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ERROR ANALYSIS FOR IMP D&E

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DECEMBER 1964

GODDARD SPACE FLIGHT CENTER
GREENBELT, MD.

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SUMMARY

This report presents a study of the propagation of errors during the transfer and lunar parking orbit phases of the IMP D&E Missions.

Two error analyses consider the effects of injection errors separately and also the combined effects of injection and tracking system errors. For the latter study, tracking of the satellite in its transfer orbit is simulated by a tracking network composed of four range and range rate trackers at Rosman, Santiago, Carnarvon, and Tananarive. For both studies the RMS errors in spacecraft position and velocity are propagated up to lunar distance. Tracking of the satellite for the second study is simulated for five-minute periods at the end of three-hour periods of no tracking.

A third error analysis study considers tracking the satellite in its lunar parking orbit. This study combines the effects of insertion and tracking system errors. Tracking is simulated by three Range and Range Rate Systems at Carnarvon, Rosman, and Tananarive for five-minute periods at the end of nine-hour periods of no tracking. The study considers tracking the satellite for 110 hours, or approximately 4.6 days.

INTRODUCTION

Three separate but related error analysis studies are made for the IMP D&E Missions. The purpose of these studies is to evaluate the effects of such error sources as injection errors, tracking system errors, and tracking system location errors. Neglected are errors in the equation of motion and bias errors in the tracking system measurements. The latter error sources will be considered at a later time.

Two of the error analysis studies consider the effects of injection errors separately and injection errors combined with tracking system location errors for the transfer trajectory. For lack of information about the errors at the time of insertion of the satellite into a lunar parking orbit, very pessimistic values for these errors are used in the third study.

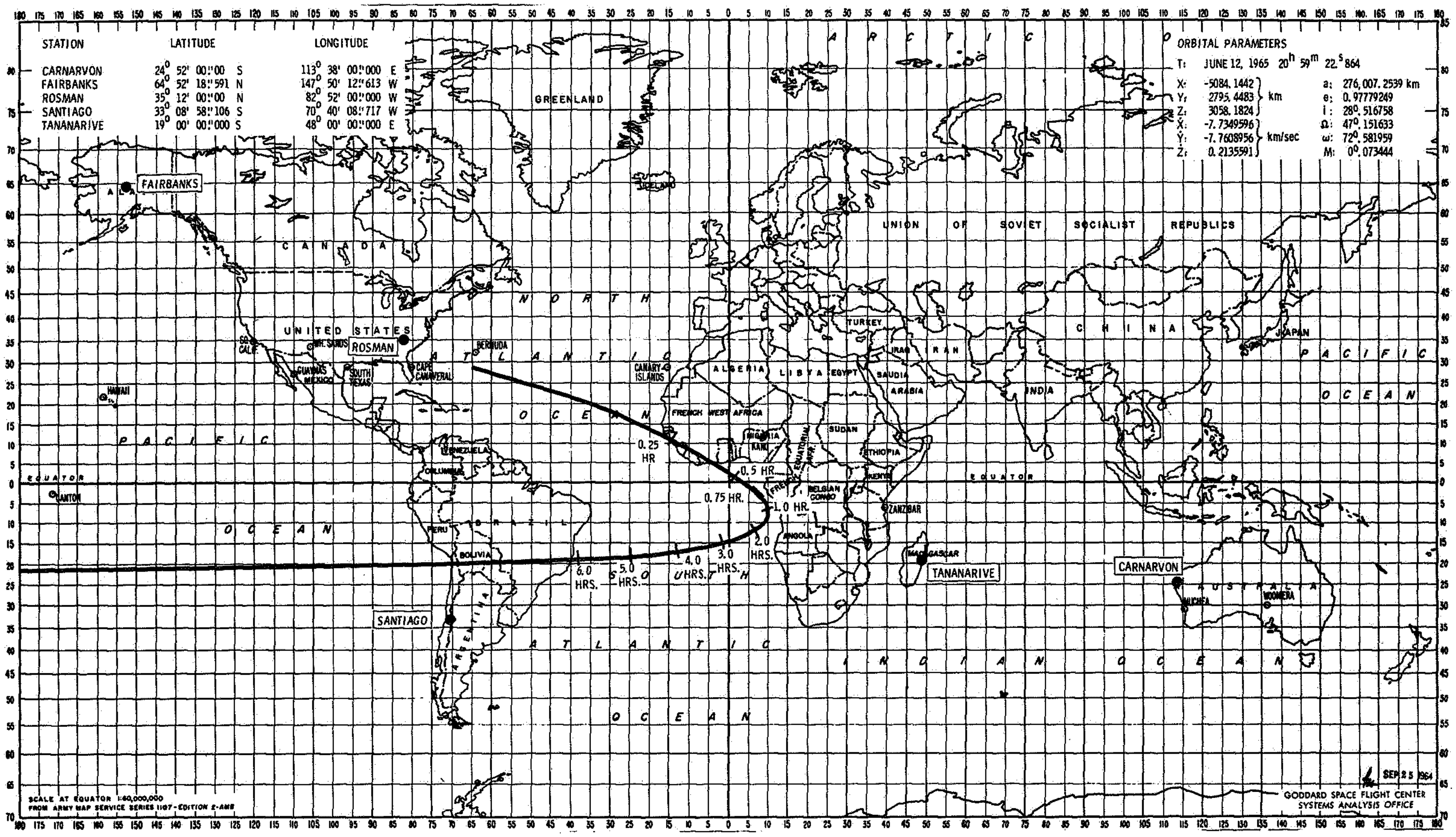
The Tracking Network for IMP D&E Missions

Each of the IMP D&E vehicles will carry a range and range rate transponder whose frequency response is to a carrier frequency of 136 mc/sec. The Goddard Space Flight Center R & $\dot{R}$ system operating at that carrier frequency is to be used for tracking the satellites.

A tracking network of range and range rate systems to be used for IMP D&E Missions is composed of the following tracking stations:

Station Name	Latitude	Longitude
Carnarvon	24° 52' 00″.0 S	113° 38' 00″.0 E
Fairbanks	64° 52' 18″.591 N	147° 50' 12″.613 W
Rosman	35° 12' 00″.0 N	82° 52' 00″.0 W
Santiago	33° 08' 58″.106 S	70° 40' 08″.717 W
Tananarive	19° 00' 00″.0 S	48° 00' 00″.0 E

For the transfer trajectory chosen for this study, which is shown in Figure 1, the tracker at Fairbanks does not "see" the spacecraft, see Fig. 2. Therefore, the error analysis study considers a tracking network composed of the four remaining stations. This network provides continuous tracking coverage of the satellite from shortly after injection up to lunar distance. Simulataneous tracking is possible by at least two trackers over all but a few hours of the entire transfer trajectory which can also be seen from Figure 2.



STATION	LATITUDE	LONGITUDE
CARNARVON	24° 52' 00."00 S	113° 38' 00."000 E
FAIRBANKS	64° 52' 18."591 N	147° 50' 12."613 W
ROSMAN	35° 12' 00."00 N	82° 52' 00."000 W
SANTIAGO	33° 08' 58."106 S	70° 40' 08."717 W
TANANARIVE	19° 00' 00."000 S	48° 00' 00."000 E

ORBITAL PARAMETERS

T: JUNE 12, 1965 $20^h\ 59^m\ 22.^s864$

X:	-5084.1442	km	a:	276,007.2539 km
Y:	2795.4483	km	e:	0.97779249
Z:	3058.1824	km	i:	28°.516758
$\dot{X}$:	-7.7349596	km/sec	Ω:	47°.151633
$\dot{Y}$:	-7.7608956	km/sec	ω:	72°.581959
$\dot{Z}$:	0.2135591	km/sec	M:	0°.073444

SCALE AT EQUATOR 1:60,000,000
FROM ARMY MAP SERVICE SERIES 1107-EDITION 2-AMS

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Figure 1-Sub-Orbital Plot of Trajectory for IMP D&E

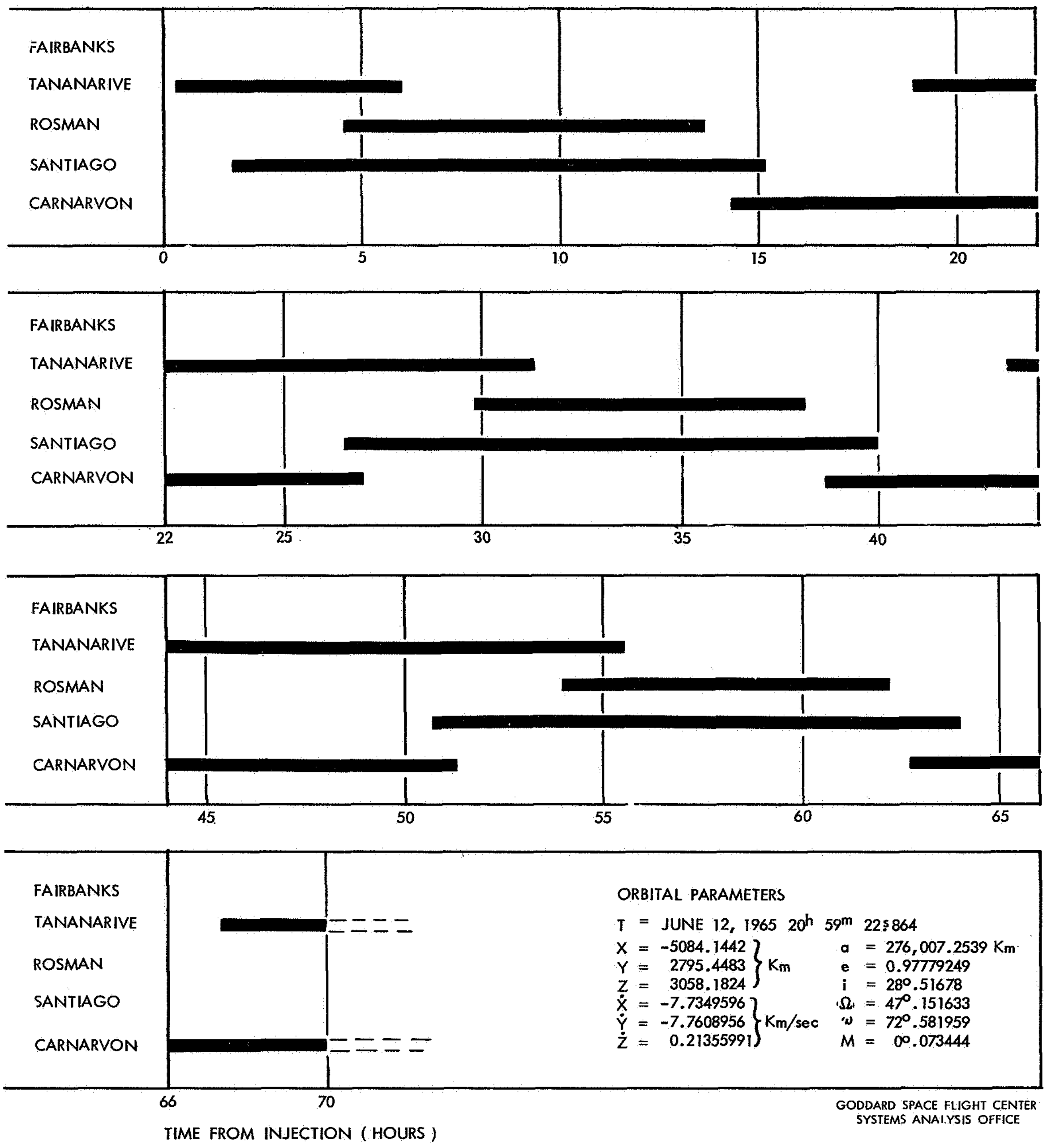

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Figure 2–Station Contact Times for IMP D&E to Lunar Distance ($\epsilon \geq 5^\circ$)

Description of The Tracking Mode

IMP D&E vehicles will be tracked from the ground by the Range and Range Rate System using a maximum of five minutes (at a sampling rate of 1 measurement/sec) at the end of an extended period of no tracking. Figure 3 illustrates the tracking geometry which is considered. The solid segments represent the five minute tracking periods separated by a tracking gap time T_G, which is the period during which no tracking occurs.

Projection of Injection Errors

Injection errors used in this report correspond to the errors at burnout of the third stage of the launch vehicle. For convenience, these are restated as follows:

Burnout Velocity Error = ± 16.2 m/s

Burnout Altitude Error = ± 1.9 km.

Burnout Flt. Path Angle Error = $\pm 0\overset{\circ}{.}1$

Burnout Azimuth Angle Error = $\pm 0\overset{\circ}{.}1$

In order to fully describe the errors at injection of the state vector, errors in right ascension and declination were assumed to be $\pm 0\overset{\circ}{.}017$. Each of these errors corresponds to a positional error of ± 1.9 km.

The results of the analysis show that the injection errors projected to lunar distance result in RMS errors in satellite position and velocity of ± 27,000 km and ± 160 meters/sec, respectively. Growth of the RMS errors in the state vector along the transfer trajectory resulting from injection errors is illustrated by curve A, Figures 4 and 5, and by Figure 6. In the latter illustration the 95% probability error ellipsoid in satellite position is shown as it grows along the transfer trajectory in a three dimensional presentation in order to aid in a better physical understanding of error propagation. The orientation of this ellipsoid indicates that the maximum error is laterally directed to the trajectory.

From a parametric analysis (curves B, C, & D, Figures 4 & 5) it was determined that the major influence on the growth of the RMS error in the state vector is the large error in the injection velocity magnitude. The results obtained from the analysis were verified by a separate study using the Monte Carlo technique.

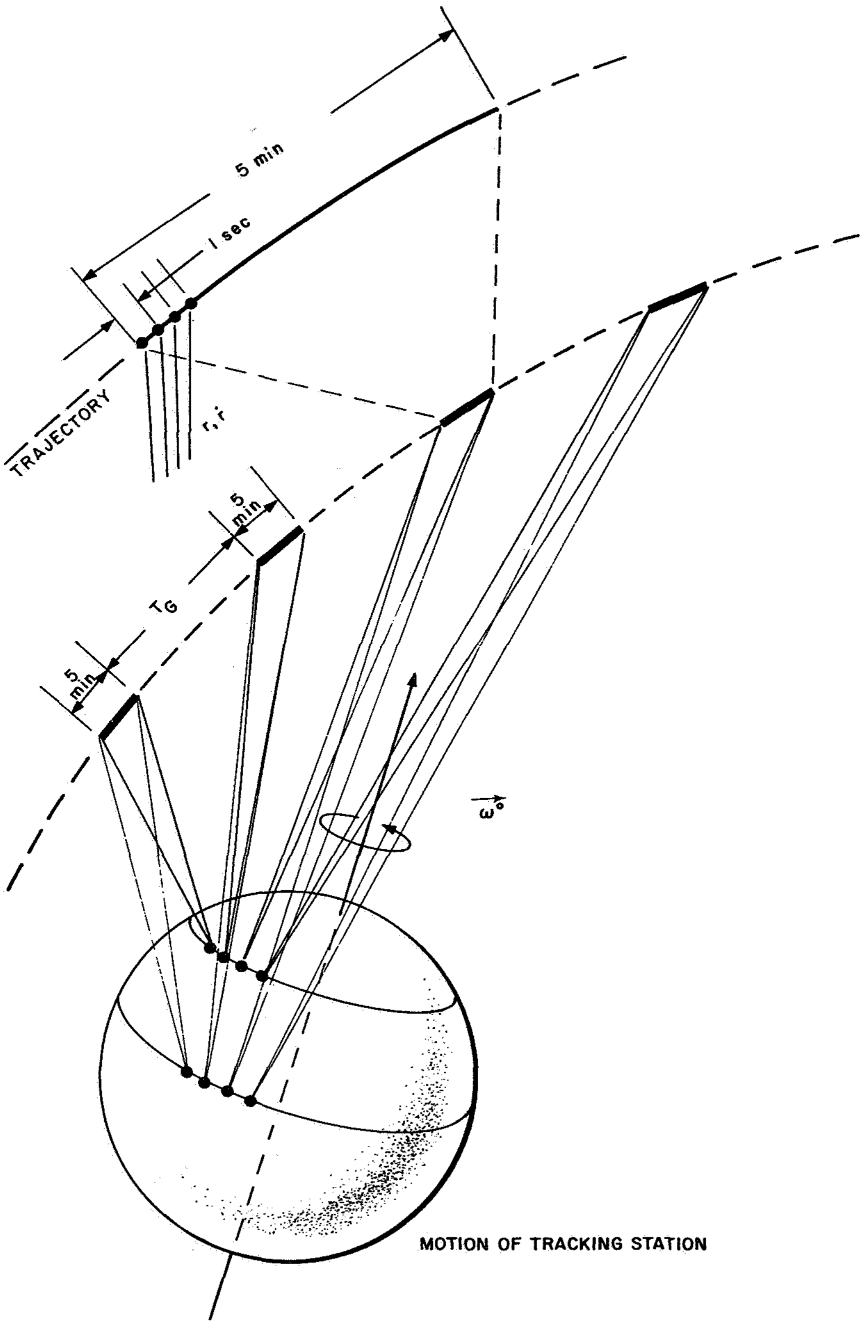
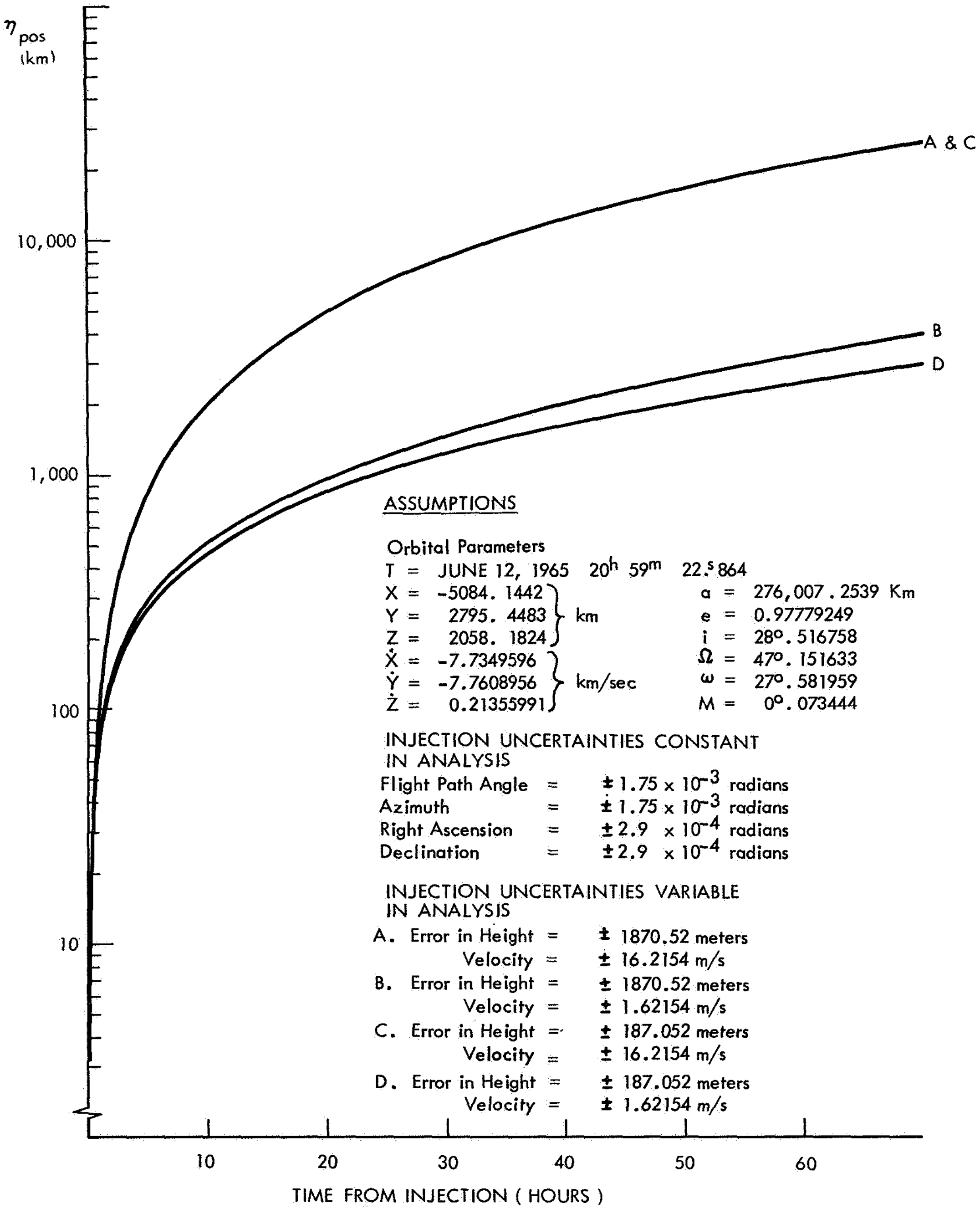


Figure 3–Tracking Geometry



ASSUMPTIONS

Orbital Parameters
T = JUNE 12, 1965 20^h 59^m $22.^s864$
X = -5084.1442 km a = 276,007.2539 Km
Y = 2795.4483 km e = 0.97779249
Z = 2058.1824 km i = 28°.516758
$\dot{X}$ = -7.7349596 km/sec Ω = 47°.151633
$\dot{Y}$ = -7.7608956 km/sec ω = 27°.581959
$\dot{Z}$ = 0.21355991 km/sec M = 0°.073444

INJECTION UNCERTAINTIES CONSTANT IN ANALYSIS
Flight Path Angle = $\pm 1.75 \times 10^{-3}$ radians
Azimuth = $\pm 1.75 \times 10^{-3}$ radians
Right Ascension = $\pm 2.9 \times 10^{-4}$ radians
Declination = $\pm 2.9 \times 10^{-4}$ radians

INJECTION UNCERTAINTIES VARIABLE IN ANALYSIS
A. Error in Height = ± 1870.52 meters
Velocity = ± 16.2154 m/s
B. Error in Height = ± 1870.52 meters
Velocity = ± 1.62154 m/s
C. Error in Height = ± 187.052 meters
Velocity = ± 16.2154 m/s
D. Error in Height = ± 187.052 meters
Velocity = ± 1.62154 m/s

Figure 4-Projection of Injection Errors to Lunar Distance with Varying Injection Errors in Distance and Velocity for IMP D&E

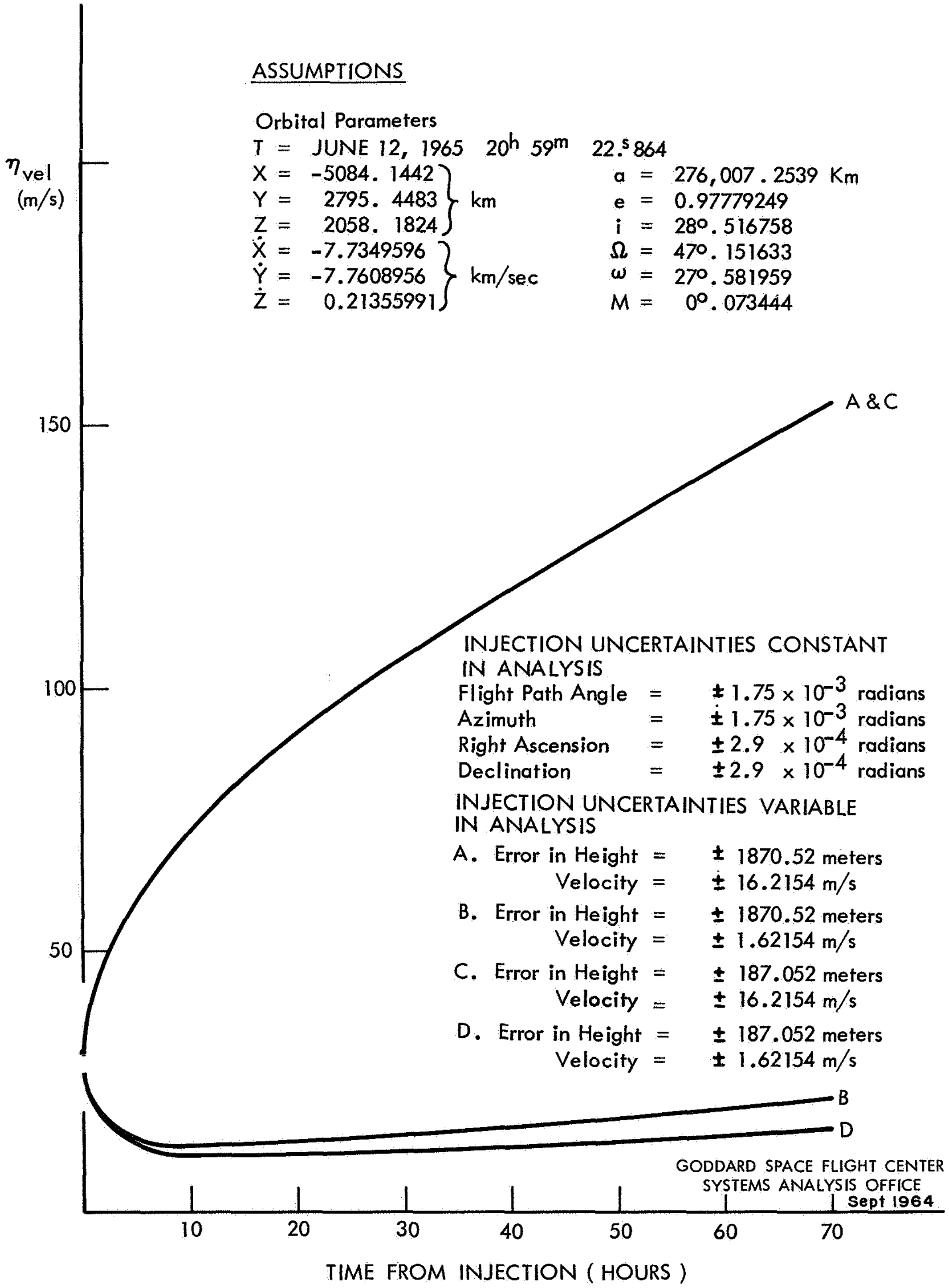


Figure 5-Projection of Injection Errors to Lunar Distance with Varying Injection Errors in Distance and Velocity IMP D&E

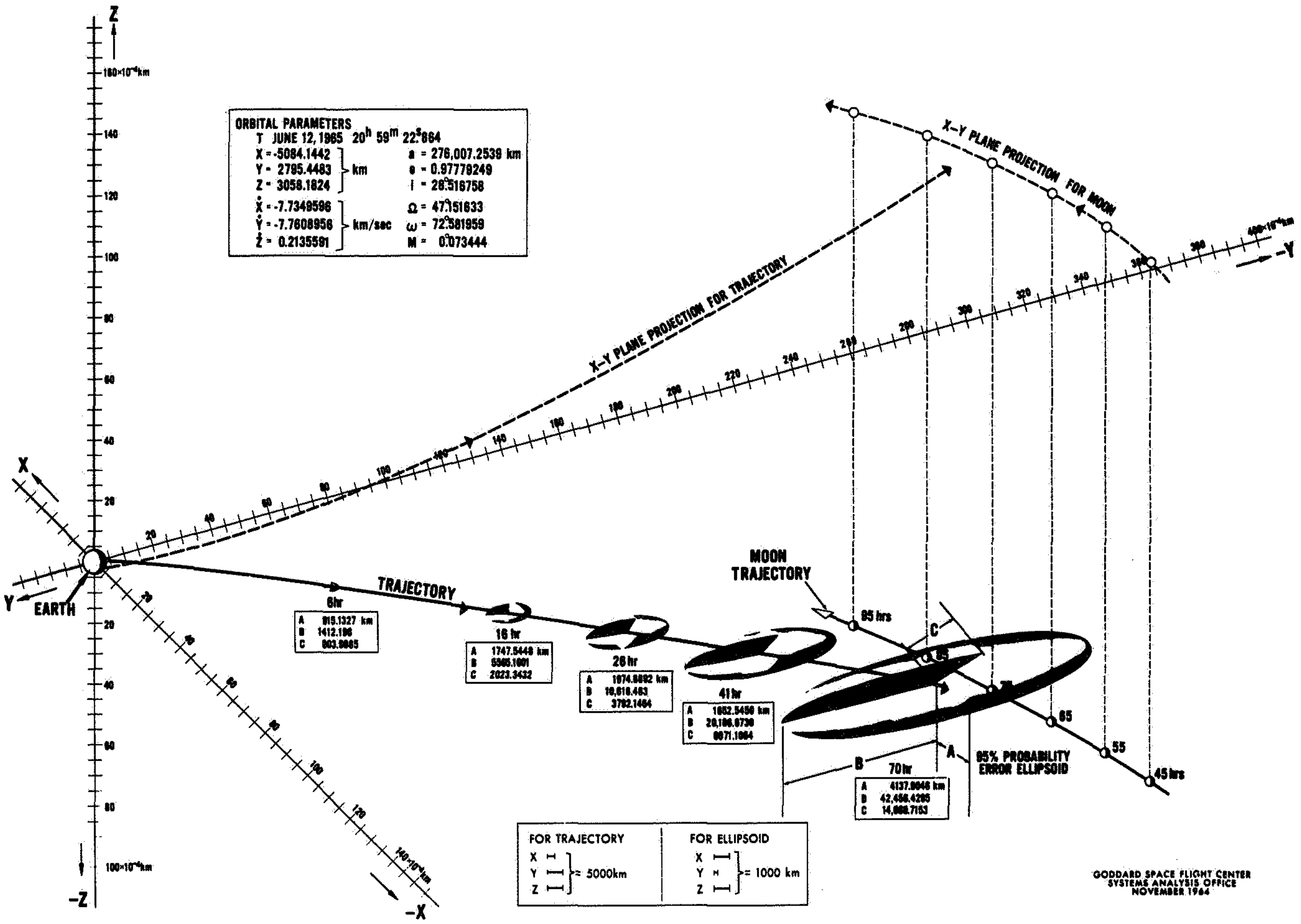


Figure 6-Position Error Ellipsoids for IMP D&E Lunar Transfer

Error Analyses During Tracking Mode

A. Transfer Trajectory

Tracking of the transfer trajectory is simulated by this error analysis study. A network of four range and range rate trackers (Rosman, Carnarvon, Tananarive, and Santiago) track the spacecraft from shortly after injection up to lunar distance. Injection errors, tracking system errors, and tracking system location errors, as well as a bias error due to the uncertainty in the velocity of light, are considered. Neglected are errors in the equations of motion and bias errors in the measurements. In Figures 7 and 8, the RMS errors in position and velocity are propagated along the transfer trajectory. Tracking is simulated in the manner described above for three-hour periods of no tracking. Longer periods of no tracking may be considered, based on results of this error analysis study.

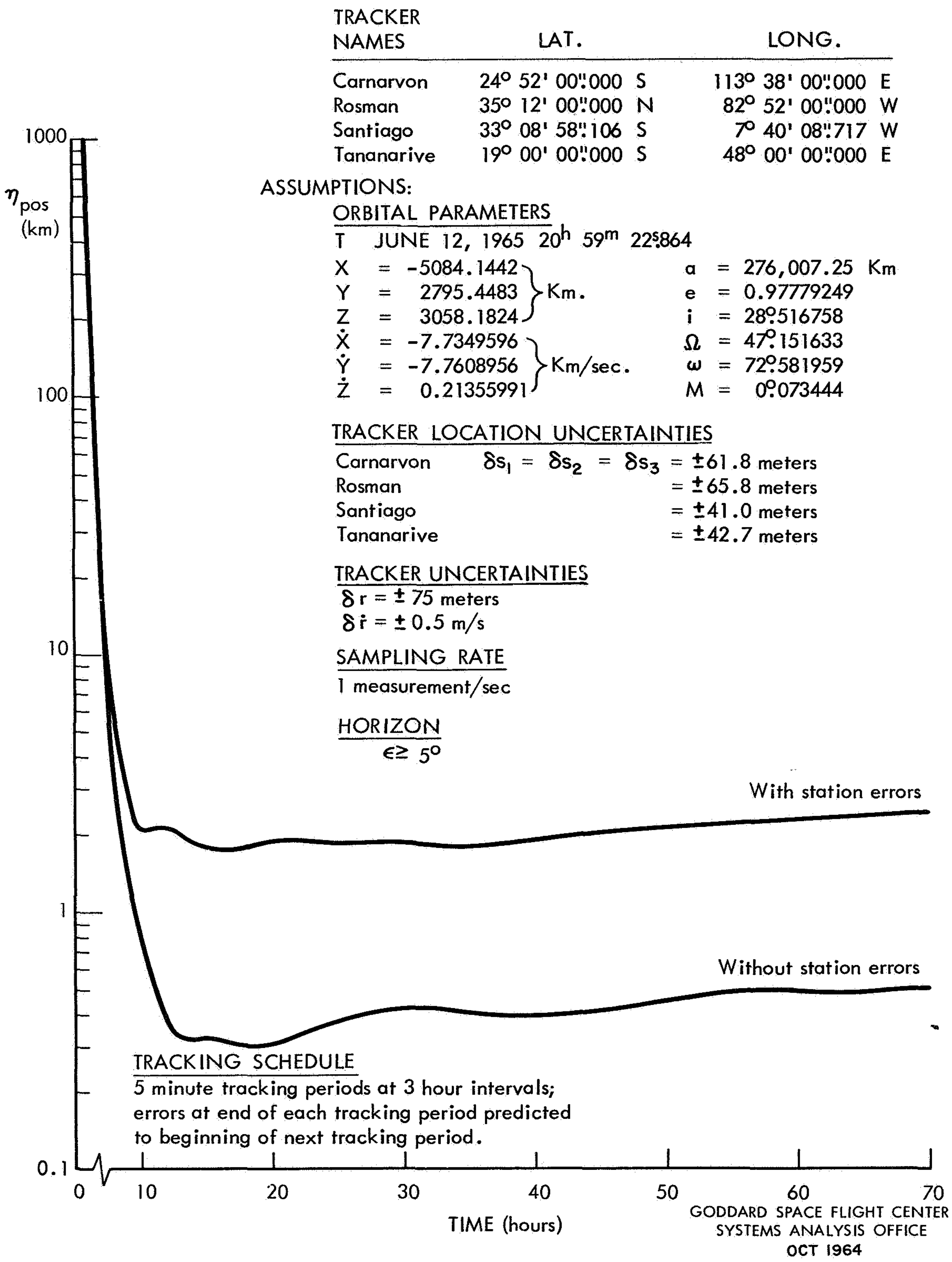
Because errors in the equations of motion, such as the combined effects of uncertainties in the earth's gravitational constant, radiation pressure and solar winds etc., are not considered, the RMS error in satellite velocity (see Figure 8, velocity curve including station errors) becomes smaller than is realistically possible. When the effects of uncertainties in the equation of motion are included, the RMS error in satellite velocity at lunar distance should be in the order of ± 4 cm/sec.

From tracking data obtained with the Range and Range Rate System the initial transfer trajectory is corrected. The corrected transfer trajectory, commonly referred to as the "updated transfer trajectory," has smaller RMS errors associated with its state vector than the initial transfer trajectory. Results presented in Figures 7 and 8 therefore correspond only to the "updated transfer trajectory."

Since IMP D&E vehicles do not have mid-course correction capability and hence the initial transfer trajectory cannot be corrected (updated), the RMS errors in the state vector resulting from projecting the injection errors to lunar distance remain unchanged. Consequently, it is incorrect to assume that tracking the IMP satellite will decrease or perhaps completely offset the effects of the large injection errors. (See Figure 6).

B. Lunar Parking Orbit

Tracking of the IMP satellite in a lunar parking orbit is simulated for a tracking network consisting of range and range rate trackers located at Carnarvon, Rosman, and Tananarive. For this error analysis study, range and range rate measurements are made for five-minute periods at the end of nine-hour periods



TRACKER NAMES	LAT.	LONG.
Carnarvon	24° 52' 00".000 S	113° 38' 00".000 E
Rosman	35° 12' 00".000 N	82° 52' 00".000 W
Santiago	33° 08' 58".106 S	7° 40' 08".717 W
Tananarive	19° 00' 00".000 S	48° 00' 00".000 E

ASSUMPTIONS:

ORBITAL PARAMETERS

T JUNE 12, 1965 $20^h\ 59^m\ 22^s.864$

$X = -5084.1442$ Km.
$Y = 2795.4483$ Km.
$Z = 3058.1824$ Km.
$\dot{X} = -7.7349596$ Km/sec.
$\dot{Y} = -7.7608956$ Km/sec.
$\dot{Z} = 0.21355991$ Km/sec.

$a = 276{,}007.25$ Km
$e = 0.97779249$
$i = 28^\circ.516758$
$\Omega = 47^\circ.151633$
$\omega = 72^\circ.581959$
$M = 0^\circ.073444$

TRACKER LOCATION UNCERTAINTIES

Carnarvon $\delta s_1 = \delta s_2 = \delta s_3 = \pm 61.8$ meters
Rosman $= \pm 65.8$ meters
Santiago $= \pm 41.0$ meters
Tananarive $= \pm 42.7$ meters

TRACKER UNCERTAINTIES

$\delta r = \pm 75$ meters
$\delta \dot{r} = \pm 0.5$ m/s

SAMPLING RATE

1 measurement/sec

HORIZON

$\epsilon \geq 5^\circ$

TRACKING SCHEDULE

5 minute tracking periods at 3 hour intervals; errors at end of each tracking period predicted to beginning of next tracking period.

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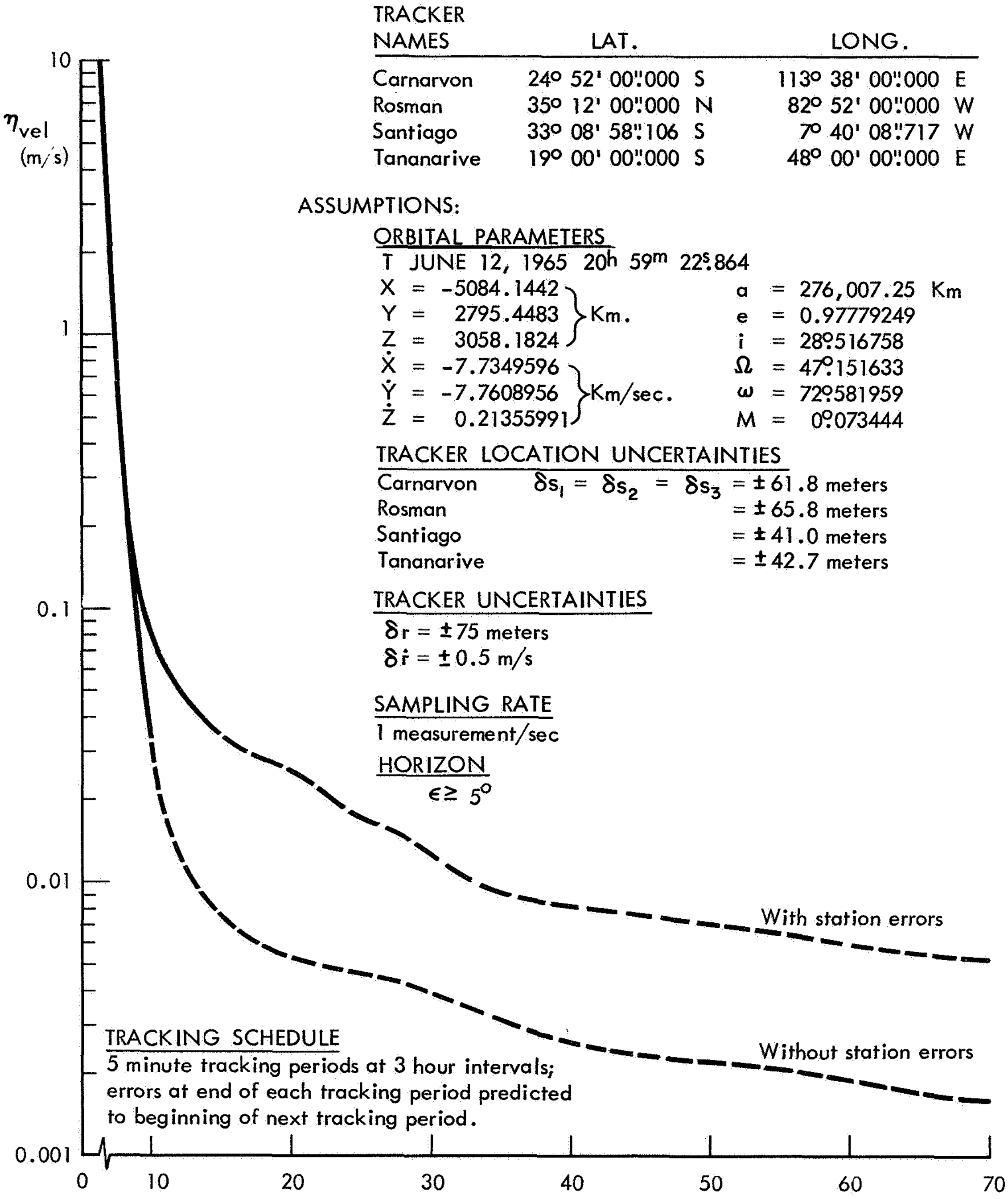
Figure 7-Propagation of Errors in Spacecraft Position IMP D&E



TRACKER NAMES	LAT.	LONG.
Carnarvon	24° 52' 00".000 S	113° 38' 00".000 E
Rosman	35° 12' 00".000 N	82° 52' 00".000 W
Santiago	33° 08' 58".106 S	7° 40' 08".717 W
Tananarive	19° 00' 00".000 S	48° 00' 00".000 E

ASSUMPTIONS:

ORBITAL PARAMETERS

T JUNE 12, 1965 $20^h\ 59^m\ 22^s.864$

$X = -5084.1442$, $Y = 2795.4483$, $Z = 3058.1824$ Km.

$\dot{X} = -7.7349596$, $\dot{Y} = -7.7608956$, $\dot{Z} = 0.21355991$ Km/sec.

$a = 276{,}007.25$ Km

$e = 0.97779249$

$i = 28^\circ.516758$

$\Omega = 47^\circ.151633$

$\omega = 72^\circ.581959$

$M = 0^\circ.073444$

TRACKER LOCATION UNCERTAINTIES

	$\delta s_1 = \delta s_2 = \delta s_3$
Carnarvon	= ± 61.8 meters
Rosman	= ± 65.8 meters
Santiago	= ± 41.0 meters
Tananarive	= ± 42.7 meters

TRACKER UNCERTAINTIES

$\delta r = \pm 75$ meters

$\delta \dot{r} = \pm 0.5$ m/s

SAMPLING RATE

1 measurement/sec

HORIZON

$\epsilon \geq 5^\circ$

TRACKING SCHEDULE

5 minute tracking periods at 3 hour intervals; errors at end of each tracking period predicted to beginning of next tracking period.

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Figure 8-Propagation of Errors in Spacdcraft Velocity IMP D&E

of no tracking. The analysis is terminated 110 hours, or 4.6 days, after the spacecraft is inserted into orbit. A similar analysis over longer time spans will be made at a later date.

Values for the insertion errors correspond to the errors at the end of burn of the IMP vehicle's fourth stage. These errors are ± 1.1 km and ± 14 meters/sec.

Other error sources included in this study are the same as those used for the transfer trajectory.

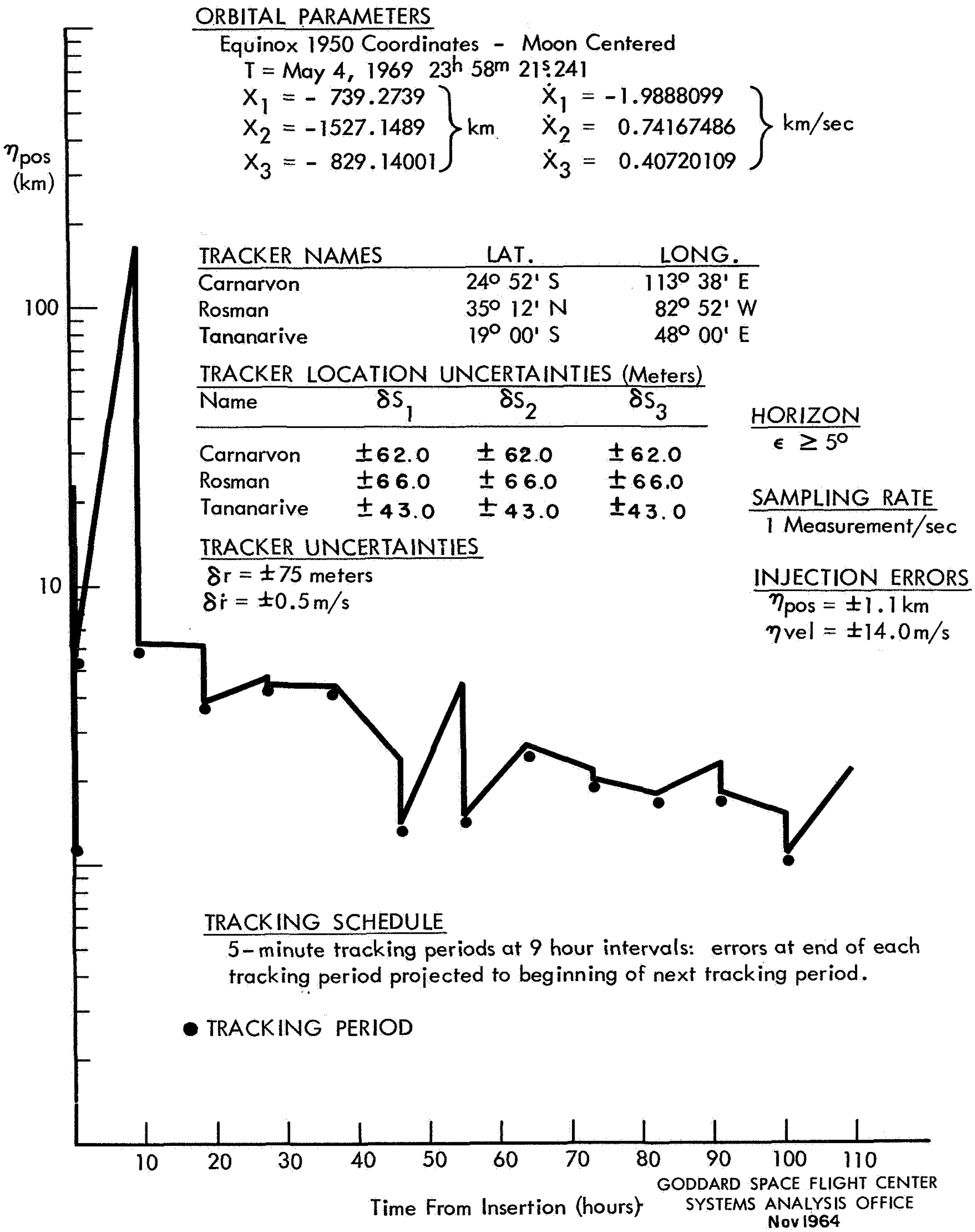
Characteristics of the lunar parking orbit chosen for the error study are as follows:

Height of Pericynthion:	1888.43 km
Height of Apocynthion:	16,995.87 km
Orbital Period:	24.2 Hours
Orbital Inclination to Lunar Equator:	151°.46

During each orbit the spacecraft is occulted by the moon for approximately four hours. At this time no tracking of the spacecraft will be possible.

Results from the error analysis study as given in Figures 9 and 10 show the RMS error in position and velocity decreasing at the end of every five-minute tracking period. Predictions of these errors over a nine-hour period of no tracking causes these errors to grow until tracking from the next five-minute tracking period again reduces the RMS errors in the state vector. After a sufficient number of five-minute tracking periods, the growth of the RMS errors in both position and velocity will be inhibited while being projected over the nine-hour periods of no tracking to the next five-minute tracking period.

By the end of the 4.6 day tracking period, the RMS errors in position and velocity are ± 2.2 km and ± 6 cm/sec respectively. If tracking were to be continued for an additional 3.4 days (a total tracking period of 8 days), the RMS errors in satellite position and velocity will then be ± 1.1 km and ± 6 cm/sec. The latter results show that the RMS errors in the state vector change very slowly, even though data from the additional tracking periods is used.



ORBITAL PARAMETERS

Equinox 1950 Coordinates - Moon Centered

T = May 4, 1969 $23^h\ 58^m\ 21^s.241$

$X_1 = -\ 739.2739$	km	$\dot{X}_1 = -1.9888099$
$X_2 = -1527.1489$	km	$\dot{X}_2 = 0.74167486$
$X_3 = -\ 829.14001$	km	$\dot{X}_3 = 0.40720109$

$\dot{X}_1, \dot{X}_2, \dot{X}_3$ in km/sec

TRACKER NAMES	LAT.	LONG.
Carnarvon	24° 52' S	113° 38' E
Rosman	35° 12' N	82° 52' W
Tananarive	19° 00' S	48° 00' E

TRACKER LOCATION UNCERTAINTIES (Meters)

Name	δs_1	δs_2	δs_3
Carnarvon	±62.0	± 62.0	±62.0
Rosman	±66.0	± 66.0	±66.0
Tananarive	± 43.0	± 43.0	±43.0

TRACKER UNCERTAINTIES

$\delta r = \pm 75$ meters

$\delta \dot{r} = \pm 0.5$ m/s

HORIZON

$\epsilon \geq 5°$

SAMPLING RATE

1 Measurement/sec

INJECTION ERRORS

$\eta_{pos} = \pm 1.1$ km

$\eta_{vel} = \pm 14.0$ m/s

TRACKING SCHEDULE

5-minute tracking periods at 9 hour intervals: errors at end of each tracking period projected to beginning of next tracking period.

● TRACKING PERIOD

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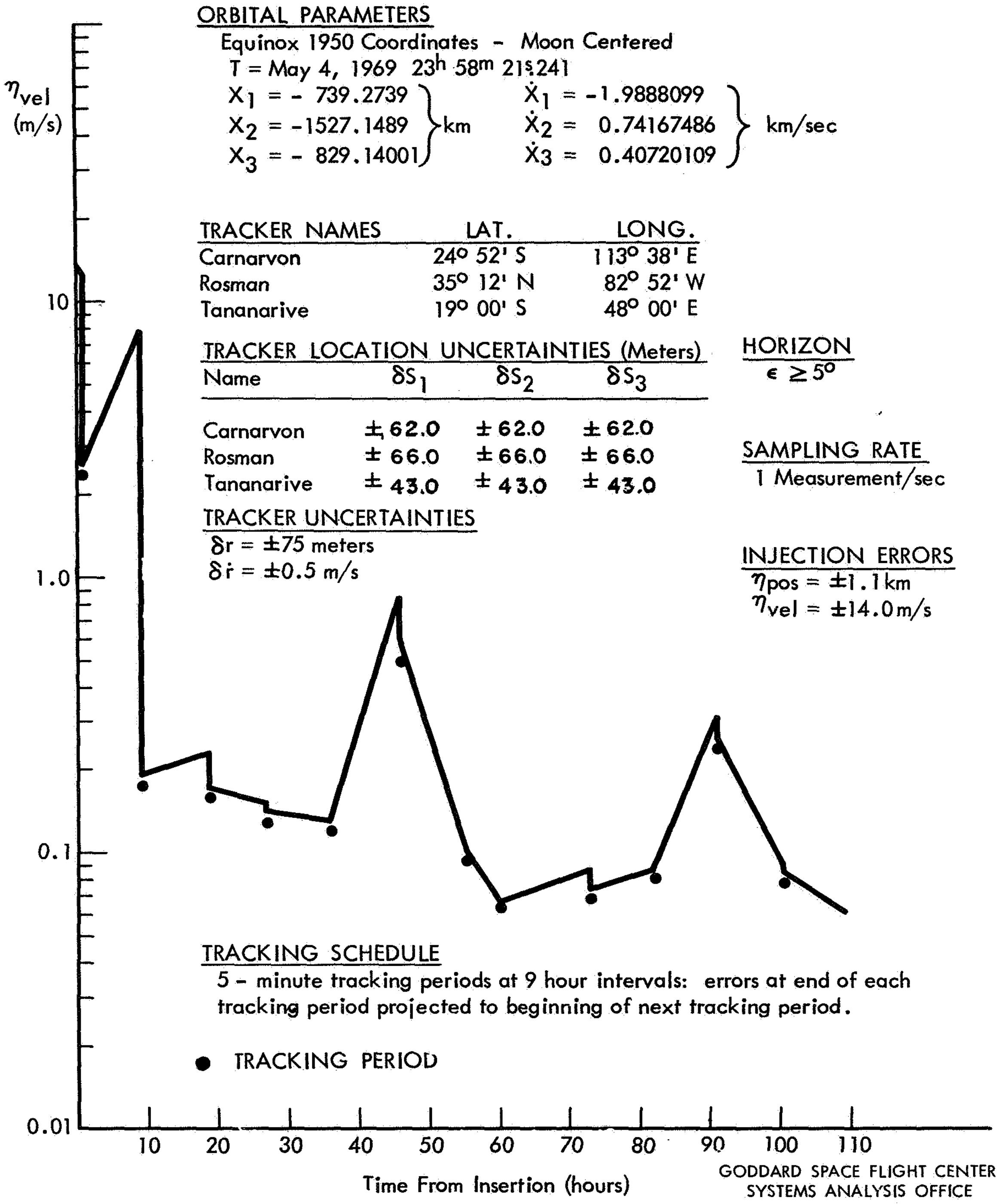
Figure 9-Error Propagation in Position after Insertion of IMP D&E into Lunar Parking Orbit



ORBITAL PARAMETERS

Equinox 1950 Coordinates - Moon Centered

T = May 4, 1969 $23^h\,58^m\,21^s.241$

$$\left.\begin{aligned} X_1 &= -739.2739 \\ X_2 &= -1527.1489 \\ X_3 &= -829.14001 \end{aligned}\right\}\text{km} \qquad \left.\begin{aligned} \dot{X}_1 &= -1.9888099 \\ \dot{X}_2 &= 0.74167486 \\ \dot{X}_3 &= 0.40720109 \end{aligned}\right\}\text{km/sec}$$

TRACKER NAMES	LAT.	LONG.
Carnarvon	24° 52' S	113° 38' E
Rosman	35° 12' N	82° 52' W
Tananarive	19° 00' S	48° 00' E

TRACKER LOCATION UNCERTAINTIES (Meters)

Name	δS_1	δS_2	δS_3
Carnarvon	± 62.0	± 62.0	± 62.0
Rosman	± 66.0	± 66.0	± 66.0
Tananarive	± 43.0	± 43.0	± 43.0

TRACKER UNCERTAINTIES

$\delta r = \pm 75$ meters
$\delta \dot{r} = \pm 0.5$ m/s

HORIZON

$\epsilon \geq 5°$

SAMPLING RATE

1 Measurement/sec

INJECTION ERRORS

$\eta_{pos} = \pm 1.1$ km
$\eta_{vel} = \pm 14.0$ m/s

TRACKING SCHEDULE

5 - minute tracking periods at 9 hour intervals: errors at end of each tracking period projected to beginning of next tracking period.

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Figure 10-Error Propagation in Velocity after Insertion of IMP D&E into Lunar Parking Orbit

CONCLUSION

The large injection errors for IMP D&E Missions projected to lunar distance result in large RMS errors in satellite position and velocity; i.e., ± 27,000 km and ± 160 meters/sec. These large errors will not be changed as a result of tracking the satellite. Since tracking data does, however, provide a good estimate of the current state vector (± 2 km and ± 4 cm/sec), it is possible to insert a satellite into a lunar parking orbit by initiating the retrofire maneuver at some optimized time along the transfer trajectory. A study by R. T. Groves verifying this fact is now in preparation.[6]

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6. Groves, R. T. Private Communication.