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THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME II

BUCKLING OF CURVED ISOTROPIC SKIN PANELS; AXIAL COMPRESSION

GENERAL DYNAMICS
Convair Division

93 Pages

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VOLUME II

BUCKLING OF CURVED ISOTROPIC SKIN PANELS; AXIAL COMPRESSION

Prepared for the
GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama

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During the overall effort, programming for the digital computer was accomplished mainly by Mrs. L. S. Fossum, Mrs. E. A. Muscha, and Mrs. N. L. Fraser, all of the Technical Programming Group. Mr. J. R. Anderson of the Guidance and Trajectory Programming Group also contributed.

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All six volumes of this report were typed by Mrs. F. C. Jaeger of the Convair Structural Analysis Group.

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ABSTRACT

This is the second of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of documents was prepared under NASA Contract NAS8-11181. This particular volume presents a practical method for the determination of design values for the critical buckling stresses of axially compressed, curved isotropic skin panels having various degrees of edge restraint. For wide panels which behave essentially as full cylinders, the method provides lower-bound predictions. As the panel width decreases, a transitional relationship is employed which, for narrow panels, approaches standard flat plate values. In order to facilitate ready application, this volume includes several groups of design curves as well as a digital computer program which provides either automatic plotting or single-point solutions, as desired.

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DEFINITION OF SYMBOLS

Symbol	Definition
a	Axial length of curved isotropic skin panel; Spacing between rings.
b	Circumferential arc length of curved isotropic skin panel; Spacing between stringers.
C	Factor defined by equation (2-8).
E	Young's modulus.
K	Buckling coefficient for a flat plate.
K_c	Buckling coefficient for an isotropic flat plate having the loaded edges simply supported and the unloaded edges fully clamped.
K_s	Buckling coefficient for an isotropic flat plate having all edges simply supported.
R	Radius to middle surface of basic cylindrical skin.
t	Skin thickness.
ν	Poisson's ratio.
σ_{CL}	Classical theoretical value for the compressive buckling stress of an isotropic cylinder $\left(= .606 \frac{Et}{R} \right)$.
σ_{cr}	Critical compressive stress.
σ_p	Critical compressive stress for the buckling of an isotropic flat plate.
σ_R	Critical compressive stress for the buckling of an isotropic cylinder.

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SECTION 1

INTRODUCTION

This is the second of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of volumes was prepared under NASA Contract NAS8-11181. This particular volume deals with the buckling of the curved isotropic skin panels which are bounded by stringers and rings. Although the buckling of these skin panels is usually not catastrophic, the related critical stress values must be established in order to compute effective skin widths for use in the investigation of other modes of instability such as so-called panel instability (see GLOSSARY, Volume I [1]) and general instability (see GLOSSARY, Volume I [1]). In addition, although the overall stiffened cylinder can usually continue to support loading beyond the point at which the isotropic skins buckle, this condition will sometimes be undesirable from other viewpoints (fatigue, excessive deformation, etc.)

Numbers in brackets [ ] in the text denote references listed in SECTION 6.

SECTION 2

EQUATIONS

2.1 GENERAL

The procedures of this volume are based on the Schapitz [2] criterion for the buckling of curved isotropic skin panels. This criterion supplies a means for evaluating the effects of skin panel geometry as the transition is made from wide panels behaving essentially as full cylinders to smaller panels which approach the behavior of flat plates. In the former case the fixity effects at the longitudinal edges are negligible. However, for the narrow skin panels this fixity has a marked influence on the critical buckling stress.

In the application of the Schapitz criterion, three different empirical foundations were considered for the case where the longitudinal stiffeners are widely spaced. One of these results in typical strength predictions which would primarily be of use in making comparisons against test data. The other two give design levels of high reliability recognizing the wide scatter which exists in actual strengths. The recommended design procedure was selected from these latter two alternatives and is consistent with the isotropic cylinder criterion of reference 3.

It is helpful to note that the character of flat plate buckling is much different from that exhibited by wide curved skin panels and isotropic cylinders. The flat plate can continue to support steadily increasing external loads well into the postbuckling region. This is in contrast to the sudden drop-off in load usually observed for wide skin panels and isotropic cylinders. Consequently, the Schapitz criterion [2] utilizes full theoretical predictions as the boundary case of a flat plate is approached. Therefore, within this region, one might expect that the test data will display some small degree of scatter on both sides of the proposed design curves. However, because of the physical behavior cited above, this generally will not lead to any serious structural deficiencies.

2.2 SCHAPITZ CRITERION

The Schapitz criterion [2] is embodied in the following simple relationships which have been verified by the rederivation of reference 4:

When $\sigma_R \leq 2\sigma_p$ (2-1)

then $$\sigma_{cr} = \sigma_p + \frac{\sigma_R^2}{4\sigma_p} \tag{2-2}$$

and

when $\sigma_R > 2\sigma_p$ (2-3)

then $$\sigma_{cr} = \sigma_R \tag{2-4}$$

where,

σ_p = Critical stress for buckling of a flat isotropic skin panel.

σ_R = Critical stress for buckling of an isotropic cylindrical shell.

σ_{cr} = Critical stress for buckling of a curved isotropic skin panel.

Equation (2-2) supplies the transitional relationship. To clarify discussions which follow, it is pointed out here that the notation of Figure 1 will be employed in the application of the foregoing criterion. Note that the dimension b is the true circumferential arc length of the curved panel.

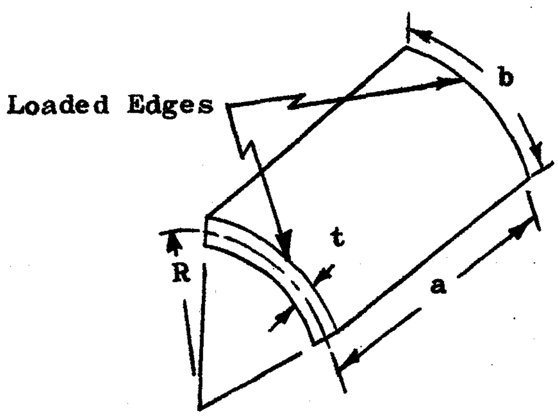


Figure 1 - Geometry of Isotropic Skin Panel

2.2.1 FLAT PLATE EQUATION

From equations (2-1) through (2-4), it can be seen that application of the Schapitz criterion [2] requires the availability of suitable means for computation of the flat plate critical buckling stress σ_p. The following well-known formula [5] will be used for this purpose:

$$\sigma_p = K \frac{E}{(1-\nu^2)} \left(\frac{t}{b} \right)^2 \tag{2-5}$$

where

K = Flat plate buckling coefficient which depends on aspect ratio (a/b), boundary conditions, and type of loading.

E = Young's modulus.

ν = Poisson's ratio.

In this volume it will always be conservatively assumed that the loaded edges are simply supported. No advantage will be taken of any rotational restraint along these particular boundaries. However, for the longitudinal edges, both the simply supported case and the fully clamped condition will be covered. Therefore, the following notation may be used to identify the buckling coefficients K of interest here:

K_s = Buckling coefficient for flat plate having all four edges simply supported.

K_c = Buckling coefficient for flat plate having the loaded edges simply supported and the remaining two edges fully clamped.

Thus, for the purposes of this volume, it is convenient to rewrite equation (2-5) in the form

$$\sigma_p = K_{s[c]} \frac{E}{(1-\nu^2)} \left(\frac{t}{b} \right)^2 \tag{2-6}$$

to identify the nature of the selected boundary conditions.

2.2.2 FULL CYLINDER CRITERIA

From equations (2-1) through (2-4), it can be seen that application of the Schapitz criterion [2] also requires the availability of suitable means for computation of the critical buckling stress σ_R of an isotropic cylinder. It must be kept in mind, however, that the test data for axially compressed isotropic cylinders usually fall far below classical theoretical values and display a high degree of scatter. Therefore, design stress levels for such cylinders are normally established by applying empirical reductions to the classical theoretical predictions. In this volume, three different empirical criteria are considered. These alternatives are identified below as OPTIONS 1, 2, and 3.

OPTION 1) The lower-bound approach of Seide, et al. [6]:

$$\sigma_R = CE \frac{t}{R} \tag{2-7}$$

where

$$C = 0.605 - 0.546 \left(1 - e^{-\frac{1}{16}\sqrt{\frac{R}{t}}} \right) \tag{2-8}$$

OPTION 2) The "best-fit" (or "mean-expected") relations of reference 7 which may be rewritten in terms of the parameters of interest here as follows:

$$\frac{a}{b}\frac{b}{R} < 1.0 \rightarrow \sigma_R = 11.28 \left(\frac{R}{t}\right)^{-1.6} + 0.109E\left(\frac{R}{t}\ \frac{a}{b}\ \frac{b}{R}\right)^{-1.3} \tag{2-9}$$

$$\frac{a}{b}\frac{b}{R} \geq 1.0 \rightarrow \sigma_R = 11.28 \left(\frac{R}{t}\right)^{-1.6} + 0.109E\left(\frac{R}{t}\ \frac{a}{b}\ \frac{b}{R}\right)^{-1.3}$$

$$-1.418 \left(\frac{R}{t}\right)^{-1.6} \mathrm{Log}_e\left(\frac{a}{b}\ \frac{b}{R}\right) \tag{2-10}$$

OPTION 3) The statistical level [7] which represents 90% probability with 95% confidence; i.e., there is 95% confidence that 90% of specimens tested would exceed the buckling levels given by:

$$\frac{a}{b}\,\frac{b}{R} < 1.0 \rightarrow \sigma_R = 8.01E\left(\frac{R}{t}\right)^{-1.6} + 0.076E\left(\frac{R}{t}\,\frac{a}{b}\,\frac{b}{R}\right)^{-1.3} \tag{2-11}$$

$$\frac{a}{b}\,\frac{b}{R} \geq 1.0 \rightarrow \sigma_R = 7.52E\left(\frac{R}{t}\right)^{-1.6} + 0.072E\left(\frac{R}{t}\,\frac{a}{b}\,\frac{b}{R}\right)^{-1.3} - 1.418\left(\frac{R}{t}\right)^{-1.6} \text{Log}_e\left(\frac{a}{b}\,\frac{b}{R}\right) \tag{2-12}$$

In view of its relative simplicity and apparent reliability (see SECTION 3, "TEST DATA COMPARISONS"), it is recommended here that OPTION 1 be used in the analysis of curved isotropic skin panels bounded by stringers and rings. However, it is further recommended that the OPTION 1 application be restricted to cases where $(a/R) \leq 1$. Since practical configurations will almost always satisfy this condition, it does not constitute a severe limitation. This restriction has been imposed here in view of the situation depicted in Figure 2 where the lower-bound curve of Seide, et al. [6] is compared against a 90% probability curve $(a/R = 1.0)$ of reference 7. A comparison could be made against the corresponding 99% probability curve of reference 7 but it is thought here that this latter statistical level is excessively conservative for multiple load-path structures such as the stiffened cylinders under consideration. The curves for 99% probability are more suitable to single load-path configurations such as pure monocoques. The comparison given in Figure 2 shows that the curve of Seide, et al. [6] corresponds reasonably well to the reference 7 curve for $(a/R) = 1$. The

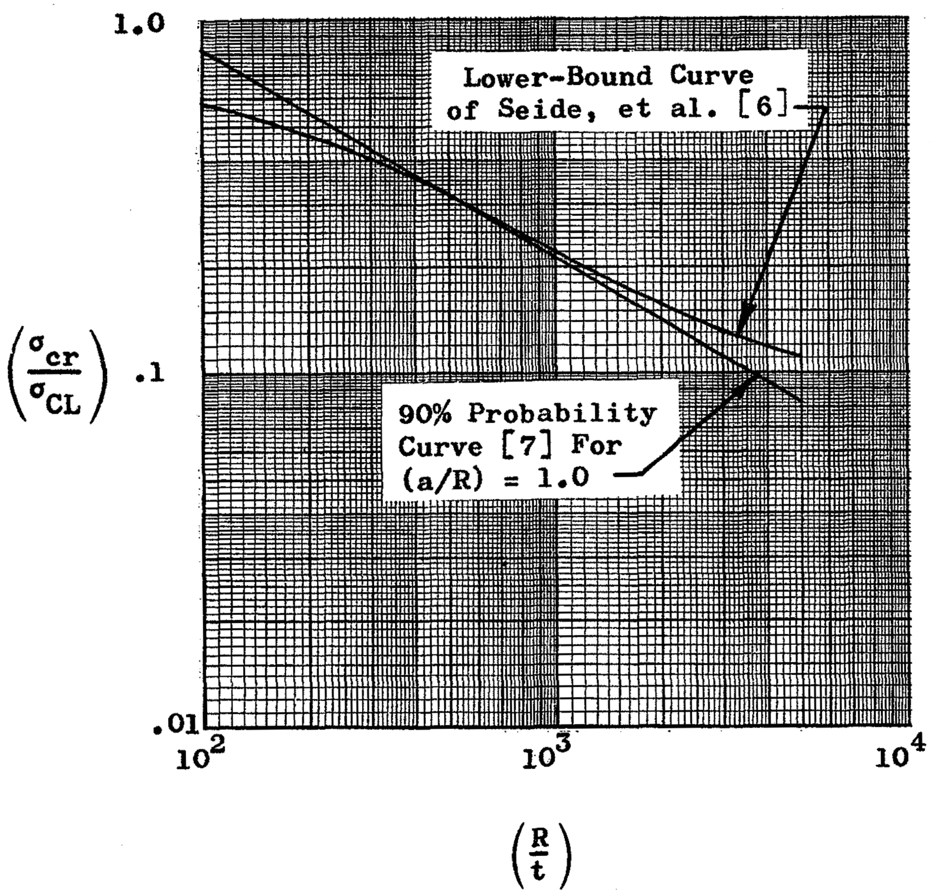


Figure 2 - Comparison of Lower-Bound Curve of Seide, et al. [6] Against 90% Probability Curve of Reference 7

Seide criterion is somewhat more conservative in the lower (R/t) region. On the other hand, for (R/t) values in excess of 1000, the Seide criterion is the less conservative of the two alternatives but in this high (R/t) region the differences in terms of lbs/in.^{2} are of rather small magnitudes.

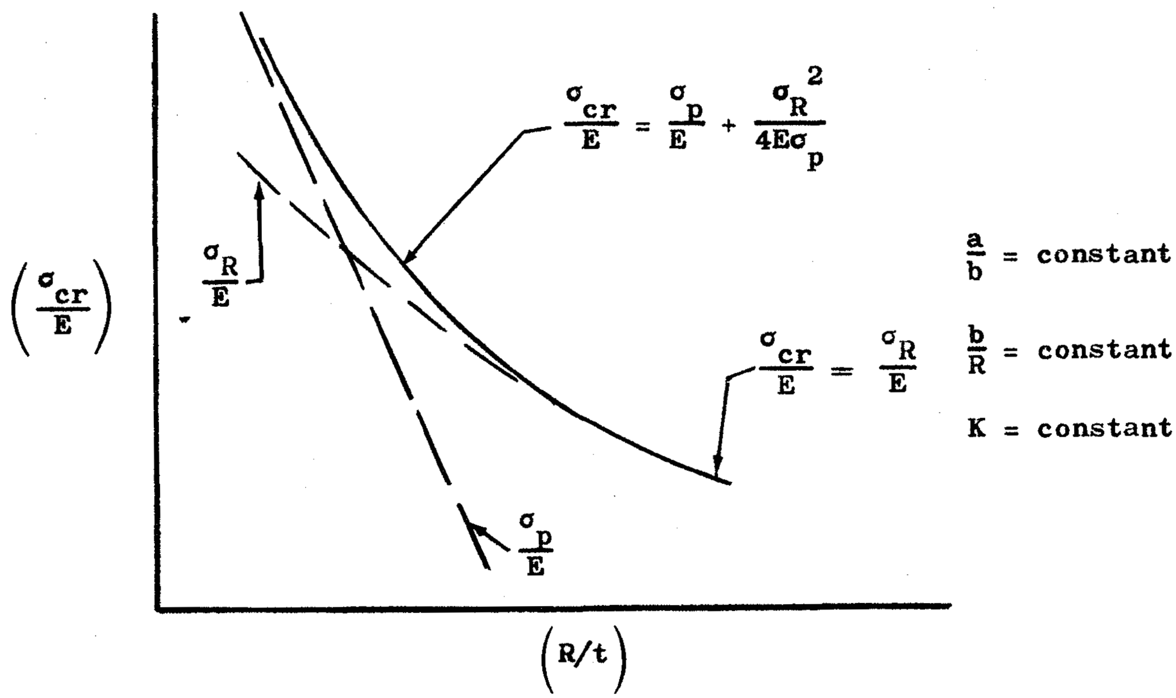
It should be noted that, unlike OPTION 1, the equations for OPTIONS 2 and 3 include a statistical effect from the cylinder length. This added influence complicates the graphical presentation of the Schapitz criterion [2] to such a degree that the associated buckling curves are considered here only as supplements to the recommended curves given in SECTION 4, "DESIGN CURVES". These supplementary plots are furnished in Appendix A. The buckling curves based on OPTION 2 could be of some use in the evaluation of test results while the curves based on OPTION 3 might be used to obtain design values for cases where (a/R) > 1.

2.2.3 GRAPHICAL REPRESENTATION

In order to permit a graphical representation of the buckling criterion of equations (2-1) through (2-4), equation (2-6) may be rewritten as

$$\sigma_p = K_s[c] \frac{E}{(1-\nu^2)} \frac{1}{\left(\frac{R}{t}\right)^2 \left(\frac{b}{R}\right)^2} \tag{2-13}$$

Within a nondimensional, logarithmic format, equations (2-1) through (2-4) can then be presented as shown in Figure 3. The transition curve defined by equation (2-2) becomes tangent to the full cylinder curve when $\sigma_R = 2\sigma_p$. For all (R/t) values greater than that of the tangency point, the skin panel behaves as a complete isotropic cylinder. For all other (R/t) values, the transitional relationship applies. Note that the transition curve asymptotically approaches the curve for σ_p.



<u>Figure 3 - Nondimensional Logarithmic Plot of Buckling Criteria for Isotropic Skin Panels</u>

SECTION 3

TEST DATA COMPARISONS

In order to evaluate the reliability of the recommended methods of this volume, comparisons were made between predicted critical stresses and experimental data appearing in the literature. In particular, the test data of references 8, 9, 10, and 11 were compared against Schapitz criterion [2] predictions which employ the OPTION 1 full cylinder equation. The results obtained from this investigation are summarized in Table I. All predicted values were obtained using digital computer program 3875 which is given in SECTION 5 of this volume. It should be recalled that the OPTION 1 equation describes the lower-bound curve of Seide, et al. [6] for σ_R [see equations (2-1) through (2-4)]. Hence one would expect all the predicted critical stresses to be conservative with respect to the actual test results, particularly in the transition and full cylinder regions of behavior. Considerable scatter would also be expected of the test data in these regions so that the degree of conservatism would vary rather widely from specimen to specimen. Inspection of Table I shows that, for most of the data considered, the predictions (based on OPTION 1) were indeed conservative and that considerable scatter did exist in the test results. However out of a total of 127 test points, 24 fell below the supposedly conservative predictions. On the surface, this would appear to be an unsatisfactory condition and would tend to discredit the reliability of the proposed analysis methods. However, further scrutiny of the test data shows that, except for 4 points, the troublesome data came from specimens which fell into the flat plate region of behavior. Therefore, in general, the discrepant prediction values could only be significantly lowered by applying a reduction factor to the classical flat plate values. This would be contrary to time-tested current design and analysis practices which have usually proven to be perfectly adequate. It has therefore been

TABLE I - Buckling of Isotropic Skin Panels

REF.	R in.	t in.	a in.	b in.	K	$\left(\frac{a}{b}\right)$	$\left(\frac{b}{R}\right)$	$\left(\frac{R}{t}\right)$	Test $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	Calculated $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
8	23.9	.0249	17.0	2.0	5.1	8.5	.0838	961	.914	.870	.95
		.0244	20.0			10.0		980	.924	.837	.91
		.0246	25.0			12.5		973	.924	.849	.92
		.0248	30.0			15.0		964	.952	.865	.91
		.0248	30.0			15.0		964	.838	.865	1.03
		.0246	34.0	2.0		17.0	.0838	973	.790	.849	1.07
		.0248	17.0	3.1		5.4	.131	964	.457	.366	.80
		.0256	20.0			6.4		934	.486	.389	.80
		.0256	25.0			8.0		934	.409	.389	.95
		.0240	30.0			9.6		997	.495	.342	.69
		.0247	34.0	3.1		10.9	.131	968	.438	.363	.83
		.0247	17.0	5.0		3.4	.21	968	.391	.173	.44
		.0248	20.0			4.0		964	.410	.174	.42
		.0244	25.0			5.0		980	.391	.169	.43
		.0237	30.0			6.0		1,010	.286	.160	.56
		.0258	30.0			6.0		927	.343	.187	.54
8	23.9	.0247	34.0	5.0	5.1	6.8	.21	968	.419	.173	.41
9	18	.067	24	12	5.4	2	.667	269	1.71	.948	.55
				12		2	.667		1.37	.948	.69
				10		2.4	.556		1.63	.948	.58
				10		2.4	.556		1.67	.948	.57
				8		3	.444		1.30	.948	.73
				8		3	.444		1.59	.948	.60
				6		4	.333		1.33	1.04	.78
9	18	.067	24	6	5.4	4	.333	269	1.31	1.04	.80

TABLE I - Buckling of Isotropic Skin Panels
(Continued)

REF.	R in.	t in.	a in.	b in.	K	$\left(\frac{a}{b}\right)$	$\left(\frac{b}{R}\right)$	$\left(\frac{R}{t}\right)$	Test $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	Calculated $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
9	18	.067	24	4	5.4	6	.222	269	1.57	1.80	1.15
		.067		4		6	.222	269	1.54	1.80	1.17
		.038		12		2	.667	473	.681	.421	.62
				12		2	.667		----	.421	----
				12		2	.667		.879	.421	.48
				12		2	.667		.937	.421	.45
				10		2.4	.556		.789	.421	.53
				10		2.4	.556		.885	.421	.48
				8		3	.444		1.04	.421	.41
				6		4	.333		1.11	.425	.38
				6		4	.333		1.16	.425	.37
				4		6	.222		.806	.621	.77
		.038		4		6	.222	473	.985	.621	.63
		.030		12		2	.667	600	----	.295	----
				10		2.4	.556		.529	.295	.56
				8		3	.444		.521	.295	.57
				6		4	.333		.622	.295	.47
		.030		4		6	.222	600	.241	.400	1.66
		.024		12		2	.667	750	.391	.210	.54
				10		2.4	.556		.431	.210	.49
				8		3	.444		.466	.210	.45
				6		4	.333		.685	.210	.31
		.024		4		6	.222	750	.477	.266	.56
		.018		12		2	.667	1,000	.158	.135	.85
				10		2.4	.556		.175	.135	.77
				8		3	.444		.190	.135	.71
				6		4	.333		.265	.135	.51
9	18	.018	24	4	5.4	6	.222	1,000	----	.158	----

TABLE I - Buckling of Isotropic Skin Panels
(Continued)

REF.	R in.	t in.	a in.	b in.	K	$\left(\frac{a}{b}\right)$	$\left(\frac{b}{R}\right)$	$\left(\frac{R}{t}\right)$	Test $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	Calculated $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
9	36	.067	24	12	5.4	2	.333	537	.489	.349	.71
				10		2.4	.278		.510	.380	.75
				8		3	.222		.585	.490	.84
				6		4	.167		.839	.779	.93
		.067		4		6	.111	537	1.19	1.69	1.42
		.038		12		2	.333	947	.239	.147	.61
				12		2	.333		.386	.147	.38
				10		2.4	.278		.256	.148	.58
				8		3	.222		.148	.174	1.18
				6		4	.167		.458	.260	.57
				6		4	.167		.379	.260	.69
				4		6	.111		.686	.547	.80
9	36	.038	24	4	5.4	6	.111	947	.626	.547	.87
10	106.9	.1274	24	8.2	4.4	2.93	.077	839	1.071	1.165	1.09
	97.4	.1256					.084	776	1.045	1.147	1.10
	90.0	.1251					.091	719	1.042	1.141	1.09
	88.0	.1268					.093	694	1.026	1.173	1.14
	76.0	.1238					.107	614	.911	1.138	1.25
	66.6	.1270					.123	524	1.160	1.192	1.03
	57.0	.1268					.144	450	1.242	1.196	.96
	45.8	.1285					.179	356	1.425	1.276	.90
	34.0	.1268					.241	268	1.545	1.355	.88
	27.8	.1276					.295	218	2.003	1.512	.75
	16.8	.1270		8.2		2.93	.488	132	2.886	2.46	.85
10	121.5	.0990	24	8.25	4.4	2.91	.068	1,227	.775	.698	.90

TABLE I - Buckling of Isotropic Skin Panels
(Continued)

REF.	R in.	t in.	a in.	b in.	K	$\left(\frac{a}{b}\right)$	$\left(\frac{b}{R}\right)$	$\left(\frac{R}{t}\right)$	Test $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	Calculated $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
10	97.1	.1015	24	8.25	4.4	2.91	.085	957	.740	.738	1.00
	96.2	.1029					.086	935	.810	.755	.93
	91.5	.0993					.090	922	.681	.711	1.04
	77.1	.1014					.107	760	.714	.746	1.04
	73.4	.1003					.112	732	.703	.736	1.05
	75.1	.1030					.110	729	.726	.768	1.06
	63.4	.1007					.130	630	.858	.747	.87
	54.7	.0993					.151	551	.866	.739	.85
	37.5	.1015					.220	370	1.172	.854	.73
	26.2	.1017					.315	258	1.387	1.076	.78
	16.2	.0996		8.25		2.91	.509	163	2.70	1.870	.69
	98.7	.0818		8.4		2.86	.085	1,206	.509	.466	.91
	59.9	.0813					.140	737	.564	.480	.85
	46.3	.0815					.181	568	.990	.514	.52
	42.0	.0818					.200	513	.933	.535	.57
	34.5	.0808					.243	427	.858	.583	.68
	32.7	.0814					.257	402	1.172	.611	.52
	28.2	.0810		8.4		2.86	.298	348	1.369	.691	.50
	24.2	.0807		8.25		2.91	.341	300	1.304	.820	.63
	22.1	.0810		8.4		2.86	.380	273	1.558	.928	.60
	17.1	.0807		8.4		2.86	.491	212	2.313	1.315	.57
	15.2	.0810		8.25		2.91	.543	188	2.393	1.547	.65
	11.7	.0817		8.25		2.91	.705	143	3.146	2.22	.71
	97.0	.0639		8.5		2.82	.088	1,518	.366	.276	.75
10	55.7	.0639	24	8.5	4.4	2.82	.153	872	.400	.297	.74

TABLE I - Buckling of Isotropic Skin Panels
(Continued)

REF.	R in.	t in.	a in.	b in.	K	$\left(\frac{a}{b}\right)$	$\left(\frac{b}{R}\right)$	$\left(\frac{R}{t}\right)$	Test $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	Calculated $10^3 x \left(\frac{\sigma_{cr}}{E}\right)$	$\left(\frac{\text{Calculated } \sigma_{cr}}{\text{Test } \sigma_{cr}}\right)$
10	48.1	.0640	24	8.5	4.4	2.82	.177	752	----	.313	----
	43.1	.0640					.197	673	.925	.331	.36
	40.2	.0642					.211	626	.376	.346	.92
	35.0	.0640					.243	547	.371	.379	1.02
	29.7	.0662					.286	449	1.375	.470	.34
	25.7	.0645		8.5		2.82	.331	398	.796	.543	.68
	24.2	.0646		8.4		2.86	.347	374	.697	.594	.85
	16.7	.0644		8.5		2.82	.509	259	1.780	.999	.56
	14.7	.0647		8.4		2.86	.571	227	1.725	1.198	.69
10	11.4	.0646	24	8.4	4.4	2.82	.737	176	2.604	1.689	.65
11	24.0	.018	18	3.40	6.0	5.29	.1417	1,333	*.305	.1948	.64
	24.0			5.30		3.40	.221	1,333	*.275	.1004	.36
	24.0			7.40		2.43	.308	1,333	*.265	.0861	.32
	48.0			3.40		5.29	.0708	2,670	*.1773	.1858	1.05
	48.0			5.30		3.40	.1104	2,670	*.1483	.0789	.53
	48.0	.018		7.40		2.43	.1542	2,670	*.1380	.0448	.32
	24.0	.031		3.10		5.81	.1292	774	*.619	.675	1.09
	24.0			5.10		3.53	.213	774	*.504	.284	.56
	24.0			7.10		2.54	.296	774	*.471	.205	.44
	48.0			3.10		5.81	.0646	1,548	*.587	.661	1.13
	48.0			5.10		3.53	.1063	1,548	*.285	.248	.87
	48.0	.031		7.10		2.54	.1479	1,548	*.253	.1351	.53
	24.0	.038		2.90		6.21	.1208	632	*1.009	1.148	1.14
	24.0			4.80		3.75	.200	632	*.651	.458	.70
	24.0			7.00		2.57	.292	632	*.590	.290	.49
	48.0			2.90		6.21	.0604	1,263	*1.009	1.135	1.12
	48.0			4.80		3.75	.1000	1,263	*.382	.419	1.10
11	48.0	.038	18	7.00	6.0	2.57	.1458	1,263	*.322	.206	.64

*In reference 11, the results from individual tests are not tabulated. Therefore the values identified above by an asterisk (*) were computed using the lower limit criterion shown on page 7 of reference 11. This criterion simply describes a lower-bound curve to the actual test data. Only the second-series data of reference 11 were used here. The first-series data of that reference are of questionable validity due to test procedure difficulties.

concluded that, in general, the discrepancies under discussion here are probably due to the inability to select precisely correct values for the buckling coefficient K [see equation (2-5)].This coefficient is strongly dependent upon the actual degree of restraint afforded to the skin panel by the boundary members. Further uncertainty is introduced through the usual experimental difficulties surrounding the decision as to when initial buckling has in fact occurred in an imperfect flat plate.

For the test data of reference 8, the K value was taken equal to 5.1. This constitutes an average value obtained from the curves developed by Peterson and Whitley [12] for long skin panels having Z-shaped stringers as longitudinal boundary members. In studying the curved specimens of reference 9, use was made of the flat plate data which was concurrently obtained for control-type purposes. These flat plates were subjected to boundary conditions which were similar to those of the corresponding curved skin panels. The experimental critical stress for each flat plate was substituted into the equation

$$K = \sigma_p \frac{(1-\nu^2)}{E} \left(\frac{b}{t} \right)^2 \qquad (3\text{-}1)$$

which is obtained from a simple transformation of equation (2-5). The average of these several values was computed to be 5.4. As noted in Table I, this furnished the basis for analysis of the reference 9 curved skin panels. This same procedure was followed for the specimens of reference 10. In this case, the average K value was found to be 4.4. As stated in the footnote to Table I, reference 11 did not include any listing of specific test results for the individual specimens. However, sufficient information was provided to ascertain that an average K value of 6.0 would be applicable there. Hence the related predictions of Table I were based upon this value.

In order to further clarify the preceding discussion, the qualitative presentation of Figure 4 is offered. This figure shows the general characteristics and relative positioning for each of the following when transformed into the desired nondimensional logarithmic format:

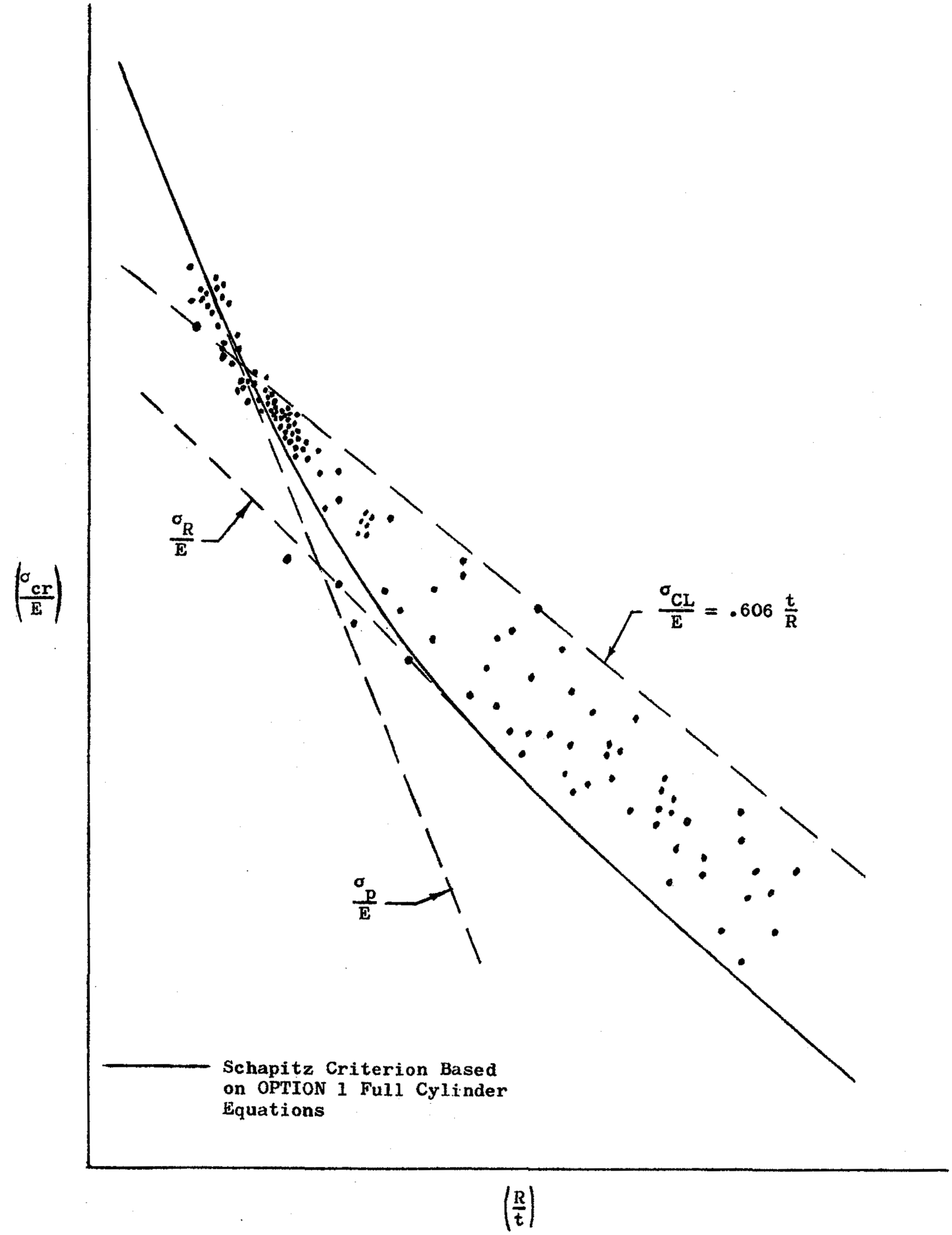


Figure 4 - Schematic Nondimensional Logarithmic Plot of Test Data for Curved Isotropic Skin Panels

(a) The flat plate relationship given as equation (2-5).

(b) The recommended design curve based upon the substitution of OPTION 1 cylinder expressions into the Schapitz criterion [2].

(c) The classical small-deflection buckling equation for isotropic cylinders.

Also shown in Figure 4 are the individual approximate locations of all test points listed in Table I. In the course of the study under NASA Contract NAS8-11181, quantitative plots were made for each specimen and the corresponding test point was accurately located on the appropriate plot. Based on these many different plots, the test points were inserted in Figure 4 in approximation to their actual position relative to the several basic curves and regions of behavior. In general, this figure shows that

(a) Except for the region of flat plate behavior, the test data fall above the recommended design curve but below the value which would have been predicted if σ_R were computed from the classical small-deflection equation.

(b) All but 4 of the unconservative predictions occur in the region of flat plate behavior.

In view of the foregoing, it is concluded that the use of OPTION 1 equations in conjunction with the Schapitz criterion provides reliable critical stress values which can be safely employed for the design of isotropic skin panels in stiffened circular cylinders.

SECTION 4

DESIGN CURVES

All of the buckling curves presented in Figures 5(a) through 5(l) are based upon the use of the OPTION 1 full cylinder equations [see equations (2-7) and (2-8)] in the Schapitz criterion [2]. These curves were obtained by using digital computer program numbered 3875 and an automatic plotter. This program is described in SECTION 5 and can be used to obtain additional plots and/or single-point solutions, as desired. Since the automatic plotting machine does not provide any capability for the print-out of lower case letters, the ratio (a/b) appears on all the given buckling curves as (A/B).

It should be recalled that the Schapitz criterion [2] considers two regions of response dependent upon the two values σ_p and σ_R. When $\sigma_R \leq 2\sigma_p$, the critical stress for the curved skin panel is established by a relationship which provides a smooth transition between flat plate theory and the full cylinder behavior. For $\sigma_R > 2\sigma_p$ the critical stress is simply taken as $\sigma_{cr} = \sigma_R$. To establish the point of separation between these two regions, this section includes curves 6(a) through 6(d) which show b/R versus R/t for the condition $\sigma_R = 2\sigma_p$. Here again, the plotting machine converted lower case notation into upper case print-outs. All of the curves in Figure 6 are based on the OPTION 1 full cylinder criterion.

The buckling curves of this section are to be applied only where the condition $(a/R) \leq 1$ is satisfied. For longer skin panels, unconservative predictions can result. Whenever $(a/R) > 1$, it is recommended that the predicted critical stresses be based upon the use of the OPTION 3 full cylinder equation in conjunction with the Schapitz criterion [2]. A selection of design curves based on this option are given in the appendix.

All of the design curves given in this volume apply only where the behavior is elastic. Conventional plasticity reduction factors should be used whenever the stresses exceed the elastic limit. In addition, all curves are based on a Poisson's ratio of 0.30. The curves for aluminum employ $E = 10 \times 10^6$ psi while the curves for steel use $E = 30 \times 10^6$ psi.

In order to use the design curves of this volume, one must first select an appropriate value for the buckling coefficient K. Table II lists such values for the two boundary conditions of interest here. Tables III and IV list the various families of curves provided in this section.

TABLE II - Buckling Coefficients For Isotropic Skin Panels Subjected To Edge Compression

BOUNDARY CONDITIONS		$\left(\frac{a}{b}\right)$	K
Unloaded Edges	Loaded Edges		
Clamped	Simply Supported	0.4	$K_c = 7.0$
Clamped	Simply Supported	$0.6 \to \infty$	$K_c = 5.7$
Simply Supported	Simply Supported	0.4	$K_s = 7.0$
Simply Supported	Simply Supported	0.6	$K_s = 4.0$
Simply Supported	Simply Supported	$0.8 \to \infty$	$K_s = 3.29$

TABLE III - Table of Contents for the Design Curves "Buckling of Isotropic Panels"

TABLE IV - Table of Contents for the Design Curves "b/R vs. R/t for $\sigma_R = 2\sigma_p$"

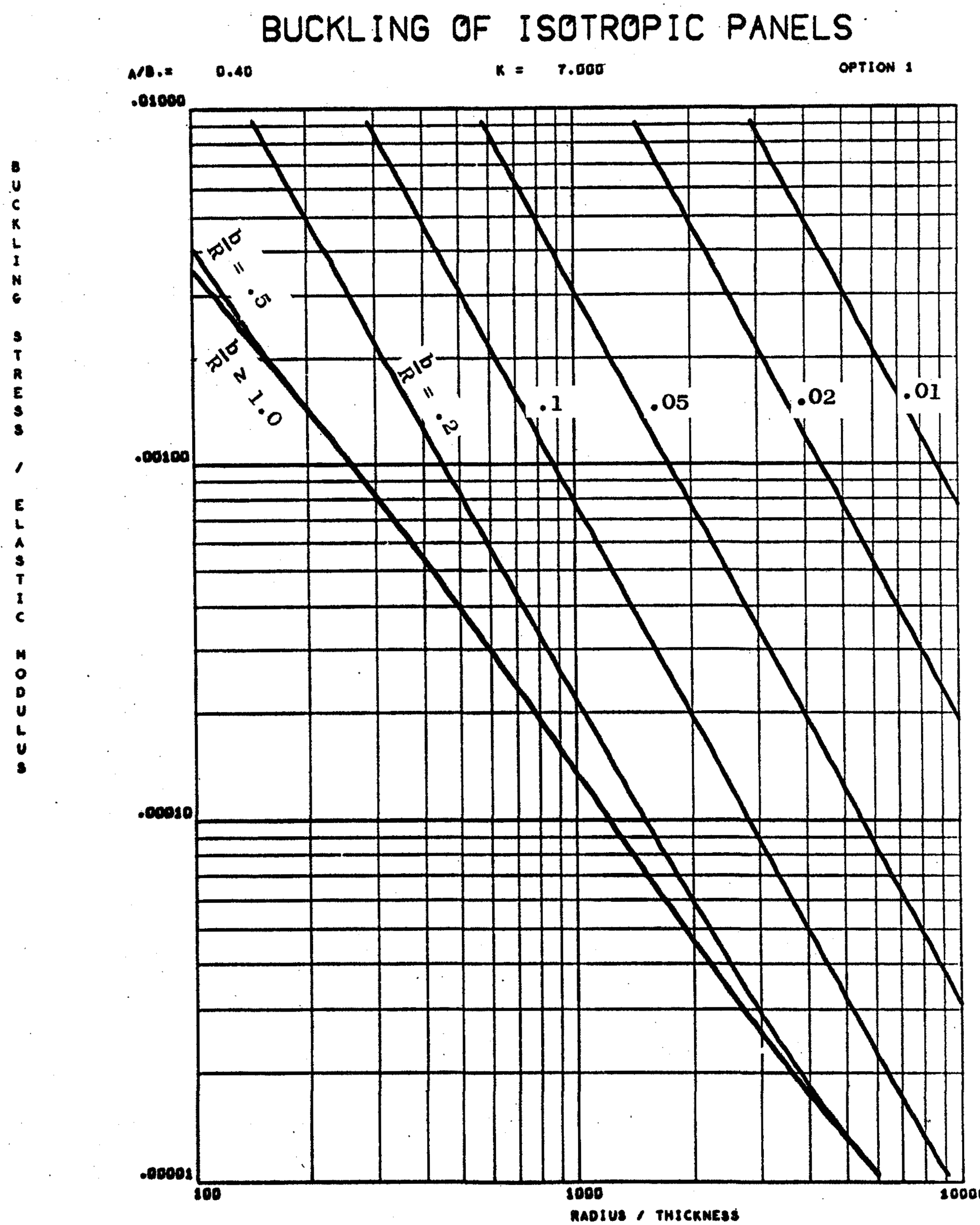

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.40
K = 7.000
OPTION 1
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
b/R = .5
b/R ≥ 1.0
b/R = .2
.1
.05
.02
.01

Figure 5(a)

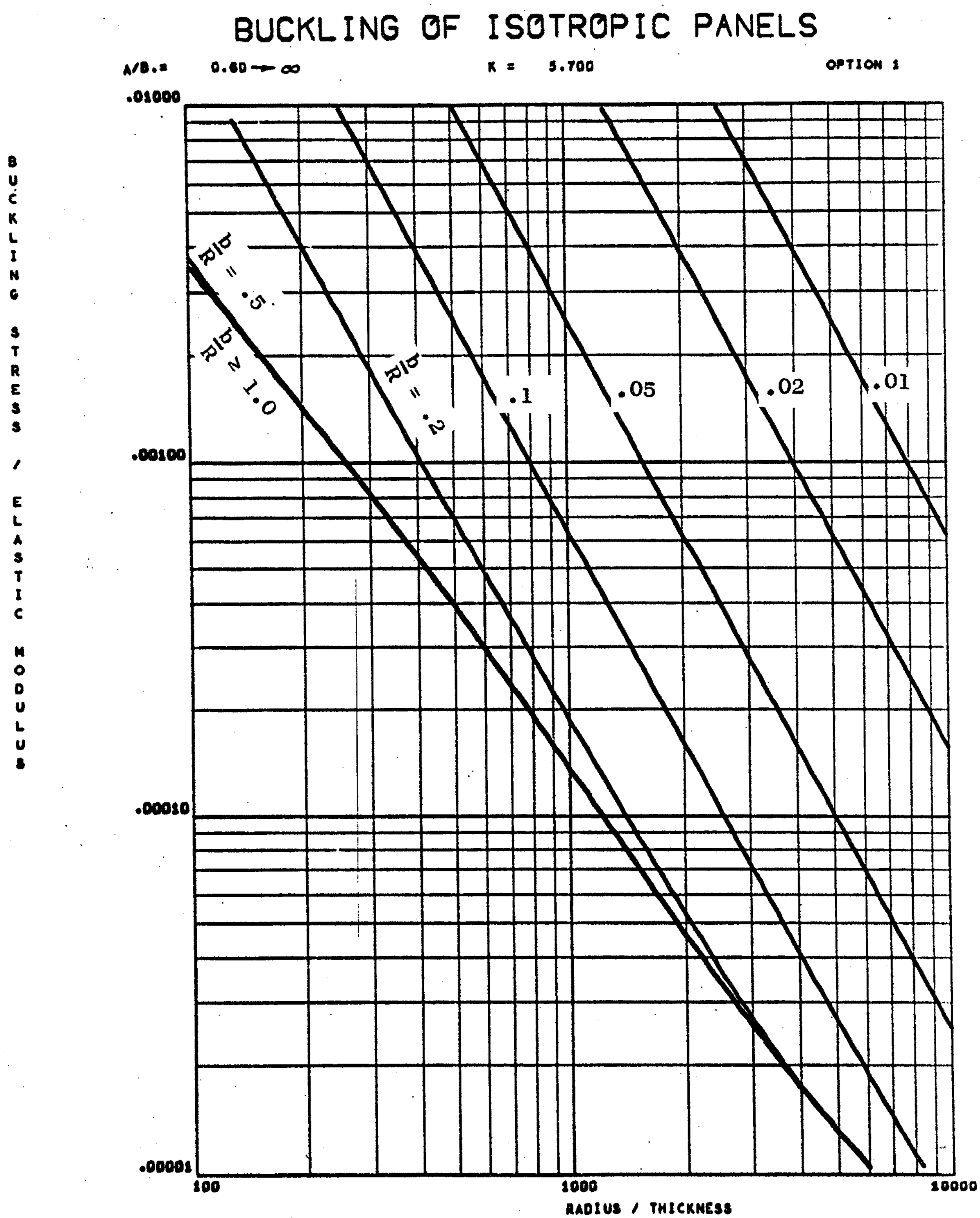

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60 → ∞
K = 5.700
OPTION 1
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
b/R = .5
b/R ≥ 1.0
b/R = .2
.1
.05
.02
.01

Figure 5(b)

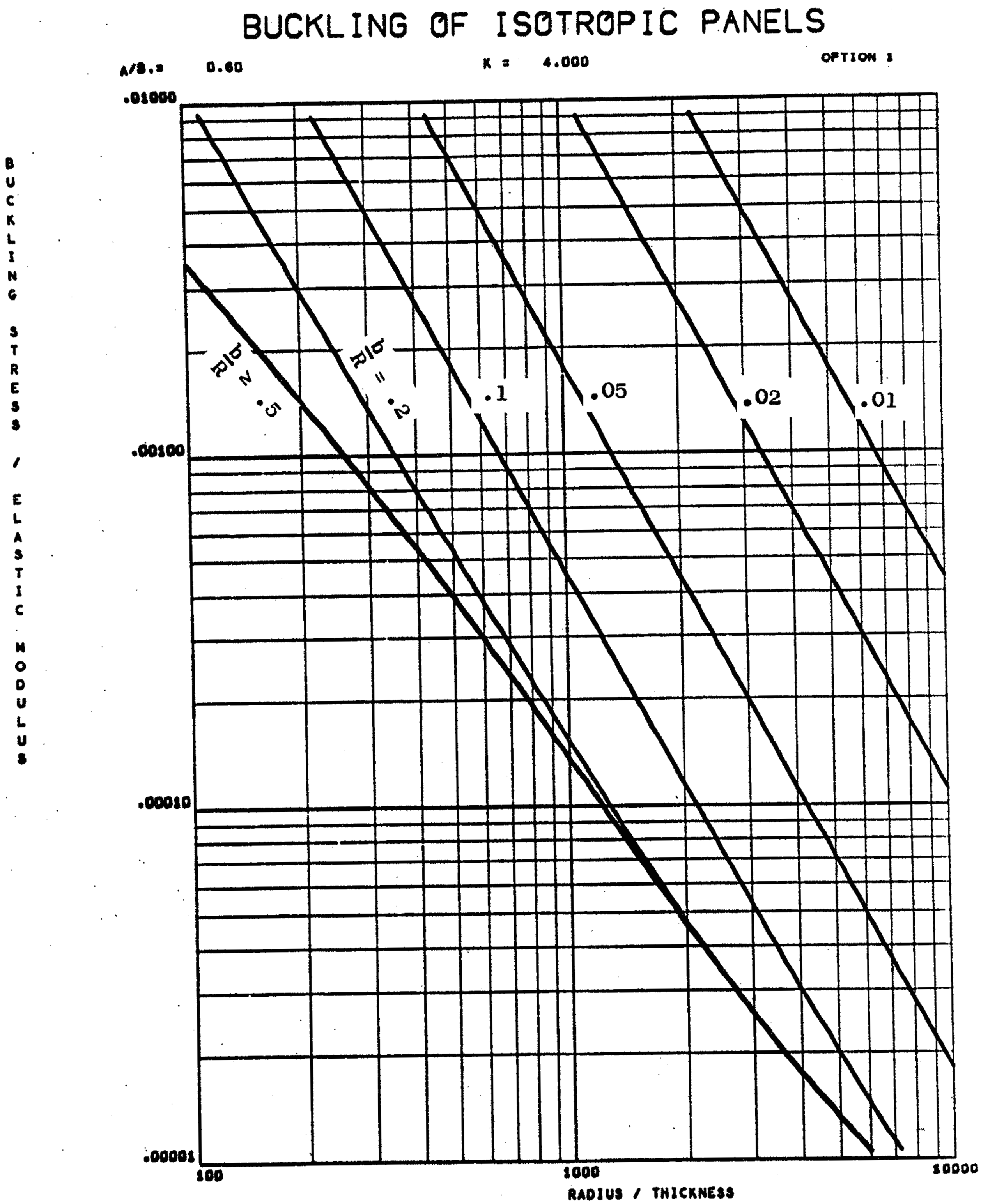

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 4.000
OPTION 1
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
b/R = .5
b/R = .2
.1
.05
.02
.01
100
1000
10000
RADIUS / THICKNESS

Figure 5(c)

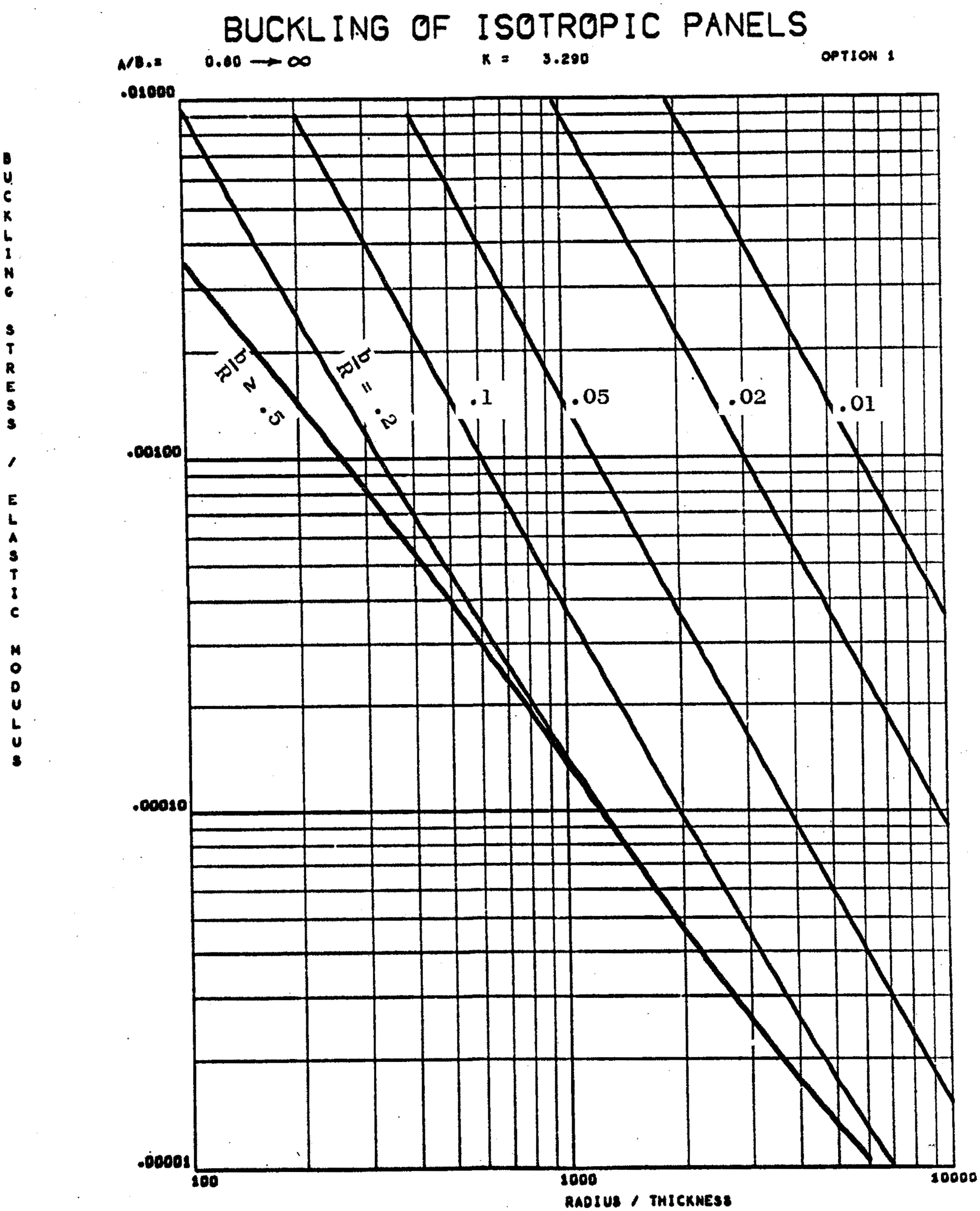

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60 → ∞
K = 3.290
OPTION 1
.01000
.00100
.00010
.00001
b/R ≥ .5
b/R = .2
.1
.05
.02
.01
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS

Figure 5(d)

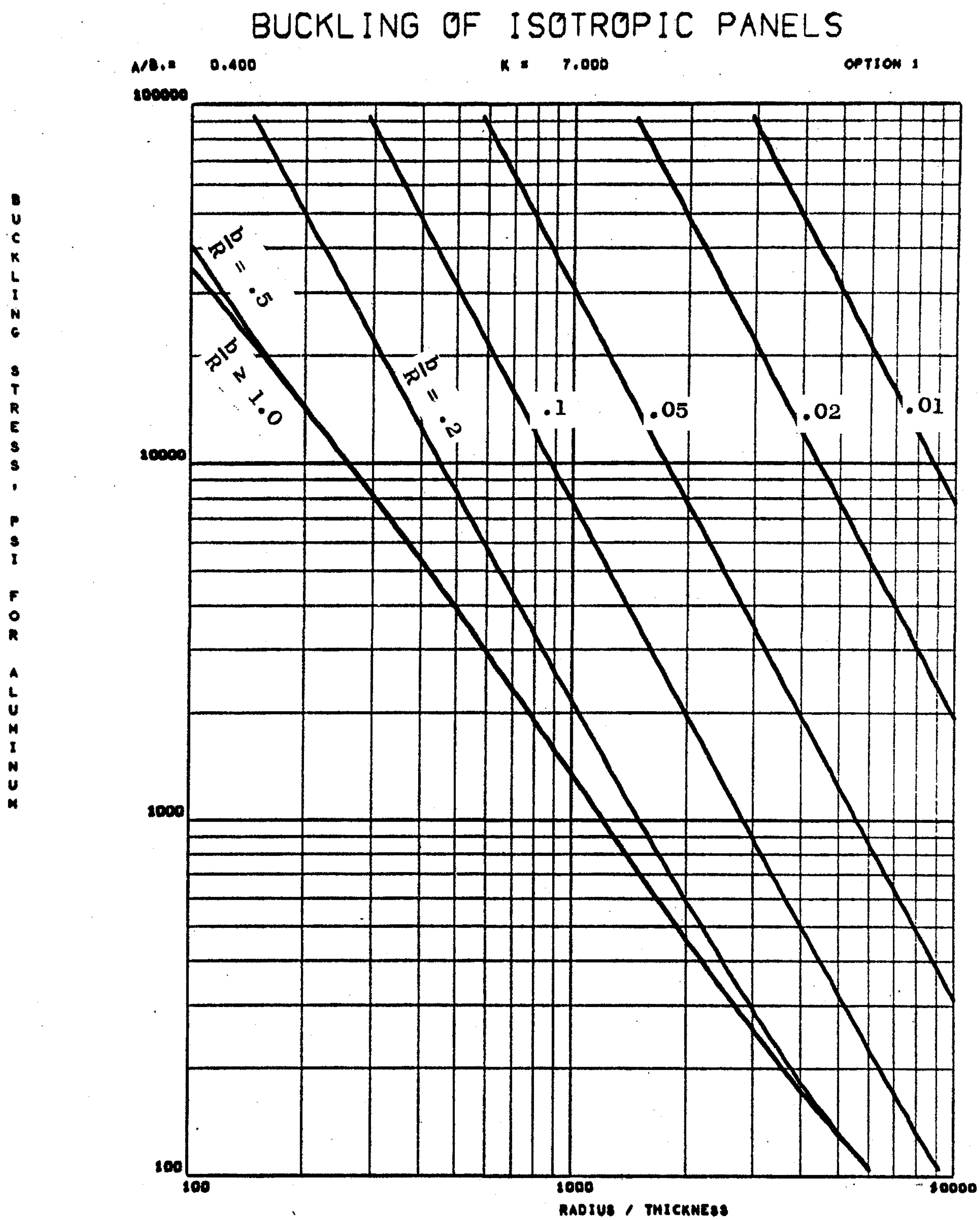

BUCKLING OF ISOTROPIC PANELS
A/B = 0.400
K = 7.000
OPTION 1
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS, PSI FOR ALUMINUM
b/R = .5
b/R ≥ 1.0
b/R = .2
.1
.05
.02
.01

Figure 5(e)

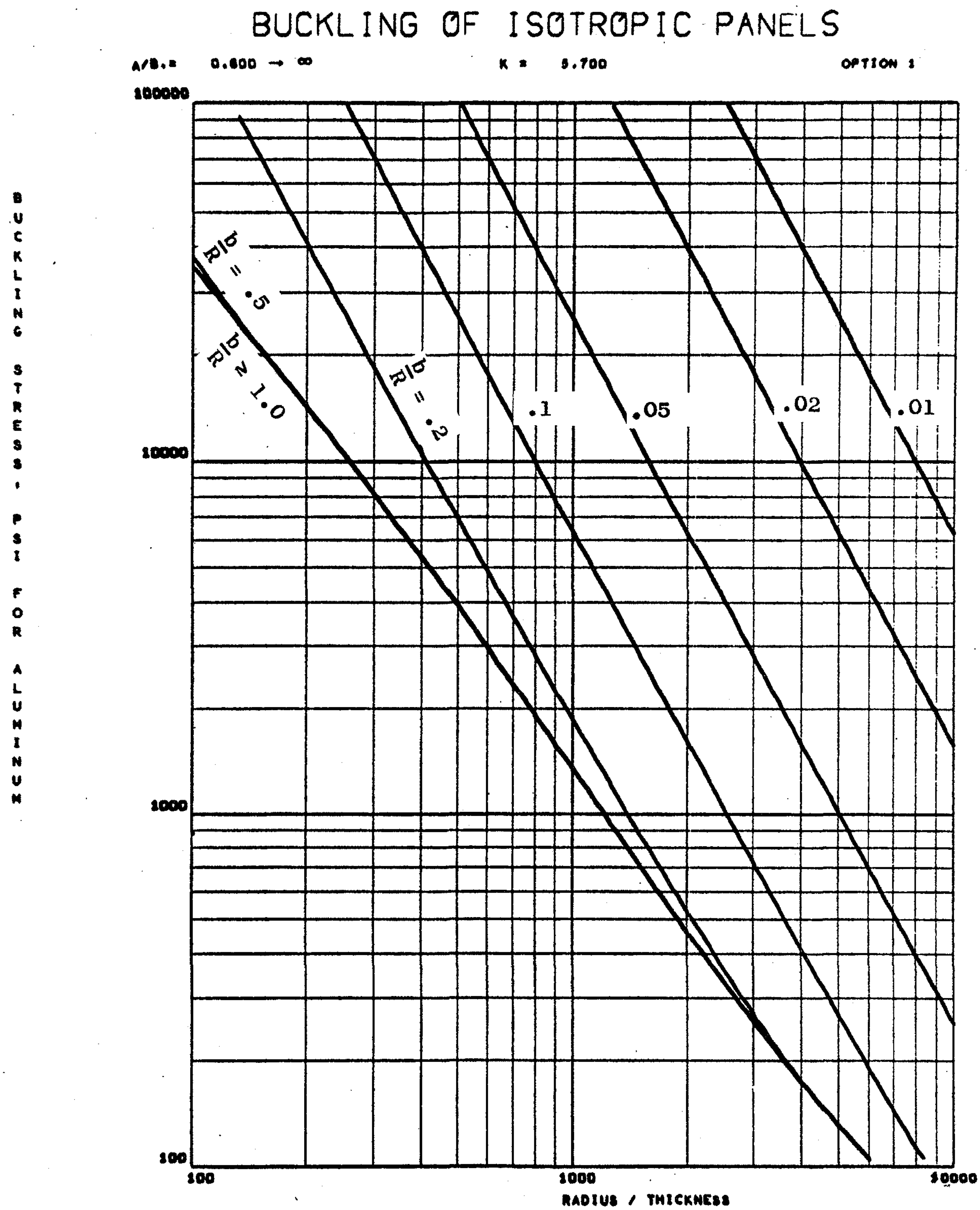
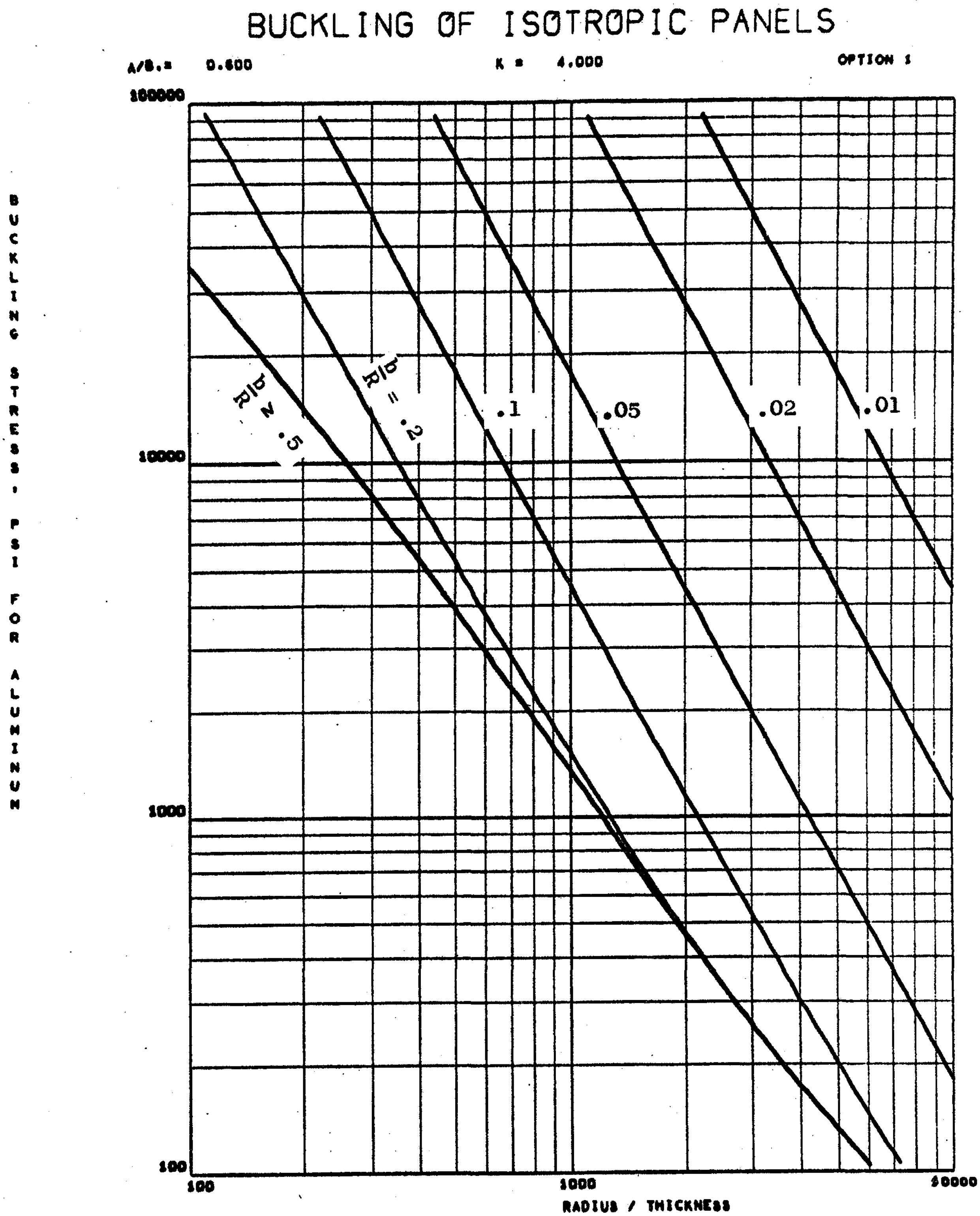

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.600 → ∞
K = 5.700
OPTION 1
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS, PSI FOR ALUMINUM
b/R = .5
b/R ≥ 1.0
b/R = .2
.1
.05
.02
.01

Figure 5(f)



BUCKLING OF ISOTROPIC PANELS
A/B.= 0.600
K = 4.000
OPTION 1
BUCKLING STRESS, PSI FOR ALUMINUM
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
b/R = .5
b/R = .2
.1
.05
.02
.01

Figure 5(g)

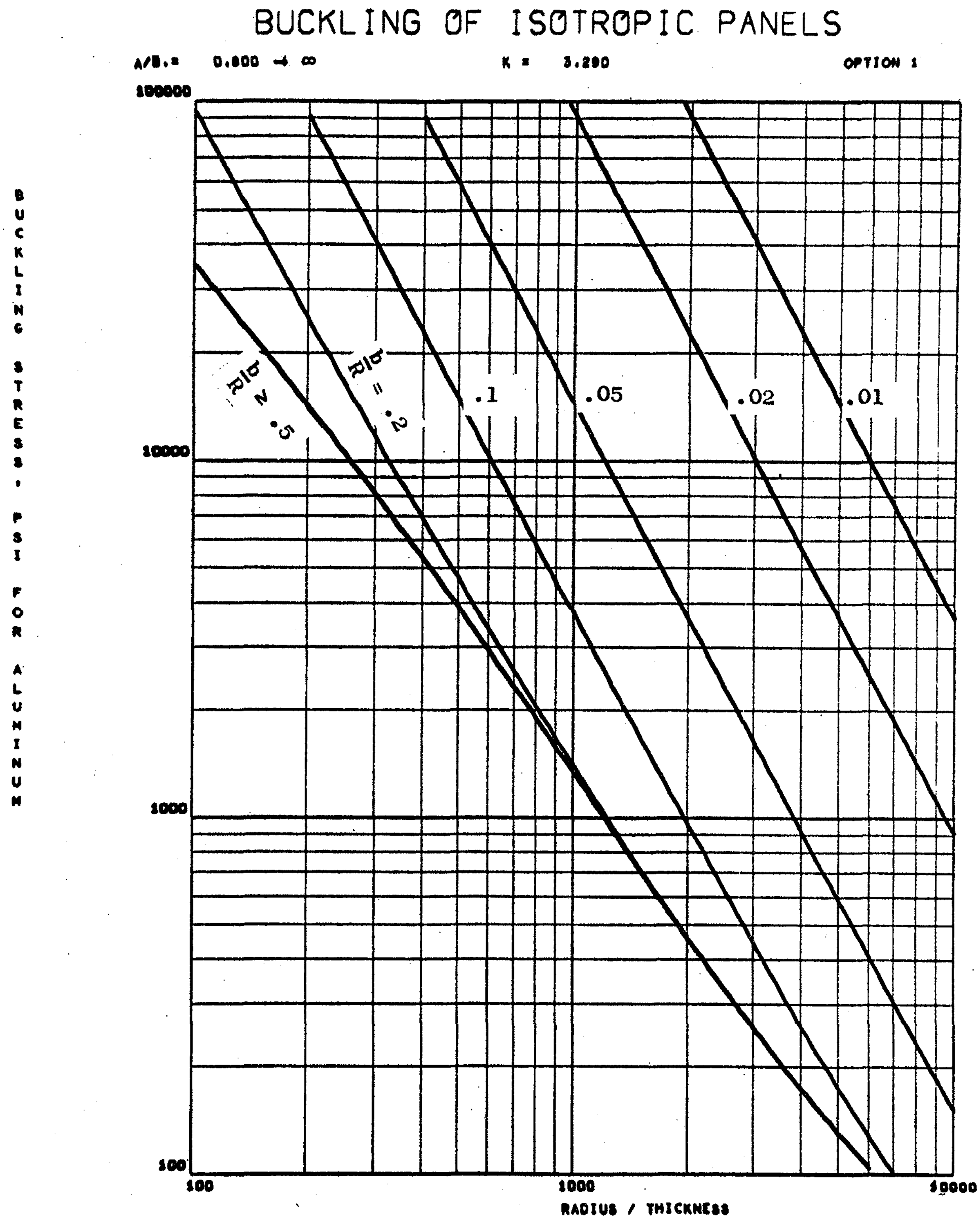

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.800 → ∞
K = 3.290
OPTION 1
BUCKLING STRESS, PSI FOR ALUMINUM
RADIUS / THICKNESS
100000
10000
1000
100
100
1000
10000
b/R = .5
b/R = .2
.1
.05
.02
.01

Figure 5(h)

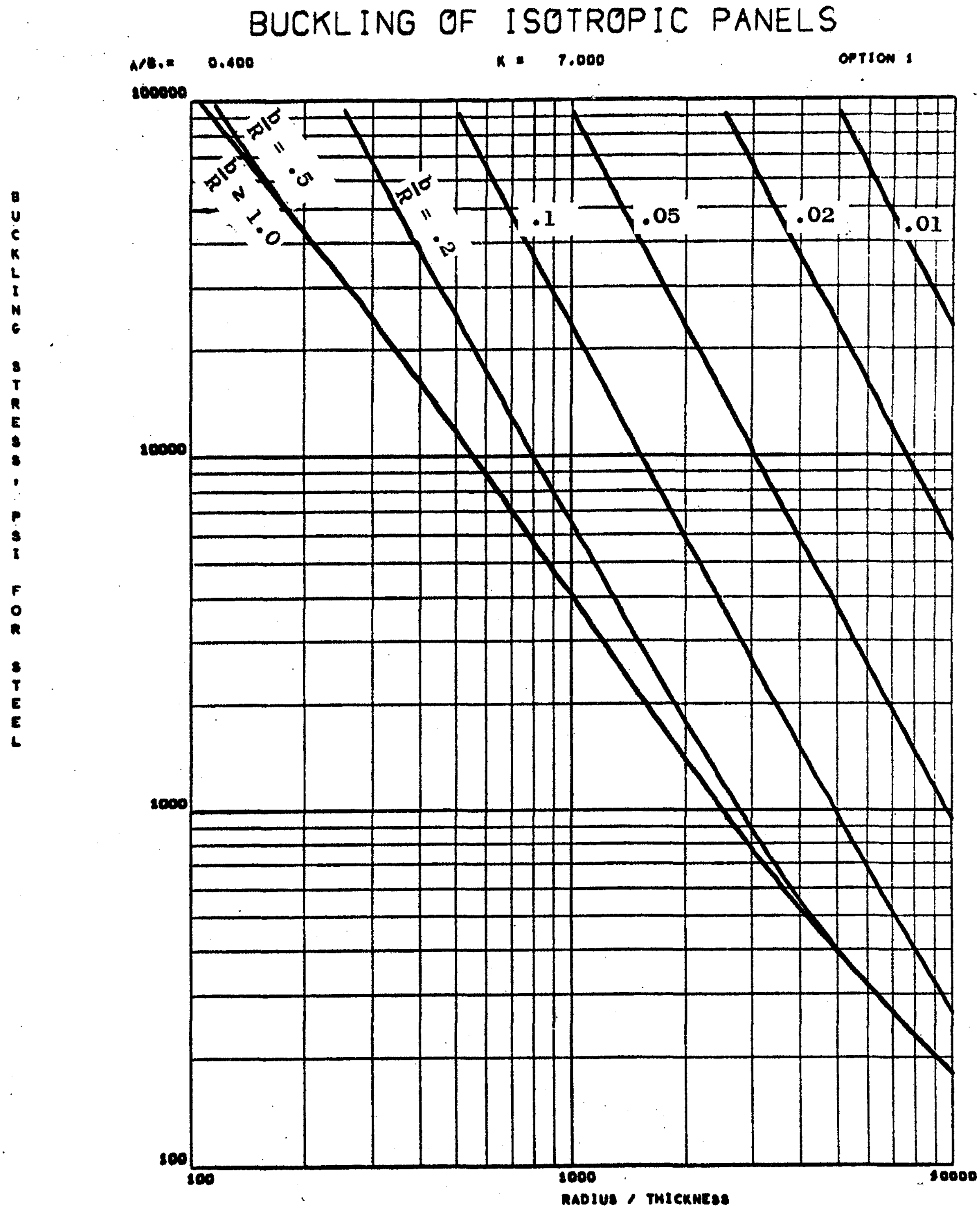

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.400
K = 7.000
OPTION 1
BUCKLING STRESS, PSI FOR STEEL
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
b/R ≥ 1.0
b/R = .5
b/R = .2
.1
.05
.02
.01

Figure 5(i)

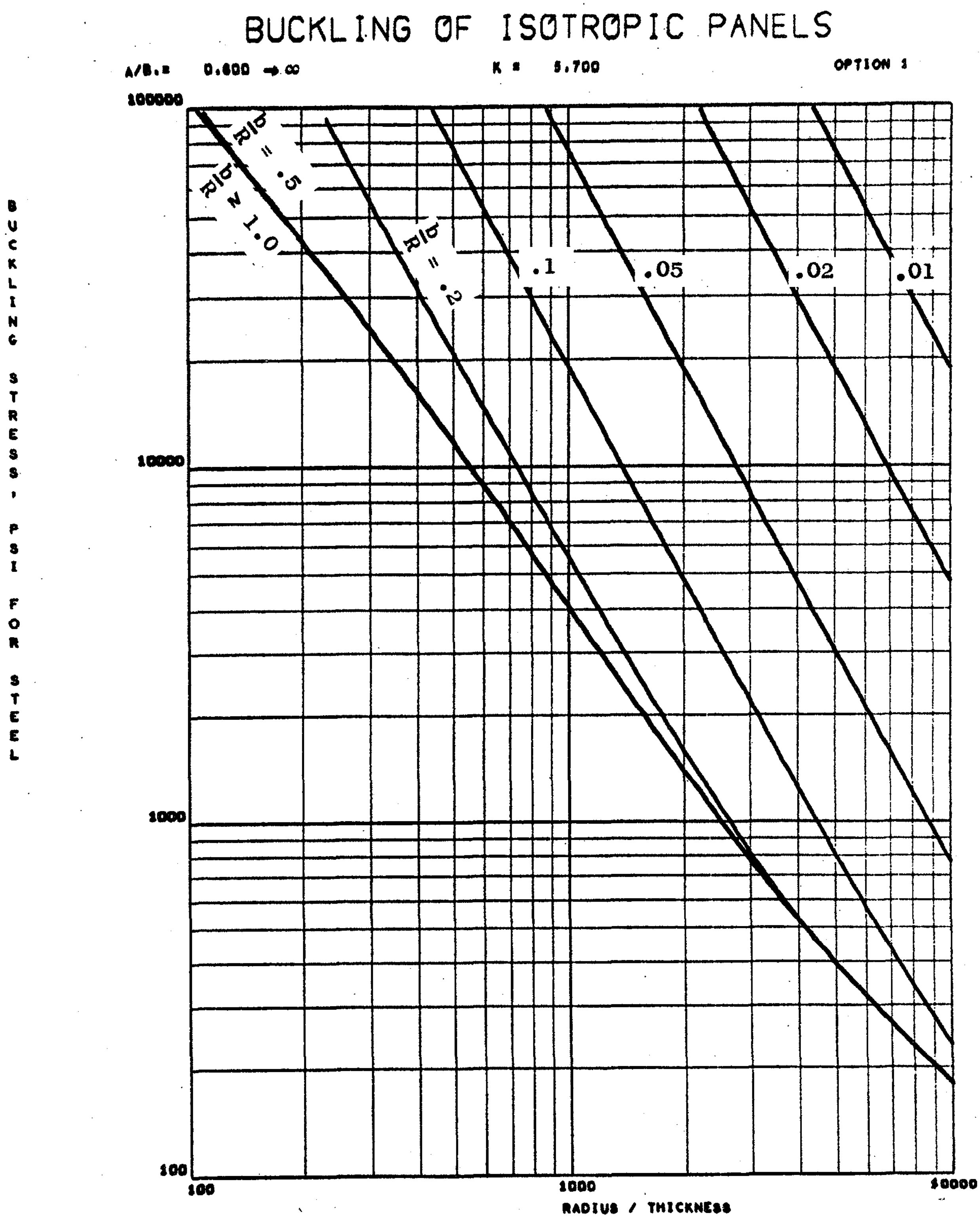

BUCKLING OF ISOTROPIC PANELS
A/B. = 0.600 → ∞
K = 5.700
OPTION 1
b/R = .5
b/R ≥ 1.0
b/R = .2
.1
.05
.02
.01
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS, PSI FOR STEEL

Figure 5(j)

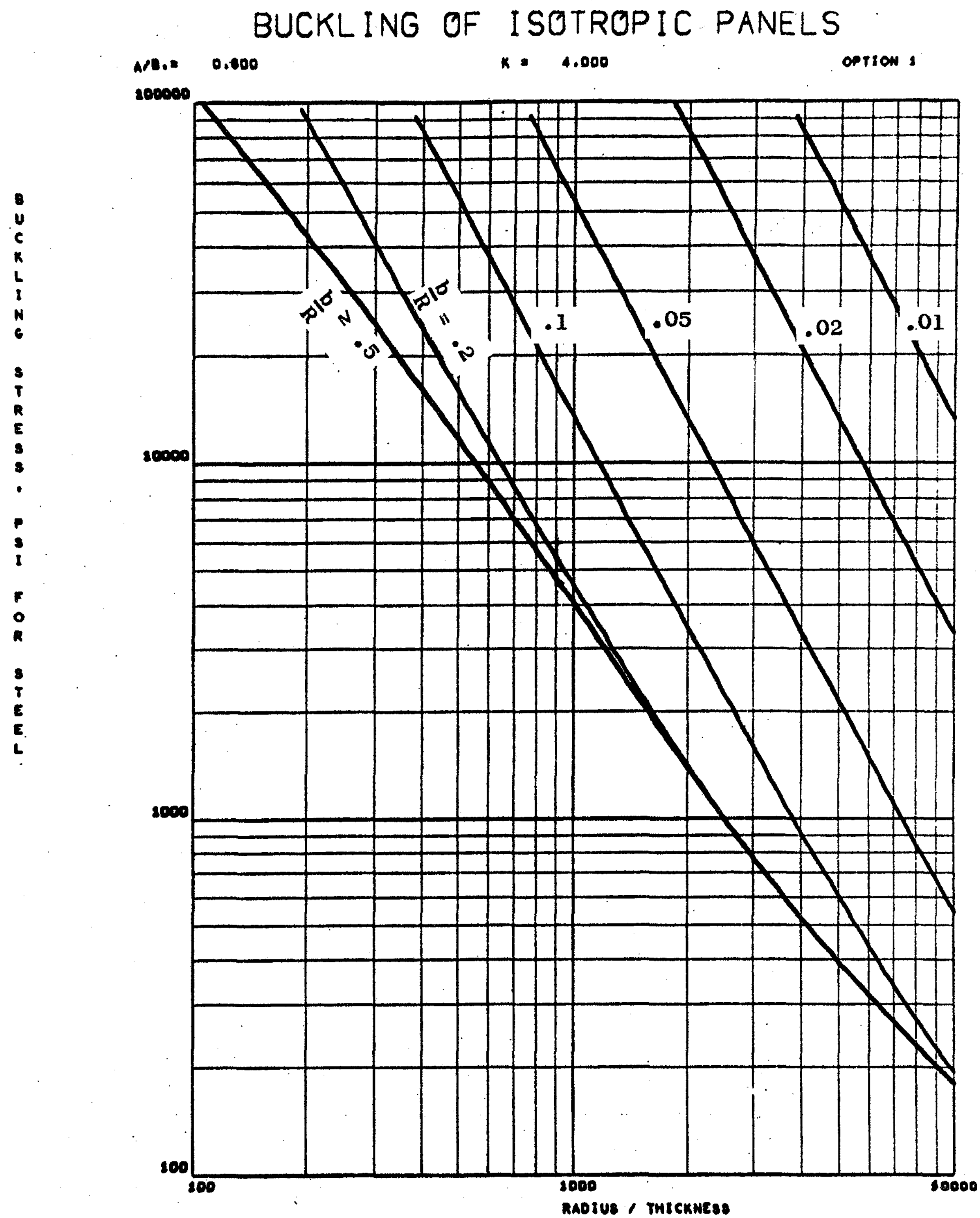

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.600
K = 4.000
OPTION 1
100000
10000
1000
100
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS, PSI FOR STEEL
b/R = .5
b/R = .2
.1
.05
.02
.01

Figure 5(k)

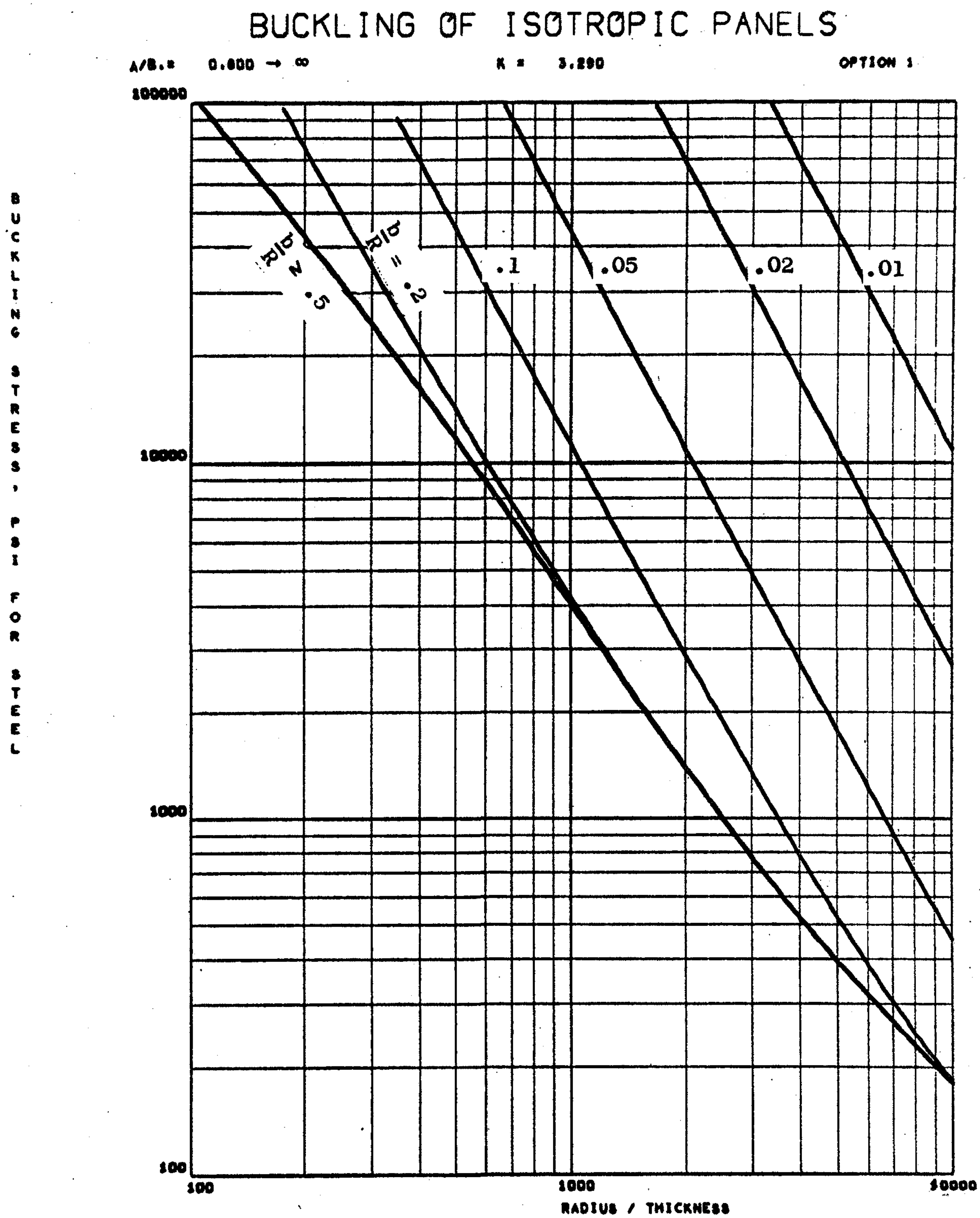

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.800 → ∞
K = 3.290
OPTION 1
BUCKLING STRESS, PSI FOR STEEL
RADIUS / THICKNESS
100000
10000
1000
100
100
1000
10000
b/R = .5
b/R = .2
.1
.05
.02
.01

Figure 5(1)

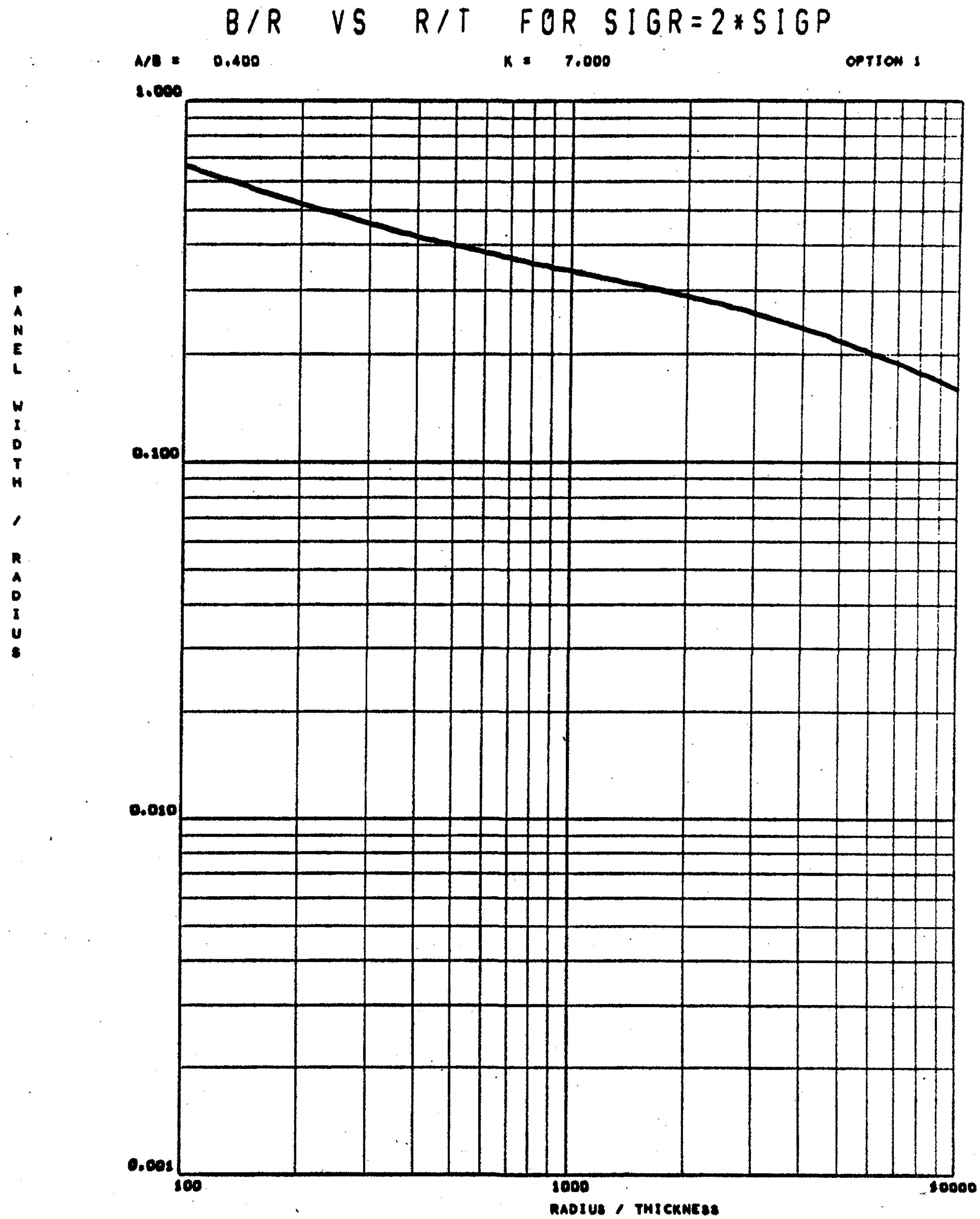

B/R VS R/T FOR SIGR=2*SIGP
A/B = 0.400
K = 7.000
OPTION 1
1.000
0.100
0.010
0.001
PANEL WIDTH / RADIUS
100
1000
10000
RADIUS / THICKNESS

Figure 6(a)

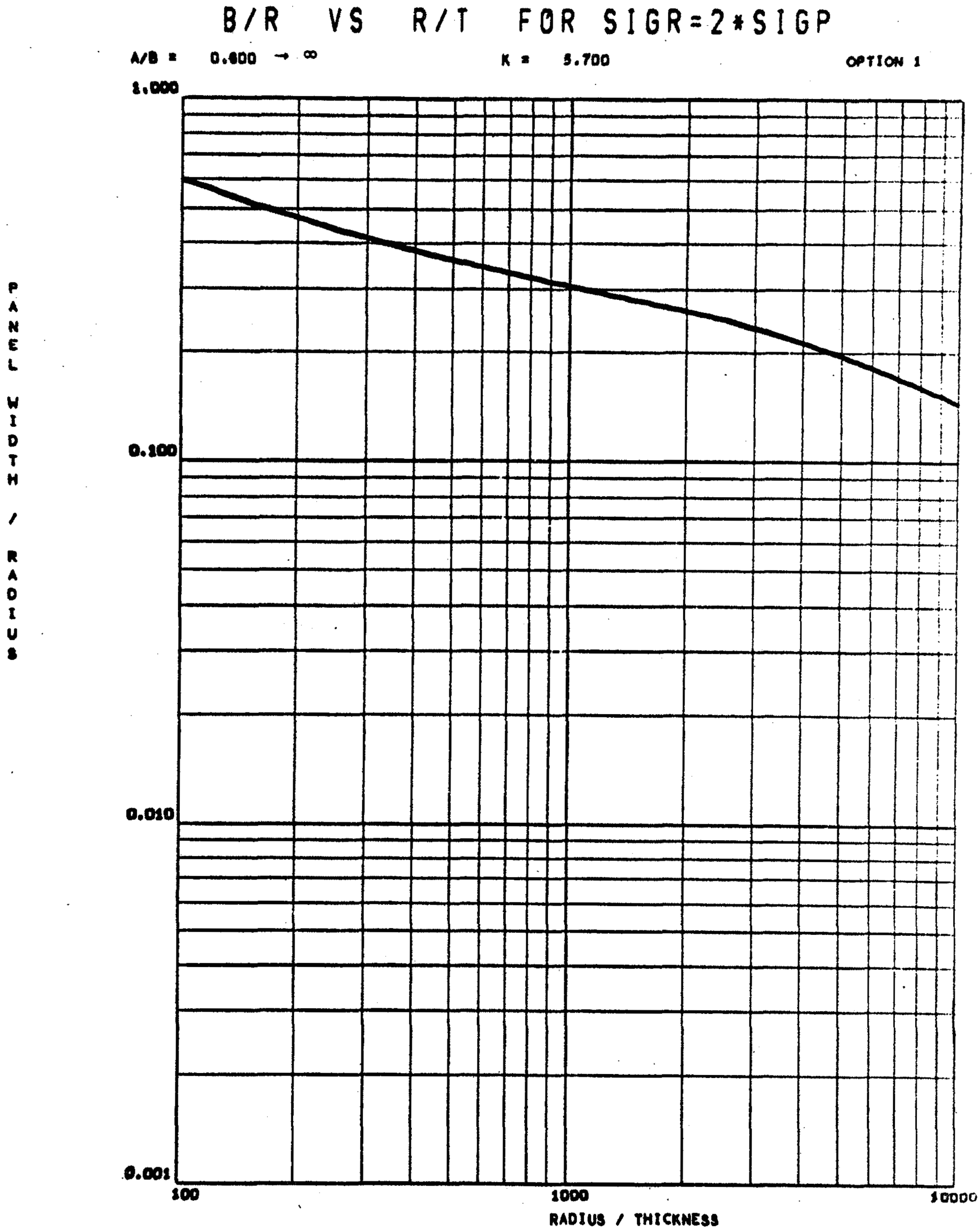

B/R VS R/T FOR SIGR=2*SIGP
A/B = 0.600 → ∞
K = 5.700
OPTION 1
1.000
0.100
0.010
0.001
PANEL WIDTH / RADIUS
100
1000
10000
RADIUS / THICKNESS

Figure 6(b)

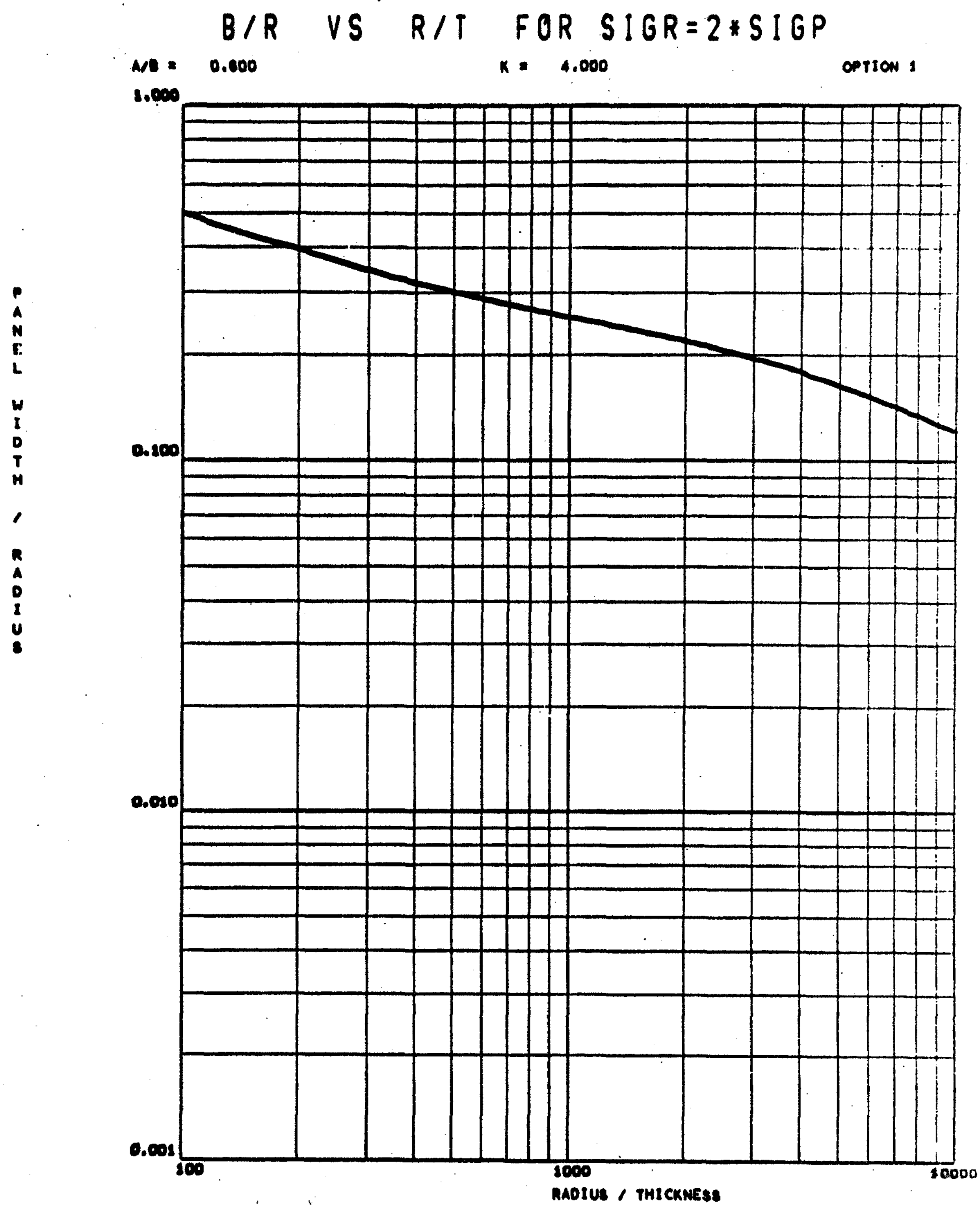

B/R VS R/T FOR SIGR=2*SIGP
A/B = 0.600
K = 4.000
OPTION 1
1.000
0.100
0.010
0.001
PANEL WIDTH / RADIUS
100
1000
10000
RADIUS / THICKNESS

Figure 6(c)

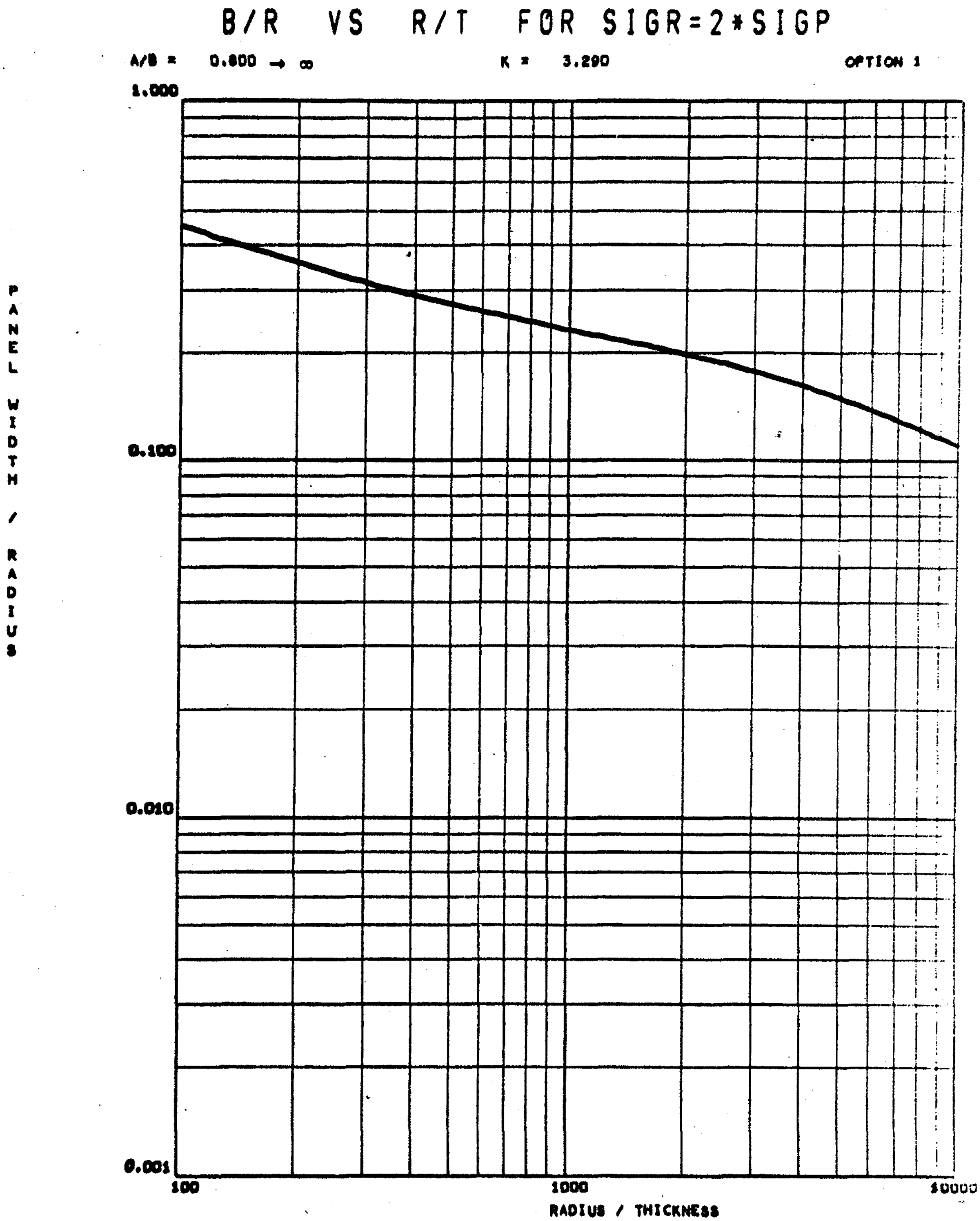

B/R VS R/T FOR SIGR=2*SIGP
A/B = 0.800 → ∞
K = 3.290
OPTION 1
1.000
0.100
0.010
0.001
100
1000
10000
RADIUS / THICKNESS
PANEL WIDTH / RADIUS

Figure 6(d)

SECTION 5

DIGITAL COMPUTER PROGRAM

This section presents the essential features of General Dynamics Convair digital computer program numbered 3875. This program was developed for the analysis of compressive buckling in curved isotropic skin panels which are bounded by the stringers and rings of stiffened cylinders. The solution is based upon the theoretical and empirical considerations presented in SECTION 2. The reliability of the output values is demonstrated by the test data comparisons given in SECTION 3. The output can be obtained in the form of automatically plotted buckling curves or single-point solutions, as desired. All of the buckling curves presented in this volume were obtained by using the automatic plotting option of this program. In addition to its primary application to curved plates, the program also includes capability for the analysis of flat plates and/or complete isotropic cylindrical shells.

The input format is shown in Figure 7 . Symbols are listed in Table V . A detailed, card-by-card discussion of the input follows below. Runs may be stacked.

CARD TYPE 1: One card per run. Each entry must be integer(s) right adjusted in appropriate columns.

Enter NAB (the number of a/b ratios to be entered on cards TYPE 2) in columns 1-5 (30 maximum).
Enter NBR (the number of b/R ratios to be entered on cards TYPE 3) in columns 6-10 (20 maximum).

FORTRAN CODING AND DATA FORM

PROBLEM BUCKLING ØF ISØTRØPIC PANELS | AWO | EWO
CODER L.S. FØSSUM | EXT. | DATE | PAGE 1 | OF 1

CARD TYPE:	1–10	11–20	21–30	31–40	41–50	51–60
1	NAB NBR	NBT NRTIN	NØP IBUG	K1 K2	K3	
2	AB_i	$CKNM_i$	$CAPK_i$			
3	BR_1	BR_2	BR_3	....	BR_{NBR-1}	BR_{NBR}
4	RT_1	RT_2	RT_3	....	$RT_{NRTIN-1}$	RT_{NRTIN}
5	YBSIG	YTSIG	YBBR	YTBR	YBSIGE	YTSIGE
6	BT_1	BT_2	BT_3	....	BT_{NBT-1}	BT_{NBT}

FORM A032 (4-58) CONVAIR | ASTRONAUTICS

Figure 7 - Input Format-Program 3875

Enter NBT (the number of b/t ratios to be entered on cards TYPE 6) in columns 11-15 (20 maximum). Enter NRTIN (the number of R/t ratios to be entered on cards TYPE 4) in columns 16-20 (99 maximum). Enter NOP (determines which OPTIONS are used) in column 25:

OPTIONS	NOP
1	4
2	2
3	1
1 and 2	6
1 and 3	5
2 and 3	3
1,2,and 3	7

As discussed in Section 2.2.2, OPTION 1 utilizes the lower bound relation of Seide, et al. [6] for σ_R and would be generally employed for design. OPTION 2 uses mean expected results for σ_R and OPTION 3 uses 90% probability, 95% confidence values for σ_R. If more than one OPTION is called for (NOP = 3,5,6, or 7), complete sets of output for each desired OPTION are obtained.

Enter IBUG (debugging and printout option) in column 30. If IBUG = 0 (or is blank), only calculations for the R/t values input on cards TYPE 2 will be printed out. If IBUG = 1 (or $\neq 0$), calculations will be run and printed for 101 values of R/t generated within the program whose logarithms are equally spaced.

Enter K1 (plotting option) in column 35. If K1 = 0 (or is blank), calculations will be run and logarithmic plots will be made using 101 values of R/t generated within the program whose logarithms are equally spaced. If K1 = 1, no plots will be made.

Enter K2 (header card option) in columns 36-40. This option allows the use of one TYPE 1 header card if applicable to a number of runs. If K2 is blank, 0, or 1, a TYPE 1 header card will be used as usual. If K2 > 1, say "n", then "n" runs may be stacked without repeating the header card TYPE 1. When "n" runs have been read, the program will then begin reading TYPE 1 cards, if additional runs are present.

Disregard K3 (not used).

CARD TYPE 2: There will be NAB cards per run.

Enter AB_i (i^{th} value of the panel length/width ratio a/b to be input) in columns 1-10 (E10.5).

Enter $CKNM_i$ (heading for i^{th} output listing and/or i^{th} plot) right adjusted ending in column 20: Hollerith characters KS= or KC= or KS=KC= (maximum of 6 characters).

Enter $CAPK_i$ (i^{th} value of KS or KC) in columns 21-30 (E10.5). Note that these three entries must correspond; i.e., the i^{th} TYPE 2 card shows a/b and K_s or K_c values which are related since K_s and K_c are functions of a/b [see equation (2-13)].

CARD TYPE 3: There will be NBR/6 (rounded to higher whole number) cards per run.

Enter BR values (panel width to radius ratio b/R, 6 to a card (6E10.5).

CARD TYPE 4: There will be NRTIN/6 (rounded to higher whole number) cards per run.

Enter RT values (panel radius to thickness ratio R/t), 6 to a card (6E10.5).

CARD TYPE 5: One card per run. (If K1 = 1 omit this card)

Enter YBSIG (lower limit of σ_{cr} for plots of Buckling Stress, psi vs. R/t) in columns 1-10 (E10.5).

Enter YTSIG (upper limit of σ_{cr} for plots of Buckling Stress, psi vs. R/t) in columns 11-20 (E10.5).

Disregard YBBR (not used).

Disregard YTBR (not used).

Enter YBSIGE (lower limit of σ_{cr}/E for plots of Buckling Stress/E vs. R/t) in columns 41-50 (E10.5).

Enter YTSIGE (upper limit of σ_{cr}/E for plots of Buckling Stress/E vs. R/t) in columns 51-60 (E10.5).

CARD TYPE 6: There will be NBT/6 (rounded to higher whole number) cards per run.

Enter BT values (panel width to thickness ratio b/t for flat panels only), 6 to a card (6E10.5). Omit this card if NBT = 0. These values will be used only if b/R = 0 (flat plate).

A sample input coding form is shown in Figure 8.

Approximate execution times at the General Dynamics Convair 7094 DCS installation were 1.5 seconds per a/b and b/R combination for NOP = 7 and K1 = 0.

The program output consists of a listing of all input in addition to calculated results. The calculated results (σ_{cr}/E, σ_{cr} for E = 10×10^6 psi and σ_{cr} for E = 30×10^6 psi) are tabulated (and/or plotted) for the input values of R/t and the 101 internally determined values of R/t, if desired. Separate tables (and/or plots) are formed for each a/b and $K_{s[c]}$, b/R, and OPTION combination selected on input. A column having the heading "BASIS" is also printed. The symbol I in this column indicates that the buckling stress was determined by equation (2-2) of Section 2.2. The symbol II in this column indicates that the buckling stress was determined by equation (2-4) of Section 2.2. The latter case indicates behavior of the panel to be similar to that of a complete cylinder of the same radius, thickness, and length.

A sample output listing is given in Figure 9 where the first and two typical pages are shown for the sample problem of Figure 8. Output plots from this sample solution are included in SECTION 4. A basic flow diagram for the program is given in Figure 10. Table VI presents a Fortran listing of the program, including input data from the sample problem.

TABLE V - Program 3875 Notation

PROGRAM NOTATION	REPORT NOTATION	DESCRIPTION
AB	a/b	Panel length/width ratio.
NAB	-	Number of a/b values input.
BR	b/R	Panel width/radius ratio.
NBR	-	Number of b/R values input.
BT	b/t	Panel width/thickness ratio.
NBT	-	Number of b/t values input.
CAPK	K_s or K_c	Buckling coefficient.
E1, E2	E	Elastic modulus, $10x10^6$ and $30x10^6$ psi respectively.
FMU	ν	Poisson's Ratio, $\nu = 0.3$.
NOP	-	Option No. (1, 2, or 3; or 4 meaning 1, 2, and 3).
RT	R/t	Radius/thickness ratio.
NRTIN	-	Number of R/t values input.
NRT	-	Total number of R/t values, both input and calculated.
SIGRE	σ_R/E	Nondimensional buckling parameter.
SIGPE	σ_p/E	Nondimensional buckling parameter
SIGCRE	σ_{cr}/E	Nondimensional buckling parameter
SIGCR1	σ_{cr} (for $E = 10x10^6$ psi)	Buckling stress (psi) for aluminum where $E = 10x10^6$ psi.
SIGCR2	σ_{cr} (for $E = 30x10^6$ psi)	Buckling stress (psi) for steel where $E = 30x10^6$ psi.

TABLE V - Program 3875 Notation (Cont'd.)

PROGRAM NOTATION	REPORT NOTATION	DESCRIPTION
YBSIGE	-	Lower limit of σ_{cr}/E for plots.
YTSIGE	-	Upper limit of σ_{cr}/E for plots.
YBSIG	-	Lower limit of SIGCR1 and SIGCR2 for plots,psi.
YTSIG	-	Upper limit of SIGCR1 and SIGCR2 for plots,psi.
IB	-	Basis number (1 or 2); e.g., BASIS(IB) = BASIS II for IB = 2, and $\sigma_{cr} = \sigma_R$.

GENERAL DYNAMICS CONVAIR DIVISION

5-10

GDC-DDG-67-006

FORTRAN CODING AND DATA FORM

PROBLEM: BUCKLING ØF ISØTRØPIC PANELS
CODER: L.S. FØSSUM
EXT.: DATE: AWO: EWO:
PAGE 1 OF 1

CARD TYPE: 1 (line 1); 2 (lines 2–5); 3 (lines 6–7); 4 (line 8); 5 (line 9)

```
    4    7    0    2    4    1
    .4        KS=KC=   7.0
    .6           KS=   4.0
    .6           KC=   5.7
    .8           KS=   3.29
    .01       .02       .05       .1        .2        .5
   1.0
 100.0    1000.0
 100.0    100000.0                          .00001    .01
```

FORM A032 (4-58) CONVAIR | ASTRONAUTICS

Figure 8 - Sample Input Data-Program 3875

```
                         LOCAL BUCKLING OF ISOTROPIC PANELS

NAB =   4   NBR =   7   NBT =   0   NRTIN =   2   NOP =   4   IBUG =   1   K1 =  -0   K2 =  -0   K3 =  -0

               PANEL ASPECT RATIO =   0.400            KS=KC=   7.000
               PANEL ASPECT RATIO =   0.600               KS=   4.000
               PANEL ASPECT RATIO =   0.600               KC=   5.700
               PANEL ASPECT RATIO =   0.800               KS=   3.290

                              PANEL WIDTH / RADIUS
     0.010000        0.020000        0.050000        0.100000        0.200000
     0.500000        1.000000

                              RADIUS / THICKNESS
       100.00         1000.00

YBSIG=   1.00E 02  YTSIG=   1.00E 05  YBBR=  -0.      YTBR=  -0.      YBSIGE=   1.00E-05  YTSIGE=   1.00E-02
```

Figure 9 - Sample Output Listing-Program 3875

OPTION 1

PANEL ASPECT RATIO= 0.80 KS= 3.290 PANEL WIDTH/RADIUS= 0.2000

RADIUS/THICKNESS	BUCKLING STRESS/E	BUCKLING STRESS,PSI E = 10,000,000	BUCKLING STRESS,PSI E = 30,000,000	BASIS
100.	9.3797E-03	9.3797E 04	2.8139E 05	I
1000.	1.4054E-04	1.4054E 03	4.2161E 03	I
100.	9.3797E-03	9.3797E 04	2.8139E 05	I
105.	8.5763E-03	8.5763E 04	2.5729E 05	I
110.	7.8428E-03	7.8428E 04	2.3529E 05	I
115.	7.1733E-03	7.1733E 04	2.1520E 05	I
120.	6.5619E-03	6.5619E 04	1.9686E 05	I
126.	6.0037E-03	6.0037E 04	1.8011E 05	I
132.	5.4940E-03	5.4940E 04	1.6482E 05	I
138.	5.0284E-03	5.0284E 04	1.5085E 05	I
145.	4.6032E-03	4.6032E 04	1.3810E 05	I
151.	4.2148E-03	4.2148E 04	1.2644E 05	I
158.	3.8600E-03	3.8600E 04	1.1580E 05	I
166.	3.5357E-03	3.5357E 04	1.0607E 05	I
174.	3.2394E-03	3.2394E 04	9.7183E 04	I
182.	2.9686E-03	2.9686E 04	8.9059E 04	I
191.	2.7211E-03	2.7211E 04	8.1632E 04	I
200.	2.4948E-03	2.4948E 04	7.4843E 04	I
209.	2.2878E-03	2.2878E 04	6.8634E 04	I
219.	2.0985E-03	2.0985E 04	6.2956E 04	I
229.	1.9254E-03	1.9254E 04	5.7762E 04	I
240.	1.7670E-03	1.7670E 04	5.3010E 04	I
251.	1.6220E-03	1.6220E 04	4.8661E 04	I
263.	1.4893E-03	1.4893E 04	4.4680E 04	I
275.	1.3679E-03	1.3679E 04	4.1036E 04	I
288.	1.2566E-03	1.2566E 04	3.7699E 04	I
302.	1.1548E-03	1.1548E 04	3.4643E 04	I
316.	1.0614E-03	1.0614E 04	3.1843E 04	I
331.	9.7589E-04	9.7589E 03	2.9277E 04	I
347.	8.9749E-04	8.9749E 03	2.6925E 04	I
363.	8.2561E-04	8.2561E 03	2.4768E 04	I
380.	7.5968E-04	7.5968E 03	2.2791E 04	I
398.	6.9921E-04	6.9921E 03	2.0976E 04	I
417.	6.4372E-04	6.4372E 03	1.9312E 04	I
437.	5.9280E-04	5.9280E 03	1.7784E 04	I
457.	5.4604E-04	5.4604E 03	1.6381E 04	I
479.	5.0310E-04	5.0310E 03	1.5093E 04	I
501.	4.6366E-04	4.6366E 03	1.3910E 04	I
525.	4.2742E-04	4.2742E 03	1.2823E 04	I
550.	3.9411E-04	3.9411E 03	1.1823E 04	I
575.	3.6349E-04	3.6349E 03	1.0905E 04	I
603.	3.3533E-04	3.3533E 03	1.0060E 04	I
631.	3.0944E-04	3.0944E 03	9.2831E 03	I
661.	2.8561E-04	2.8561E 03	8.5683E 03	I
692.	2.6369E-04	2.6369E 03	7.9106E 03	I
724.	2.4351E-04	2.4351E 03	7.3052E 03	I
759.	2.2493E-04	2.2493E 03	6.7479E 03	I
794.	2.0783E-04	2.0783E 03	6.2348E 03	I
832.	1.9207E-04	1.9207E 03	5.7622E 03	I
871.	1.7756E-04	1.7756E 03	5.3269E 03	I
912.	1.6420E-04	1.6420E 03	4.9260E 03	I

Figure 9 - Sample Output Listing-Program 3875 (Cont'd.)

955.	1.5188E-04	1.5188E 03	4.5565E 03	I
1000.	1.4054E-04	1.4054E 03	4.2161E 03	I
1047.	1.3008E-04	1.3008E 03	3.9024E 03	I
1096.	1.2044E-04	1.2044E 03	3.6133E 03	I
1148.	1.1156E-04	1.1156E 03	3.3469E 03	I
1202.	1.0338E-04	1.0338E 03	3.1013E 03	I
1259.	9.5832E-05	9.5832E 02	2.8750E 03	I
1318.	8.8878E-05	8.8878E 02	2.6664E 03	I
1380.	8.2469E-05	8.2469E 02	2.4741E 03	I
1445.	7.6562E-05	7.6562E 02	2.2969E 03	I
1514.	7.1119E-05	7.1119E 02	2.1336E 03	I
1585.	6.6102E-05	6.6102E 02	1.9831E 03	I
1660.	6.1480E-05	6.1480E 02	1.8444E 03	I
1738.	5.7221E-05	5.7221E 02	1.7166E 03	I
1820.	5.3298E-05	5.3298E 02	1.5990E 03	I
1905.	4.9685E-05	4.9685E 02	1.4906E 03	I
1995.	4.6348E-05	4.6348E 02	1.3904E 03	II
2089.	4.3253E-05	4.3253E 02	1.2976E 03	II
2188.	4.0384E-05	4.0384E 02	1.2115E 03	II
2291.	3.7723E-05	3.7723E 02	1.1317E 03	II
2399.	3.5256E-05	3.5256E 02	1.0577E 03	II
2512.	3.2968E-05	3.2968E 02	9.8904E 02	II
2630.	3.0847E-05	3.0847E 02	9.2542E 02	II
2754.	2.8881E-05	2.8881E 02	8.6642E 02	II
2884.	2.7057E-05	2.7057E 02	8.1171E 02	II
3020.	2.5365E-05	2.5365E 02	7.6096E 02	II
3162.	2.3796E-05	2.3796E 02	7.1387E 02	II
3311.	2.2339E-05	2.2339E 02	6.7017E 02	II
3467.	2.0987E-05	2.0987E 02	6.2960E 02	II
3631.	1.9730E-05	1.9730E 02	5.9191E 02	II
3802.	1.8563E-05	1.8563E 02	5.5690E 02	II
3981.	1.7478E-05	1.7478E 02	5.2434E 02	II
4169.	1.6469E-05	1.6469E 02	4.9406E 02	II
4365.	1.5529E-05	1.5529E 02	4.6587E 02	II
4571.	1.4654E-05	1.4654E 02	4.3962E 02	II
4786.	1.3838E-05	1.3838E 02	4.1514E 02	II
5012.	1.3077E-05	1.3077E 02	3.9231E 02	II
5248.	1.2366E-05	1.2366E 02	3.7099E 02	II
5495.	1.1702E-05	1.1702E 02	3.5107E 02	II
5754.	1.1081E-05	1.1081E 02	3.3244E 02	II
6026.	1.0500E-05	1.0500E 02	3.1499E 02	II
6310.	9.9550E-06	9.9550E 01	2.9865E 02	II
6607.	9.4439E-06	9.4439E 01	2.8332E 02	II
6918.	8.9641E-06	8.9641E 01	2.6892E 02	II
7244.	8.5132E-06	8.5132E 01	2.5540E 02	II
7586.	8.0890E-06	8.0890E 01	2.4267E 02	II
7943.	7.6895E-06	7.6895E 01	2.3069E 02	II
8318.	7.3130E-06	7.3130E 01	2.1939E 02	II
8710.	6.9578E-06	6.9578E 01	2.0873E 02	II
9120.	6.6223E-06	6.6223E 01	1.9867E 02	II
9550.	6.3053E-06	6.3053E 01	1.8916E 02	II
10000.	6.0054E-06	6.0054E 01	1.8016E 02	II

Figure 9 - Sample Output Listing-Program 3875 (Cont'd.)

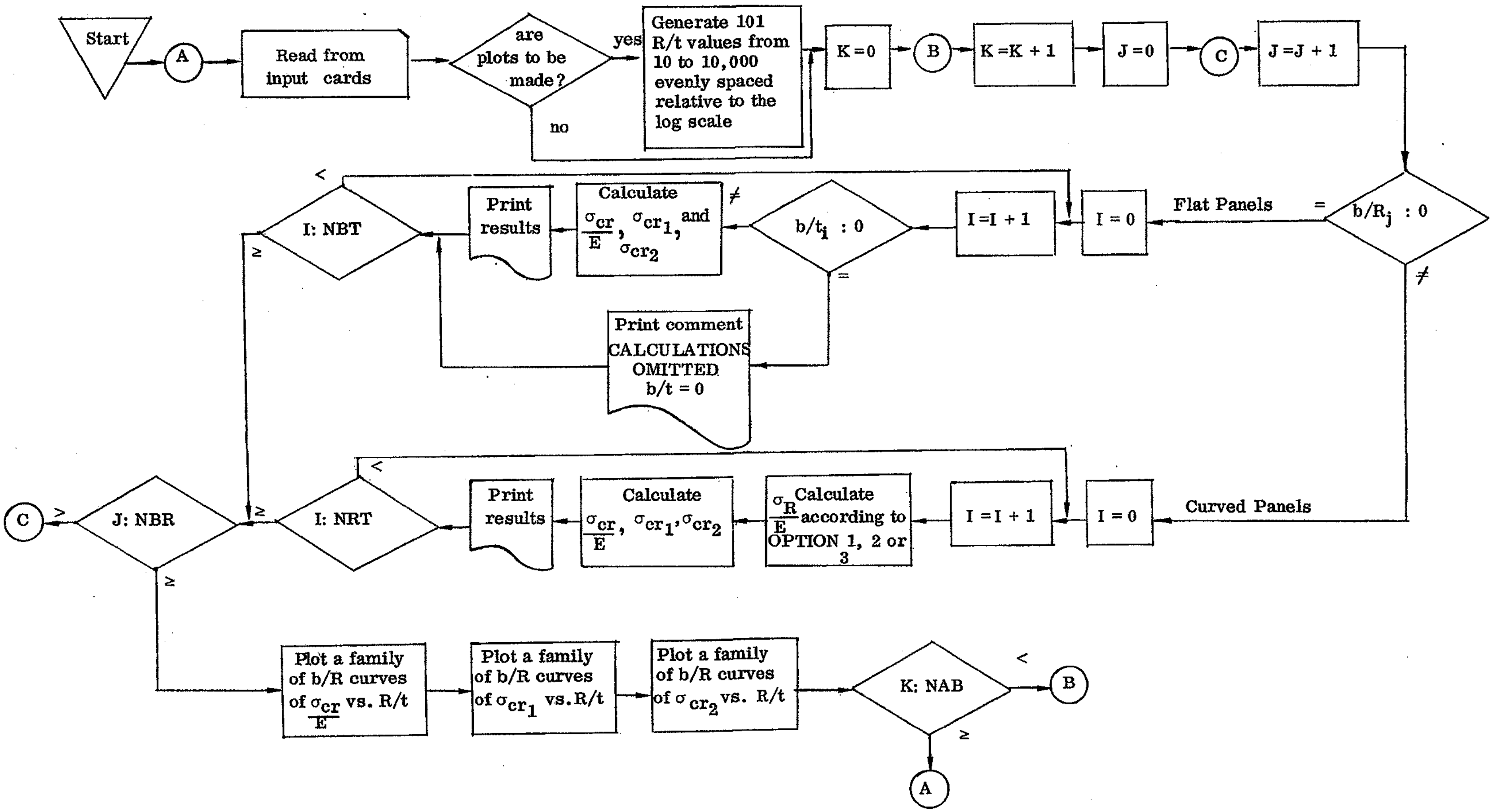


Figure 10 - Flow Diagram-Program 3875

TABLE VI - Fortran Listing-Program 3875

```
$SETUP LB4      DISK,PLOT01,SAVE
$EXECUTE        IBJOB
$IBJOB
$IBFTC MAIN
C       MAIN PROGRAM 3875 - LOCAL BUCKLING OF ISOTROPIC PANELS
C
  100 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
     1   E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
     2   RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
     3   YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
      COMMON /COMPUT/ C, IB, N, NRT, NRTC, OM1MU2, SIGRE, SIGPE,
     1   SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
  150 SKIP = 0.
  200 WRITE (6,201)
  201 FORMAT (1H1 /// 49X,34HLOCAL BUCKLING OF ISOTROPIC PANELS ////)
  250 CALL DATAIN (SKIP)
  300 OM1MU2 = 1.-FMU**2
  325 IF (K1.NE.1) GO TO 350
  326 NRT = NRTIN
  327 NRTC = 0
  340 GO TO 400
  350 CALL RTC (RT,NRTIN,NRT,NRTC)
      CALL SETMIV (288,0,24,24)
      CALL SMXYV (1,1)
  400 GO TO ( 650, 550, 550, 450, 450, 450, 450 ), NOP
  450 CALL CALC(1)
  500 IF ( NOP .EQ. 4 ) GO TO 700
  501 IF ( NOP .EQ. 5)  GO TO 650
  550 CALL CALC(2)
  600 IF (NOP.EQ.2) GO TO 700
  601 IF ( NOP .EQ. 6)  GO TO 700
  650 CALL CALC(3)
  700 CONTINUE
  720 GO TO 200
  750 END
$IBFTC BLKDAT
  110 BLOCK DATA
  111 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
     1   E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
     2   RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
     3   YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
  120 DATA  TITLE(1),   YLAB1(1),   YLAB2(1),   YLAB3(1),   XLAB1(1)
     1
     2  /     36HLOCAL BUCKLING OF ISOTROPIC PANELS   ,
     3        36H  BUCKLING STRESS, PSI FOR ALUMINUM        ,
     4                        36HBUCKLING STRESS / ELASTIC MODULUS    ,
     5        36H  BUCKLING STRESS, PSI FOR STEEL           ,
     6                        36H            RADIUS / THICKNESS          /
  121 DATA    (BASIS(IB), IB=1,2)  /6H  I    ,6H II       /,
     *        E1, E2, FMU  /1.E7, 3.E7, 0.30 /
  122 END
$IBFTC DATAIN
      SUBROUTINE DATAIN (SKIP)
C     SUBROUTINE TO READ IN AND PRINT OUT INPUT DATA.
```

TABLE VI - Fortran Listing-Program 3875 (Cont'd.)

```
C
   50 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
     1    E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
     2    RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
     3    YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
   51 COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
     1    SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
   75 IF (SKIP.NE.0.) GO TO 299
  100 READ (5,101) NAB, NBR, NBT, NRTIN, NOP, IBUG,  K1,  K2,  K3
  101 FORMAT (9I5)
  200 WRITE (6,201) NAB, NBR, NBT, NRTIN, NOP, IBUG,  K1,  K2,  K3
  201  FORMAT (4X,5HNAB =I5,4X,5HNBR =I5,4X,5HNBT =I5,4X,7HNRTIN =I5,4X,
     *    5HNOP =I5,4X,6HIBUG =I5,4X,5H K1 =I5,4X,5H K2 =I5,4X,5H K3 =I5)
  250 IF (K2.EQ.0) GO TO 300
  251 SKIP = K2 - 1
  252 GO TO 300
  299 SKIP = SKIP - 1.
  300 READ (5,301) (AB(K),CKNM(K),CAPK(K),K=1,NAB)
  301 FORMAT (E10.5,A10,E10.5)
  400 WRITE (6,401) (AB(K),CKNM(K),CAPK(K),K=1,NAB)
  401 FORMAT ( /// (33X,20HPANEL ASPECT RATIO =F10.3,16X,A10,F10.3) )
  500 READ (5,501) (BR(J),J=1,NBR)
  501 FORMAT (6E10.5)
  600 WRITE (6,601) (BR(J),J=1,NBR)
  601 FORMAT   ( /// 56X,20HPANEL WIDTH / RADIUS //(5F23.6))
  700 READ (5,701) (RT(I),I=1,NRTIN)
  701 FORMAT (6E10.5)
  800 WRITE (6,801) (RT(I),I=1,NRTIN)
  801 FORMAT   ( /// 57X,16HRADIUS / THICKNESS // (5F23.2) )
  850 IF (K1.EQ.1) GO TO 1050
  900 READ (5,901) YBSIG,YTSIG,YBBR,YTBR,YBSIGE,YTSIGE
  901 FORMAT ( 6E10.5 )
 1000 WRITE (6,1001) YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
 1001 FORMAT ( // 4X, 6HYBSIG=1PE12.2,   3X, 6HYTSIG=1PE12.2,
     1    3X, 5HYBBR=1PE12.2,   3X, 5HYTBR=1PE12.2,
     2    3X, 7HYBSIGE=1PE12.2,    3X, 7HYTSIGE=1PE12.2  )
 1050 IF (NBT.EQ.0) GO TO 2000
 1100 READ (5,1101) (BT(I),I=1,NBT)
 1101 FORMAT (6E10.5)
 1200 WRITE (6,1201) (BT(I),I=1,NBT)
 1201 FORMAT (/// 54X,23HPANEL WIDTH / THICKNESS // (6F23.5) )
 2000 RETURN
 3000 END
$IBFTC RTC
      SUBROUTINE RTC (RT,NRTIN,NRT,NRTC)
C     TO GENERATE RT(I)S FOR I FROM N1 TO N2,
C     EVENLY SPACED RELATIVE TO LOGID  SCALE.
      DIMENSION RT(200)
  100 N1 = NRTIN + 1
  150 NRT = NRTIN + 101
  200 RT(N1) = 100.
  250 RT(NRT) =10000.
  300 DLOG = .02
  350 ONE = 2.0
  400 NRTC = 101
  450 DO 650 I=1,99
  500 N1 = N1 +  1
  550 RTLOG = ONE + FLOAT(I)*DLOG
```

TABLE VI - Fortran Listing-Program 3875 (Cont'd.)

```
  600 RT(N1) = 10.**RTLOG
  650 CONTINUE
  700 RETURN
  750 END
$IBFTC CALC
      SUBROUTINE CALC(NN)
C     SUBROUTINE TO CALCULATE SIGMA CRITICAL AND SIGMA CRITICAL / E .
C     SUBROUTINE NOVA IS CALLED TO MAKE PLOTS.
C
   10 COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
    1   E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
    2   RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
    3   YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
   11 COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
    1   SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
      N =NN
   50 DO 1090 K=1,NAB
   70 DO 1030 J=1,NBR
   71 IF (BR(J).NE.0.0) GO TO 90
   72 WRITE (6,73) N, AB(K), CKNM(K), CAPK(K), BR(J)
   73 FORMAT (1H1,62X,7HOPTION ,I1 /// 14X,19HPANEL ASPECT RATIO=F8.2,
    1   14X,A10,F8.3,15X,19HPANEL WIDTH/RADIUS=F10.4 ///
    2   11X,21HPANEL WIDTH/THICKNESS 9X,17HBUCKLING STRESS/E 13X,
    3   19HBUCKLING STRESS,PSI 11X,19HBUCKLING STRESS,PSI/
    4  73X,14HE = 10,000,000  16X,14HE = 30,000,000  /// )
   74 DO 84  I=1,NBT
   75 IF (BT(I).EQ.0.0) GO TO 82
   76 SIGCRE(I,J) = CAPK(K) / OMIMU2 * (1./BT(I))**2
   77 SIGCR1(I,J) = SIGCRE(I,J) * E1
   78 SIGCR2(I,J) = SIGCRE(I,J) * E2
   79 WRITE (6,80) BT(I), SIGCRE(I,J), SIGCR1(I,J), SIGCR2(I,J)
   80 FORMAT (1H  F25.4, 3(1PE30.4))
   81 GO TO 84
   82 WRITE (6,83) BT(I)
   83 FORMAT (1H  F25.4, 10X, 31H CALCULATIONS OMITTED, B/T = 0.)
   84 CONTINUE
   85 GO TO 1030
   90 WRITE (6,91) N, AB(K), CKNM(K), CAPK(K), BR(J)
   91 FORMAT (1H1,62X,7HOPTION ,I1 /// 14X,19HPANEL ASPECT RATIO=F8.2,
    1   14X,A10,F8.3,15X,19HPANEL WIDTH/RADIUS=F10.4 ///
    2   7X,16HRADIUS/THICKNESS,11X,17HBUCKLING STRESS/E,12X,
    3   19HBUCKLING STRESS,PSI,11X,19HBUCKLING STRESS,PSI,6X,5HBASIS /
    4   66X,14HE = 10,000,000 16X,14HE = 30,000,000 // )
   92 NOP = NOP
  100 DO 1020 I=1,NRT
  120 C=0.605-(0.546*(1.-EXP(-1./16.*SQRT(RT(I)))))
  130 GO TO (150,210,350),N
  150 SIGRE = C*(1./RT(I))
  190 GO TO 490
  210 IF (AB(K)*BR(J).GE.1.0) GO TO 290
  230 SIGRE = 11.28*RT(I)**(-1.6)  +  0.109*(RT(I)*AB(K)*BR(J))**(-1.3)
  270 GO TO 490
  290 SIGRE = 11.28*RT(I)**(-1.6)  +  0.109*(RT(I)*AB(K)*BR(J))**(-1.3)
    *  -  1.418 * RT(I)**(-1.6) * ALOG (AB(K)*BR(J))
  330 GO TO 490
  350 IF (AB(K)*BR(J).GE.1.0) GO TO 430
  370 SIGRE = 8.01*RT(I)**(-1.6)  +  0.076*(RT(I)*AB(K)*BR(J))**(-1.3)
  410 GO TO 490
```

TABLE VI - Fortran Listing-Program 3875 (Cont'd.)

```
  430 SIGRE = 7.52 * RT(I)**(-1.6) + 0.072 * (RT(I)*AB(K)*BR(J))**(-1.3)
     *   - 1.418 *RT(I)**(-1.6) * ALOG (AB(K)*BR(J))
  490 SIGPE = CAPK(K) / OMIMU2 * (1./(RT(I)**2 * BR(J)**2))
  630 IF (SIGRE .LE. 2.*SIGPE) GO TO 730
  650 IB = 2
  670 SIGCRE(I,J) = SIGRE
  710 GO TO 770
  730 IB = 1
  750 SIGCRE(I,J) = SIGPE   +   SIGRE**2  /  (4. * SIGPE)
  770 CONTINUE
  780 SIGCR1(I,J) = SIGCRE(I,J) * E1
  790 SIGCR2(I,J) = SIGCRE(I,J) * E2
  810 IF (IBUG.NE.0) GO TO 1010
  830 IF (I.LE.NRTIN) GO TO 1010
  850 GO TO 1020
 1010 WRITE (6,1011) RT(I),SIGCRE(I,J),SIGCR1(I,J),SIGCR2(I,J),BASIS(IB)
 1011 FORMAT (1H ,F17.0,3(1PE30.4),A16)
 1020 CONTINUE
 1030 CONTINUE
 1050 IF (K1.EQ.1) GO TO 1090
 1070 CALL NOVA(K)
 1090 CONTINUE
 2010 RETURN
 2030 END
$IBFTC NOVA
      SUBROUTINE NOVA (K)
      COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
     1   E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
     2   RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
     3   YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
      COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
     1   SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
C
C     TO PLOT THREE FRAMES WITH NBR CURVES EACH FRAME
C        LOG-LOG SCALE FOR ALL PLOTS
C
C       PLOT 1 --- SIGCR1 VS R/T    (E=10,000,000)
C       PLOT 2 --- SIGCR2 VS R/T    (E=30,000,000)
C       PLOT 3 --- SIGCRE VS R/T
C
       CALL SETMIV(288,0,24,60)
C
C
C
      CALL SUBPLT (SIGCR1, YLAB1, YBSIG, YTSIG, K )
      CALL SUBPLT (SIGCR2, YLAB3, YBSIG, YTSIG, K )
      CALL SUBPLT (SIGCRE, YLAB2, YBSIGE, YTSIGE, K )
      RETURN
      END
$IBFTC SUBPLT
       SUBROUTINE SUBPLT (F, YNAME, YB,YT,K)
      COMMON /INPUT/ AB(30),BASIS(2),BR(20),BT(20),CAPK(30),CKNM(30),
     1   E1, E2, FMU, IBUG,  K1,  K2,  K3, NAB, NBR, NOP, NRTIN, NBT,
     2   RT(200), TITLE(12), XLAB1(12), YLAB1(12), YLAB2(12), YLAB3(12),
     3   YBSIG, YTSIG, YBBR, YTBR, YBSIGE, YTSIGE
      COMMON /COMPUT/ C, IB, N, NRT, NRTC, OMIMU2, SIGRE, SIGPE,
     1   SIGCRE(200,20), SIGCR1(200,20), SIGCR2(200,20)
      DIMENSION  OPT (2,3)
```

TABLE VI - Fortran Listing-Program 3875 (Cont'd.)

```
      DIMENSION F(200, 20) , YNAME(12)
      DATA (OPT(1,I),I=1,3)/ 12H OPTION 1    , 12H OPTION 2    ,
     *  12H OPTION 3          /
C
      AA= AB(K)
      CC=CAPK(K)
  102 XR = 1.E4
  103 XL =100.
       N=N
      CALL GRID1V(4,XL,XR,YB,YT, 1., 1., 0, 0, 0, 0, 5, 6)
C      PRINT TOP TITLE, VERT,  HORIZ
      CALL APRNTV (0, -16,  36, YNAME,   200, 900)
      CALL PRINTV (36, XLAB1,  600, 12)
      CALL PRINTV(-6, 5HA/B.= ,300, 980 )
      CALL PRINTV(-6, 5H  K = ,600, 980 )
      CALL PRINTV(12,OPT(1,N) ,900, 980 )
      CALL LABLV ( AA,360,980, 6, 1,3)
      CALL LABLV  (CC, 660,980,6,1,2)
      CALL RITE2V (380,1012,1023,90,1, 36, -1, 36HLOCAL BUCKLING OF ISOT
     *ROPIC PANELS   , NLAST )
      DO  150    J=1,NBR
      NB=NRTIN+1
      IF (BR(J)) 150,150,101
  101 DO 100      I=NB,NRT
      IF (F(I,J) .LE. YT) GO TO 105
  100 CONTINUE
      GO TO 150
  105 NB=I+1
      DO 106  I=NB,NRT
      IY = NRT+NB -I
      IF (F(IY,J) .GE. YB) GO TO 110
  106 CONTINUE
      GO TO 150
  110 NE=IY
      RTT= RT(NB-1)
      FF = F (NB-1,J)
      IX1 = NXV(RTT)
      IY1 = NYV( FF)
      DO 140 I=NB,NE
       RTT=   RT(I)
      FF=  F(I,J)
      IX2 = NXV(RTT)
      IY2 = NYV( FF)
      DO 130 NF= 1,5
      CALL LINEV  (IX1, IY1, IX2, IY2)
  130 CONTINUE
      IX1=IX2
      IY1=IY2
  140 CONTINUE
  150 CONTINUE
  200 RETURN
      END
@        END OF FILE OR DATA CARD  DATA FOLLOWS
    4    7    0    2    4    1
   .4        KS=KC=   7.0
   .6           KS=   4.0
   .6           KC=   5.7
   .8           KS=   3.29
```

TABLE VI - Fortran Listing-Program 3875 (Cont'd.)

```
    .01        .02        .05        .1         .2         .5
   1.
100.      1000.
100.      100000.                              .00001     .01
```

SECTION 6

REFERENCES

1. Smith, G. W. and Spier, E. E., "The Stability of Eccentrically Stiffened Circular Cylinders, Volume I - General", Contract NAS8-11181, General Dynamics Convair Division Report No. GDC-DDG-67-006, 20 June 1967.
2. Schapitz, E., Festigkeitslehre für den Leichtbau, 2 Aufl., VDI-Verlag GmbH Düsseldorf, 1963.
3. Anonymous, "Buckling of Thin-Walled Circular Cylinders", NASA Space Vehicle Design Criteria, NASA SP-8007, September 1965.
4. Spier, E. E. and Smith, G. W., "The Schapitz Criterion for the Elastic Buckling of Isotropic Simply Supported Cylindrical Skin Panels Subjected to Axial Compression", Contract NAS8-11181, General Dynamics Convair Division Memo AS-D-1029, 31 January 1967.
5. Gerard, G. and Becker, H., "Handbook of Structural Stability, Part I - Buckling of Flat Plates", NACA TN 3781, July 1957.
6. Seide, P., Weingarten, V. I., and Morgan, E. J., "Final Report on the Development of Design Criteria for Elastic Stability of Thin Shell Structures", STL-TR-60-0000-19425, 31 December 1960.
7. Schumacher, J. G., "Development of Design Curves for the Stability of Thin Pressurized and Unpressurized Circular Cylinders", General Dynamics Convair Report No. AZS-27-275, Revision B, 22 July 1960.
8. Peterson, J. P., Whitley, R. O., and Deaton, J. W., "Structural Behavior and Compressive Strength of Circular Cylinders with Longitudinal Stiffening", TN-D-1251, May 1962.
9. Cox, H. L. and Clenshaw, W. J., "Compression Tests on Curved Plates of Thin Sheet Duralumin", British A.R.C. Technical Report R. & M. No. 1894, November 1941.

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10. Crate, H. and Levin, L. R., "Data on Buckling Strength of Curved Sheet in Compression", NACA WR L-557, 1943.

11. Jackson, K. B. and Hall, A. H., "Curved Plates in Compression", National Research Council of Canada Aeronautical Report AR-1, 1947.

12. Peterson, J. P. and Whitley, R. O., "Local Buckling of Longitudinally Stiffened Curved Plates", NASA TN D-750, April 1961.

APPENDIX A

SUPPLEMENTARY OPTIONS

A.1 OPTION 2

A.1.1 GENERAL

All of the curves presented in Figure 11 are based upon substitution of the OPTION 2 full cylinder equations [see equations (2-9) and (2-10)] into the Schapitz criterion [2]. These curves were obtained by using digital computer program 3875 and an automatic plotter. The program is described in SECTION 5 and can be used to obtain additional plots and/or single-point solutions, as desired. Since the automatic plotting machine does not provide any capability for the print-out of lower case letters, the ratio (a/b) appears on all the given buckling curves as (A/B). In view of the fact that the OPTION 2 full cylinder equations constitute a mean-expected criterion, the curves given here may be of value in the comparison of predictions versus test data. In addition, when used in conjunction with the buckling curves based on OPTIONS 1 or 3, these plots give an indication of the dispersion which might be expected among a large array of test data.

A.1.2 BUCKLING CURVES

Table VII lists the families provided here. All of these curves are based on elastic behavior and a Poisson's ratio of 0.30.

TABLE VII - Table of Contents for the Supplementary Buckling Curves Based on OPTION 2

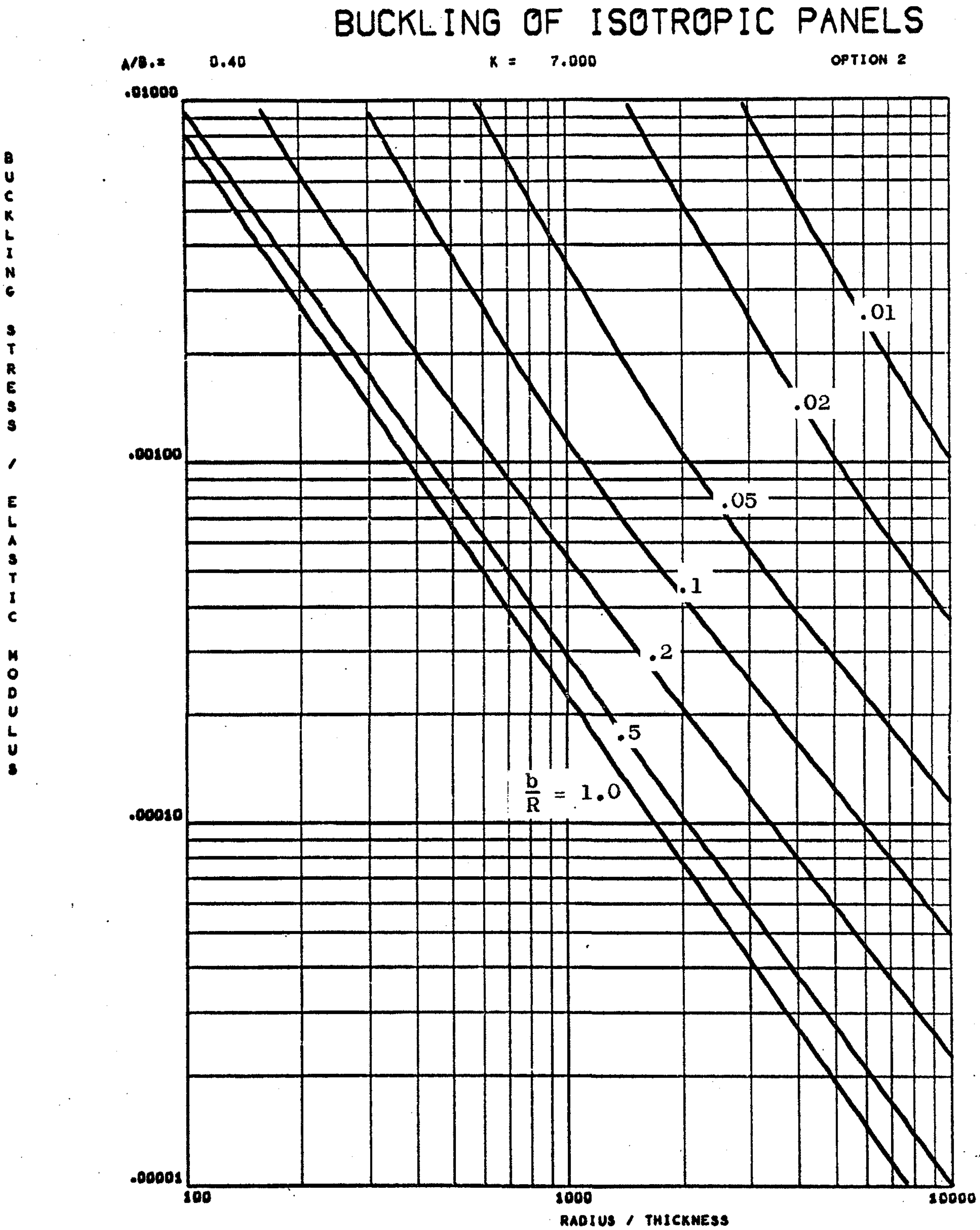

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.40
K = 7.000
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 11(a)

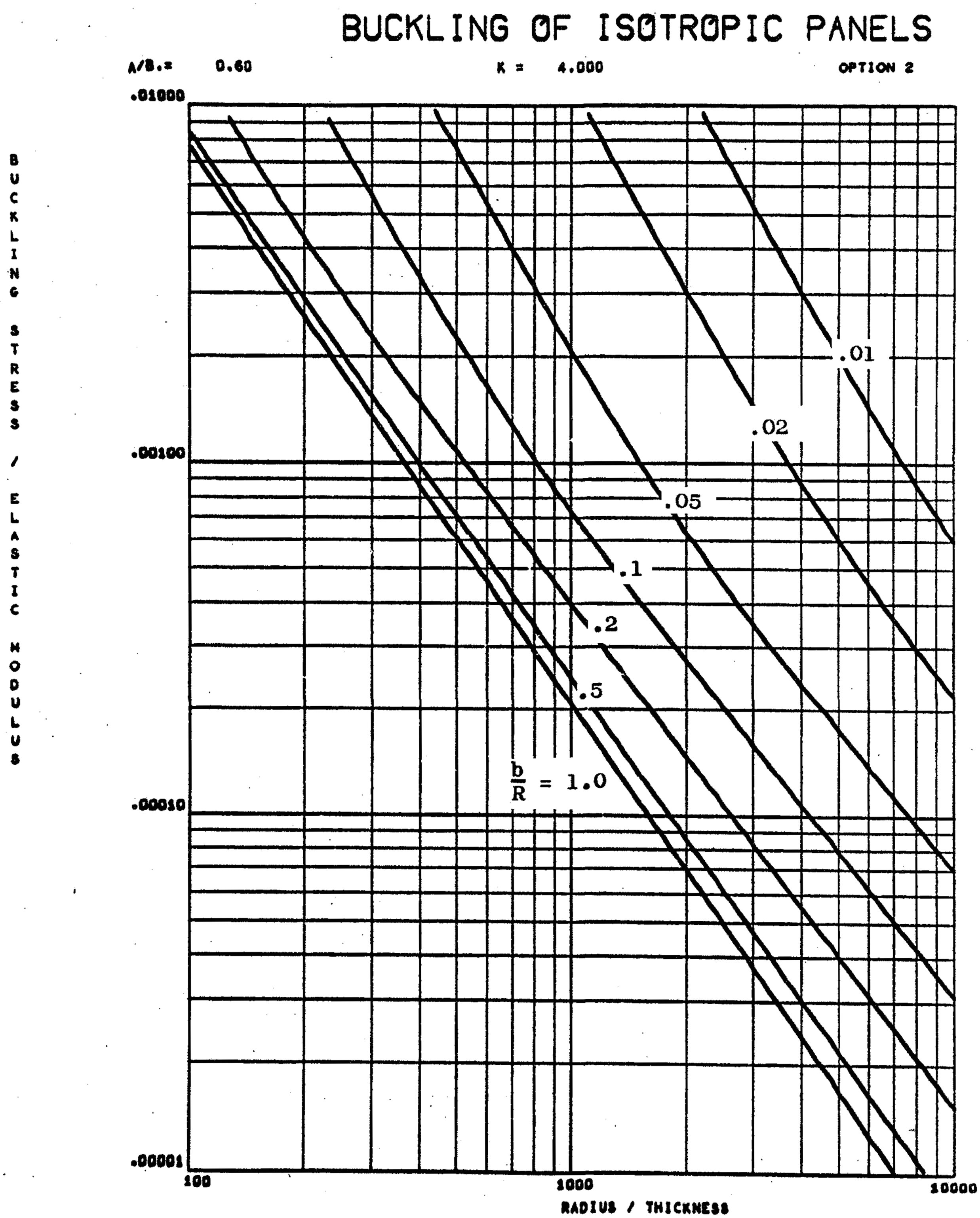

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 4.000
OPTION 2
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(b)

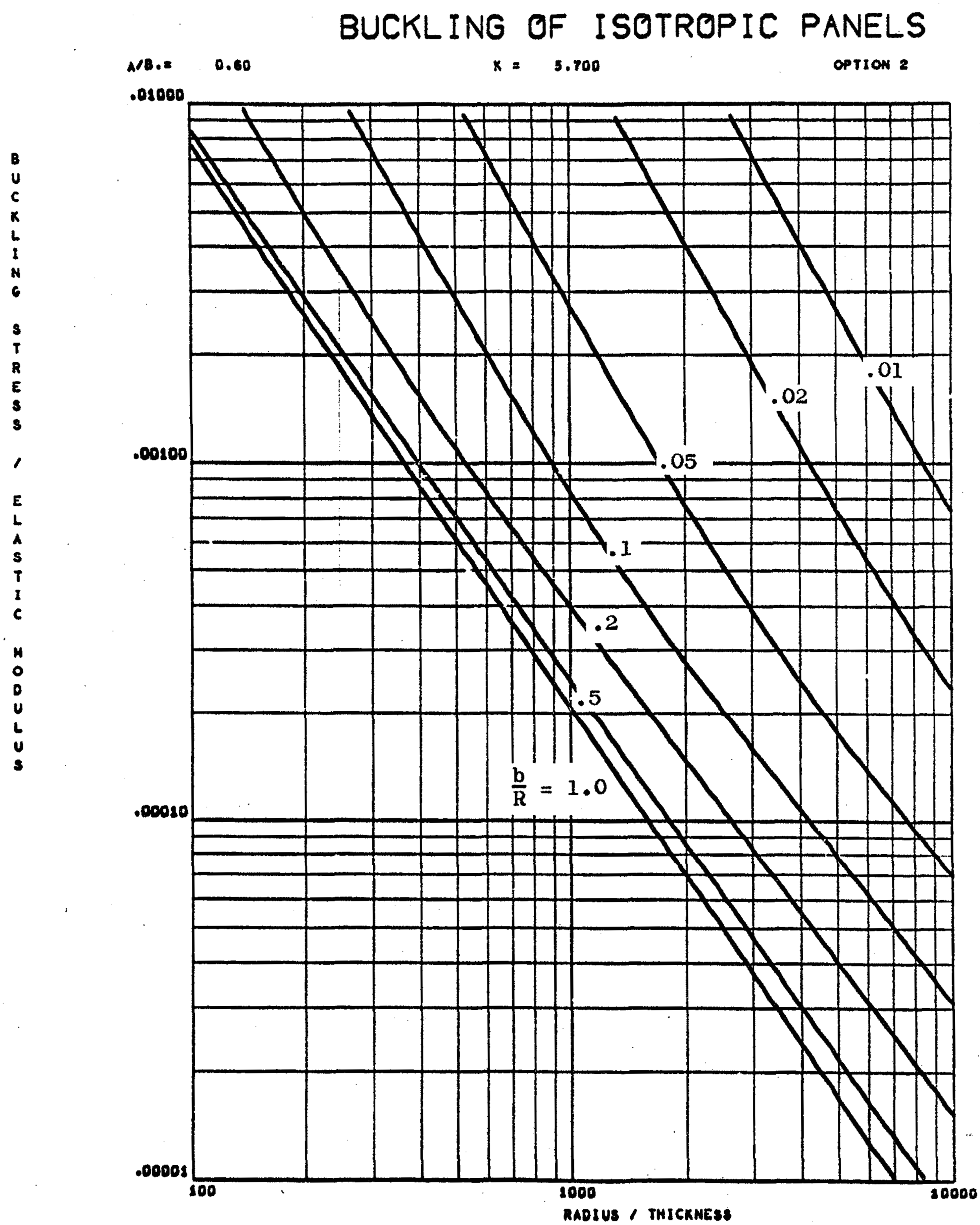

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 5.700
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(c)

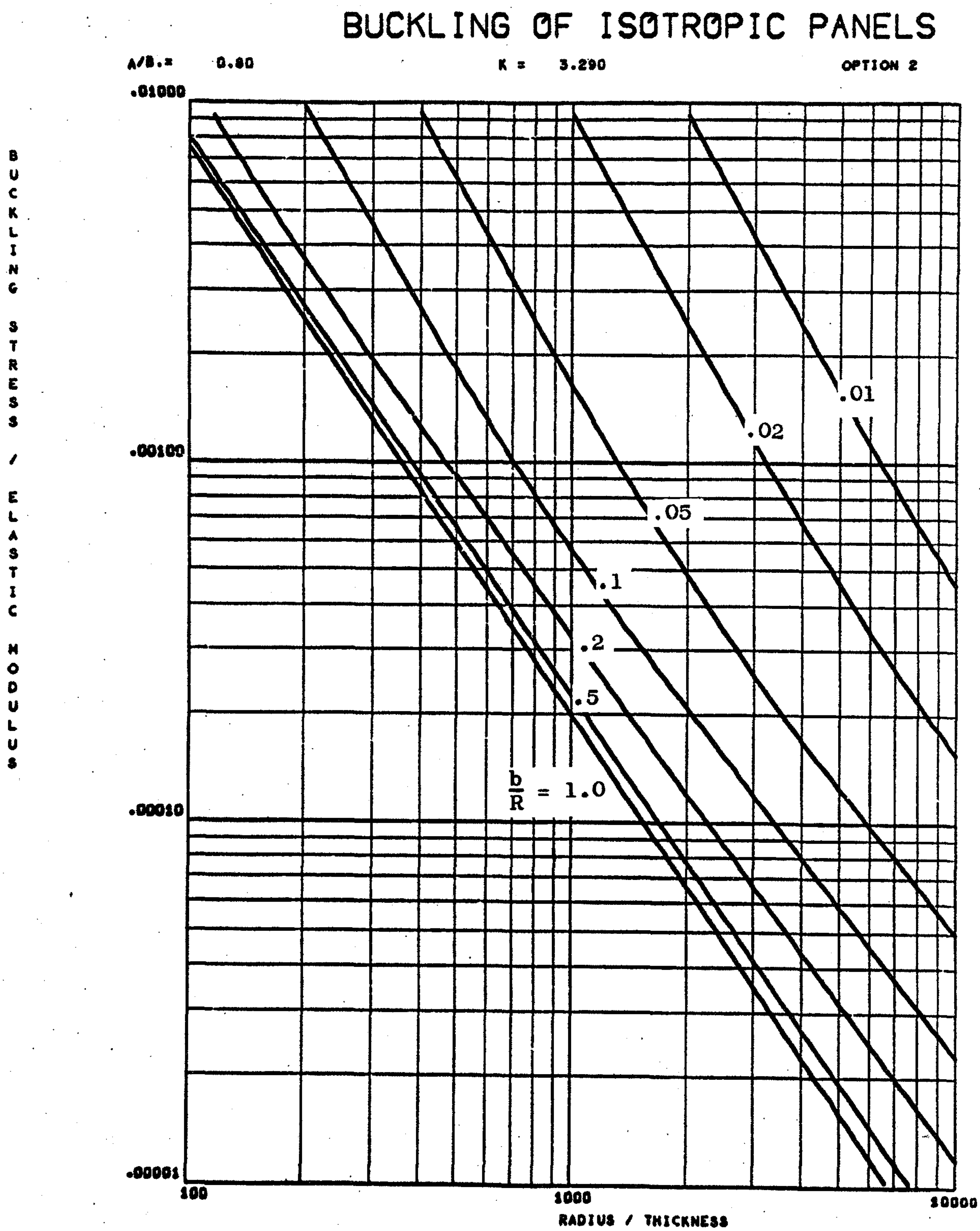

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.80
K = 3.290
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(d)

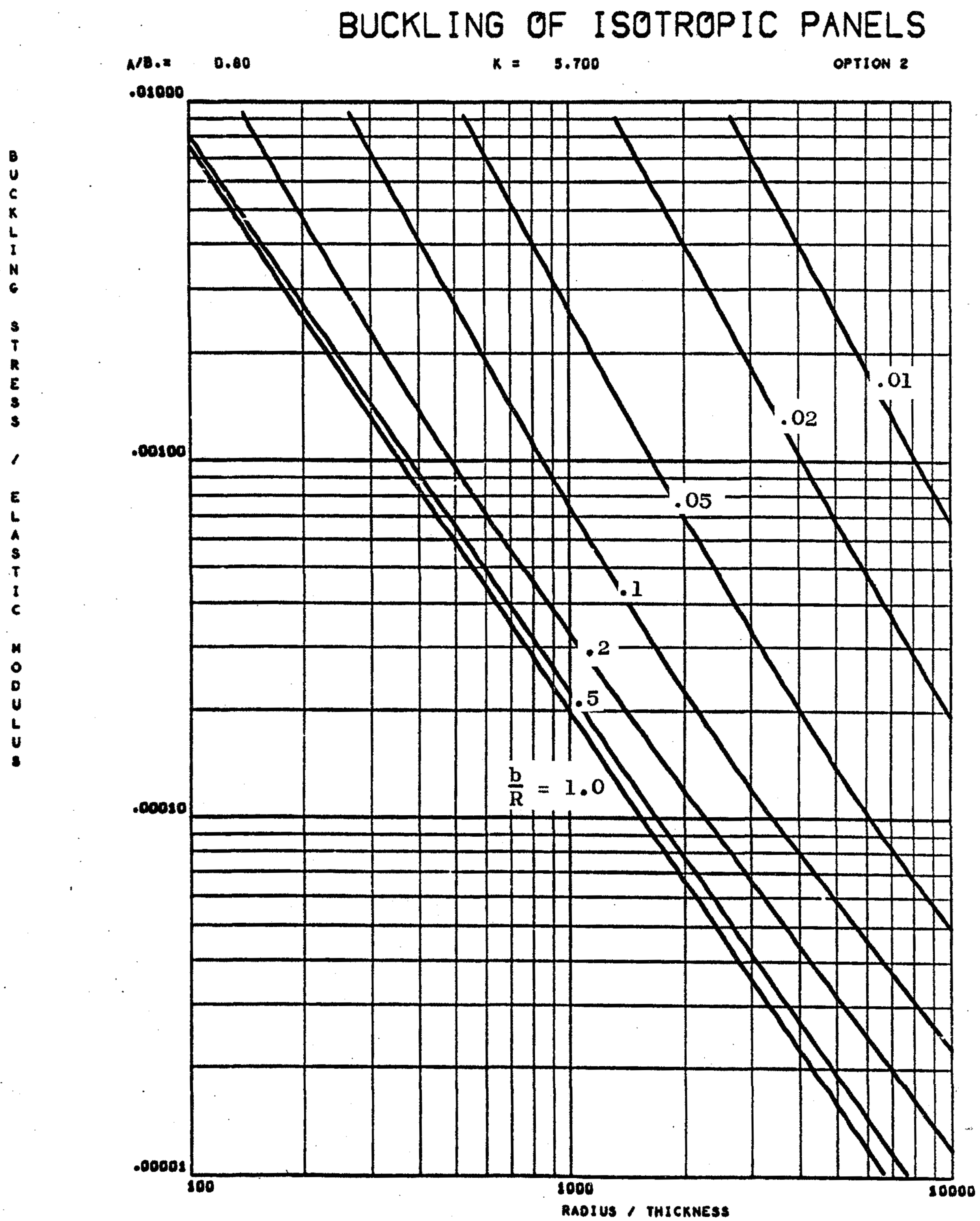

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.80
K = 5.700
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(e)

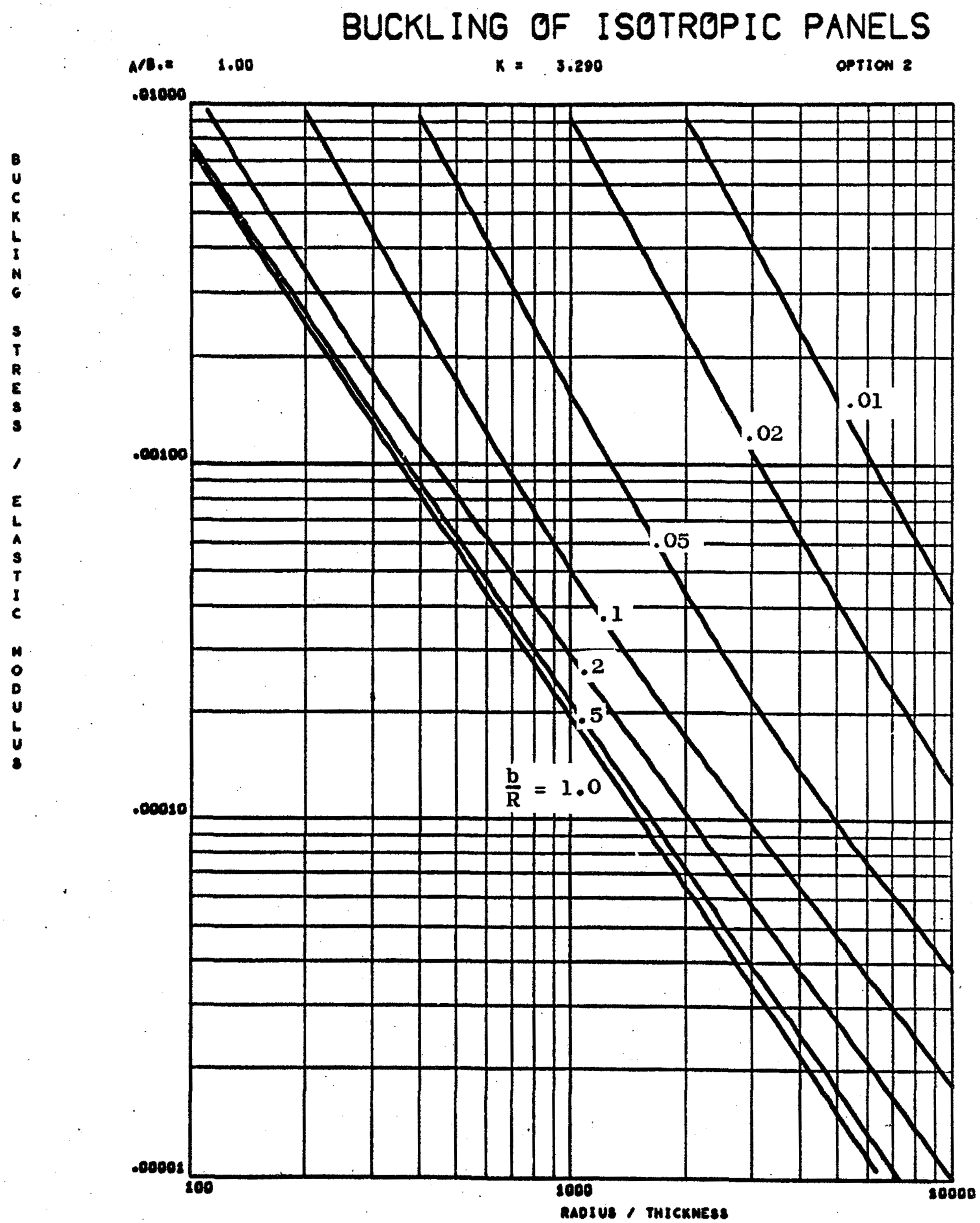
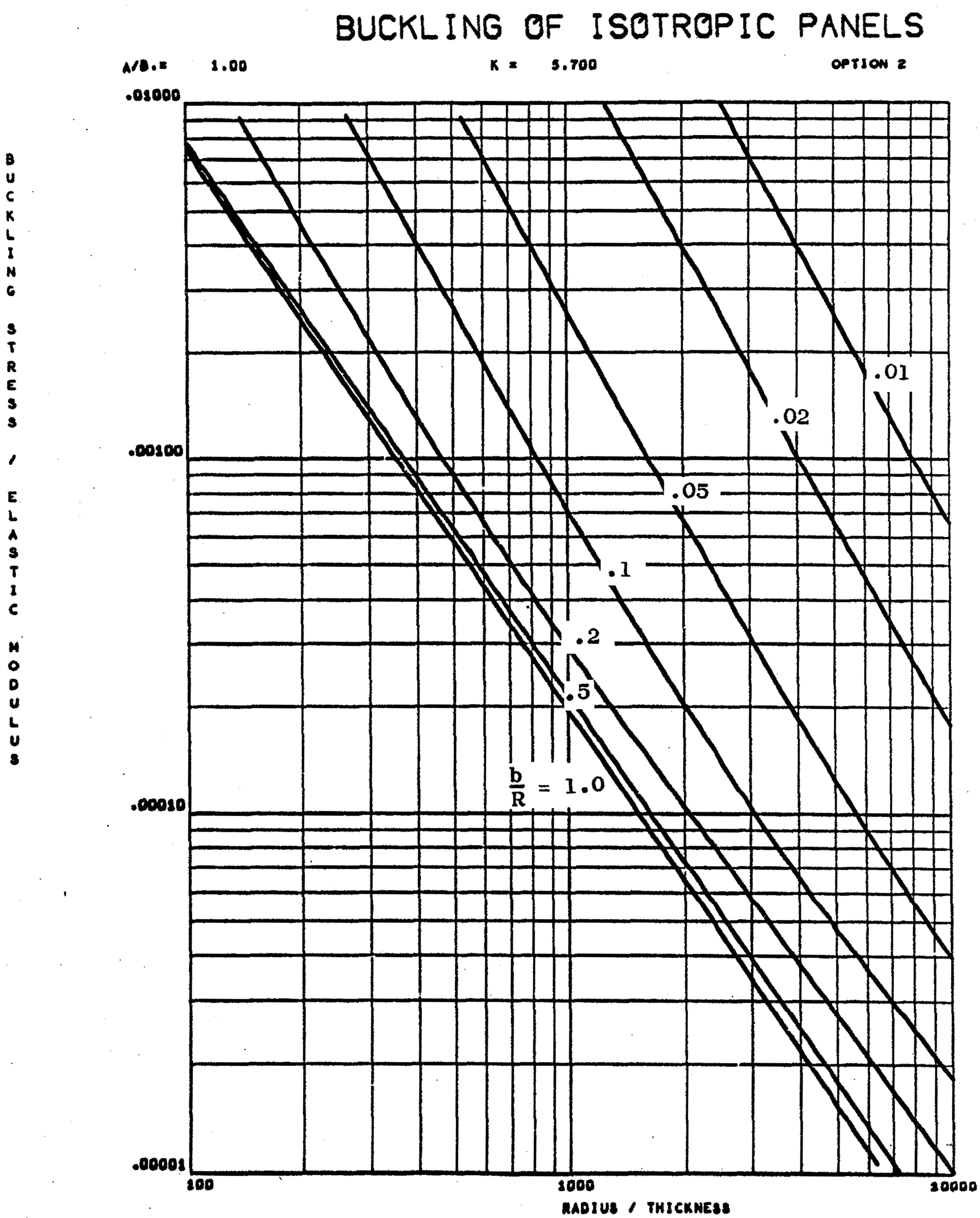
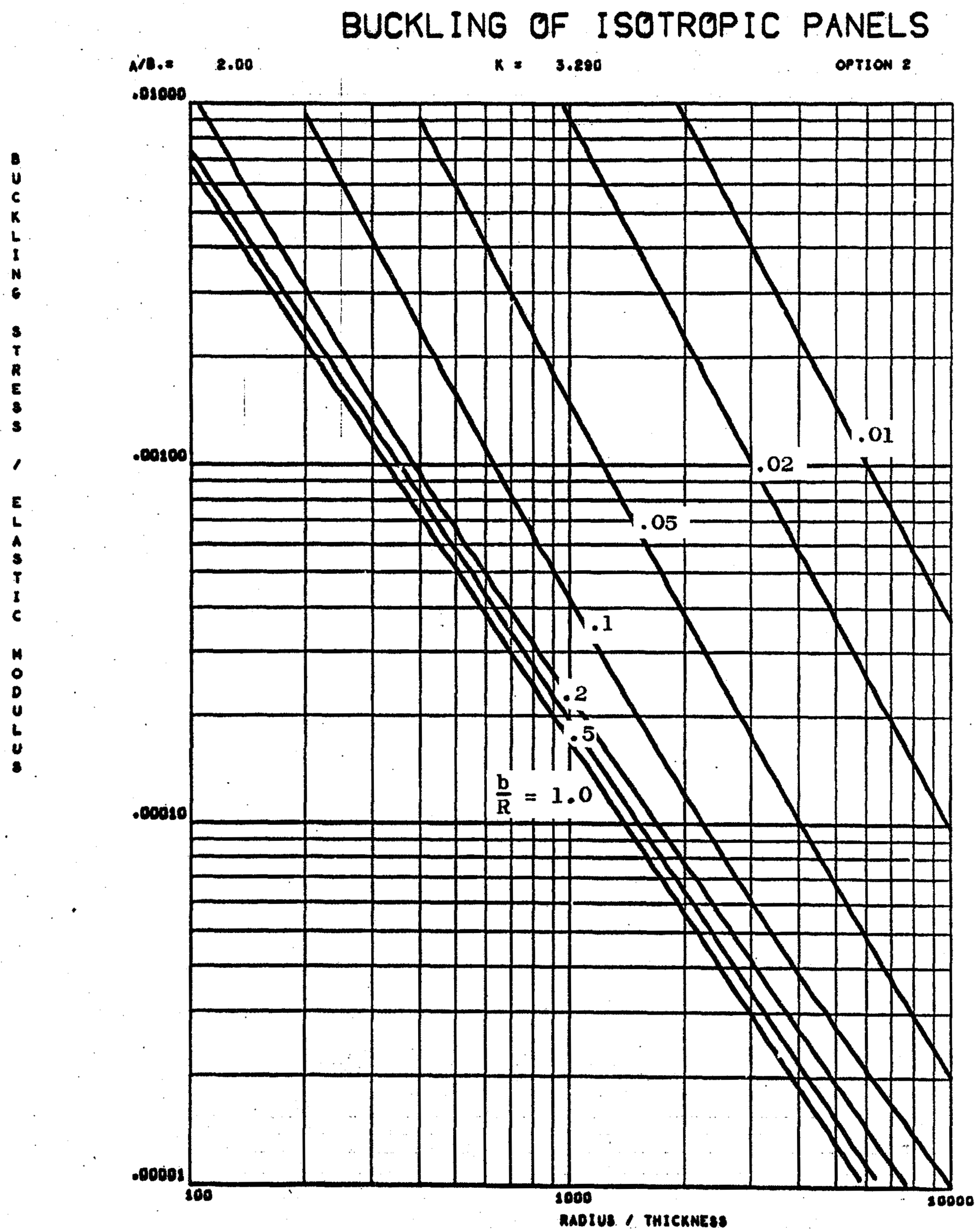

BUCKLING OF ISOTROPIC PANELS
A/B.= 1.00
K = 3.290
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(f)



BUCKLING ØF ISØTRØPIC PANELS
A/B.= 1.00
K = 5.700
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(g)



BUCKLING OF ISOTROPIC PANELS
A/B.= 2.00
K = 3.290
OPTION 2
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(h)

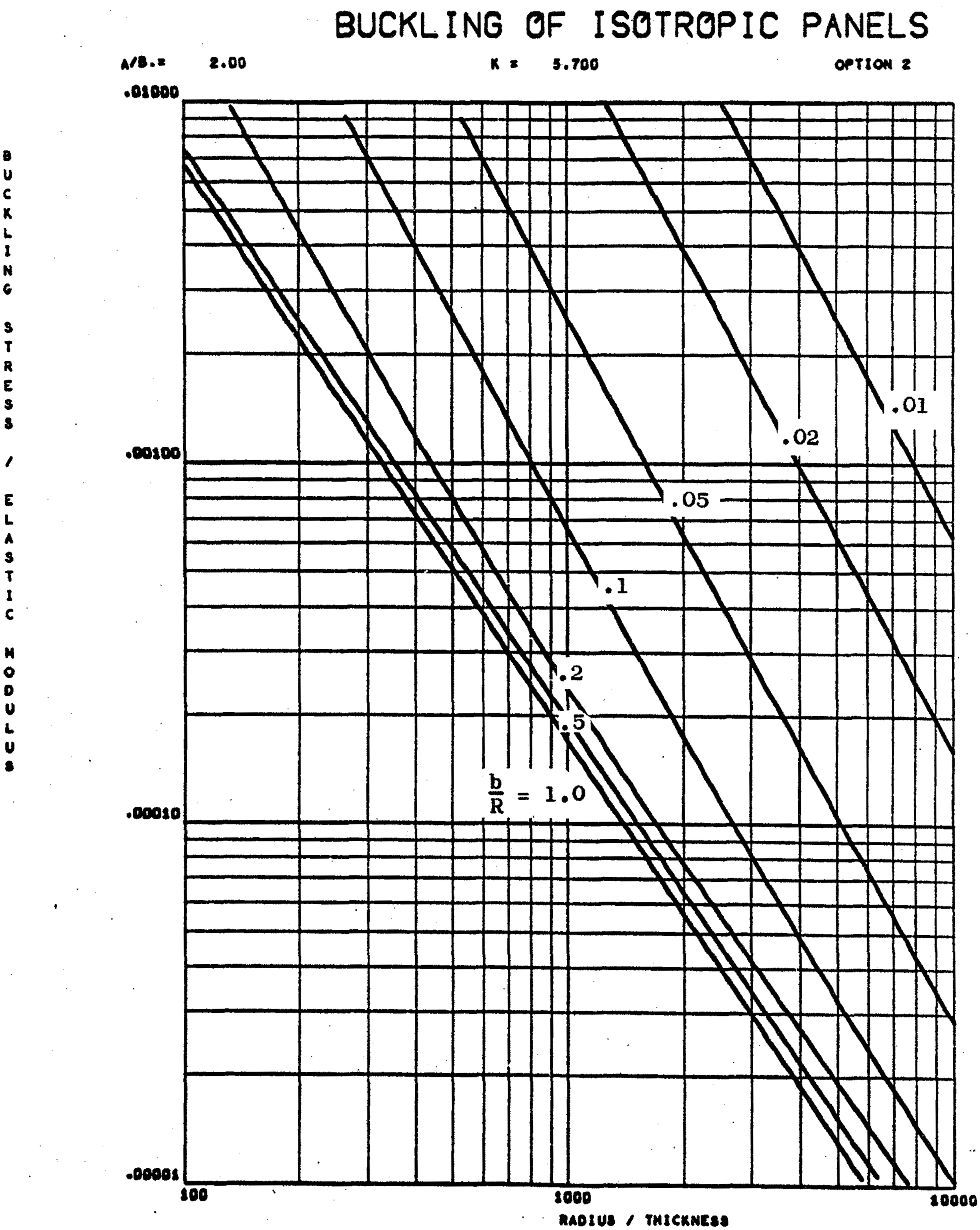

BUCKLING OF ISOTROPIC PANELS
A/B.= 2.00
K = 5.700
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 11(i)

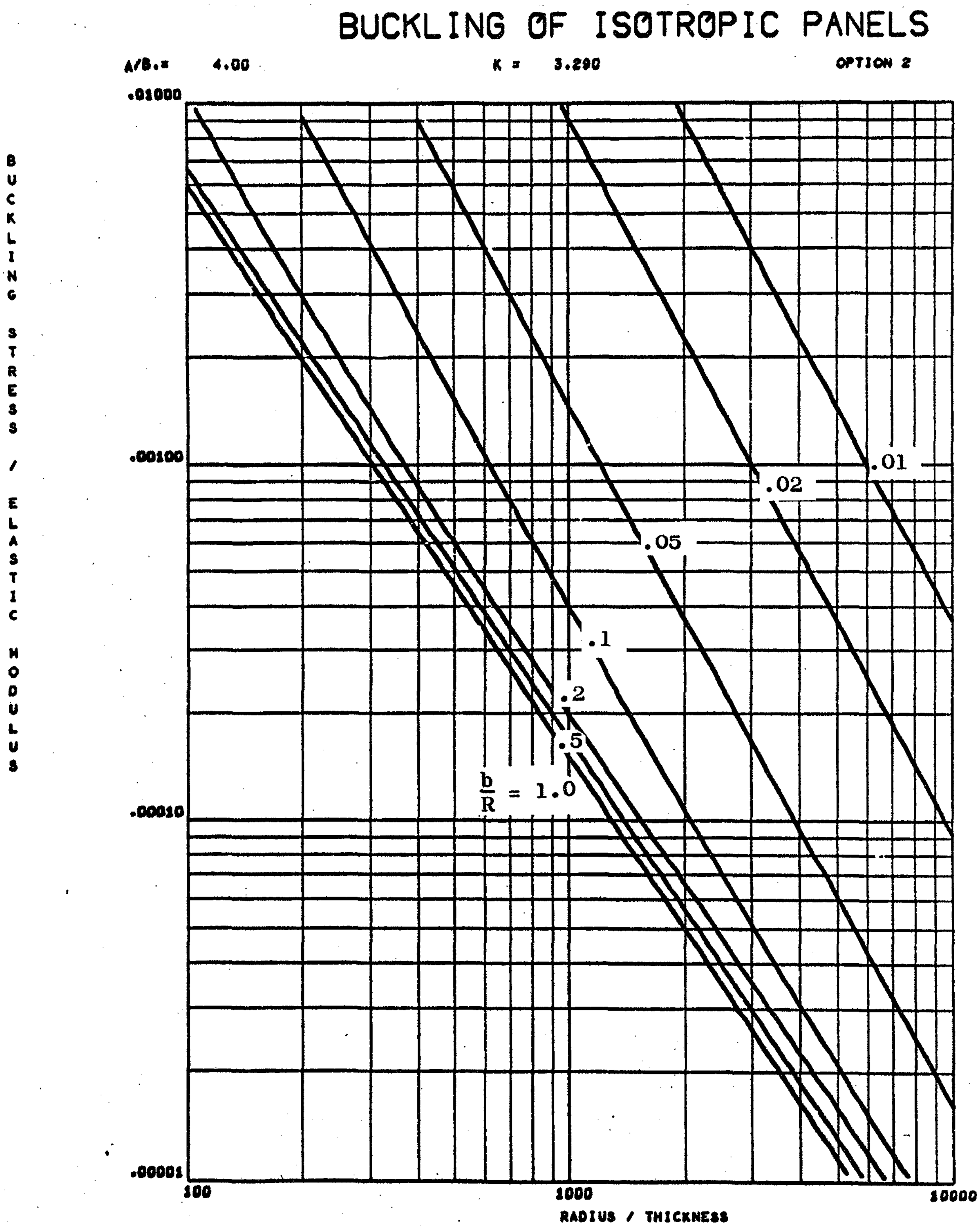

BUCKLING OF ISOTROPIC PANELS
A/B.= 4.00
K = 3.290
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(j)

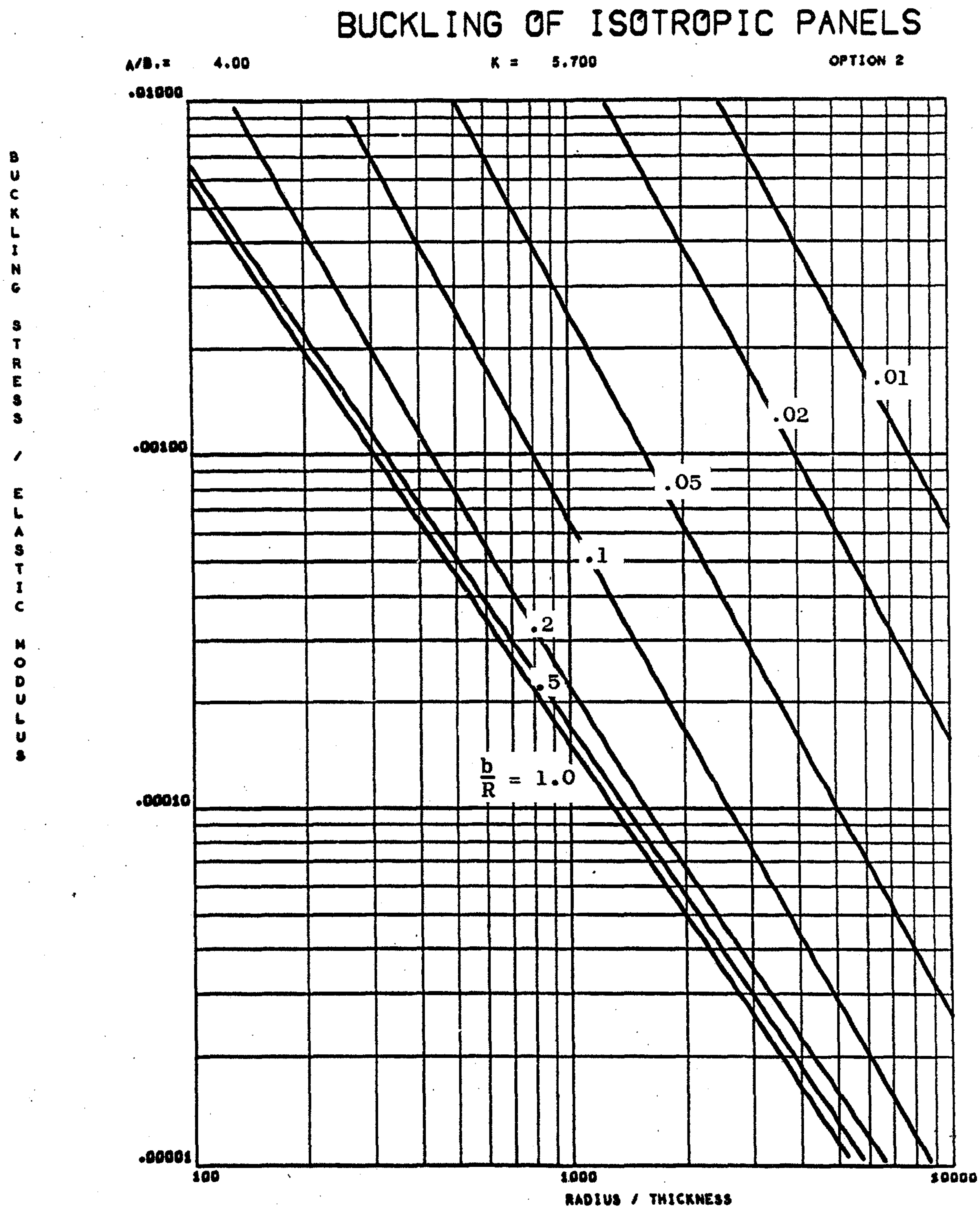

BUCKLING OF ISOTROPIC PANELS
A/B.= 4.00
K = 5.700
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 11(k)

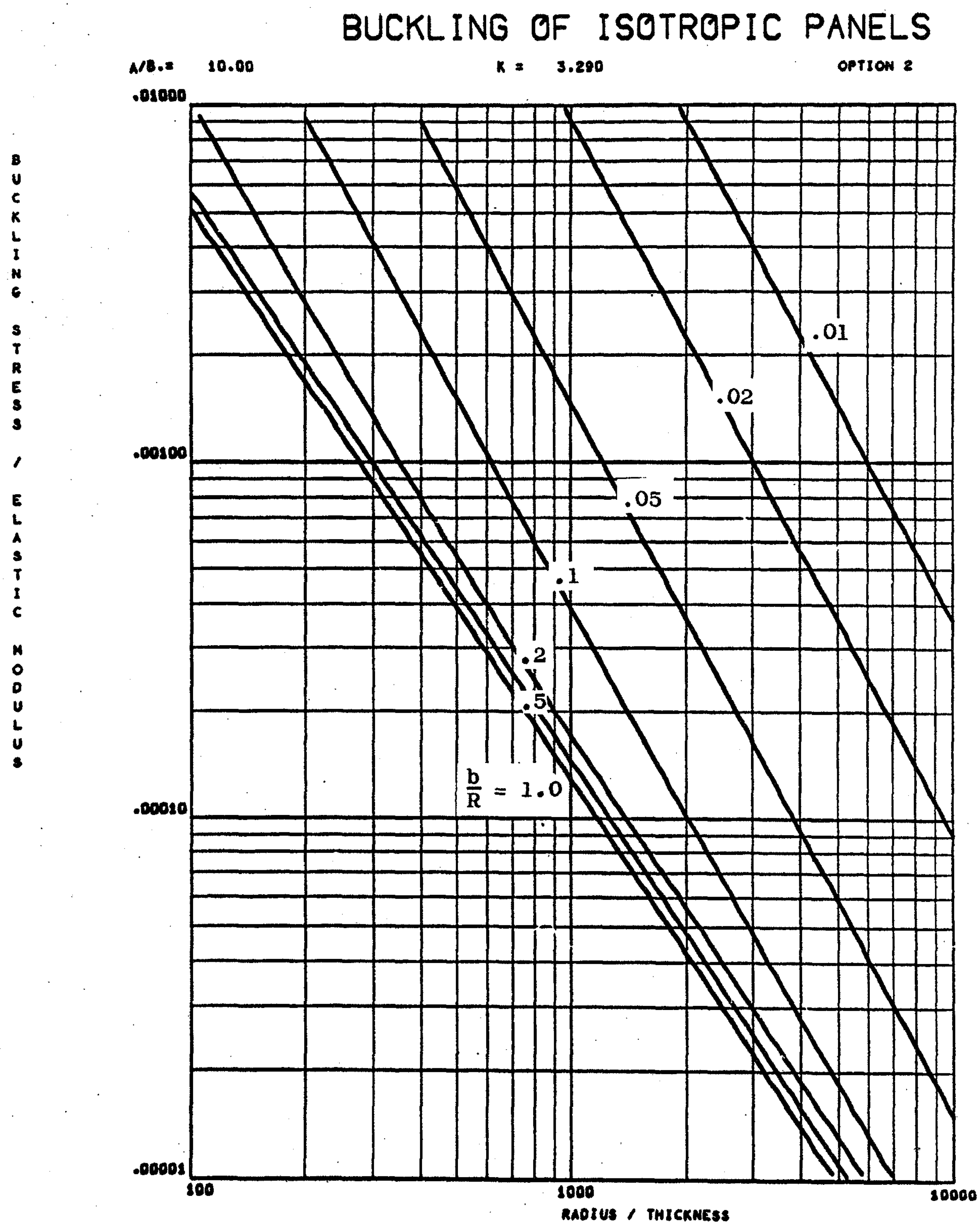

BUCKLING OF ISOTROPIC PANELS
A/B.= 10.00
K = 3.290
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
b/R = 1.0
100
1000
10000
RADIUS / THICKNESS

Figure 11(1)

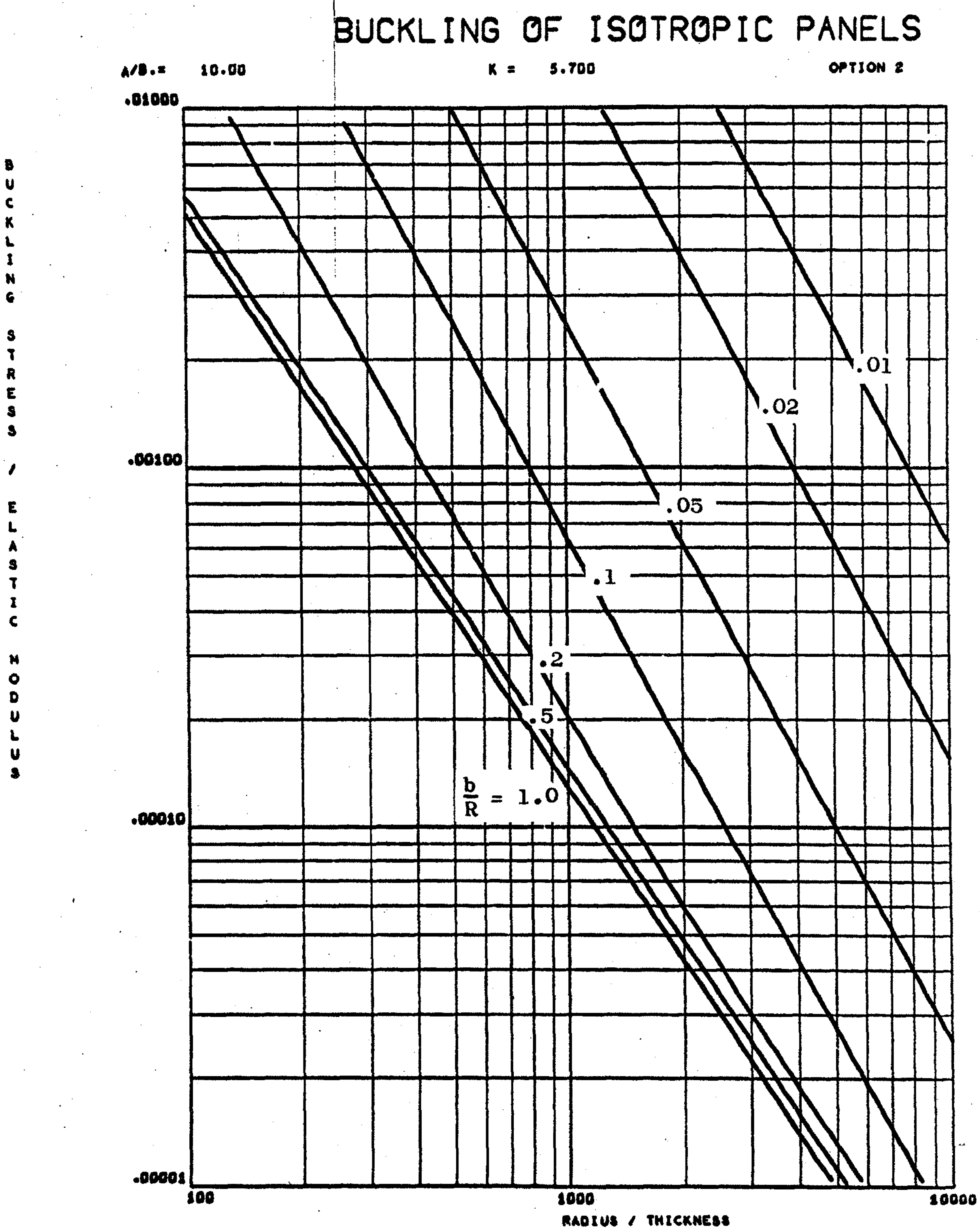

BUCKLING OF ISOTROPIC PANELS
A/B.= 10.00
K = 5.700
OPTION 2
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 11(m)

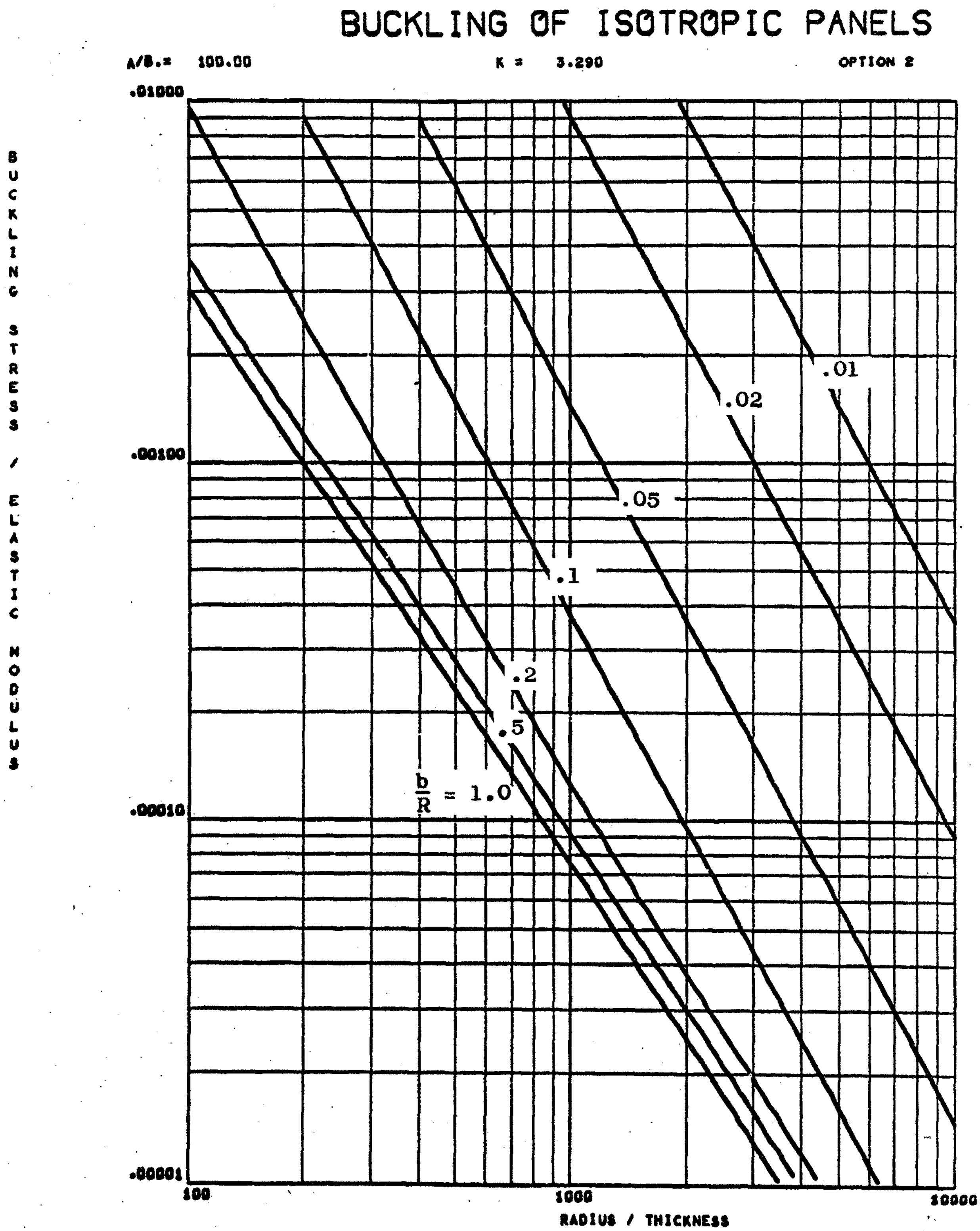

BUCKLING OF ISOTROPIC PANELS
A/B.= 100.00
K = 3.290
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(n)

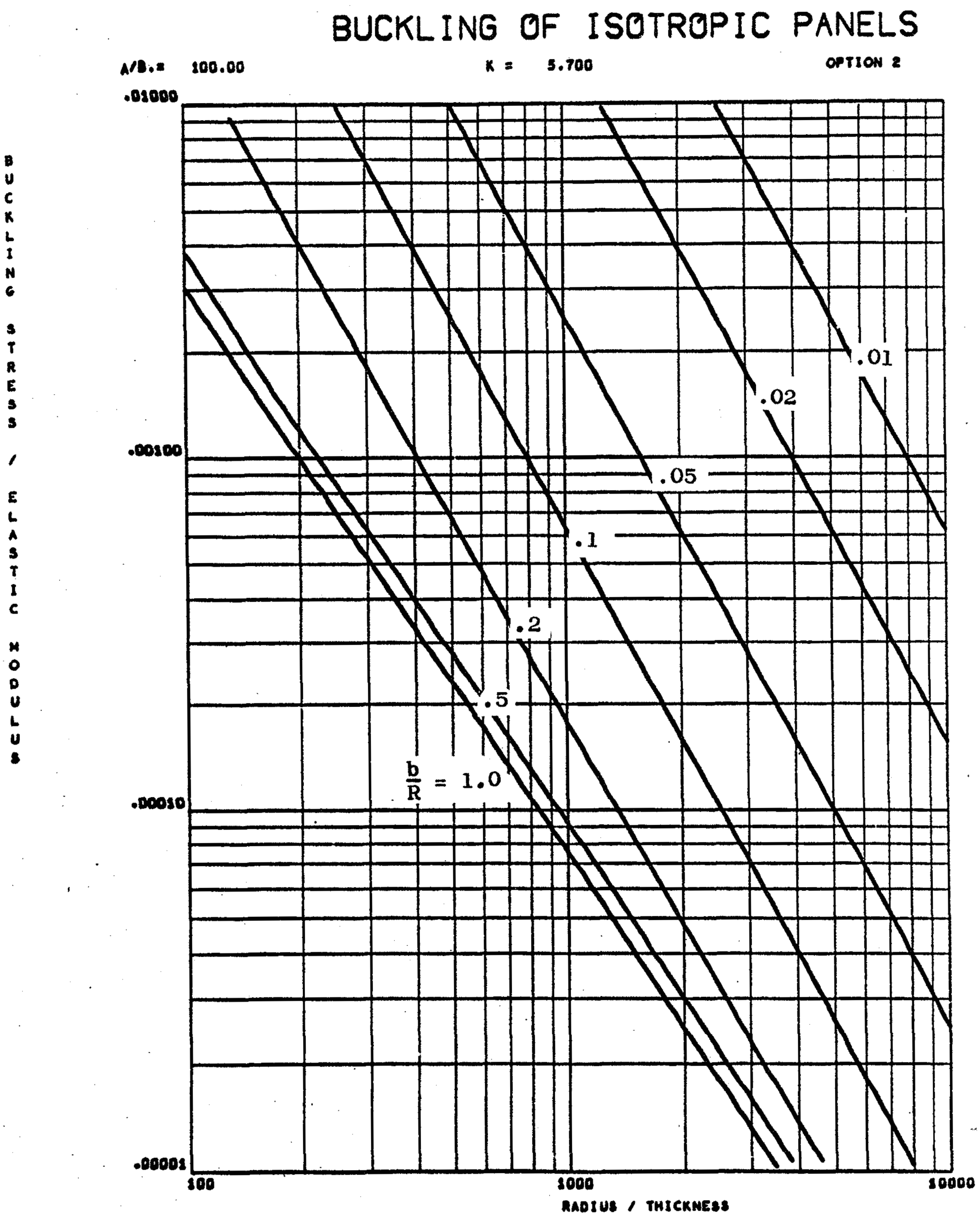

BUCKLING OF ISOTROPIC PANELS
A/B.= 100.00
K = 5.700
OPTION 2
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 11(o)

A.2 OPTION 3

A.2.1 GENERAL

All of the curves presented in Figure 12 are based upon substitution of the OPTION 3 full cylinder equations [see equations (2-11) and (2-12)] into the Schapitz criterion [2]. These curves were obtained by using digital computer program 3875 and an automatic plotter. The program is described in SECTION 5 and can be used to obtain additional plots and/or single-point solutions, as desired. Since the automatic plotting machine does not provide any capability for the print-out of lower case letters, the ratio (a/b) appears on all the given buckling curves as (A/B). It is recommended that these plots be used whenever $(a/R) > 1$ since, in this range, the curves of SECTION 4 can give unconservative design values.

A.2.2 BUCKLING CURVES

Table VIII lists the families provided here. All of these curves are based on elastic behavior and a Poisson's ratio of 0.30.

TABLE VIII - Table of Contents for the Supplementary Buckling Curves Based on OPTION 3

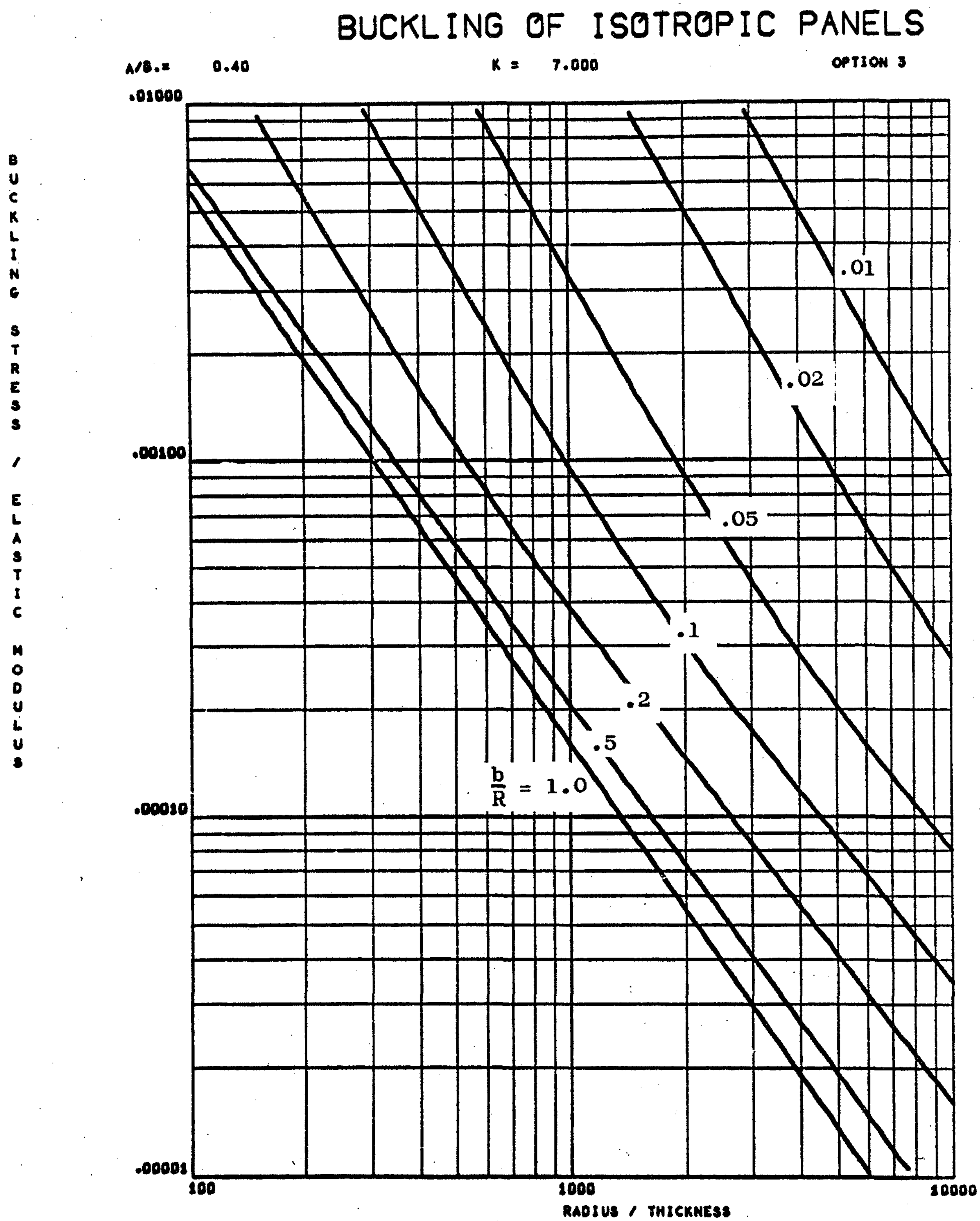

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.40
K = 7.000
OPTION 3
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(a)

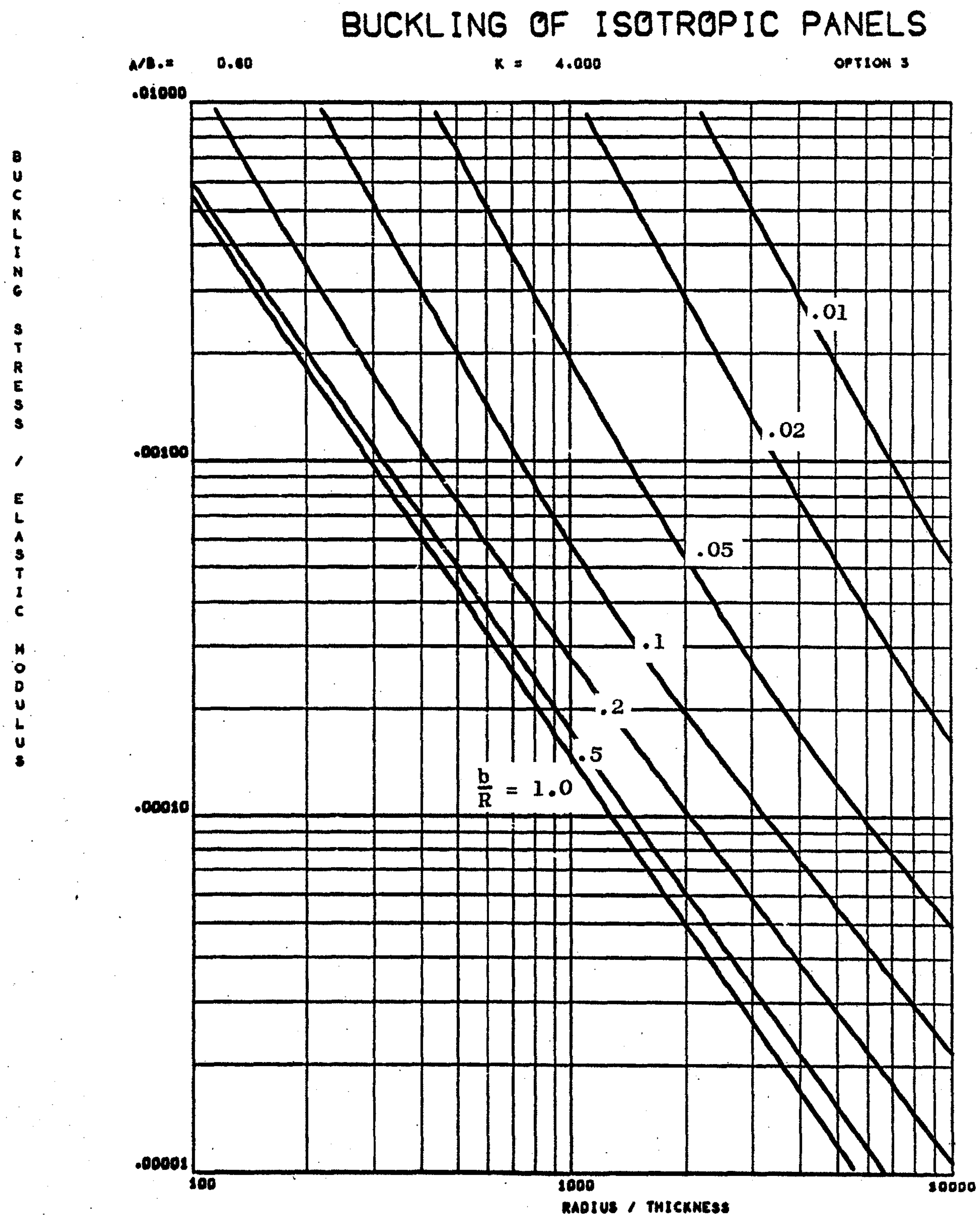

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 4.000
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 12(b)

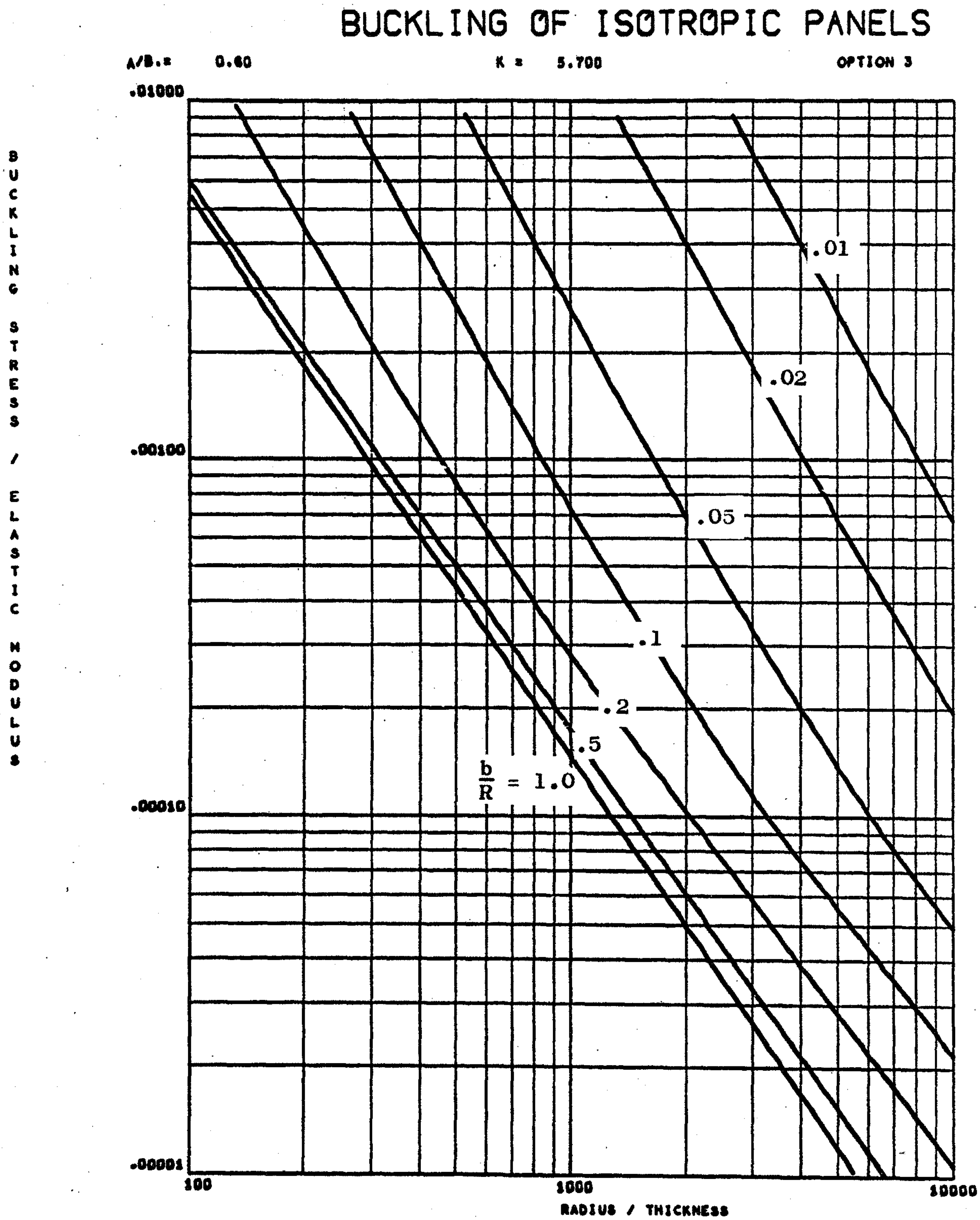

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 5.700
OPTION 3
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(c)

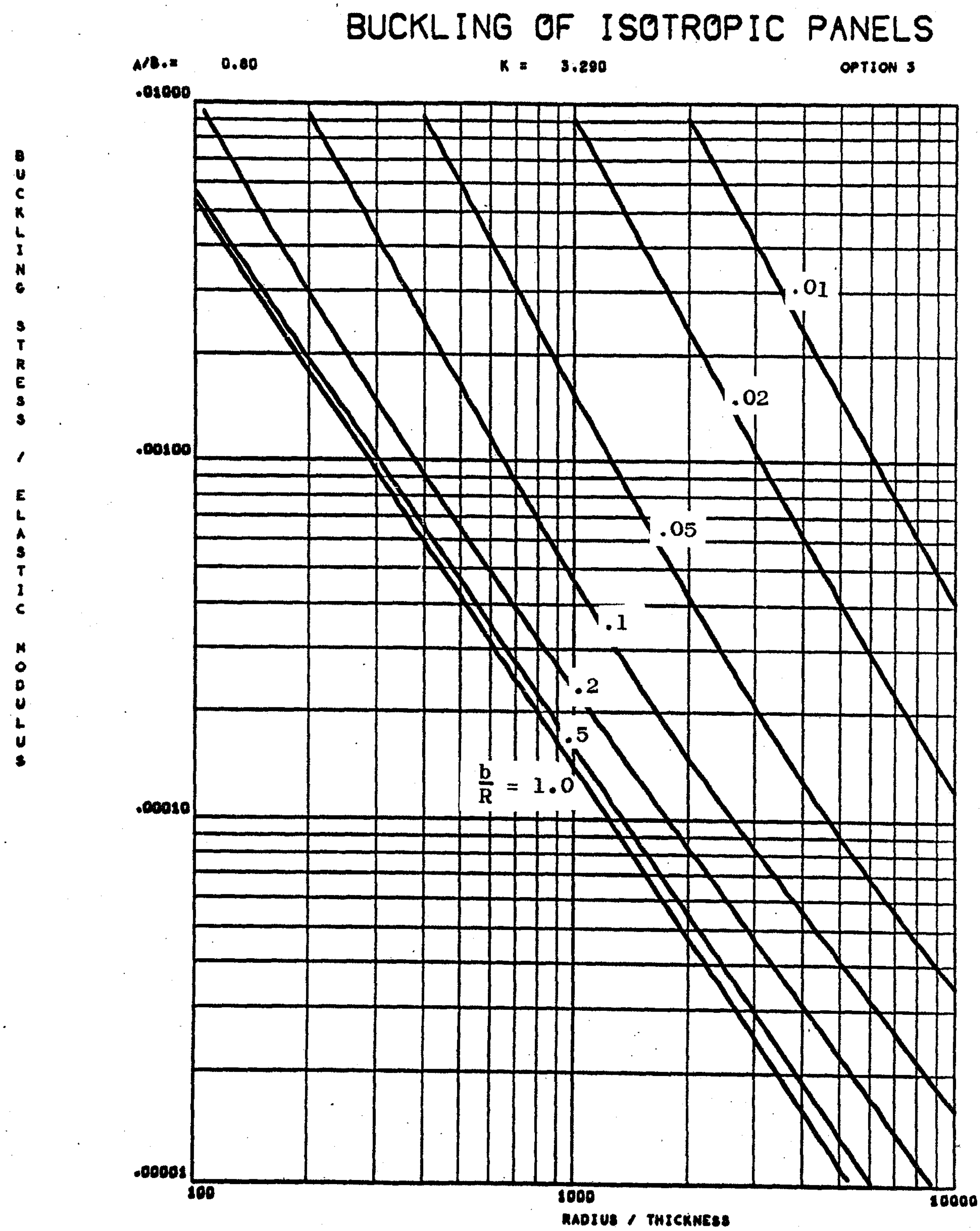

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.80
K = 3.290
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(d)

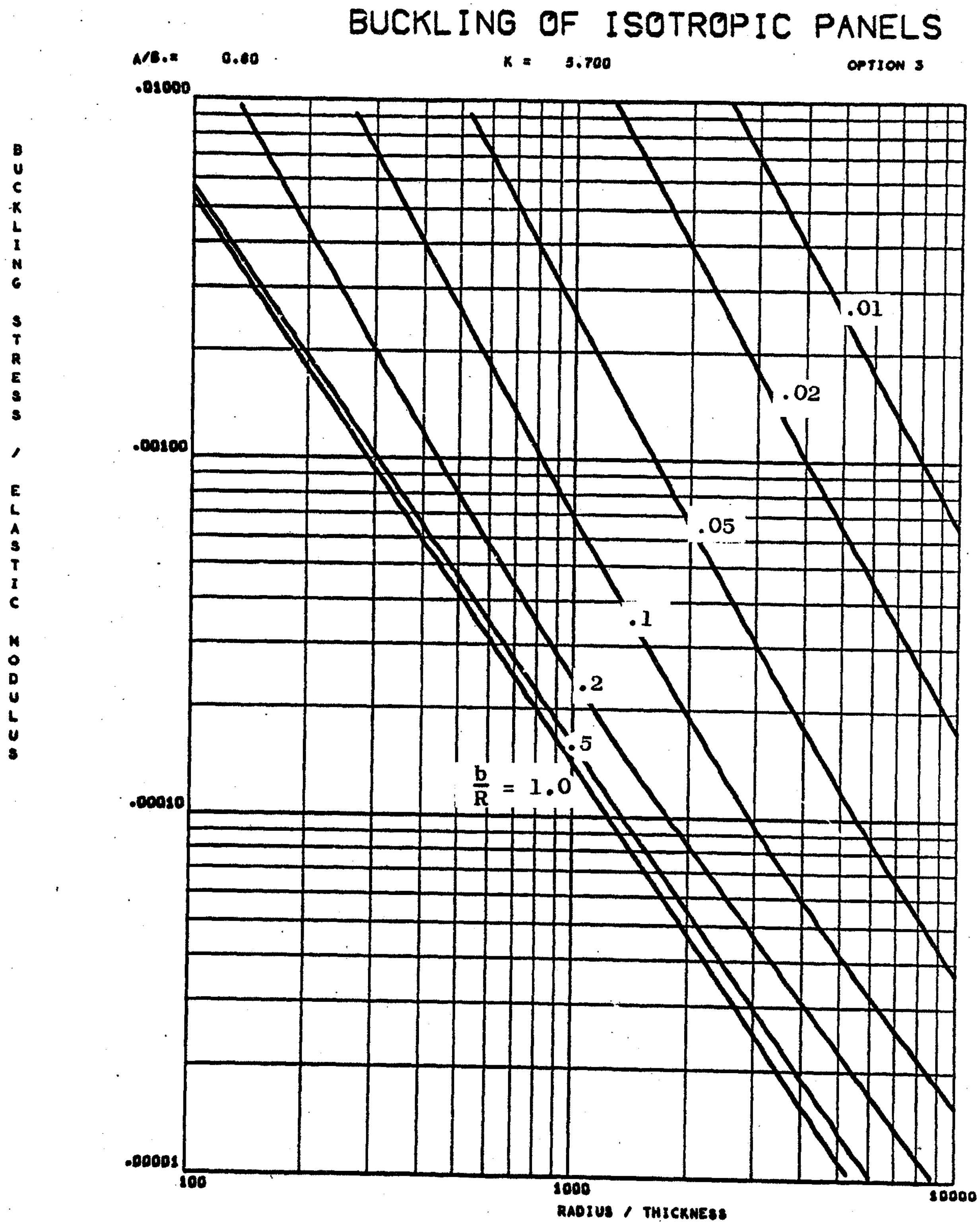

BUCKLING OF ISOTROPIC PANELS
A/B.= 0.60
K = 5.700
OPTION 3
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 12(e)

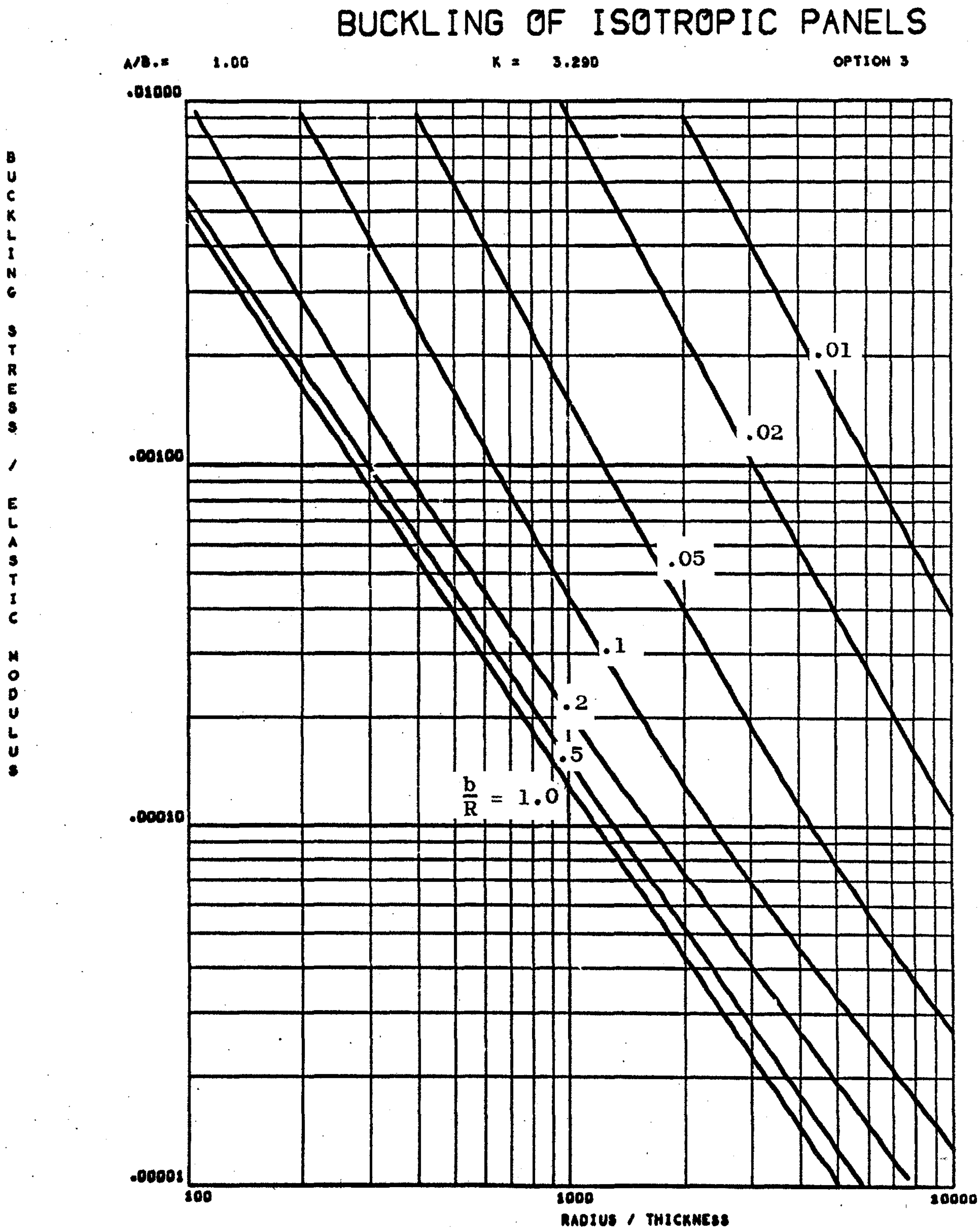

BUCKLING OF ISOTROPIC PANELS
A/B.= 1.00
K = 3.290
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(f)

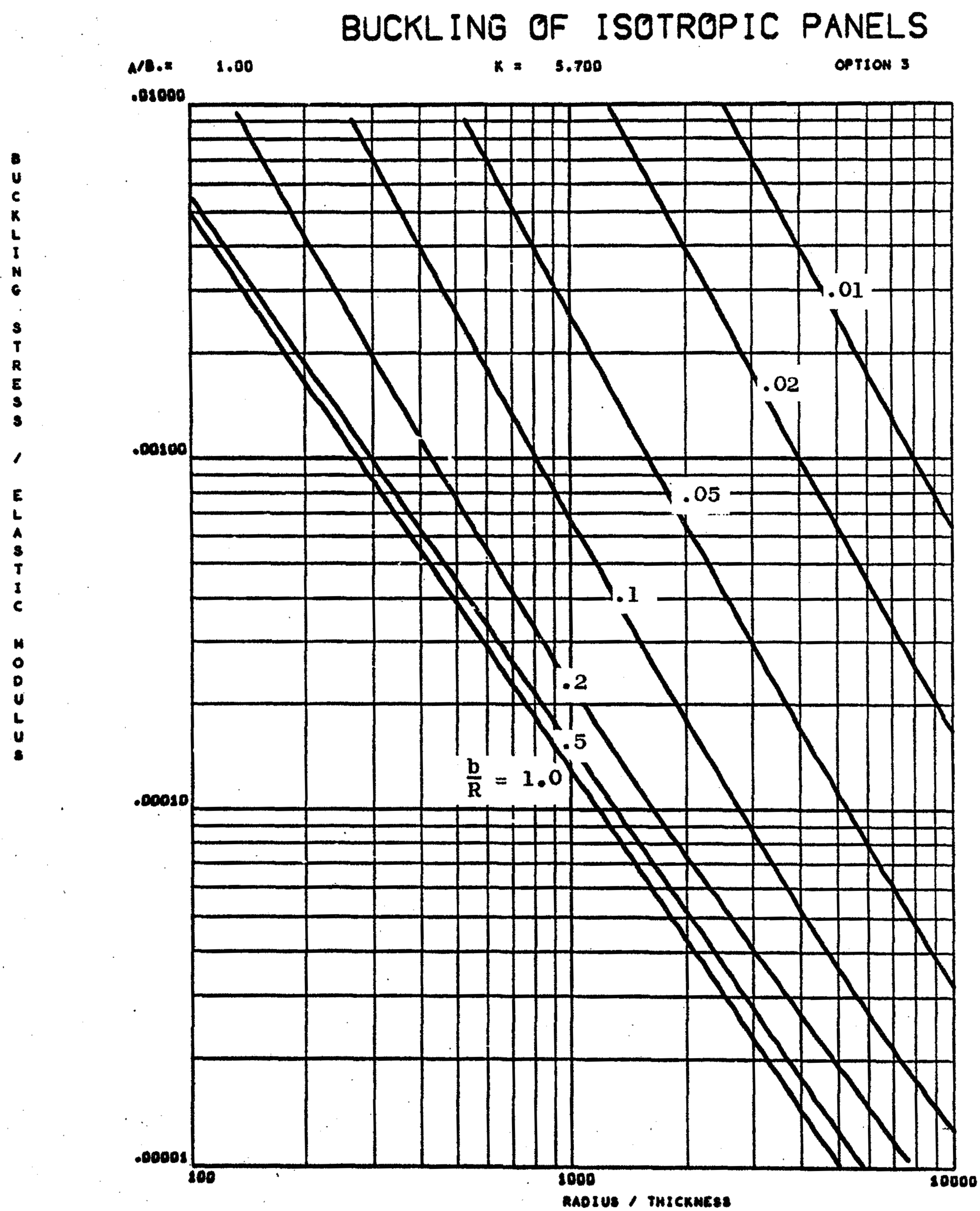

BUCKLING OF ISOTROPIC PANELS
A/B.= 1.00
K = 5.700
OPTION 3
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(g)

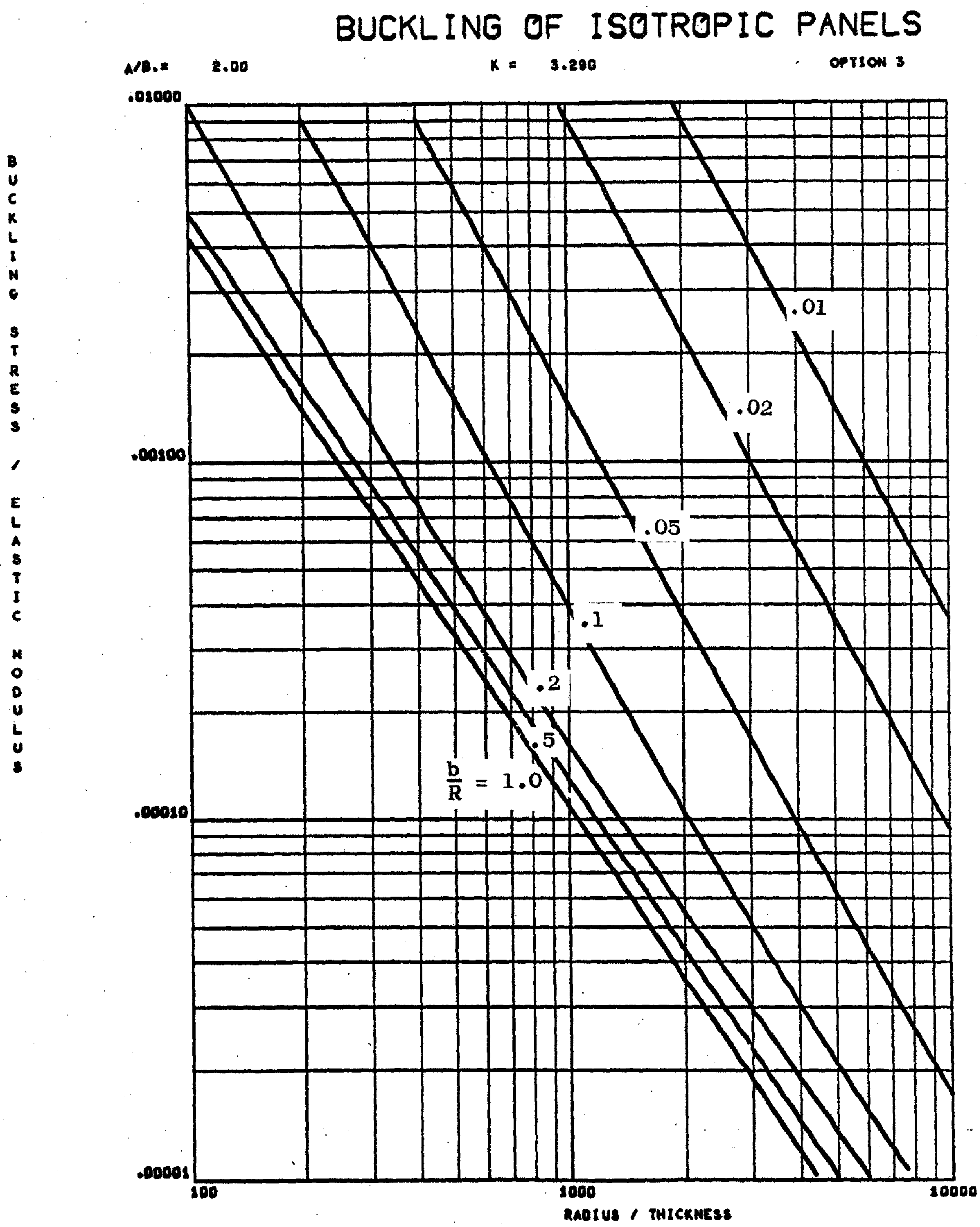

BUCKLING OF ISOTROPIC PANELS
A/B.= 2.00
K = 3.290
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 12(h)

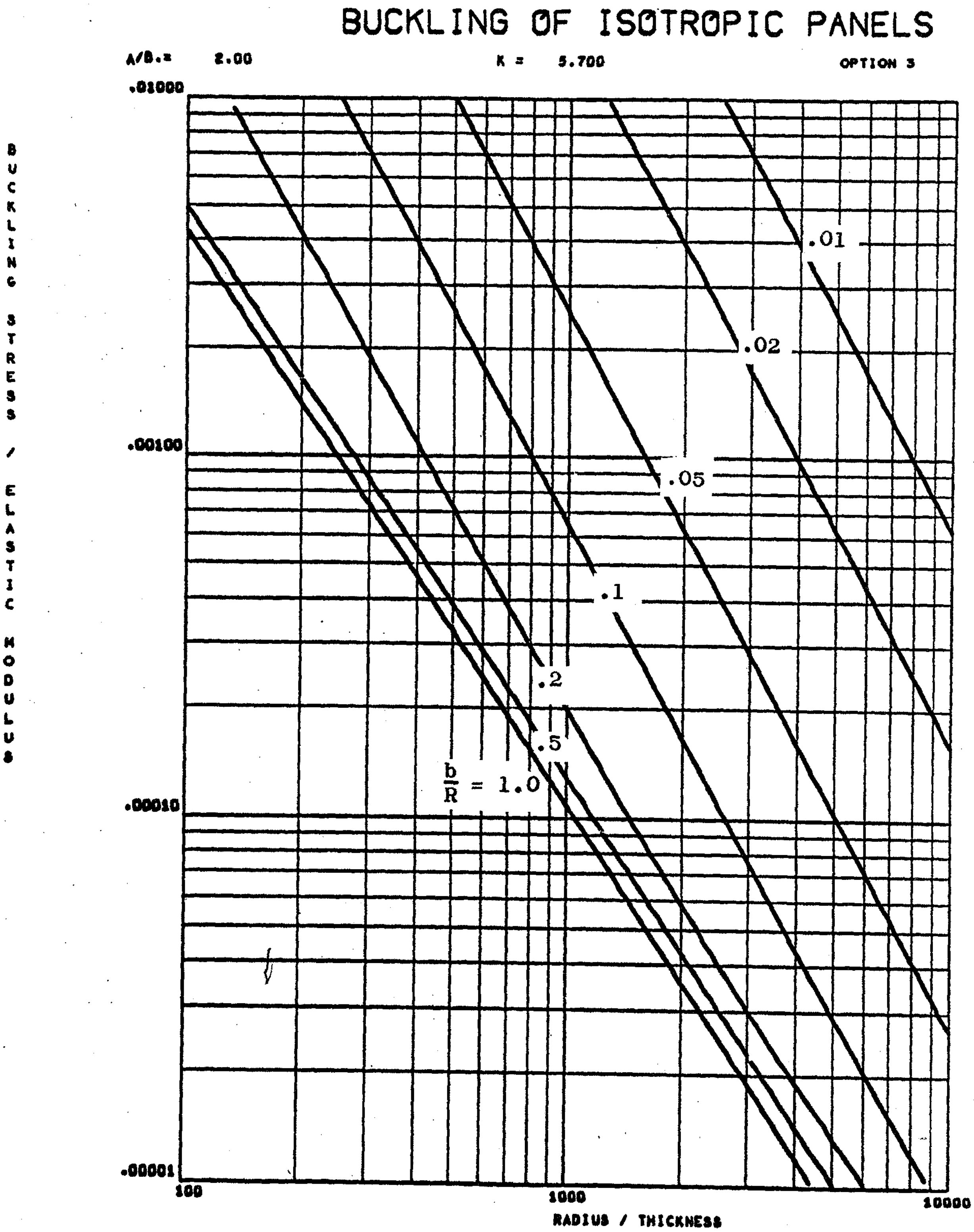

BUCKLING OF ISOTROPIC PANELS
A/B.= 2.00
K = 5.700
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(i)

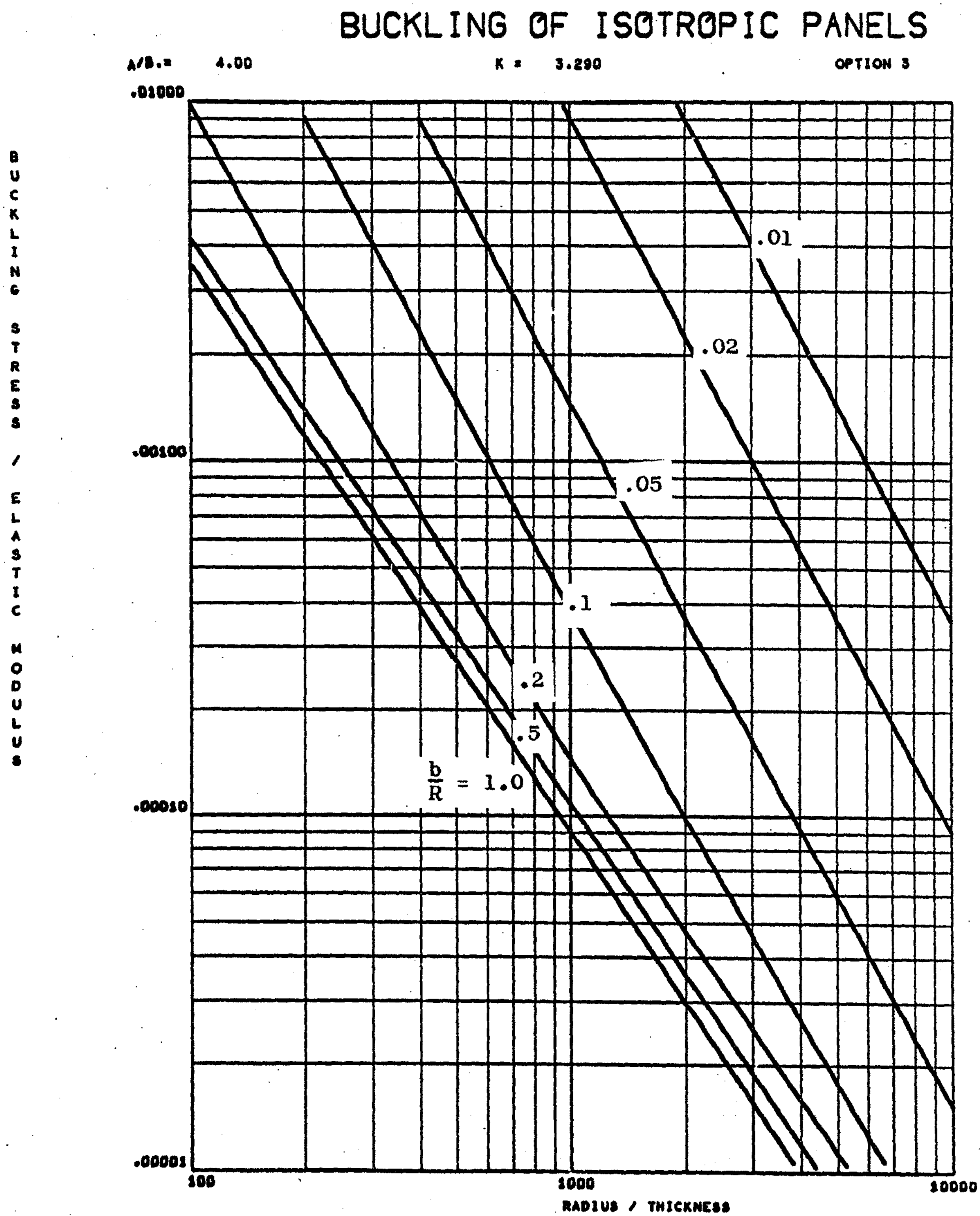

BUCKLING OF ISOTROPIC PANELS
A/B.= 4.00
K = 3.290
OPTION 3
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(j)

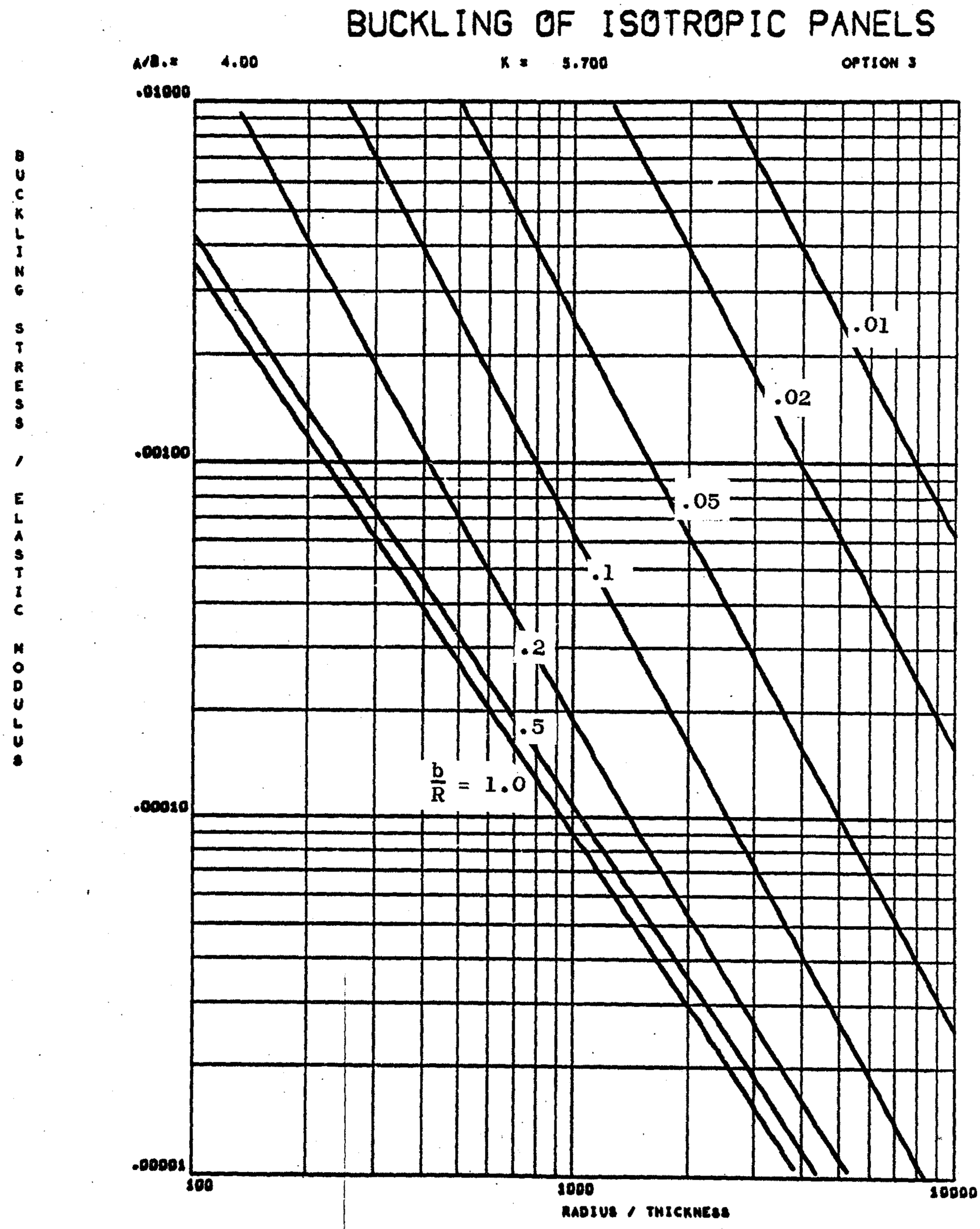

BUCKLING OF ISOTROPIC PANELS
A/B.= 4.00
K = 5.700
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(k)

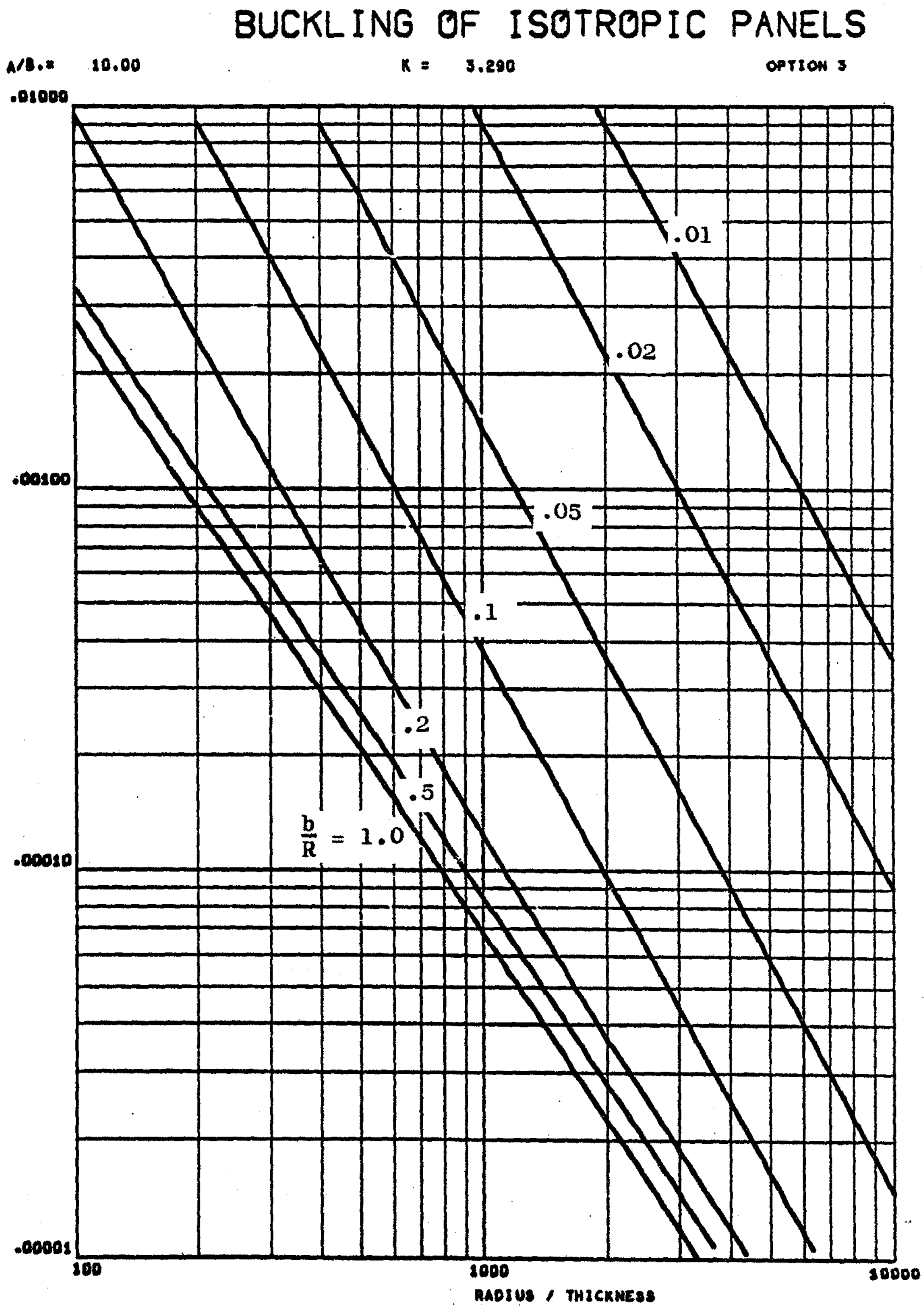

BUCKLING OF ISOTROPIC PANELS
A/B.= 10.00
K = 3.290
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(1)

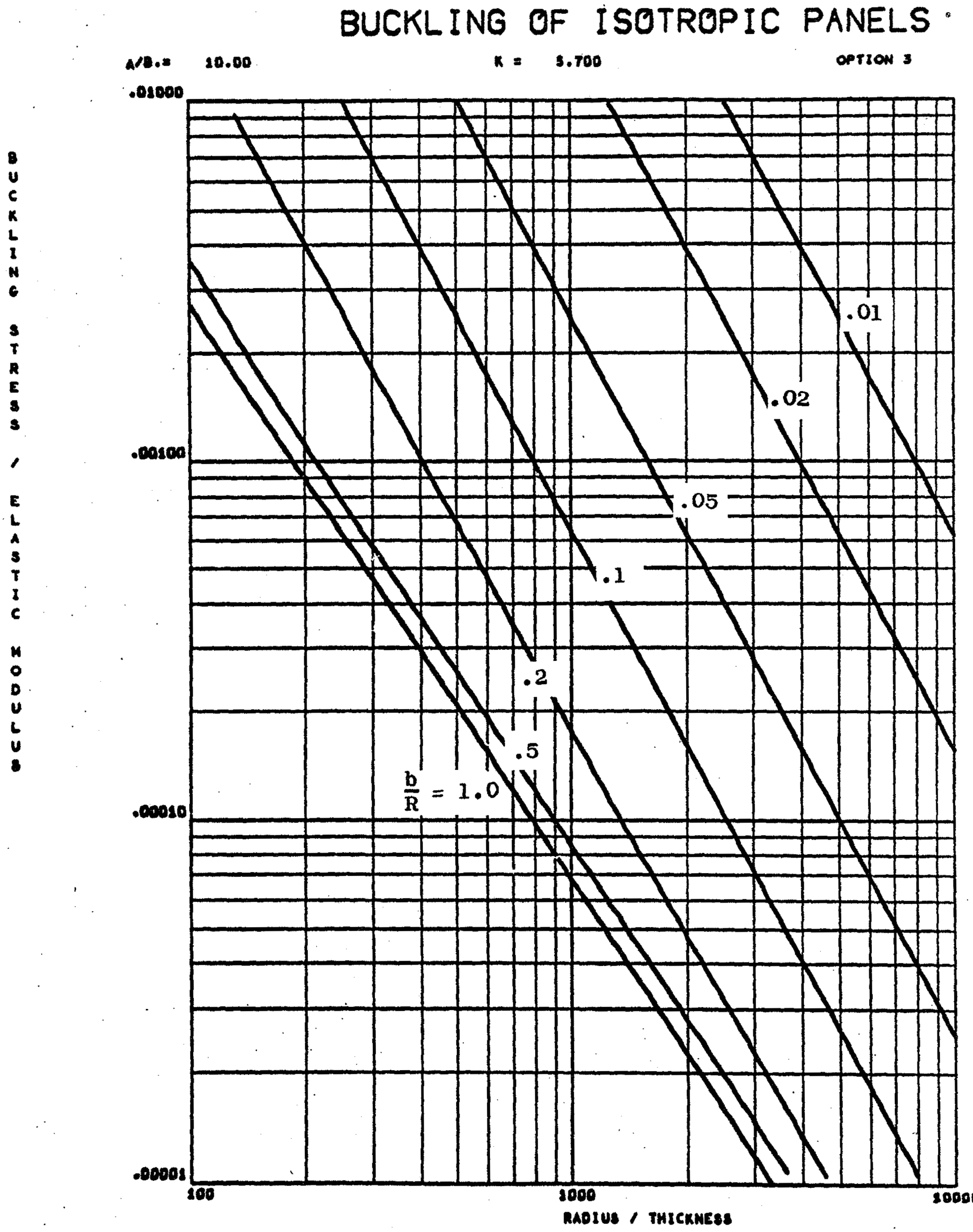

BUCKLING OF ISOTROPIC PANELS
A/B.= 10.00
K = 5.700
OPTION 3
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
BUCKLING STRESS / ELASTIC MODULUS
.01
.02
.05
.1
.2
.5
$\frac{b}{R}$ = 1.0

Figure 12(m)

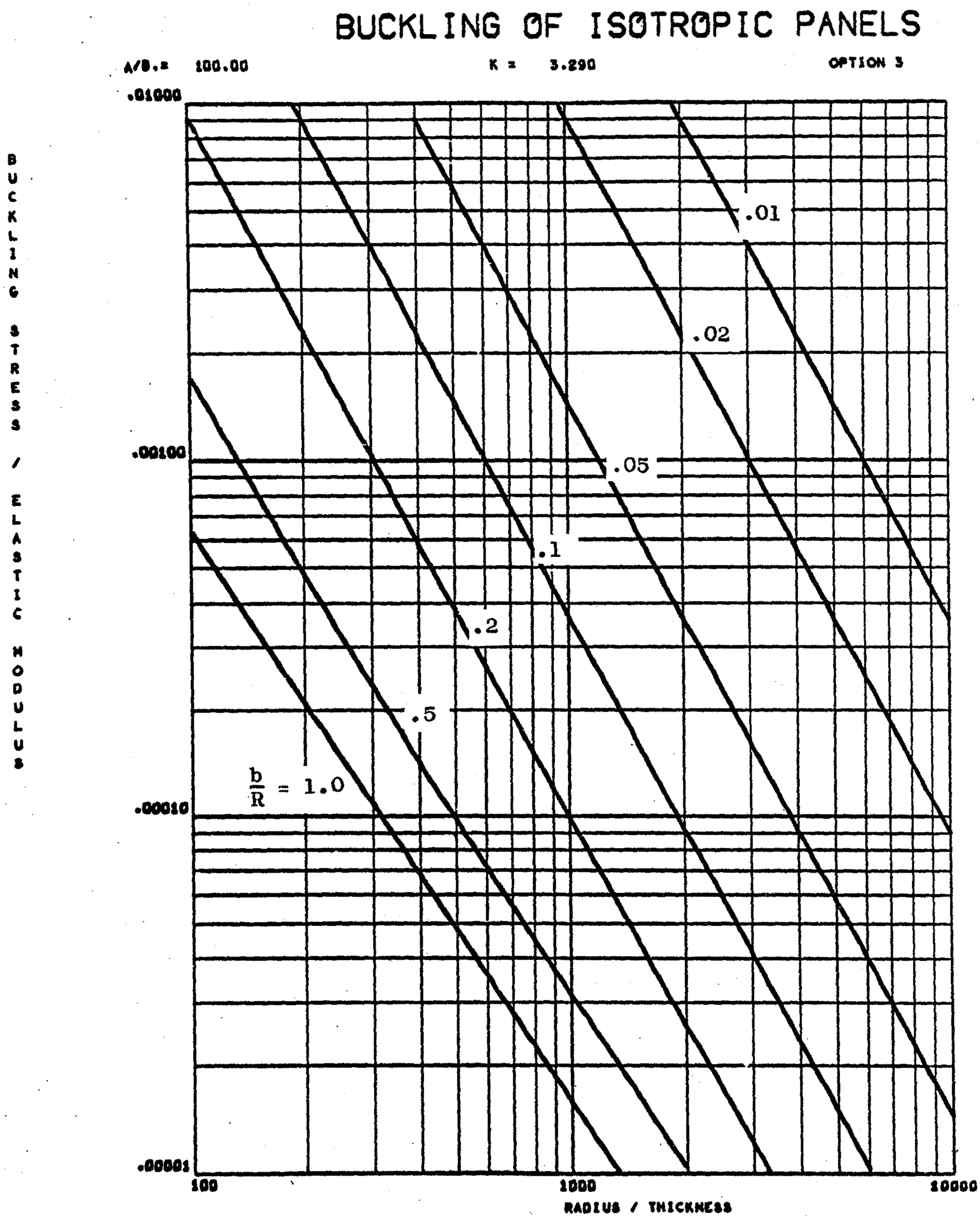

BUCKLING OF ISOTROPIC PANELS
A/B.= 100.00
K = 3.290
OPTION 3
.01000
.00100
.00010
.00001
BUCKLING STRESS / ELASTIC MODULUS
100
1000
10000
RADIUS / THICKNESS
$\frac{b}{R}$ = 1.0
.5
.2
.1
.05
.02
.01

Figure 12(n)

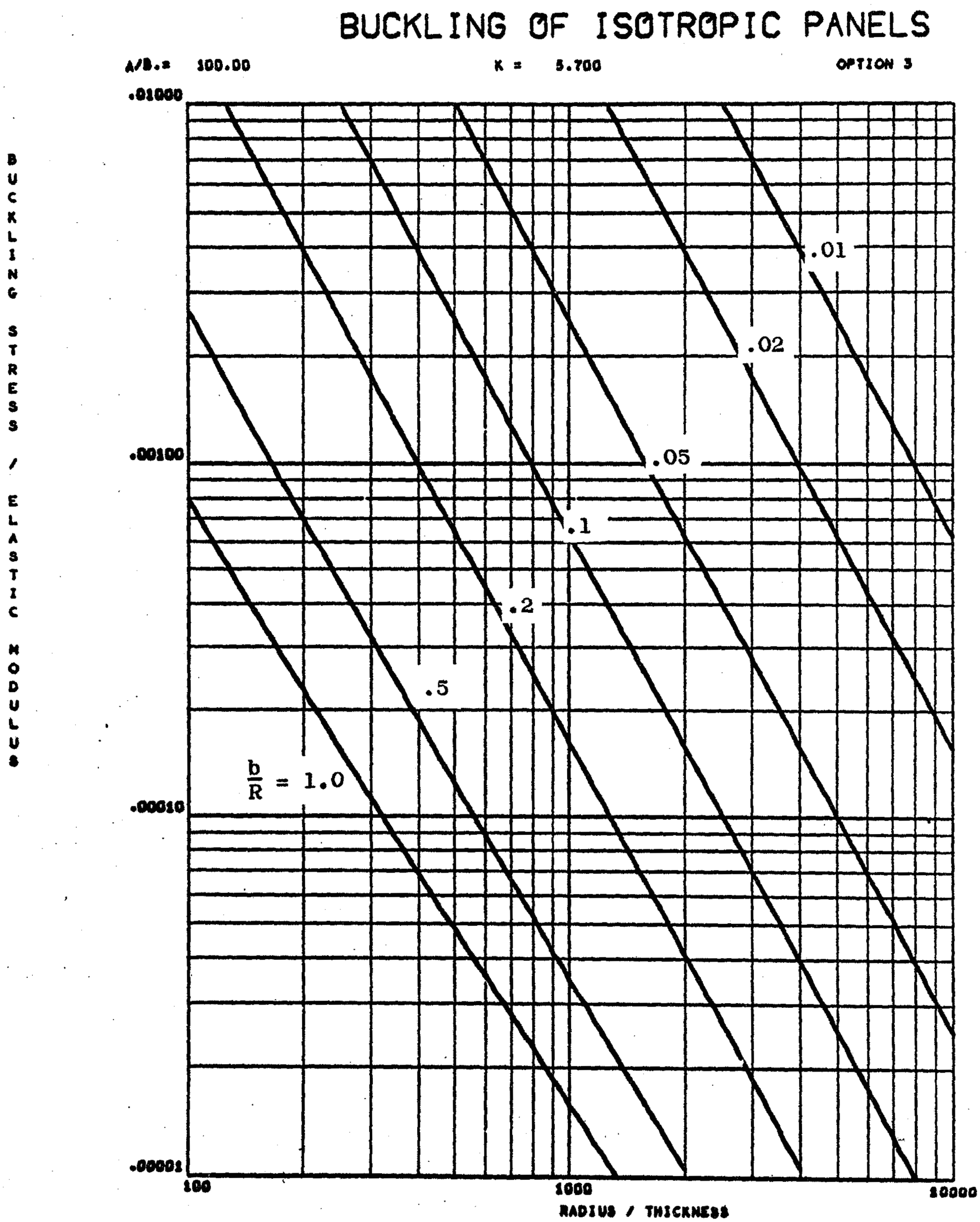

BUCKLING OF ISOTROPIC PANELS
A/B.= 100.00
K = 5.700
OPTION 3
BUCKLING STRESS / ELASTIC MODULUS
.01000
.00100
.00010
.00001
100
1000
10000
RADIUS / THICKNESS
.01
.02
.05
.1
.2
.5
b/R = 1.0

Figure 12(o)