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## SUMMARY

Critical experiments with fully enriched (93.2 percent uranium 235) uranyl fluoride - water solution reactors that contain arrays of large-diameter void tubes have been analyzed satisfactorily. This study evaluates a calculational method that involves the direct application of widely used multigroup computer programs and techniques to cases of extreme heterogeneous voids.

Experimental critical solution heights for cores that contain no void tubes and for 19, 31, and 37 void tubes with a 7.658-centimeter diameter were obtained by using the NASA Zero Power Reactor-II facility. Both unreflected cores and cores radially reflected with 15.24 centimeters of water were considered. The void arrays with triangular lattice pitches of either 9.652 or 10.922 centimeters were centrally located in the 76.2-centimeter-diameter core tank. The critical heights of the voided reactors ranged from 21 to about 84 centimeters.

The calculational method consists of first computing axial leakage rates from axially finite cylindrical cells that contain the void tube and a proportional amount of fuel solution. The cell dimensions and fuel are obtained from the corresponding critical reactors. Two-dimensional (r-z) $S_4P_1$ transport calculations with five energy groups of finite height cylindrical cells are used. The axial leakage rates per source neutron, obtained from the cell calculations, are incorporated into one-dimensional radially finite reactor calculations by defining an axial leakage cross section for each energy group to account for axial neutron streaming out of the voided region of the reactor. Disadvantage factors, which are also obtained from the two-dimensional cells, are important especially for the shorter height reactors that contain the more concentrated fuel solutions. The calculational method is satisfactory for the reactors examined and is readily adapted for use with other reactor configurations provided that two-dimensional (r-z) cells can be defined appropriately.

## INTRODUCTION

The insertion or incorporation of void or gas tubes in nuclear reactors has pronounced reactivity effects. The core materials are displaced by the voids to regions of different statistical importance, which results in reactivity changes. In addition, for voids oriented axially in cylindrical cores, a more subtle effect, and one that is more difficult to compute, is that neutrons may stream preferentially out of the reactor through the void passage and thus not be available for sustaining the nuclear chain reaction. Gas-cooled reactors with their low-density gas-coolant passages are examples of such reactors in which axial streaming effects may be important.

This study was conducted to determine if widely used multigroup digital computer programs can be applied to the understanding of the effects of the neutron streaming phenomenon. Experiments conducted in the NASA Zero Power Reactor-II (ZPR-II) (refs. 1 and 2) facility yield experimental data for a wide range of solution reactors that represent a more severe test of calculational methods than do the normally larger gas-cooled reactors.

The problem of voids in a nuclear reactor has been considered by many people in the past few years. References 3 to 9 are examples of the approaches taken by other authors. Several authors considered only single, centrally located void tubes.

Critoph and Pearce (ref. 3) used a perturbation method to compute the change in the reactor critical height $\Delta H$ required to compensate for neutron streaming out of a single centrally located cylindrical void tube inserted into a critical reactor. Calculations and experiments with tubes up to about 7.6 centimeters were done for the ZEEP reactor. The critical height of the reactor was about 200 centimeters. The calculations did not always fall within the rather large estimates of experimental error, approximately $\pm 20$ percent in $\Delta H$.

Zimmerman (ref. 4) derived a boundary condition at the void-fuel interface for annular reactors consisting of a single centrally located cylindrical void in a cylindrical reactor. The boundary condition for each energy group can be used to define an effective diffusion coefficient and absorption cross section for the void region. Use is made of the void diameter, core height, and transport mean free path of the surrounding medium. Fieno, et al. (ref. 5) show the results of using this method for several NASA ZPR-I reactors with single void tube insertions having diameters up to 5.08 centimeters. The results show an increasing discrepancy between experimental and calculated values of the change in critical height of the reactors due to void insertion with increasing void diameter. A general overestimation of the leakage is observed. One might expect that the method based on isolated voids would be even less applicable for arrays of closely spaced voids.

However, a method for treating multiple void-tube arrays must be capable of correctly treating the special case of a single void tube in a reactor. The approach taken in this report is to consider the treatment of void tube arrays as an evolution of a method for the solution of reactors with single void tubes in which two-dimensional (r-z) multigroup transport calculations with the reactor explicitly represented are used. By merely changing the radial boundary conditions for the solution of a bare cylindrical reactor (containing a centrally located cylindrical void tube), a solution for an axially finite, radially infinite lattice of void tubes is obtained. The calculation of the reactor with a void tube array may then be performed by utilizing axial leakage parameters evaluated from the solution of the infinite lattice.

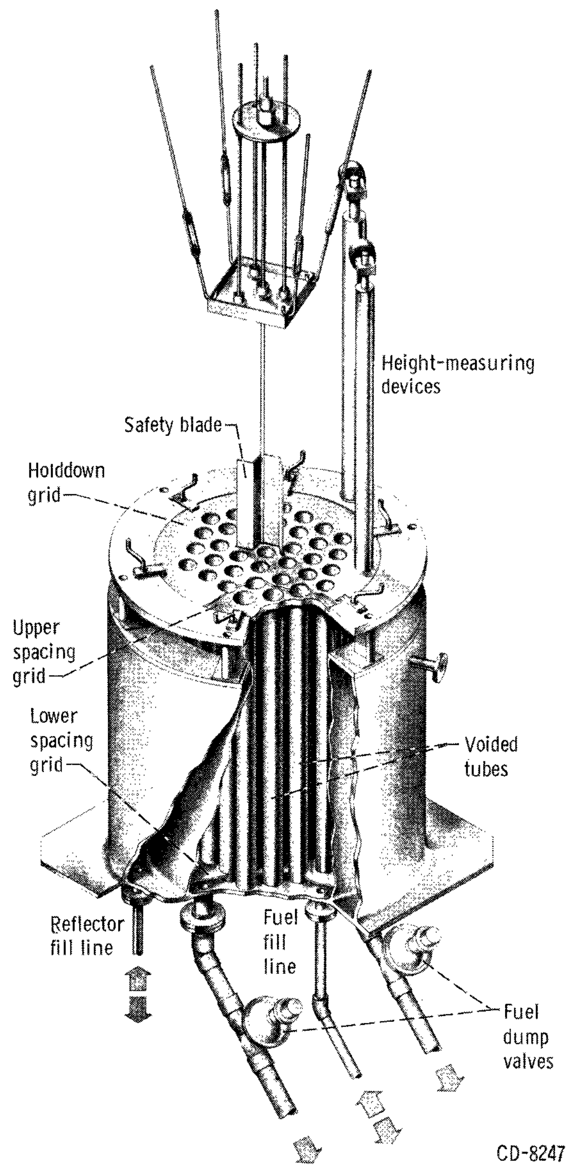
The use of two-dimensional transport theory solutions for the axially finite, radially infinite, regular lattice of void tubes to determine axial leakage parameters for use in gross reactor calculations is the major difference between the method presented herein and previous methods for treating multiple voids. For example, Behrens (ref. 6) treats the effect of multiple voids in terms of an increase in the migration area. The equations derived for infinitely long, parallel, cylindrical voids indicate substantial ansiotropy in the diffusion properties of the medium. The anisotropic term parallel to the voids is twice that perpendicular to the voids. Benoist (ref. 7) derives a similar equation but includes terms to account for the interference between adjacent void channels.

Peak and Cohen (ref. 8) used both Behrens' and Benoist's methods to compute the multiplication factor $K_{eff}$ for a critical uranyl fluoride solution reactor penetrated axially by an array of parallel cylindrical voids. Both methods gave a $K_{eff}$ of 0.94, which indicated a large overestimation of the axial leakage from the voided region.

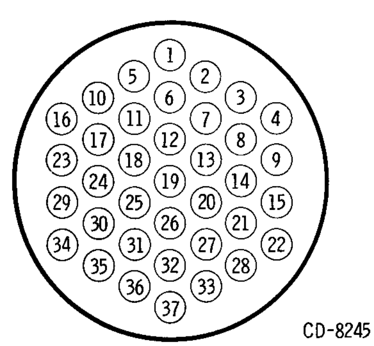
Marti and Schneeberger (ref. 9) considered an axially finite, radially infinite, regular array of cylindrical void tubes in a multiplying medium and derived an expression for the reduction in the infinite multiplication factor of the medium required for criticality. Behrens' formulation is recast to yield the same change in infinite multiplication factor, and a comparison is made for a theoretical lattice with tube diameters of 0.5 to 5.0 centimeters with a height of 200 centimeters. The comparison of the results of reference 9 with Behrens' results for the larger diameter voids indicates greater axial leakage. The calculation in reference 8 indicated that Behrens' method overestimated the leakage, thus the method of reference 9 could be expected to do worse and, consequently, was not applied to ZPR-II reactors.

## DESCRIPTION OF CRITICAL EXPERIMENTS

The NASA Zero Power Reactors (I and II) utilize aqueous uranyl fluoride - water solution as fuel. The uranium is enriched to 93.2 percent in uranium 235. Criticality is achieved by pumping fuel solution into the cylindrical core tank until the critical height is reached. The height is measured to ±0.001 centimeter by a micrometer screw




(a) Cutaway view.




(b) Plan view of 37-tube array (typical).

Figure 1. - ZPR-II tube array.

with a platinum electrical continuity probe attached. The uranium concentration may be varied by adding water to the solution or by removing water in an evaporator. A convenient measure of the fuel concentration is the ratio of the atomic densities of hydrogen to uranium 235, referred to as H/X.

## Reactors Containing Void Arrays

A sketch of the NASA Zero Power Reactor-II (ZPR-II) with void tubes is shown in figure 1(a), and the arrangement of the voids is shown in figure 1(b). Up to 37 void tubes with outside diameters of 7.658 centimeters may be accommodated. Omitting the six corner tubes from the 37 void tube array gives a 31-tube configuration, while deleting the entire outer ring gives a 19-tube assembly. The void tubes are held in position in the cylindrical core tank by top and bottom grid plates. The positioning holes in two pairs of grid plates form hexagonal lattices with pitches (center-to-center tube spacing) of either 9.652 or 10.922 centimeters. Reactors reflected radially with 15.24 centimeters of water may be operated by filling the reflector tank as indicated in figure 1(a). Figure 2 presents a cutaway view of a 37-void tube ZPR-II reactor with the void tube array shown more clearly than in figure 1(a). The experimental procedures and the corrections made to the measured critical height to account for small deviations from $20^{\circ}$ C and for fuel solution evaporation during the experiments are described in references 1 and 2. Table I gives the ZPR-II reactor component dimensions, and table II

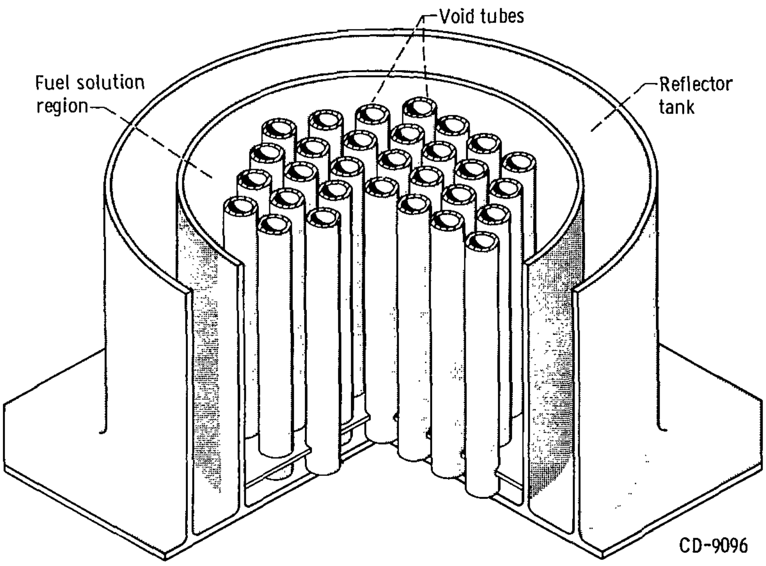



Figure 2. - Cutaway view of ZPR-II reactor tank with 37-void-tube array.

TABLE I. - ZPR-II REACTOR COMPONENT DIMENSIONS

| Component | Outside diameter, cm | Inside diameter, cm | Wall thickness, cm | Bottom thickness, cm | Pitch, cm | Hole diameter, cm |
|---|---|---|---|---|---|---|
| Core tank | 77.559±0.025 | 76.152±0.013 | 0.704±0.018 | 1.27±0.064 | ------------- | 7.691±0.0025 |
| Aluminum tubes | 7.658±0.023 | 6.965±0.023 | .347±0.023 | ---------- | ------------- | ------------ |
| Reflector tank | 110.5±0.64 | 107.95±0.64 | 1.27 | 1.27±0.157 | ------------- | ------------ |
| Spacer grids: | | | | | | |
| 10.922 cm | ------------- | ------------- | ----------- | ---------- | 10.922±0.0025 | 7.691±0.0025 |
| 9.652 cm | ------------- | ------------- | ----------- | ---------- | 9.652±0.0025 | 7.706±0.0025 |

TABLE II. - DESCRIPTION OF CRITICAL ZPR-II

REACTORS WITH VOID ARRAYS

| Case | Fuel concentration parameter, H/X | Void tubes | Pitch, cm | Reflector | Critical height, H, cm |
|---|---|---|---|---|---|
| 1 | 324 | 19 | 9.652 | Water | 20.98 |
| 2 | | 19 | 9.652 | None | 23.20 |
| 3 | | 31 | 10.922 | Water | 28.43 |
| 4 | | 37 | 9.652 | None | 40.76 |
| 5 | 789 | 19 | | None | 41.92 |
| 6 | | 37 | | Water | 64.72 |
| 7 | | 31 | | None | 67.94 |
| 8 | 1082 | 19 | | None | 67.13 |
| 9 | 1082 | 31 | 10.922 | Water | 83.69 |

lists nine voided ZPR-II configurations analyzed in this study. In the following sections, these reactors are referred to by the case numbers assigned in table II.

## Additional Reactor Configurations

Several additional critical solution reactors are considered for specific purposes in developing the calculational model and in emphasizing particular features of it:

(1) Six unvoided ZPR-II reactors, corresponding to the voided cases presented in table II, are computed as reference cases with which to compare criticality calculations of the reactors with voids. Figure 3 shows the ZPR-II reactor tank with no void tubes. The experimental data are reproduced from reference 1 in table III.

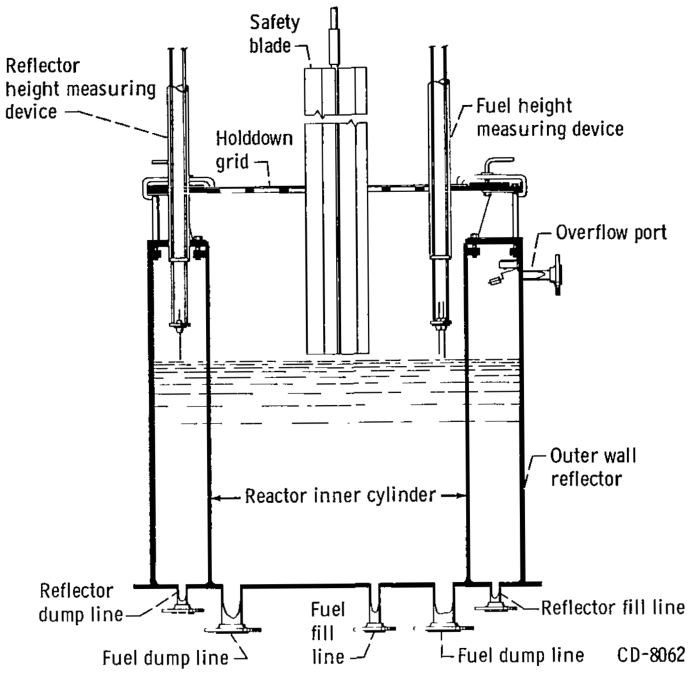



Figure 3. - ZPR-II reactor tank.

TABLE III. - CRITICAL UNVOIDED

ZPR-II REACTORS

| Fuel concentration parameter | Reflector | Critical height, H, cm |
|---|---|---|
| 324 | Water | 15.47 |
| 324 | None | 15.67 |
| 789 | None | 22.81 |
| 789 | Water | 22.35 |
| 1082 | None | 29.97 |
| 1082 | Water | 29.06 |

(2) Three NASA ZPR-I unreflected solution reactor configurations are computed to establish the validity of the axial leakage computations. The cylindrical ZPR-I core tank is 30.48 centimeters in inside diameter (0.3175-cm wall thickness). Experiments were performed at H/X = 454 with a single centrally located, axially oriented 3.175-centimeter-diameter (0.159-cm-wall-thickness) aluminum void tube, with a 3.175-centimeter-diameter aluminum rod, and with no insert. The experimental critical heights were 48.732, 48.725, and 43.152 centimeters, respectively. Note the almost identical experimental critical heights with the aluminum rod insert as compared with

TABLE IV. - CRITICAL ZPR-II 37-TUBE

REFLECTED REACTORS

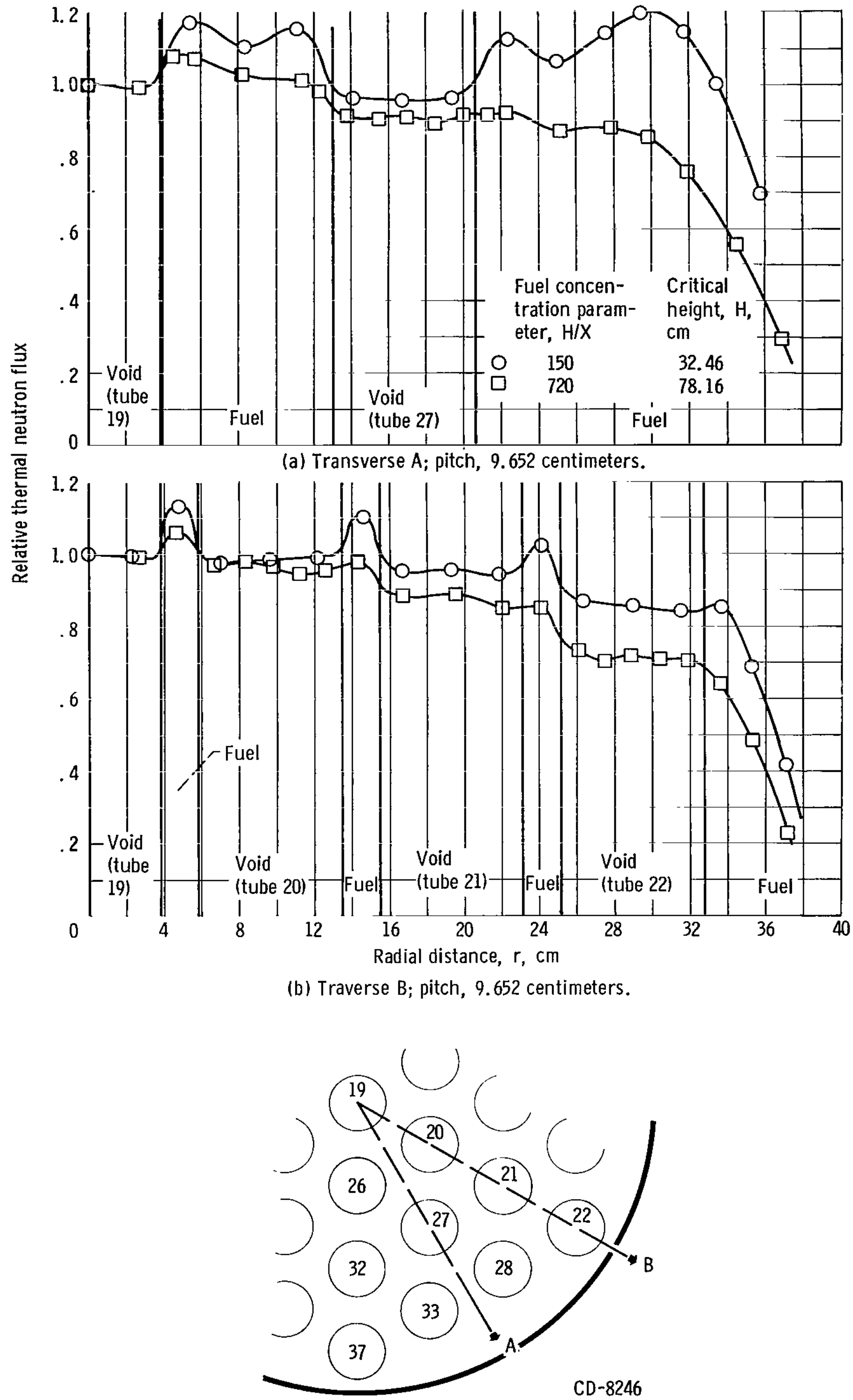
| Fuel concentration parameter, H/X | Absorber tube | | Critical height, H, cm | Pitch, cm |
|---|---|---|---|---|
| | Thickness, cm | Outside diameter, cm | | |
| 634 | 0.051 | 4.684 | 76.91 | 9.652 |
| 865 | None | None | 75.34 | 9.652 |

the void tube insert. For these critical experiments, the compensatory effects of the absorptions and transport mean free path in the aluminum rod make it almost equivalent to the void.

(3) Two 37-tube ZPR-II critical experiments, which are very similar except that one contains a tungsten tube inside each of 37 void tubes, are computed as an additional verification of the calculational model and illustrate its extension to more complex reactors. The experimental data for these two reactors are given in table IV.

## Experimental Thermal Flux Profiles

Flux distributions were measured for several ZPR-II reactor configurations that contain void tubes (ref. 2). Figures 4(a) and (b), obtained from reference 2, shows experimental thermal neutron-flux distributions as measured by dysprosium foil activations. Figure 4(c) shows the radial traverses, in 37 tube reactors, corresponding to the flux profiles shown in figures 4(a) and (b). The critical heights and fuel solution concentrations of the corresponding critical reactors are indicated in figure 4(a). Note that the flux peaking in the fuel solution is much more pronounced at H/X = 150 than for the taller reactor with an H/X = 720. The flux peaking results from the high axial leakage rate in the shorter reactors and from the shorter mean free path in the more concentrated fuel solution. Additional experiments (ref. 2) showed that the magnitude of the flux peaking in the fuel solution varies with the core height at a given fuel concentration. A seven-tube reactor at an H/X = 150 with a height of 16 centimeters shows an increase of 14 percent in the peak flux compared with the corresponding flux peak around the center tube of the reactor shown in figure 4(a). The flux profiles measured in ZPR-II voided reactors indicate that fuel disadvantage factors will be required in the analysis of the critical experiments to account for the reactivity effect of the flux peaking in the fuel solution.




(c) Radial flux traverses.

Figure 4. - Experimental thermal neutron-flux distributions for bare 37-void-tube arrays.

# ANALYTICAL METHODS

The calculational model is first summarized, and then the various techniques, approximations, and calculational details are discussed more thoroughly. The calculational model presented consists of two basic steps:

(1) The first step is the evaluation of axial leakage parameters for the core region that contains void tubes from an axially finite, radially infinite, regular lattice of void tubes in a homogeneous multiplying medium. This evaluation is made with the use of two-dimensional (r-z) five energy group transport calculations in the $S_4P_1$ approximation to the Boltzmann transport equation. The calculational geometry equivalent to the radially infinite lattice is a cylindrical cell centered on a void tube with an annulus of fuel around it chosen so that the volume of fuel associated with the void tube is conserved. Axial symmetry is assumed.

(2) The second step is a solution of the critical experiments. The calculation is performed with the use of the five energy group diffusion theory in the radial dimension. The axial leakage rates from the two-dimensional cell solution are incorporated into the finite reactor calculation by defining axial leakage cross sections for each energy group that yields the proper leakage rate out of the voided region of the reactor.

An adequate solution of reactors that contain voids must properly account for the neutron leakage out of the voided region and for the geometric rearrangement of the fuel solution due to void insertion. The calculational method presented herein is predicated upon the use of rather standard digital computer programs to account for both these problem areas adequately. The first of these, the neutron streaming, and to a lesser extent, the fuel displacement and geometric representation, may be tested by computing a small-diameter unreflected cylindrical reactor with a single centrally located void tube in it. Such a reactor may be represented explicitly in two-dimensional (r-z) geometry and solved with the TDSN transport program (ref. 10).

## ZPR-I Single Tube Analysis

Experiments were done in the NASA Zero Power Reactor-I (ZPR-I) facility expressly for these TDSN calculations. Five group $S_4P_0$ transport calculations with the reactor explicitly represented were done. Table V presents the critical heights for each experiment and the calculated multiplication factors. The first calculation with the void tube insert includes both the neutron-streaming effect and the fuel-displacement effect. The second case, in which the void tube is replaced with a solid aluminum rod of the same diameter, has the same geometric fuel solution arrangement and, therefore, the same fuel-displacement effect. The unvoided critical reactor has neither the fuel displacement

TABLE V. - ZPR-I CRITICAL EXPERIMENTS

AND CALCULATIONS

| Reactor insert | Insert outside diameter, | Critical height, H, cm | Multiplication factor, $K_{eff}$ |
|---|---|---|---|
| Void tube | 3.175 | 48.732 | 1.001 |
| Aluminum rod | 3.175 | 48.725 | 1.001 |
| None | ----- | 43.152 | 1.002 |

nor the streaming. The calculated multiplication factors for the three critical experiments in table V are in very good agreement with the experimental multiplication factor of 1. The capability of accurately computing the neutron leakage rate out of the void tube, as demonstrated by the results in table V for a single tube, suggests that incorporation of this capability into arrays of void tubes could yield satisfactory solutions for critical experiments with multiple voids.

## Equivalence of Voided and Unvoided Reactors

It should be pointed out that, although the calculated multiplication factors in table V are in good agreement with the experimental value of $K_{eff} = 1.0$, the intercomparison of the calculated values is even better (0.1 percent). The infinite medium multiplication factor $K_\infty$ and the total leakage rate and absorption rate per source neutron of critical reactors composed of a given homogeneous fuel penetrated by voids are constant, regardless of the void arrangement and content. Thus, the calculation of unvoided reactors, which represent simple systems for calculational purposes, provides an excellent basis for the comparison of complex voided reactors utilizing the same fuel. The small number of absorptions in the aluminum void tubes of the ZPR-II voided reactors relative to the absorption in the fuel solution does not alter the situation significantly. Therefore, if various minor details, common to both the voided and unvoided reactors, are omitted of necessity or for convenience, valid comparisons may still be made between the calculated multiplication factors, although they may be displaced significantly from 1.

## Evolution of Lattice Cell

The two-dimensional bare cylindrical reactor calculation just discussed may be converted calculationally into an axially finite, radially infinite lattice of void tubes by

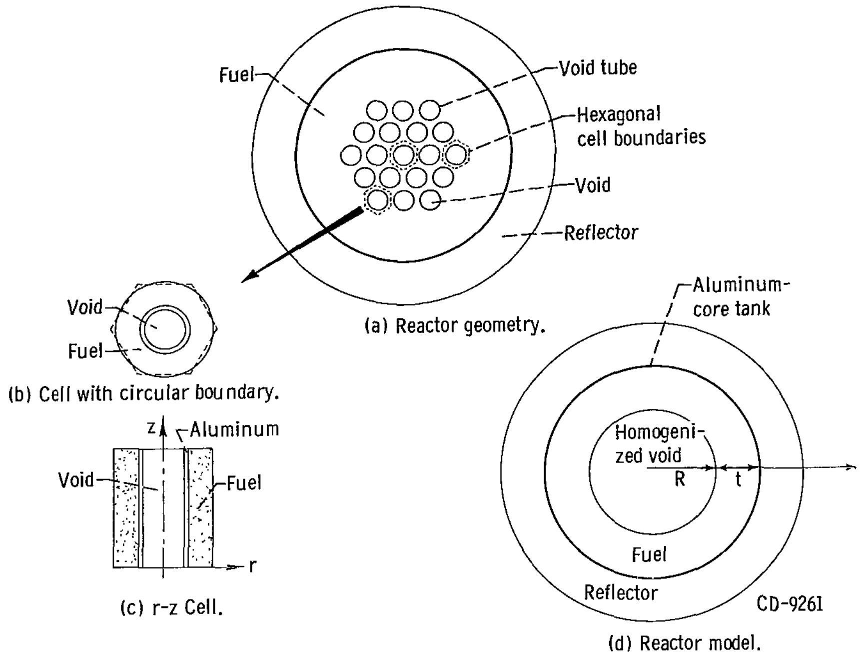



Figure 5. - Calculational geometry for voided ZPR-II reactors.

changing the boundary conditions on the angular fluxes from a "no-return-current" boundary condition to an "isotropic-return-current" boundary condition at the outer radius. The calculational geometry then becomes that of a two-dimensional (r-z) cell that is part of an infinite lattice. The lattices actually calculated are based on the experimental configurations of the reactors listed in table II.

The geometric approximations used in defining the cells are discussed with the aid of figure 5, which presents a sketch of a ZPR-II reactor containing 19 void tubes in a hexagonal array. The aluminum core tank and the reflector are shown. The dotted lines around the center void tube are lines of symmetry. It is assumed that the entire array may be represented by these hexagonal cells centered on each void tube. Figure 5(b) shows one of the cells in more detail with an equivalent (in the sense that fuel area is conserved) circular boundary. The circular boundary is a necessary expedient required to define the r-z cell shown in figure 5(c). The solution of this cell, obtained using an isotropic-return-current boundary condition at the curved surface, constitutes a solution of the axially finite, radially infinite lattice.

## Geometric Representation of Critical Experiments

Figure 5(d) shows the calculational geometry for the finite reactor calculations. The homogenized void-fuel region is obtained by homogenizing the cell of figure 5(b) and

TABLE VI. - CALCULATIONAL GEOMETRY

AND DIMENSIONAL PARAMETERS FOR

ZPR-II REACTORS

(a) Core dimensions

| Number of void tubes | Pitch, cm | | | |
|---|---|---|---|---|
| | 10.92 | | 9.65 | |
| | Radius of voided region, R, cm | Thickness of driver, t, cm | Radius of voided region, R, cm | Thickness of driver, t, cm |
| 0 | 0 | 38.076 | 0 | 38.076 |
| 19 | 24.996 | 13.080 | 22.089 | 15.987 |
| 31 | 31.928 | 6.148 | 28.216 | 9.860 |
| 37 | 34.881 | 3.195 | 30.825 | 7.251 |

(b) Cell dimensions

| Pitch, cm | Outside diameter, cm | Fuel thickness, cm |
|---|---|---|
| 10.922 | 11.472 | 1.907 |
| 9.652 | 10.135 | 1.239 |

(c) Cell volume fractions

| Pitch, cm | Fuel | Aluminum | Void |
|---|---|---|---|
| 10.922 | 0.5541 | 0.0771 | 0.3688 |
| 9.652 | .4291 | .0987 | .4722 |

choosing the radius R so that the area represents that of 19 cells. The annular thickness t of the remaining void-free fuel region is thus specified. The one-dimensional radial reactor calculations consist of a homogenized void region of radius R, an annular fuel-driver region of thickness t, an aluminum core tank, and a reflector. Table VI gives the calculational geometry and the dimensional parameters for both the reactor calculations and the cell calculations. Three types of diffusion theory calculations are done by using the geometry of figure 5(d):

(1) The first type is for unvoided reactors (R=0).

(2) The second type is for homogenized voids as shown where the axial leakage is based on the diffusion theory with group-dependent extrapolation distances at each end of the reactor. The homogenized void-fuel central region is obtained by volume averaging.

(3) The third type, referred to as the r-z cell method, treats the axial leakage out of the voided central region with the use of group-dependent, axial leakage cross sections computed from the solution of the two-dimensional cell of figure 5(c). The axial leakage cross sections are defined as

$$\Sigma_g = \frac{\text{Leakage rate per source neutron for group } g}{\text{Volume integral of group } g \text{ flux}} \quad (\text{cm}^{-1})$$

The homogenized void-fuel central region is obtained by spatial flux-weighting with fluxes from the r-z cell solution.

## Fuel Disadvantage Factors

The experimental thermal flux profiles presented in figure 4(a) show significant flux peaking in the interstitial regions between void tubes for the shorter height, more heavily loaded reactor. The flux peaking results from the high leakage rate in the shorter reactors and from the shorter mean free path in the more concentrated fuel solutions. The flux peaking effect, in which the average flux in the fueled regions is greater than the average flux in the cell, must be accounted for in the calculations of the critical experiments. A disadvantage factor for each region of the two-dimensional cell may be defined as

$$d_{gk} = \frac{V_{cell} \int_k \varphi_{gk} \, dV}{V_k \int_{cell} \varphi_{gk} \, dV}$$

where $V$ is the indicated cell volume, and $\varphi_{gk}$ is the flux for group $g$ and region $k$ obtained from the $2D\text{-}S_4P_1$ cell solution. The cross sections for the homogenized cell containing disadvantage factors are calculated for each group $g$ as

$$\Sigma_{g,\,cell} = \sum_k \Sigma_{kg} d_{gk} \frac{V_k}{V_{cell}}$$

where $\Sigma$ is a general cross section. For example, $\Sigma$ may be the macroscopic absorption cross section or the scattering cross section. The incorporation of cell disad-

TABLE VII. - ENERGY GROUP STRUCTURE AND

NORMALIZED FISSION SPECTRUM

| Group | Lower energy[a] | | Lower lethargy[b] | Normalized fission spectrum, $\chi_g$ |
|---|---|---|---|---|
| | eV | J | | |
| 1 | $0.821\times10^{6}$ | $1.31\times10^{-13}$ | 2.5 | 0.7538 |
| 2 | $5.531\times10^{3}$ | $8.85\times10^{-16}$ | 7.5 | .2462 |
| 3 | 3.06 | $4.90\times10^{-19}$ | 15.0 | 0 |
| 4 | .414 | $6.62\times10^{-20}$ | 17.0 | 0 |
| 5 | 0.0 | 0.0 | $\infty$ | 0 |

[a]Upper starting energy, $14.9\times10^{6}$ eV ($2.39\times10^{-12}$ J).
[b]Upper starting lethargy, -0.4.

vantage factors into the cell cross sections by this method is equivalent to averaging the group cross sections by spatial flux weighting.

## Computer Programs and Neutron Cross Sections

The TDSN program with the $S_4P_1$ approximation is used for all two-dimensional cell calculations. All diffusion theory calculations utilized RP1, a NASA one-dimensional, multigroup digital program. Five energy groups are used for all calculations with the exception of a special case discussed in connection with reactors containing tungsten. The group structure and the normalized fission neutron spectrum are presented in table VII. The GAM-II program (ref. 11) is used in the B1 approximation to compute the neutron spectrum that results from the interaction of fission neutrons with the slowing down medium. Macroscopic cross sections for the first four groups for each region shown in figure 5(d) are averaged over the resulting spectrum of the respective regions. The TEMPEST program (ref. 12) is used similarly for group 5. A thermalization calculation that uses the Wigner-Wilkins option is performed, and one-group macroscopic cross sections are averaged over the resulting spectrum.

# RESULTS FOR AXIALLY FINITE, RADIALLY INFINITE LATTICES

The results of the two-dimensional cell calculations representing solutions of axially finite, radially infinite lattices may not be compared directly with experimental critical height data. A discussion of these calculations is in order, however, because the results have a direct bearing on the critical experiment calculations in the next section.

For example, a comparison with homogenized void cell calculations shows that significant discrepancies in finite reactor multiplication factors could be expected with the use of homogenized cells because of two deficiencies: (1) an underestimation of the axial leakage, and (2) homogenization alone does not properly account for the flux depression in the void region of the cell due to axial streaming.

## Multiplication Factors for Cells

Table VIII gives the multiplication factors for the finite height cells for each of the nine cases presented in table II. The 2D-$S_4P_1$ calculations of the r-z cell were done with the TDSN program. The homogenized cell calculations were done with the RP1 program. All the calculations assumed axial symmetry; that is, the 1.27-centimeter-thick aluminum-core tank bottom was not included. The reflector effect of the tank bottom is assumed to be a separable effect that can be incorporated as an additive correction to the multiplication factors.

The multiplication factors from the two-dimensional cells range from 0.624 to 1.152. All 19 tube cases show $K_{eff} < 1.0$, which indicates that the neutron production-rate distribution in the corresponding reactor must be decreasing radially toward the center of

TABLE VIII. - COMPARISON OF CALCULATED CELL MULTIPLICATION FACTOR FOR CRITICAL ZPR-II VOIDED REACTOR

[Axially symmetric, finite height calculations.]

| Case | Cell description | | | | | Two-dimensional cell solution, r-z cell, 2D-$S_4P_1$ | Homogenized cell | |
|---|---|---|---|---|---|---|---|---|
| | Fuel concentration parameter, H/X | Number of void tubes | Pitch, cm | Reflector | Critical height, H, cm | | One-dimension diffusion | One-dimension diffusion with disadvantage factor dg |
| 1 | 324 | 19 | 9.652 | Water | 20.98 | 0.624 | 0.624 | 0.672 |
| 2 | | 19 | 9.652 | None | 23.20 | .688 | .704 | .744 |
| 3 | | 31 | 10.922 | Water | 28.43 | 1.024 | 1.061 | 1.088 |
| 4 | | 37 | 9.652 | None | 40.76 | 1.098 | 1.147 | 1.161 |
| 5 | 789 | 19 | | None | 41.92 | .860 | .935 | ----- |
| 6 | | 37 | | Water | 64.72 | 1.093 | 1.155 | ----- |
| 7 | | 31 | | None | 56.94 | 1.116 | 1.174 | ----- |
| 8 | 1082 | 19 | | None | 67.13 | .988 | 1.045 | ----- |
| 9 | 1082 | 31 | 10.922 | Water | 83.69 | 1.152 | 1.189 | ----- |

the core. The driver region of the fuel solution supplies the additional neutrons required for criticality. The neutron production-rate distribution for case 8 with $K_{eff} = 0.988$ will be almost constant radially through the voided region since few neutrons are required from the driver region. On the other hand, $K_{eff}$ is greater than 1 for all the 31- and 37-tube cases which indicates that neutrons must leak radially to result in critical reactors.

The eighth column of table VIII results from one-dimensional diffusion theory calculations in which the cell is homogenized by volume averaging. Homogenizing the cell implies that there is no anisotropy in the radial and axial neutron migration within the cell. Thus, no attempt is made to account for preferential streaming axially out of the voids; however, the average mean free path is increased over that of the full-density fuel solution by the homogenization procedure. The increase in the mean free path that results is equivalent to the isotropic correction to the migration area given by Behrens (ref. 6) as a lower limit. The additional terms in Behrens' equation increase the amount of leakage and, as pointed out in reference 8, overestimate the leakage for these reactors. It is noted that all of the homogenized void multiplication factors in the eighth column of table VIII are higher, as expected, than those from the two-dimensional cells, except for case 1. This case is the shortest reactor lattice examined (20.98 cm). Agreement in this case is fortuitous; the large positive reactivity effect associated with the fuel disadvantage factors is not included in the homogenized cell results.

Examination of the thermal flux profiles reported in reference 2 for ZPR-II voided reactors shows significant flux peaking for the shorter height reactors in the interstitial regions between void tubes. Figure 4(a), from reference 2, shows that flux peaking is negligible for the taller cores with the lower density fuel solutions. It is instructive to see how the fuel disadvantage factors affect the homogenized cell multiplication factors.

The ninth column of table VIII shows the calculated multiplication factors from homogenized cells containing disadvantage factors. The reactivity difference (eighth and ninth columns) due to the disadvantage factors decreases rapidly with increasing core height. The reactivity difference is 11.5 percent for case 1 (20.98 cm) but only 1 percent for case 4 (40.76 cm).

## Axial Leakage Rates from Cells

Table IX gives axial leakage rates per source neutron for cases 5 and 7. These cases are representative of all the cases examined. The second column shows by energy group the axial leakage rate per source neutron as computed from the two-dimensional (r-z) $S_4P_1$ cell solution. This leakage rate is compared with that of the third column obtained from diffusion theory solutions for the homogenized cell. The total axial leakage rate per source neutron predicted by the 2D cell solution is substantially greater

TABLE IX. - AXIAL LEAKAGE RATES PER SOURCE NEUTRON FROM AXIALLY SYMMETRIC CELL CALCULATIONS

| Energy group | Two-dimensional cell solution, 2D-$S_4P_1$ (r-z) | One-dimensional homogeneous cells[a] | |
|---|---|---|---|
| | | Diffusion | $S_4P_1$ |
| Case 5 | | | |
| 1 | 0.10581 | 0.13601 | 0.12375 |
| 2 | .09629 | .09713 | .09047 |
| 3 | .07061 | .05413 | .05780 |
| 4 | .01665 | .01238 | .01336 |
| 5 | .10635 | .04253 | .05591 |
| Total | 0.39571 | 0.34218 | 0.34129 |
| Case 7 | | | |
| 1 | 0.05219 | 0.07129 | 0.06149 |
| 2 | .04952 | .04868 | .04494 |
| 3 | .03843 | .02713 | .02952 |
| 4 | .00921 | .00625 | .00688 |
| 5 | .06692 | .02097 | .02961 |
| Total | 0.21627 | 0.17432 | 0.17244 |

[a]No fuel disadvantage factors are included in these calculations.

than that from the homogenized cell. This results from greater streaming from the lower energy groups, primarily from the thermal (group 5) neutrons. In the explicit void of the two-dimensional case, neutrons traveling within the solid angle subtended by the cross-sectional exit area of the void have a probability of 1 of leaking out regardless of their energy. Contrast this to the homogenized void in which variations of the mean free path with energy tend to reduce thermal neutron leakage. As pointed out earlier, the reduction in neutron leakage accounts for the lower critical height of the single-tube ZPR-1 reactor when an aluminum rod is inserted in place of the void tube (table V).

The fourth column of table IX shows the axial leakage rate per source neutron for homogenized cells of cases 5 and 7 as computed with one-dimensional $S_4P_1$ transport calculations. These results, compared with those calculated with diffusion theory, show some variation among the groups but show essentially the same total leakage. An intercomparison of the second to fourth columns leads to the conclusion that the axial leakage must be calculated with the two-dimensional explicit cell geometric approximation rather than with a one-dimensional homogenized cell. The degree of sophistication of the one-dimensional calculations ($S_4P_1$ against diffusion theory) is irrelevant. The one-dimensional calculations are incapable of estimating the axial leakage properly.

## RESULTS OF ANALYSES OF CRITICAL EXPERIMENTS

The axially finite, radially infinite lattice (finite height cells) discussed in the previous section must be incorporated into finite reactor calculations of the experimental configuration to draw any conclusions as to the validity of the method. Two gross assumptions are put to the test in the finite reactor calculations:

(1) That the axial leakage cross sections, as defined from the two-dimensional cells and the homogenized void region cross sections, adequately represent the center voided region.

(2) That converting the complex outer boundary of the voided region (figs. 5(a) and (d)) into a circular boundary to permit one-dimensional radial calculations is valid.

### Comparison of Voided Reactor Calculations With Reference Unvoided Cases

Table X gives the calculated multiplication factors for each of the nine reactors

TABLE X. - COMPARISON OF VOIDED AND UNVOIDED REACTOR MULTIPLICATION FACTORS

[One-dimensional radial diffusion theory and r-z cell method calculations.]

| Case | Description | | | | | | Calculated multiplication factor, $K_{eff}$ | |
|---|---|---|---|---|---|---|---|---|
| | Fuel concentration parameter, H/X | Number of void tubes (a) | Pitch, cm (a) | Reflector effect | Critical height, H, cm | | | |
| | | | | | Voided | Unvoided | Voided | Unvoided |
| 1 | 324 | 19 | 9.652 | Reflected | 20.98 | 15.47 | 0.999 | 0.999 |
| 2 | | 19 | 9.652 | Bare | 23.20 | 15.67 | 1.004 | 1.002 |
| 3 | | 31 | 10.922 | Reflected | 28.43 | 15.47 | 1.003 | .999 |
| 4 | | 37 | 9.652 | Bare | 40.76 | 15.67 | 1.014 | 1.002 |
| 5 | 789 | 19 | | Bare | 41.92 | 22.81 | 1.010 | 1.008 |
| 6 | 789 | 37 | | Reflected | 64.72 | 22.35 | 1.016 | 1.007 |
| 7 | 789 | 31 | | Bare | 67.94 | 22.81 | 1.010 | 1.008 |
| 8 | 1082 | 19 | | Bare | 67.13 | 29.97 | 1.009 | 1.006 |
| 9 | 1082 | 31 | 10.922 | Reflected | 83.69 | 29.06 | 1.016 | 1.006 |

[a]The number of void tubes and pitch apply only to voided reactors.

presented earlier in table II. The description of both the voided and unvoided reactors for each case is given. The critical height of the voided and corresponding unvoided reactor at the same value of H/X and with the same reflector is given along with the multiplication factors. The effectiveness of the r-z cell calculational method for treating the voided reactors is better seen by comparing the pairs of multiplication factors in table X than by comparing them with the experimental $K_{eff} = 1.0$. Differences of less than 0.5 percent $\Delta K$ are noted, except for the reactors that contain 37 void tubes (cases 4 and 6) and for the tallest reactor (case 9), where differences of from 0.9 to 1.2 percent $\Delta K$ are observed. The definition of the interface between the fuel solution driver region and the voided region in the calculations may be responsible for these discrepancies. A contributing factor is the relatively high multiplication factor (table VIII) of the voided region, which indicates substantial radial leakage into the driver region. It is further noted that the thicknesses of the driver regions for the 31-tube cases (10.922-cm pitch) and the 37-tube cases (9.652-cm pitch) are about the same (table VI). The driver thicknesses are 6.148 and 7.251-centimeters, respectively. The effect of the high lattice multiplication factors can be seen by comparing cases 3 and 9, which have the same number of void tubes and pitch. The value of 1.024 for case 3, compared with 1.152 for case 9, indicates a relatively flat fission neutron source distribution through the voided region, which results in little interaction of the voided region and driver region. A more accurate evaluation of the multiplication factor is also indicated. Cases 2, 5, and 8 differ only in H/X and critical heights. The multiplication factors in table X for these cases indicate that the calculational model is not sensitive to the length of the reactors.

All the multiplication factors in table X were corrected additively for the reactivity worth of the aluminum core tank bottom. The 1.27-centimeter-thick tank bottom is a moderately effective reflector, especially for the reactors with H/X = 324. One-dimensional axial calculations ($S_4P_0$) indicate reactivity worths of 1.8 percent $\Delta K/K$ at H/X = 324 but only 0.8 percent at H/X = 789 and 0.4 percent at H/X = 1082 for the unreflected, unvoided reactors. Radial-axial buckling-synthesis calculations of the bare voided (case 2) and unvoided reactors at H/X = 324 showed that the same tank bottom reactivity worth carried over to the voided cases. Inclusion of the tank bottom reactivity worth does not alter the comparisons between the voided and unvoided reactors.

## Comparison of r-z Cell Method and Homogenized Cell Method

The multiplication factors for cells were given in table VIII and discussed previously in the section on radially infinite lattices. A comparison of calculated reactor multiplication factors made with the r-z cell and the homogenized cell methods shows the magnitude of discrepancies that may be expected if the axial leakage is not properly accounted for. The homogenized cell method utilizes volume-averaged cross sections and the diffusion theory axial leakage in the homogenized void-fuel region of the gross reactor calculations. The r-z cell method utilizes spatially flux-weighted cross sections, which, therefore, contain fuel disadvantage factors, and axial leakage cross sections in the gross reactor calculations.

Table XI shows the multiplication factors computed by the two methods. The reactors and case numbers are the same as discussed previously. The data of the second column are repeated from table X as computed by the r-z cell method. The third column gives the multiplication factors computed with the homogenized cell method. The data in the third column are all larger than those in the second column by as much as 3.7 percent $\Delta K$ (case 7), which indicates less axial leakage. The observed differences for the first four cases would be increased if the fuel disadvantage factors had been included in the calculations for the third column. Values of 0.6, 0.7, 1.1, and 0.8 percent $\Delta K$ for cases 1 to 4 were obtained for the effect of the disadvantage factors in the finite reactors. These values are much lower than those obtained in the infinite lattice case because the homogenized void region constitutes only a part of the total reactor fuel. The importance of including the disadvantage factors should not be underestimated. They are required for consistency in interpreting the results for these voided reactors, especially for the short cores with many void tubes. In addition, for more practical gas-cooled reactors, materials, which may absorb neutrons but increase the effect of the disadvantage factors

TABLE XI. - MULTIPLICATION FACTORS FOR VOIDED REACTORS CALCULATED WITH DIFFERENT METHODS FOR CELL SOLUTION

[One-dimensional radial diffusion theory calculations.]

| Case | Multiplication factor, $K_{eff}$ | |
|---|---|---|
| | r-z Cell method | Homogenized cell method |
| 1 | 0.999 | 1.000 |
| 2 | 1.004 | 1.030 |
| 3 | 1.003 | 1.007 |
| 4 | 1.014 | 1.045 |
| 5 | 1.010 | 1.033 |
| 6 | 1.016 | 1.052 |
| 7 | 1.010 | 1.047 |
| 8 | 1.009 | 1.028 |
| 9 | 1.016 | 1.045 |

even though the core is much taller, may be placed within the void tubes. Axial streaming may preclude accurate calculation of the disadvantage factors unless the two-dimensional cells are used. The succeeding section presents an example to emphasize these points.

### ZPR-II Reactor With Tungsten Tubes Inside Void Tubes

Two ZPR-II experiments are available that are very similar except that one had a 0.051-centimeter-thick tungsten tube inserted into each of 37 void tubes. The experimental data are given in table IV. As part of a NASA contract with General Atomic, Peak and Cohen calculated these ZPR-II criticals. The results of their analysis is reported in reference 8. The void tube case was computed similarly to the calculations for the homogenized void reactors in this report, except that four-group $S_4P_1$ transport radial calculations were done. The resulting $K_{eff}$ was 1.025 for the voided reactor. (This is the same voided reactor for which both Behrens' and Benoist's methods gave a $K_{eff}$ of 0.94, as calculated by Peak and Cohen.) Calculations with different numbers of energy groups did not show any appreciable difference. For the tungsten case, one-dimensional-transport-cell solutions were used to compute collision probability tables for resonance calculations of the effective cross sections of tungsten. Disadvantage factors for each material region were also obtained from the one-dimensional cells. The resulting $K_{eff}$ for the reactor containing tungsten was 0.971. The difference in $K_{eff}$ for these two cases is 5.4 percent.

The cross sections for four energy groups were sent to the Lewis Research Center by Peak and Cohen for use in the two-dimensional (r-z) cell method. The reactor multiplication factors computed by this method were 1.010 for the voided reactor and 0.998 for the reactor containing tungsten. Thus, the previous difference of 5.4 percent $\Delta K$ is reduced to 1.2 percent $\Delta K$. The disadvantage factors from the r-z cell contributed 1.3 percent $\Delta K$ to the multiplication factor for the reactor with tungsten inserts. The one-dimensional model could not compute the axial leakage or the disadvantage factors properly.

## CONCLUSIONS

The model used to estimate axial leakage parameters based on the two-dimensional transport solution of cells that contain discrete voids is accurate for the wide range of critical experiments calculated.

The results indicate that the implicit assumption of radial and axial separability of fluxes is valid. The geometric approximations made in defining the central cylindrical voided cell appear valid. Annularizing the fuel-driver region outside the voided core region works rather well for cases that have a thick driver region. A small but noticeable discrepancy appears to be associated with the representation of the complex interface between void tubes and the fuel-driver region by a circular boundary.

Homogenization of the voided region by spatial flux-weighting for use in the radial full-core calculations and the subsequent results indicate little or no significant anisotropy in the radial direction. The axial leakage does indicate anisotropy and is treated adequately with the use of the method described.

Fuel disadvantage factors are significant in the shorter reactors with the greater fuel concentrations and may be obtained from the two-dimensional cell solutions. For the same reactors, the reflector effect of the core tank bottom must be accounted for in predicting reactor criticality. This is not necessary for determining the validity of the method, however, since the leakage-modified homogenized void calculations based on axial symmetry may be compared with the clean critical results also based on axial symmetry.

The model is based on a direct calculation with widely used computer programs. The techniques are not limited to voided core applications, except that an equivalent cell must be defined for each particular geometry to account for the axial leakage and associated disadvantage factors. The full-core radial calculations are not restricted to one-dimensional solutions. Two-dimensional calculations (x-y or r-$\theta$ geometry) diffusion theory or transport theory may be used for the radial calculations.

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