BISSETT
BERMAN

MJO 3116

FINAL REPORT
ON
LUNAR MODULE RELAY EXPERIMENT
TRACKING ACCURACY STUDY
July 1967

Contract No. NASW-1591

To: National Aeronautics and Space Administration
Washington, D. C. 20546

Attention: Mr. Samuel W. Fordyce
Code MLA

CONTENTS

1. SUMMARY

This final report summarizes the efforts which took place under NASA Headquarters Contract No. NASW-1591 (Lunar Module Relay Experiment Tracking Accuracy Study) by The Bissett-Berman Corporation, Santa Monica, California. Technical progress under this contract was indicated by LM Relay notes and informal monthly letter progress reports. The LM Relay notes indicated detailed technical results and the approach as they evolved during this program. The monthly letter reports summarized the results of the LM Relay notes and indicated highlights of meetings attended by Bissett-Berman.

Earth-based tracking stations are severely restricted in coverage which can be provided to spacecraft during near-earth phases. On the other hand, one synchronous relay can provide essentially hemispherical earth coverage. This makes the concept of a synchronous satellite network attractive from both communication and navigation capability. The LM Relay Experiment proposes both to demonstrate this enhanced capability and gain operational experience with such a system. The tracking accuracy studies were performed to provide quantitative results which would be indicative for a number of spacecraft mission phases.

Conservative error estimates were made throughout. Parameter and measurement error sources were taken directly from the Apollo Navigation Working Group, Technical Report No. AN-1.2. These values are conservative for even present-day applications, and substantial improvement should be expected for the period of interest for the LM Relay Experiment and a Synchronous Relay Network. In addition, results obtained for tracking a CSM assumed a simple rather than more optimal data processor. The results obtained indicated that one could expect high quality tracking accuracy with a small number of relays comprising a network.

This study was limited to an accuracy study to serve as one input for NASA cost-effectiveness studies. The final desired number of relays would be influenced by mission navigation accuracy requirements. As

few as two relays could considerably enhance navigational (and communications) capability.

Three mission phases were considered during this effort: one, earth insertion; two, translunar injection; and, three, transearth. Apollo Mission reference trajectories were used throughout and taken as indicative. The results obtained indicated that the relay orbit inclination angle would not markedly affect CSM tracking accuracy during these mission phases. A comparison was made for CSM accuracy between tracking by one Relay and a ship. The ship would perform better for short tracking intervals. On the other hand, the comparison between a ship and two relays indicates that the two relays would perform significantly better.

An additional study was performed regarding the location of spacecraft subsequent to earth touchdown. This study was suggested to BBC just prior to the conclusion of the study effort. Results were obtained for one relay tracking the downed spacecraft. The results were encouraging and further indicated that spacecraft drift associated with the surface ocean currents would be the major error source. Further studies were made regarding geometric and multipath limitations on tracking coverage.

The remaining sections of this report deal with the system concept, implications of a synchronous relay network, the technical approach, and the quantitative results.

2. SYSTEM CONCEPT

The purpose of this section is to provide a general background into the elements involved in a LM Relay Experiment and later Synchronous Relay Networks. The LM Relay Experiment would consist of a communications and tracking relay in an earth synchronous orbit linking two terminals (see Figure 1). One of these terminals could be a station of the Manned Space Flight Network (MSFN); the other a manned spacecraft in a low altitude earth orbit (e. g. , a CSM). The radio frequency links could use equipment similar to the Unified S-Band Equipment (USBE) developed on the Apollo Program or repeater type equipment developed for communications satellites applications.

The use of USBE type equipment would allow coherent type tracking. Appropriate tracking measurables are then two-way doppler or two-way doppler and range. The repeater type equipment would use only doppler as a tracking measurable. Either type equipment would be required to have the necessary communication capability including communications initiation from either the CSM or the Mission Control Center (MCC).

The enhanced coverage (or visibility) capability associated with a relay is indicated in Figure 2. The visibility contours correspond to a CSM in a 200 n. mi. circular orbit. This figure includes ground-based stations, tracking ships, and a synchronous relay. Only 2. 7% of the earth's surface can be seen from this altitude of 200 n. mi. , while 42% is visible from the synchronous altitude. In addition, the CSM can be seen over 59% of its orbital sphere from a synchronous relay.
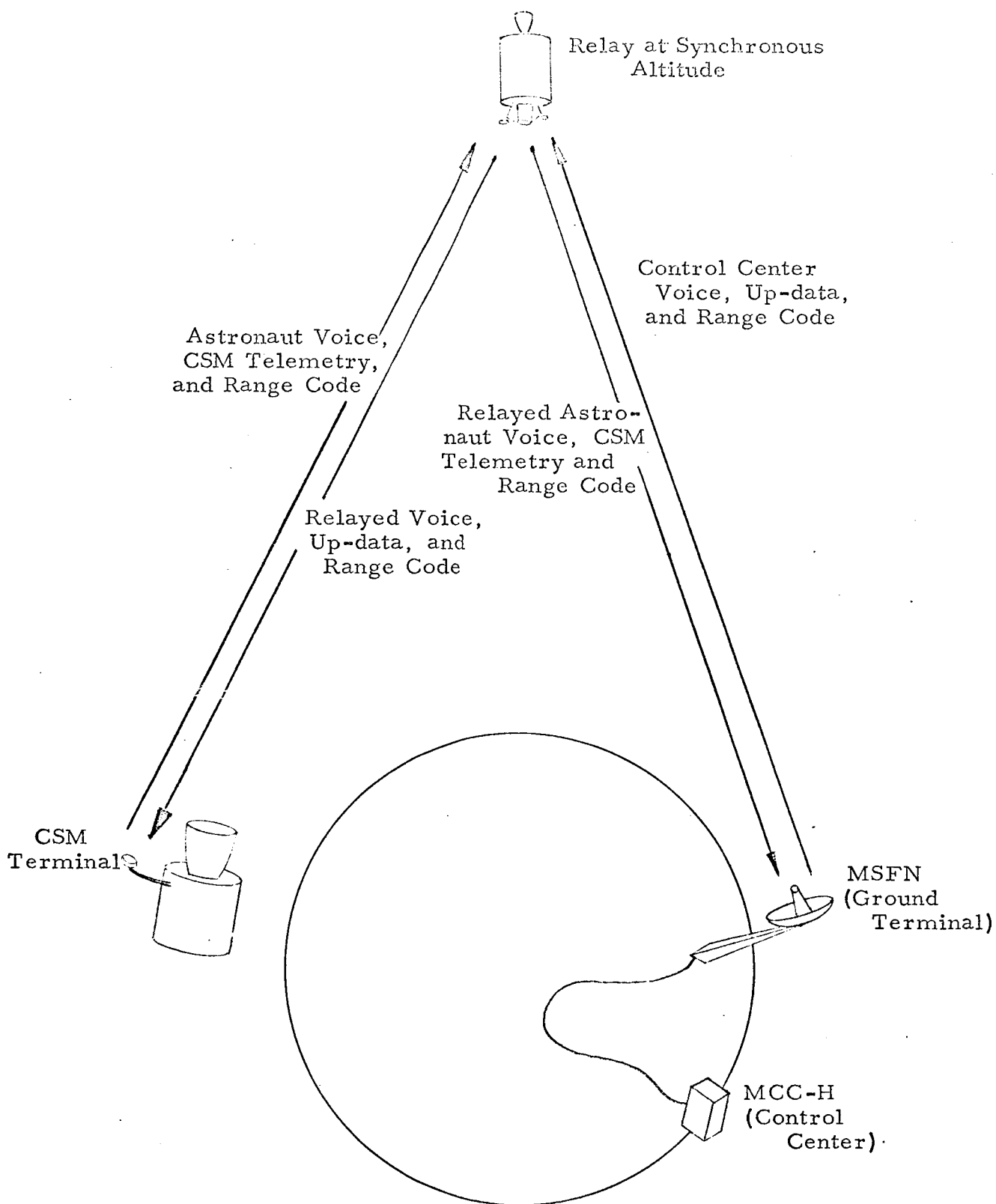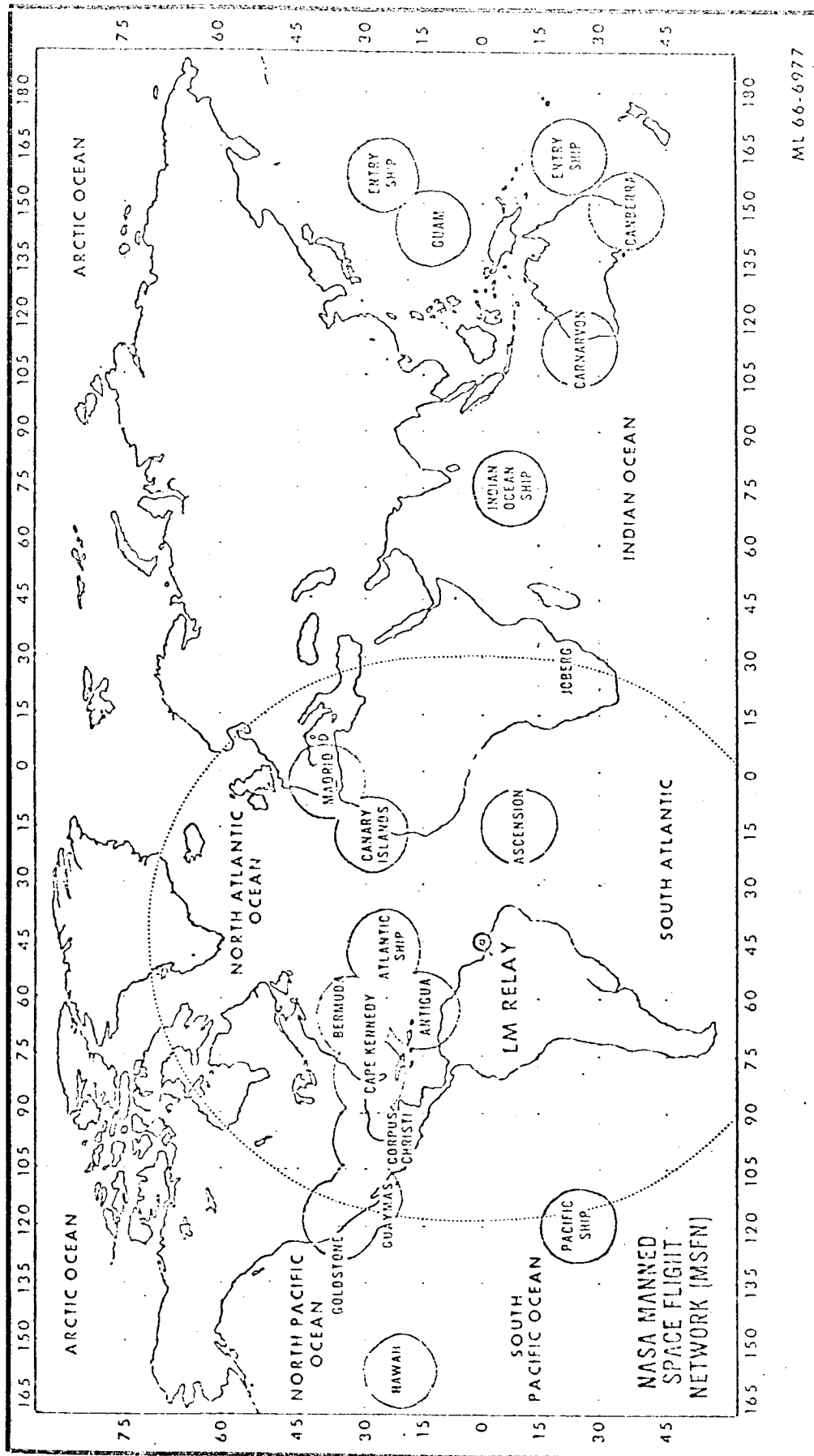
Figure 1. An Illustration of the "LM Relay" Experiment Concept

The diagram shows a "Relay at Synchronous Altitude" at the top, with communication links to a "CSM Terminal" on the lower left and an "MSFN (Ground Terminal)" on the lower right, with the "MCC-H (Control Center)" shown on the Earth.

Labels on the diagram:

- Relay at Synchronous Altitude
- Control Center Voice, Up-data, and Range Code
- Astronaut Voice, CSM Telemetry, and Range Code
- Relayed Astronaut Voice, CSM Telemetry and Range Code
- Relayed Voice, Up-data, and Range Code
- CSM Terminal
- MSFN (Ground Terminal)
- MCC-H (Control Center)

Figure 2. A Comparison of Visibility Contours of the MSFN and a LM Relay at Synchronous Altitude

3. IMPLICATIONS OF SYNCHRONOUS RELAY NETWORK

A Synchronous Relay Network could provide the continuous communications and tracking capability to the MSFN not presently available during near-earth phases.

1. Substantial improvements could be achieved for crew safety and mission success. Timely detection of malfunctions could be effected to initiate necessary remedial actions, thus enhancing the control capability of the Mission Control Center (MCC).

2. Constraints imposed on performing critical operations within view of an earth-based tracking station could be relieved. This could widen the tolerance on allowable parameters during various mission phases.

3. The network would augment or supplant elements of the MSFN and NASCOM. Many elements of the present MSFN are predicated by the necessity for communication and tracking during earth phases. Aircraft, tracking ships, and perhaps even elements of the ground station (in the order indicated) associated with communications and tracking requirements could be eliminated.

4. An extension of the powered flight monitoring capability could be made. For example, during Gemini Missions a number of powered flight phases were not directly observable from earth-based stations.

5. Additional cruise flight tracking data could be made available to establish deviations from expected performance levels on a more timely basis.

6. Elimination of communication and tracking black-out during the reentry phase could be achieved. This could arise from a higher radio frequency being used for this purpose and/or a lower election density sheath between the CSM and the Relay than between the CSM and an earth-based station.

6

7. The relay network could provide assistance in the location of downed spacecraft subsequent to earth touchdown. This could allow earlier recovery of downed spacecraft.

4. TECHNICAL APPROACH

A number of elements were involved in the technical approach. It was necessary that guidelines and ground rules be formulated that could be implemented in a quantitative fashion. An analytical technique had to be adopted which would allow meaningful numerical results to be obtained with a minimum modification to an existing Orbit Error Analysis Program (OEAP). This initial task was a combined NASA-BBC effort.

A. Guidelines

1. The tracking accuracy studies should present typical results for a number of mission phases. Apollo reference trajectories for earth insertion, translunar injection, and transearth phases were adopted.

2. Conservative values for error sources should be used throughout. The values used are conservative for even present-day studies.

3. Only ground-based USB stations would track the synchronous relays. No station being required to operate for more than one shift (eight hours) per day for this purpose.

4. Spacecraft equipment must be compatible with initiation of tracking at either the CSM or a ground station.

5. Results would be obtained for spacecraft equipment configurations compatible with both the dual coherent mode and the repeater mode.

6. Computing time would not be critical in the establishment of the relay ephemeris. This allows a more sophisticated data processor at the MCC for this purpose.

7. Computing time and reliability would be important constraints in the establishment of the CSM ephemeris. A simple data processor which only estimated the state vector (no systematic errors estimated) was used for this purpose. This processor has less capability than the Apollo Real-Time Orbit Determination Program (RTODP).

B. Ground Rules

The intent here was to use established navigational requirements on the various mission phases. The requirements which were available are indicated below.

1. Earth Insertion (Go, No-Go Decision)

    (a) Insertion State Vector Uncertainty

Three sigma speed	16 fps
Three sigma flight path angle	0.16 deg.
Three sigma altitude	2.4 n. mi.

    (b) Propagated Insertion State Vector

Three sigma altitude	30 n. mi.

2. Translunar Injection

Minimum of 10 minutes of tracking data during the first 20 minutes following injection.

3. Reentry Phase

Telemetry data only.

C. Orbit Error Analysis Program (OEAP)

The analysis technique was based on a version of an OEAP developed by BBC for MSC under Contract No. NAS 9-4435. This particular program is a patched conic version which can treat inter-vehicle measurements (e. g. , Relay-to-CSM) in addition to measurements from earth-to-spacecraft. Patched conic programs minimize computing time and costs. This program treats data only during cruise flight conditions.

9

Three different covariance matrices can be obtained on a given computer run. The first indicates computed error quantities with no systematic error sources present. The second covariance matrix pertains to results which include the effect of non-estimated parameters on parameters estimated. The third covariance matrix indicates results for optimal processing. These results can be interpreted to indicate whether random noise errors, the data processing technique, or systematic error sources are the limiting factor on accuracy. The error sources considered are indicated in Table 1 below.

Systematic Errors	Noise Type Errors
Measurement Bias	Clock
Station Location	Quantization
Clock Parameters	Doppler Random Phase
Earth's Gravitational Constant	Range Noise
	Angle Noise

Table 1. Sources of Uncertainty Considered

10

5. ANALYTICAL RESULTS

The analytical results are presented in the appendices. These appendices are:

Appendix	LM Relay Note No.	Title
A	9	Tracking the LM Relay
B	10	Determining Earth Parking Orbit Insertion Conditions with the LM Relay
C	11	Tracking the Translunar Trajectory with the LM Relay and a Ship
D	12	Tracking the Transearth Trajectory with the LM Relay and a Ship
E	13	Location of Spacecraft Following Touchdown on Earth
F	14	Tracking the Apollo Mission Insertion, Injection, and Reentry Phases with Two LM Relays
G	15	Supplement to the Two Relay Tracking Analysis of LM Relay Note No. 14

LM RELAY NOTE NO. 9

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TRACKING THE LM RELAY

5 June 1967

Summary

A theoretical study of LM Relay Tracking data obtained by a Master station and two Slave stations has been made to determine the accuracy of real time orbit determination solutions. The LM Relay is in a synchronous orbit (inclination $0°$ and $28.5°$) positioned initially on the equator at various longitudes in the Atlantic and Pacific Ocean areas.

For 16 hours of tracking (0-8 hrs. and 24-32 hrs. after insertion) with range-rate, range, and angles by the Master station and range-rate by the Slave stations, the following typical accuracies can be obtained for the LM Relay state vector:

	1-σ Position	1-σ Velocity
All parameters estimated	1000 -3000 feet	0.1-0.3 ft/sec.
Only state vector components estimated	2000 -30,000 ft.	1.2-2.0 ft/sec.

Generally, the state vector estimates for the LM Relay are worst when the relay is initially positioned at the same longitude as the Master tracking station, especially for synchronous equatorial orbits.

Error Analysis Inputs

The Atlantic Relay is tracked by MAD (Master station), ANG, and ACN, while the Pacific Relay is tracked by CNB (Master station), GWM, and HAW. Tracking measurements are accumulated at a one minute sample rate above $5°$ elevation during the periods 0-8 hours

and 24-32 hours after insertion. At some initial longitude positions, the relay is not always in view of the tracking stations considered during the above periods. Table 1 shows the hours of tracking data accumulated in these situations for the various trajectories.

In every case, the Master station collects range-rate, range, and X-Y angle data, while the Slave stations collect range-rate data only. Measurement errors and bias, taken from Reference 1, are shown in Table 2, as well as uncertainties in station location and other pertinent parameters.

A complete list of parameters (25 in all) contains 6 state vector components, 9 station locations, 3 clock offset parameters, 3 clock rate parameters, 3 measurement biases (Master station range and angles). These constitute all the parameters considered in this analysis.

Error Analysis Results

Figures 1 through 10 show the results obtained for the accuracy in position and velocity of the LM Relay state vector. Here, the standard deviation in position (velocity) is defined as the square root of the sum of the diagonal position (velocity) elements in the covariance matrix.

In each figure, the curve labelled 1 represents the result obtained if only the six state vector components are estimated in the orbit determination, with remaining 19 parameters, non-estimated, having uncertainties as shown in Table 2 and contributing to the total errors. Curve 2 corresponds to estimating all the parameters with the apriori values as shown in Table 2, while curve 3 corresponds to estimating only the six state vector components (with apriori) and here only measurement noise contributes to the total errors.

2

In all cases, the LM Relay initial position is on the equator and the orbit designation 28.5° ascending (descending) means that the spacecraft is in a 28.5° inclined orbit proceeding northward (southward).

Figures 9 and 10 are not as extensive as the others and are mainly to portray the negligible difference in results for tracking different portions of the synchronous orbit.

TABLE 1 - REDUCED TRACKING INTERVALS*

Relay	Orbit	Initial Longitude	Stations	Tracking Intervals (Hrs. from Insertion)
Pacific	28.5° Ascending	150°W	CNB GWM HAW	0 to 2.8 24 to 26.8
Pacific	28.5° Ascending	165°W	CNB GWM HAW	0 to 5.5 7.7 to 8.0 24 to 29.5 31.7 to 32
Pacific	28.5° Ascending	120°E	HAW	3.5 to 8 27.5 to 32
Atlantic	Equatorial	30° E	ANG	none
Atlantic	Equatorial	15° E	ANG	none
Atlantic	28.5° Ascending	30° E	ANG	none
Atlantic	28.5°Descending	60° W	MAD ANG ACN	0 to 2 24 to 26
Pacific	Equatorial	120°E	HAW	none

*Nominal tracking is from 0-8 and 24-32 hours from insertion.

4

TABLE 2 - MEASUREMENT ERRORS AND APRIORI UNCERTAINTIES

1. MEASUREMENT ERRORS (one sigma)

Measurement	Noise	Bias
Range rate (2-way)	0. 004 fps	NA
Range (1-way)	30 ft.	60 ft.
X-angle	. 8 milli-rad	1. 6 milli-rad
Y-angle	. 8 milli-rad	1. 6 milli-rad

2. STATION LOCATION UNCERTAINTIES (one sigma)

Station	Symbol	Altitude (ft.)	East-West(ft)	North-South(ft)
Madrid	MAD	141. 1	92. 8	101. 3
Antigua	ANG	137. 8	116. 4	111. 6
Ascension	ACN	105. 0	351. 7	344. 9
Canberra	CNB	216. 5	182. 3	192. 5
Guam	GWM	105. 0	651. 0	649. 2
Hawaii	HAW	141. 1	150. 5	142. 0

3. OTHER UNCERTAINTIES (one sigma)

Master Clock Offset	$3. 0 \times 10^{-3}$ sec
Slave Clock Offset	$3. 0 \times 10^{-3}$ sec
Master Clock Rate	$2. 0 \times 10^{-10}$ sec/sec
Slave Clock Rate	$2. 0 \times 10^{-10}$ sec/sec
Primary Gravitational Constant	$1. 06 \times 10^{11}$ ft^3/sec^2
Initial State Vector Position Components	60, 000 feet
Initial State Vector Velocity Components	100 ft/sec

5
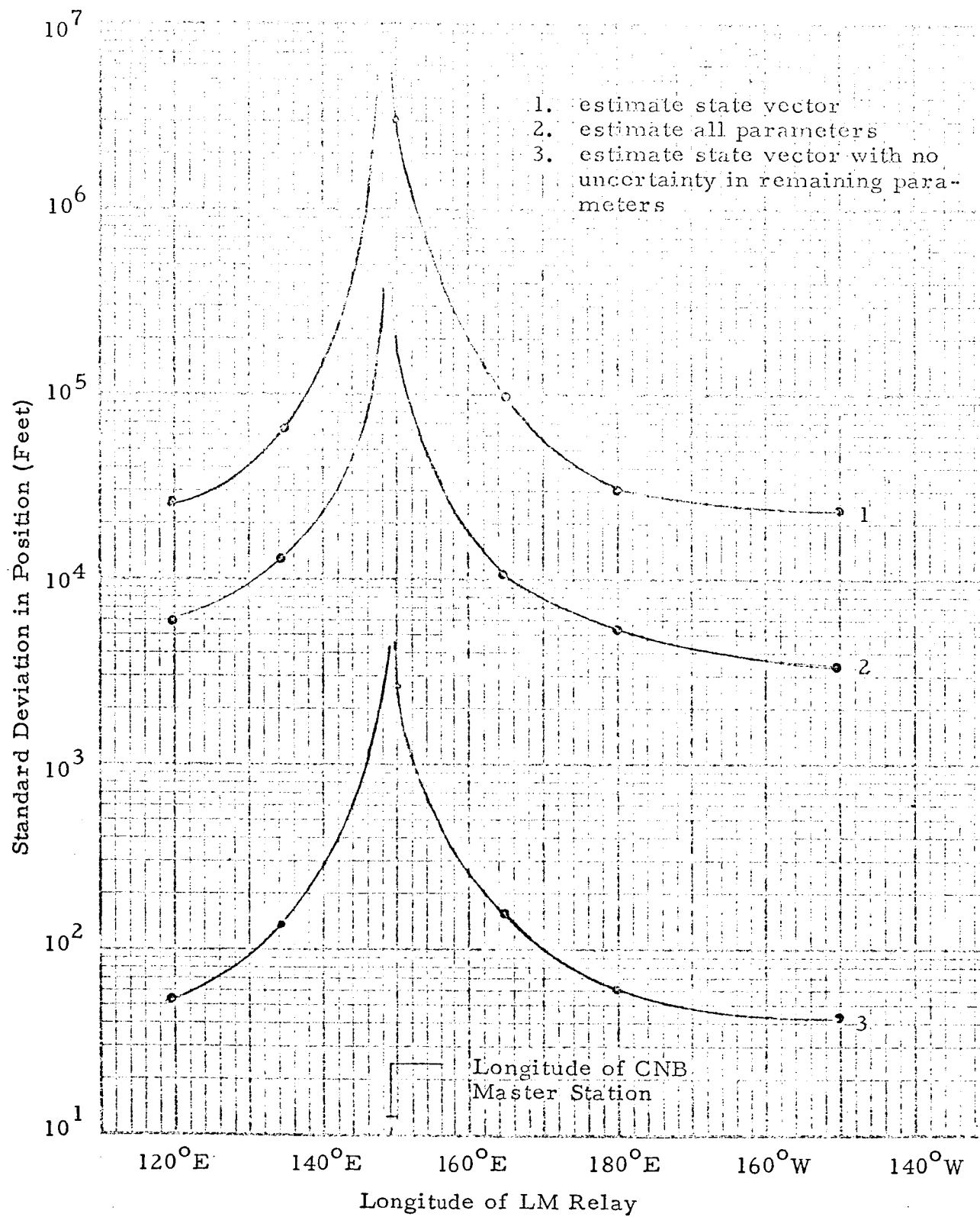
Figure 1　　Position Uncertainty at 32 Hours for Pacific LM Relay
In Equatorial Orbit
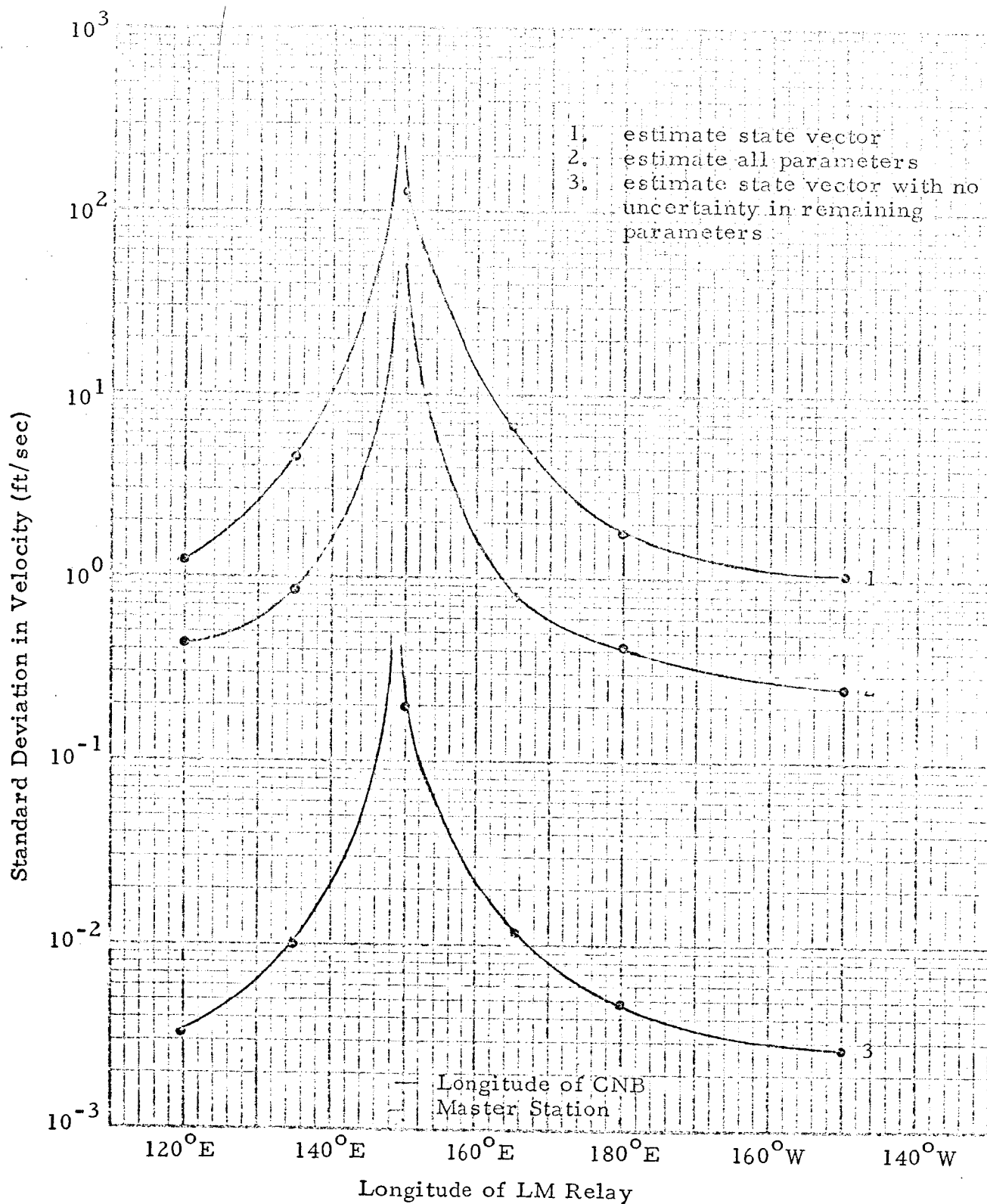
Figure 2   Velocity Uncertainty at 32 Hours for Pacific LM Relay
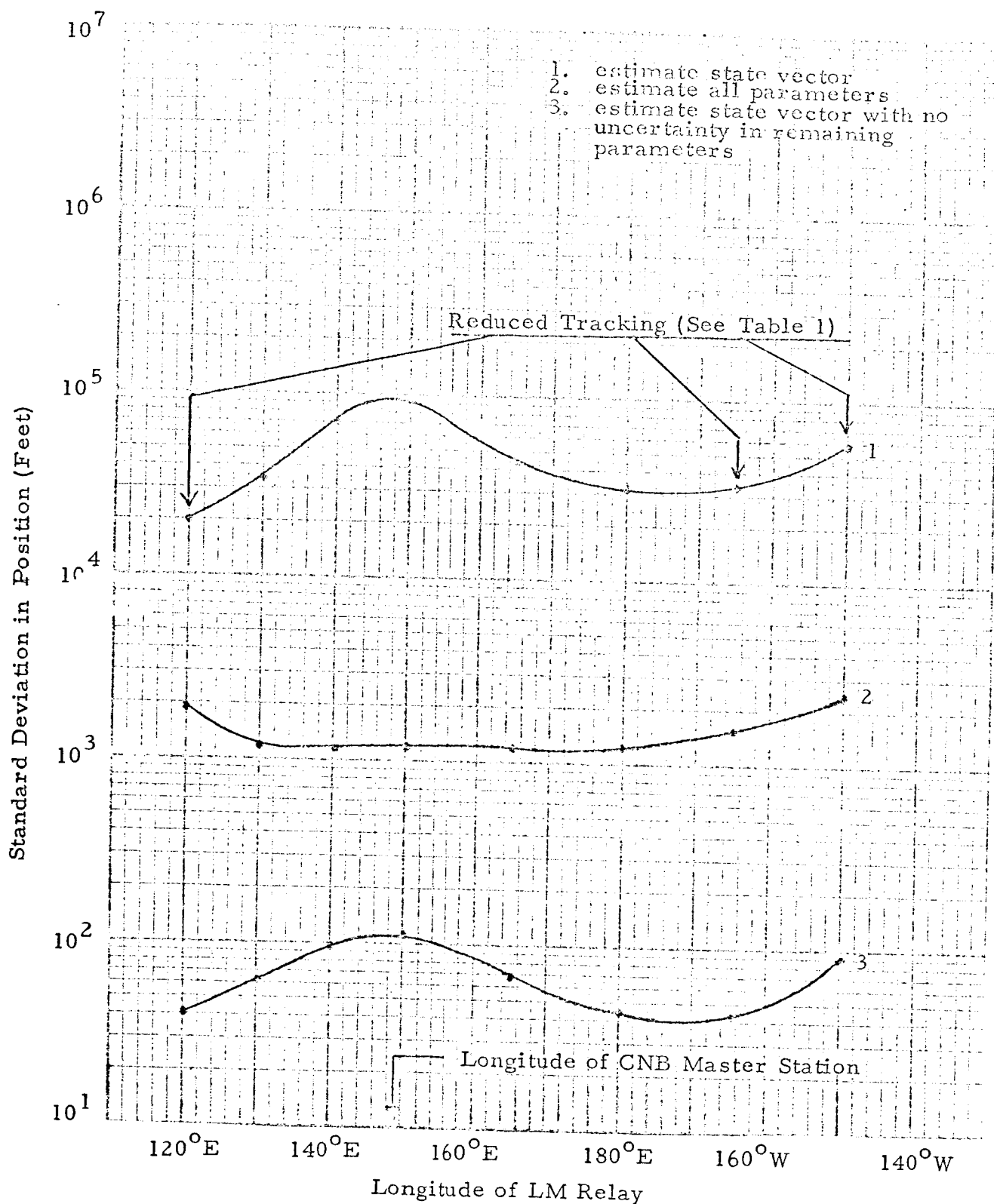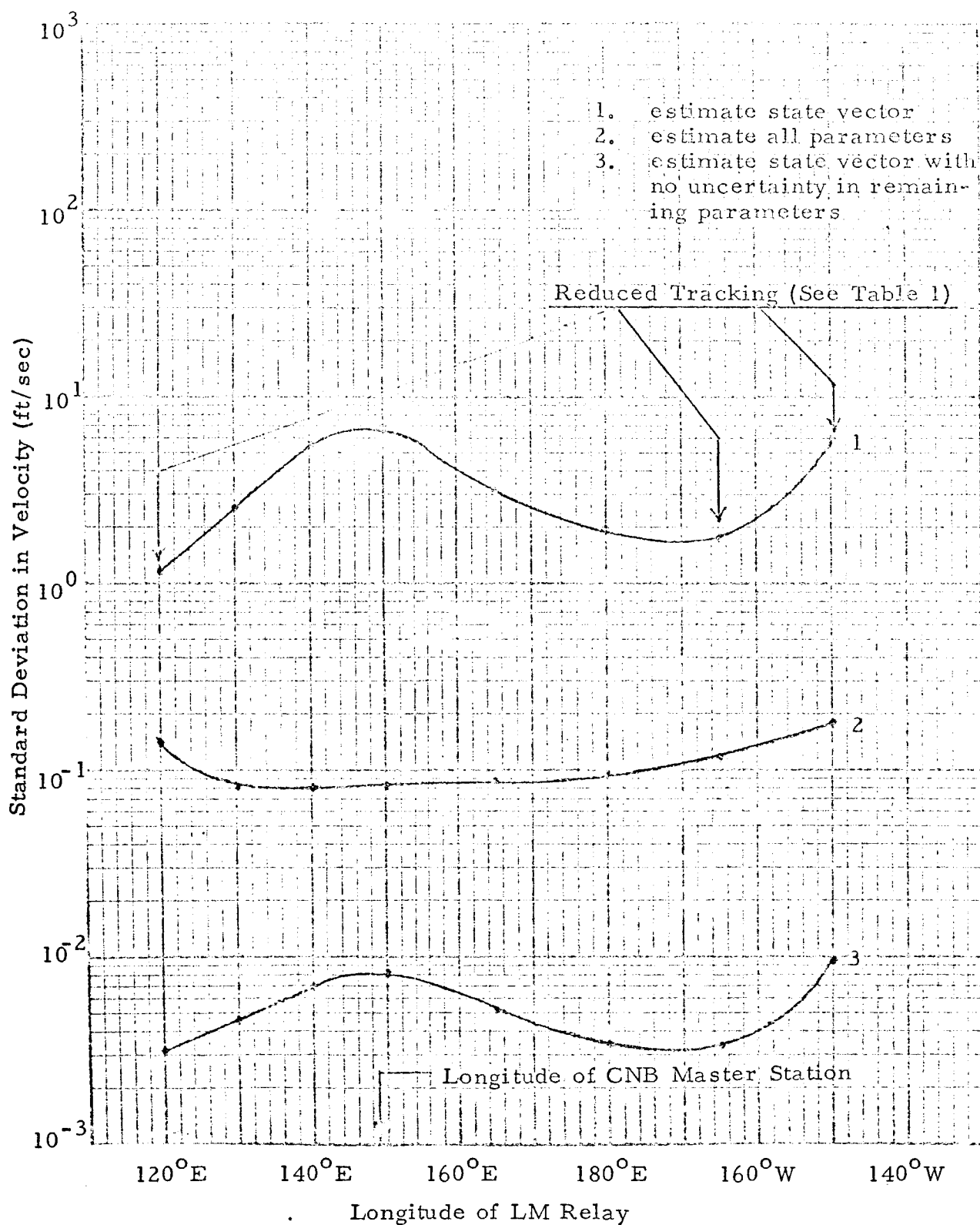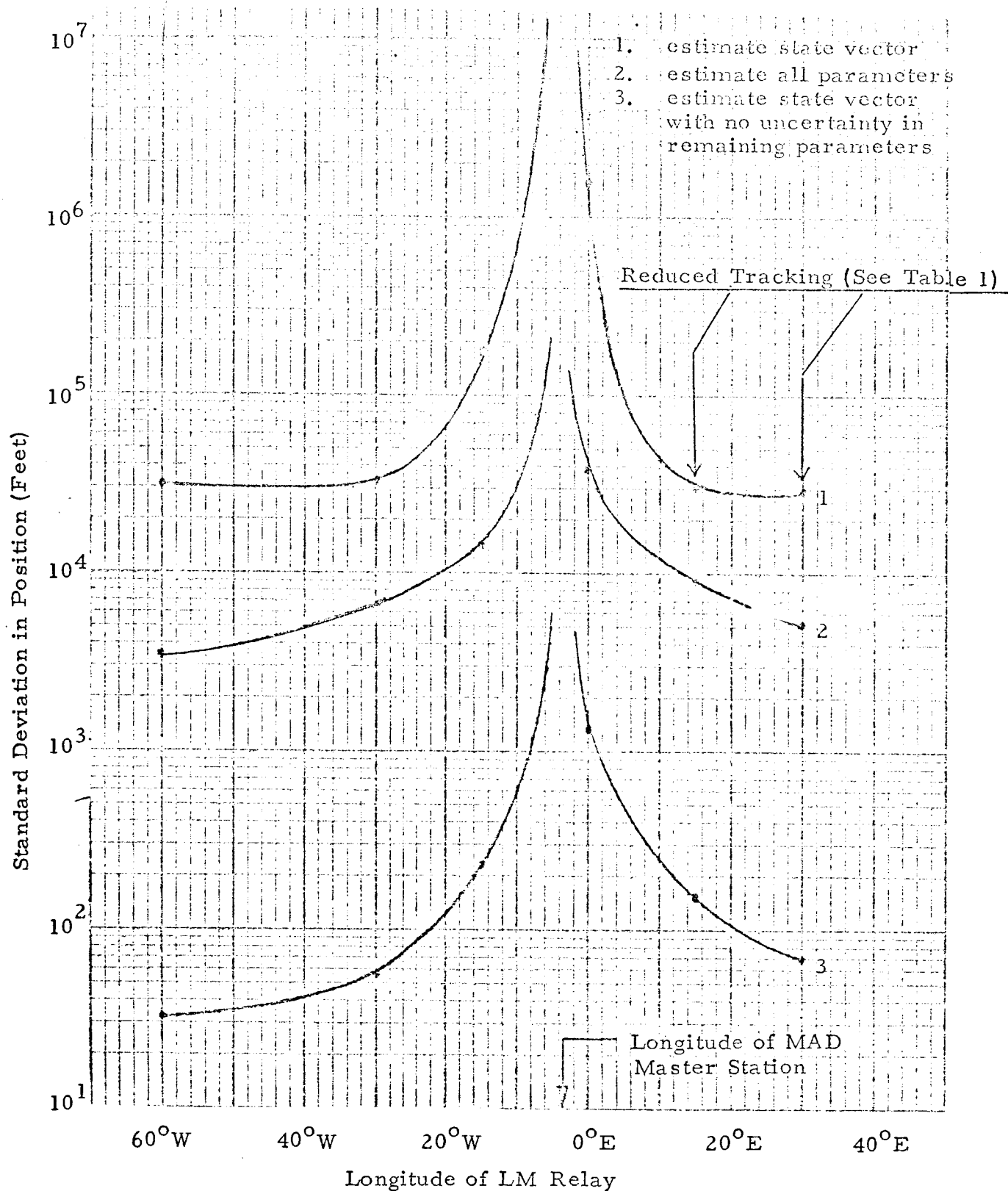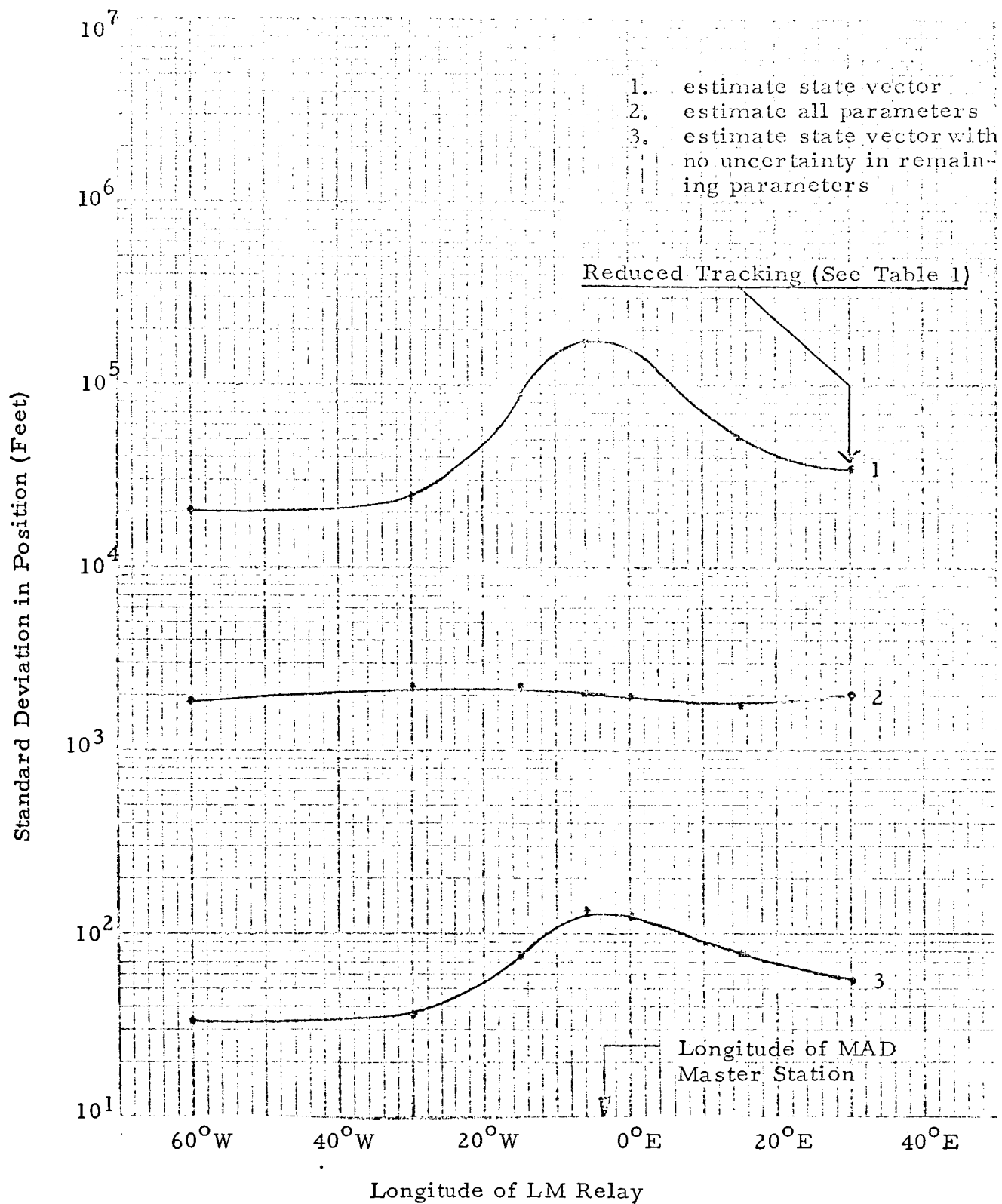In Equatorial Orbit

The figure shows a log-scale plot with "Standard Deviation in Velocity (ft/sec)" on the y-axis (from 10^{-3} to 10^3) and "Longitude of LM Relay" on the x-axis (from 120°E to 140°W).

Legend:
1. estimate state vector
2. estimate all parameters
3. estimate state vector with no uncertainty in remaining parameters

Longitude of CNB Master Station

10^7

1. estimate state vector
2. estimate all parameters
3. estimate state vector with no uncertainty in remaining parameters

10^6

Reduced Tracking (See Table 1)

10^5

Standard Deviation in Position (Feet)

10^4

1

2

10^3

10^2

3

Longitude of CNB Master Station

10^1

120°E   140°E   160°E   180°E   160°W   140°W

Longitude of LM Relay

Figure 3   Position Uncertainty at 32 Hrs. for Pacific
LM Relay in 28. 5° Orbit

(Ascending)

8

Figure 4   Velocity Uncertainty at 32 Hrs. for Pacific LM
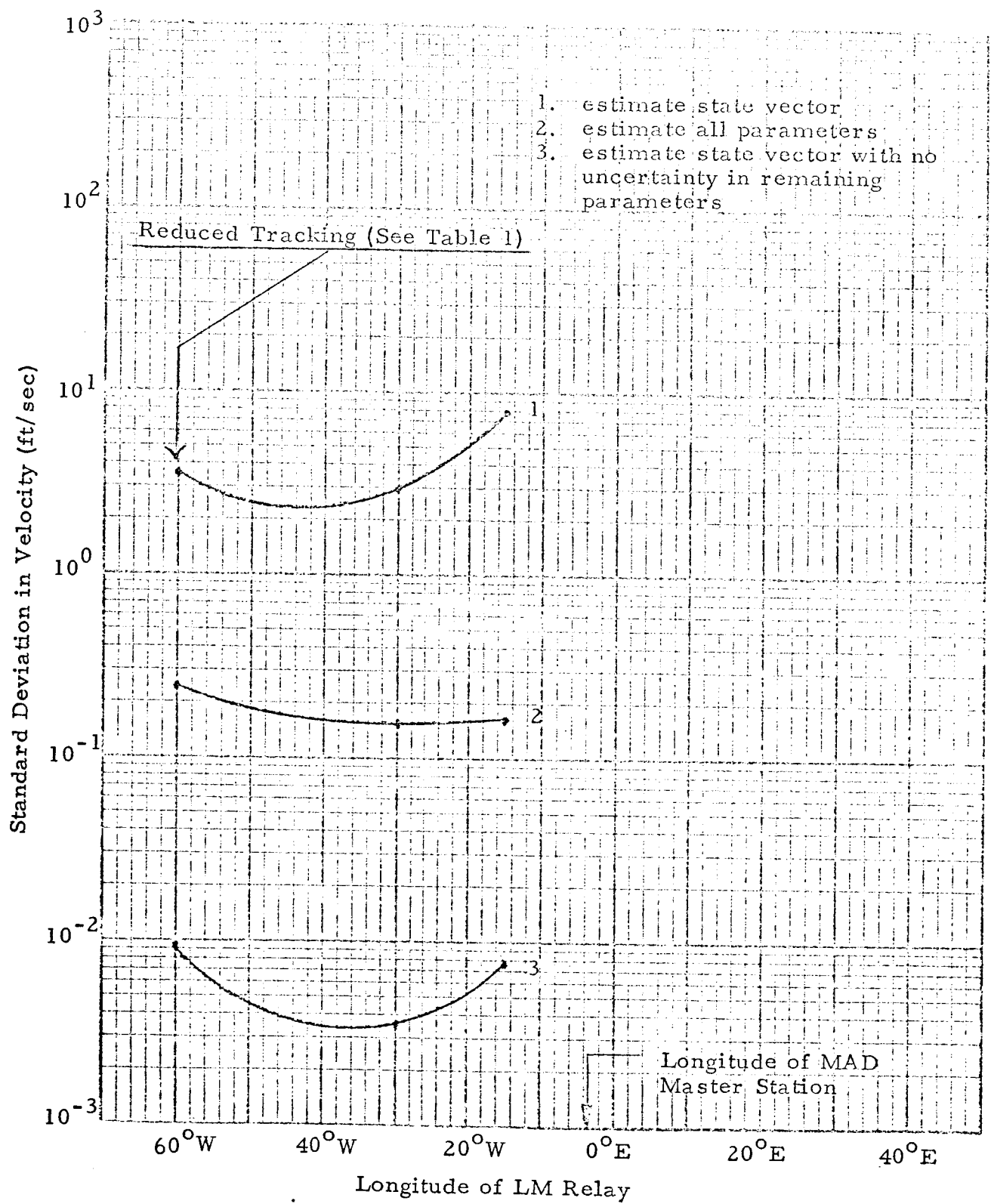Relay in 28. 5° Orbit (Ascending)

Figure 5　Position Uncertainty at 32 Hrs. for Atlantic
LM Relay in Equatorial Orbit
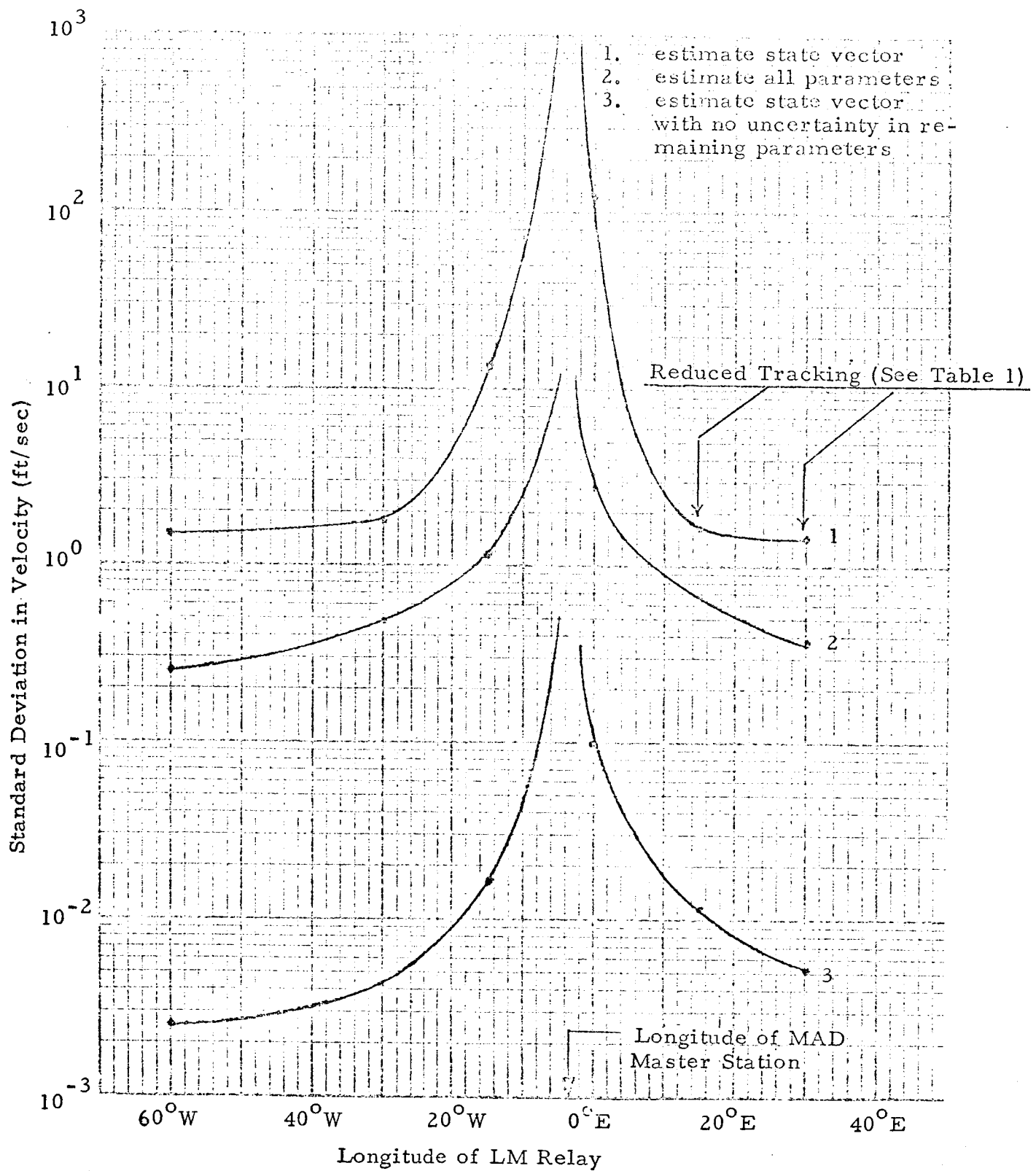
Figure 6   Velocity Uncertainty at 32 Hrs. for Atlantic
LM Relay in Equatorial Orbit

1. estimate state vector
2. estimate all parameters
3. estimate state vector with no uncertainty in remaining parameters

Reduced Tracking (See Table 1)

Longitude of MAD Master Station

Figure 7   Position Uncertainty at 32 Hrs. for Atlantic LM Relay
in 28.5° Orbit (Ascending)
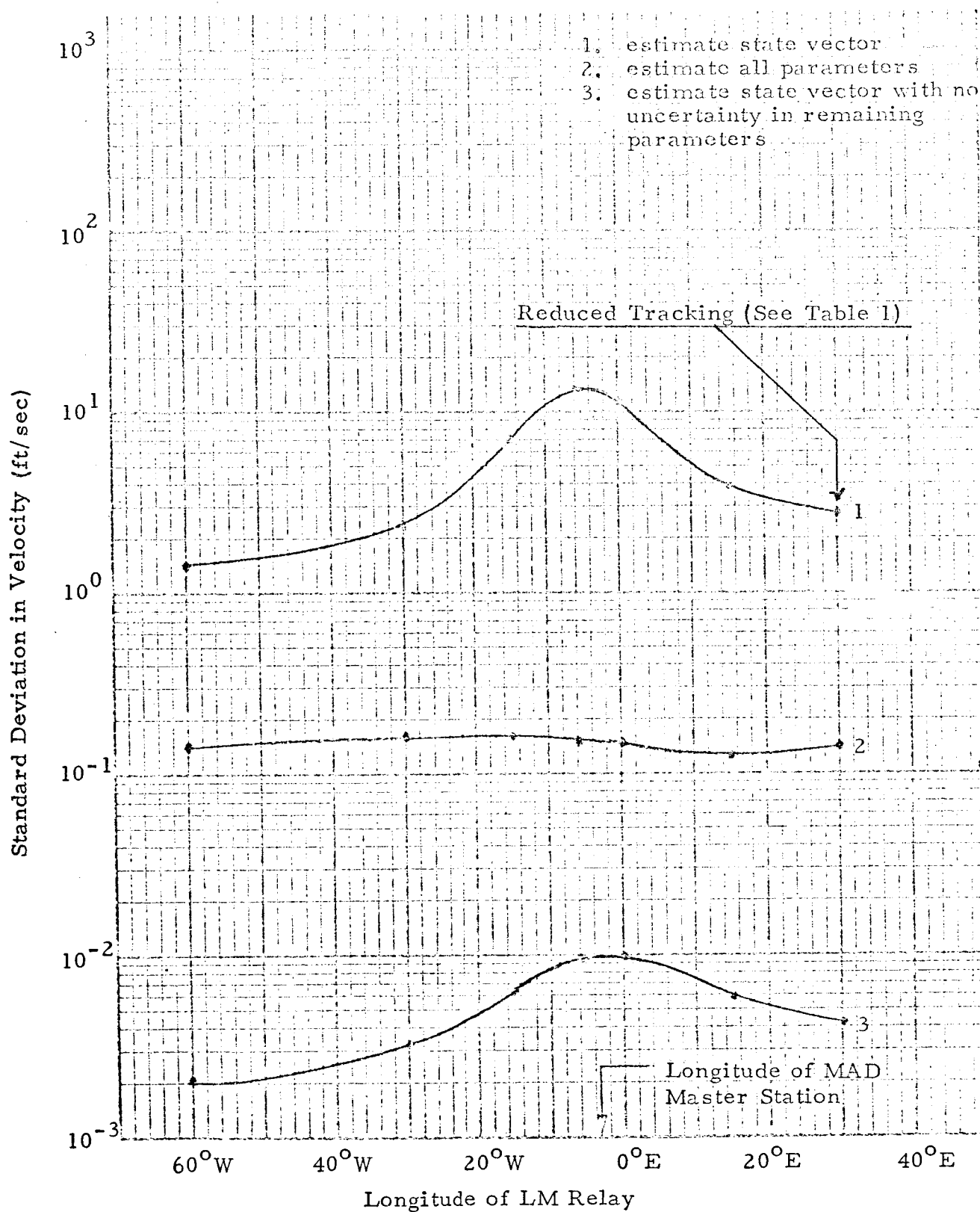
Figure 8   Velocity Uncertainty at 32 Hrs. for Atlantic LM Relay

in 28.5° Orbit (Ascending)
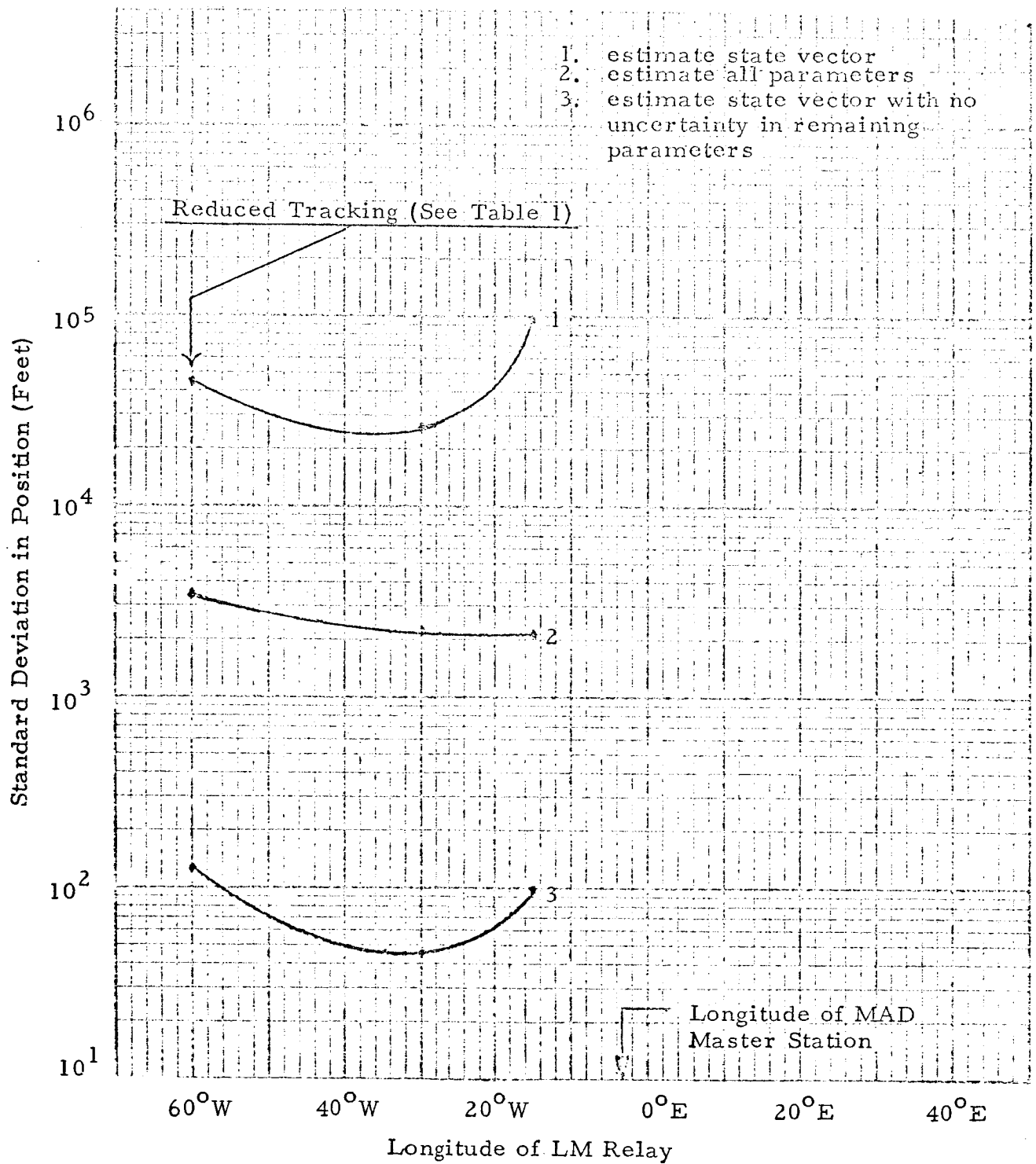
Figure 9   Position Uncertainty at 32 Hrs. for Atlantic LM Relay
in 28. 5° Orbit (Descending)

14

Figure 10   Velocity Uncertainty at 32 Hrs. for Atlantic LM Relay
In 28.5° Orbit (Descending)

REFERENCES

1.   ANWG Technical Report No. AN-1.1, "Apollo Missions and
Navigation Systems Characteristics," April 4, 1966.
National Aeronautics and Space Administration, GSFC,
Greenbelt, Md., and MSC, Houston, Texas.

H. Epstein
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A. Burkow
5 June 1967

LM RELAY NOTE NO. 10

DETERMINING EARTH PARKING ORBIT INSERTION CONDITIONS WITH THE LM RELAY

Summary

A theoretical study has been made of the accuracy of Earth parking orbit insertion conditions for a LM Relay, in an equatorial synchronous orbit, tracking the CSM with 2-way and 1-way Doppler measurements. For comparative purposes, results are also shown for ship tracking data.

The 2-way Doppler mode with range and range-rate data yields the best results for the LM Relay. In the case of range-rate data only, the 1-way and 2-way Doppler modes yield almost identical results.

Accuracy requirements for parking orbit insertion conditions and maximum altitude deviation during one revolution require the following tracking durations for the 2-way Doppler mode with range and range-rate data:

Parameter	1-σ Requirement	Tracking Required
Altitude at insertion	4800 ft.	less than 1 minute
Speed at insertion	5. 3 ft/sec.	4. 5 minutes
Path angle at insertion	.93 milli-rads.	less than 1 minute
Maximum altitude uncertainty	60,000 ft.	3. 0 minutes

Generally, the duration of ship tracking required to meet insertion accuracy requirements is less than the values indicated above.

Introduction

The objectives in this investigation are to determine the comparative accuracies with which various LM Relay tracking systems and a tracking ship can predict Earth parking orbit insertion conditions for a CSM spacecraft.

The LM Relay tracking systems are assumed to include a 2-way Doppler mode (transmitter and receiver in the LM and a transponder in the CSM) and a 1-way Doppler mode (transmitter in the CSM and receiver in the LM). In both the above cases, measurement data (both range and range-rate) is relayed to a ground station for data processing. Past studies have indicated that adding range data to range-rate data from the 1-way Doppler mode increases prediction errors when estimating state vector components only. Therefore, the 1-way Doppler mode is examined using range-rate data only. In the case of 2-way Doppler data, the effects of both range-rate data only and range plus range-rate data is examined.

Hypothetically, the sequence of events is like the following. The LM Relay is inserted into an equatorial synchronous orbit at 44°W longitude. A Master station (Madrid) and two Slave stations (Antigua and Ascension) track the relay for sixteen hours in two eight hour shifts (0-8 hrs. and 24-32 hrs. after insertion). Tracking data consists of range-rate measurements for the Slaves and range-rate plus range plus X-Y angle measurements for the Master. All pertinent parameters are estimated in obtaining the LM Relay state vector at 32 hours after insertion (see LM Relay Note No. 9). At approximately this time, the CSM is inserted into a nominal 100 n. mi. circular parking orbit, and at insertion, the CSM is tracked by the LM Relay or the tracking ship. Visibility of the CSM for the tracking ship (32°N, 44°W) is approximately 5.2 minutes, while for the LM Relay, the visibility period is about 30 minutes. The CSM parking orbit is described in Reference 1.

2

Error Analysis Inputs

All measurements are taken at 6 second intervals with the tracking ship obtaining range-rate, range, X-Y angle data above 5° elevation, and the LM Relay obtaining range-rate and range data. Measurement noise is .04 fps, range-rate, 30 ft. range, and 1 milliradian for X-Y angles.

The parameters considered in the analysis are listed in Table 1 for each case, together with their apriori values. Most of the above values are taken from Reference 1. Only the state vector components of the CSM are estimated and errors to the estimate result from measurement noise and bias, and uncertainties in location of the tracking vehicle, clock offset and rate, and primary gravitational constant.

In the case of the LM Relay apriori values, a complete 7 x 7 covariance matrix was used for the 6 state vector components, and primary gravitational constant. These results are obtained by the 16 hours of tracking by the Master and two Slave stations prior to CSM insertion.

Error Analysis Results

Insertion conditions are described by the parameters altitude, speed, and flight path angle. The resulting uncertainties in these parameters for various tracking durations is shown in Figures 1, 2, and 3. Requirements for a GO-NO-GO condition are also illustrated and are obtained from Reference 2. Figures 4 and 5 show the uncertainty in position and velocity at insertion. Here position (velocity) is defined as the square root of the sum of the three position (velocity) diagonal components in the resulting covariance matrix.

The covariance matrix at insertion is mapped forward in time over one orbit revolution and the altitude uncertainty at various times, and for various tracking durations is shown in Figures 6, 7 and 8. Requirements in the maximum altitude uncertainty are also obtained from Reference 2.

3

Generally, LM Relay tracking durations must exceed those of the tracking ship for comparable uncertainties in CSM state vector estimates. To match insertion condition requirements, the parameter requiring the longest tracking duration (4.5 minutes) for the LM Relay is speed at insertion.

TABLE 1 – LIST OF PARAMETERS AND APRIORI VALUES

Parameter	Ship	2-way Doppler	1-way Doppler	Nominal Value
1. CSM position component	X	X	X	25,000 ft.
2. CSM velocity component	X	X	X	50 fps.
3. Ship R bias	X			60 ft.
4. Ship angle bias (X-Y)	X			2.0 milli-rad.
5. Ship clock offset	X			3 milli-secs.
6. Ship clock rate	X			2×10^{-10} sec/sec
7. Ship location Up	X			100 ft.
8. Ship location E-W	X			2000 ft.
9. Ship location N-S	X			2000 ft.
10. LM state vector		X	X	(2)
11. LM range bias		X(1)		60 ft.
12. LM clock offset		X	X	3 milli-secs.
13. LM clock rate		X	X	2×10^{-10} sec/sec
14. CSM clock offset			X	3 milli-secs
15. CSM clock rate			X	2×10^{-10} sec/sec
16. Gravitational constant	X	X	X	1.06×10^{11} ft^3/sec^2
TOTAL NUMBER	15	16(1)	17	

(1) Range bias not included in RD only case, therefore total parameters are 15 in this case.

(2) Marginal distribution from 16 hours tracking by master and 2 slaves.

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Figure 1 - Uncertainty in Altitude at Insertion

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
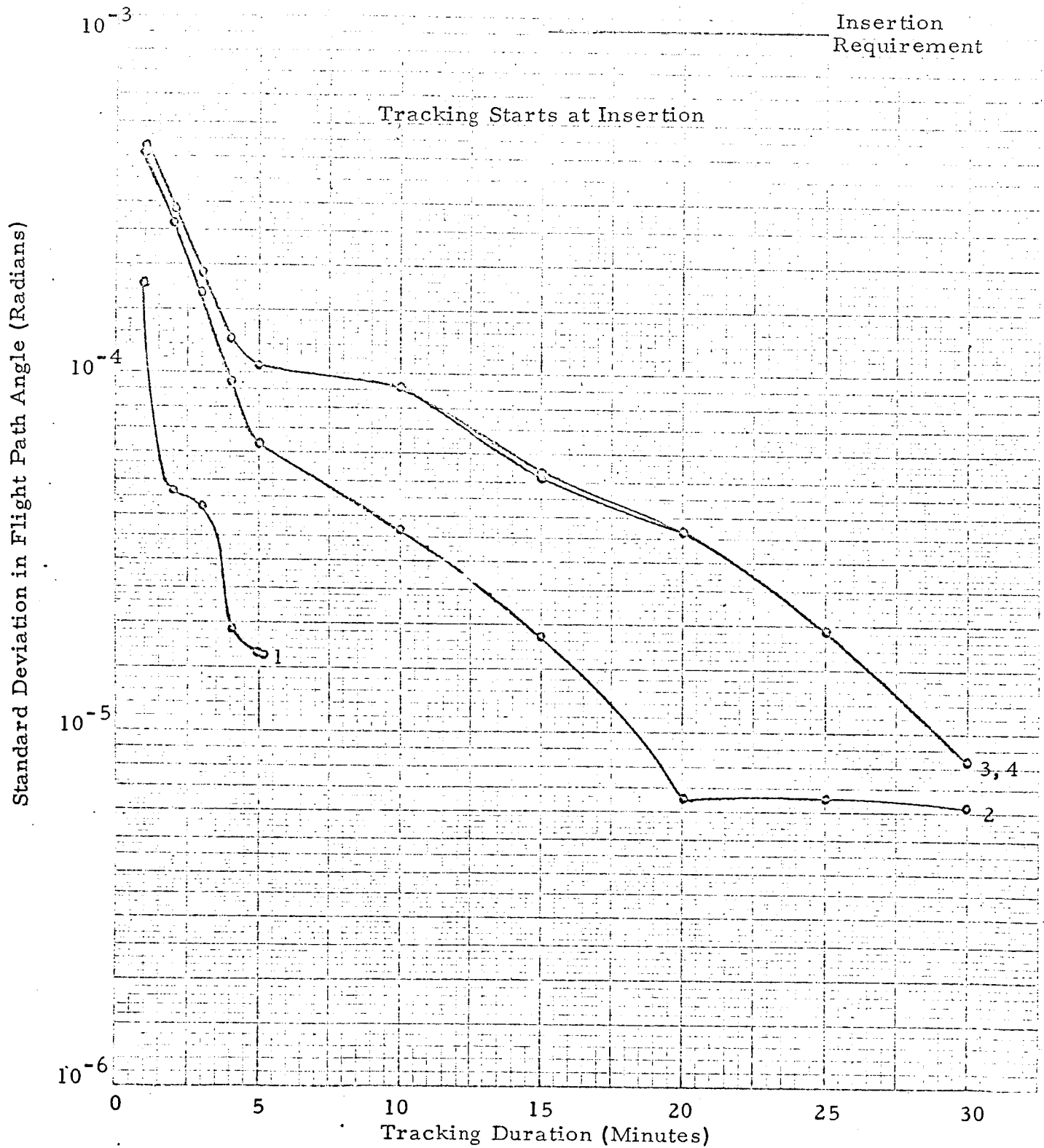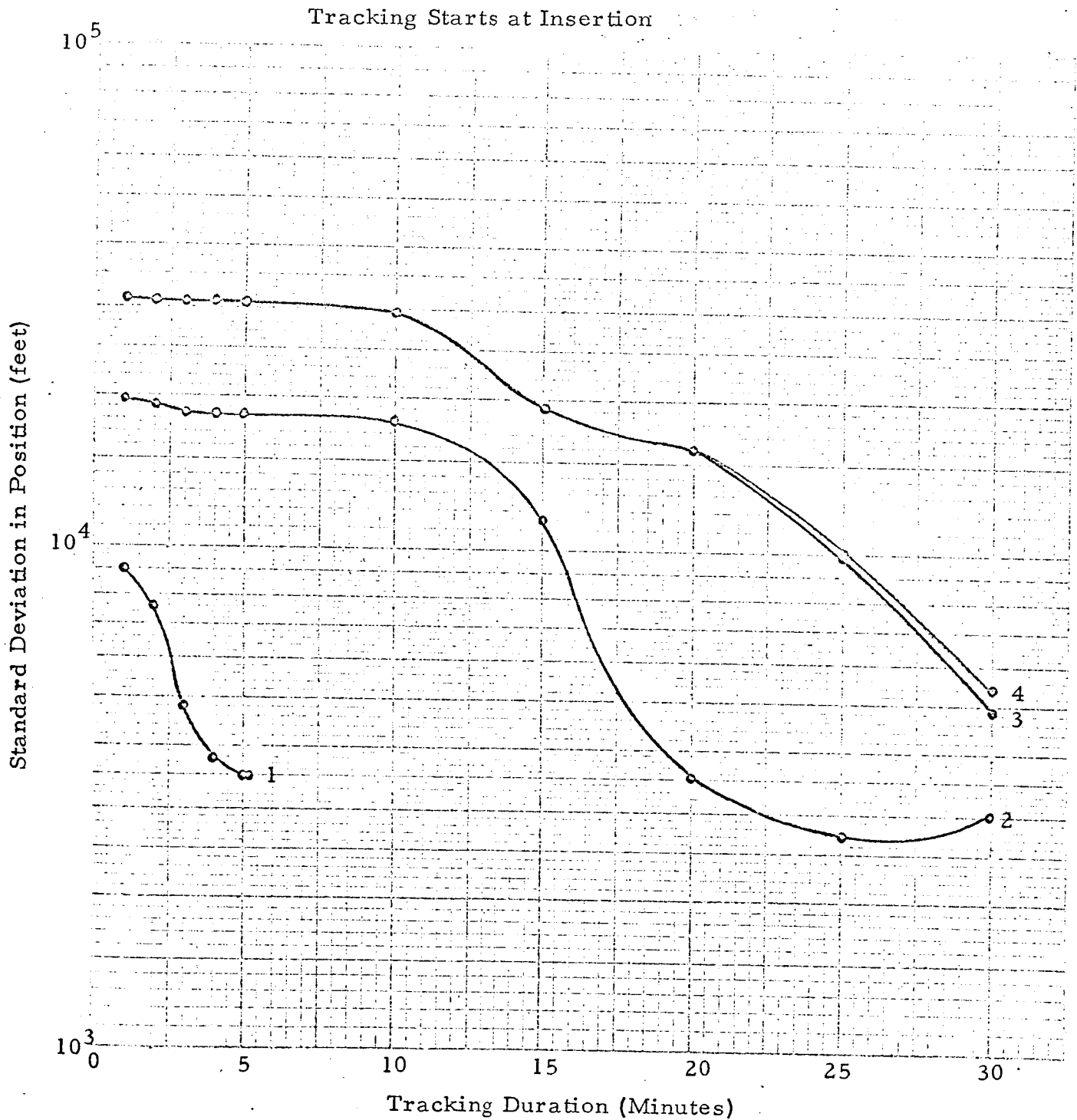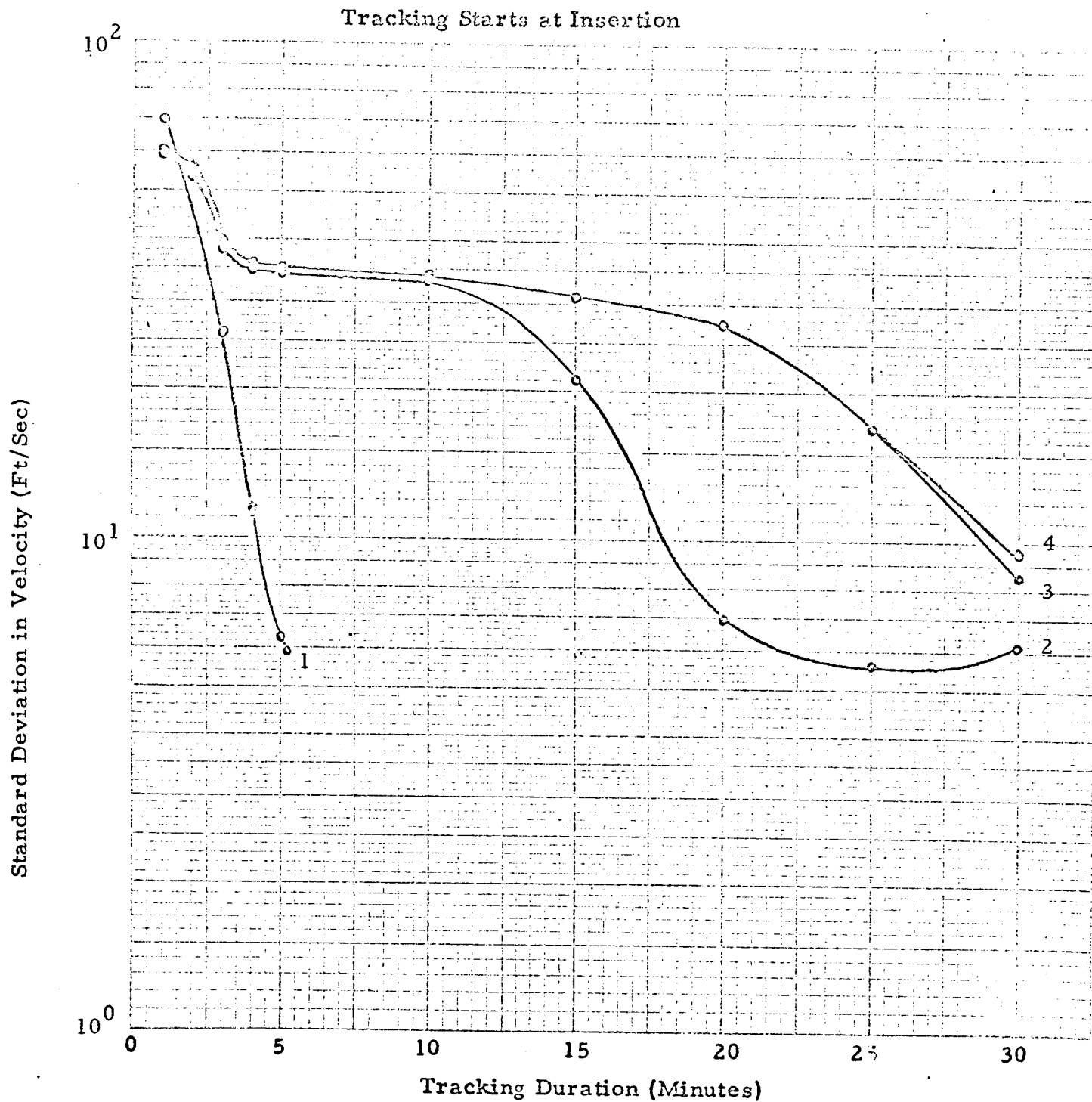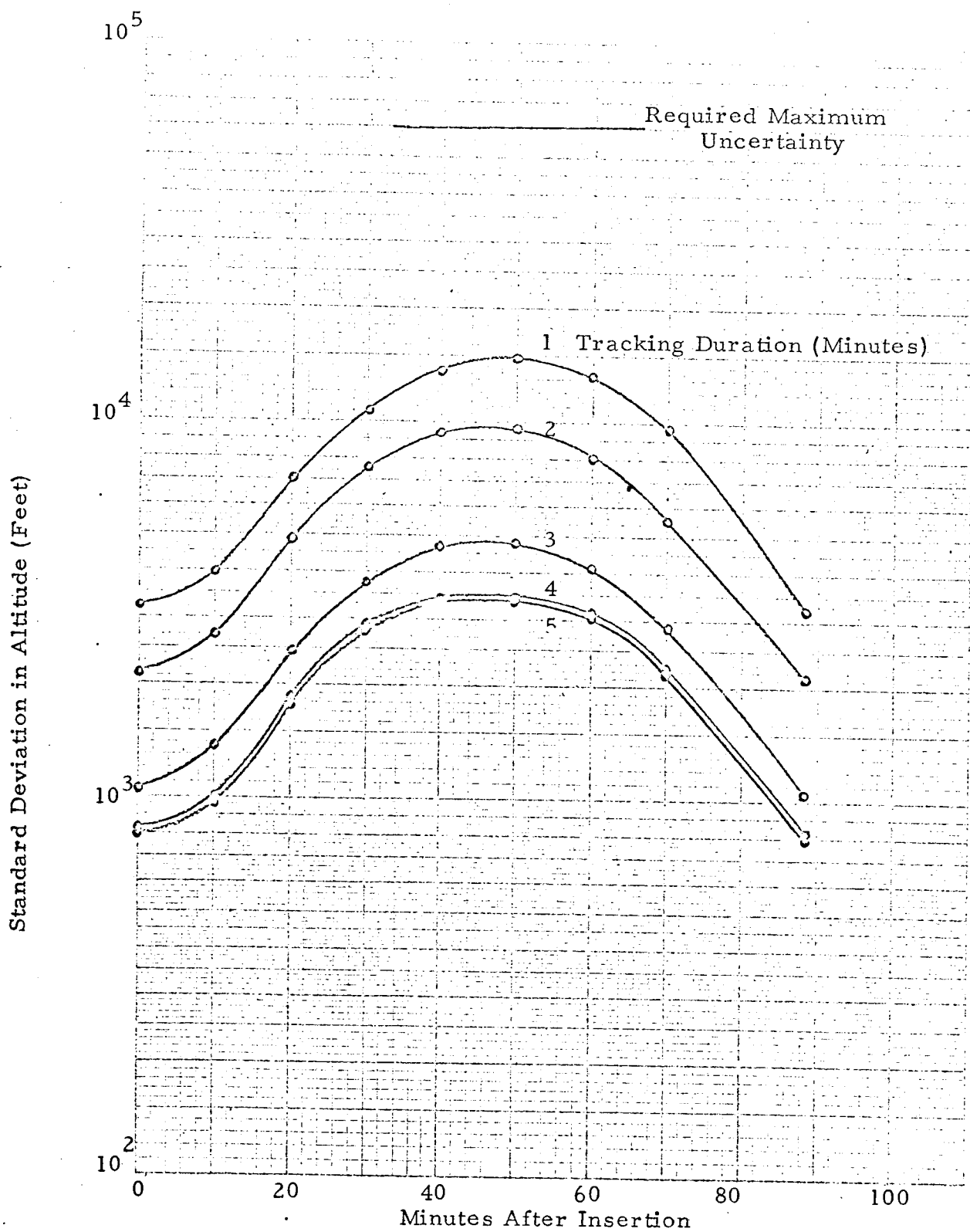4. LM Relay 1-way Doppler (RD ONLY)

Tracking Starts at Insertion

Insertion Requirement

Figure 2 — Uncertainty in Speed at Insertion

Figure 3 - Uncertainty in Flight Path Angle at Insertion

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Tracking Starts at Insertion

Figure 4 — Uncertainty in Position at Insertion

9

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Tracking Starts at Insertion

Figure 5 - Uncertainty in Velocity at Insertion

10

Figure 6 - Uncertainty in Altitude at Various Times After Insertion for Ship

11

Figure 7 - Uncertainty in Altitude at Various Times after Insertion for LM Relay 2-way Doppler (R+RD)

Figure 8 - Uncertainty in Altitude at Various Times after
Insertion for LM Relay - RD Data only, 1-way
or 2-way Doppler

13

REFERENCES

(1)   ANWG Technical Report No. AN-1.1, "Apollo Missions
        and Navigation Systems Characteristics," April
        4, 1966.   National Aeronautics and Space Adminis-
        tration, GSFC, Greenbelt, Md., and MSC, Houston,
        Texas.

(2)   Memorandum to FM/Chief, Missions Planning and Analysis
        Division from FM4/Mathematical Physics Branch,
        by Emil R. Schiesser, dated March 15, 1966.

LM RELAY NOTE NO. 11

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TRACKING THE TRANSLUNAR TRAJECTORY
WITH THE LM RELAY AND A SHIP

5 June 1967

Summary

A theoretical study has been made of the accuracy in predicting translunar injection conditions by a LM Relay, in a synchronous equatorial orbit, tracking in 1-way and 2-way Doppler modes. For comparative purposes, a tracking ship is also included.

For tracking data accumulated during the first twenty minutes after translunar injection, results indicate that the best LM Relay tracking system is the 2-way Doppler mode obtaining range-rate and range measurements. When obtaining range-rate measurements only, the 1-way and 2-way Doppler modes yield identical results during the period of interest.

The 2-way Doppler mode, using range-rate and range data, yields root sum square position and velocity uncertainties at injection of 60,000 feet and 5 feet per second for 20 minutes of tracking data. In order to obtain injection uncertainties comparable to ship tracking, the above LM Relay tracking system requires 18 minutes of tracking for comparable position uncertainty, and 13 minutes of tracking for comparable velocity uncertainty.

Introduction

This report represents a continuation of the study of the potential of LM Relay tracking systems. LM Relay Note No. 10 presented results for tracking an Earth parking orbit. This note also contained a description of the factors which form the basis of this report. In effect, the primary difference in this analysis is that the reference orbit has been changed from a parking orbit to a translunar trajectory, the orbit elements of which are given in Reference 1.

For convenience, Table 1 of the previous note is reproduced here, in order to show the parameters being considered in the analysis, and also their apriori values.

The 2-way Doppler mode assumes a transmitter and receiver in the LM Relay and a transponder in the translunar spacecraft. In the 1-way Doppler mode, the LM Relay has a receiver and the spacecraft a transmitter. All data is relayed to a ground station by the LM for data processing.

As before, the tracking ship is positioned at 32° North latitude and 44° West longitude. In this location, the ship starts viewing the spacecraft 5 minutes after injection, which takes place over the northwest section of Texas.

At the present time, to our knowledge, there is no definitive statement of the uncertainty requirements at translunar injection for a GO-NO-GO decision. Thus, this aspect of comparison in the error analysis result cannot be shown

Error Analysis Results

Figures 1 and 2 show the position and velocity uncertainties at translunar injection for various tracking duartions by the tracking systems being considered. Here, the position (velocity) uncertainty is defined as the square root of the sum of the position (velocity) diagonal elements in the resulting covariance matrix. It is noted that, after about 8-10 minutes of ship tracking, the ship results show an increase as a result of errors introduced by the non-estimated parameters. There is no apparent difference in the results for 1-way and 2-way Doppler modes when using range-rate data only.

Figures 3 and 4 show the result of mapping the covariance matrix at insertion to two hours after insertion. This result is for illustrative purposes only, the particular time being a possible opportunity for a mid-course correction boost.

2

TABLE 1 - LIST OF PARAMETERS AND APRIORI VALUES

Parameter	Ship	2-way Doppler	1-way Doppler	Nominal Value
1. CSM position component	X	X	X	25,000 ft.
2. CSM velocity component	X	X	X	50 fps.
3. Ship R bias	X			60 ft.
4. Ship angle bias (X-Y)	X			2.0 milli-rad.
5. Ship clock offset	X			3 milli-secs.
6. Ship clock rate	X			2×10^{-10} sec/sec.
7. Ship location Up	X			100 ft.
8. Ship location E-W	X			2000 ft.
9. Ship location N-S	X			2000 ft.
10. LM state vector		X	X	(2)
11. LM range bias		X(1)		60 ft.
12. LM clock offset		X	X	3 milli-secs.
13. LM clock rate		X	X	2×10^{-10} sec/sec
14. CSM clock offset			X	3 milli-secs
15. CSM clock rate			X	2×10^{-10} sec/sec
16. Gravitational constant	X	X	X	1.06×10^{11} ft^3/sec^2
TOTAL NUMBER	15	16(1)	17	

(1) Range bias not included in RD only case, therefore total parameters are 15 in this case.
(2) Marginal distribution from 16 hours tracking by master and 2 slaves.

1. Ship
2. LM Relay 2-way Doppler (R+RD)
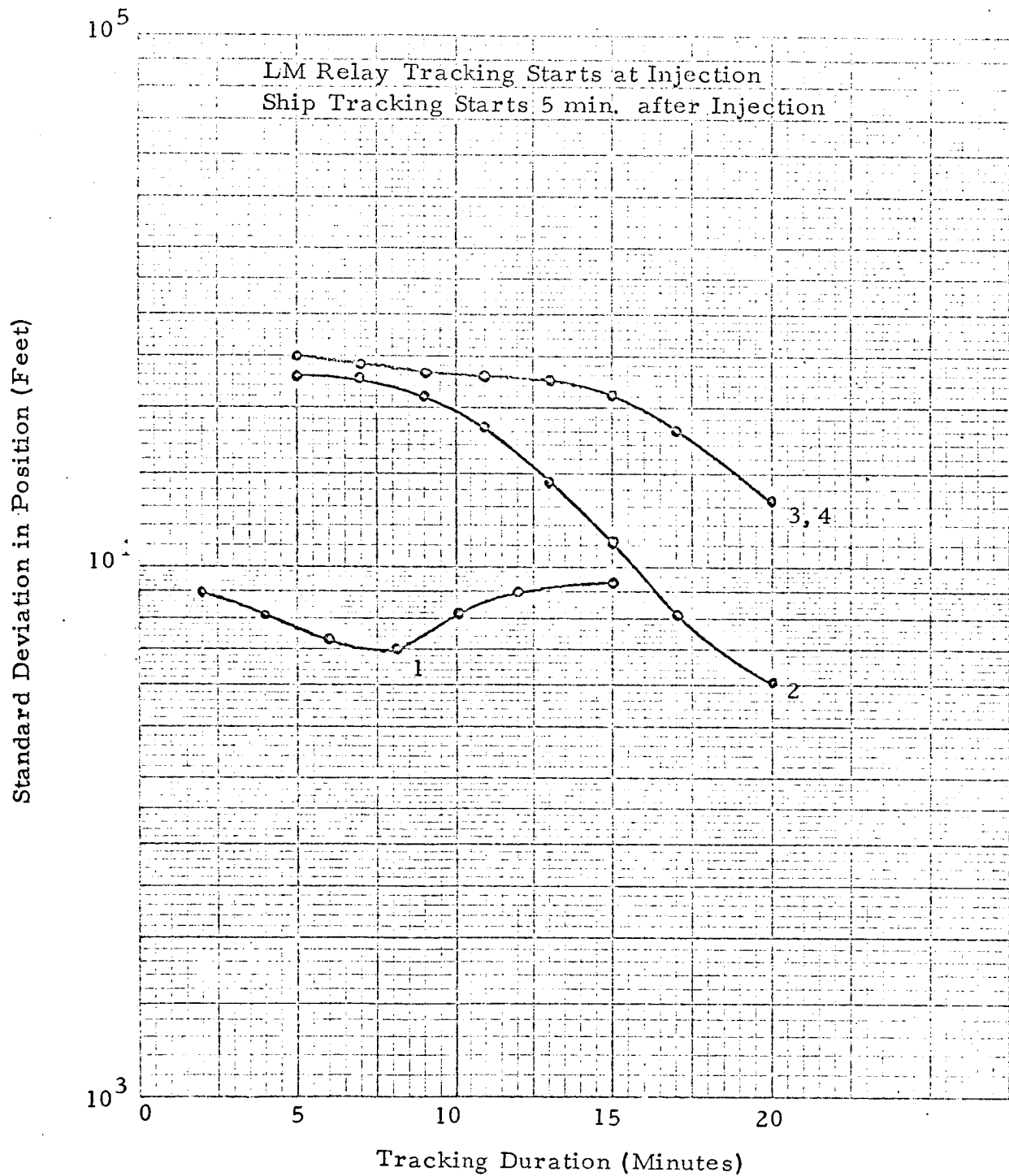3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Figure 1 – Uncertainty in Position at Translunar Injection

1. Ship
2. LM Relay 2-way Doppler (R≠RD)
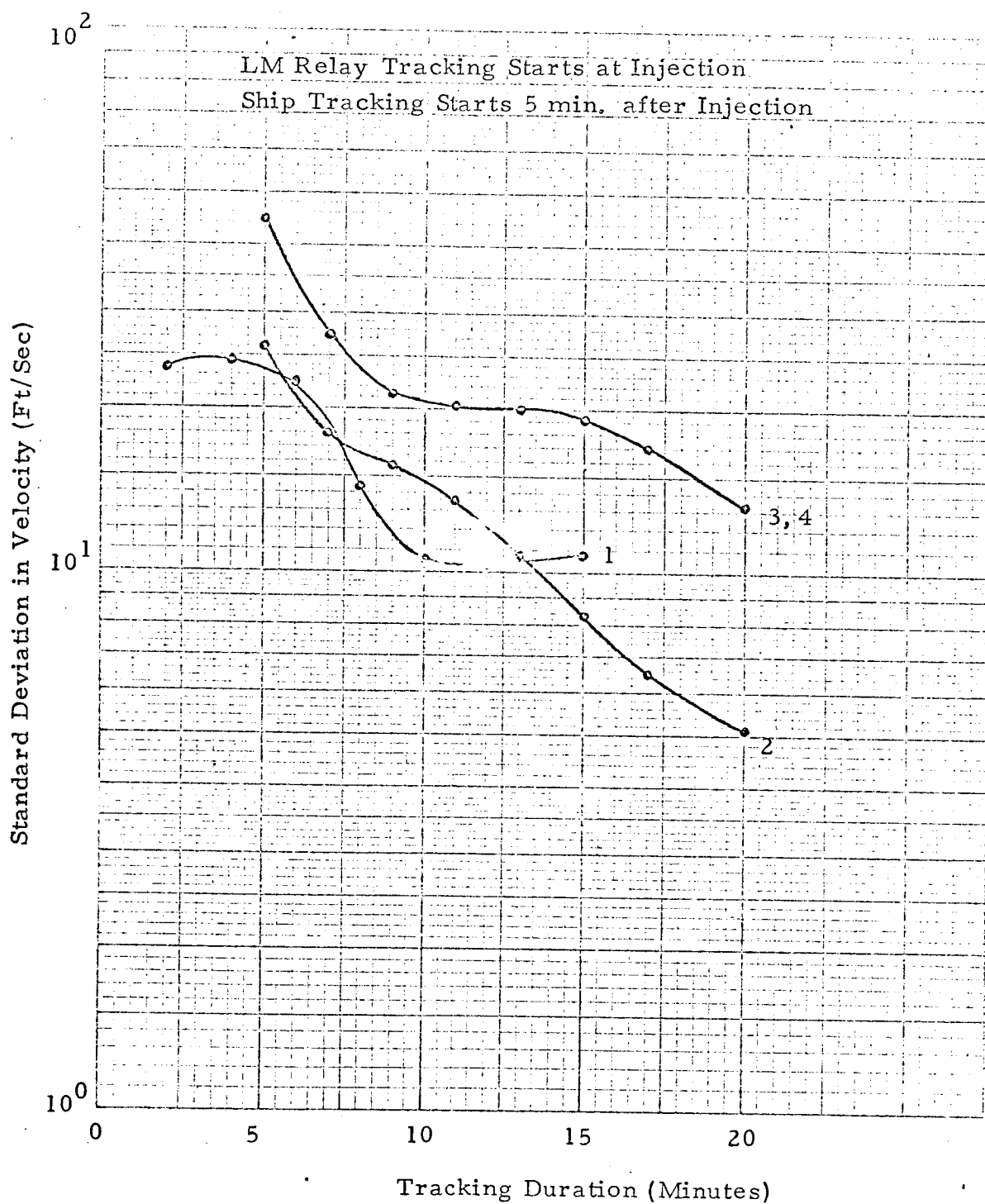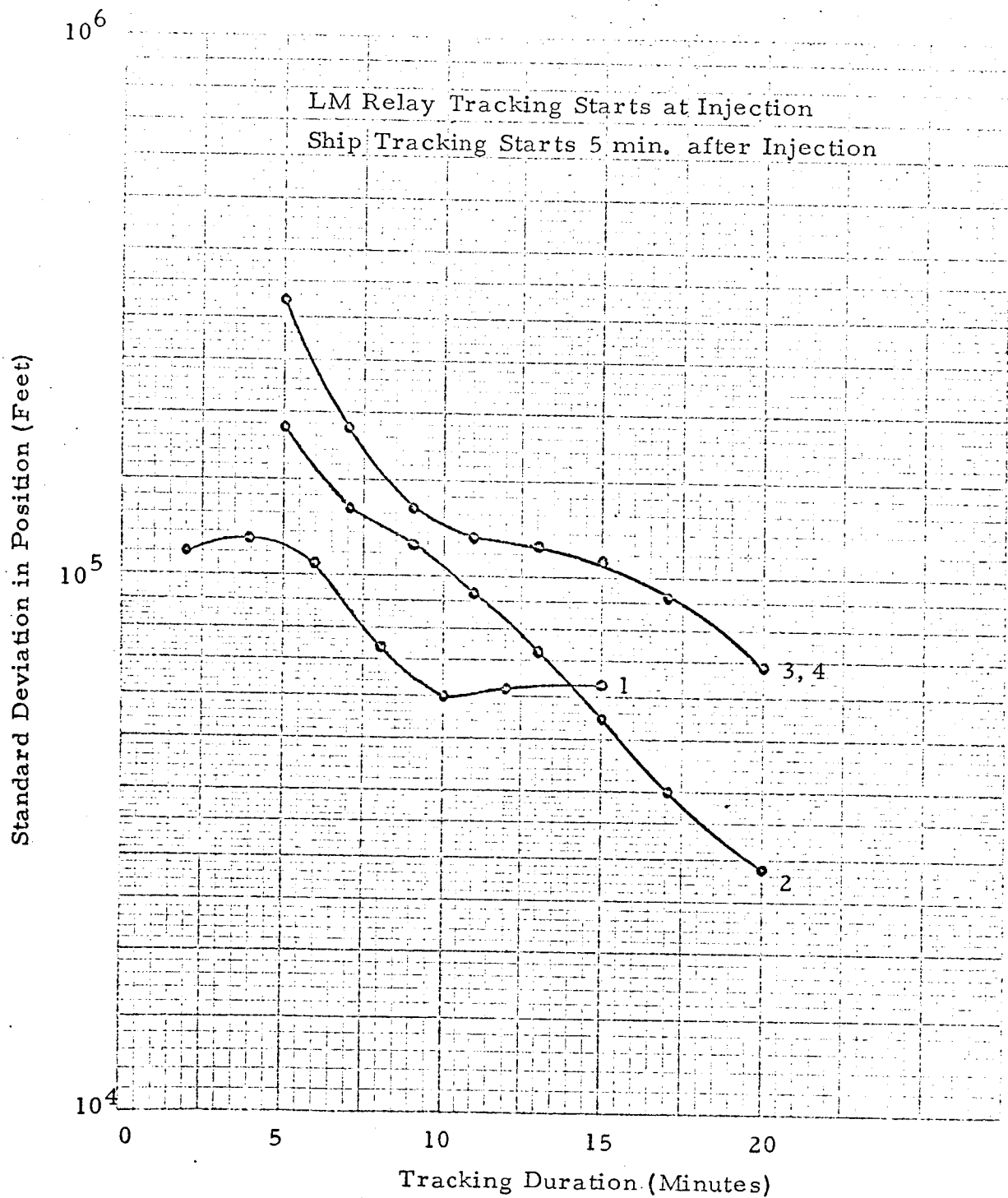3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Figure 2 - Uncertainty in Velocity at Translunar Injection

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

Figure 3 - Uncertainty in Position at Two Hours
After Translunar Injection

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)
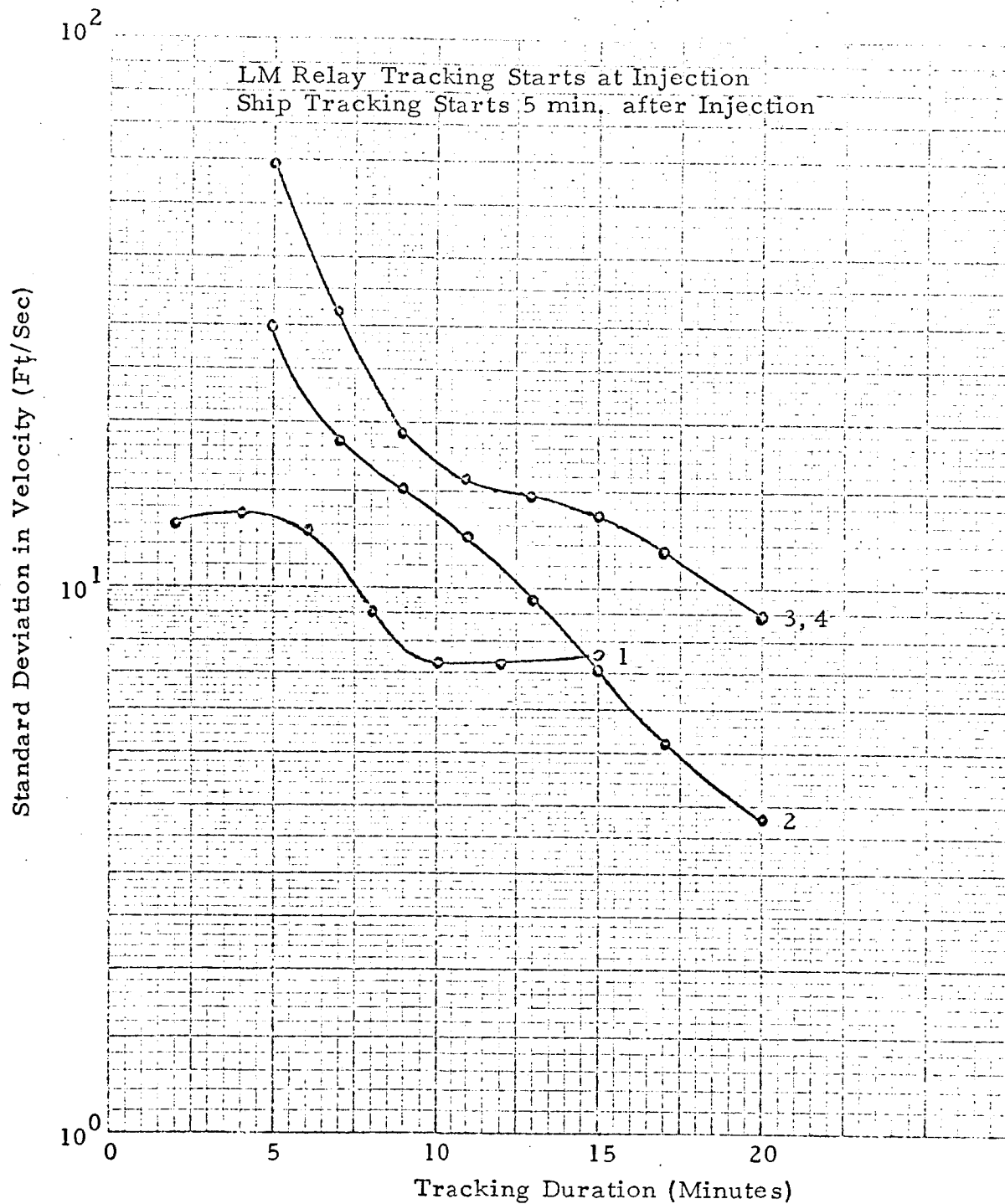
Figure 4 - Uncertainty in Velocity at Two Hours
after Translunar Injection

REFERENCES

(1)   ANWG Technical Report No. AN-1.1, "Apollo Missions and
        Navigation Systems Characteristics," April 4, 1966.
        National Aeronautics and Space Administration, GSFC,
        Greenbelt, Md., and MSC, Houston, Texas.

H. Epstein
C. Siska
A. Burkow
5 June 1967

LM RELAY NOTE NO. 12

TRACKING THE TRANSEARTH TRAJECTORY WITH THE LM RELAY AND A SHIP

Summary

A theoretical study has been made of the accuracy with which re-entry conditions can be predicted with a LM Relay, in a synchronous equatorial orbit, tracking in 1-way and 2-way Doppler modes. Hypothetical ship tracking is also included for comparative purposes.

For 20 minutes of tracking just prior to re-entry, there is no significant difference in the results for 1-way or 2-way Doppler measurements. The resulting uncertainties are re-entry speed, 1 ft/sec., re-entry angle, .01 degrees, and total miss distance on the re-entry sphere, 25,000 feet.

Ship tracking data yields about an order of magnitude better accuracies than the LM Relay.

Introduction

This report represents a continuation of the study of the potential of LM Relay tracking systems. LM Relay Notes No. 10 and 11 report results of tracking an Earth parking orbit and a translunar trajectory.

The case under consideration is examined for the sake of completeness, and it is somewhat academic, since at the time of re-entry, there will be many hours of ground station tracking data available for processing.

In somewhat the same vein, the ship is positioned at 30° N latitude and 160°E longitude to yield a viewing period of the last ten minutes of the transearth orbit prior to re-entry at 400,000 feet altitude. In actual practice, a ship will be stationed so as to view the

pull-up section of the atmospheric phase of re-entry, but in this situation, it will not be able to view the last part of the coasting phase. Nevertheless, ship tracking data does provide a basis for comparing LM Relay tracking data.

Re-entry occurs somewhat to the west of Hawaii (the orbit elements of the trajectory are given in Reference 1), and consequently, the LM Relay is positioned on the Equator at 135° E longitude. In all other respects the input data for the analysis is the same as for the previous studies in the aforementioned notes, except that tracking periods are suitably changed for the re-entry phase. For convenience in defining the pertinent parameters considered, and their apriori values, Table 1 from Note No. 10 is reproduced here.

In the 2-way Doppler mode, the LM Relay has a transmitter and receiver, while the re-entering spacecraft has a transponder. In the 1-way Doppler mode the spacecraft has a transmitter and the LM Relay a receiver. All data is relayed by the LM to a ground station for data processing.

Error Analysis Results

Figures 1, 2, and 3 show the uncertainty at re-entry in speed, angle, and miss distance for various tracking durations. Total miss distance is defined as the root mean square of the in-plane uncertainty and the out-of-plane uncertainty on the re-entry sphere. The primary requirement in re-entry is the re-entry angle uncertainty. To determine if the spacecraft is safely in the re-entry certainties in path angle must be less than 0.4^O (.007 radians) for $L/D \approx .4$, and less than 0.1^O (.0017 radians) for $L/D \approx 0$, when the nominal re-entry angle is about 6.5^O. Figure 2 indicates that a 5 minute tracking interval for the LM Relay is sufficient to meet these requirements.

Figure 4 and 5 show the position and velocity uncertainties at re-entry. Here, position (velocity) is defined as the square root of the sum of the position (velocity) diagonal elements in the covariance matrix.

2

TABLE 1 - LIST OF PARAMETERS AND APRIORI VALUES

Parameter	Ship	2-way Doppler	1-way Doppler	Nominal Value
1. CSM position component	X	X	X	25,000 ft.
2. CSM velocity component	X	X	X	50 fps.
3. Ship R bias	X			60 ft.
4. Ship angle bias (X-Y)	X			2.0 milli-rad
5. Ship clock offset	X			3 milli-sec.
6. Ship clock rate	X			2×10^{-10} sec/sec
7. Ship location Up	X			100 ft.
8. Ship location E-W	X			2000 ft.
9. Ship location N-S	X			2000 ft.
10. LM state vector		X	X	(2)
11. LM range bias		X(1)		60 ft.
12. LM clock offset		X	X	3 milli-secs.
13. LM clock rate		X	X	2×10^{-10} sec/sec
14. CSM clock offset			X	3 milli-secs
15. CSM clock rate			X	2×10^{-10} sec/sec
16. Gravitational constant	X	X	X	1.06×10^{11} ft^3/sec^2
TOTAL NUMBER	15	16(1)	17	

(1) Range bias not included in RD only case, therefore total parameters are 15 in this case.

(2) Marginal distribution from 16 hours tracking by master and 2 slaves.

1. Ship
2. LM Relay 2-way Doppler (R+RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

LM Relay Tracking Starts 20 minutes prior to re-entry
Ship Tracking Starts 10 minutes prior to re-entry

Figure 1 - Uncertainty in Speed at Re-entry for
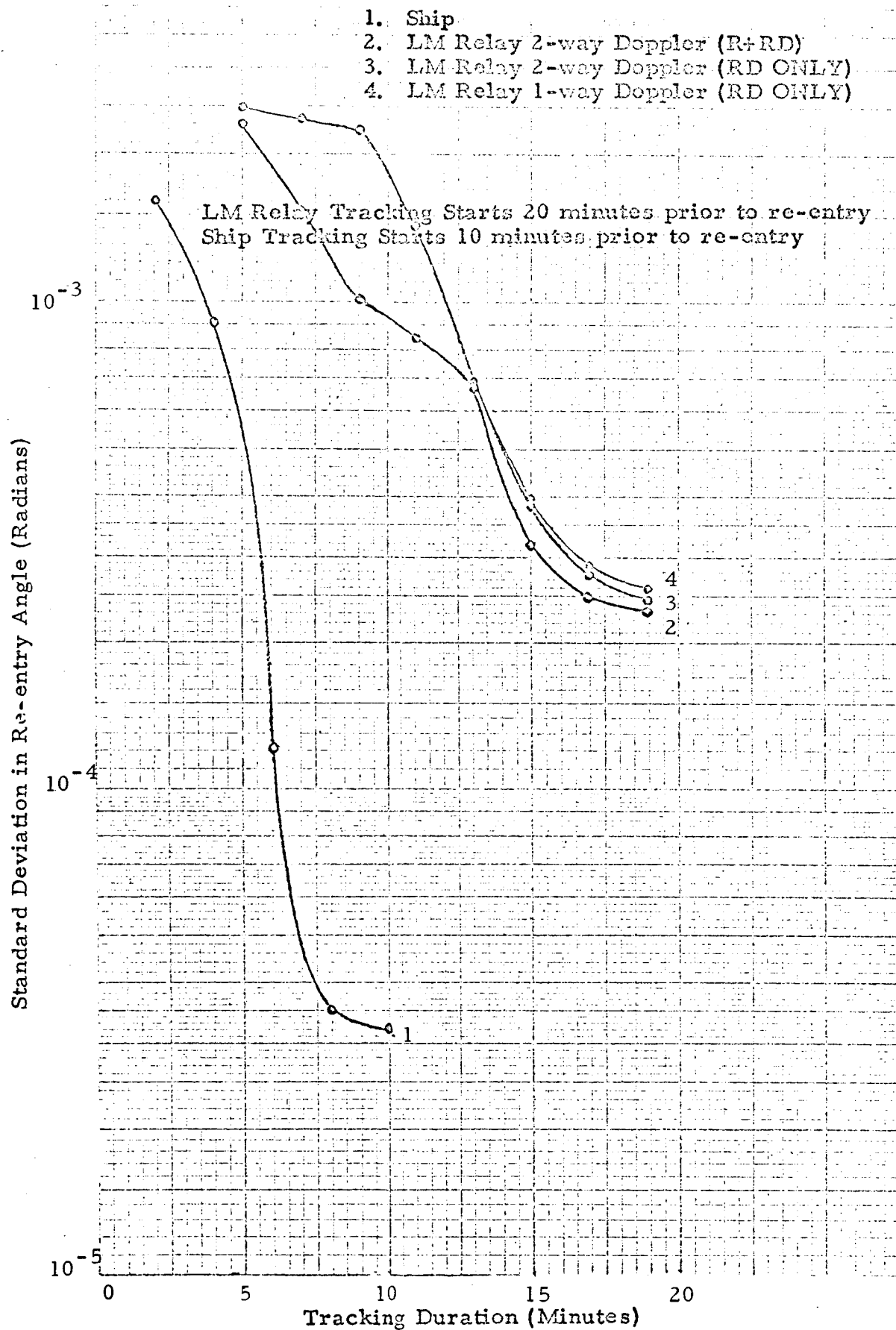Transearth Trajectory

Figure 2 - Uncertainty in Re-entry Angle for
Transearth Trajectory

1. Ship
2. LM Relay 2-way Doppler (R+RD)
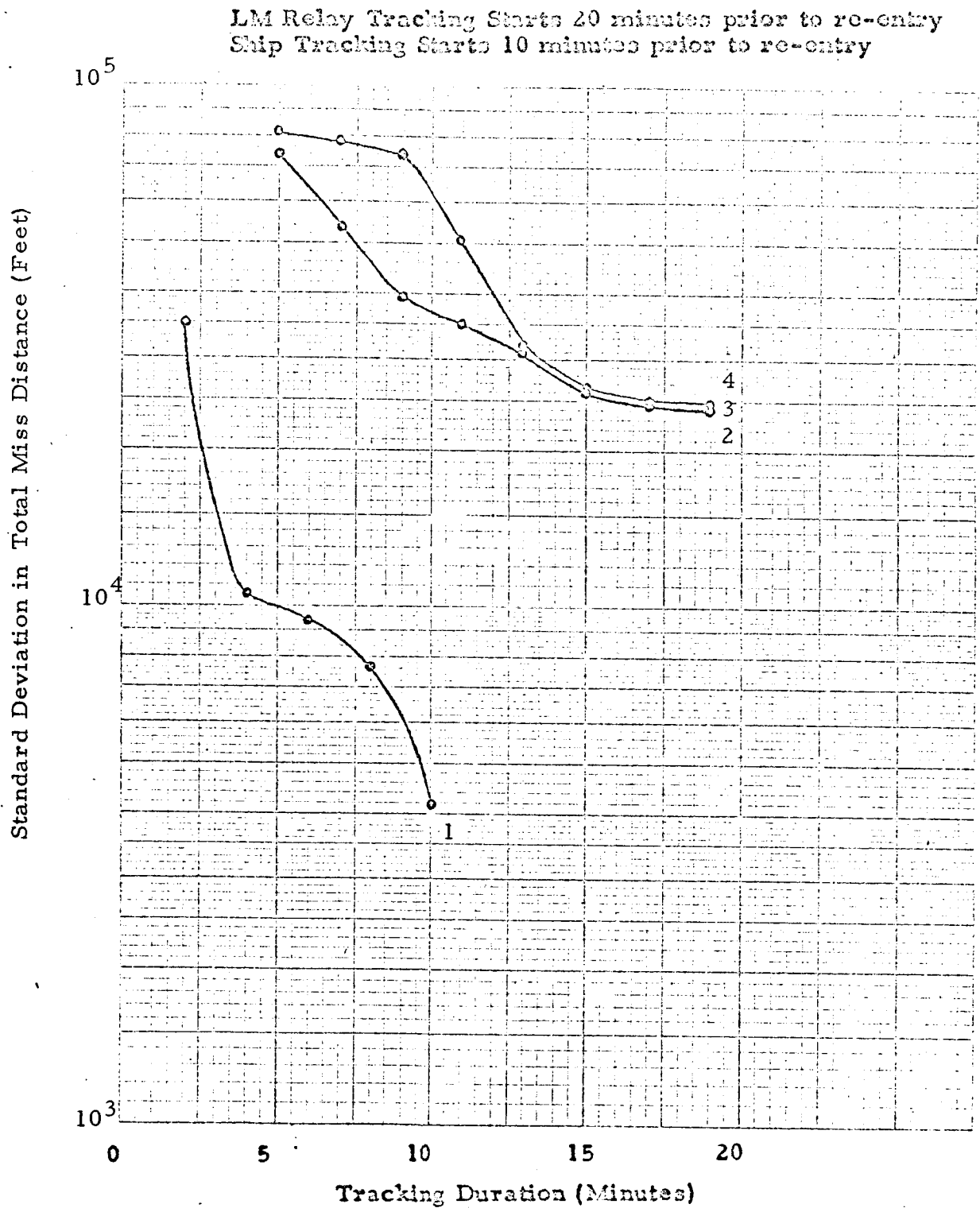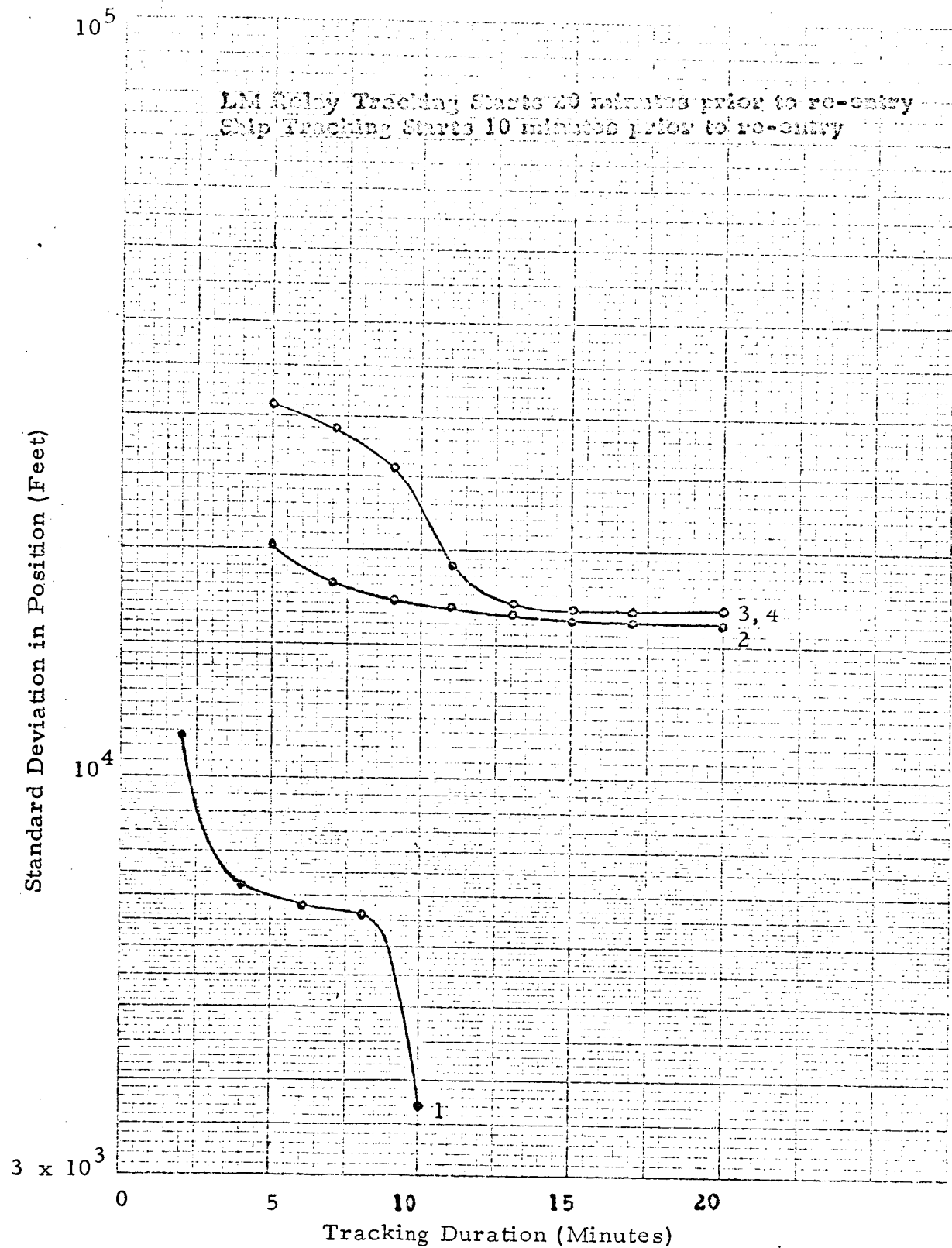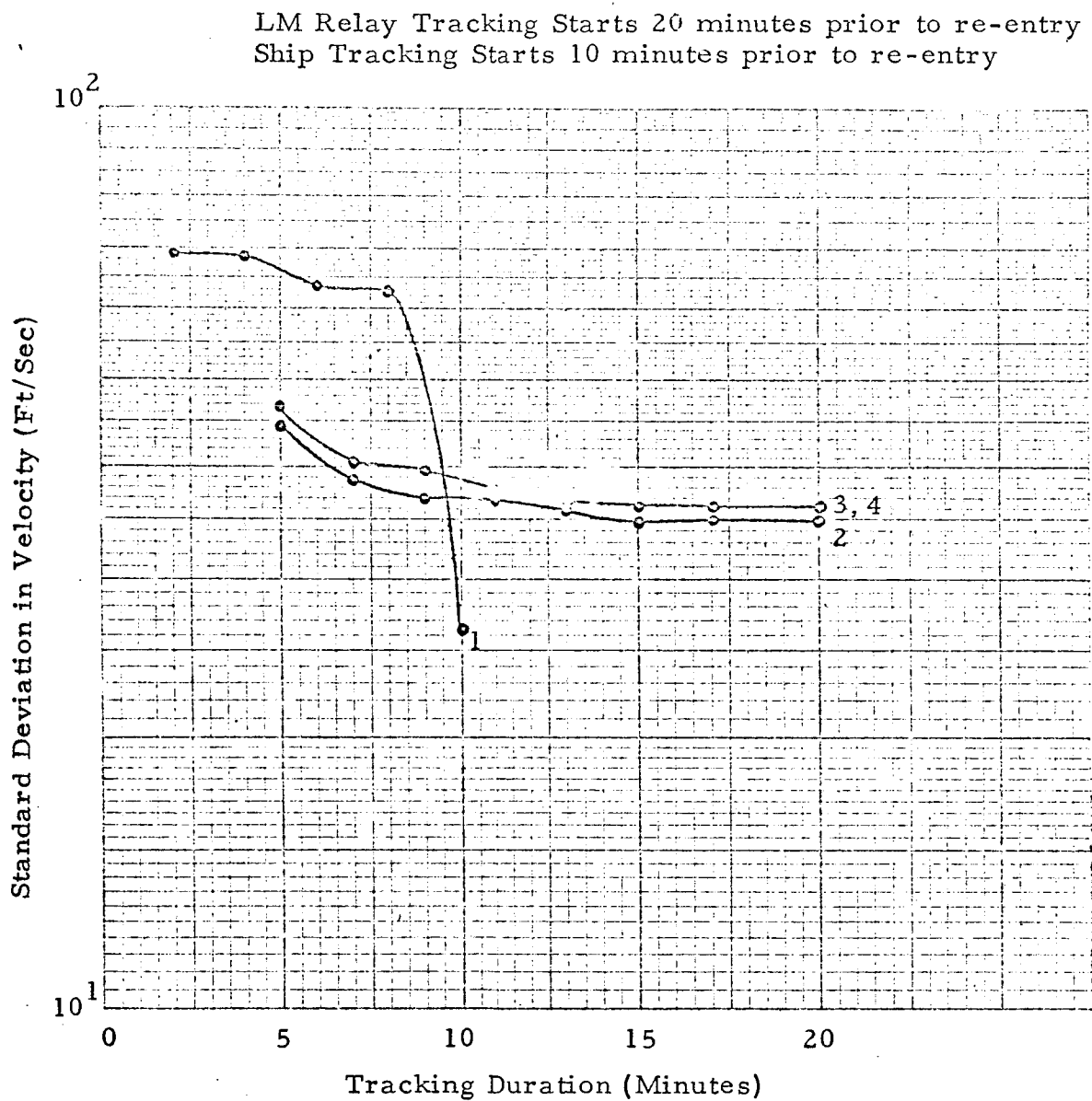3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

LM Relay Tracking Starts 20 minutes prior to re-entry
Ship Tracking Starts 10 minutes prior to re-entry

Figure 3 - Uncertainty in Total Miss Distance at
Re-entry for Transearth Trajectory

LM Relay Tracking Starts 20 minutes prior to re-entry
Ship Tracking Starts 10 minutes prior to re-entry

Figure 4 - Uncertainty in Position at Re-entry
for Transearth Trajectory

7

1. Ship
2. LM Relay 2-way Doppler (R + RD)
3. LM Relay 2-way Doppler (RD ONLY)
4. LM Relay 1-way Doppler (RD ONLY)

LM Relay Tracking Starts 20 minutes prior to re-entry
Ship Tracking Starts 10 minutes prior to re-entry

Figure 5 - Uncertainty in Velocity at Re-entry for
Transearth Trajectory

REFERENCES

(1)   ANWG Technical Report No. AN-1.1, "Apollo Missions and
        Navigation Systems Characteristics," April 4, 1966.
        National Aeronautics and Space Administration, GSFC,
        Greenbelt, Md., and MSC, Houston, Texas.

LM RELAY NOTE NO. 13

LOCATION OF SPACECRAFT
FOLLOWING TOUCHDOWN ON EARTH

H. Epstein
C. Siska
A. Burkow
13 June 1967

The location of spacecraft following touchdown on earth may not be well known.   Therefore, it appeared desirable to make an investigation of the value of navigational data obtained from synchronous satellites on the downed spacecraft.   Good navigational data on the downed spacecraft can minimize the search process associated with the recovery operation.

This note will indicate some numerical results which have been obtained on the uncertainty in location of downed spacecraft.   The downed spacecraft is treated as a tracking ship by the error analysis program to enable results to be obtained without modifications to the OEAP.   Results can then be obtained for touchdown on water or land. Pacific Ocean or Atlantic Ocean recovery operations certainly are of primary importance for the near future.   A drift velocity of the downed spacecraft associated with ocean currents needs to be taken into account Present numerical results indicate the uncertainty in the drift velocity as the most important error source in this navigational data when tracking by one relay occurs.

The downed spacecraft has been characterized by position and velocity components.   The altitude uncertainty was considered as being 200 feet, (1 σ) and the altitude rate uncertainty negligible.   Additional position uncertainties are treated by the OEAP as N-S and E-W components.   An a priori uncertainty of 100 n. mi. (1 σ) was assumed on each component to correspond to very poor a priori knowledge on these components.   Where applicable, a priori uncertainties in velocity of one (1) foot per second (1 σ) were assumed on the N-S and E-W components.

The spacecraft is treated then as a moving master station by the OEAP whose station location components are to be estimated.   It is necessary to estimate the synchronous relay state vectors to arrive

at the station location errors with the present program. Prior tracking of the synchronous relay as in LM Relay Note 9 should insure that the results will be in good agreement with results when only spacecraft parameters are estimated.

Two-way doppler and range measurements are made by the synchronous relay as is available in the dual coherent mode. The predominant error source will later be shown as associated with the drift velocity. Some insight in the numerical results can be simply obtained. Consider the following simplified model with:

(1) Synchronous relay ephemeris perfectly known.
(2) No altitude uncertainty.
(3) No uncertainty in the range measurement.
(4) A perfect spherical earth.

This combination would lead to the spacecraft being located on a circle of known radius and center. The doppler or range-rate data is then used to determine where along the circle the spacecraft is located. The present OEAP does not perform this rotation to allow this result to be obtained directly. Numerical results to be presented at this time indicate only N-S and E-W error components.

The synchronous relay orbit was taken to be inclined 28.5° with a nominal longitude of 30°W. The synchronous relay latitude is about 25°N at the initiation of tracking. Two downed spacecraft locations were used (30°N, 70°W and 0°N, 70°W). Numerical results were obtained for the location accuracy at the commencement of tracking as a function of the tracking period.

Figures 1 and 2 pertain to a downed spacecraft location of 30°N and 70°W while Figures 3 and 4 pertain to the 0°N and 70°W location. Figure 1 indicates the north uncertainty. Three curves are shown. The upper curve indicates a five to six mile North Uncertainty when E and N components are estimated with a one foot per second $\dot{E}$ and $\dot{N}$ uncertainty.

2

The estimation of $\dot{E}$ and $\dot{N}$ in addition to E and N show an accuracy improvement. Uncertainties in N are reduced to values varying from about 3 miles to less than a mile as the tracking period increases from 5 minutes to 60 minutes. This estimation process is for all practical purposes the same result as was obtained for optimum processing (full update). The bottom curve indicates results in the absence of the velocity uncertainties in the downed spacecraft. Uncertainties of only a few hundred feet can be noted. These results indicate the importance of uncertainties in surface ocean currents on downed spacecraft accuracy.

Figure 2 indicates the corresponding results for East uncertainties In this case, drift velocity components decrease the East accuracy as the tracking period is increased.

Figures 3 and 4 indicate N and E uncertainties for the alternate spacecraft location. The results are shown for the two estimation techniques to be used in the presence of drift velocity uncertainties. Numerical values similar to the previous case, are obtained for the North error component while the East error component is increased.

The use of a rotated co-ordinate system would indicate only one large error component. This error component is perpendicular to the plane formed by the center of the earth, the downed spacecraft location, and the synchronous relay location.

Better accuracy should be expected for the relay at zero latitude when tracking commences. This would arise due to the greater relay velocity. The results obtained to date indicate promise with regard to relays providing navigational data on downed spacecraft.

Figure 1.

Uncertainty in North Co-Ordinate for
Spacecraft at 30°N, 70°W

E, N Estimated

E, N, $\dot{E}$, $\dot{N}$ Estimated

North - 1σ Uncertainty (feet)

E, N Estimated (No Spacecraft Drift)

Tracking Period (minutes)

4

Figure 2.

Uncertainty in East Co-Ordinate for
Spacecraft at 30°N, 70°W

E, N Estimated

E, N, Ė, Ṅ Estimated

E, N Estimated (No Spacecraft Drift)

East - 1σ Uncertainty (feet)

Tracking Period (minutes)

Figure 3.

Uncertainty in North Co-Ordinate for
Spacecraft at $0°N$, $70°W$
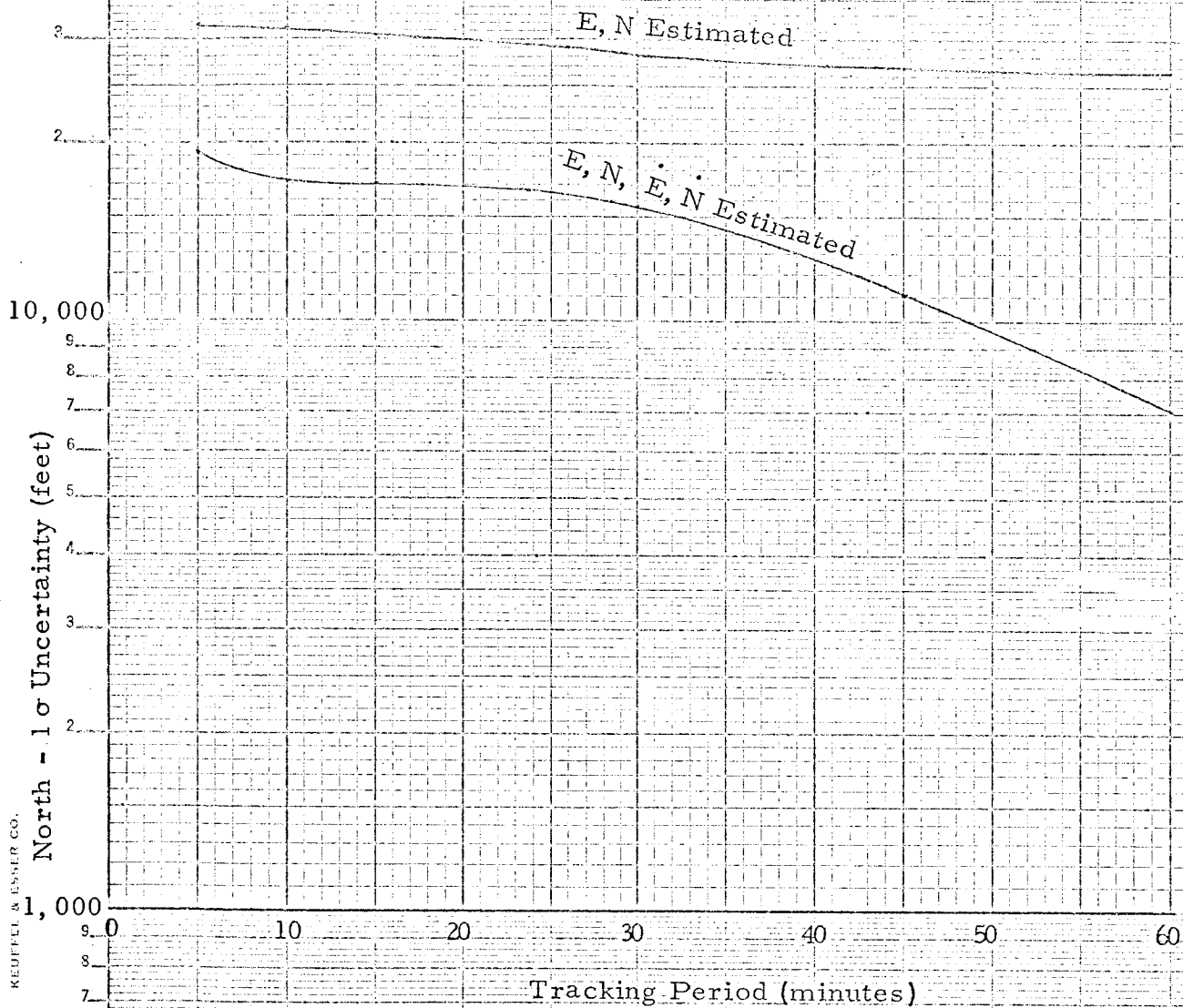
E, N Estimated

E, N, $\dot{E}$, $\dot{N}$ Estimated

North - 1σ Uncertainty (feet)

Tracking Period (minutes)

6

Figure 4.

Uncertainty in East Co-Ordinate for
Spacecraft at 0°N, 70°W

E, N Estimated

E, N, Ė, Ṅ Estimated

East - 1 σ Uncertainty (feet)

Tracking Period (minutes)

LM RELAY NOTE NO. 14

C. P. Siska
15 June 1967

TRACKING THE APOLLO MISSION INSERTION, INJECTION, AND RE-ENTRY PHASES WITH TWO LM RELAYS

SUMMARY

A theoretical analysis has been made of the accuracy with which 2-way Doppler tracking data from two synchronous LM Relays can predict spacecraft orbits during various near-Earth phases of the Apollo mission.

Generally, the results indicate that with 5-7 minutes of tracking, two LM Relays can assist in obtaining prediction accuracies much better than a tracking ship, and comparable to several MSFN stations.

In particular, insertion accuracy requirements for altitude, speed, and flight path angle, can be achieved with less than 30 seconds of tracking data, and the maximum altitude uncertainty requirement during one revolution can be attained in less than 60 seconds after insertion. Mission plan verification can be determined within five to seven minutes after insertion, the accuracy being better than that obtained by tracking with 3 C-band and 2 USB radars from the Canary Islands, Tanavarive, and Carnarvon. The one-sigma position and velocity uncertainties at insertion are 350 feet and 0.53 ft/sec.

Position and velocity uncertainties (one-sigma) obtained at translunar injection for 10 minutes of tracking are 450 feet and 0.8 ft/sec.

One-sigma position and velocity uncertainties at re-entry for a 10 minute tracking interval (starting 20 minutes prior to re-entry) are 1900 ft. and 1.5 ft/sec. This is about a factor of three improvement in accuracy over that obtained by MSFN tracking of the transearth phase up to 72 minutes prior to re-entry.

INTRODUCTION

This report represents a continuing study of the potential of LM Relay synchronous satellites in the role of tracking stations during Apollo mission phases. Previous studies on this subject are reported in LM Relay Note No's. 9, 10, 11, and 12. These studies dealt with the situation where only one LM Relay was used for tracking purposes for any particular Apollo mission phase. In this investigation, the effect of using two LM Relays is examined with tracking data obtained by the 2-way Doppler mode and consisting of both range-rate and range measurements every six seconds.

Basically, the parameters involved, their numerical values, and the calculation procedures are the same as reported in the above Notes, except that two relays are now considered. If we consider that each relay is tracked by a different set of three ground stations, then the total list of parameters for two relays is one less than twice as large as the list for one relay - the primary gravitational constant being common to both relays. This is the situation considered in this analysis. That is to say, even though the same three ground stations are tracking both relays, the data is processed as if each relay is tracked by a different set of three stations. The net result of this assumption is conservative, since the more parameters which are introduced, the greater the possible resulting error in estimators.

Figure 1 illustrates the geometrical relations between the LM Relays, ground stations, and Apollo spacecraft. LM Relay positions at the time they start tracking the Apollo spacecraft are 40 hours subsequent to the time (zero hour) when the ground stations started tracking the LM Relays. The tracking intervals are then 0-8, 24-32 hours for the one relay and 8-16, 32 - 40 hours for the other relay. State vector estimates are the result of estimation of all parameters (full update), the position and velocity uncertainties being substantially the same as those reported in LM Relay Note No. 9, for the corresponding LM Relay longitudes. A 13 x 13 marginal distribution (two state vectors + gravitational constant) of the full update result is then used as apriori when the two LM Relays start tracking the Apollo spacecraft (which starts 40 hours after zero hour).

In order to better assess the tracking capability of two LM Relays, the resulting Apollo spacecraft uncertainties will be compared to those obtained from a single tracking ship, and also to those obtained from MSFN tracking. These results pertain to estimating the spacecraft state vector only, the remaining parameters not being estimated (Relay position and velocity, range bias, clock offset and rate, gravitational constant).

RESULTS FOR INSERTION PHASE

The uncertainty requirements at insertion are generally given in terms of speed, altitude, flight path angle at insertion and/or maximum altitude deviation during one revolution (see Reference 1).

Figures 2, 3, 4 and 5 show the resulting uncertainties obtained for these parameters, the requirements being taken from Reference 1. Results indicate that 2 Relays tracking 90 seconds can meet the altitude, speed, and flight path angle requirements, with speed being the most sensitive parameter. If maximum altitude deviation during one revolution is the criterion, less than 60 seconds of tracking is required.

Ship data from Reference 2 (Figs. 3.2d, 3.3d, 3.4d) is for six second samples over 90 seconds with bias and station location as non-estimated parameters. The 3 MSFN station results (CYI, TAN, CRO) from Reference 3 are for 3 C-band and 2 USB radars and include S-IVB venting as a non-estimated parameter in addition to measurement bias, station location, and gravitational constant.

Figures 6 and 7 show the root sum square position and velocity components. At 5-7 minutes of tracking, the 2 Relays results are slightly better than the 3 MSFN stations results of Reference 3 and much better than the the ship result of Ref. 2 (Figs. 3-5d, 3.6d). This implies that the 2 Relays can periodically update the spacecraft on-board guidance computer (required because of S-IVB venting) and furthermore, the Relays can provide mission plan verification much sooner than the MSFN stations (see pp. 4-6 and 4-7 of Ref. 2).

RESULTS FOR TRANSLUNAR INJECTION PHASE

At the present time, there is no definitive statement concerning injection requirements, so that only a comparison between Relay tracking capability and the capability of standard tracking stations can be made.

Figures 8 and 9 show the Relay results for various tracking durations. The results at 2 hours and LSOI are obtained by projecting the covariance matrix at injection to the appropriate orbit times. The values from Reference 2 are obtained from Table 5.1 (Plan 1) for the 2 hour value, Table 5.4 for LSOI value, and from the covariance matrix of Appendix on page 5-9 for the 20 minute value. These are the result of 2-way Doppler tracking with CYI (6 sec. RD samples from 0-4 min.), ASC (6 sec. RD samples plus one R sample from 5-19 min.), and CNB (1 min. RD samples plus one R sample from 1-2 hours). Only CIY and ASC obtain tracking data for the 20 minute value indicated.

A comparison of results indicates that 2 Relays tracking the first ten or fifteen minutes after injection (prior to transposition and docking) can importantly augment the tracking data obtained by MSFN stations. This is especially the case when determining injection conditions (for a possible GO-NO-GO decision).

RESULTS FOR TRANSEARTH RE-ENTRY PHASE

Since Relay tracking is being restricted in this analysis to near-Earth Apollo mission phases, only the very last portion of the transearth trajectory is pertinent.

In this situation, however, it is difficult to make comparisons with MSFN tracking, since this tracking starts shortly after transearth injection. Thus, the MSFN stations accumulate about 88 hours of tracking before the last portion of the transearth phase is reached.

Disregarding the above situation, a comparison of uncertainties at re-entry are shown in Figures 10, 11, 12, 13, and 14. The two LM Relays start tracking the spacecraft 20 minutes prior to re-entry. Reference 2 values are obtained from Table 8.3 and pertain to 3-way Doppler data

(Master and two slave stations) gathered over almost the entire transearth leg to the 3rd midcourse correction position (which occurs at 72 minutes prior to re-entry). The MSFN stations obtain 1 minute RD samples and 1 hour R samples during tracking.

Results indicate that the 2 Relay tracking data can certainly aid the MSFN stations in establishing better re-entry initial conditions for the re-entry guidance computer on-board the spacecraft.

CONCLUDING REMARKS

The results in this report for two LM Relay tracking assumed that, initially, each Relay was tracked by a different set of ground stations. A test case was completed where both Atlantic Relays were tracked by the same three stations. As expected, the all parameters updated result for Relay position and velocity was improved by a factor of three over those appearing in LM Relay Note. No. 9. The numerical values are as follows.

LM Relay	Position	Velocity
60°W	675 feet	.049 ft/sec
15°W	658 feet	.051 ft/sec

It is interesting to note that station location uncertainties in this test case are considerably decreased from the standard apriori values used.

Station	Altitude (feet)	East (feet)	North (feet)
Madrid	141.1*	92.8*	101.3*
	62.1	66.5	64.4
Ascension	105.0*	351.7*	344.9*
	74.0	89.5	72.5
Antigua	137.8*	116.4*	111.6*
	82.0	83.8	68.6

*Apriori values - see LM Relay Note No. 8

REFERENCES

(1)   Memorandum to FM/Chief, Mission Planning and Analysis
       Division from FM4/Mathematical Physics Branch, by
       Emil R. Schiesser, dated March 15, 1966.

(2)   Apollo Navigation Working Group Technical Report No.
       AN-2. 1, "Apollo Navigation Ground and Onboard
       Capabilities, " September 1, 1966.   National Aero-
       nautics and Space Administration, GSFC, Greenbelt,
       Md., and MSC, Houston, Texas.

(3)   "Error Analysis for the AS-503A Spacecraft Refer ...
       Trajectory, " (to be published by Mission Planning
       and Analysis Division, NASA, MSC, Houston, Texas.
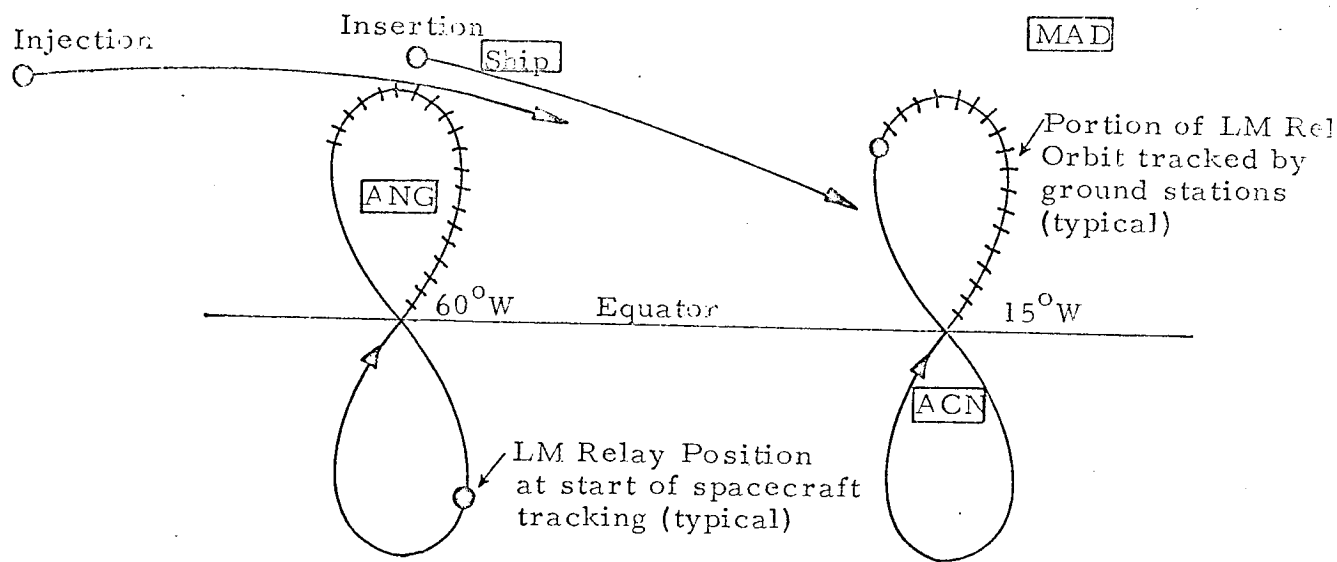
(a) ATLANTIC RELAYS
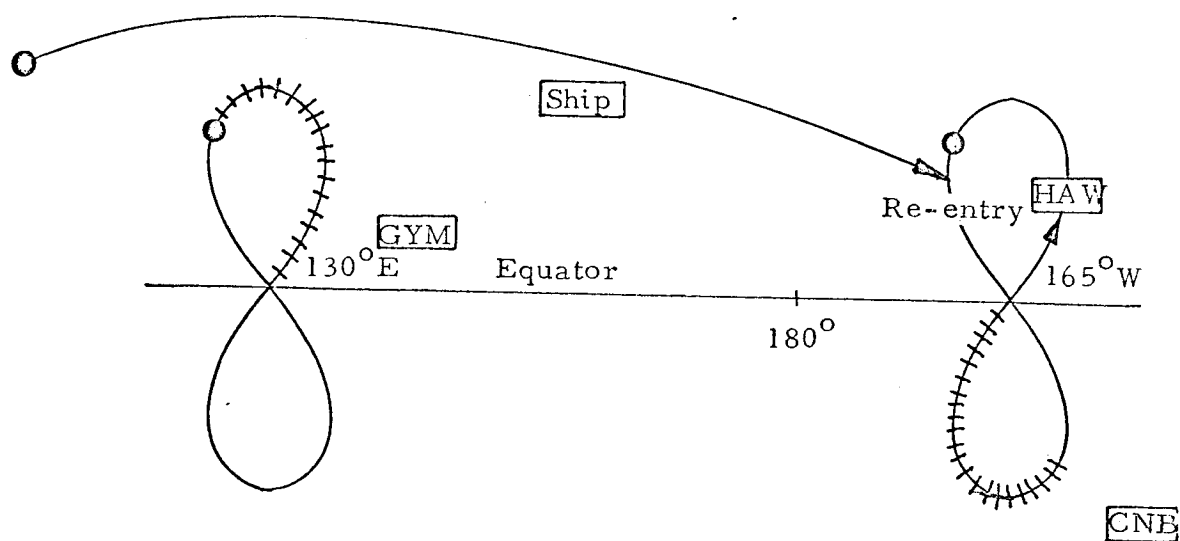
(b) PACIFIC RELAYS

Figure 1 - SCHEMATIC DIAGRAM OF LM RELAY
-SPACECRAFT-STATION GEOMETRY
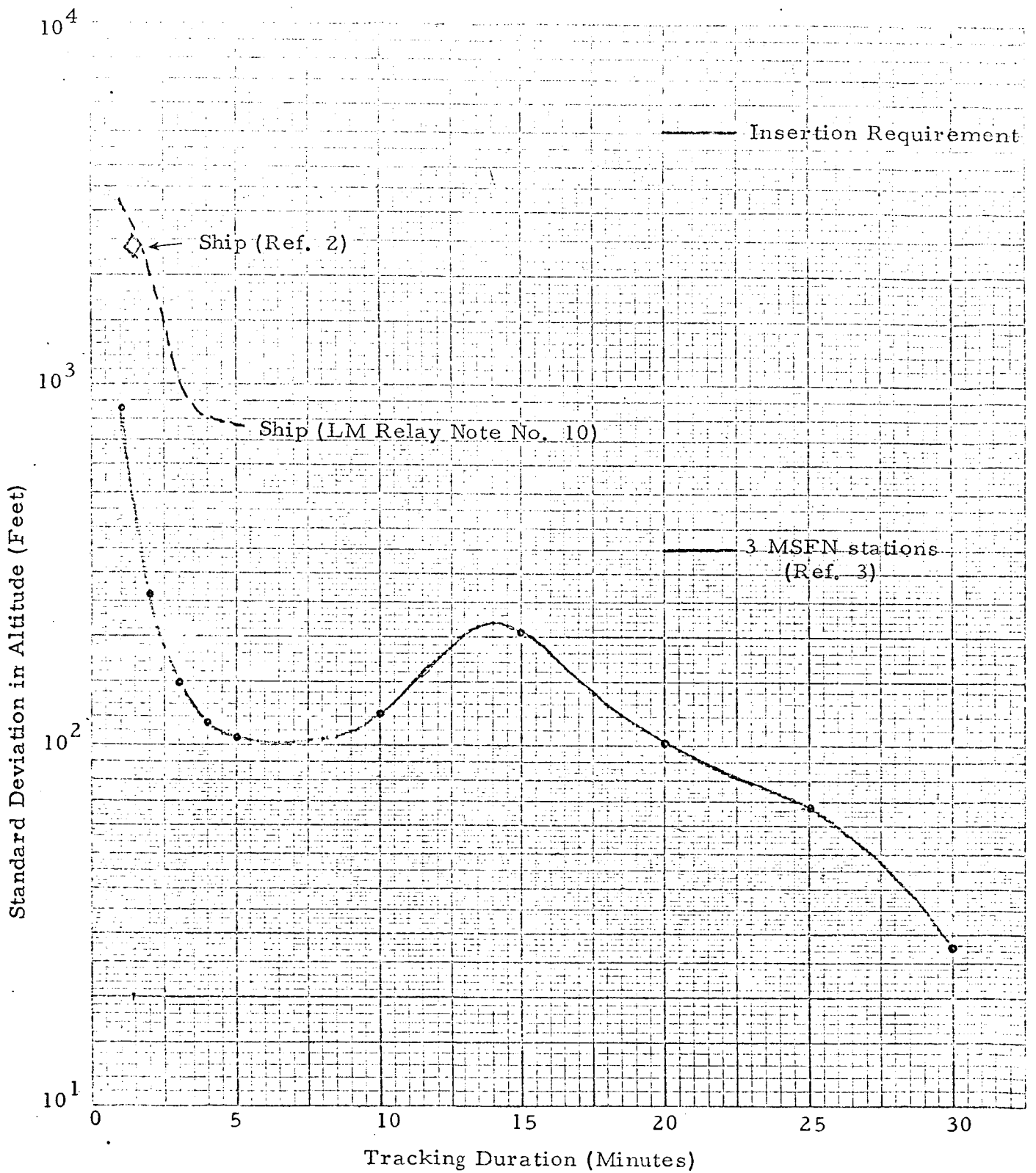
Figure 2.   Uncertainty in Altitude at Insertion for Two LM Relay Tracking
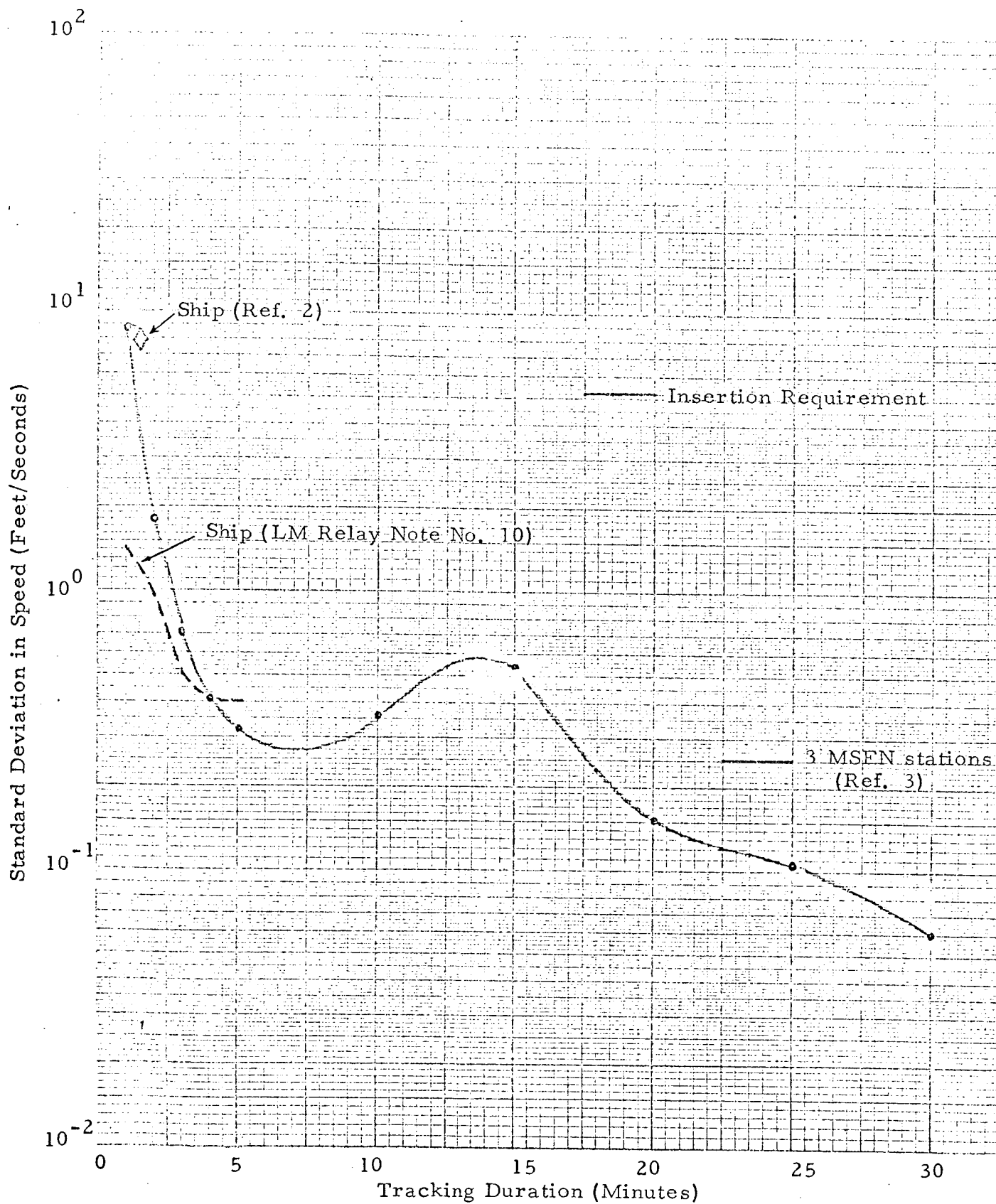
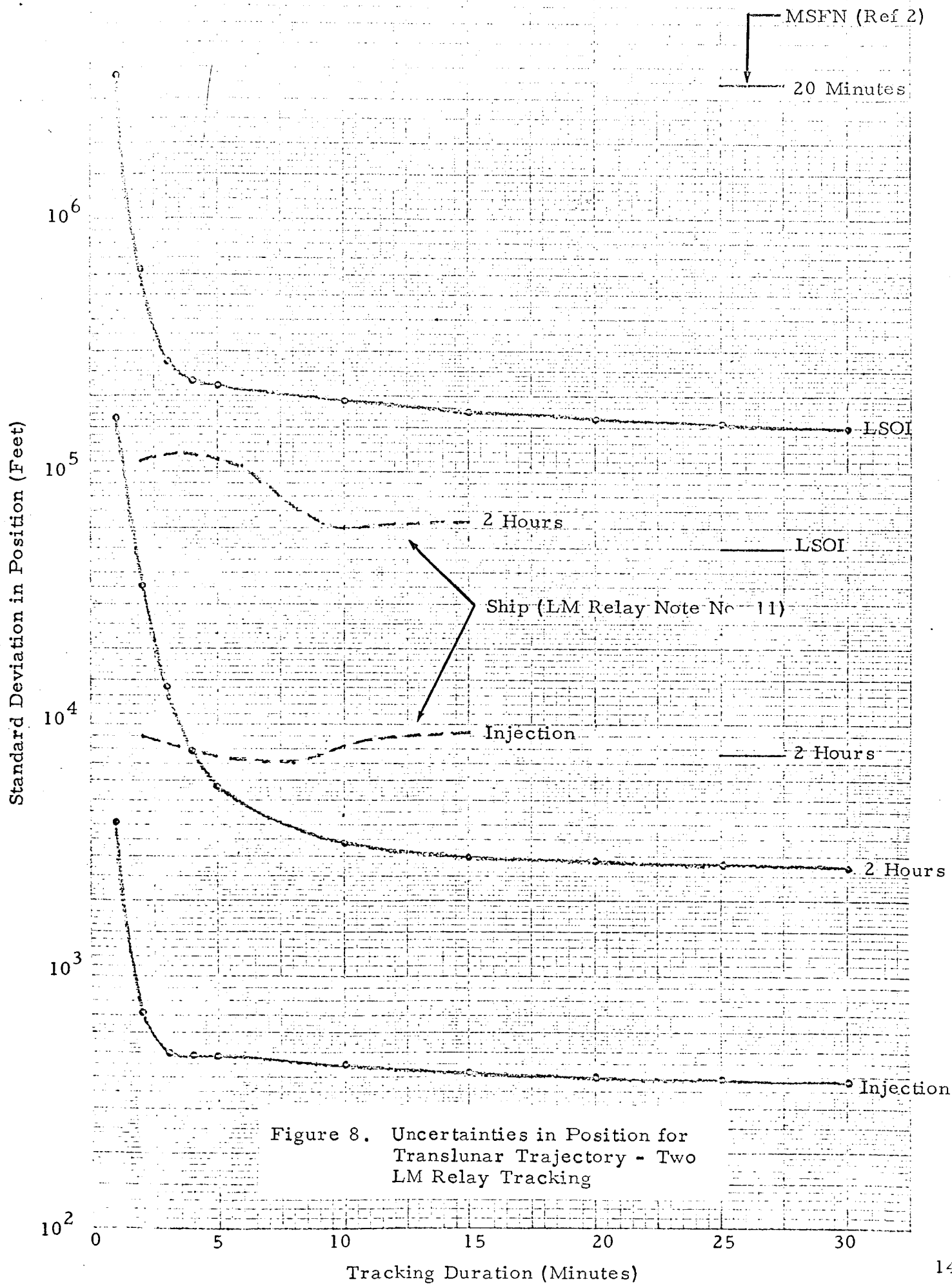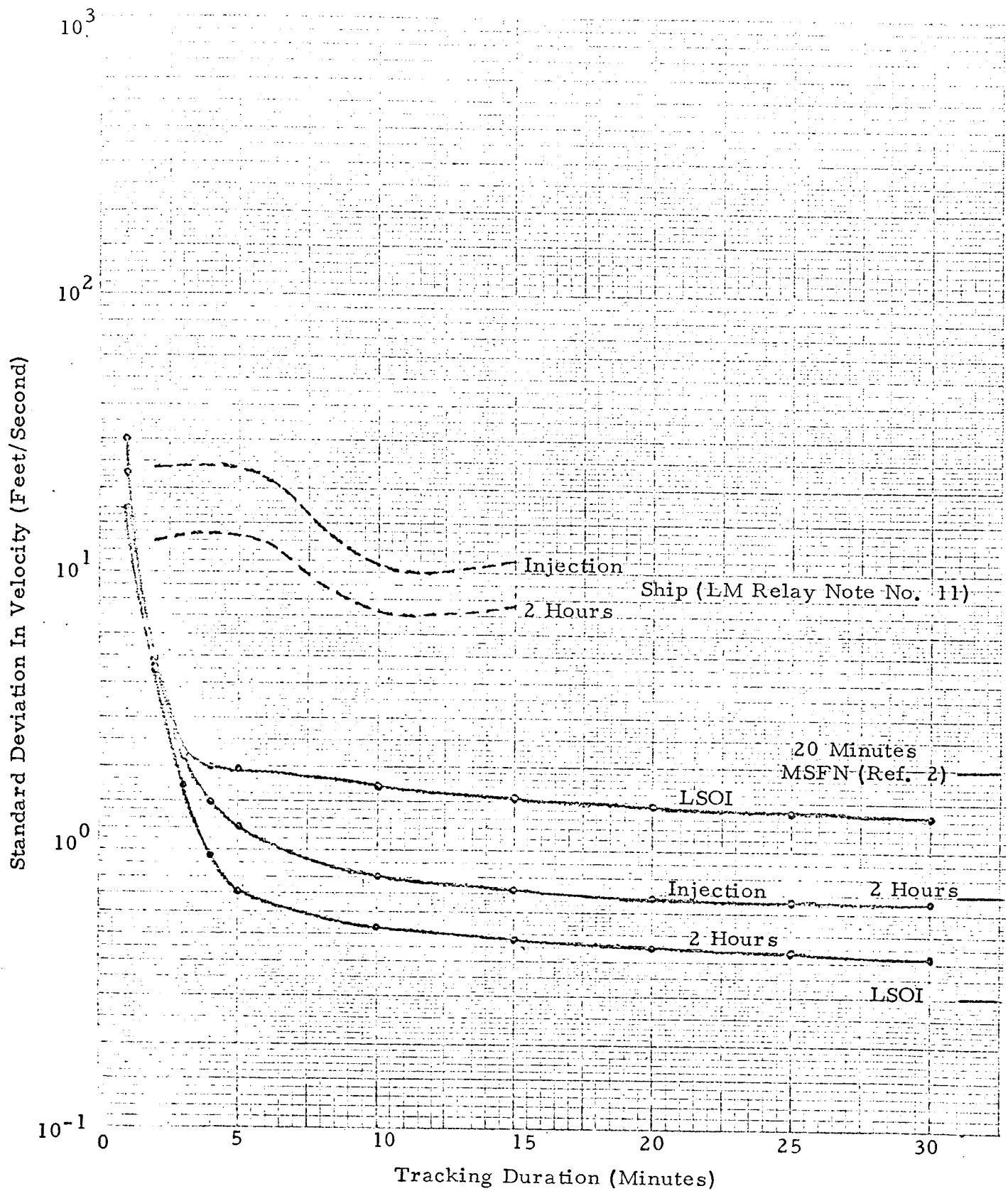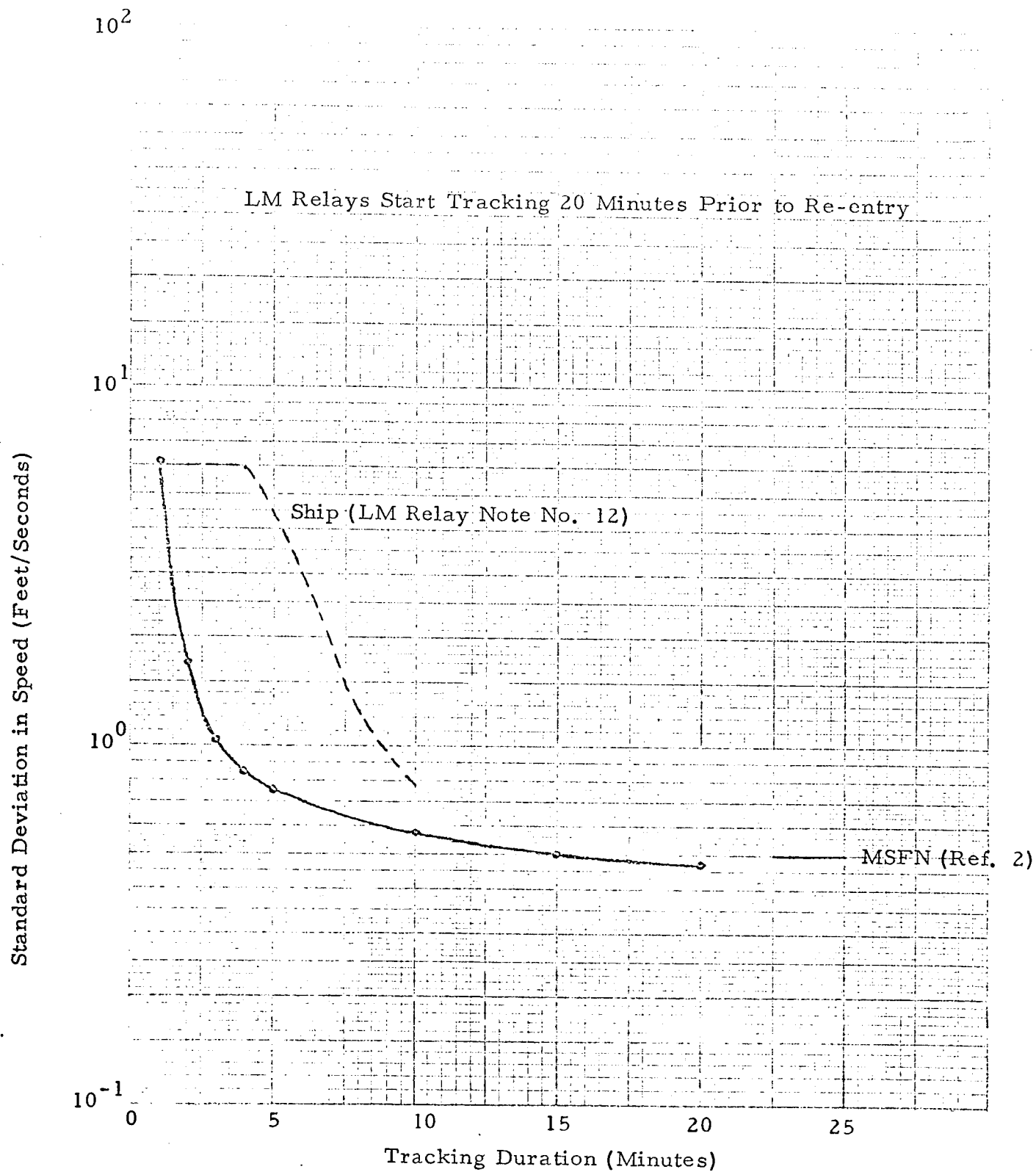Figure 3. Uncertainty in Speed at Insertion for Two LM Relay Tracking
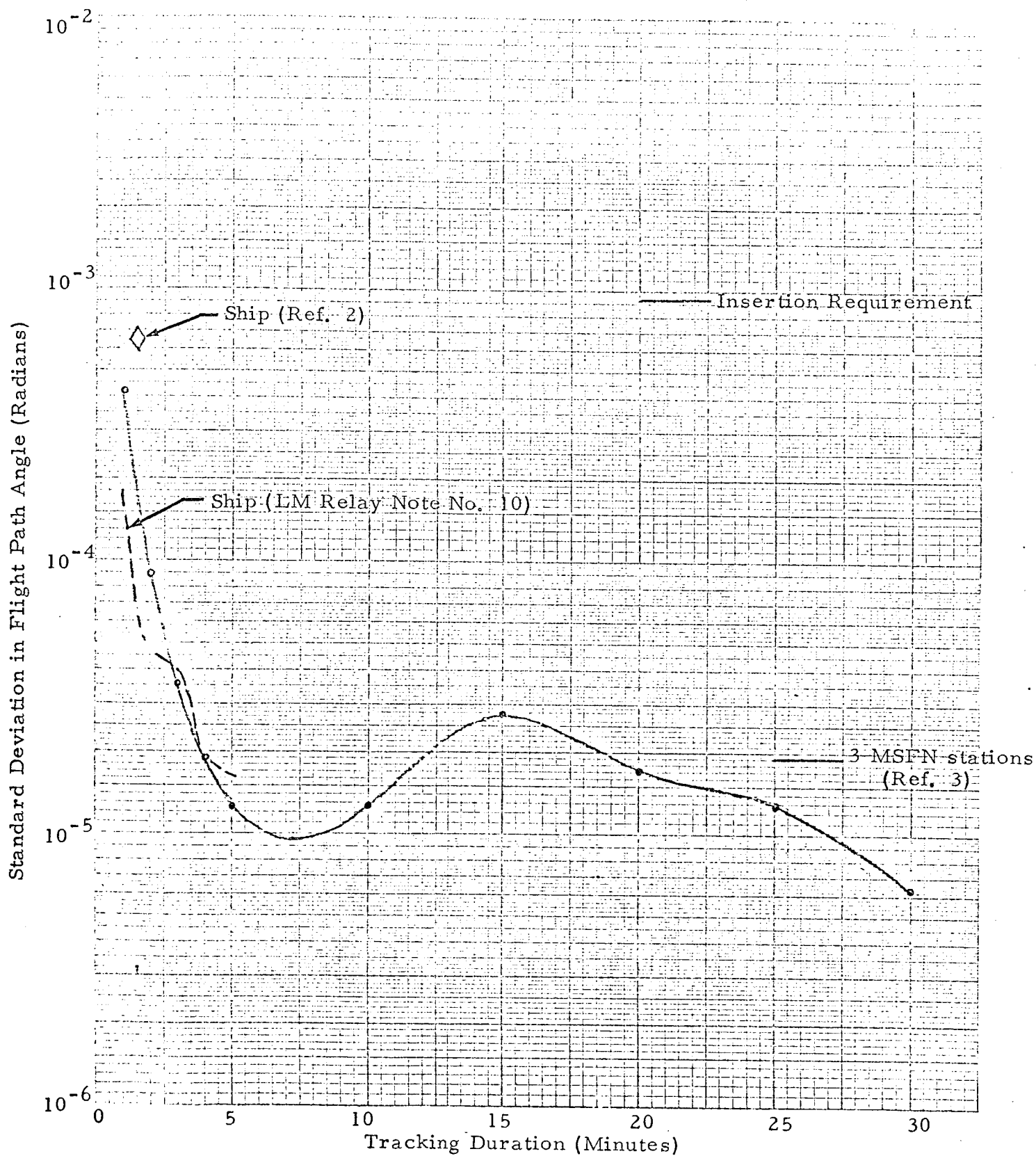
Figure 4.   Uncertainty in Flight Path Angle at Insertion for Two LM
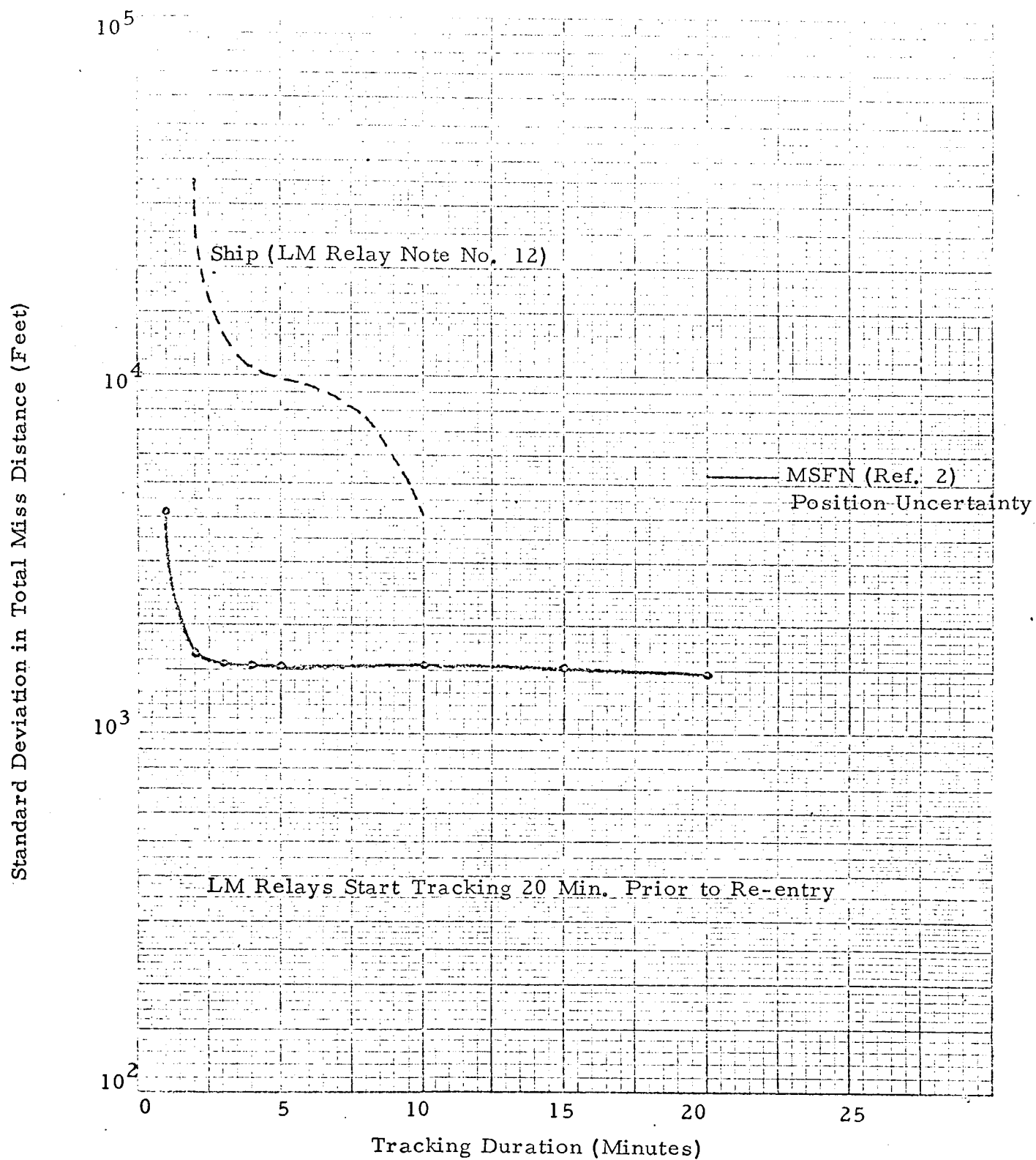Relay Tracking.
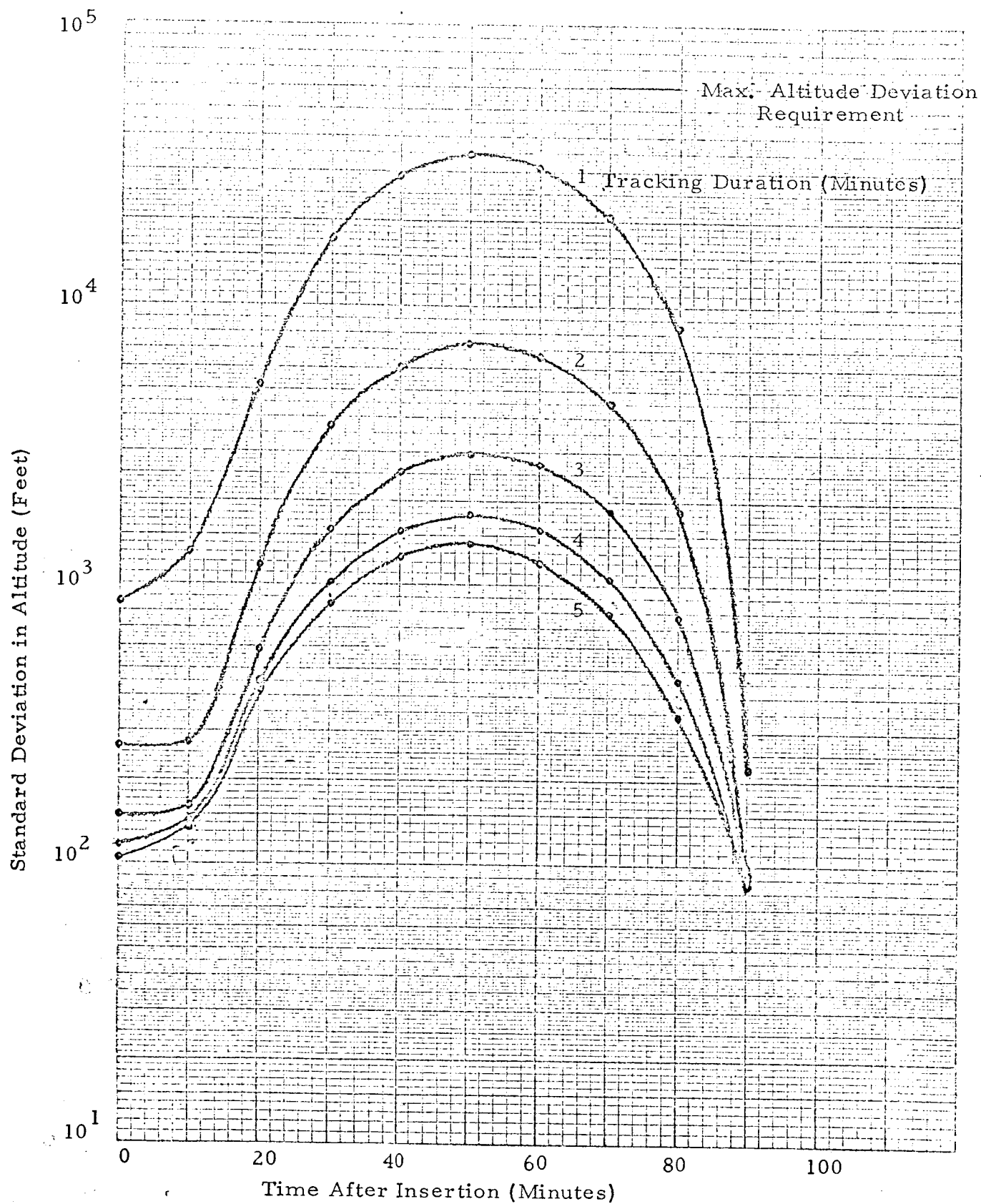
Figure 5.   Uncertainty in Altitude During one Revolution for Two LM Relay Tracking.
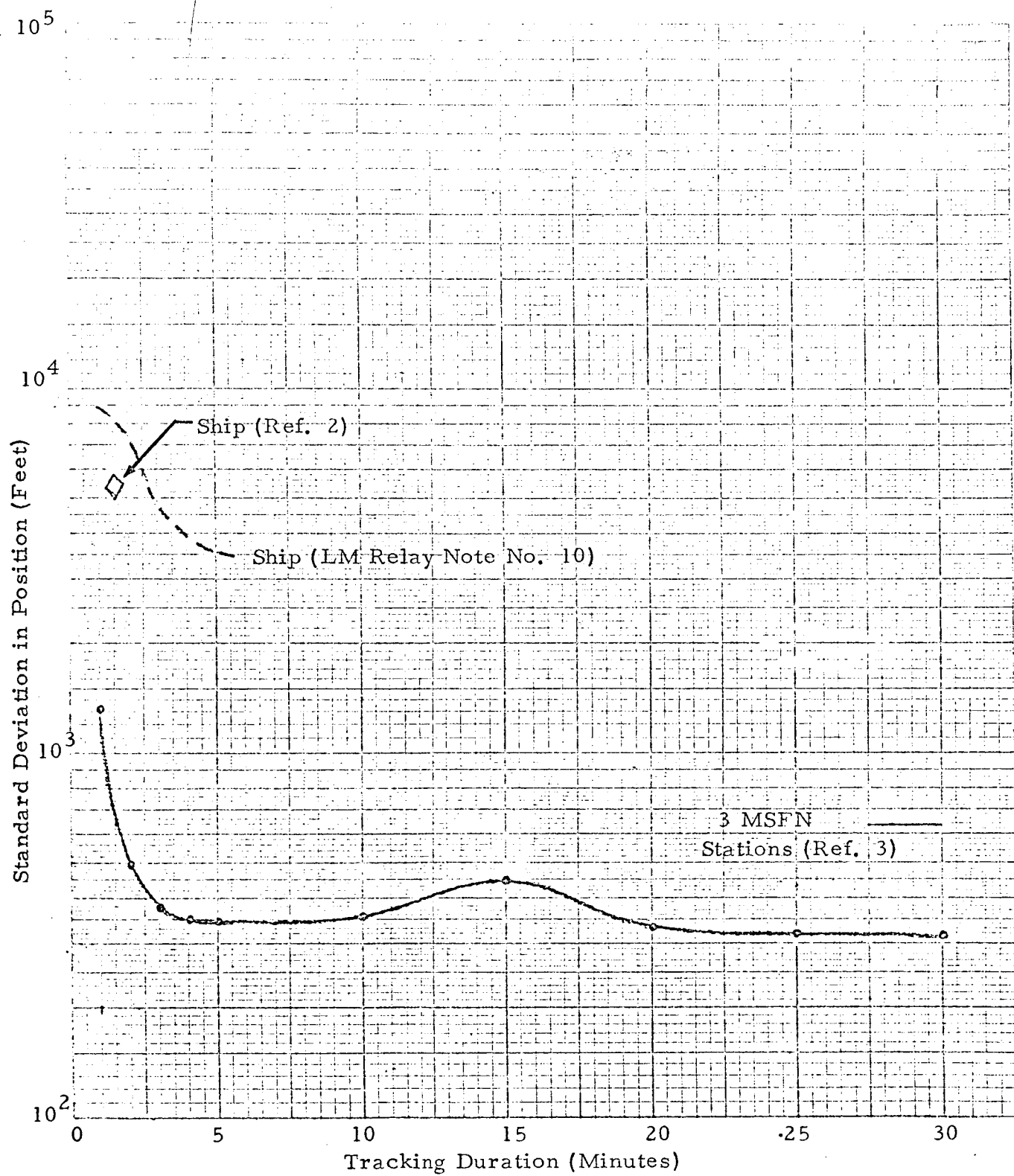
Figure 6.   Uncertainty in Position at Insertion for Two LM Relay Tracking.

The y-axis is labeled "Standard Deviation in Position (Feet)" with values 10^2, 10^3, 10^4, 10^5. The x-axis is labeled "Tracking Duration (Minutes)" with values 0, 5, 10, 15, 20, 25, 30.

Curves are labeled: Ship (Ref. 2), Ship (LM Relay Note No. 10), 3 MSFN Stations (Ref. 3).

Figure 7.   Uncertainty in Velocity at Insertion for Two LM Relay Tracking.
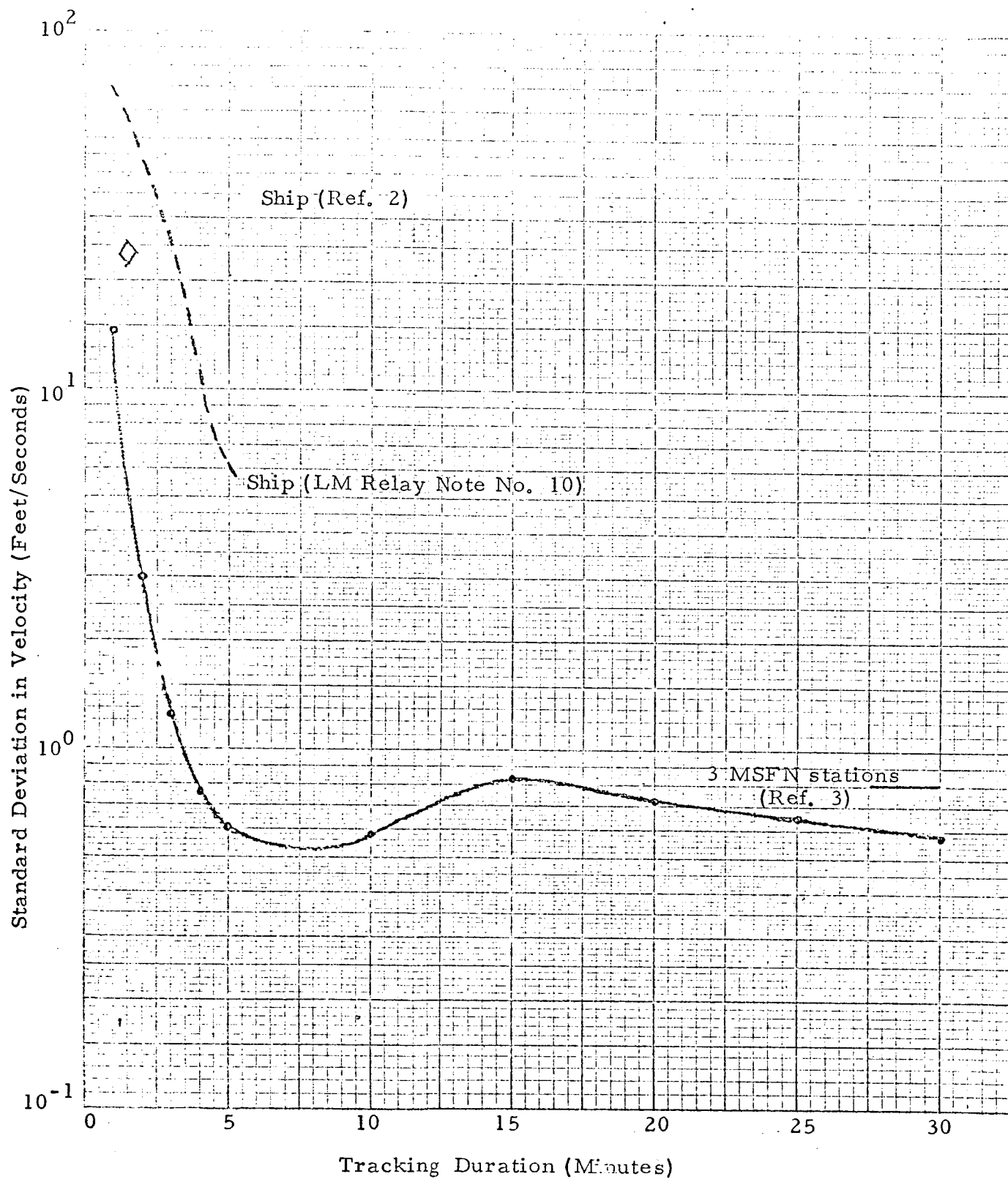
Figure 8. Uncertainties in Position for Translunar Trajectory - Two LM Relay Tracking

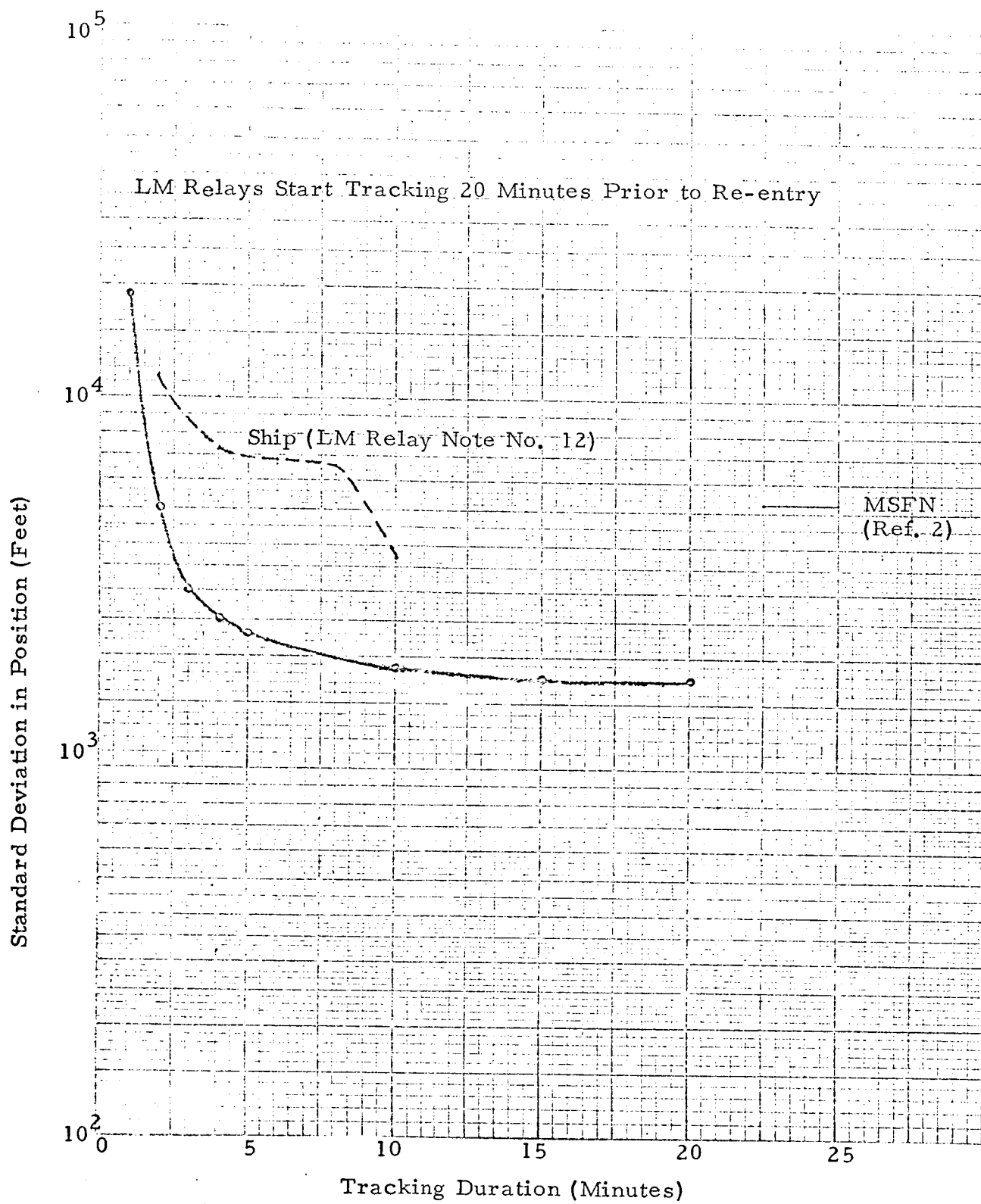Figure 9. Uncertainty in Velocity for Translunar Trajectory - Two LM Relay Tracking

Figure 10.   Uncertainty in Speed at Re-entry for Two LM Relay Tracking
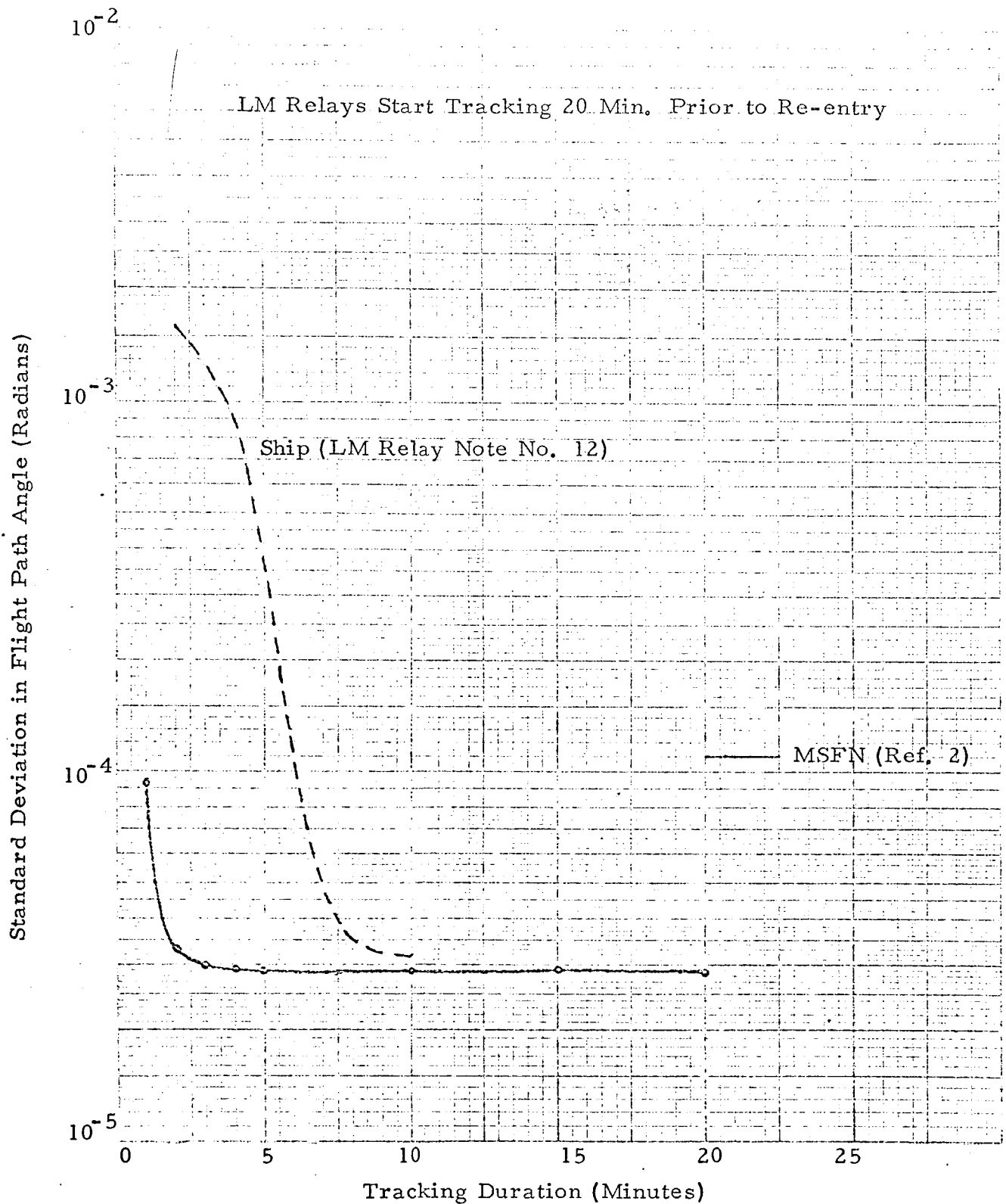
Figure 11. Uncertainty in Flight Path Angle at Re-entry for Two
LM Relay Tracking

Figure 12.   Uncertainty in Total Miss Distance on Re-entry Sphere
for Two LM Relay Tracking.
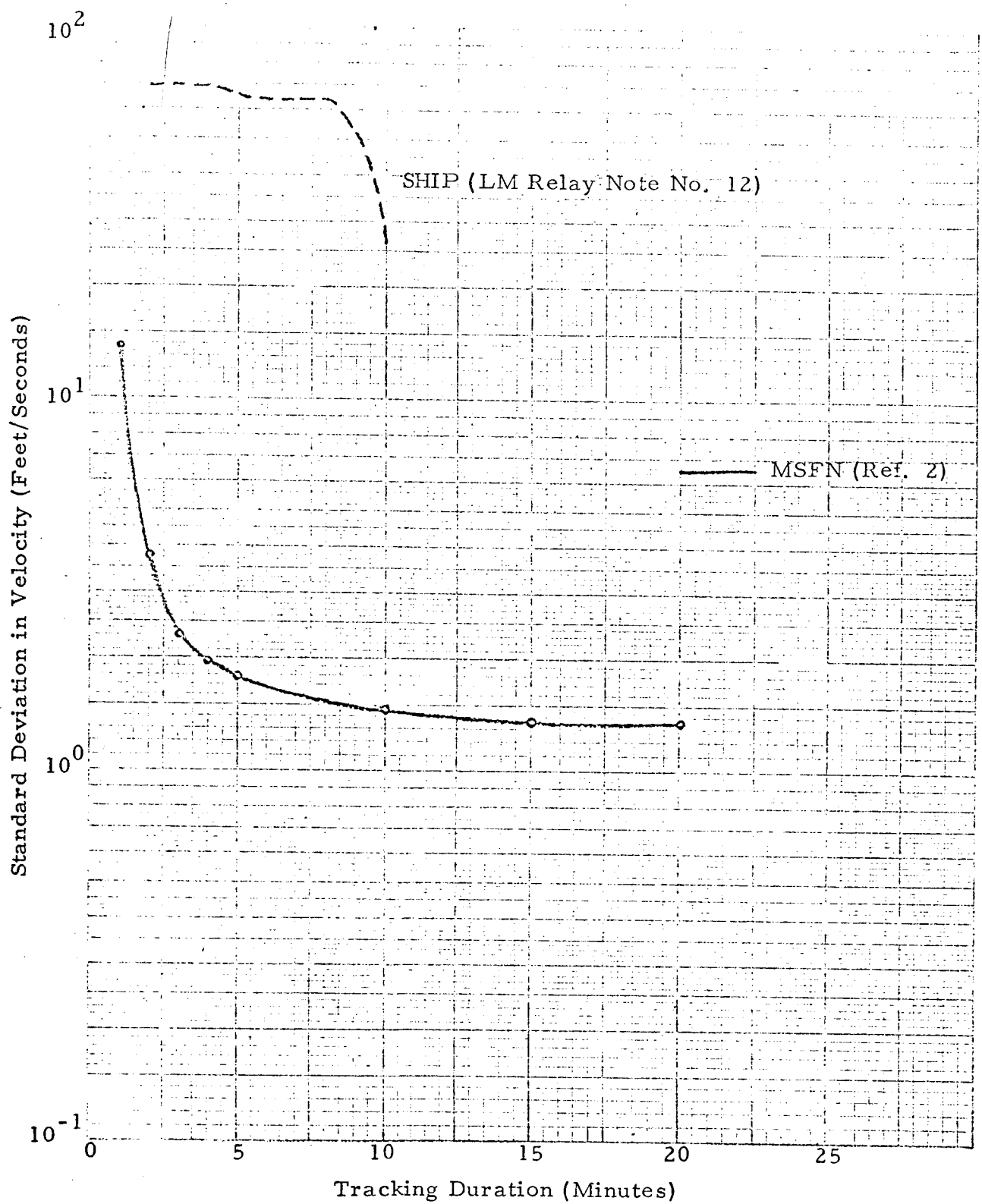
Figure 13. Uncertainty in Position at Re-entry for Two LM Relay
Tracking

Figure 14. Uncertainty in Velocity at Re-entry for Two LM Relay
Tracking

LM RELAY NOTE NO. 15
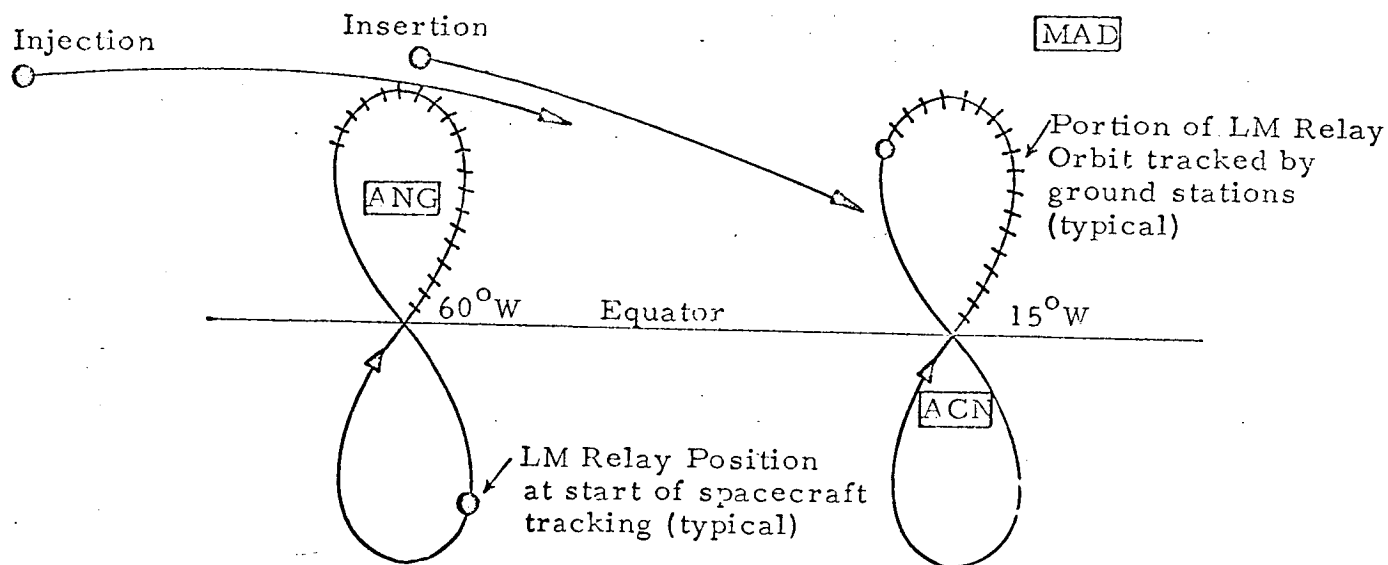
C. P. Siska
29 June 1967

SUPPLEMENT TO THE TWO RELAY TRACKING ANALYSIS OF LM RELAY NOTE NO. 14

Introduction

In LM Relay Note No. 14, the uncertainty in state vector estimates for the two relays was obtained by assuming that a different set of three ground stations (Master and two Slaves) tracked each Relay, even though the same three ground stations were involved in each case. The effect of this assumption was felt to be conservative, since the number of estimated parameters was greater than necessary (6 stations instead of 3), thus presumably leading to greater errors in state vector estimates. The above artifice was adopted principally because of difficulties in setting up the desired error analysis runs on the computer.

However, since that time, minor modifications to the computer program have enabled us to analyze the problem where the same set of three stations track each Relay. To substantiate the contention that the results of LM Relay Note No. 14 are conservative, the insertion and injection Apollo mission phases have been investigated in this report.

To recapitulate briefly, the ground stations track each Relay for a total of sixteen hours (2 eight hour shifts) and forty hours after tracking has started, the Apollo spacecraft is inserted or injected into the appropriate orbit and is then tracked by the two LM Relays with range and range-rate measurements. Figure 1 illustrates the relative geometry of the situation.

Injection Insertion MAD

Portion of LM Relay
Orbit tracked by
ground stations
(typical)

ANG

60°W Equator 15°W

ACN

LM Relay Position
at start of spacecraft
tracking (typical)

ATLANTIC RELAYS

Figure 1. Schematic Diagram of LM Relay - Spacecraft-Station
Geometry.

Results

Table 1 shows the uncertainties in the estimated parameters after processing range-rate, range, and angle data from Madrid (Master station) and range-rate from Ascension and Antigua (Slave stations).

Figures 2 through 8 show a comparison of the present results with those of LM Relay Note No. 14, for the parking orbit insertion and trans-lunar injection phases of the Apollo mission.

*In the Orbit plane normal to the radius vector

Estimated Parameters	Apriori One Sigma Uncertainty	Updated One Sigma Uncertainties	
1. Radius (ft.) - 60°W. Relay	2.828 + 06	SQRT(1, 1)	9.3953080E 01
2. *	2.828 + 06	SQRT(2, 2)	6.3293388E 02
3. Out-of-plane position (ft.)	2.828 + 06	SQRT(3, 3)	2.1358657E 02
4. Radial velocity (f/s)	5.000 + 02	SQRT(4, 4)	4.5724092E-02
5. *	5.000 + 02	SQRT(5, 5)	6.0887283E-03
6. Out-of-plane velocity (f/s)	5.000 + 02	SQRT(6, 6)	1.7458513E-02
7. MU (ft³/sec²)	1.060 + 11	SQRT(7, 7)	2.5158005E 10
8. Radius (ft.) - 15°W. Relay	2.828 + 06	SQRT(8, 8)	7.8941982E 01
9. *	2.828 + 06	SQRT(9, 9)	6.3420784E 02
10. Out-of-plane position (ft.)	2.828 + 06	SQRT(10, 10)	1.5906070E 02
11. Radial Velocity (f/s)	5.000 + 02	SQRT(11, 11)	4.6506984E-02
12. *	5.000 + 02	SQRT(12, 12)	7.0674595E-03
13. Out-of-plane velocity (f/s)	5.000 + 02	SQRT(13, 13)	1.9475015E-02
14. Mad Clock Rate (sec/sec)	2.000 + 10	SQRT(14, 14)	1.1488847E-10
15. ACN Altitude (ft.)	1.050 + 02	SQRT(15, 15)	7.3980908E 01
16. ACN E-W Displacement (ft)	3.517 + 02	SQRT(16, 16)	8.9473342E 01
17. ACN N-S Displacement (ft)	3.449 + 02	SQRT(17, 17)	7.2518364E 01
18. ACN Clock offset (sec)	3.000 - 03	SQRT(18, 18)	2.7362820E-03
19. ACN Clock Rate (sec/sec)	2.000 - 10	SQRT(19, 19)	1.1487822E-10
20. ANG Altitude (ft)	1.378 + 02	SQRT(20, 20)	8.1986306E 01
21. ANG E-W Displacement (ft)	1.164 + 02	SQRT(21, 21)	8.3798667E 01
22. ANG N-S Displacement (ft)	1.116 + 02	SQRT(22, 22)	6.8601656E 01
23. ANG Clock Offset (sec)	3.000 - 03	SQRT(23, 23)	2.9981507E-03
24. ANG Clock rate (sec/sec)	2.000 - 10	SQRT(24, 24)	1.1490469E-10
25. MAD Range Bias (ft)	6.000 + 01	SQRT(25, 25)	5.9112052E 01
26. MAD X-Angle Bias (rad.)	1.600 - 03	SQRT(26, 26)	1.8321062E-05
27. MAD Y-Angle Bias (rad.)	1.600 - 03	SQRT(27, 27)	1.8675689E-05
28. MAD Altitude (ft)	1.411 + 02	SQRT(28, 28)	6.2065574E 01
29. MAD E-W Displacement (ft)	9.280 + 01	SQRT(29, 29)	6.6536392E 01
30. MAD N-S Displacement (ft)	1.013 + 02	SQRT(30, 30)	6.4394312E 01
31. MAD Clock Offset (sec)	3.000 - 03	SQRT(31, 31)	1.3464596E-04

Table 1. Uncertainties in Updated Parameters for Atlantic LM Relays
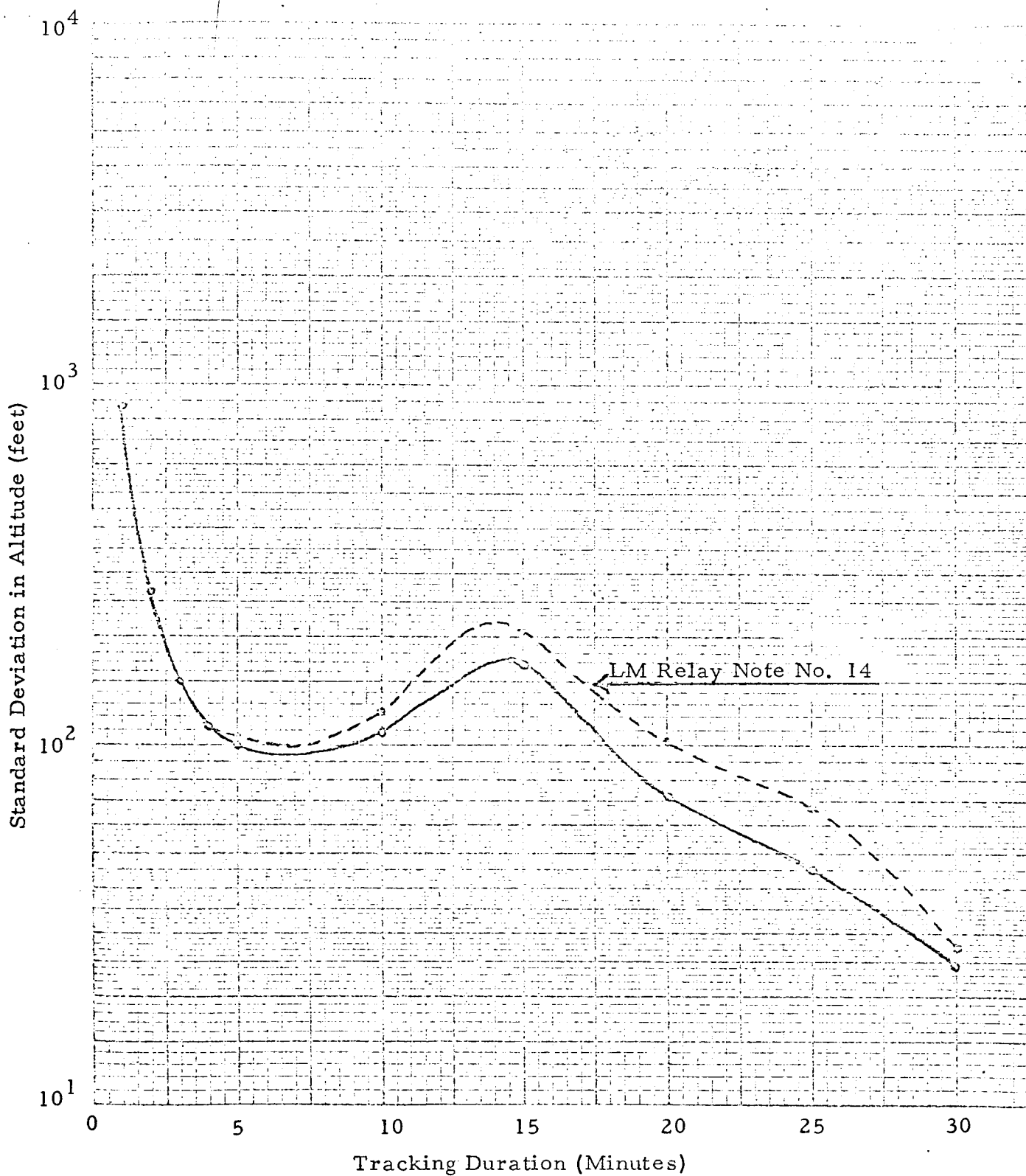
4

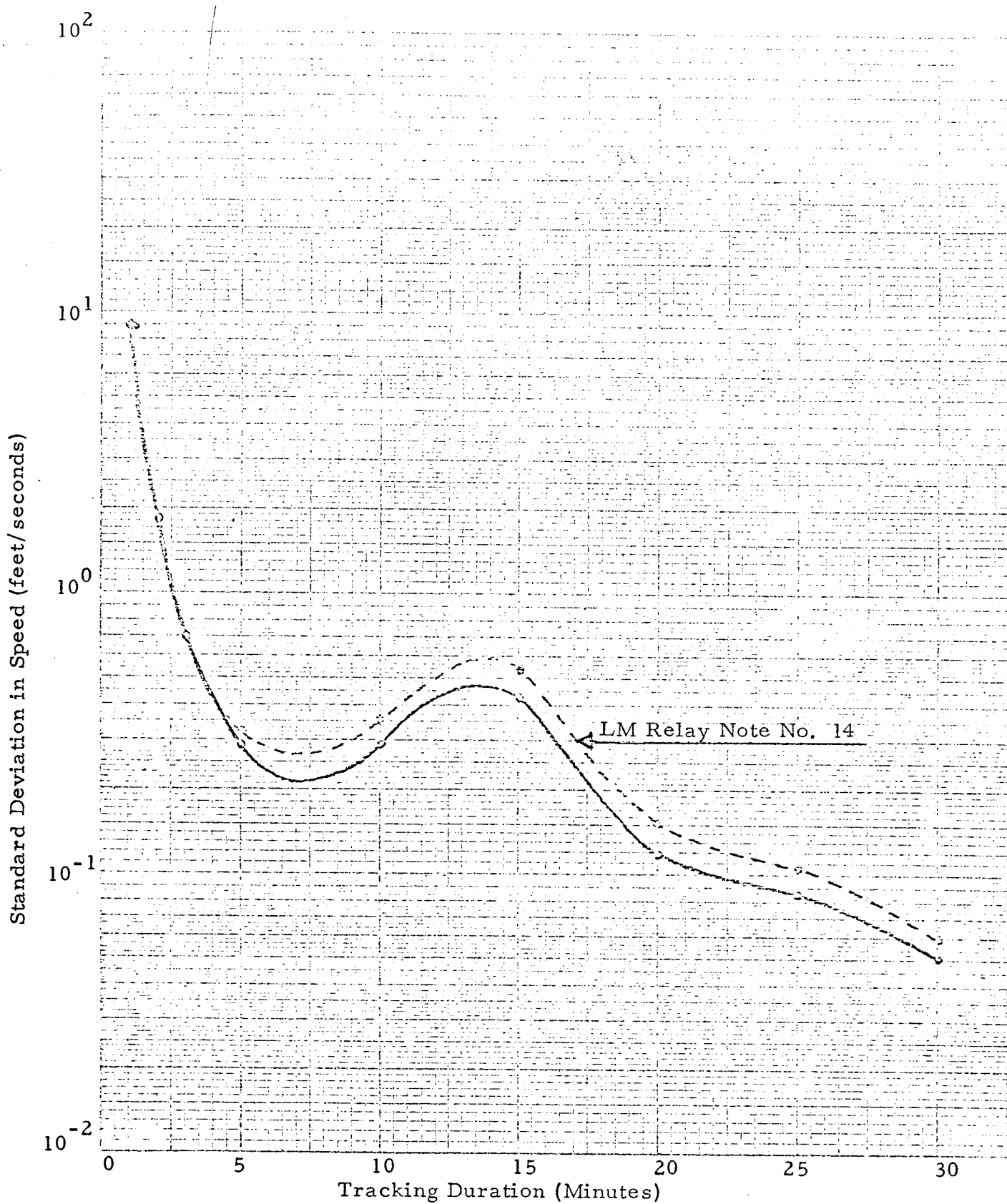Figure 2. Uncertainty in Altitude at Insertion for Two LM Relay Tracking

Figure 3.   Uncertainty in Speed at Insertion for Two LM Relay Tracking.
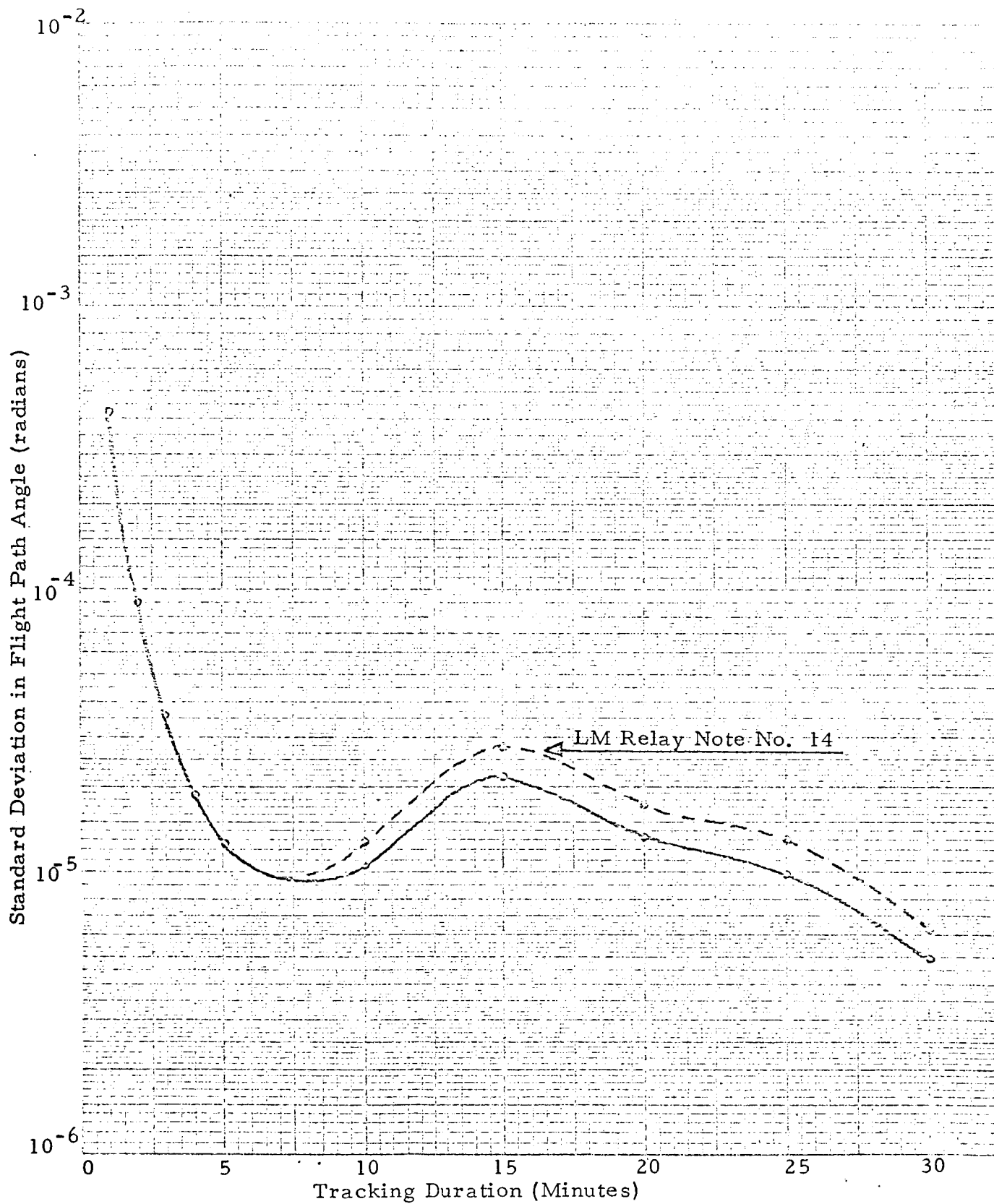
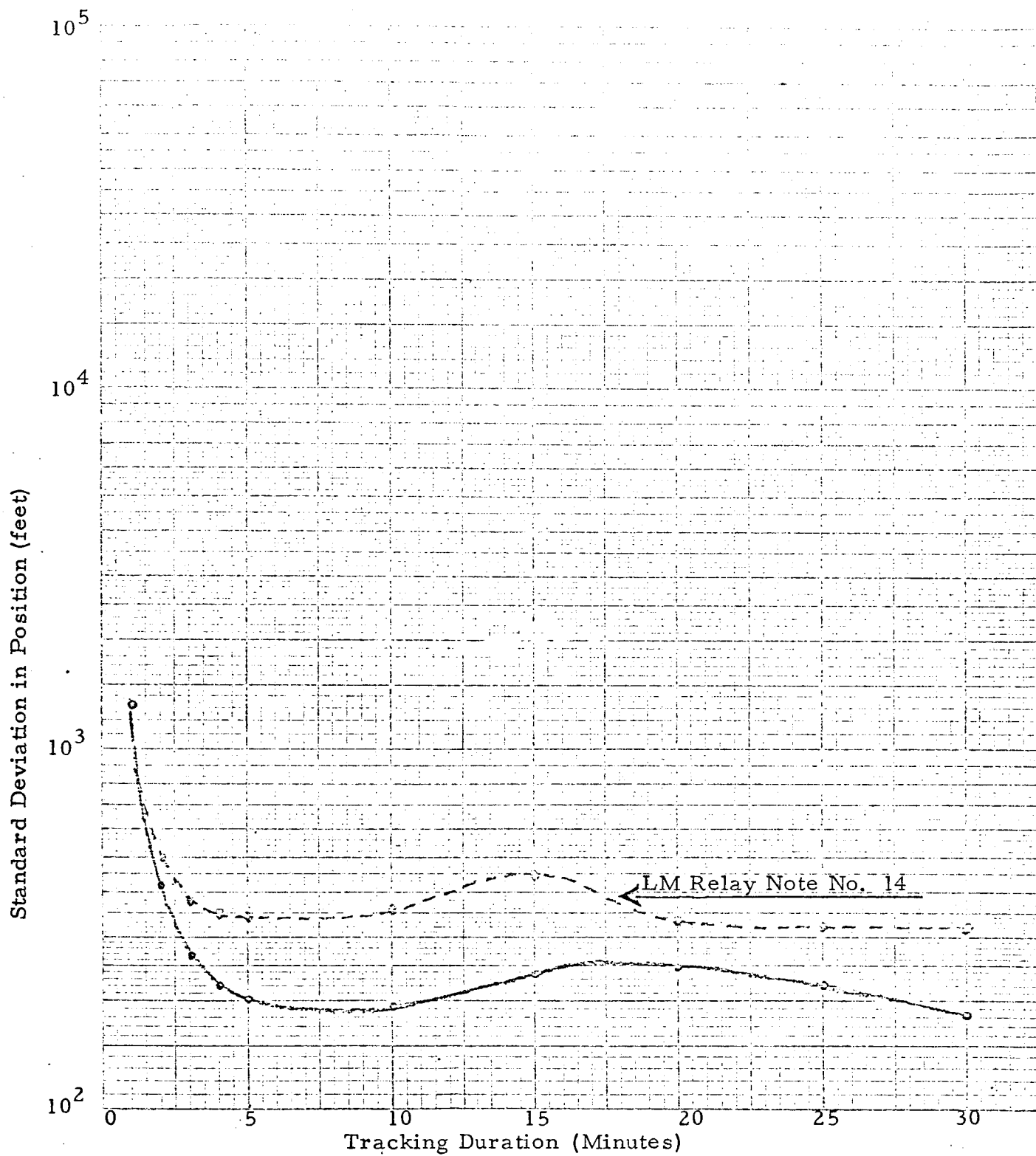Figure 4.   Uncertainty in Flight Path Angle at Insertion for Two LM Relay Tracking.

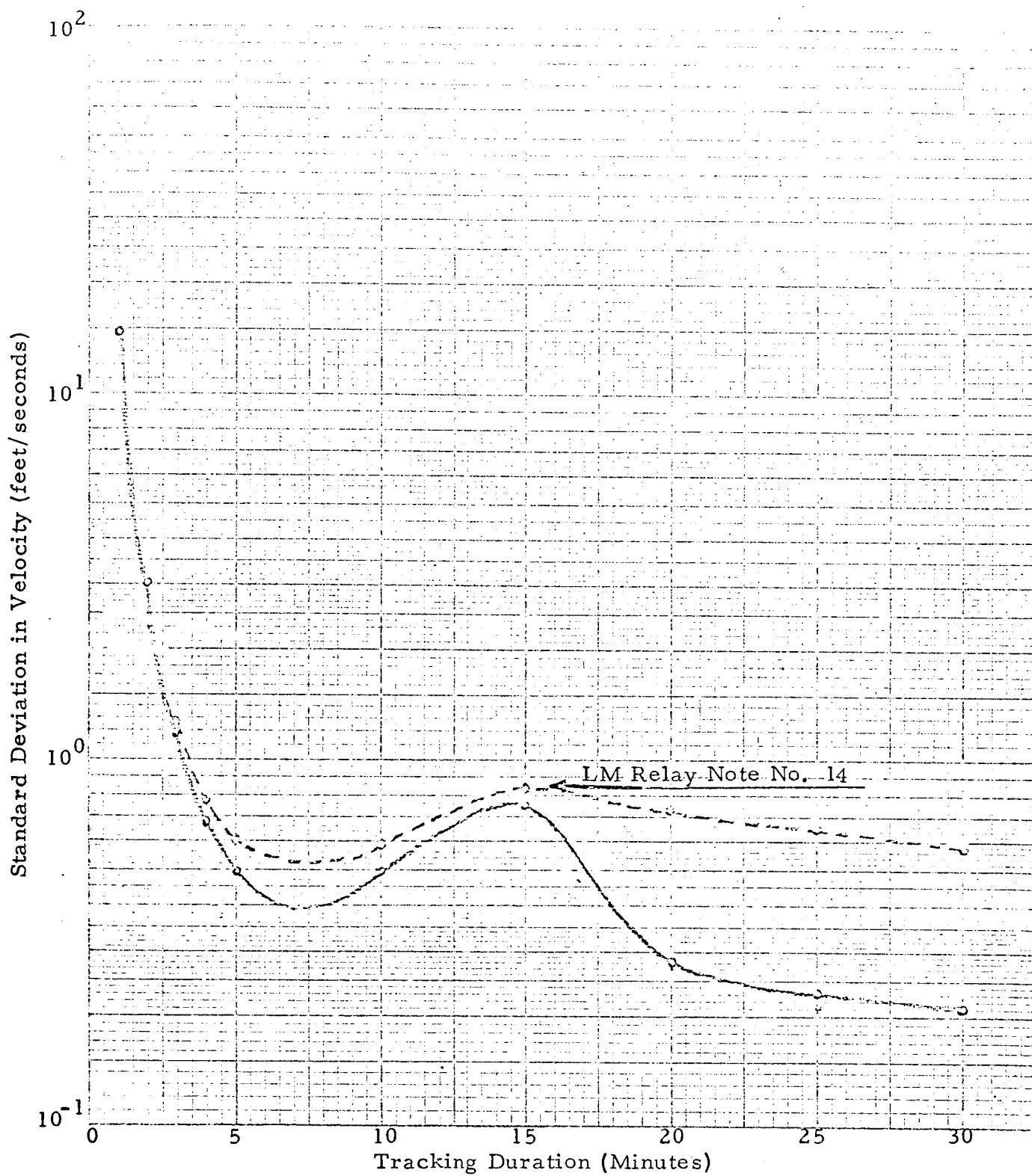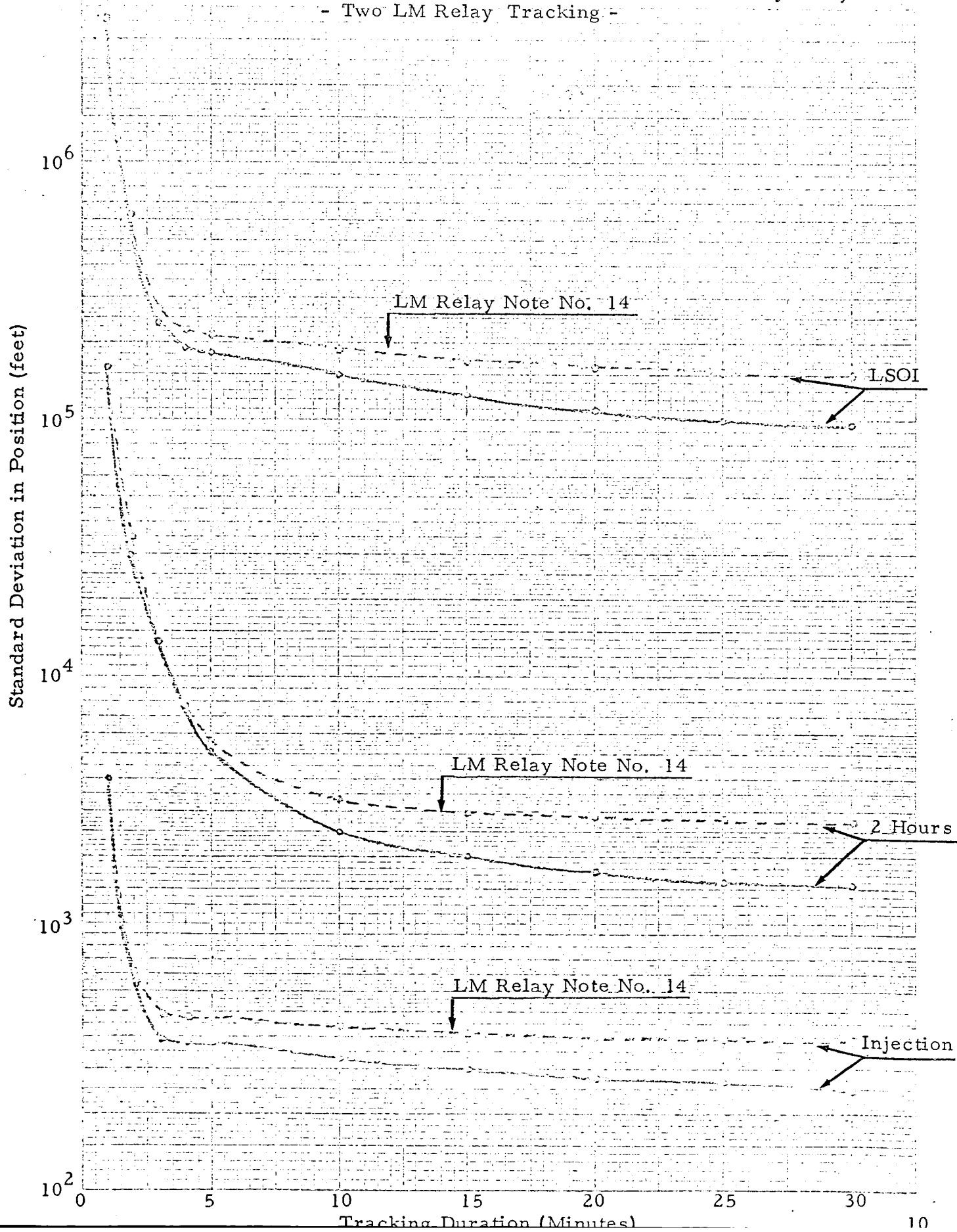Figure 5. Uncertainty in Position at Insertion for Two LM Relay Tracking.

Figure 6. Uncertainty in Velocity at Insertion for Two LM Relay Tracking.

Figure 7.   Uncertainties in Position for Translunar Trajectory
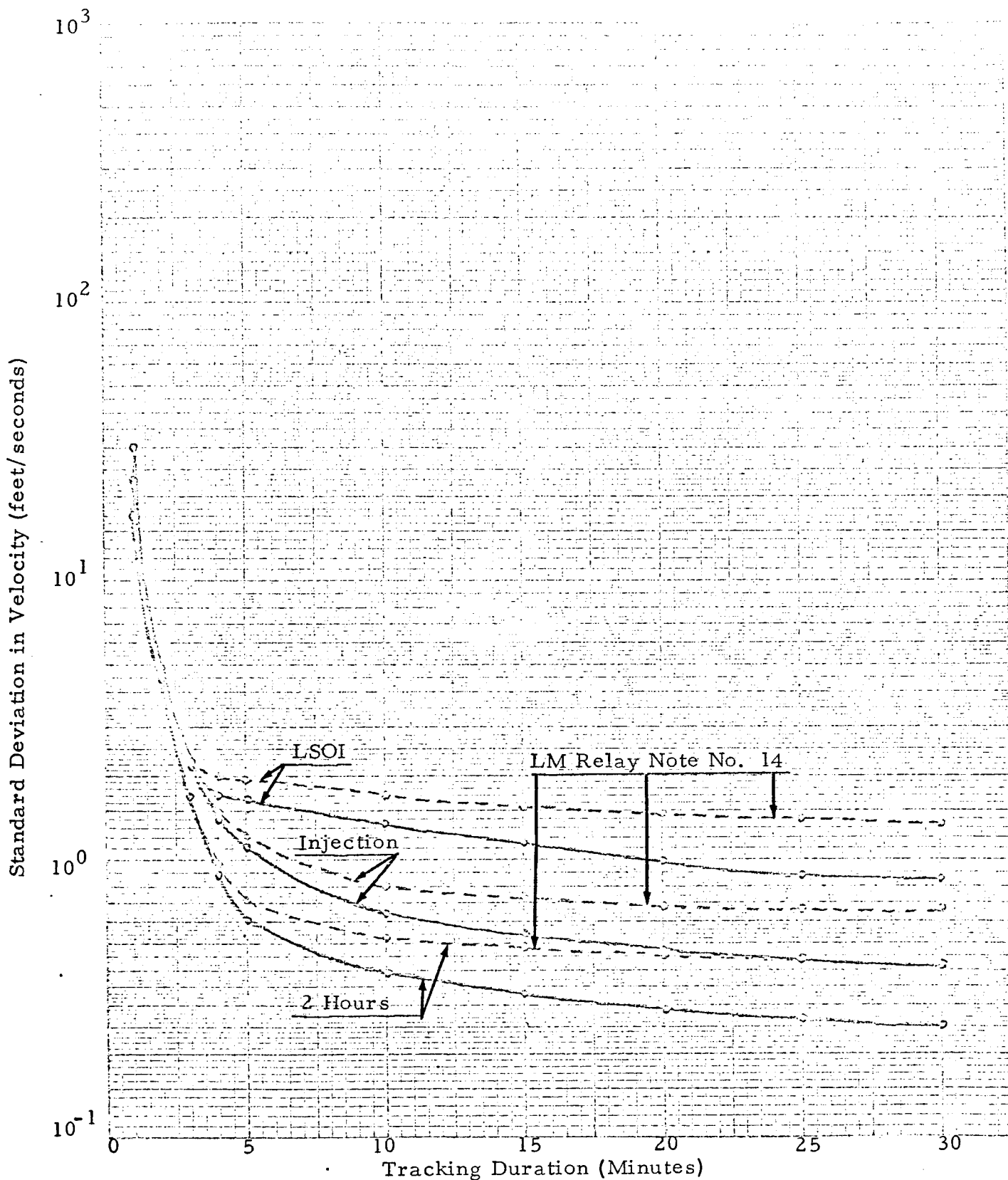- Two LM Relay Tracking -

Figure 8.   Uncertainty in Velocity for Translunar Trajectory - Two LM
Relay Tracking.

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