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Yearly Report on Nonlinear Analysis of Shells of Revolution

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By

James A. Stricklin
Principal Investigator

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Texas Engineering Experiment Station
Texas A&M University
College Station, Texas

ABSTRACT

This report presents a summary of the research accomplishments and a description of the work in progress. The development of a computer code for the analysis of shells of revolution with symmetrical stiffness properties is 95% complete. A computer code for the linear analysis of shells of revolution with asymmetrical stiffness properties is complete and the computer code for the nonlinear analysis is 75% complete. Results from the computer codes have been compared with experimental and other theoretical results. Excellent agreement was obtained. A paper entitled "Nonlinear Analysis of Shells of Revolution by the Matrix Displacement Method" which pertains to this research was presented at the 6th AIAA Aerospace Sciences Meeting, New York, January 24, 1968. Another paper and two reports are in preparation. All research efforts are on or ahead of schedule.

For the coming year refinements of the present analysis techniques are planned. The following additional studies are planned.

1. Include thermal stresses in the computer codes.
2. Apply the computer codes to the analysis of a test structure, and compare the theoretical results with experimental results being obtained by Dr. Stebbins of the Manned Spacecraft Center.
3. Apply the computer codes to the stability analysis of pressure vessels and compare the theoretical results with experimental results being obtained by Mr. MacDougall of Sandia Corporation.

4. Develop finite elements for the analysis of shells of arbitrary shape.
5. Initiate development of finite elements for the analysis of bodies of revolution and three dimensional solids.
6. Study the dynamic response of shells of revolution under impulsive loadings.

INTRODUCTION

In January of 1967 the National Aeronautics and Space Administration awarded Texas A&M University a research grant for the analysis of shells of revolution with asymmetrical stiffness properties. In April of the same year Sandia Corporation awarded a contract for the non-linear analysis of orthotropic shells of revolution. The research efforts are aimed at two different objectives. In particular the objectives of the Sandia Contract are to develop a computer code which may be used to analyze filament wound pressure vessels and to use the computer code to explain the discrepancies between linear theoretical results and experimental results. The NASA grant has the objective of developing computer codes which are useful in the analysis of present and future space vehicles. The research efforts do, however, have many features in common and consequently have been treated as a large research effort with joint support. This arrangement has proven highly beneficial to all parties.

Concurrent with the theoretical study, Mr. Hugh MacDougall of Sandia Corporation and Dr. Fred Stebbins of the Manned Spacecraft Center undertook experimental investigations to obtain data for the purpose of checking the theoretical results. Thus, the research effort involves coordination among Texas A&M University, NASA, and Sandia Corporation.

The theoretical analysis is based on the earlier work presented in Refs. 1-4. Grafton and Strome[1] used the matrix displacement method in conjunction with conical frustum elements to analyze shells of revolution under axisymmetric loading. Percy, Pian, Klein, and Navaratna[2] extended the research of Grafton and Strome to include the analysis of shells of revolution under arbitrary loadings. Stricklin, Navaratna, and Pian[3] introduced the curved element used here and Haisler and Stricklin[4] conducted studies on the behavior of the curved element.

There are currently investigations on curved elements being conducted at many research centers. While results will undoubtedly vary with the problems under consideration, the results for shells of revolution are typified by the results given in Ref. 3 and shown in Fig. 1. In this figure the meridional bending moment for a parabolic shell under uniform loading is shown as obtained using a conical frustum element, the curved element, and asymptotic integration. It is noted that the curved element results and the results obtained through asymptotic integration agree quite well while the conical frustum element yields some extraneous points. Furthermore, the number of elements required using curved elements is considerably less than the number required using conical frustum elements. This latter consideration is important in non-linear problems where the computer time required is a major factor. The conclusion is that the curved element is both necessary and practical for shells of revolution.

While the present research effort has concentrated on the analysis of shells of revolution, there are many other types of problems of interest to NASA and Sandia Corporation which can be solved by the direct stiffness method. Two such problems are the analysis of bodies of revolution and arbitrary shells. The analysis of bodies of revolution is of interest to a group at Sandia Corporation and is of interest to the Manned Spacecraft Center as applied to the earth landing of space vehicles. Research in this area is currently underway in the Civil Engineering Department at Texas A&M University and their efforts are being coordinated with planned efforts under this research support. A description of all the planned research is presented in this report.

RESULTS AND PLANNED RESEARCH

Non-linear Analysis of Shells of Revolution

The computer code for the non-linear analysis is operational and a paper[5] has been presented on this topic. Typical results from this paper are presented in Figs. 1A and 1B. Copies of the reprint will be forwarded later.

In this study the non-linear terms are treated as additional generalized forces and the resulting equations of equilibrium solved by iteration. The theoretical results were compared with experimental results and excellent agreement was obtained. This computer code has been delivered to Sandia Corporation.

Future work on this program pertains to simplifying the input data and writing a report giving the details of the theory and a listing of the computer code. At present simple shapes such as a spherical cap, torus, or a cylinder may be input quite simply whereas other shapes require a detailed description of the shell structure. Research is currently being conducted to permit the analysis of parabolic and conical sections using the simplified input format. With this inclusion it is believed that a detailed description of the structure will be needed in only rare cases.

Analysis of Shells of Revolution with Asymmetrical Stiffness Properties

The computer code for the linear analysis of shells of revolution with asymmetrical stiffness properties is operational and has been used for the analysis of several problems. The basic difference between this analysis and the one discussed in the previous section is that all the Fourier harmonics are coupled in this analysis and consequently the analysis is much more complicated.

The theoretical details will be presented in a report which is in preparation. Typical theoretical results for the Apollo aft heat shield are shown for a 150 PSI loading over a circle with a 20" radius and centered at a 15° angle from the apex. This loading is encountered during water impact. The results show that the shear resultant is slightly below the allowable values for the honeycomb core. The computer code for the linear analysis has been presented to the Manned Spacecraft Center and is currently in use at this center.

The development of the theory for the non-linear analysis of shells of revolution with asymmetrical stiffness properties is complete and has been programmed. Figures 3 and 4 present some typical results from the analysis. Figure 3 shows the linear and non-linear shear resultants in the aft heat shield under a loading of 300 PSI centered at 15° from the apex and covering a circle with a radius of 20". Figure 4 presents the meridional bending moment of the same structure under the same loading.

It is noted that the structure does not show a high degree of non-linearity. It is believed that the lack of non-linearity is due to the large thickness (2") of the aft heat shield. Other problems have shown non-linearities which increase the stress by 25%.

The limiting case of a shell of revolution with asymmetrical stiffness properties is a shell of revolution with a cut-out. Since this is a very important problem a considerable effort is being spent to see if the numerical method can be used to analyze "cut-out" problems. Present results are promising but this study is expected to continue for the next six months.

In summary, the feasibility of this type of analysis has been demonstrated. Future efforts will be devoted to refining the computer code and testing its range of applications.

Thermal Loadings

The inclusion of thermal loadings in the numerical programs is progressing quite well. The basic theory is well understood and work is underway to determine the equivalent thermal loadings. The thermal loadings will first be incorporated in the computer code for the linear analysis and then be incorporated in the non-linear program. The results from the linear analysis will be compared with those obtained by Cwiertny and the non-linear program will be used to study thermal buckling.

Test Structure Analysis

Many structures are composed of shells with beam reinforcements and consequently there is a need for a numerical program to analyze such configurations.

such structure is currently being tested experimentally and analyzed by Dr. Stebbins of the Manned Spacecraft Center. Mr. Jose DeAndrade, a graduate student at Texas A&M University, will modify the numerical program to analyze shells of revolution with beam reinforcements. Then, during the summer, he plans to work at the Manned Spacecraft Center and apply the numerical program to the analysis of the structure being tested.

Static Stability Analysis

The computer code for the non-linear analysis of shells of revolution yields buckling loads as a limiting case. Several problems have been analyzed for stability but no comparison with experimental data has been made. It is planned to compare theoretical and experimental results, especially the experimental results being obtained by Mr. MacDougall of Sandia Corporation.

Arbitrary Shell Analysis

Many finite element representations have been developed for use in the analysis of arbitrary shells. However, practically all of the current representations have theoretical defects. The most common are the lack of slope compatibility or invariance of the form of the displacement function under translation of coordinates.

A formulation has been developed at Texas A&M which satisfies all of the essential conditions. For example it satisfies the condition set forth by Key.[7]

In essence the formulation consists of using a strain energy expression which depends on the displacements and their first derivatives only. With this formulation it is possible, in accordance with the calculus of variations, to use a displacement function with continuous displacements between elements but with discontinuous first derivatives. Many displacement functions exist which satisfy both this condition and the condition that all terms of a certain order should be included. The latter condition is equivalent to the condition of invariance under translation of coordinates.

The development of a computer code for the analysis of arbitrary shells will proceed in two phases. In the first phase the concept will be tested on beams, plates and shells of revolution. This will consist of using displacement functions of different orders and the inclusion or deletion of curvature terms. It is expected that the first phase will be completed and reported on during this calendar year. The second phase will be the development of a general numerical program and its application to some unsolved problems.

Non-linear Dynamic Analysis

The non-linear dynamic analysis of shells of revolution is of interest to Sandia Corporation for impulsive loadings and is of interest to NASA for impact loadings. The dynamic analysis program is greatly simplified by the development of non-linear static analysis techniques. It is necessary to include the dynamic forces and modify the method used to solved the equations of equilibrium.

It was originally planned to solve the equations by using finite difference relations in time and some progress has been made along these lines. However, an inherent difficulty exists in the use of finite difference relations in that divergent solutions may be obtained if a proper time interval is not chosen. Consequently, other less sensitive methods are currently being stuided.

After a method is selected it is planned to apply the computer code to the impulsive buckling of shells of revolution under axisymmetric loadings, permitting the shell to buckle asymmetrically. It is planned to compare theoretical results with any available experimental results.

PERSONNEL

The overall research is under the direction of Dr. James A. Stricklin with assistance from Dr. Jose E. Martinez. Biographical data sheets for Dr. Stricklin and Dr. Martinez are attached. The planned research efforts are designed to give the students working on the project a topic leading to a thesis, dissertation and/or AIAA publication. These students, and their topics are:

Jose DeAndrade	Test Structure Analysis
Walter Haisler	Arbitrary Shells
Joseph Beck	Thermal Stresses
Steven Huzar	Dynamic Analysis
John Abshier	Stability Analysis
Charles Hoover	Bodies of Revolution

Several undergraduate students are assisting in the analyses.

SUMMARY

During the past year two computer codes have been developed and used in the analysis of problems of interest to Sandia Corporation and NASA. Future areas of investigation have been presented.

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4. Haisler, W.E. and Stricklin, J.A., "Rigid-body Displacements of Curved Elements in the Analysis of Shells by the Matrix Displacement Method," AIAA Journal, Vol. 5, No. 8, August 1967, pp. 1525-1527.

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6. Cwiertny, A.J., "Thermal Stress Analysis of Single-Layered and Soft-Bonded Double-Layered Shells of Revolution," Aeroelastic and Structures Research Lab., Report TR 139-7, Massachusetts Institute of Technology, Cambridge, Massachusetts.

7. Key, S.W., "A Convergence Investigation of the Direct Stiffness Method," Ph.D. Thesis, University of Washington, 1964.

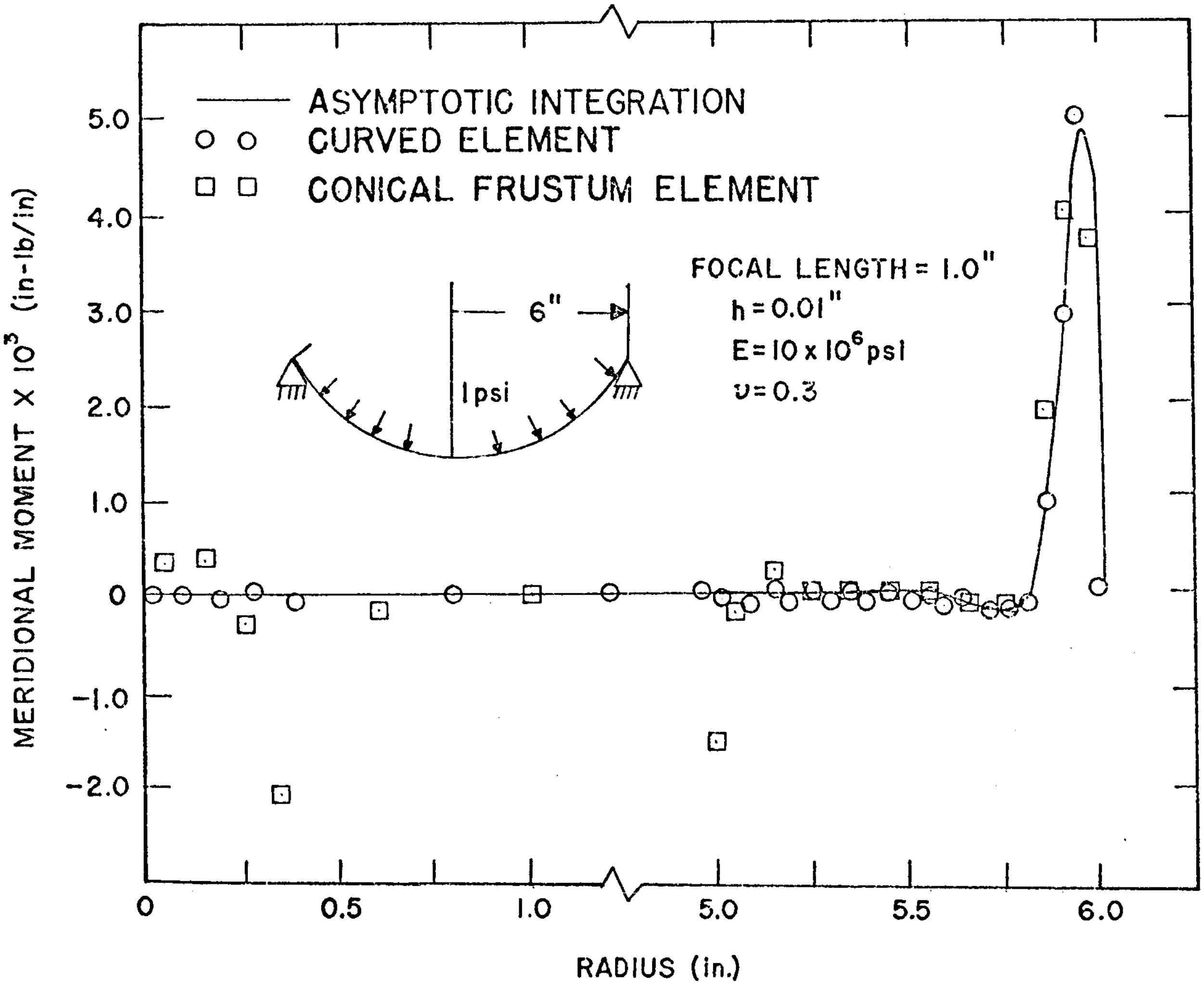

Fig. 1 Meridional bending moment in simply supported parabolic shell under uniform internal pressure

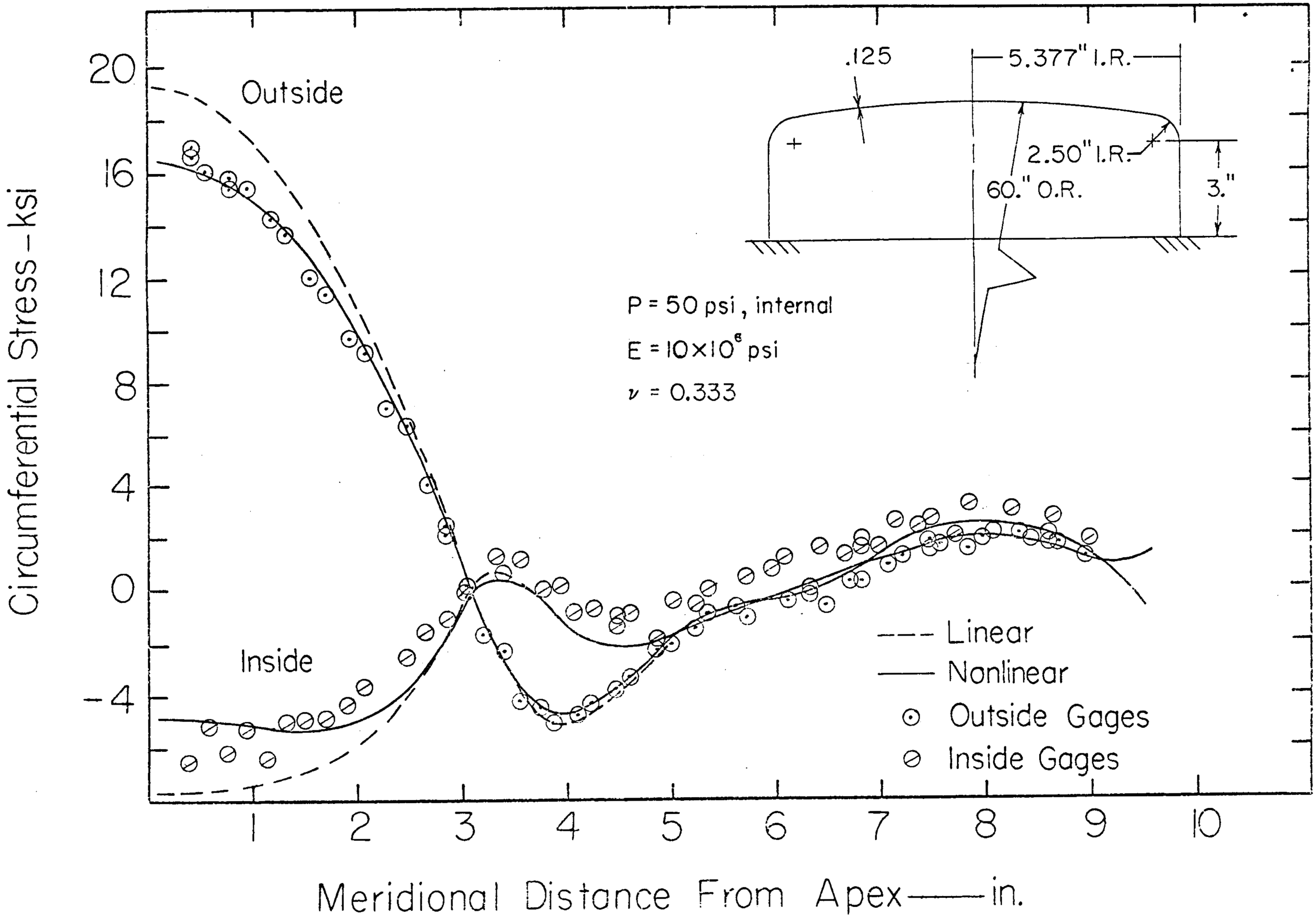

FIG.1A CIRCUMFERENTIAL STRESS ALONG ARC LENGTH

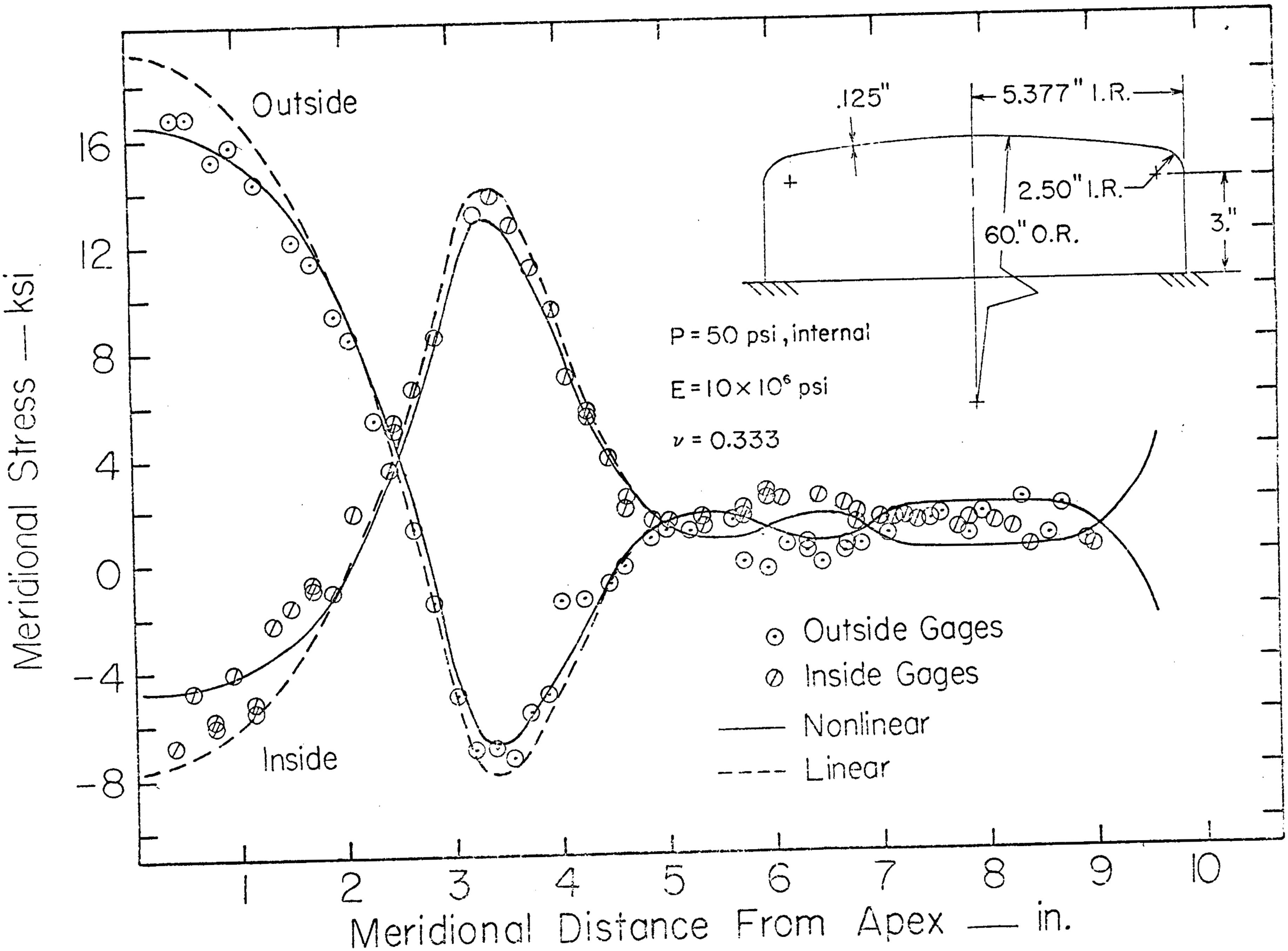

FIG. 1B MERIDIONAL STRESS ALONG ARC LENGTH

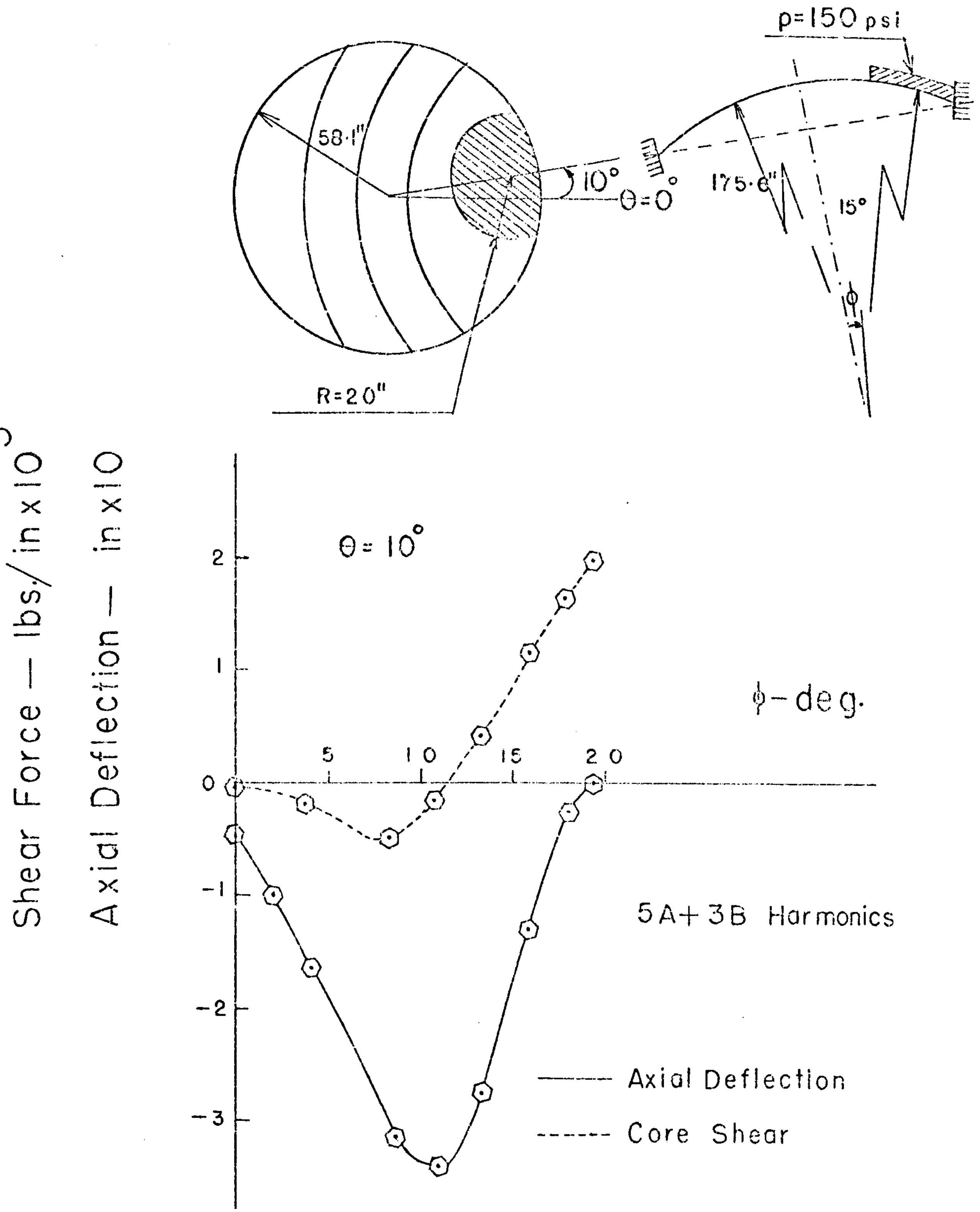

Fig. 2 AXIAL DEFLECTION AND SHEAR

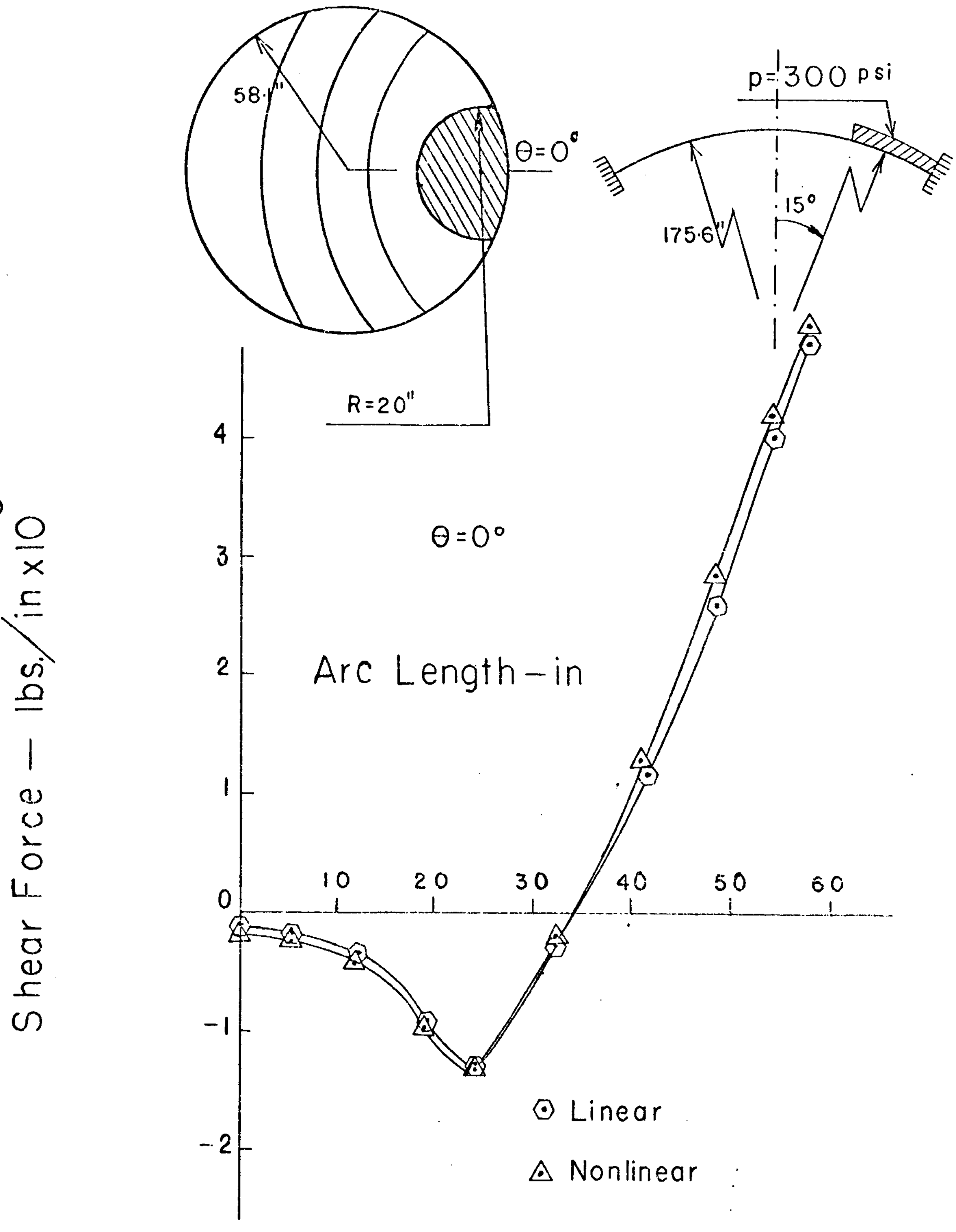

Fig. 3 SHEAR ALONG ARC LENGTH

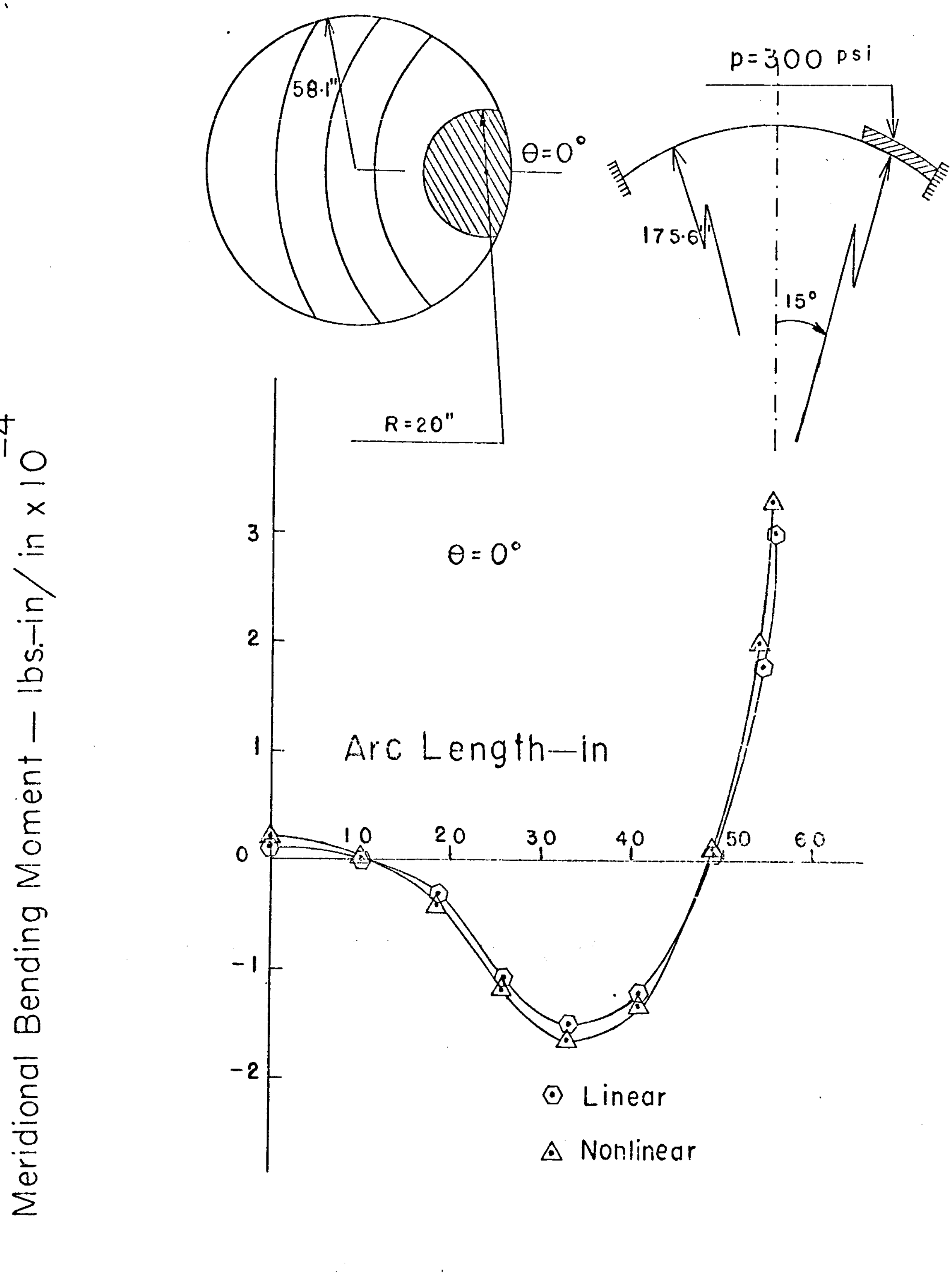

Fig. 4 BENDING MOMENT VERSUS ARC LENGTH

B I O G R A P H I C A L D A T A S H E E T

JAMES A. STRICKLIN

Associate Professor, Aerospace Engineering, Texas A&M University.
Associate Member, Graduate Faculty, Texas A&M University.

EDUCATION:

Yazoo City High School, Yazoo City, Mississippi, 1950.
B. S. in Aerospace Engineering, Mississippi State University, 1955.
M. S. in Aerospace Engineering, Georgia Institute of Technology, 1958.
Ph.D. in Aeronautics and Astronautics, Massachusetts Institute of Technology, 1964.

EXPERIENCE:

Industrial

Associate Engineer, Flutter and Vibrations, Lockheed - Georgia Co., 1955-56.
Engineer, Flutter and Vibrations, North American Aviation, 1958.
Consultant, Natroval Research Corp., 1961.
Research Assistant, Aeroelastic and Structures Laboratory, Massachusetts Institute of Technology, 1962-63.
Consultant, National Aeronautics and Space Administration, 1965.
Staff Research Engineer, Aeroelastic and Structures Laboratory, Massachusetts Institute of Technology, 1965.

Educational

Instructor, Aerospace Engineering, Georgia Institute of Technology, 1956-58.
Assistant Professor, Aerospace Engineering, Georgia Institute of Technology, 1959-65.
Associate Professor, Aerospace Engineering, Texas A&M University, 1965- .

PROFESSIONAL LICENSES AND SOCIETY MEMBERSHIPS:

American Institute of Aeronautics and Astronautics
Tau Beta Pi
Sigma Xi
Sigma Gamma Tau

HONORS AND AWARDS:

Delta Airlines Scholarship presented to outstanding senior in Aerospace Engineering, 1954. (Mississippi State)
Ford Foundation Fellowship, 1961-1963. (Georgia Tech.)
M.I.T. Whitney Fellowship, 1961-1962.
Elected to Tau Beta Pi as Distinguished Engineer in 1967, by Mississippi Alpha Chater, Mississippi State University.

JAMES A. STRICKLIN

PUBLICATIONS:

1) "C-130A Rigid Pylon Tank Flutter Data", Lockheed Aircraft Report ER 1308, 1955.

2) "C-130A Ground Vibration Test", Lockheed Aircraft Report, ER 1229, Part II, 1955.

3) "Additional Information on the Calculation of Natural Modes of Free-Free Structures," Institute of Aerospace Sciences Journal, Vol. 28, No. 11, 1961.

4) "Notes on Basic Research on Static and Dynamic Nonlinear Structural Problems", M. I. T. Aeroelastic and Structures Laboratory, TR-109-1, 1963.

5) "Large Elastic, Plastic and Creep Deflections of Curved Beams and Axisymmetric Shells", Ph.D. Thesis, M.I.T., 1964.

6) "Large Elastic, Plastic, and Creep Deflections of Curved Beams and Axisymmetric Shells", AIAA Journal, Vol. 2, No. 9, 1964, pp. 1613-1621.

7) "Computation of Stress Resultants from the Element Stiffness Matrix", AIAA Journal, Vol. 4, No. 6, 1966.

8) "Improvements on the Analysis of Shells of Revolution by the Matrix Displacement Method", AIAA Journal, Vol. 4, No. 11, 1966, pp. 2069-2072.

9) "Consistent Stiffness Matrices in the Analysis of Shells", (to be published in AIAA Journal).

10) "Rigid-Body Displacements of Curved Elements in the Analysis of Shells by the Matrix-displacement Method", AIAA Journal, Vol. 5, No. 9, August 1967, pp. 1525-1527.

FACULTY ADVISOR:

Faculty advisor to the following students on AIAA papers.

1) Banchetti, J. "Introduction and Partial Evaluation of Photo-stress", third place winner, Southeast AIAA student paper competition, 1960.

2) Stadles, W., and Hooper, E., "Vibration Analysis of Multicomponent Systems", first place winners, Southeast AIAA student paper competition, 1965.

3) Haisler, W., "Conditions for Convergence in the Analysis of Shells Using Curved Elements", third place winner, Southwest AIAA student paper competition, 1967.

JAMES A. STRICKLIN

LECTURES AND PRINTED SPEECHES:

1) "Large Elastic, Plastic, and Creep Deflections of Curved Beams and Axisymmetric Shells", presented at the First Aerospace Science Meeting, New York, January 1964.

2) "Nonlinear Analysis of Shells of Revolution", presented to National Aeronautics and Space Administration, Langley Field, VA., 1965.

3) "Consistent Stiffness Matrices In the Analysis of Shells", presented at 8th AIAA/ASME Structures, Structural Dynamics and Materials Conference, Palm Springs, California, March 30, 1967.

4) "Nonlinear Analysis of Shells of Revolution By the Matrix Displacement Method", presented at the AIAA 6th Aerospace Sciences Meeting, New York, January 23, 1968.

PATENTS:

1) "Method and Means for Augmenting Hypervelocity Flight" (pending).

B I O G R A P H I C A L D A T A S H E E T

JOSE E. MARTINEZ October 1967

Instructor, Civil Engineering, Texas A&M University.
Research Assistant, Texas Transportation Institute, Texas A&M University.

EDUCATION:

B. S. in Mechanical Engineering, Texas A&M University, 1963.
M. E. in Mechanical Engineering, Texas A&M University, 1965.
Ph.D. in Mechanical Engineering, Texas A&M University, 1967.

EXPERIENCE:

Industrial

Structures Engineer, General Dynamics, Fort Worth, 1963-1964.

Educational

Graduate Assistant, Mechanical Engineering Department, Texas A&M University, 1965-1966.
Research Assistant, Texas Transportation Institute, Texas A&M University, 1966- .
Instructor, Civil Engineering, Texas A&M University, 1967- .

PROFESSIONAL LICENSES AND SOCIETY MEMBERSHIPS:

Humble Oil Company Fellowship, Texas A&M University, 1964-1965
Humble Oil Company Fellowship, Texas A&M University, 1965-1966
NASA Fellowship, Texas A&M University, 1966 (did not accept it; preferred to work for TTI instead)
Finalist American Oil Company Fellowship, 1966
Graduate Council, 1965-1966
Sigma Xi
Tau Beta Pi
Pi Tau Sigma
Texas Society of Professional Engineers

PAPERS:

1) "Evaluation of the Torsional Rigidity of a Non-circular Cross-section", Master of Engineering Report, Texas A&M University, 1965.

2) "Linear Analysis of Thin Shallow Shells", Master of Engineering Report, Texas A&M University, 1965.

3) "An Investigation of the Impact Behavior of a Rigid Body", unpublished Doctoral Dissertation, Texas A&M University, 1967.

4) "An Analytical Solution of the Impact Behavior of Luminaire Support Assemblies", Research Report Number 75-9, Texas Transportation Institute, Texas A&M University, August 1967.