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# THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

## VOLUME V

## EFFECTS OF INITIAL IMPERFECTIONS; AXIAL COMPRESSION AND PURE BENDING

Prepared for the
GEORGE C. MARSHALL SPACE FLIGHT CENTER
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ABSTRACT

This is the fifth of six volumes, all bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of documents was prepared under NASA Contract NAS8-11181. In this particular volume, practical design curves are presented for empirical knock-down factors. These factors are needed for the reduction of classical theoretical strength values to safe design levels. In view of the relative scarcity of test data for stiffened cylinders, the design curves are based on the use of an effective thickness concept in conjunction with test data from isotropic cylinders.

## TABLE OF CONTENTS



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## LIST OF ILLUSTRATIONS

## DEFINITION OF SYMBOLS

| Symbol | Definition |
| --- | --- |
| $A$ | Area |
| $A_{11}, A_{22}$ | Elastic constants (see Volume I [12]) |
| $a$ | Ring spacing; Postbuckling variable defined by Figures 9 and 10. |
| $b$ | Postbuckling variable defined by Figures 9 and 10. |
| $C_F$ | Fixity factor. |
| $D_{11}, D_{22}$ | Elastic constants (see Volume I [12]) |
| $E$ | Young's modulus. |
| $E_{tan}$ | Tangent modulus in compression. |
| $e$ | Base of natural logarithms. |
| $L$ | Overall length of cylinder. |
| $\bar{N}_x$ | Applied longitudinal compressive running load (acting at centroid of effective skin-stringer combination). |
| $(\bar{N}_x)_{CL}$ | Classical theoretical value for the critical longitudinal compressive running load (acting at centroid of effective skin-stringer combination). |
| $(\bar{N}_x)_{cr}$ | Critical longitudinal compressive running load (acting at centroid of effective skin-stringer combination). |
| $(\bar{N}_x)_{MIN}$ | Minimum longitudinal compressive running load (acting at centroid of effective skin-stringer combination) for the postbuckling equilibrium path (see Figures 9 and 10). |
| $(\bar{N}_x)_{wc}$ | Wide-column critical longitudinal compressive running load (acting at centroid of effective skin-stringer combination). |
| $R$ | Radius to middle surface of basic cylindrical skin. |
| $t$ | Thickness of basic cylindrical skin. |
| $t_{eff}$ | Equivalent thickness. |
| $\bar{t}_x$ | Wall thickness for a monocoque cylinder of same total cross-sectional area as actual composite wall (including all of the effective skin and stringer material). |

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## DEFINITION OF SYMBOLS (Continued)

| Symbol | Definition |
|---|---|
| $\Gamma$ | Knock-down factor which accounts for effects of initial imperfections plus other uncertainties. |
| $\Gamma_{Axial}$ | Knock-down factor for circular cylinder subjected to pure axial load. |
| $\Gamma_{Bend}$ | Knock-down factor for circular cylinder subjected to pure bending. |
| $\rho$ | Local radius of gyration of effective skin-stiffener combination. |
| $\rho_x$ | Local radius of gyration of effective skin-stringer combination. |
| $\rho_y$ | Local radius of gyration of effective skin-ring combination. |
| $\sigma_{cc}$ | Crippling stress. |
| $\phi$ | Parameter defined by equation (2-3). |

# SECTION 1

# INTRODUCTION

For isotropic cylinders under axial compression, it is widely known that large disparities exist between test results and the predictions from classical small-deflection theory. This phenomenon is usually attributed to the influences of initial imperfections and to the shape of the postbuckling equilibrium path. It should be noted, however, that recent theoretical and experimental investigations have established the fact that a significant portion of the differences results from test boundary conditions which differ from those assumed in the classical analysis. For isotropic cylinders, the present design practice is to consolidate these various influences, along with other uncertainties, into a single empirical knock-down factor. The procedures presented in the sections to follow are based on the use of this same approach for the analysis of eccentrically stiffened cylinders. For stiffened shells in general, the limited available test data tend to indicate that the predictions from classical small-deflection theory will be more nearly approached than in the case of thin-walled isotropic cylinders. This undoubtedly is the result of the stiffened wall configurations being effectively "thick". Therefore, a currently popular viewpoint is to consider classical theory to be directly applicable to most practical stiffened shells. Nevertheless, to account for uncertainties and to guard against reckless extrapolation into extreme parameter ranges, it is suggested here that a knock-down factor be retained in the buckling analysis of stiffened cylinders. This should result in conservative strength estimates which can be employed with confidence in the design of actual hardware.

# SECTION 2

# KNOCK-DOWN CRITERIA

## 2.1 EFFECTIVE THICKNESS

For the design of axially compressed isotropic cylinders, the allowable load intensities are usually computed by the equation

$$\left(\bar{N}_x\right)_{cr} = \Gamma \left(\bar{N}_x\right)_{CL} \tag{2-1}$$

where $\Gamma$ is the empirical knock-down factor. It might be noted however, that, in 1950, a more fundamental approach was proposed by Donnell and Wan [1] involving the use of a so-called unevenness factor. This factor was to provide a measure of imperfection magnitudes. Donnell and Wan published their paper primarily in the hope of demonstrating how a theoretically-based imperfection analysis might eventually be accomplished. Although their presentation has since become a classical source of insight into the influences of initial imperfections, the type of analysis which they proposed has never been widely used as a practical working method. In general, the designer and analyst have insufficient information to select suitable values for unevenness factors or other quantitative measures of imperfection magnitudes. As a result, one usually resorts to the catch-all knock-down factor of equation (2-1). This factor is generally recognized to be a function of the ratio (R/t). Various sources have proposed different relationships in this regard. The differences usually arise out of the chosen statistical criteria and/or out of the particular test data selected as the empirical basis. One of the most popular relationships proposed to date is the lower-bound criterion of Seide, et al. [2] which is schematically depicted in Figure 1. Note that this criterion may be formulated as follows:

$$\Gamma = 1 - 0.901 \left(1 - e^{-\phi}\right) \tag{2-2}$$

$$\phi = \frac{1}{16} \sqrt{\frac{R}{t}} \tag{2-3}$$

---

Numbers in brackets [ ] in the text denote references listed in SECTION 4.

Incidentally, this same relationship is incorporated into the OPTION 1 analysis of Volume II [3] for the buckling of curved isotropic skin panels. Furthermore, it is pointed out that this particular criterion is recommended in reference 4.

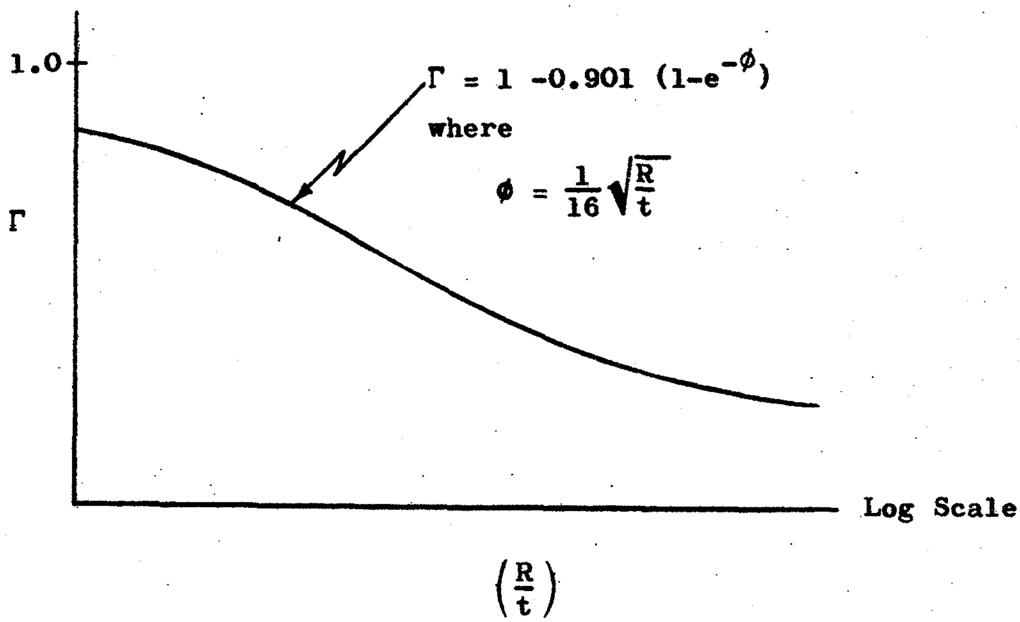



Figure 1 - Semi-Logarithmic Plot of $\Gamma$ vs R/t For Isotropic Cylinders Under Axial Compression

It is desired that empirical means, such as that given in Figure 1, also be provided for the design of eccentrically stiffened cylinders. One of the major obstacles to the achievement of this goal is the lack of sufficient stiffened cylinder test data for a thorough empirical determination. Faced with this deficiency, one therefore finds it necessary to employ the data from isotropic cylinders in conjunction with an effective thickness concept. For example, the curve of Figure 1 might be applied to stiffened cylinders if the variable (R/t) is replaced by an appropriate $(R/t_{eff})$ ratio. The crux of the problem then reduces to the choice of a suitable method for determination of the effective thickness $t_{eff}$. Toward this end, note that for a monocoque shell the local radius of gyration of the shell wall can be expressed as follows:

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{t^3}{12} \div t} = \frac{t}{\sqrt{12}} \tag{2-4}$$

This gives the following relationship:

$$t = \sqrt{12}\, \rho \tag{2-5}$$

It is easily recognized that equation (2-5) gives the monocoque wall thickness that will provide a given value for the local radius of gyration. This simple relationship is the basis for most of the effective thickness concepts used for relating stiffened shell imperfection influences to the behavior of monocoque cylinders. That is, it is usually assumed that equal sensitivity to imperfections results from equivalence of the local radii of gyration. However, this equivalence is rather difficult to establish since, for stiffened cylinders, the $\rho$ values generally are not the same in the longitudinal and circumferential directions. This requires the use of some type of averaging technique. The methods available for this purpose are those proposed by Peterson in reference 5 and by Almroth in reference 6. It should be noted that the former method is specified in the criterion of reference 4.

The effective thickness proposed by Peterson bases the desired equivalence on the geometric mean of the longitudinal and circumferential local radii of gyration ($\rho_x$ and $\rho_y$, respectively) for the stiffened cylinder. This leads to the following expression:

$$t_{eff} = \sqrt{12} \left[\rho_x \rho_y\right]^{1/2} \tag{2-6}$$

This may be rewritten in terms of the elastic constants as

$$t_{eff} = \sqrt{12} \left[\sqrt{D_{11}A_{11}} \sqrt{D_{22}A_{22}}\right]^{1/2} \tag{2-7}$$

or

$$t_{eff} = \left[\left(144\right)\left(D_{11}A_{11}\right)\left(D_{22}A_{22}\right)\right]^{1/4} \tag{2-8}$$

The effective thickness proposed by Almroth [6] bases the desired equivalence on a stiffened shell radius of gyration which involves the arithmetic mean of the longitudinal and circumferential flexural stiffnesses. This leads to the expression

$$t_{eff} = \sqrt{12} \sqrt{\left(\frac{D_{11} + D_{22}}{2}\right) A_{11}} \tag{2-9}$$

or

$$t_{eff} = \sqrt{6 \left(D_{11} + D_{22}\right) A_{11}} \tag{2-10}$$

The $A_{ij}$ and $D_{ij}$ values of equations (2-7) through (2-10) may be computed from the appropriate formulas of Volumes III [7] and IV [8]. However, when computing $D_{22}$, one should always set $\nu = 0$ .

Equations (2-8) and (2-10) both reduce to $t_{eff} = t$ in the special case of an isotropic cylinder. In addition, for stiffened cylinders having $A_{11} = A_{22}$ and $D_{11} = D_{22}$, equations (2-8) and (2-10) will give identical results. However, for all other geometries the two approaches will yield differing effective thicknesses. A choice between the two methods must be rather arbitrary in view of the lack of rigor in both. Therefore, in the interest of conservatism, and to conform with the criteria of reference 4, it is recommended here that equations (2-6) through (2-8) be used in applying the analysis methods of this multiple-volume report.

## 2.2 DESIGN CURVES

The simplicity of the lower-bound criterion of Seide, et al. [2], along with its prior selection for the applications of reference 4, makes it an attractive basis for the knock-down procedures of this volume. However, one should note that other criteria have been published [6, 9] which tend toward greater conservatism in all but the extremely low (R/t) ranges. Furthermore, references 6 and 9 both provide several design curves for differing degrees of statistical probability whereas the Seide [2] formulation is simply a lower-bound to a selected array of test data. Note that 50% probability curves can be quite useful for the comparison of predictions against actual experimental values whereas 90% and 99% probability curves provide reasonable design levels. In addition, unlike the other criteria cited here, the curves of reference 9 recognize a statistical influence from the parameter (L/R).

In view of the several factors noted in the preceding paragraph, the following recommendations are made at this time:

(a) For axially compressed circular cylinders having shallow stiffeners (such as in waffle configurations), use the knock-down curves given in Figure 2, which embody the 99% probability results of reference 9. These same curves may be used for monocoque orthotropic cylinders as well as for isotropic cylinders.

(b) For heavily stiffened circular cylinders subjected to axial compression, use the knock-down curve given in Figure 5 which presents the criterion of Seide, et al. [2].

For informational purposes, these axial compression design curves are supplemented by the 90% and 50% probability curves shown in Figures 3 and 4 which embody additional results of reference 9.

All of the foregoing discussions of this volume have been solely concerned with the case of axial compression. However, it should be observed that both references 2 and 9 also provide design curves to be used for cases of pure bending. Under bending, only a small portion of the cylinder's circumference experiences stress levels which initiate the buckling process. Because of the consequent reduced probability for peak stresses to coincide with the location of an imperfection, it is to be expected that the knock-down factors for pure bending will be somewhat higher than the corresponding factors for axial load. However, the bending criterion proposed by Seide, et al. [2] seems to give unreasonably high values for the ratio ($\Gamma_{Bend} / \Gamma_{Axial}$), particularly in the high (R/t) range. It is customarily expected that this ratio of knock-down factors should be in the neighborhood of 1.30. The curves by Seide, et al. [2] can give much higher values and a study of this discrepancy casts suspicion on the $\Gamma_{Bend}$ values. Therefore, the following recommendations are made at this time:

(a) For circular cylinders having shallow stiffeners (such as in waffle configurations), and subjected to pure bending, use the knock-down curves given in Figure 6 which embody the results of reference 9. These same curves may be used for monocoque orthotropic cylinders as well as for isotropic cylinders.

(b) For heavily stiffened circular cylinders subjected to pure bending, use the knock-down curve given in Figure 5 together with a correction factor as follows:

$$\Gamma_{Bend} = \Gamma_{Figure\ 5} \left( \frac{\Gamma_{Bend}}{\Gamma_{Axial}} \right)_{\substack{Figures\ 6 \\ and\ 2}} \tag{2-11}$$

For informational purpose, the pure bending design curves are supplemented by the 90% and 50% probability curves shown in Figures 7 and 8 which embody additional results of reference 9.

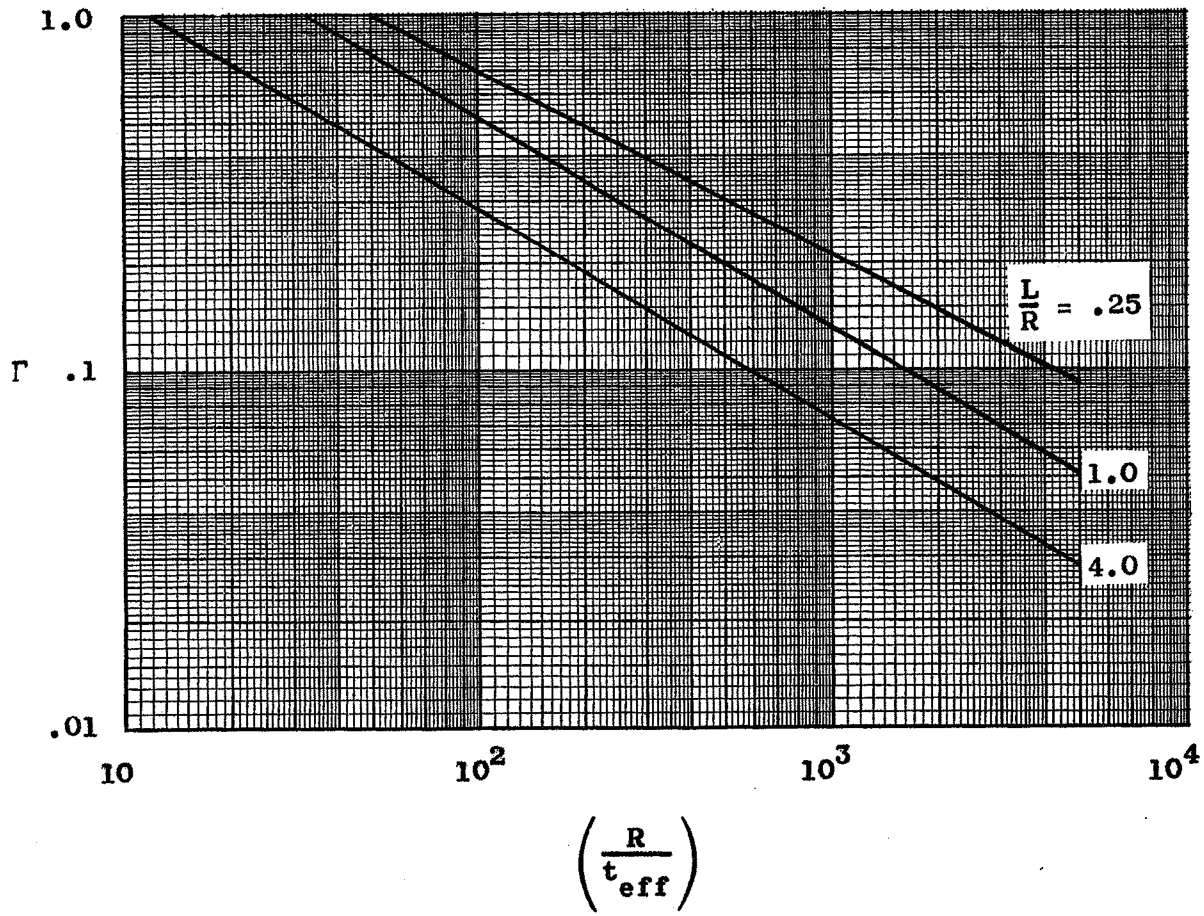



Figure 2 - Knock-Down Factors for Lightly Stiffened Cylinders Under Axial Compression (99% probability criterion of reference 9)

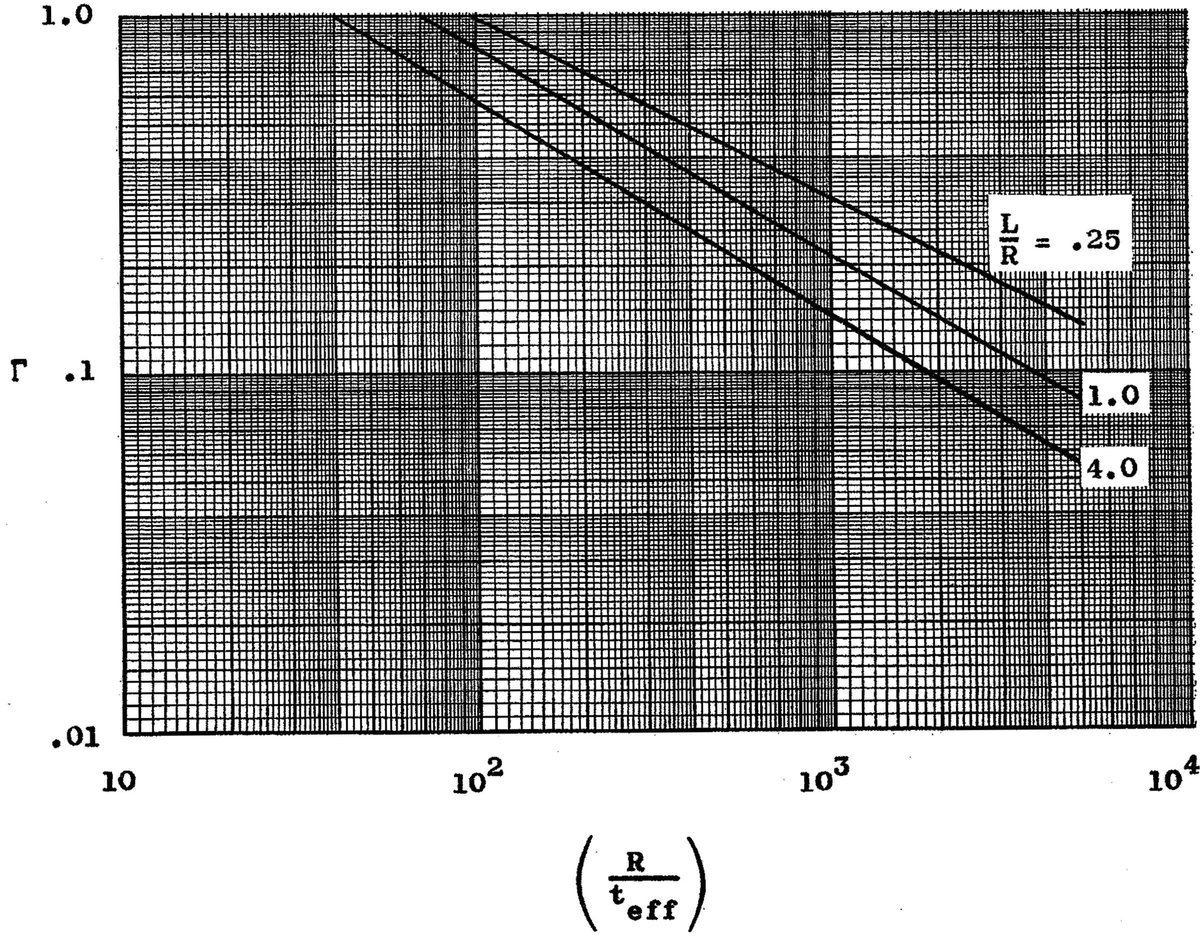



Figure 3 - Knock-Down Factors for Lightly Stiffened Cylinders Under Axial Compression (90% probability criterion of reference 9)

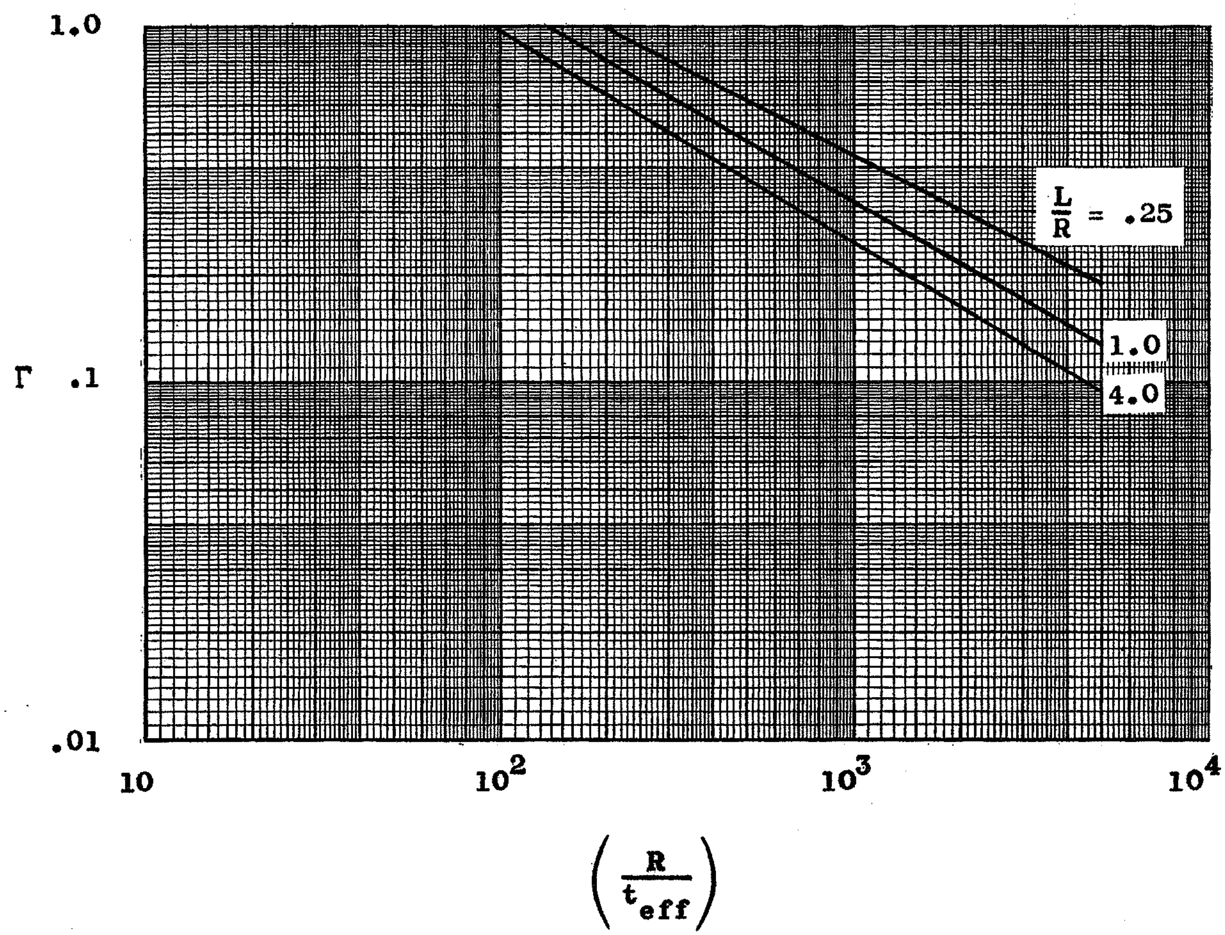



**Figure 4 - Knock-Down Factors for Lightly Stiffened Cylinders Under Axial Compression (50% probability criterion of reference 9)**

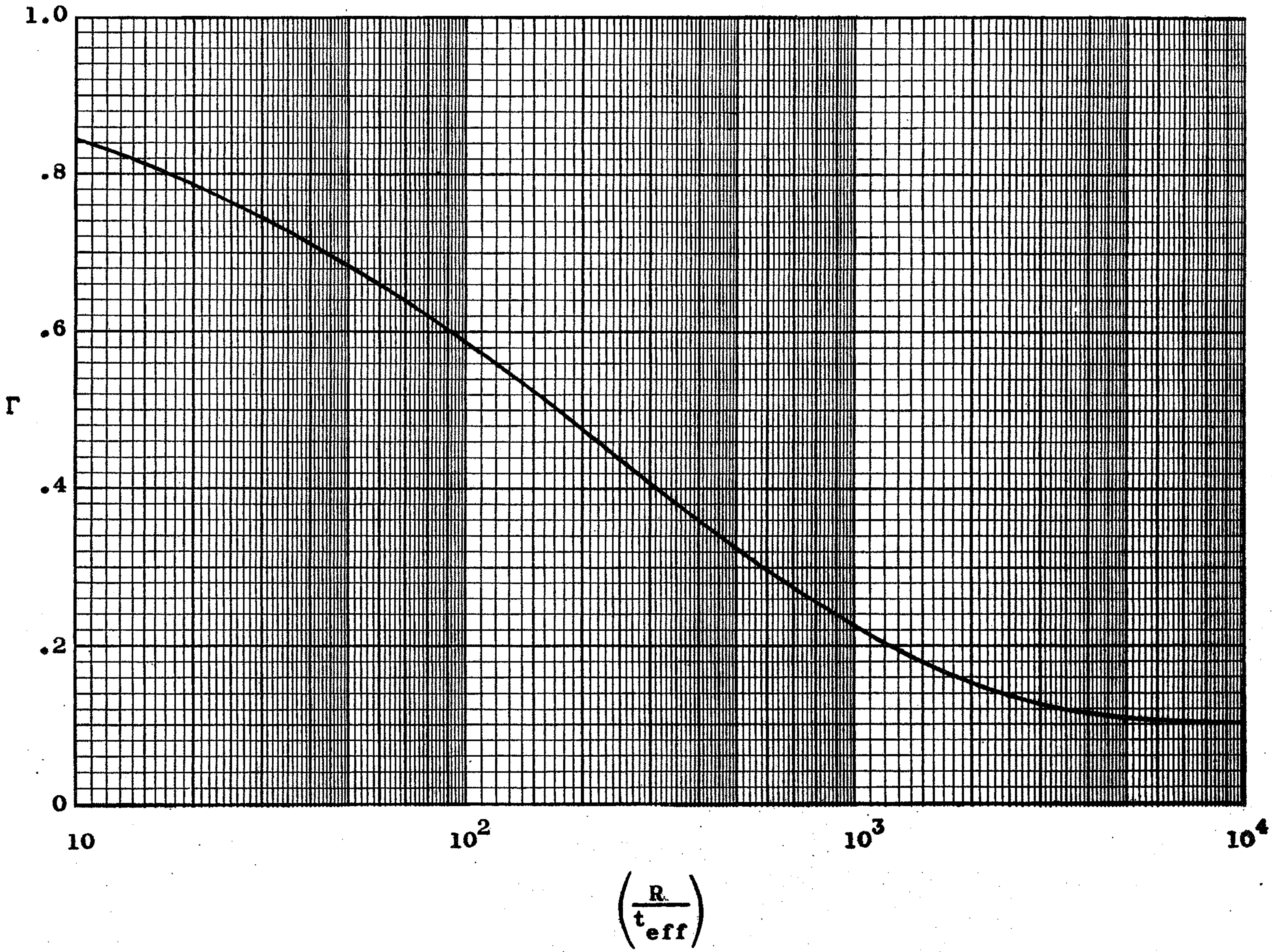



**Figure 5 - Knock-Down Factors for Heavily Stiffened Cylinders Under Axial Compression (Lower-bound criterion of reference 2)**

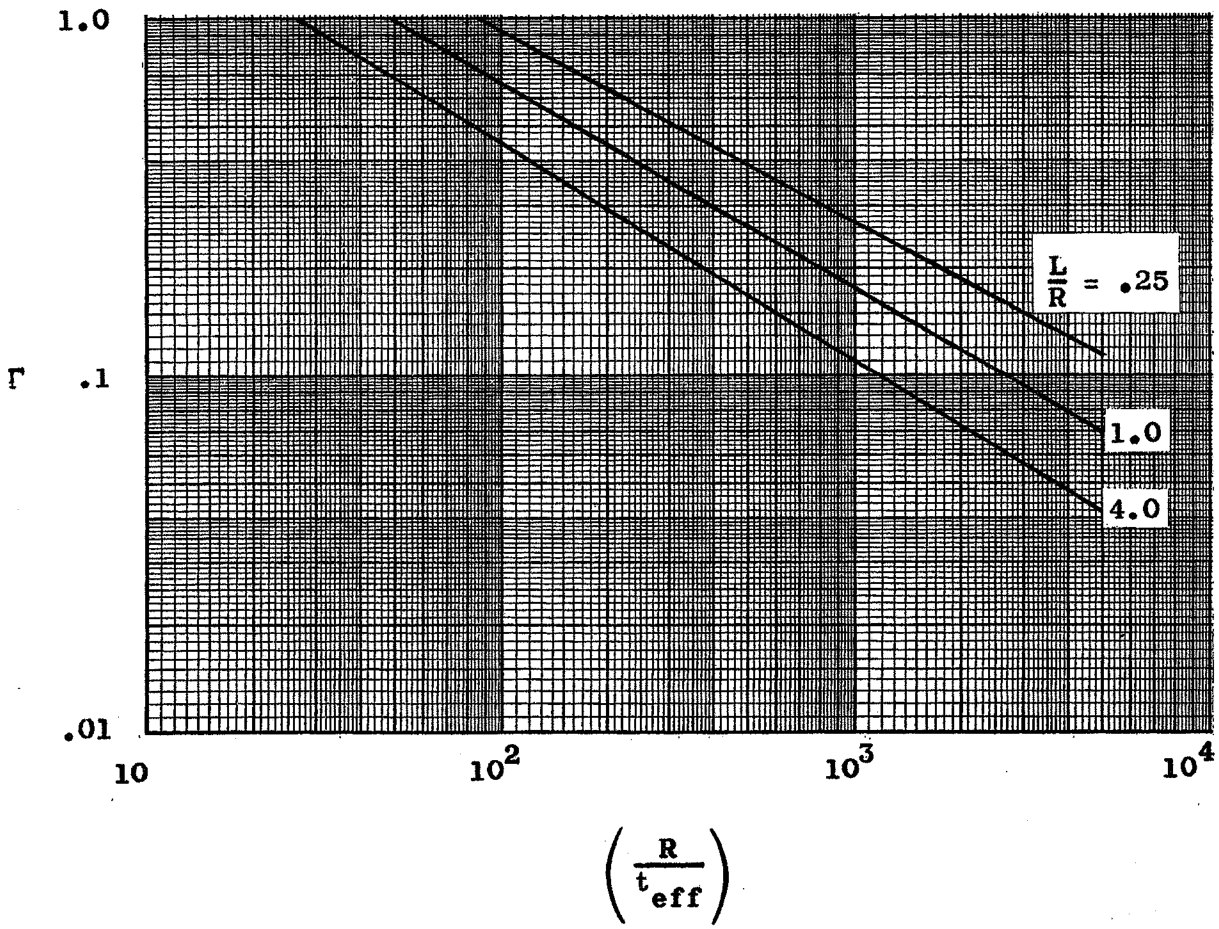
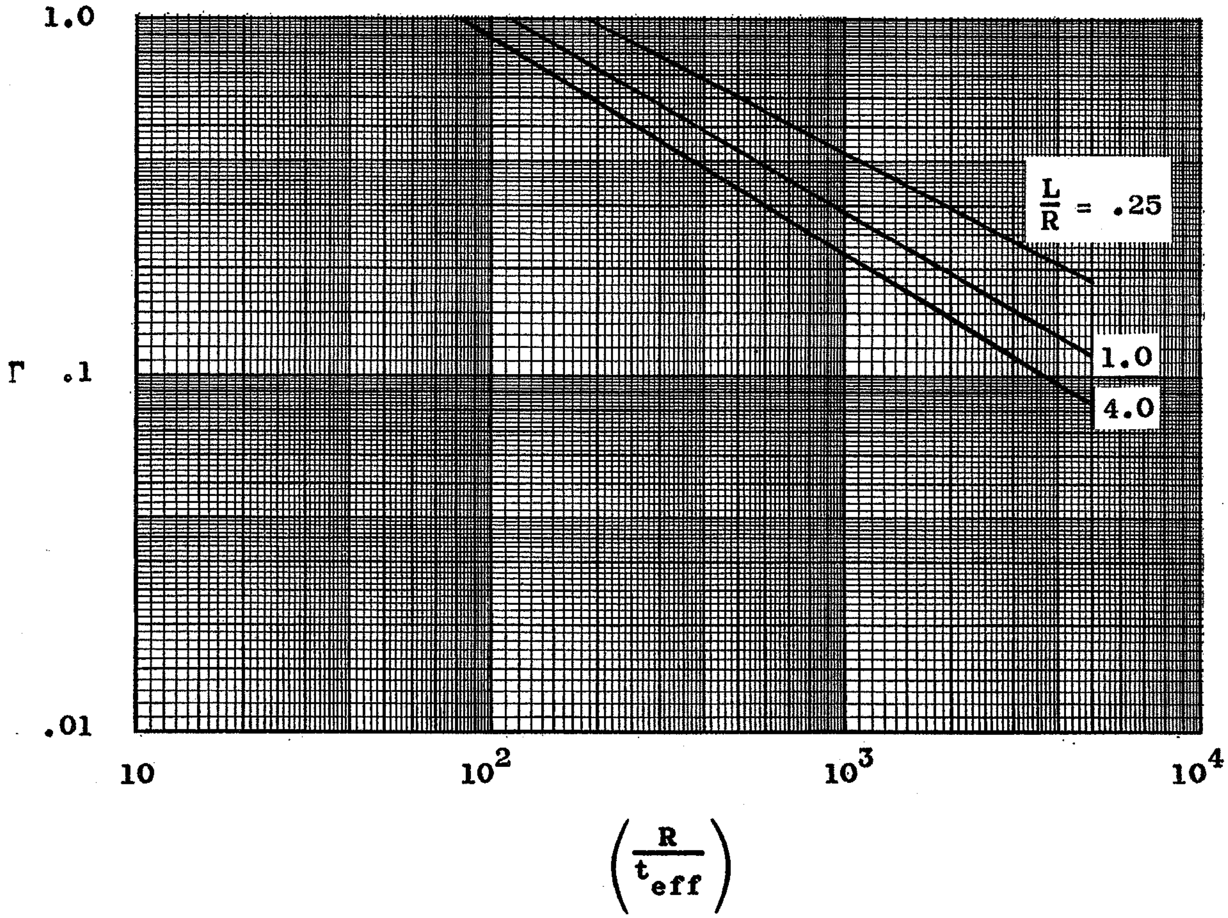
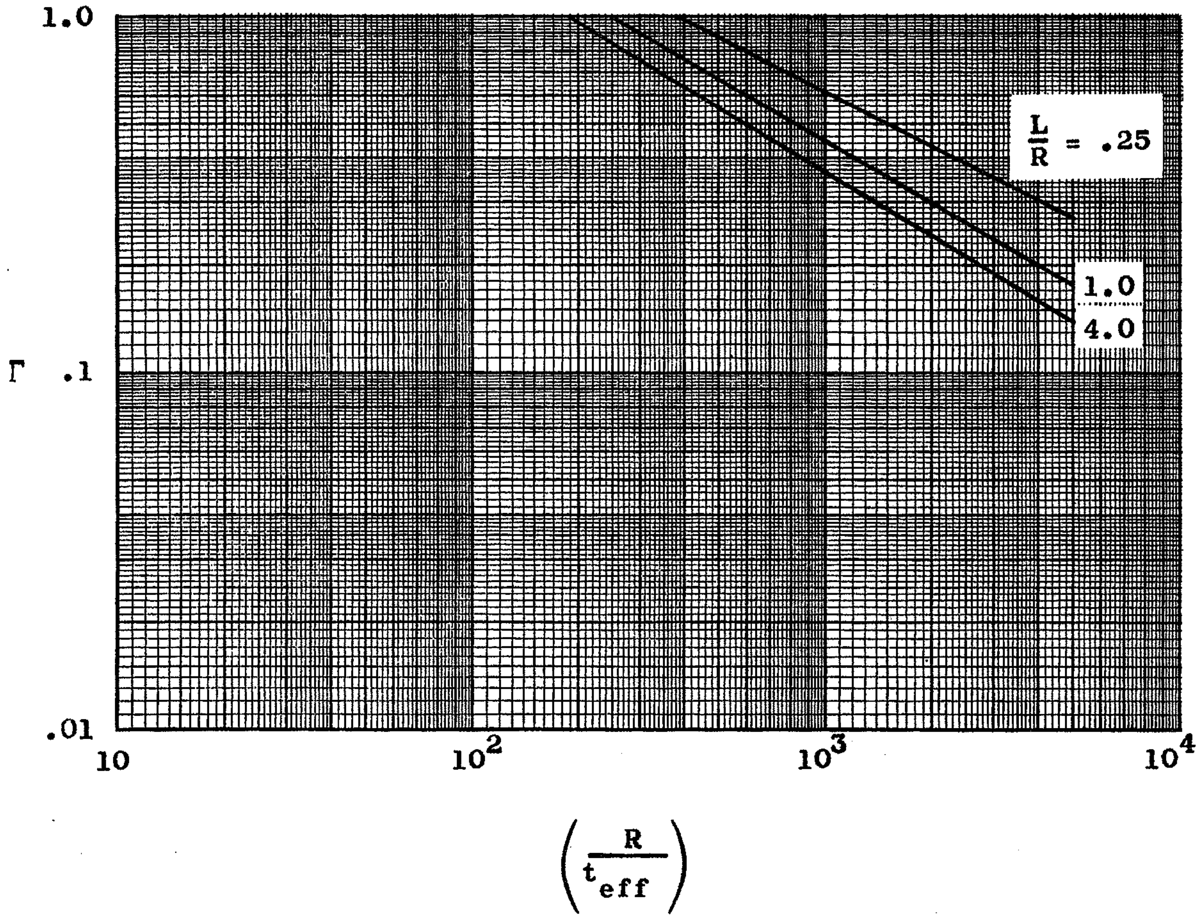



**Figure 6 - Knock-Down Factors for Lightly Stiffened Cylinders Under Pure Bending (99% probability criterion of reference 9)**




<u>**Figure 7 - Knock-Down Factors for Lightly Stiffened Cylinders Under Pure Bending (90% probability criterion of reference 9)**</u>




<u>Figure 8 - Knock-Down Factors for Lightly Stiffened Cylinders Under Pure Bending (50% probability criterion of reference 9)</u>

# SECTION 3

# APPLICATION

## 3.1 GENERAL

Once having used the appropriate $(R/t_{eff})$ ratio to find a numerical value for $\Gamma$, one must then decide upon the means by which this correction should be injected into the stiffened cylinder analysis. To shed some light on this question, reference is made to a presentation by Almroth [6]. Assuming that the shape of the postbuckling equilibrium path is of primary importance to this issue, Almroth suggests that this shape be reflected in the way $\Gamma$ is introduced. In particular, he proposes that the postbuckling curve be used to establish a correctable fraction of the total theoretical strength. This concept is illustrated in the nondimensional load-displacement curves shown in Figures 9 and 10. As implied by these figures, the postbuckling behavior of stiffened cylinders can be quite different from that of unstiffened cylinders. The exact nature of the postbuckling curve

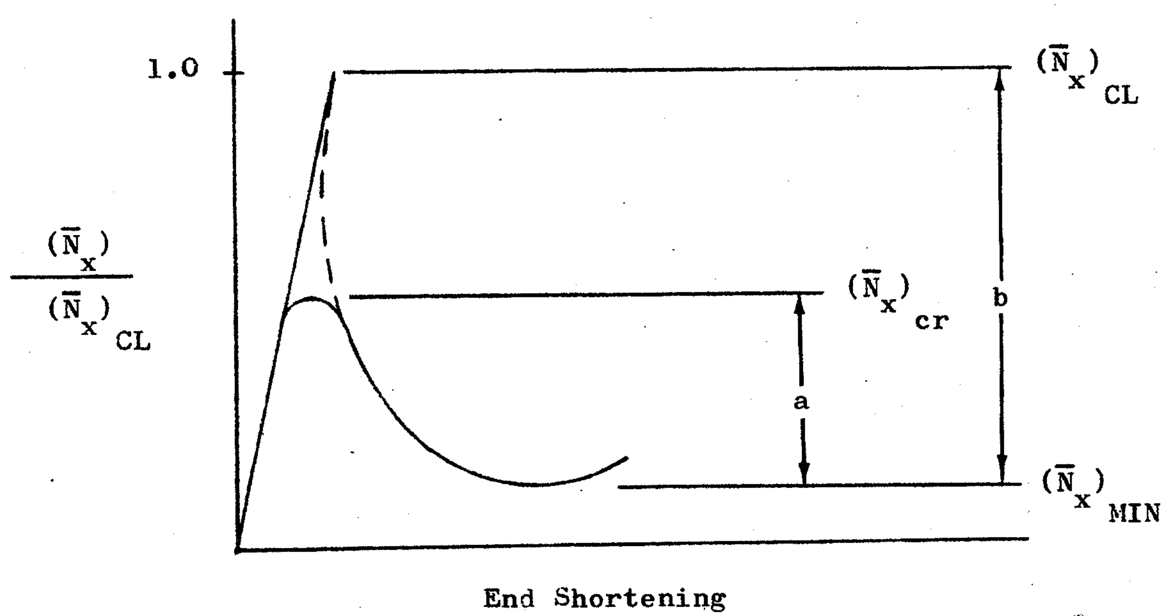
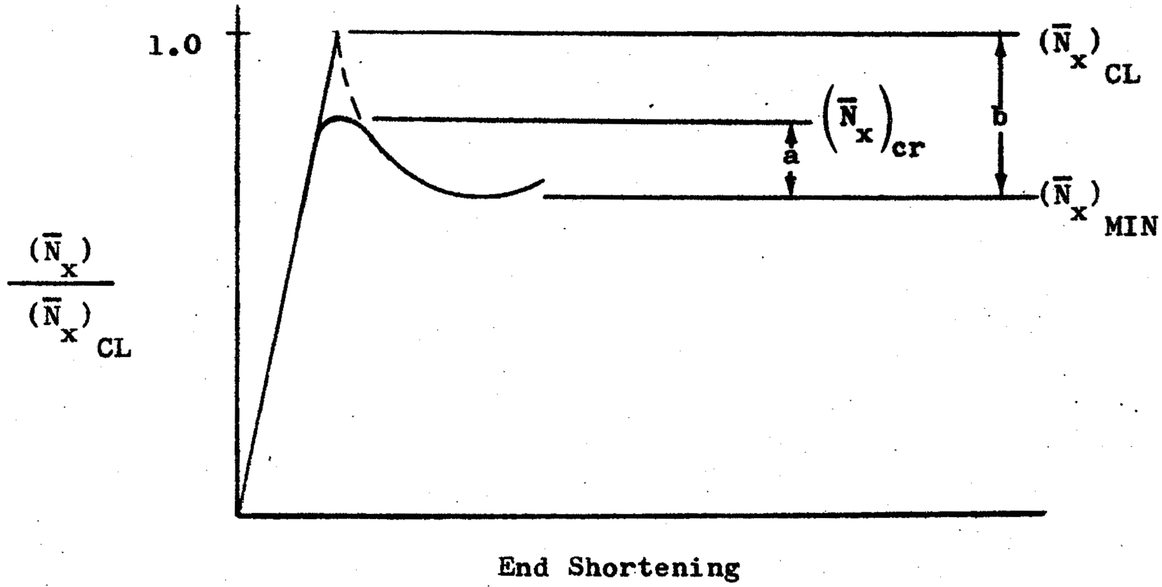



Figure 9 - Load-Displacement Curve for Example Monocoque Cylinder




<u>Figure 10 - Load-Displacement Curve for Example Orthotropic Cylinder</u>

for any particular configuration will depend upon the type and degree of stiffening. Almroth's proposal is that all cylinders which have the same $(R/t_{eff})$ values might be assumed to have identical values for the ratio

$$\frac{\left(\bar{N}_x\right)_{cr} - \left(\bar{N}_x\right)_{MIN}}{\left(\bar{N}_x\right)_{CL} - \left(\bar{N}_x\right)_{MIN}} = \frac{a}{b} \tag{3-1}$$

By assuming the minimum postbuckling strength for an isotropic cylinder to be zero, one may rewrite equation (3-1) as follows:

$$\left(\bar{N}_x\right)_{cr} = \left(\bar{N}_x\right)_{MIN} + \Gamma\left[\left(\bar{N}_x\right)_{CL} - \left(\bar{N}_x\right)_{MIN}\right] \tag{3-2}$$

All of the $\bar{N}$ 's in this equation refer to the stiffened configuration. The formula presented by Almroth in reference 6 is slightly more complicated because of his assumption that the minimum postbuckling load for isotropic cylinders is 0.12 times the classical theoretical value. This choice was based on an earlier Almroth paper [10]. However, Hoff, et al. [11] have subsequently concluded that the theoretical $(\bar{N}_x)_{MIN}$ value for isotropic cylinders can be reduced below Almroth's value by increasing the number of terms used in the trigonometric series for the radial displacements. Hoff, et al. interpret their own results to imply "that the minimal value of the compressive load, under which a large-displacement equilibrium is possible, is zero".

To properly apply equation (3-2), one must perform a postbuckling analysis of the stiffened cylinder to establish its applicable $(\bar{N}_x)_{MIN}$ value. However, this is considered to be beyond the scope and degree of complexity intended for the methods of this report. Therefore, as an engineering approximation, it will be assumed here that $(\bar{N}_x)_{MIN}$ for a stiffened cylinder can be taken equal to the wide-column strength $(\bar{N}_x)_{wc}$ chosen as follows:

Whenever the applicable local slenderness ratio satisfies the condition

$$\left(\frac{L}{\rho_x}\right) \geq \left(\sqrt{2C_F}\right)(\pi)\left(\sqrt{\frac{E}{\sigma_{cc}}}\right) \tag{3-3}$$

then use

$$\left(\bar{N}_x\right)_{wc} = \left[\frac{C_F\pi^2E_{tan}}{\left(\frac{L}{\rho_x}\right)^2}\right](\bar{t}_x) \tag{3-4}$$

Whenever the applicable local slenderness ratio satisfies the condition

$$\left(\frac{L}{\rho_x}\right) < \left(\sqrt{2C_F}\right)\left(\pi\right)\left(\sqrt{\frac{E}{\sigma_{cc}}}\right) \tag{3-5}$$

then use the lower of the two values obtained from equations (3-4) and (3-6).

$$\left(\bar{N}_x\right)_{wc} = \left[\sigma_{cc} - \frac{\sigma_{cc}^{\,2}\left(\frac{L}{\rho_x}\right)^2}{4C_F\pi^2E}\right]\left(\bar{t}_x\right) \tag{3-6}$$

These several equations are to be applied to the wide column obtained by unfolding the composite circular wall into a flat configuration, while retaining equivalent boundary constraint. Equation (3-2) may then be re-written as follows:

$$\left(\bar{N}_x\right)_{cr} = \left(\bar{N}_x\right)_{wc} + \Gamma\left[\left(\bar{N}_x\right)_{CL} - \left(\bar{N}_x\right)_{wc}\right] \tag{3-7}$$

For the analysis of general instability (see GLOSSARY, Volume I [12]), the entire overall length is used in equations (3-3) through (3-6) regardless of the ring spacing a . In such cases the $(\bar{N}_x)_{wc}$ value will usually be small and its influence in equation (3-7) will not be very significant. However, for cylinders which are stiffened only in the longitudinal direction, the situation will usually be quite different. Although these structures still employ the overall length L in the wide column computation, the $(\bar{N}_x)_{wc}$ component will usually comprise a major part of the total compressive strength. The remaining possibility of interest to this report is the situation encountered in the analysis of so-called panel instability (see GLOSSARY, Volume I [12]) in cylinders that incorporate both longitudinal and circumferential stiffening. In this case, one is concerned with the behavior of longitudinally stiffened sections that lie between rings and the wide-column component is calculated by inserting $L = a$ into equations (3-3) through (3-6). Here again, the usual result is that $(\overline{N}_x)_{wc}$ comprises a major portion of the total resistance to instability.

## 3.2 SUMMARY PROCEDURE

To expedite ready application to practical problems, the recommendations of this volume are summarized in the following concise procedure:

(a) First obtain classical theoretical values for the critical longitudinal compressive running load $(\overline{N}_x)_{CL}$, acting at the centroid of the effective skin-stringer combination. The methods of Volumes III [7] and IV [8] are used for this purpose.

(b) Then compute the equivalent thickness $t_{eff}$ from the equations

$$t_{eff} = \sqrt{12}\left[\rho_x \rho_y\right]^{1/2} \tag{3-8}$$

$$= \left[(144)(D_{11}A_{11})(D_{22}A_{22})\right]^{1/4} \tag{3-9}$$

(c) Compute the knock-down factor $\Gamma$ as follows:

| Lightly Stiffened Cylinders | Heavily Stiffened Cylinders |
|---|---|
| Axial Compression - Use Figure 2. | Axial Compression - Use Figure 5. |
| Pure Bending - Use Figure 6. | Pure Bending - Use the equation: $\Gamma_{Bend} = (\Gamma)_{Figure\ 5}\left(\frac{\Gamma_{Bend}}{\Gamma_{Axial}}\right)_{Figures\ 6\ and\ 2}$ (3-10 |

(d) For the analysis of cylinders having only longitudinal stiffening and for the analysis of general instability (see GLOSSARY, Volume I [12]) in cylinders having intermediate rings, compute the wide-column strength $(\bar{N}_x)_{wc}$ as follows:

Whenever

$$\left(\frac{L}{\rho_x}\right) \geq \left(\sqrt{2C_F}\right)\left(\pi\right)\left(\sqrt{\frac{E}{\sigma_{cc}}}\right) \tag{3-11}$$

then,

$$(\bar{N}_x)_{wc} = \left[\frac{C_F \pi^2 E_{tan}}{\left(\frac{L}{\rho_x}\right)^2}\right] (\bar{t}_x) \tag{3-12}$$

Whenever

$$\left(\frac{L}{\rho_x}\right) < \left(\sqrt{2C_F}\right)\left(\pi\right)\left(\sqrt{\frac{E}{\sigma_{cc}}}\right) \tag{3-13}$$

then use the lower of the two values obtained from equations (3-12) and (3-14).

$$(\bar{N}_x)_{wc} = \left[\sigma_{cc} - \frac{\sigma_{cc}^2 \left(\frac{L}{\rho_x}\right)^2}{4C_F \pi^2 E}\right] (\bar{t}_x) \tag{3-14}$$

It should be noted that the logic embodied in equations (3-11) through (3-14) is automatically executed by digital computer program 4196 [7] if the input N* value is set equal to zero. Hence, one can find $(\bar{N}_x)_{wc}$ by using that program or the critical stress curves (for N* = 0) furnished in Volume III [7].

(e) For the analysis of panel instability (see GLOSSARY, Volume I [12]) in cylinders having both stringers and intermediate rings, simply make the substitution $L = a$ in equations (3-11) through (3-14), where $a$ is the ring spacing.

(f) Then compute the critical longitudinal compressive running load $(\bar{N}_x)_{cr}$, acting at the centroid of the effective skin-stringer combination. The following equation is used for this purpose:

$$(\bar{N}_x)_{cr} = (\bar{N}_x)_{wc} + \Gamma \left[ (\bar{N}_x)_{CL} - (\bar{N}_x)_{wc} \right] \qquad (3\text{-}15)$$

# SECTION 4

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