frac{21}{16}\frac{\mu_0^{10}}{2k^6}+\frac{33}{16}\frac{\mu_0^{11}}{2k^7}-\frac{429}{128}\frac{\mu_0^{12}}{2k^8}$$
$$+\frac{8}{35}k^4-\cancel{\mu_0^4}+\frac{4}{5}\frac{\mu_0^5}{2k}-\frac{\mu_0^6}{2k^2}+\frac{10}{7}\frac{\mu_0^7}{2k^3}-\frac{35}{16}\frac{\mu_0^8}{2k^4}+\frac{7}{2}\frac{\mu_0^9}{2k^5}-\frac{231}{40}\frac{\mu_0^{10}}{2k^6}+\frac{39}{4}\frac{\mu_0^{11}}{2k^7}-\frac{2145}{128}\frac{\mu_0^{12}}{2k^8}+\cdots\Big]$$

$$I_4=\frac{k^4}{9}\sqrt{\frac{k}{A}}\left[\frac{8}{35}+\frac{9}{5}\left(\frac{\mu_0}{2k}\right)^5-\frac{3}{2}\left(\frac{\mu_0}{2k}\right)^6+\frac{27}{14}\left(\frac{\mu_0}{2k}\right)^7-\frac{45}{16}\left(\frac{\mu_0}{2k}\right)^8+\frac{35}{8}\left(\frac{\mu_0}{2k}\right)^9-\frac{567}{80}\left(\frac{\mu_0}{2k}\right)^{10}+\frac{189}{16}\left(\frac{\mu_0}{2k}\right)^{11}-\frac{1287}{64}\left(\frac{\mu_0}{2k}\right)^{12}+\cdots\right]$$

$$I_5 = \frac{1}{11}\sqrt{\frac{k}{A}}\left[\cancel{\frac{5}{0}} + \frac{\mu_0^6}{k^6} - \frac{1}{2}\frac{\mu_0^7}{k^7} + \frac{1}{2}\frac{\mu_0^8}{k^8} - \frac{5}{8}\frac{\mu_0^9}{k^9} + \frac{7}{8}\frac{\mu_0^{10}}{k^{10}} - \frac{21}{16}\frac{\mu_0^{11}}{k^{11}} + \frac{33}{16}\frac{\mu_0^{12}}{k^{12}}\right.$$

$$\left. - \frac{8}{63}k^5 - \cancel{\mu_0^5} + \frac{5}{6}\frac{\mu_0^6}{k^6} - \frac{15}{14}\frac{\mu_0^7}{k^7} + \frac{25}{16}\frac{\mu_0^8}{k^8} - \frac{175}{72}\frac{\mu_0^9}{k^9} + \frac{63}{16}\frac{\mu_0^{10}}{k^{10}} - \frac{105}{16}\frac{\mu_0^{11}}{k^{11}} + \frac{715}{64}\frac{\mu_0^{12}}{k^{12}} - \cdots\right]$$

$$\underline{I_5 = \frac{k^5}{11}\sqrt{\frac{k}{A}}\left[-\frac{8}{63} + \frac{11}{6}\left(\frac{\mu_0}{k}\right)^6 - \frac{11}{7}\left(\frac{\mu_0}{k}\right)^7 + \frac{33}{16}\left(\frac{\mu_0}{k}\right)^8 - \frac{55}{18}\left(\frac{\mu_0}{k}\right)^9 + \frac{77}{16}\left(\frac{\mu_0}{k}\right)^{10} - \frac{63}{8}\left(\frac{\mu_0}{k}\right)^{11} + \frac{847}{64}\left(\frac{\mu_0}{k}\right)^{12} - \cdots\right]}$$

$$I_6 = \frac{1}{13}\sqrt{\frac{k}{A}}\left[\cancel{\mu_0^6} + \frac{\mu_0^7}{k^7} - \frac{1}{2}\frac{\mu_0^8}{k^8} + \frac{1}{2}\frac{\mu_0^9}{k^9} - \frac{5}{8}\frac{\mu_0^{10}}{k^{10}} + \frac{7}{8}\frac{\mu_0^{11}}{k^{11}} - \frac{21}{16}\frac{\mu_0^{12}}{k^{12}}\right.$$

$$\left. + \frac{16}{231}k^6 - \cancel{\mu_0^6} + \frac{6}{7}\frac{\mu_0^7}{k^7} - \frac{9}{8}\frac{\mu_0^8}{k^8} + \frac{5}{3}\frac{\mu_0^9}{k^9} - \frac{21}{8}\frac{\mu_0^{10}}{k^{10}} + \frac{189}{44}\frac{\mu_0^{11}}{k^{11}} - \frac{231}{32}\frac{\mu_0^{12}}{k^{12}} + \cdots\right]$$

$$\underline{I_6 = \frac{k^6}{13}\sqrt{\frac{k}{A}}\left[\frac{16}{231} + \frac{13}{7}\left(\frac{\mu_0}{k}\right)^7 - \frac{13}{8}\left(\frac{\mu_0}{k}\right)^8 + \frac{13}{6}\left(\frac{\mu_0}{k}\right)^9 - \frac{13}{4}\left(\frac{\mu_0}{k}\right)^{10} + \frac{455}{88}\left(\frac{\mu_0}{k}\right)^{11} - \frac{273}{32}\left(\frac{\mu_0}{k}\right)^{12} + \cdots\right]}$$

$$I_7 = \frac{1}{15}\sqrt{\frac{k}{A}}\left[\cancel{\mu_0^7} + \frac{\mu_0^8}{k^8} - \frac{1}{2}\frac{\mu_0^9}{k^9} + \frac{1}{2}\frac{\mu_0^{10}}{k^{10}} - \frac{5}{8}\frac{\mu_0^{11}}{k^{11}} + \frac{7}{8}\frac{\mu_0^{12}}{k^{12}} - \cdots\right.$$

$$\left. - \frac{7}{33}k^7 - \cancel{\mu_0^7} + \frac{7}{8}\frac{\mu_0^8}{k^8} - \frac{7}{6}\frac{\mu_0^9}{k^9} + \frac{7}{4}\frac{\mu_0^{10}}{k^{10}} - \frac{245}{88}\frac{\mu_0^{11}}{k^{11}} + \frac{147}{32}\frac{\mu_0^{12}}{k^{12}} - \cdots\right]$$

$$\underline{I_7 = \frac{k^7}{15}\sqrt{\frac{k}{A}}\left[-\frac{7}{33} + \frac{15}{8}\left(\frac{\mu_0}{k}\right)^8 - \frac{5}{3}\left(\frac{\mu_0}{k}\right)^9 + \frac{9}{4}\left(\frac{\mu_0}{k}\right)^{10} - \frac{75}{22}\left(\frac{\mu_0}{k}\right)^{11} + \frac{175}{32}\left(\frac{\mu_0}{k}\right)^{12} - \cdots\right]}$$

$$I_8 = \frac{1}{17}\sqrt{\frac{k}{A}}\left[\cancel{\mu_0^8} + \frac{\mu_0^9}{k^9} - \frac{1}{2}\frac{\mu_0^{10}}{k^{10}} + \frac{1}{2}\frac{\mu_0^{11}}{k^{11}} - \frac{5}{8}\frac{\mu_0^{12}}{k^{12}} + \cdots\right.$$

$$\left. + \frac{56}{495}k^8 - \cancel{\mu_0^8} + \frac{8}{9}\frac{\mu_0^9}{k^9} - \frac{6}{5}\frac{\mu_0^{10}}{k^{10}} + \frac{20}{11}\frac{\mu_0^{11}}{k^{11}} - \frac{35}{12}\frac{\mu_0^{12}}{k^{12}} + \cdots\right]$$

$$\underline{I_8 = \frac{k^8}{17}\sqrt{\frac{k}{A}}\left[\frac{56}{495} + \frac{17}{9}\left(\frac{\mu_0}{k}\right)^9 - \frac{17}{10}\left(\frac{\mu_0}{k}\right)^{10} + \frac{51}{22}\left(\frac{\mu_0}{k}\right)^{11} - \frac{85}{24}\left(\frac{\mu_0}{k}\right)^{12} + \cdots\right]}$$

$$I_9 = \frac{1}{19}\sqrt{\frac{k}{A}}\left[\cancel{\mu_0^9} + \frac{\mu_0^{10}}{k^{10}} - \frac{1}{2}\frac{\mu_0^{11}}{k^{11}} + \frac{1}{2}\frac{\mu_0^{12}}{k^{12}} - \cdots\right.$$

$$\left. - \frac{56}{935}k^9 - \cancel{\mu_0^9} + \frac{9}{10}\frac{\mu_0^{10}}{k^{10}} - \frac{27}{22}\frac{\mu_0^{11}}{k^{11}} + \frac{45}{24}\frac{\mu_0^{12}}{k^{12}} - \cdots\right]$$

$$\underline{I_9 = \frac{k^9}{19}\sqrt{\frac{k}{A}}\left[-\frac{56}{935} + \frac{19}{10}\left(\frac{\mu_0}{k}\right)^{10} - \frac{19}{11}\left(\frac{\mu_0}{k}\right)^{11} + \frac{19}{8}\left(\frac{\mu_0}{k}\right)^{12} - \cdots\right]}$$

$$I_{10} = \frac{1}{21}\sqrt{\frac{k}{A}}\left[\cancel{\mu_0^{10}} + \frac{\mu_0^{11}}{k^{11}} - \frac{1}{2}\frac{\mu_0^{12}}{k^{12}} + \cdots + \frac{112}{3553}k^{10} - \cancel{\mu_0^{10}} + \frac{10}{11}\frac{\mu_0^{11}}{k^{11}} - \frac{5}{4}\frac{\mu_0^{12}}{k^{12}} + \cdots\right]$$

$$\underline{I_{10} = \frac{k^{10}}{21}\sqrt{\frac{k}{A}}\left[\frac{112}{3553} + \frac{21}{11}\left(\frac{\mu_0}{k}\right)^{11} - \frac{7}{4}\left(\frac{\mu_0}{k}\right)^{12} + \cdots\right]}$$

Let $\Delta^n \equiv M_0^n(d) - M_0^n(c)$ where $M_0(d)$ is the upper limit of integration over E and $M_0(c)$ is the lower,

Also let $\Delta I_i \equiv I_i[M_0(d)] - I_i[M_0(c)]$. Then:

$$\Delta I_0 = \frac{1}{\sqrt{1+A^2}}\left\{\Delta - \frac{\Delta^2}{2k} + \frac{\Delta^3}{2k^2} - \frac{5\Delta^4}{8k^3} + \frac{7\Delta^5}{8k^4} - \frac{21\Delta^6}{16k^5} + \frac{33\Delta^7}{16k^6} - \frac{429}{128}\frac{\Delta^8}{k^7} + \frac{2145}{384}\frac{\Delta^9}{k^8} - \frac{7293}{768}\frac{\Delta^{10}}{k^9} + \frac{12597}{768}\frac{\Delta^{11}}{k^{10}} - \frac{88179}{3072}\frac{\Delta^{12}}{k^{11}} + \cdots\right\}$$

$$\Delta I_1 = \frac{1}{3}\frac{1}{\sqrt{1+A^2}}\left\{\frac{3}{2}\Delta^2 - \frac{\Delta^3}{k} + \frac{9\Delta^4}{8k^2} - \frac{3\Delta^5}{2k^3} + \frac{35\Delta^6}{16k^4} - \frac{27\Delta^7}{8k^5} + \frac{693}{128}\frac{\Delta^8}{k^6} - \frac{143}{16}\frac{\Delta^9}{k^7} + \frac{3861}{256}\frac{\Delta^{10}}{k^8} - \frac{3315}{128}\frac{\Delta^{11}}{k^9} + \frac{46189}{1024}\frac{\Delta^{12}}{k^{10}} - \cdots\right\}$$

$$\Delta I_2 = \frac{1}{5}\frac{1}{\sqrt{1+A^2}}\left\{\frac{5\Delta^3}{3} - \frac{5\Delta^4}{4k} + \frac{3\Delta^5}{2k^2} - \frac{25}{12}\frac{\Delta^6}{k^3} + \frac{25}{8}\frac{\Delta^7}{k^4} - \frac{315}{64}\frac{\Delta^8}{k^5} + \frac{385}{48}\frac{\Delta^9}{k^6} - \frac{429}{32}\frac{\Delta^{10}}{k^7} + \frac{2925}{128}\frac{\Delta^{11}}{k^8} - \frac{60775}{1536}\frac{\Delta^{12}}{k^9} + \cdots\right\}$$

$$\Delta I_3 = \frac{1}{7}\frac{1}{\sqrt{1+A^2}}\left\{\frac{7\Delta^4}{4} - \frac{7}{5}\frac{\Delta^5}{k} + \frac{7}{4}\frac{\Delta^6}{k^2} - \frac{5}{2}\frac{\Delta^7}{k^3} + \frac{245}{64}\frac{\Delta^8}{k^4} - \frac{49}{8}\frac{\Delta^9}{k^5} + \frac{1617}{160}\frac{\Delta^{10}}{k^6} - \frac{273}{16}\frac{\Delta^{11}}{k^7} + \frac{15015}{512}\frac{\Delta^{12}}{k^8} - \cdots\right\}$$

$$\Delta I_4 = \frac{1}{9}\frac{1}{\sqrt{1+A^2}}\left\{\frac{9\Delta^5}{5} - \frac{3}{2}\frac{\Delta^6}{k} + \frac{27}{14}\frac{\Delta^7}{k^2} - \frac{45}{16}\frac{\Delta^8}{k^3} + \frac{35}{8}\frac{\Delta^9}{k^4} - \frac{567}{80}\frac{\Delta^{10}}{k^5} + \frac{189}{16}\frac{\Delta^{11}}{k^6} - \frac{1287}{64}\frac{\Delta^{12}}{k^7} + \cdots\right\}$$

$$\Delta I_5 = \frac{1}{11}\frac{1}{\sqrt{1+A^2}}\left\{\frac{11\Delta^6}{6} - \frac{11}{7}\frac{\Delta^7}{k} + \frac{33}{16}\frac{\Delta^8}{k^2} - \frac{55}{18}\frac{\Delta^9}{k^3} + \frac{77}{16}\frac{\Delta^{10}}{k^4} - \frac{63}{8}\frac{\Delta^{11}}{k^5} + \frac{847}{64}\frac{\Delta^{12}}{k^6} - \cdots\right\}$$

$$\Delta I_6 = \frac{1}{13}\frac{1}{\sqrt{1+A^2}}\left\{\frac{13\Delta^7}{7} - \frac{13}{8}\frac{\Delta^8}{k} + \frac{13}{6}\frac{\Delta^9}{k^2} - \frac{13}{4}\frac{\Delta^{10}}{k^3} + \frac{455}{88}\frac{\Delta^{11}}{k^4} - \frac{273}{32}\frac{\Delta^{12}}{k^5} + \cdots\right\}$$

$$\Delta I_7 = \frac{1}{15}\frac{1}{\sqrt{1+A^2}}\left\{\frac{15\Delta^8}{8} - \frac{5}{3}\frac{\Delta^9}{k} + \frac{9}{4}\frac{\Delta^{10}}{k^2} - \frac{75}{22}\frac{\Delta^{11}}{k^3} + \frac{175\Delta^{12}}{32k^4} - \cdots\right\}$$

(Recall $\Delta^2 \neq \Delta\cdot\Delta$)

$$\Delta I_8 = \frac{1}{17}\frac{1}{\sqrt{1+A^2}}\left\{\frac{17\Delta^9}{9} - \frac{17}{10}\frac{\Delta^{10}}{k} + \frac{51}{22}\frac{\Delta^{11}}{k^2} - \frac{85}{24}\frac{\Delta^{12}}{k^3} + \cdots\right\}$$

$$\Delta I_9 = \frac{1}{19}\frac{1}{\sqrt{1+A^2}}\left\{\frac{19\Delta^{10}}{10} - \frac{19}{11}\frac{\Delta^{11}}{k} + \frac{19}{8}\frac{\Delta^{12}}{k^2} - \cdots\right\}$$

$$\Delta I_{10} = \frac{1}{21}\frac{1}{\sqrt{1+A^2}}\left\{\frac{21\Delta^{11}}{11} - \frac{7}{4}\frac{\Delta^{12}}{k} + \cdots\right\}$$

By Observation:

$$\Delta I_n = \frac{1}{2n+1}\frac{1}{\sqrt{1+A^2}}\left\{\frac{2n+1}{n+1}\frac{\Delta^{n+1}}{1} - \frac{2n+1}{n+2}\frac{\Delta^{n+2}}{k} + \frac{3(2n+1)}{2(n+3)}\frac{\Delta^{n+3}}{k^2} - \frac{5}{2}\frac{(2n+1)}{(n+4)}\frac{\Delta^{n+4}}{k^3} + \frac{35(2n+1)}{8(n+5)}\frac{\Delta^{n+5}}{k^4} - \frac{63(2n+1)}{8(n+6)}\frac{\Delta^{n+6}}{k^5} + \frac{231}{16}\frac{(2n+1)}{(n+7)}\frac{\Delta^{n+7}}{k^6} - \frac{429}{16}\frac{(2n+1)}{(n+8)}\frac{\Delta^{n+8}}{k^7} + \cdots\right\}$$

OR:

$$\Delta I_n = \frac{1}{\sqrt{1+A^2}}\left\{\frac{\Delta^{n+1}}{n+1} - \frac{\Delta^{n+2}}{(n+2)}\frac{1}{k} + \frac{3}{2}\frac{\Delta^{n+3}}{(n+3)}\frac{1}{k^2} - \frac{5}{2}\frac{\Delta^{n+4}}{(n+4)}\frac{1}{k^3} + \frac{35}{8}\frac{\Delta^{n+5}}{(n+5)}\frac{1}{k^4} - \frac{63}{8}\frac{\Delta^{n+6}}{(n+6)}\frac{1}{k^5} + \frac{231}{16}\frac{\Delta^{n+7}}{(n+7)}\frac{1}{k^6} - \frac{429}{16}\frac{\Delta^{n+8}}{(n+8)}\frac{1}{k^7} + \cdots\right\}$$

Part IIb

Rather than use a truncated series let us retain all the terms of Equation 5. Then

$$\Delta I_o = \sqrt{\frac{k}{A}} \sum_{i=1}^{\infty} a_i \frac{\Delta^i}{k^i}$$

Let $i = j+1$ Then

$$\Delta I_o = \sqrt{\frac{k}{A}} \left\{ \frac{\Delta}{k} + \sum_{j=1} a_{j+1} \frac{\Delta^{j+1}}{k^{j+1}} \right\}$$

$$\Delta I_o = \sqrt{\frac{k}{A}} \frac{1}{k} \left\{ \Delta + \sum_{j=1}^{j=1} (-1)^{j+2} \frac{(2j-1)(2j-3)(2j-5)\cdots 1}{(j+1)\,j!} \frac{\Delta^{j+1}}{k^j} \right\}$$

OR $$\Delta I_o = \frac{1}{k}\sqrt{\frac{k}{A}} \sum_{j=0} (-1)^j c_j \frac{\Delta^{j+1}}{(j+1)} \frac{1}{k^j}$$

WHERE $c_0 = 1$, AND $c_j = \dfrac{(2j-1)(2j-3)(2j-5)\cdots 1}{j!}$ $\quad j \geq 1$.

$$I_1 = \frac{\mu_0}{3} I_o - \frac{k}{3} I_o = \frac{1}{3}\sqrt{\frac{k}{A}} \sum_{i=0} \left(\mu_0 a_i \xi^i - k a_i \xi^i \right), \quad \mu_0 = k\xi,$$

OR $$I_1 = \frac{k}{3}\sqrt{\frac{k}{A}} \sum_{i=0} a_i \left(\xi^{i+1} - \xi^i \right),$$

WHERE $a_i = (-1)^{i+1} \dfrac{(2i-3)(2i-5)\cdots 1}{i!}$ $\quad i = 2,3,4\cdots$ AND $a_0 = a_1 = 1$

$$\left.\begin{array}{l} a_0 \xi^1 - a_0 \\ a_1 \xi^2 - a_1 \xi^1 \\ a_2 \xi^3 - a_2 \xi^2 \\ a_3 \xi^4 - a_3 \xi^3 \end{array}\right\} (a_i - a_{i+1}) \xi^{i+1} \quad i \geq 1$$

$$\therefore\ I_1 = \frac{k}{3}\sqrt{\frac{k}{A}} \left\{ -a_0 + \sum_{i=1} (a_i - a_{i+1}) \xi^{i+1} \right\}$$

Now $(a_i - a_{i+1}) = \left[1 - (-1)\frac{2i-1}{i+1}\right] a_i \qquad i \geq 2$

$= \left[\frac{i+1+2i-1}{i+1}\right] a_i = \frac{3i}{i+1} a_i \qquad i \geq 2$

And $a_1 - a_2 = 1 + \frac{1}{2} = \frac{3}{2} \qquad i = 1$

So $I_1 = \frac{k}{3}\sqrt{\frac{k}{A}}\left\{-a_0 + \frac{3}{2}\xi^2 + \sum_{i=2}^{1} \frac{3i}{i+1} a_i \xi^{i+1}\right\}$ which can be compared with Eq 9 of this Appendix.

Putting In The Limits: $\Delta I_1 = \sqrt{\frac{k}{A}}\left\{\frac{\Delta^2}{2k} + \sum_{i=2}^{} \frac{i}{i+1} a_i \frac{\Delta^{i+1}}{k^i}\right\}$.

Let $i = j + 1$, then

$$\Delta I_1 = \sqrt{\frac{k}{A}}\left\{\frac{\Delta^2}{2k} + \sum_{j=1} \frac{j+1}{j+2} (-1)^{j+2} \frac{(2j-1)(2j-3)\cdots 1}{(j+1)\, j!} \frac{\Delta^{j+2}}{k^{j+1}}\right\}$$

OR $\Delta I_1 = \frac{1}{k}\sqrt{\frac{k}{A}} \sum_{j=0} (-1)^j C_j \frac{\Delta^{j+2}}{(j+2)} \frac{1}{k^j}$. C_j has been derived.

$$I_2 = \frac{\mu_0^2}{5} I_0 - \frac{2}{5} k I_1$$

$$I_2 = \frac{k^2}{5}\sqrt{\frac{k}{A}} \sum_{i=0} a_i \xi^{i+2} - \frac{2}{5}\left\{-a_0 \frac{k^2}{3}\sqrt{\frac{k}{A}} + \frac{k^2}{2}\sqrt{\frac{k}{A}}\xi^2 + k^2\sqrt{\frac{k}{A}} \sum_{i=2} \frac{i}{i+1} a_i \xi^{i+1}\right\}$$

$$I_2 = \frac{k^2}{5}\sqrt{\frac{k}{A}}\left\{\frac{2}{3}a_0 + \xi^2 + \sum_{i=1} \left(a_i \xi^{i+2} - \frac{2i}{i+1} a_i \xi^{i+1}\right)\right\}$$

$\frac{2}{3} a_0 + \xi^2$

$a_1 \xi^3 - a_1 \xi^2$

$a_2 \xi^4 - \frac{4}{3} a_2 \xi^3$

$a_3 \xi^5 - \frac{6}{4} a_3 \xi^4$

$\Big\}\ (a_i - \frac{2i+2}{i+2} a_{i+1}) \xi^{i+2}_{j} \quad i \geq 1$

$$I_2 = \frac{k^2}{5}\sqrt{\frac{k}{A}}\left\{\frac{2}{3}a_0 + \sum_{i=1}\left(a_i - \frac{2i+2}{i+2}a_{i+1}\right)\xi^{i+2}\right\}$$

$$i \geq 2 : \quad a_i - \frac{2i+2}{i+2}a_{i+1} = \left(1 + \frac{2(i+1)(2i-1)}{(i+2)(i+1)}\right)a_i = \frac{i+2+4i-2}{i+2}a_i$$

$$i = 1 : \quad = 1 - \frac{4}{3}\frac{1}{2} = \frac{5}{3}$$

$$\text{So} \quad I_2 = k^2\sqrt{\frac{k}{A}}\left\{\sum_{i=1}\frac{i}{i+2}a_i\,\xi^{i+2} + \frac{2}{15}a_0\right\}$$

$$\Delta I_2 = \sqrt{\frac{k}{A}}\sum_{i=1}\frac{i}{i+2}a_i\frac{\Delta^{i+2}}{k^i}$$

Let $i = j+1$

$$\Delta I_2 = \sqrt{\frac{k}{A}}\left\{\frac{1}{3}\frac{\Delta^3}{k} + \sum_{j=1}\frac{j+1}{j+3}(-1)^{j+2}\frac{(2j-1)(2j-3)\cdots 1}{(j+1)\,j!}\frac{\Delta^{j+3}}{k^{j+1}}\right\}$$

$$\Delta I_2 = \frac{1}{k}\sqrt{\frac{k}{A}}\sum_{j=0}(-1)^j c_j\frac{\Delta^{j+3}}{(j+3)}\frac{1}{k^j}$$

$$I_3 = \frac{\mu_0^3}{7}I_0 - \frac{3}{7}kI_2$$

$$I_3 = \frac{k^3}{7}\sqrt{\frac{k}{A}}\sum_{i=0}a_i\xi^{i+3} - \frac{3}{7}k^3\sqrt{\frac{k}{A}}\sum_{i=1}\left\{\frac{2}{15}a_0 + \frac{i}{i+2}a_i\xi^{i+2}\right\}$$

$$I_3 = \frac{k^3}{7}\sqrt{\frac{k}{A}}\sum_{i=1}\left\{\xi^3 + a_1\xi^{i+3} - \frac{3i}{i+2}a_i\xi^{i+2}\right\} - \frac{2}{35}k^3\sqrt{\frac{k}{A}}a_0$$

$$\left.\begin{array}{l} a_0\xi^3 \\ a_1\xi^4 - a_1\xi^3 \\ a_2\xi^5 - \frac{6}{4}a_2\xi^4 \\ a_3\xi^6 - \frac{9}{5}a_3\xi^5 \end{array}\right\} \left(a_i - \frac{3i+3}{i+3}a_{i+1}\right)\xi^{i+3}, \quad i \geq 1$$

$$I_3 = \frac{k^3}{7}\sqrt{\frac{k}{A}}\sum_{i=1}\left(1 + \frac{3i+3}{i+3}\frac{2i-1}{i+1}\right)a_i\xi^{i+3} - \frac{2}{35}k^3\sqrt{\frac{k}{A}}a_0$$

$$1+\frac{(3i+3)(2i-1)}{(i+3)(i+1)}=\frac{i^2+4i+3+6i^2+3i-3}{(i+3)(i+1)}=\frac{7i}{(i+3)}$$

$$I_3=h^3\sqrt{\frac{h}{A}}\sum_{i=1}\frac{i}{i+3}a_i\xi^{i+3}-\frac{2}{35}h^3\sqrt{\frac{h}{A}}\quad\text{so}\quad\Delta I_3=\sqrt{\frac{h}{A}}\sum_{i=1}\frac{i}{i+3}a_i\frac{\Delta^{i+3}}{h^{i+3}}$$

$$\text{Let } i=j+1:\ \Delta I_3=\sqrt{\frac{h}{A}}\left[\frac{1}{4}\frac{\Delta^4}{h^4}+\sum_{j=1}\frac{j+1}{j+4}(-1)^j\frac{(2j-1)(2j-3)\cdots1}{j!}\frac{\Delta^{j+4}}{h^{j+4}}\right]$$

$$\text{OR}\quad\Delta I_3=\frac{1}{4}h\sqrt{\frac{h}{A}}\sum_{j=0}(-1)^j\frac{\Delta^{j+4}}{(j+4)}\frac{1}{h^4}C_j$$

$$I_4=\frac{\mu_0}{9}^4I_0-\frac{4}{9}hI_3$$

$$I_4=\frac{h^4}{9}\sqrt{\frac{h}{A}}\sum_{i=0}a_i\xi^{i+4}-\frac{4}{9}h\left\{h^3\sqrt{\frac{h}{A}}\sum_{i=1}\frac{i}{i+3}a_i\xi^{i+3}-\frac{2}{35}h^3\sqrt{\frac{h}{A}}\right\}$$

$$I_4=\frac{h^4}{9}\sqrt{\frac{h}{A}}\left\{\xi^4+\sum_{i=1}\left(a_i\xi^{i+4}-\frac{4i}{i+3}a_i\xi^{i+3}\right)+\frac{8}{35}\right\}$$

$$\left.\begin{array}{l}\cancel{\xi^4}\\ a_1\xi^5-\cancel{a_1\xi^4}\\ a_2\xi^6-\frac{8}{5}a_2\xi^5\\ a_3\xi^7-\frac{12}{6}a_3\xi^6\end{array}\right\}\left(a_i-\frac{4i+4}{i+4}a_{i+1}\right)\xi^{i+4}$$

$$I_4=\frac{h^4}{9}\sqrt{\frac{h}{A}}\sum_{i=1}\left(1+\frac{4i+4}{i+4}\frac{2i-1}{i+1}\right)a_i\xi^{i+4}+\frac{8}{9\cdot35}h^4\sqrt{\frac{h}{A}}$$

$$1+\frac{(4i+4)(2i-1)}{(i+4)(i+1)}=\frac{i^2+5i+4+8i^2+4i-4}{(i+4)(i+1)}=\frac{9i}{i+4}$$

$$I_4=h^4\sqrt{\frac{h}{A}}\sum_{i=1}\frac{i}{i+4}a_i\xi^{i+4}+\frac{8}{315}h^4\sqrt{\frac{h}{A}}$$

$$\Delta I_4 = \sqrt{\frac{h}{A}} \sum_{i=1} \frac{i}{i+4} a_i \frac{\Delta^{i+4}}{h^i}$$

Let $i = j+1$

$$\Delta I_4 = \sqrt{\frac{h}{A}} \left\{ \frac{1}{5} \frac{\Delta^5}{h} + \sum_{j=1} \frac{(j+1)}{j+5} (-1)^j \frac{(2j-1)(2j-3)\cdots 1}{j!\,(j+1)} \left(\frac{\Delta^{j+5}}{h^{j+1}} \right) \right\}$$

OR

$$\Delta I_4 = \frac{1}{h} \sqrt{\frac{h}{A}} \sum_{j=0} (-1)^j C_j \frac{\Delta^{j+5}}{j+5} \frac{1}{h^j}$$

⋮

etc

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REFERENCES

(1) G. A. Long and C. A. Stevens, WANL-TME-689, DOODLES: A Fortran Program for Computing Elastic Transfer Matrices for GAM-I Program (1963).

(2) e.g. C. W. Craven, Jr. and J. L. Lucius, ORNL-TM-1537, TRANSFER: A Program to Calculate P_n Multigroup Scattering Matrices (1966).

(3) E. Jahnke and F. Emde, Tables of Functions, Dover Publications, New York (1945).

(4) e.g. See A. M. Weinberg and E. P. Wigner, The Physical Theory of Neutron Chain Reactors, University of Chicago Press, Chicago, Illinois (1958).