AD/COM

Final Report
Task III
Oscillator Stability in a Tracking System

Contract NAS 5-9742

RADIO COMMUNICATIONS STUDY ON NOISE THRESHOLD REDUCTION

26 August 1965

Prepared for

Goddard Space Flight Center
Greenbelt, Maryland

Submitted by

ADCOM, Inc.
808 Memorial Drive
Cambridge, Massachusetts 02139

[redacted]

ADVANCED COMMUNICATIONS • RESEARCH AND DEVELOPMENT

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Ahmad F. Ghais

Approved by Elie J. Baghdady

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Technical Director

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ABSTRACT

The effects of oscillator jitter on the performance of a tracking system are studied. First, a suitable characterization of the jitter is sought. It is found that the power spectrum of the frequency (or phase) fluctuations is a meaningful and useful characterization of oscillator jitter. Next, the mean-square error introduced by an oscillator in the measurement of range or range-rate is related to this spectrum. Simple relationships result, enabling the identification of those parts of the jitter spectrum which represent "short-term" and "long-term" instabilities respectively. The dividing line may be drawn roughly at a frequency $1/\tau$ where τ is the round-trip propagation time. Thus, long and short term instabilities are distinguishable only in the context of a given application.

The Goddard Range and Range-Rate System contains many possible sources of jitter. The contributions of the various jitter sources to the overall range-rate error are evaluated. It was not possible to arrive at accurate numerical results on errors because of the complete absence of information on jitter spectra. It is concluded that oscillator specifications for future systems should be made on the basis of jitter spectra, and accurate measurement techniques should be developed for evaluation of oscillator stability.

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I. INTRODUCTION

1.1 Purpose of the Report

This report comprises the first final task report on the results of a program of investigations carried out at ADCOM, Inc. under Contract No. NAS 5-9742 for NASA. This work was conducted in close coordination with, and in direct support of,activities of members of the RF Systems Branch, Advanced Development Division of the Goddard Space Flight Center.

The effort during the quarter 27 August – 30 November 1964 was allocated exclusively to performing the third task of the study program. Since the task was completed in one quarter, no quarterly reports are contractually required for the first quarter.

1.2 Scope of the Report

The first task undertaken in the study program was chosen to be task III listed in the task requirements of the subject contract. This task reads as follows:

> Determine through an analysis the effect of short-term carrier phase jitter on the performance of the GSFC Range and Range-Rate Tracking System. This analysis should relate oscillator performance to system performance characteristics such as maximum attainable range, system accuracy, etc. The analysis should take into consideration all system signal sources including the transmitter, receiver,local oscillators and spacecraft oscillators.

The main body of this report consists of three chapters, each treating one aspect of the task described above. Chapter II is devoted to the meaningful characterization of oscillator jitter. The power density spectrum of the phase (or frequency) fluctuations is found to be a particularly useful characterization. Chapter III establishes the fundamental relationships between the performance of a Range and Range-Rate Tracking System and the jitter properties of its master oscillator. Chapter IV identifies the range-rate errors contributed by the jitters in various system signal sources.

The results of this task were presented orally to the technical staff of the RF Systems Branch on two occasions:

Date of Presentation	ADCOM Staff Participating
26 October 1964	Ahmad F. Ghais Richard N. Lincoln
30 November 1964	Ahmad F. Ghais Bert D. Nelin

In addition, most of the contents of Chapters II and III were presented in the form of an informal technical memorandum (Technical Memorandum No. G-63-1) to the technical representative of the contracting officer.

A technical paper covering some of the results of this effort was prepared by members of the ADCOM staff and presented at the Symposium on the Definition and Measurement of Short-Term Frequency Stability, held on November 23-24, 1964 at the Goddard Space Flight Center. This paper was subsequently published in the IEEE Proceedings (see Ref. 1).

II. CHARACTERIZATION OF OSCILLATOR JITTER

2.1 Spectral Characterization of Jitter

In any CW tracking system, a ranging tone derived from a stable master oscillator is transmitted from the ground to the target, which in turn retransmits it back to the ground. The phase of the tone received on the ground is then compared with the phase of the transmitted tone (or of the master oscillator), the difference phase being proportional to the range of the target. Any fluctuations in the phase of the master oscillator would introduce errors in the measurement of range and range-rate, whence it is clear that the oscillator phase-jitter characteristics are central to the evaluation of the effects of oscillator jitter on a tracking system.

It is convenient to express the oscillator output as

$$e_1(t) = (A(t) + 1) \cos \left[\bar{\omega} t + \phi(t)\right]. \tag{2.1}$$

$A(t)$ represents the amplitude jitter, $\phi(t)$ represents the phase jitter, and $\bar{\omega}$ is the mean angular frequency of the oscillator. We may think of the bracket $\left[\bar{\omega} t + \phi(t)\right] = \theta(t)$ as the "instantaneous phase" of the oscillator, and its derivative as the "instantaneous angular frequency" (see Ref. 2, p. 449)

$$\omega_i(t) \equiv \frac{d\theta}{dt} = \bar{\omega} + \dot{\phi}(t). \tag{2.2}$$

$\dot{\phi}(t)$ then represents the frequency jitter. Now, four possible spectral characterizations of oscillator jitter immediately come to mind, namely,

a) the power spectrum $S_e(\omega)$ of the oscillator output,

b) the power spectrum $S_\phi(\omega)$ of the oscillator phase fluctuations,

c) the power spectrum $S_{\dot{\phi}}(\omega)$ of the oscillator frequency fluctuations, and

d) the power spectrum $S_A(\omega)$ of the oscillator amplitude fluctuations.

Since the frequency fluctuation is the derivative of the phase fluctuation, it can easily be shown (see Ref. 3, p. 252) that

$$S_{\phi}(\omega) = \frac{1}{\omega^2} S_{\dot{\phi}}(\omega) \tag{2.3}$$

so that characterizations b) and c) are entirely equivalent. Furthermore, all CW tracking systems employ amplitude limiters at the receiver, consequently amplitude fluctuations of the oscillator (if they exist) do not affect the performance of the system. Thus, the amplitude characterization d) is of no significance in the present context; the characterization we need must be simply and uniquely related to the phase fluctuations.

We are left with essentially two possible spectral characterizations, namely, a) and b) (recall that b) and c) are equivalent). We now claim that the oscillator output spectrum $S_e(\omega)$ is not suitable for our purposes, for two reasons:

a) It can be shown that any given $S_e(\omega)$ may be the result of a specific amplitude fluctuation spectrum $S_A(\omega)$, or of a phase fluctuation having a spectrum $S_{\phi}(\omega)$ which is one of a large class of possible spectra (differing in rms phase deviation and frequency of highest spectral component), or of a large class of combinations of amplitude and phase fluctuations. Thus, the oscillator output spectrum is neither simply nor uniquely related to the phase fluctuations that affect the performance of a tracking system.

b) As will be shown in Chapter III, there exist simple analytical relationships between the phase spectral density $S_{\phi}(\omega)$ and suitable measures of system performance. No such relationships involving the oscillator output spectral density exist.

We conclude that the power spectrum $S_{\phi}(\omega)$ of the phase fluctuation (or the equivalent $S_{\dot{\phi}}(\omega)$) is the most useful and meaningful characterization of oscillator performance for our purposes.

2.2 Sources of Oscillator Phase Jitter

We recognize three distinct sources of phase instability, namely:

a) Noise added to the oscillator signal in the amplifier following the oscillator loop. This noise often has a uniform (flat) power spectrum of density N_o watts/radian per sec over the amplifier bandwidth $2B_a$ rad/sec around the oscillator frequency. Since we are dealing with very stable oscillators, we may assume that the total noise power $2B_aN_o$ is much smaller than the oscillator signal power P, in which case the resultant power spectrum of the phase jitter is also uniform over a band B_a with density N_o/P radians^2/radian per sec. We can thus write (see Ref. 1 for details)

$$S_\phi(\omega) = \begin{cases} N_o/P & \text{for } 0 < \omega < B_a \\ 0 & \text{for } \omega > B_a \end{cases} \tag{2.4}$$

or equivalently, by using Eq. (2.3)

$$S_{\dot\phi}(\omega) = \begin{cases} N\omega^2/P & \text{for } 0 < \omega < B_a \\ 0 & \text{for } \omega > B_a. \end{cases} \tag{2.5}$$

b) Noise added to the oscillator signal inside the oscillator loop, and occupying a narrowband spectrum centered around the oscillator frequency. The effect of this type of noise is to induce fluctuations of the oscillator frequency. As in a) above, the amplifier following the oscillator will limit the spectrum of the frequency fluctuations to the spectral region $0 < \omega < B_a$. The power spectrum of these fluctuations is found (see Ref. 1) to be approximately uniform:

$$S_{\dot\phi}(\omega) = \begin{cases} \alpha & \text{for } 0 < \omega < B_a \\ 0 & \text{for } \omega > B_a \end{cases} \tag{2.6}$$

and the corresponding phase power spectrum is

$$S_\phi(\omega) = \begin{cases} \alpha/\omega^2 & \text{for } 0 < \omega < B_a \\ 0 & \text{for } \omega > B_a . \end{cases} \tag{2.7}$$

c) Slow noise processes that directly frequency modulate the oscillator. Examples of such processes are: current and voltage fluctuations (flicker noise), mechanical vibrations, temperature variations, etc. All these processes, and hence the resultant frequency fluctuations have power spectra restricted to very low frequencies. The most common of these noise processes is flicker noise, which has a power spectrum inversely proportional to frequency. The spectrum of the resultant frequency fluctuations is thus given by

$$S_{\dot{\phi}}(\omega) = \beta/\omega \tag{2.8}$$

where β is some constant and correspondingly

$$S_{\phi}(\omega) = \beta/\omega^3 . \tag{2.9}$$

Clearly, $S_{\dot{\phi}}(\omega)$ can have the form in Eq. (2.8) only down to some small but nonzero ω, otherwise the total flicker-noise power would be infinite.

It is reasonable to expect a typical oscillator to be influenced by all three sources of jitter discussed above, in which case the phase and frequency fluctuation spectra would be given by

$$S_{\phi}(\omega) = \frac{N_o}{P} + \frac{\alpha}{\omega^2} + \frac{\beta}{\omega^3} \qquad \text{for } \omega < B_a \tag{2.10}$$

$$S_{\dot{\phi}}(\omega) = \frac{N_o}{P}\,\omega^2 + \alpha + \frac{\beta}{\omega} \qquad \text{for } \omega < B_a \tag{2.11}$$

There exists some experimental evidence indicating that Eqs. (2.10) and (2.11) are indeed reasonable characterizations of oscillator performance. However, techniques for measuring these spectra accurately have so far been lacking (see Ref. 1). In particular, it would be very useful to be able to measure the four parameters (N_o, α, β, B_a) of Eqs. (2.10) and (2.11) that would characterize the jitter properties of a particular oscillator.

III. EFFECT OF OSCILLATOR JITTER ON RANGE AND RANGE-RATE MEASUREMENT

3.1 Errors in Range Measurement Caused by Oscillator Phase Jitter

The mean square error is a useful and widely employed measure of random range errors in a tracking system. In this section, we present a fundamental relationship between the mean-square range error (denoted by σ_R^2) and the power spectrum of the phase (or frequency) jitter in the master oscillator. This relationship is derived analytically in the Appendix.

If we denote the true range by R and the corresponding round-trip propagation time by τ, then the normalized mean-square range error is found to be

$$\sigma_R^2/R^2 = \frac{1}{2\pi\bar{\omega}^2} \int_0^\infty S_{\dot{\phi}}(\omega) \cdot |H(\omega)|^2 \frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2} d\omega. \tag{3.1}$$

$\bar{\omega}$ is the mean frequency of the ranging tone, and $H(\omega)$ is the transfer function of the lowpass filter that smooths the range measurement. By combining Eqs. (2.3) and (3.1) we obtain a similar expression in terms of the power spectrum $S_{\phi}(\omega)$ of the phase jitter

$$\sigma_R^2/R^2 = \frac{1}{2\pi\bar{\omega}^2} \int_0^\infty \omega^2 S_{\phi}(\omega) \cdot |H(\omega)|^2 \frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2} d\omega \tag{3.2}$$

Figure 3.1 is an illustration of the various factors appearing in the integrand of Eq. (3.1). The term $S_{\dot{\phi}}(\omega)$ represents the effect of the oscillator in determining the error, the term $|H(\omega)|^2$ represents the effect of the ranging filter, and the $\sin^2(\omega\tau/2)/(\omega\tau/2)^2$ represents the effect of the range measurement operation. The normalized mean-square range error is of course the area under the curve representing the integrand.

It can be seen from Eq. (3.1) and Fig. 3.1 that only that portion of $S_{\dot{\phi}}(\omega)$ which is passed by the lowpass filter $H(\omega)$ will contribute to the range error. Since the range measurements required in most space missions vary relatively slowly, most tracking systems employ very narrow lowpass filters $H(\omega)$ to

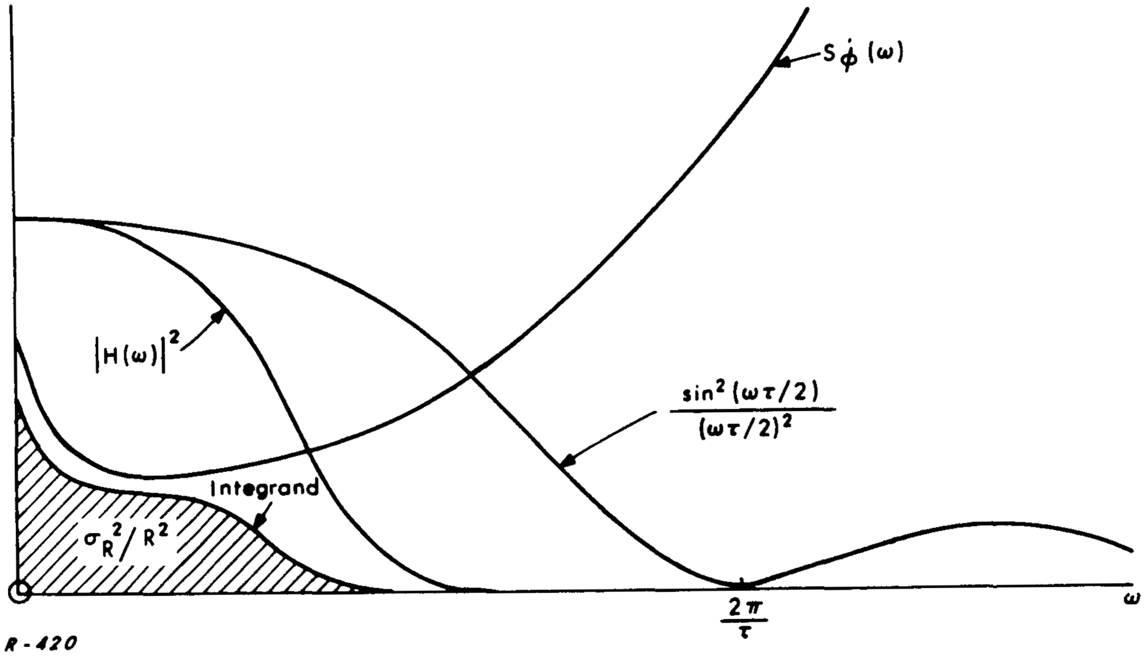

Fig. 3.1 Illustration of the integrand in Eq. (3.1).

reduce the random errors in the range reading. A reasonable choice of filter bandwidth is usually in the range 0.1 - 1 cps. In this case, the portion of $S_{\dot\phi}(\omega)$ for frequencies above 0.1 - 1 cps does not contribute to the range error; or equivalently, frequency instabilities of the oscillator over time-intervals shorter than 1-10 secs are of no significance in the ranging application.

The actual value of time-delay τ being measured determines further the portion of $S_{\dot\phi}(\omega)$ that contributes to range error. Referring to Fig. 3.1, we note that the $\sin^2 x/x^2$ filtering effect of the range measurement operation tends to emphasize the portion of $S_{\dot\phi}(\omega)$ for angular frequencies below $2\pi/\tau$, and suppresses the portion above this value. Now, the low-frequency portion of $S_{\dot\phi}(\omega)$ represents oscillator frequency-instability phenomena (called "long-term" effects) that vary slowly within the time interval τ, whereas the high-frequency portion of $S_{\dot\phi}(\omega)$ represents phenomena (called "short-term" effects) that vary rapidly within τ. Equation (3.1) then tells us that "long-term" effects will always contribute to range error, whereas the contribution of the "short-term" effects may be negligible, the dividing line being roughly $2\pi/\tau$.

A heuristic way to interpret these results is to say that the rapid short-term effects tend to be averaged out in range measurement, whereas the slow long-term effects tend to form accumulating drifts. It is noteworthy that the above conclusions are directly opposed to some commonly held views about the relative significance of short- and long-term jitter effects in tracking systems.

Equation (3.1) further indicates an interesting interaction between the lowpass filter $H(\omega)$ and the filtering effect of the range measurement operation represented by the factor $\sin^2 x/x^2$. If the longest range to be measured is such that $1/\tau$ is much greater than the cutoff frequency of $H(\omega)$, then the $\sin^2 x/x$ factor hardly influences the mean-square range error, and may be dropped from Eq. (3.1). For example, most orbital operations involve values of τ in the milliseconds, in which case $1/\tau$ is much greater than the usual filter bandwidths of 0.1 - 1 cps, and the $\sin^2 x/x^2$ factor may be dropped. On the other hand, lunar operations with $\tau \approx 5$ seconds would be more marginal.

An important conclusion to be drawn from the above results concerns the specification of oscillator performance in tracking systems. Meaningful specifications should be concerned with the power spectrum of the frequency (or phase) fluctuations, up to the cutoff frequency of the lowpass filter or $1/\tau_{max}$ whichever is smaller, where τ_{max} represents the maximum range to be measured. The spectrum beyond this point does not affect the system materially, so it is unnecessary to place specifications on it.

3.2 Errors in Range-Rate Measurement Caused by Oscillator Phase Jitter

Since range-rate $\dot{R}$ is proportional to the doppler frequency shift, the range-rate measurement is obtained (at least in principle) by differentiating the phase difference given by Eq. (A-2) of the Appendix. The term $\Delta\dot{\phi}$ thus obtained would be directly proportional to the error in range-rate (denoted by $\Delta\dot{R}$) caused by oscillator frequency jitter. Since differentiation of a function results in a multiplication of its power spectrum by ω^2, we immediately conclude that the power spectrum of $\Delta\dot{\phi}$ is found from Eq. (A-10) to be

$$S_{\Delta\dot{\phi}}(\omega) = \frac{1}{2\pi}\,\omega^2\,S_{\dot{\phi}}(\omega)\,\frac{\sin^2(\omega\tau/2)}{(\omega/2)^2}\,. \tag{3.3}$$

By following the same procedure as the one employed in the Appendix in connection with the range error, we find that the normalized mean-square range-rate error is expressed as

$$\sigma_{\dot{R}}^2/\dot{R}^2 = \left(\frac{\tau}{2\pi\varpi\tau}\right)^2 \int_0^\infty \omega^2\,S_{\dot{\phi}}(\omega)\cdot|G(\omega)|^2\,\frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2}\,d\omega \tag{3.4}$$

where $G(\omega)$ now is the transfer function of the lowpass filter that smooths the range-rate measurement.

Much the same comments made in connection with Eq. (3.1) apply to Eq. (3.4). It should be remembered, however, that in some range-rate measuring systems (of which the Goddard Range and Range-Rate System is an example) the time τ does not represent the round-trip propagation time, but rather a "cycle-counting" time associated with the digital equipment employed to measure the doppler frequency. This point is discussed further in the following section.

IV. EFFECTS OF JITTER ON THE GODDARD RANGE-RATE MEASUREMENT

4.1 The Range-Rate Measurement Technique of the GRARR System

The Goddard Range and Range-Rate System (GRARR, for short) determines the range-rate of the target by measuring the doppler frequency shift on the up-link carrier frequency.

In this section we describe the range-rate measurement technique (in accordance with Ref. 4), and in the next section identify the sources of error in this measurement that can be traced back to oscillator jitter.

The up-link carrier frequency is synthesized at the ground station from a local oscillator that is entirely independent from, and hence not coherent with, the system frequency reference.

The range tones are phase modulated onto the up-link carrier, and the modulated carrier is transmitted to the transponder on the vehicle. The transponder heterodynes the up-link carrier with a local oscillator which is coherent with the transponder output (down-link) carrier frequency to achieve low intermediate frequencies after double frequency conversion. The resultant heterodyned signal is then phase modulated onto the down-link carrier.

By reversing the transponder signal processing in the ground receiver, the up-link carrier doppler is reconstructed for range-rate measurement. This is indeed what the ground receiver does, even though the signal is not actually generated at VHF or S-band, but rather at a suitably low frequency called the bias frequency. This bias frequency is derived coherently from the system frequency reference.

Figures 4.1 and 4.2 illustrate the doppler measurement technique. Let f_D be the two-way doppler shift on the (up-link) transmitted frequency f_o, and let f_B be the bias frequency. The range-rate is measured in three steps as follows:

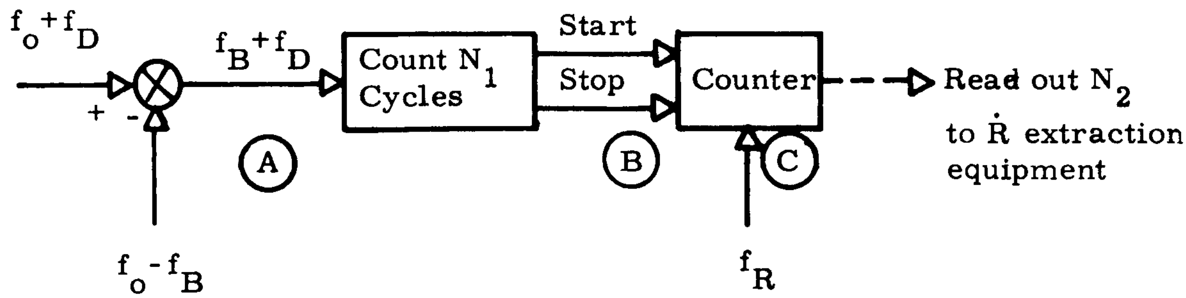

Fig. 4.1 Doppler measurement technique.

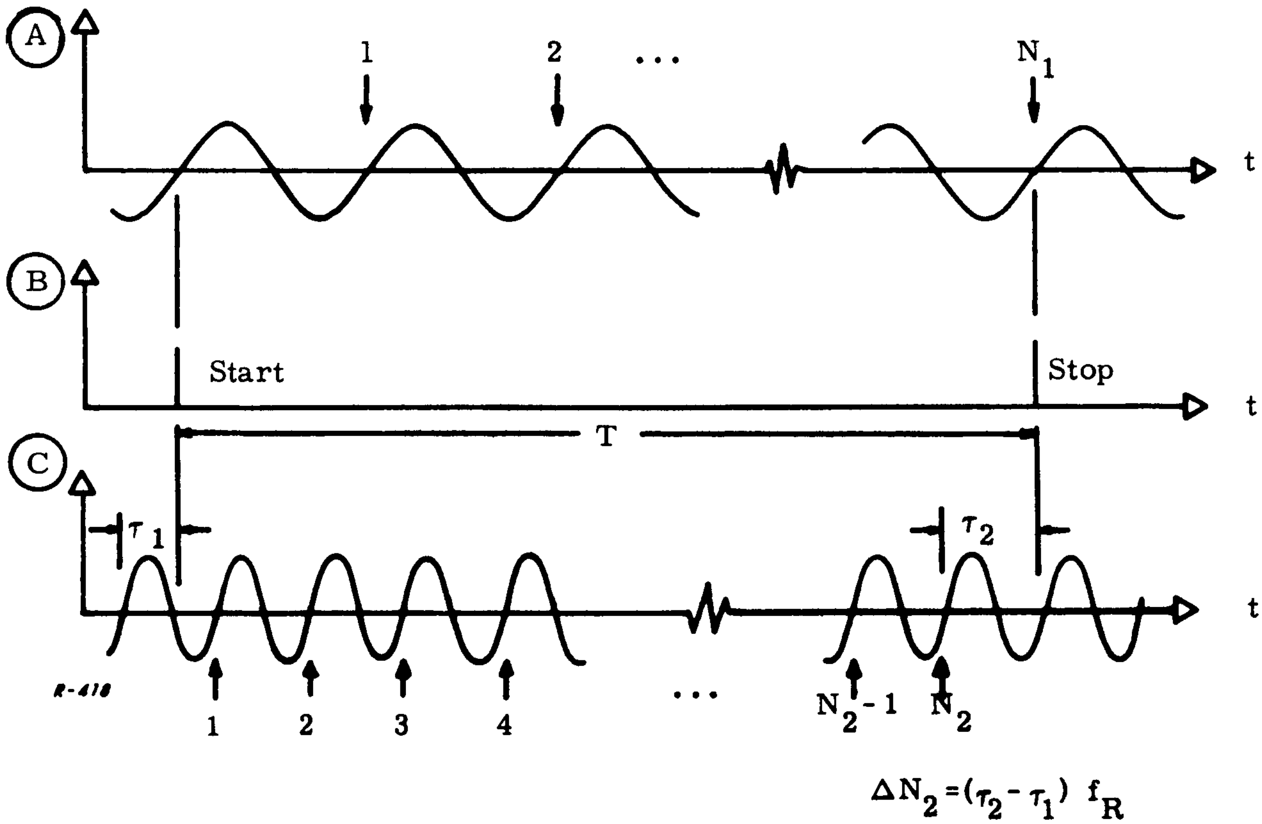

Fig. 4.2 Doppler measurement waveforms.

1. The first counter generates a pair of pulses (start and stop) separated in time by the period of a fixed number N_1 cycles of the doppler-plus-bias frequency. This time interval T is given by

$$T = \frac{N_1}{f_D + f_B} \tag{4.1}$$

from which we find the doppler to be

$$f_D = \frac{N_1}{T} - f_B . \tag{4.2}$$

2. The second counter then measures the time interval T in units of $1/f_R$ seconds, by counting the integral number N_2 of cycles of reference frequency f_R occurring between the start and stop pulses. Ideally, when the interval T spans an integral number of cycles of the reference, then

$$T = \frac{N_2}{f_R} . \tag{4.3}$$

3. Finally, the range-rate equipment converts the count N_2 to a range-rate reading. The doppler shift is found by combining Eqs. (4.2) and (4.3):

$$f_D = \frac{N_1 f_R}{N_2} - f_B \tag{4.4}$$

and the range-rate is simply related to the doppler shift by

$$\dot{R} = \frac{c}{2f_o} f_D . \tag{4.5}$$

The quantities f_o, f_R, N_1 and f_B are all fixed, and are pre-programmed into the range-rate extraction equipment.

4.2 Jitter and Quantization Errors in Range-Rate Measurement

Range-rate measurement errors are introduced at the counters by:

a) jitter in the up-link carrier oscillator f_o, from which the bias frequency f_B is derived;

b) jitter in the reference oscillator f_R;

c) jitter in the start-stop pulses generated by the first counter; and

d) quantization effect in the second (N_2) counter.

We now relate the resultant range-rate measurement error to these individual sources of error.

Equation (4. 5) indicates that errors in $\dot{R}$ result from errors in the doppler measurement f_D and in the knowledge of the up-link frequency f_o. By partial differentiation of Eq. (4. 5), we obtain a relationship between these errors:

$$\Delta\dot{R} = \frac{c}{2f_o} \left(\Delta f_D - \frac{f_D}{f_o} \Delta f_o \right). \tag{4. 6}$$

Now, referring to Eq. (4. 2), we recognize that doppler measurement errors can be caused by errors in T and f_B only, since N_1 is a fixed integer which the first counter can count without error. By partial differentiation of Eq. (4. 2), we obtain the relationship between errors:

$$\Delta f_D = - \frac{N_1}{T^2} \Delta T - \Delta f_B \tag{4. 7}$$

Next, referring to Eq. (4. 3), we recognize that errors in the measurement of T can be caused by errors in f_R (reference jitter) and N_2 (quantization in the second counter). To this must be added an error δ representing unavoidable jitter in the start-stop pulses. Again, by partial differentiation of Eq. (4. 3), we obtain

$$\Delta T = \delta + \frac{\Delta N_2}{f_R} - \frac{N_2}{f_R^2} \Delta f_R. \tag{4.8}$$

Combining Eqs. (4. 6), (4. 7) and (4. 8) we obtain for the total range-rate error

$$\Delta R = -\frac{c}{2f_o}\left[\frac{N_1}{T^2}\left(\delta + \frac{\Delta N_2}{f_R} - \frac{N_2}{f_R}\cdot\frac{\Delta f_R}{f_R}\right) + \Delta f_B + f_D\frac{\Delta f_o}{f_o}\right] \tag{4.9}$$

It is desirable to eliminate T and N_2 from this expression, in order to have $\Delta\dot{R}$ in terms of the primary system parameters f_o, f_R, f_B and N_1 and the actual doppler shift f_D. Use Eqs. (4.1) and (4.3) to get

$$\Delta R = -\frac{c}{2f_o}\left[\frac{(f_D+f_B)^2}{N_1}\left(\delta + \frac{\Delta N_2}{f_R}\right) + (f_D + f_B)\frac{\Delta f_R}{f_R} + \Delta f_B + f_D\frac{\Delta f_o}{f_o}\right] \tag{4.10}$$

Since the bias frequency is coherent with the reference frequency, then

$$\frac{\Delta f_B}{f_B} = \frac{\Delta f_R}{f_R} \tag{4.11}$$

Equation (4.10) consequently reduces to

$$\Delta\dot{R} = -\frac{c}{2f_o}\left[\frac{(f_D + f_B)^2}{N_1}\left(\delta + \frac{\Delta N_2}{f_R}\right) + f_D\left(\frac{\Delta f_R}{f_R} + \frac{\Delta f_o}{f_o}\right)\right] \tag{4.12}$$

4.3 Numerical Results

Equation (4.12) identifies the various contributions to the range-rate error, and demonstrates their relative weighting. The next step would be to apply the theory developed in Chapter III to determine the mean-square $\dot{R}$ error caused by each error source, then to add all these contributions to arrive at the overall mean-square error. Unfortunately, it is impossible to do so at this time because no information is available on the jitter spectra of the various oscillators.

There is, however, information available on "fractional jitter", usually stated in parts per billion or parts per ten billion. The inadequacy and possible ambiguity of fractional jitter as a characterization of oscillator jitter is discussed elsewhere (Ref. 1). Here, we simply use the fractional jitter information given in Ref. 5 to compute the contributions to $\dot{R}$ error from Eq. (4.12). The results are summarized in tabular form below. The oscillator jitter terms are seen to be negligible compared with the quantization term.

RANGE-RATE PEAK MEASUREMENT ERRORS

Error Source	S-band	VHF	
Drift Delay (δ)	0.0107	0.0099	m/sec
Quantization (ΔN_2)	0.0363	0.0338	m/sec
Reference Instability (Δf_R)	2.2×10^{-6}	2.2×10^{-6}	m/sec
Transmitter Instability (Δf_o)	2.8×10^{-7}	2.8×10^{-7}	m/sec
Total rms error	0.0378	0.0352	m/sec

A word of caution about these numerical results is in order. They are based on jitter specifications rather than on measured fractional jitters. Furthermore, because of the lack of information on jitter spectra, these results are not computed with the aid of the accurate theory described in Chapter III. Thus, experimentally observed errors may well differ from those computed above.

V. CONCLUSIONS AND RECOMMENDATIONS

We have shown in this report that the spectrum of the phase or frequency fluctuations is a meaningful and useful characterization of the jitter properties of an oscillator. Furthermore, this spectrum is simply related to the range and range-rate errors in a tracking system introduced by this jitter.

The Goddard Range and Range-Rate System contains many possible sources of jitter. We have shown how to determine the contributions to the overall error caused by the various jitter sources. The spectral theory can then be applied to obtain the mean square error contributions. Unfortunately, no information on the jitter spectra in the GRARR is available, so it was not possible to find accurate numerical results on errors caused by jitter. However, the method of computation and all the necessary theory were presented.

It is evident, on the basis of this work, that specification of oscillator jitter characteristics for future systems should be made on the basis of frequency or phase fluctuation spectra. Accurate experimental techniques for jitter spectrum measurement should be developed, both for acceptance testing of oscillators and for comparison of different oscillators.

PROGRAM FOR NEXT INTERVAL

The effort during the next quarter will be directed exclusively to Task II of the subject contract. This task reads as follows:

The nonlinear phase characteristic and phase distortion exhibited by conventional telemetry receivers deteriorate the fidelity of data recovery. Some of the undesirable products of this phase distortion are recognized as crosstalk between frequency multiplexed channels and degradation of bit error rates of PCM signals.

The object of this task is to determine and report on the effects of non-linear phase variations on various circuit operations. In addition to the analysis, the report should contain handbook type information, tables and graphs showing the relation between phase nonlinearity expressed in an easily interpreted fashion to such functions as the generation of intermodulation products, deterioration of bit error rates; et al.

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APPENDIX

Derivation of Equation (3.1)

The first step is to develop a relationship between the range error and the oscillator phase jitter.

If the ranging tone transmitted from the ground to the target is described analytically by Eq.(2.1), and if the round-trip propagation time is denoted by τ, then the returned signal may be expressed as

$$e_2(t) = \left[1 + A(t+\tau)\right] \cos\left[\bar{\omega}(t+\tau) + \phi(t+\tau)\right]. \tag{A-1}$$

The range-measurement portion of the tracking system extracts the instantaneous phase difference between transmitted and received tones, usually by amplitude limiting and phase-detecting the received tone. The output of the phase detector is then given by

$$\bar{\omega}\tau + \phi(t+\tau) - \phi(t) \equiv \bar{\omega}\tau + \Delta\phi. \tag{A-2}$$

The first term in Eq.(A-2) is directly proportional to the true range R, i.e.,

$$R = \frac{c\tau}{2} = \left(\frac{c}{2\bar{\omega}}\right)\bar{\omega}\tau \tag{A-3}$$

whereas the second term in Eq.(A-2) is directly proportional to the error in range (denoted by ΔR) caused by oscillator phase jitter, i.e.,

$$\Delta R = \frac{c}{2\bar{\omega}}\Delta\phi = \frac{c}{2\bar{\omega}}\left[\phi(t+\tau) - \phi(t)\right]. \tag{A-4}$$

Because $(c/2\bar{\omega})$ is a constant factor in Eq.(A-4), the mean-square range error is simply given by

$$\sigma_R{}^2 = \overline{(\Delta R)^2} = \left(\frac{c}{2\bar{\omega}}\right)^2 \overline{(\Delta\phi)^2} \tag{A-5}$$

where the bar is used to indicate the operation of obtaining the mean (or average). It is convenient to normalize the mean-square range error to the square of the range, so we combine Eqs. (A-3) and (A-5) to obtain the normalized mean-square range error

$$\frac{\sigma_R^2}{R^2} = \frac{\overline{(\Delta\phi)^2}}{(\bar{\omega}\tau)^2} \; . \qquad \text{(A-6)}$$

The next step is to relate power spectrum $S_{\Delta\phi}(\omega)$ of the phase error $\Delta\phi$ caused by jitter to the power spectrum of the phase (or frequency) fluctuations.

We note that the phase error due to jitter may be expressed in terms of the frequency fluctuations as

$$\Delta\phi = \phi(t+\tau) - \phi(t) = \int_t^{t+\tau} \dot{\phi}(x)\, dx. \qquad \text{(A-7)}$$

It is analytically convenient* to change the limits of integration in Eq. (A-7) to $(-\infty)$ and $(+\infty)$. In order to ensure that the contributions of the integral outside the interval $[t, (t+\tau)]$ remain zero, we multiply the integrand by the rectangular pulse shown in Fig. A.1a whence

$$\Delta\phi = \int_t^{t+\tau} \dot{\phi}(x)\, dx = \int_{-\infty}^{\infty} \dot{\phi}(x) \,\mathrm{Rect}\,[(x-t), (x-t-\tau)]\, dx \qquad \text{(A-8)}$$

where the Rect function is defined generally by (see Fig. A.1b)

$$\mathrm{Rect}\,[(x-a), (x-b)] = \begin{cases} 0 & \text{for } x < a \\ 1 & \text{for } a \le x \le b \\ 0 & \text{for } x > b \end{cases} \qquad \text{(A-9)}$$

* The analytical techniques employed in this appendix are all standard techniques of communication theory. A particularly good treatment of these techniques may be found in Ref. 3.

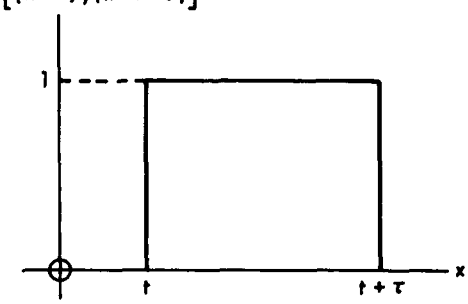

Fig. A.1a

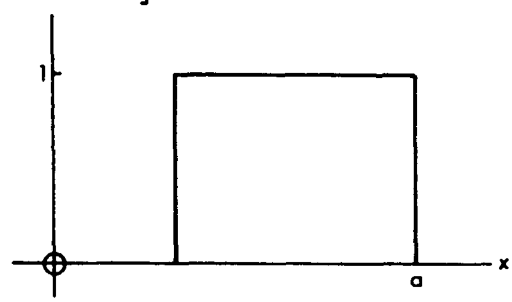

Fig. A.1b

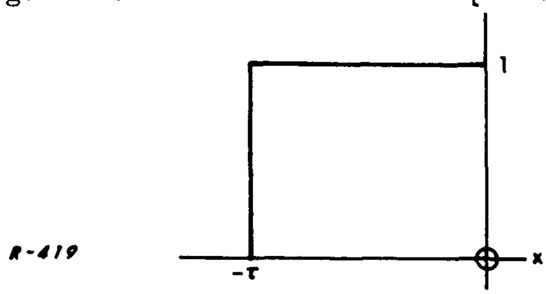

Fig. A.1c

Fig. A.1 Illustrations of Rect function.

We now recognize that the right-hand side of Eq. (A-8) represents the convolution of $\dot{\phi}(x)$ and Rect $[(x+\tau), x]$ (see Fig. A.1c). It is well known that the power spectrum of the convolution of two time-functions is the product of the power spectra of these functions divided by 2π*. Knowing that the power spectrum Rect $[(x+\tau), x]$ is $\sin^2(\omega\tau/2)/(\omega/2)^2$, we can immediately write the power spectrum of the phase error as

$$S_{\Delta\phi}(\omega) = \frac{1}{2\pi} S_{\dot{\phi}}(\omega) \frac{\sin^2(\omega\tau/2)}{(\omega/2)^2} . \tag{A-10}$$

Since the range measurement varies slowly, most tracking systems employ a very narrow lowpass filter (denoted by $H(\omega)$) to reduce the random fluctuations in the phase measurement of Eq. (A-2). The effect of this filter on the power spectrum of the phase error is to modify it to read

$$S_{\Delta\phi}(\omega) = \frac{1}{2\pi} S_{\dot{\phi}}(\omega) \cdot |H(\omega)|^2 \frac{\sin^2(\omega\tau/2)}{(\omega/2)^2} . \tag{A-11}$$

* This is discussed in detail on pp. 247-248 of Ref. 3.

Finally, the mean-square phase error $\overline{(\Delta\phi)^2}$ is the total power of the phase error, thus

$$\overline{(\Delta\phi)^2} = \int_0^\infty S_{\Delta\phi}(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\int_0^\infty S_{\dot\phi}(\omega) \cdot |H(\omega)|^2 \frac{\sin^2(\omega\tau/2)}{(\omega/2)^2}\, d\omega. \qquad \text{(A-12)}$$

Combining Eqs. (A-6) and (A-12) we obtain the normalized mean-square range error

$$\sigma_R^{\,2}/R^2 = \frac{1}{2\pi\bar\omega^2}\int_0^\infty S_{\dot\phi}(\omega) \cdot |H(\omega)|^2 \frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2}\, d\omega\,. \qquad \text{(A-13)}$$

This equation is reproduced as Eq. (3.1) in the main body of this report.