THE OWNERSHIP OF INDIVISIBLE INPUTS AND THE AGGLOMERATION OF RESOURCES

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Authors normally associate the agglomeration of resources at single geographical locations with the presence of indivisibilities; in this analysis we attempt to go beyond that association in order to investigate two factors which appear to affect the tendency of resources to agglomerate spatially. These factors are the ownership of the indivisible input and the utility functions of the entrepreneurs of the firms utilizing the indivisible inputs.

I. THE TRADITIONAL ASSOCIATION OF RESOURCE AGGLOMERATION AND THE INDIVISIBLE INPUTS

Any discussion of agglomeration seems to benefit from a benchmark reference to Alfred Weber. In this present analysis, we present Alfred Weber as the spokesman for the authors who have considered the subject of agglomeration with the acknowledgement that embellishments upon his early work have produced few significant changes.

Weber identified agglomeration economies and agglomeration diseconomies as forces which allocate industries spatially within a region.[1] According to Weber, these economies are associated with two different levels of economic organization. At the first level, specialization of productive factors or economies of scale obtainable by expanding a single plant.[2] At the second level, the agglomerative factors are advantages obtainable from the close association of several plants.[3] Of course, it is the second type of agglomerative factor with which we are concerned in this paper.

1. Alfred Weber, Theory of Location of Industries, translation C. Friedrick (Chicago: University of Chicago Press, 1929, p. 124.
2. Ibid., p. 127.
3. Ibid., p. 128.

Among the advantages derived from locating several plants in close, common spatial association, Weber identified "the development of technical equipment."[4]

> The complete technical equipment which is necessary to carry out a process of production may in highly developed industries become so specialized that minute parts of the process of production utilize specialized machines and that even quite large-scale plants are not able to make full time use of such equipment. Such specialized machines must then, together with their own parts of the process of production, be taken out of the single large plant and must work for several of them, i.e., become the basis of independent auxiliary industries.[5]
> Of course, this specialized, technical equipment described by Weber is a description of indivisibility of equipment.

The literature which considers the agglomeration of productive resources has not gone beyond the association of this phenomenon with the presence of indivisible inputs. In this analysis we consider the effects of the ownership of these inputs upon the tendency of resources to agglomerate.

II. THE SEPARATE OWNERSHIP OF THE INDIVISIBLE INPUT

We can investigate the method of allocation of the services of an indivisible input between two firms when the input is owned by a separate firm by use of an example. This separate firm, called the <u>supplier</u> of the services of the indivisible input, is motivated to allocate the services of the indivisible input in a manner such that the profits of the supplier are maximized.

For purposes of this example, we denote the profit of the supplier as π_s. We denote the revenue from selling the services of the indivisible input to

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4. <u>Ibid</u>., p. 128.
5. <u>Ibid</u>., pp. 128-129.

firm 1 as $R_1(r_1)$ which is a function of the number of units of the service sold. Likewise, we denote the revenue from sales of the service to the second firm as $R_2(r_2)$. Since the cost of supplying the service is a function of the number of services provided to both firm 1 and 2, we can form equation (1) as a profit function of the supplier.

$$\pi_s = R_1(r_1) + R_2(r_2) - C(r_1 = r_2) \tag{1}$$

For purposes of analysis, we further assume that C is a smooth differentiable function which is convex upward throughout the range relevant to this analysis.

Since the firms purchasing the services of the input would not pay the price per unit of the services of the input which exceeds the value of the marginal product, we can rewrite the profit function of the supplier as equation (2) where $X_1(r_1 \ldots)$ and $X_2(r_2 \ldots)$ are the output of firms 1 and 2, respectively, and p_1 and p_2 are the prices of these outputs.

$$\pi_s = \left(p_1 \frac{\partial X_1}{\partial r^1}\right) r_1 + \left(p_2 \frac{\partial X_2}{\partial r_2}\right) r_2 - C\ (r_1 = r_2) \tag{2}$$

From (2) the first order conditions for the maximization of the profit of the supplier are equations (3.a) and (3.b).

$$\frac{\partial \pi_s}{\partial r_1^2} = \left(p_1 \frac{\partial X_1}{\partial r_1^2}\right) r_1 + p_2 \frac{\partial X_1}{\partial r_1} - \frac{dC}{dr_1}\ (r_1 = r_2) = 0 \tag{3.a}$$

$$\frac{\partial \pi_s}{\partial r_2} = \left(p_2 \frac{\partial X_2}{\partial r_2^2}\right) r_2 + p_2 \frac{\partial X_2}{\partial r_2} - \frac{dC(r_1 + r_s)}{dr_2} = 0 \tag{3.b}$$

Since $$\frac{dC}{dr_1} = \frac{dC}{dr_2{}^2}$$

$$(4) \qquad \left(p_1 \frac{\partial X_1{}^2}{\partial r_1{}^2}\right) r_1 + p_2 \frac{\partial X_1}{\partial r_1} = p_2 \left(\frac{\partial^2 X_2}{\partial r_2{}^2}\right) r_2 + p_2 \frac{\partial X_2}{r_2} = \frac{dC}{dr_2}$$

Equation (4) indicates that the sum of the value of the marginal product of the services of the input and a weighted rate of change in the value of the marginal product must equal the marginal cost of supplying the total services of the indivisible input. From this relationship we may conclude that the two user firms would be charged the same input prices <u>only</u> if their marginal product functions were identical. That is to say, the supplier maximizes profits by price discrimination between the two firms.

Furthermore, a negative $\frac{\partial X_1{}^2}{\partial r^2}$ and $\frac{\partial X_2{}^2}{\partial r_2}$ (a necessary second order condition) leads to the immediate conclusion that separate ownership of the indivisible input leads to a lesser utilization of the indivisible input than the usage reflected in usage where the value of the marginal product equals the marginal cost of the services of the input.

III. JOINT-OWNERSHIP OF THE INDIVISIBLE INPUT

Although the traditional "theory of agglomeration" does not consider the specific problem, we can probably assume that there is some implication that authors expect joint ownership among the two user firms of the indivisible input to prevail. Since both of the user firms are profit maximizers, this possibility complicates the analysis. Not only must the user firm consider the level of output, the level of inputs (including the services of the indivisible input) in its profit maximizing calculus, but the firm must consider also the share of the costs of the indivisible input. Specifically, the problem is associated with the allocating costs of the services of the indivisible input, because the marginal cost of the services of the indivisible input is declining and a unit charge equal to the marginal cost of the last unit of service taken will not recover the total cost of the indivisible input. Some basis for allocating the total costs of the indivisible input is necessary; this allocation appears to conform readily to the two-person cooperative game.

Von Neumann and Morgenstern[6] have pointed out that so long as a cooperative payoff pairs for the two firms is at least as great as the noncooperative payoff pairs, the two firms would cooperate.. For example, in Figure 1 the noncooperative payoff pair of (u_1, u_2) is less desirable to firm 1 than (u_1', u_2'). However, any point in the shaded area which is obtainable by cooperation exceeds (u_1, u_2). Since firm 2 does not have to take any pair less desirable than (u_1, u_2), (u_1', u_2') is unobtainable by 1. Firm 1 is, therefore, motivated to cooperate with firm 2 in achieving a point on the boundary ab of the shaded area. The motivation for firm 2 is

6. Von Neumann, John and Morgenstern, Oscar, Theory of Games and Economic Behavior, (New York: John Wiley & Sons, Inc., 1964), p. 236.

similar vis a vis the pairs (u_1, u_2) and (u_1'', u_2'').

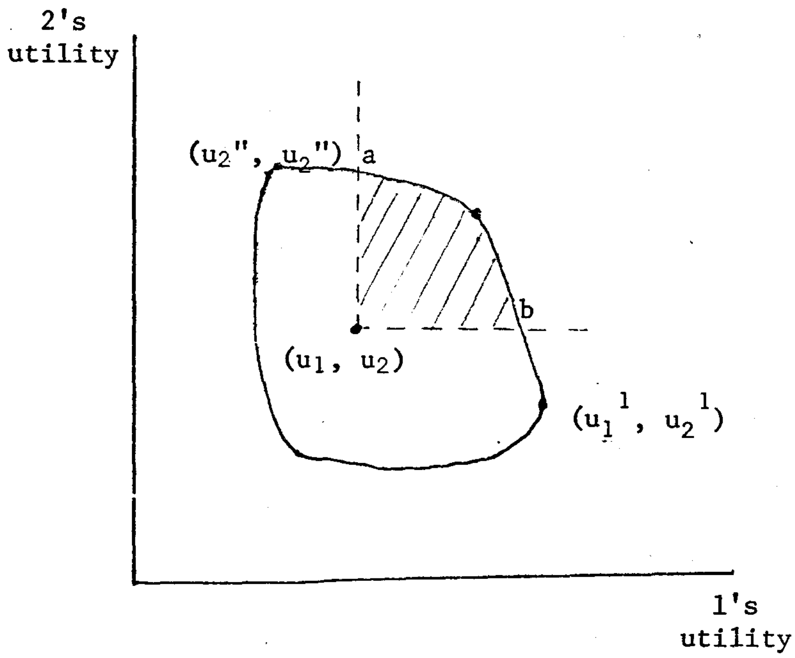

FIGURE 1

There have been several attempts to single out the point on the line segment ab that the two parties would reach through "fair" negotiations. The basis of the agreement on this point is that there should be a point where the anticipated gain by a party provides the amount of satisfaction that he should expect to get. Being rational, he agrees to this point. One such method has been developed by Nash.[7]

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7. Nash, John F., "The Bargaining Problem," Econometrica, Vol. XVIII, April, 1950.

Nash's procedure for choosing this point requires the following assumptions:

(1) u_1 and u_2 are the utility functions of two individuals

(2) the set S is compact and convex,

(3) the point of no bargain (no joint utilization) gives zero payoff, (u_1, u_2);

(4) if (u_1^1, u_2^1) in S represents the solution point, there is no point (u_1, u_2) such that $u_1 > u_1^1$ and $u_2 > u_2^1$,

(5) if T were the set of possible bargains which contains S, then if c(T) were contained in S, c(T) = c(S),

(6) if S is symmetric[8], then c(S) is a point on the line $u_1 = u_2$.

From these assumptions Nash demonstrates that the point c(S) is the point where $u_1^* u_2^*$ is the maximum of all products $u_1 u_2$. Convexity assures that this point is unique.

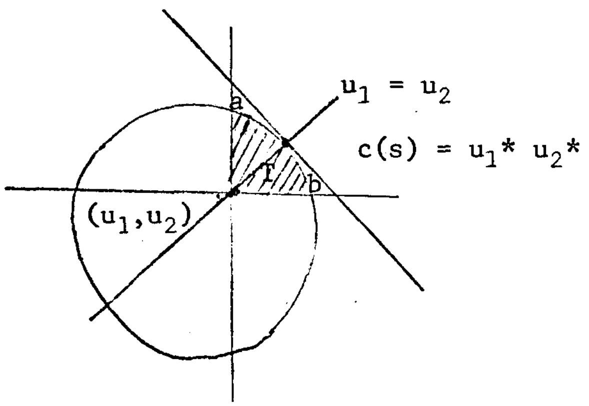

FIGURE 2

8. We define S to be symmetrical if (a,b) is in S and (b,a) is also in S.

IV. SUMMARY

Not surprisingly, we observed in our analysis of separate ownership of an indivisible input that the owner of the input would restrict the availability of the service of the input to the two user firms and discriminate between them in price. This activity would dampen the expansion of the user firms, i.e., diminish the impact of the indivisible input of the tendency of resources to agglomerate spatially.

The joint ownership of the indivisible input leads to a bargaining solution between the two firms concerning the level of utilization of the indivisible input and thereby the size of the agglomeration. A prerequisite never acknowledged in the "theory of agglomeration" is the existence of utility functions for the entrepreneurs of the user firms which leads to a stable solution.