ASPECT DETERMINATION

IN LUNAR SHADOW ON EXPLORER 35

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Introduction

Explorer 35, injected into selenocentric orbit on July 22, 1967, experiences a decrease in its moment of inertia as it passes through the lunar shadow. This is apparently due to the cooling of the spacecraft and most probably inward contraction of the solar paddles. Because angular momentum is conserved, the spin period decreases slightly (a few parts per thousand) and the psuedo-sun pulse generated onboard the spacecraft no longer accurately indicates the sunward direction. This report describes an empirical approach to solving the resulting satellite aspect problem.

A variable spin period of the form

$$\tau = a + be^{-t/t_o} \tag{1}$$

is assumed. The measured spin periods during the recovery following a shadow passage show good agreement (See Figure 1) with this analytic form. During the shadow the absence of a true sun pulse precludes the accurate determination of the spin period. However, to the extent that the azimuth angle of the magnetic field vector in the satellite spin plane remains constant throughout the shadow, the magnetic field measurements themselves give an estimate of the satellite aspect. A comparison of the apparent rotation of the magnetic field with the apparent rotation calculated using the methods to be described is shown in Figure 3. This plot and others like it for subsequent shadow passes attest to the accuracy of the calculations using a time dependent spin period of the form in 1 above.

In order that the method of analysing the spacecraft aspect in the shadow be as clear as possible it seems helpful to present a brief description of the pertinent facts of the spacecraft operation at this point. The direction of the sun relative to the spacecraft coordinate system is measured once every four telemetry sequences (1 sequence = 81.808 seconds). This is measured by counting the time from a fixed reference point in the telemetry sequence to the next "see sun" pulse from the onboard sun sensor (for a discussion of the telemetry format and optical aspect system see AIMP (IMP-D) Technical Summary Description, 1967). This time (referred to as the "sun time") together with the time between consecutive sun pulses (the raw spin period) are telemetered once every four sequences. This enables one to calculate accurately the azimuthal orientation of the spacecraft about the spin axis at any time.

Such an accurate determination of the spacecraft azimuth requires the calculation of a refined spin period. This is done by calculating the time between successive sun times and dividing by the integral number of revolutions made by the spacecraft. The number of revolutions, N, is calculated using the raw spin period. Thus the refined spin period is given by

$$\tau = \frac{C_o + C_2 - C_1}{N} \tag{2}$$

where C_o = the time between successive sun sightings.

C_1 = "sun time" at beginning of interval.

C_2 = "sun time" at end of interval.

N = integral number of revolutions between sun sightings.

N in turn is given by

$$N \pm \Delta = \frac{C_o + C_2 - C_1}{S} \tag{3}$$

where S is the raw spin period and Δ is a fraction. The fraction Δ will be less than 0.5 as long as the time C_o corresponds to less than about 40 sequences. Thus N is easily found by rounding the quantity $(N + \Delta)$ to the nearest integer.

If the spin period remained constant through the shadow it would be a simple matter to accurately determine the azimuthal orientation of the spacecraft. Actually the spacecraft spins up in the shadow, and by the end of the shadow a large error (of the order of 360^o) has accumulated in the satellite orientation computed from the optical aspect data. It is this error that must be corrected.

The time the spacecraft enters the shadow is measured to the nearest half sequence except every fourth sequence when it is accurate only to the nearest sequence. This information is telemetered in a sun flag which indicates whether the sun sensor is seeing the sun or not.

Thus the situation is as follows: The spin period and sun direction are known accurately at the time of the last sun sighting before the shadow. Then there follow 0 to 4 sequences of measurements before the satellite enters the shadow and the spin period begins to change. Upon entry into the shadow the spacecraft begins to spin up and the spin period begins to decrease. This decrease continues until the spacecraft again enters the sunlight where upon it begins to de-spin again and the spin period begins to increase, finally approaching its pre-shadow value. After leaving the shadow there will be 0-4 sequences before true solar aspect data are obtained. The total fractional change in the spin period is about 0.0015

(0.15%) over the period of the shadow, typically 40 min. (35 sequences).

With this understanding of the problem the actual calculation of the three free parameters in 1 is relatively straightforward. In carrying out the calculations the following parameters are used. Inspection of Figure 2 illustrates their significance.

Δt_i = time between the last true sun pulse and the beginning of the shadow

Δt_s = duration of the shadow

Δt_f = time between shadow end and next true sun time

τ_i = spin period before shadow

τ_1 = first spin period measured after shadow

τ_2 = second spin period measured after shadow

C_i = last "sun time" before shadow

C_f = first "sun time" after shadow

S = raw spin period

In addition the following parameters will be used:

τ_f = spin period at the end of the shadow

R(t) = Actual number of revolutions of the spacecraft since last true sun before shadow (time 0 on Figure 2)

R_o = $R\ (t = \Delta t_i + \Delta t_s + \Delta t_f)$ = Total number of spins made between true sun pulses

N_o = Integral number of rotations of spacecraft between true sun pulses.

Convenient units are as follows:

Δt's and τ's in seconds

C_i, C_f, S in 800 cps clock counts

R, R_o, N in revolutions.

It is often useful to express the Δt's in units of sequence length = 81.808 seconds and t_o will also be measured in units of sequence length.

Analysis

The problem now becomes one of fitting a spin period of the form 1 to the satellite data. There are 3 free parameters in 1 so three independent equations are required to determine these parameters. Two are obvious: the spin period before and after the shadow must be equal to that measured. The third condition is that the integrated number of revolutions between the true sun times be equal to that achieved by the spacecraft. To this end τ_f and R_o must be determined since they are not directly measured.

First τ_f may be found by using a linear interpolation from τ_1 and τ_2 at the end of the shadow. (Earlier work also fit these data to an exponential to find τ_f, but it was found to be unnecessary, the linear approximation being of more than adequate accuracy for the short interval involved.)

Thus

$$\tau_f = \tau_1 - \left(\frac{\tau_2 - \tau_1}{4}\right) (\Delta t_f + 2.). \tag{2}$$

A little thought will reveal that R_o is given by

$$R_o = N_o - \frac{C_f - C_i}{S} \tag{3}$$

where N_o is to be determined. It may be found by integration if the spin period as a function of time is known. In this work the spin period is assumed to be given by

$$\tau(t) = \begin{cases} \tau_i, & t \le \Delta t_i \\ a + b e^{-[(t - \Delta t_i)/t_o]}, & \Delta t_i < t \le L_1 \\ \tau_f + \left(\frac{\tau_2 - \tau_1}{4}\right)(t - \Delta t_s - \Delta t_i), & L_1 < t \le L_2 \end{cases} \tag{4}$$

where $L_1 = \Delta t_i + \Delta t_s$ and $L_2 = L_1 + \Delta t_f$.

Now R_o may be obtained from:

$$R_o = \int_o^{L_2} \frac{dt}{\tau(t)}$$

$$= \frac{\Delta t_i}{\tau_i} + \frac{\Delta t_s}{a} - \frac{t_o}{a} \ln \left(\frac{a + be^{-\frac{\Delta t_s}{t_o}}}{a + b} \right) + \frac{\Delta t_f}{\tau_f + \frac{(\tau_2 - \tau_1)\,\Delta t_f}{4}} \qquad (5)$$

Since initially the values of a, b, and t_o are unknown we still do not know R_o. However, the present goal is to obtain N_o so it is sufficient only to know R_o to better than $\pm$ 0.5 revolutions. It is found that the following approximate values of a, b and t_o give R_o to $\pm$ 0.1 revolutions.

$a \approx \tau_f$ - .0001 seconds

$b = \tau_i - a$

$\approx (\tau_i - \tau_1)$ + .0001 seconds

t_o = 24 sequences $\approx$ 1963 seconds

Thus by rounding R_o to the nearest integer N_o is determined. This is exact if the estimate of R_o was within $\pm$ .5 revolutions and then 3 may be used to determine R_o accurately. (The accuracy of N_o is easily verified since an error shows up in the magnetic field data as an apparent rotation of one or more complete revolutions.) With the information derived above it is a relatively straightforward matter to set up the three conditions which determine the three unknown parameters in 1, a, b and t_o.

The first two equations are obtained by requiring that the spin period be continuous at the beginning and end of the shadow. When the satellite enters the shadow $t = \Delta t_i$ and $\tau = \tau_i$ so that

$$a + b = \tau_i \qquad (6)$$

At the end of the shadow $t = L_1$ and $\tau = \tau_f$. Thus

$$a + be^{-(\Delta t_s/t_o)} = \tau_f. \qquad (7)$$

The third and final condition is set by requiring that the predicted number of revolutions between the last real sun pulse before the shadow and the first real sun pulse after the shadow is equal to the actual number of revolutions, R_o. This expressed by 5. Since some of the terms in 5 involve only the known parameters (i.e., do not involve a, b, or t_o) it is useful to define another quantity, R_s, the number of revolutions the spacecraft makes during the actual shadow.

$$R_s = R_o - \frac{\Delta t_i}{\tau_i} - \frac{\Delta t_f}{\tau_f \left(\frac{\tau_2 - \tau_1}{4}\right) \Delta t_f} \qquad (9)$$

Then the third condition may be rewritten as:

$$R_s = \frac{\Delta t_s}{a} - \frac{t_o}{a} \ln \left(\frac{a + be^{-(\Delta t_s/t_o)}}{a + b}\right) \qquad (8a)$$

Since the change in the spin period during the shadow is observed to be only about one one-thousandth of the spin period it is easily seen that $\frac{b}{a} \sim .001 \ll 1$. Thus it simplifies the problem to expand the logarithmic term in 8 in powers of $\frac{b}{a}$. This results in

$$R_s = \frac{\Delta t_s}{a} - \frac{t_o\, b}{a^2} \left(1 - e^{-(\Delta t_s/t_o)}\right) \qquad (8b)$$

where only terms up to the first power of b/a have been retained.

The next step is to solve 6, 7 and 8b simultaneously for a, b and t_o. From 8b,

$$\frac{t_o\, b}{a^2} e^{-(\Delta t_s/t_o)} = R_s - \frac{\Delta t_s}{a} + \frac{t_o b}{a^2} \qquad (10)$$

From 7,

$$\frac{t_o b}{a^2} e^{-(\Delta t_s/t_o)} = \frac{t_o}{a^2} (t_f - a)$$

and from 6,

$$b = \tau_i - a$$

Substitution of these last two expressions in 10 yields:

$$R_s - \frac{\Delta t_s}{a} + \frac{t_o}{a^2} (\tau_i - \tau_f) = 0.$$

Upon rearrangement this can be written

$$a^2 \frac{R_s}{\Delta t_s} - a + \frac{t_o}{\Delta t_s} (\tau_i - \tau_f) = 0.$$

From 6 and 7 it is seen that:

$$e^{-(\Delta t_s/t_o)} = \frac{\tau_t - a}{\tau_i - a}$$

so that

$$\ln \left(\frac{\tau_i - a}{\tau_f - a}\right) = \frac{\Delta t_s}{t_o}$$

(11)

or

$$t_o = \frac{\Delta t_s}{\ln \left(\frac{\tau_i - a}{\tau_f - a}\right)}$$

Substitution of this into 10 yields

$$a^2 \frac{R_s}{\Delta t_s} - a + \frac{\tau_i - \tau_f}{\ln\left(\frac{\tau_i - a}{\tau_f - a}\right)} = 0. \qquad (12)$$

It is possible to find an approximate solution to 12 for a by use of Newtons method [see, for example, Franklin pages 26-28]. This is an iterative method for finding the roots of an equation of the form $f(X) = 0$. In essence, an initial estimate of the root X_o is used to find an improved estimate X_1 from the equation

$$X_1 = X_o - \frac{1}{f(X_o)} \quad \frac{df(X)}{dX} \quad .$$

Thus in the present problem one can write

$$f(a) = 0 \text{ where } f(a) = \frac{a^2 R_s}{\Delta t_s} - a + \frac{\tau_i - \tau_f}{\ln\left(\frac{\tau_i - a}{\tau_f - a}\right)} \quad .$$

Then

$$\frac{df}{da} = \frac{2a\ R_s}{\Delta t_s} - 1 - \left[\frac{\tau_i - \tau_f}{\ln\left(\frac{\tau_i - a}{\tau_f - a}\right)}\right]^2 \frac{1}{(\tau_i - a)(\tau_f - a)}$$

Once a has been determined this way it is a simple matter to use 6 to determine b and 11 to obtain t_o.

Now an integration of 1 yields the actual number of rotations of the satellite, i.e.,

$$R(t) = \begin{cases} \dfrac{t}{\tau_i} & t \le \Delta t_i \\ R_1 + \dfrac{t-\Delta t_i}{a} - \dfrac{t_o\, b}{a^2} \left(1 - \exp - \left\{\dfrac{t-\Delta t_i}{t_o}\right\}\right), & \Delta t_i \le t \le L_1 \\ R_2 + \dfrac{t-L_1}{\tau_f + \left(\dfrac{\tau_2 - \tau_1}{4}\right)(t-L_1)} & L_1 \le t \le L_2 \end{cases}$$

where $R_1 = \Delta t_i/\tau_i$

and

$$R_2 = R_1 + \frac{\Delta t_s}{a} - \frac{t_o\, b}{a^2} \left(1 - \exp\left\{ - \frac{\Delta t_s}{t_o} \right\}\right) \quad .$$

Application

An application of the method described above to the pass shown in Figure 2 is useful for illustrative purposes. The input parameters are as follows:

τ_i = 2.2935 seconds

τ_f = 2.2909 seconds

Δt_s = 30.5 sequences

R_s = 1088.675 revolutions

The resulting values for a, b and t_o are:

a = 2.2898 seconds

b = .003641 seconds

t_o = 23.96 sequences

With the additional information that

τ_1 = 2.29125

τ_2 = 2.29151

Δt_i = 1.5

Δt_f = 4.0

and a knowledge of the spin period actually used by the spacecraft, the correction angles can be calculated as follows. If

$$R_u\ (t) = \int_0^t \frac{dt}{\tau_u}$$

where τ_u is the spin period used in the initial analysis, then the angle between the apparent sun direction (represented by the psuedo-sun pulse) and the true sun direction is given by

$$\begin{aligned}\psi(t) &= [R(t) - R_u\ (t)] \text{ rotations} \\ &= 360\ [R(t) - R_u(t)] \text{ degrees}\end{aligned}$$

The correction angles for the pass shown in Figure 2 are listed in Table 1. If it is assumed that the magnetic field remained constant in direction during this shadow pass, then the apparent direction would have rotated through the angle $\psi(t)$. It is on this assumption that Figure 3 was plotted. The solid curve shows the azimuthal angle of the interplanetary magnetic field obtained using the psuedo-sun pulse for orientation. The dotted curve shows the relative change $\psi(t)$.

In conclusion then it may be said that the analysis presented here is an effective method for correcting the azimuthal orientation of the Explorer 35 spacecraft during its passes behind the moon where it can no longer use the sun as a reference. The accuracy of the method is estimated to be $\pm\ 5^{\circ}$ to 10°, but can only be checked empirically if it is assumed that the magnetic field does not change direction. The largest error would occur sometime during the shadow, since at the beginning and at the end the orientation has been fitted to the true direction obtained by real sun sightings.

TABLE 1

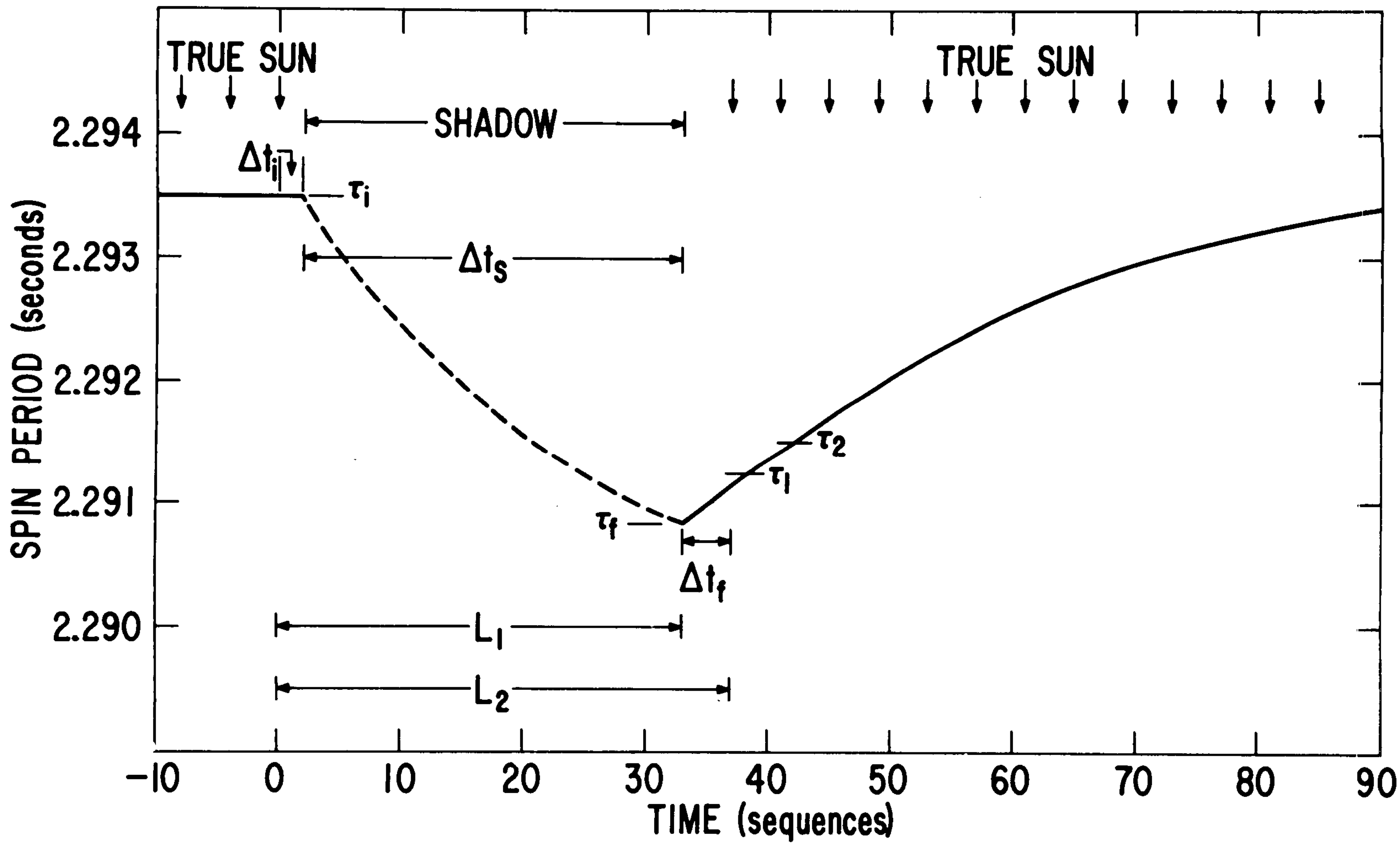
t (Sequences)	R(t) (Revolutions)	R_o (t) (Revolutions)	ψ (t) (degrees)
0	0.0	0.0	0.
1	35.6697	35.6608	3.
2	71.3395	71,3216	6
3	107.0114	106.9823	10
4	142.6856	142.6431	15
5	178.3618	178.2982	23
6	214.0401	213.9532	31
7	249.7203	249.6083	40
8	285.4023	285.2633	50
9	321.0862	320.9184	60
10	356.7717	356.5734	71
11	392.4592	392.2284	83
12	428.1482	427.8835	95
13	463.8386	463.5385	108
14	499.5305	499.1936	121
15	535.2241	534.8486	135
16	570.9187	570.5037	149
17	606.6147	606.1587	164
18	642.3118	641.8138	179
19	678.0103	677.4688	195
20	713.7100	713.1238	211
21	749.4106	748.7789	227
22	785.1123	784.4339	244
23	820.8149	820.0890	216
24	856.5188	855.7440	279
25	892.2234	891.3991	297
26	927.9287	927.0541	315
27	963.6350	962.7092	333
28	999.3425	998.3642	352
29	1035.0503	1034.0192	371
30	1070.7590	1069.6743	391
31	1106.4687	1105.3293	410
32	1142.1787	1140.9844	430
33	1177.8889	1176.7318	417
34	1213.5979	1212.4792	403
35	1249.3059	1248.2266	389

Figure Captions

Figure 1 A plot of τ-a versus time (solid line) for the recovery of the spacecraft following the first shadow pass for which complete data were received. Compared with this is the approximation derived from equation 1 (dashed line).

Figure 2 A plot of spin period versus time for the same shadow pass as in Figure 1. The times and spin periods indicated are discussed in the text. The solid part of the curve is derived from measured spin periods while the dashed portion in the shadow is the approximation of equation 1 with appropriate values of the constants a, b and t_o substituted.

Figure 3 Change of apparent azimuth of magnetic field observed (solid line) during same shadow pass as Figure 1. Compared is the predicted change (dashed line) calculated by assuming that the magnetic field did not change its direction.



TRUE SUN
SHADOW
TRUE SUN
Δt_i
τ_i
Δt_s
τ_f
τ_1
τ_2
Δt_f
L_1
L_2
2.294
2.293
2.292
2.291
2.290
SPIN PERIOD (seconds)
−10
0
10
20
30
40
50
60
70
80
90
TIME (sequences)

FIGURE 1

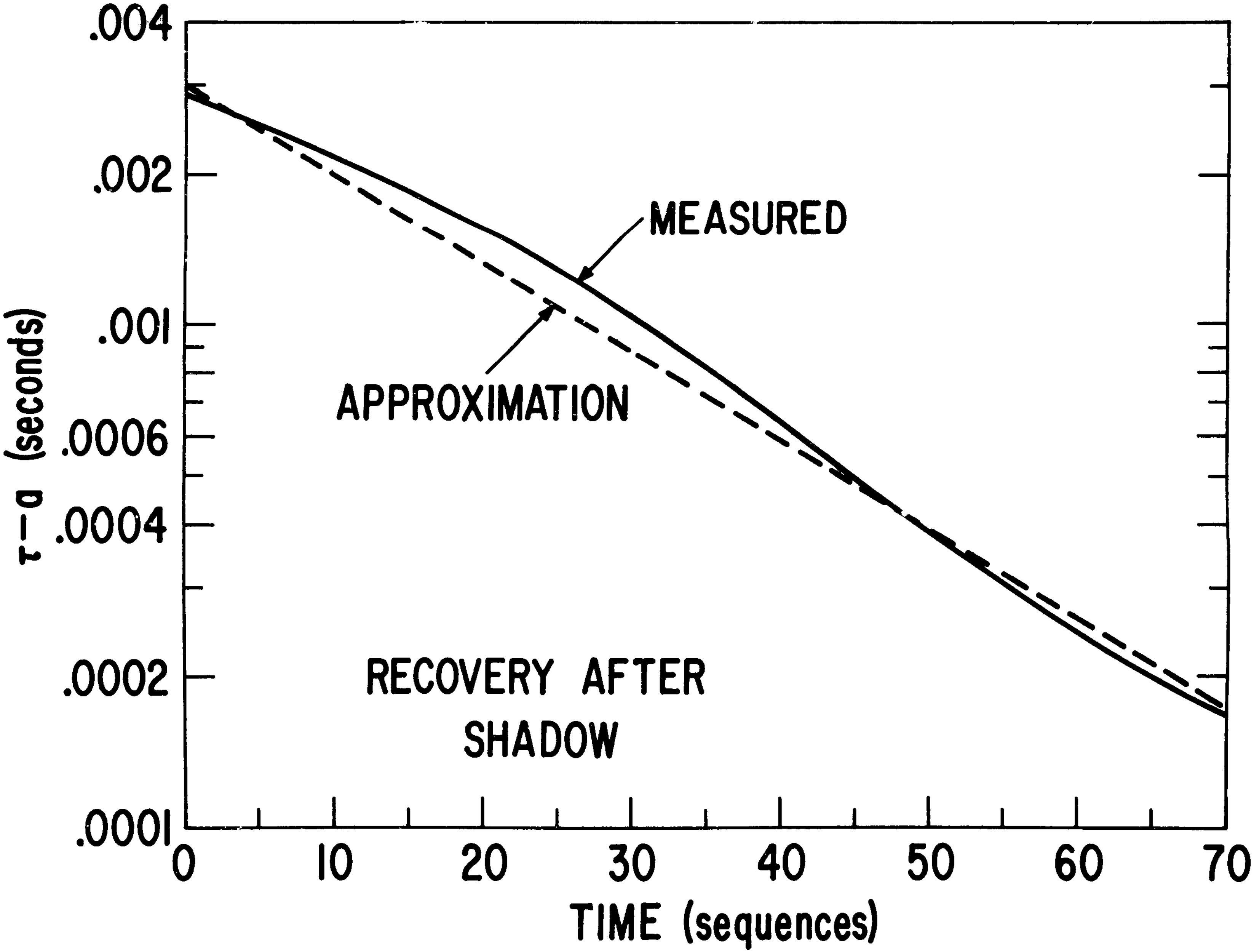

MEASURED
APPROXIMATION
RECOVERY AFTER SHADOW
.004
.002
.001
.0006
.0004
.0002
.0001
τ − a (seconds)
0
10
20
30
40
50
60
70
TIME (sequences)

FIGURE 2

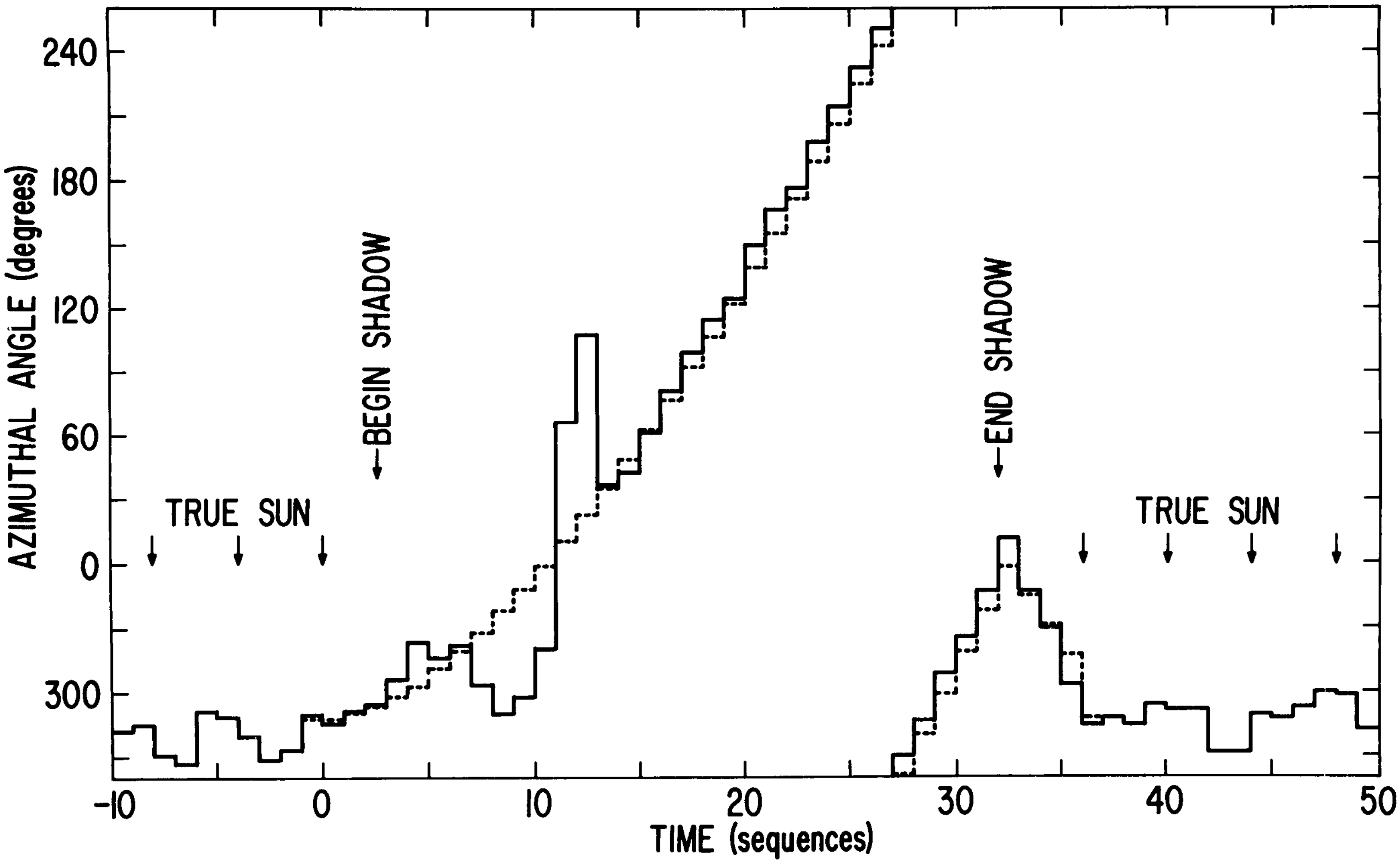

AZIMUTHAL ANGLE (degrees)
240
180
120
60
0
300
TRUE SUN
BEGIN SHADOW
END SHADOW
TRUE SUN
-10
0
10
20
30
40
50
TIME (sequences)

FIGURE 3

References

AIMP (IMP-D) Technical Summary Description, GSFC Research Report No. X-724-67-87, March 1967.

Franklin, Philip, Methods of Advanced Calculus, McGraw-Hill, New York, 1944.