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ELASTOPLASTIC ANALYSIS OF CIRCULAR CYLINDRICAL INCLUSION IN UNIFORMLY STRESSED INFINITE HOMOGENEOUS MATRIX

by Alexander Mendelson

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SUMMARY

Equations have been derived and a method has been presented for performing an elastoplastic analysis of a system consisting of a circular cylindrical inclusion in a homogeneous matrix uniformly stressed at infinity and in a condition of generalized plane strain. The material properties of the inclusion and the matrix, including their stress-strain curves, are assumed to be arbitrary and independent of each other. Several examples, including the limiting cases of a hole and a rigid inclusion, are presented. It is shown that the constraints imposed by the rigid inclusion sharply reduce both the stress and strain concentration factors over those for the hole. The results for a system roughly approximating a graphite fiber in a resin matrix indicate that the fiber acts nearly as a rigid inclusion and that strain hardening properties of the matrix play only a minor role in determining the plastic strain concentration factor for the case of plane strain.

INTRODUCTION

The importance of composite materials in aerospace applications, due primarily to their potentially high strength to weight ratios, is well known. The actual design strength of such materials is, however, frequently much lower than their potential strength. One reason for this is that there may be load components perpendicular to the fiber axes. The fibers then act as inclusions producing stress concentrations in the matrix, which is already considerably weaker than the fibers.

An elastic analysis on the effect of a cylindrical inclusion in a homogeneous matrix is presented in reference 1. Herein a more realistic elastoplastic analysis is made to determine the stress and strain concentrations due to such inclusions. A complete analysis would have to include a large number of irregularly shaped and spaced inclusions as well as the orthotropic character of the fibers. The present analysis is therefore merely a first step, wherein a single circular inclusion in an infinite homogeneous matrix, uniformly loaded at infinity, is considered.

The method used in the analysis is the successive approximation or iterative technique for solving elastoplastic problems as outlined in references 2 and 3. This technique has been successfully used for the problem of an infinite plate with a hole in references 4 and 6 and for a rigid inclusion in reference 7. In both cases, the plane stress problem of a thin plate was considered. In the present analysis, the generalized plane strain problem of an infinitely thick plate is treated for arbitrary matrix and inclusion properties. Furthermore, Hencky's total plasticity theory is used. Although the incremental theory can be used with equal ease, the results shown in reference 6 indicate that for this type of problem there is negligible difference between the total and incremental theory. By using total plasticity theory an appreciable amount of computor time can be saved.

The methods used in this report were developed by the author while at NASA Lewis Research Center. The bulk of the research was carried out while the author was on leave from Lewis at Case-Western Reserve University. The author acknowleges the sponsorship during this period of the Advanced Research Projects Agency, Department of Defense, through a contract administered by the Air Force Materials Laboratory. This report is also being issued as an Air Force Materials Laboratory Technical Report.

SYMBOLS

a	radius of inclusion
A, B	integration constants
E	elastic modulus
e	ratio of strain to yield strain
$K_1, K_2, \ldots K_8$	constants defined by eq. (18)
m	ratio of slope of linear strain-hardening curve to elastic modulus
P, Q, R	plastic strain functions defined by eq. (4)
r	radial coordinate
S	ratio of stress to yield stress
S_∞	ratio of applied uniform stress at infinity to yield stress
T	ratio of average axial stress due to end loads to yield stress
U	dimensionless radial displacement defined by eq. (1)

u radial displacement

α ratio of matrix yield stress to inclusion yield stress

β ratio of matrix elastic modulus to inclusion elastic modulus

ϵ strain

μ Poisson's ratio

ρ dimensionless radius, r/a

ρ_{max} maximum value of ρ used in computations

ρ_p radius to plastic zone boundary

σ stress

Subscripts:

e equivalent

I inclusion

M matrix

o yield

r radial

z axial

θ tangential

Superscript:

p plastic

ANALYSIS

General Relations

Consider an infinite plate uniformly loaded at infinity as shown in figure 1. Because of the symmetry, the problem is most simply cast into polar coordinates.

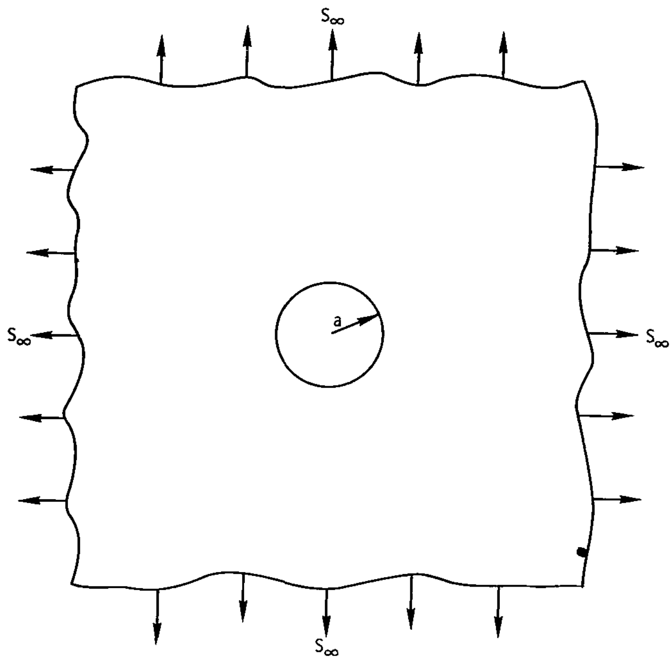


Figure 1. - Infinite body with circular inclusion.

The dimensionless quantities used in this report are defined as follows:

$$
\left.
\begin{aligned}
&\rho = \frac{r}{a} \\
&S_r = \frac{\sigma_r}{\sigma_o}, \quad S_\theta = \frac{\sigma_\theta}{\sigma_o}, \quad S_z = \frac{\sigma_z}{\sigma_o} \\
&e_r = \frac{\epsilon_r}{\epsilon_o}, \quad e_\theta = \frac{\epsilon_\theta}{\epsilon_o} \\
&U = \frac{u}{a\epsilon_o} = \frac{uE}{a\sigma_o}
\end{aligned}
\right\} \tag{1}
$$

where σ_o and ϵ_o are the yield stress and yield strain, respectively (these will be more precisely defined later), a is the radius of the inclusion, and u is the radial displacement. The equilibrium, compatibility, and stress-strain relations can be written as

$$\left.\begin{aligned}
\frac{dS_r}{d\rho} &= \frac{S_\theta - S_r}{\rho} \\
\frac{de_\theta}{d\rho} &= \frac{e_r - e_\theta}{\rho} \\
e_r &= S_r - \mu(S_\theta + S_z) + e_r^p \\
e_\theta &= S_\theta - \mu(S_r + S_z) + e_\theta^p \\
e_z &= S_z - \mu(S_r + S_\theta) + e_z^p = \text{constant}
\end{aligned}\right\} \tag{2}$$

Equations (2) can be integrated to give the following relations:

$$\left.\begin{aligned}
S_r &= A + \frac{B}{\rho^2} + \frac{1}{2(1 - \mu^2)} P(\rho) - \frac{1 - 2\mu}{2(1 - \mu^2)} Q(\rho) \\
S_\theta &= A - \frac{B}{\rho^2} + \frac{1}{2(1 - \mu^2)} P(\rho) + \frac{1 - 2\mu}{2(1 - \mu^2)} Q(\rho) + \frac{1}{1 - \mu^2} R(\rho) \\
S_z &= \mu(S_r + S_\theta) + e_r^p + e_\theta^p + e_z \\
e_r &= (1 - \mu - 2\mu^2)A + (1 + \mu)\frac{B}{\rho^2} + \frac{1}{2}\frac{1 - 2\mu}{1 - \mu}\Big[P(\rho) + Q(\rho)\Big] + \frac{1 - 2\mu}{1 - \mu} e_r^p - \mu e_z \\
e_\theta &= (1 - \mu - 2\mu^2)A - (1 + \mu)\frac{B}{\rho^2} + \frac{1}{2}\frac{1 - 2\mu}{1 - \mu}\Big[P(\rho) + Q(\rho)\Big] - \mu e_z \\
e_z &= T - 2 \lim_{\rho \to \infty} \frac{1}{\rho^2}\left[\mu \int_0^\rho (S_r + S_\theta)\rho \, d\rho + \int_0^\rho (e_r^p + e_\theta^p)\rho \, d\rho\right] \\
U &= (1 - \mu - 2\mu^2)A\rho - (1 + \mu)\frac{B}{\rho} + \frac{1}{2}\frac{1 - 2\mu}{1 - \mu}\rho\Big[P(\rho) + Q(\rho)\Big] - \mu\rho e_z
\end{aligned}\right\} \tag{3}$$

where

$$
\left.
\begin{aligned}
P(\rho) &\equiv \int_c^\rho \frac{e_r^p - e_\theta^p}{\rho}\, d\rho \\
Q(\rho) &\equiv \frac{1}{\rho^2} \int_c^\rho \rho(e_r^p + e_\theta^p) d\rho \\
R(\rho) &\equiv \mu e_r^p - (1 - \mu) e_\theta^p
\end{aligned}
\right\} \tag{4}
$$

and T is the dimensionless average axial stress due to the end loads. The constants A and B will depend on the boundary conditions as will be shown, and the lower limit on the integrals c will equal zero for the inclusion and one for the matrix.

The plastic strains are related to the total strains by the modified Prandtl-Reuss relations (ref. 2).

$$
\left.
\begin{aligned}
e_r^p &= \frac{e_p}{3e_e}(2e_r - e_\theta - e_z) \\
e_r^p &= \frac{e_p}{3e_e}(2e_\theta - e_r - e_z)
\end{aligned}
\right\} \tag{5}
$$

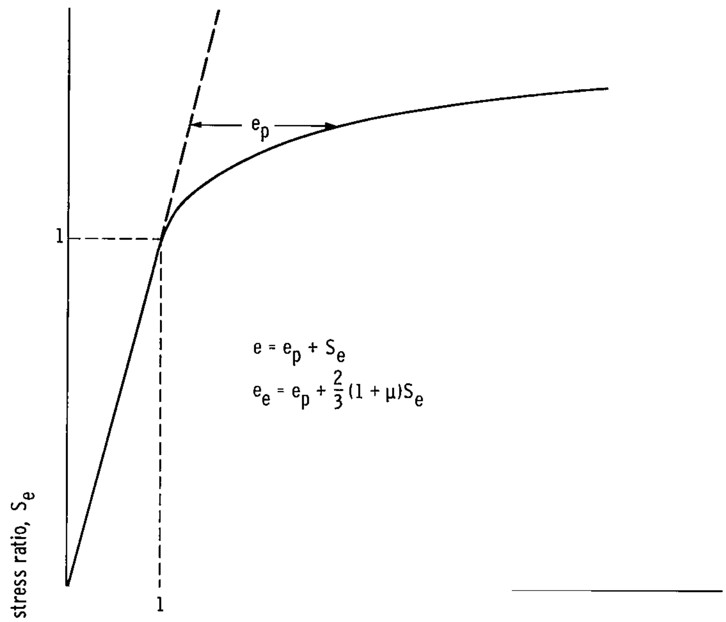
where

$$
e_e = \frac{\sqrt{2}}{3}\left[(e_r - e_\theta)^2 + (e_r - e_z)^2 + (e_\theta - e_z)^2\right]^{1/2} \tag{6}
$$

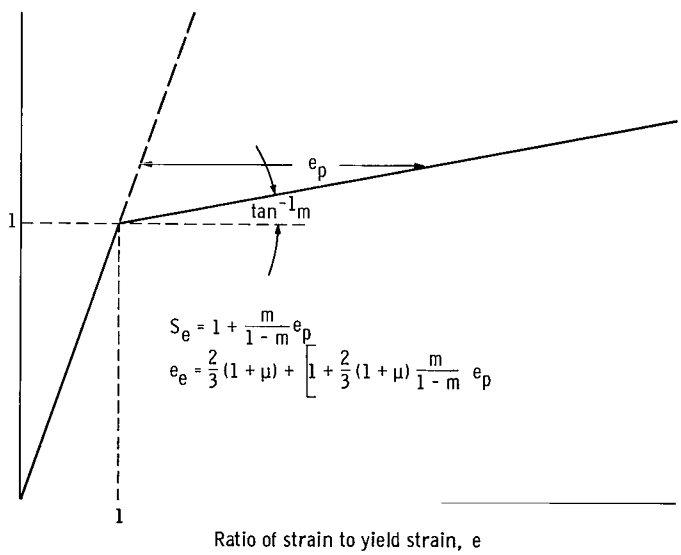
and e_p is related to e_e through the stress-strain curve and the relation

$$
e_p = e_e - \frac{2}{3}(1 + \mu)S_e \tag{7}
$$

where S_e, the dimensionless equivalent stress, is a function of the equivalent plastic strain e_p and is the ordinate of the dimensionless uniaxial stress-strain curve as shown



(a) For nonlinear strain hardening.



(b) For linear strain hardening.

Figure 2. - Dimensionless uniaxial stress-strain curve.

in figure 2(a). For the case of linear strain hardening, equation (7) can be solved for e_p to give

$$e_p = \frac{e_e - \frac{2}{3}(1 + \mu)}{1 + \frac{m}{1 - m}\frac{2}{3}(1 + \mu)} \tag{8}$$

where m is the ratio of the slope of the stress-strain curve in the plastic range to the elastic modulus, as shown in figure 2(b). For a perfectly plastic material, equations (7) and (8) reduce to

$$e_p = e_e - \frac{2}{3}(1 + \mu) \tag{8a}$$

The above equations are valid both in the inclusion and in the matrix. The constants A and B will be different for the matrix and for the inclusion as will the material properties. We will henceforth distinguish between the matrix and the inclusion by using the subscripts M and I for matrix and inclusion, respectively.

Evaluation of Constants

An evaluation of the constants makes use of the boundary conditions. As previously noted for the inclusion

$$c = 0, \qquad 0 \leq \rho \leq 1$$

and for the matrix

$$c = 1, \qquad 1 \leq \rho$$

It also follows from the first or second equation (eq. (3)) that, for finite stresses at $\rho = 0$,

$$B_I = 0 \tag{9}$$

Substituting the values of S_r and S_θ from equations (3) into the expression for e_z and taking the indicated limit gives

$$e_z = T - 2\mu_M A_M - \frac{\mu_M}{1 - \mu_M^2} P_M(\infty) \tag{10}$$

Let the dimensionless stress at $\rho = \infty$ be designated by

$$S_\infty \equiv \frac{\sigma_r(\infty)}{\sigma_{oM}}$$

where σ_{oM} is the yield stress of the matrix. Then, from the first of equations (3),

$$A_M = S_\infty - \frac{1}{2(1 - \mu_M^2)} P_M(\infty) + \frac{1 - 2\mu_M}{2(1 - \mu_M^2)} Q_M(\infty)$$

where, from equations (4),

$$\left.\begin{aligned} P_M(\infty) &= \int_1^\infty \frac{e^p_{rM} - e^p_{\theta M}}{\rho} d\rho \\ Q_M(\infty) &= 0 \end{aligned}\right\} \tag{11}$$

Hence,

$$A_M = S_\infty - \frac{1}{2(1 - \mu_M^2)} P_M(\infty) \tag{12}$$

Note that, since the plastic strains die out as ρ increases, the upper limit in equation (11) may be replaced by some arbitrary radius ρ_{max} where ρ_{max} is greater than the radius of the plastic zone. Thus,

$$P_M(\infty) = \int_1^{\rho_{max}} \frac{e^p_{rM} - e^p_{\theta M}}{\rho} d\rho \tag{13}$$

$$\rho_{max} \geq \text{radius of plastic zone}$$

The two remaining constants A_I and B_M can be determined from the conditions that the radial stress and displacement should be continuous across the matrix-inclusion interface, that is,

$$\sigma_{rI}(a) = \sigma_{rM}(a)$$

$$u_I(a) = u_M(a)$$

or

$$S_{rI}(1) = \frac{\sigma_{oM}}{\sigma_{oI}} S_{rM}(1)$$

or

$$S_{rI}(1) = \alpha S_{rM}(1) \tag{14}$$

where

$$\alpha \equiv \frac{\sigma_{oM}}{\sigma_{oI}}$$

is the ratio of the matrix yield stress to the inclusion yield stress. Note that S_{rI} is defined as the ratio of the inclusion radial stress to the inclusion yield stress and S_{rM} is defined as the ratio of the matrix radial stress to the matrix yield stress. Similarly,

$$\beta U_I(1) = \alpha U_M(1) \tag{15}$$

where

$$\beta \equiv \frac{E_m}{E_I}, \quad U_I = \frac{u_I E_I}{a\sigma_{oI}}, \quad U_M = \frac{u_M E_m}{a\sigma_{oM}} \tag{16}$$

and E_m and E_I are the elastic moduli of the matrix and inclusion, respectively.

From equations (3), (9), (12), (14), and (15) and noting that $P_M(1) = Q_M(1) = 0$,

$$\left.\begin{aligned} A_I &= K_1 S_\infty + K_2 P_I(1) + K_3 Q_I(1) + K_4 P_M(\infty) + K_8 e_z \\ B_M &= K_5 S_\infty + K_6 Q_I(1) + K_7 P_M(\infty) + \frac{1}{\alpha} K_8 e_z \end{aligned}\right\} \tag{17}$$

where

$$
\left.
\begin{aligned}
K_1 &= \frac{2(1 - \mu_M^2)\alpha}{1 + \mu_M + (1 + \mu_I)(1 - 2\mu_I)\beta} \\
K_2 &= -\frac{1}{2(1 - \mu_I^2)} \\
K_3 &= \frac{\dfrac{1 - 2\mu_I}{2(1 - \mu_I)}\left(\dfrac{1 + \mu_M}{1 + \mu_I} - \beta\right)}{1 + \mu_M + (1 + \mu_I)(1 - 2\mu_I)\beta} \\
K_4 &= \frac{-\alpha}{1 + \mu_M + (1 + \mu_I)(1 - 2\mu_I)\beta} \\
K_5 &= \frac{K_1}{\alpha} - 1 \\
K_6 &= \frac{(1 - 2\mu_I)\beta}{\alpha^2} K_4 \\
K_7 &= -\frac{K_5}{2(1 - \mu_M^2)} \\
K_8 &= \left(\mu_M - \frac{\beta}{\alpha}\mu_I\right) K_4
\end{aligned}
\right\} \tag{18}
$$

Note that the coefficients K_1 through K_8 are functions of the four material constants μ_I, μ_M, α and β.

Computation Procedure

The stresses, total strains, and plastic strains can now be computed by the successive approximation or iterative method described in reference 2. The inclusion and the matrix are each divided into finite radial intervals. For the inclusion, the stations range from zero to 1. For the matrix, the stations range from $\rho = 1$ to $\rho = \rho_{max}$ where ρ_{max} is an arbitrary dimensionless radius larger than the radius of the plastic zone. Initially, the radius of the plastic zone is not known; therefore, one must guess at a reasonable value for ρ_{max}. It may be desirable to change this value after the first iteration so that most of the stations are in the plastic zone.

The coefficients K_1 through K_8 are computed once from equation (18). Assuming all the plastic strains to be zero so that P_I, Q_I, P_M, and Q_M are all zero, A_I and B_M are computed from equation (17), A_M from equation (12), and e_z from equation (10). Equations (3) now give the complete elastic solution for the inclusion and the matrix. Beginning with the elastic solution, the iterative scheme for obtaining the elasto-plastic solution proceeds as follows:

(1) Using the values of total strains just computed, the equivalent total strain e_e is calculated at every station of the inclusion and the matrix by equation (6).

(2) The equivalent plastic strain e_p at every station is determined from equation (7) and the stress-strain curve (or eq. (8) for linear strain hardening). If e_p at any station is less than zero, there is no plastic flow at that station and the plastic strains are set equal to zero at that station.

(3) The individual plastic strains at every station are computed by means of equations (5).

(4) The plastic strain integrals P_I, Q_I, P_M, and Q_M are computed from equation (4) for every station.

(5) e_z, A_M, A_I, and B_M are calculated from equations (10), (12), and (17).

(6) The stresses and strains are then computed from equations (3).

(7) Return to step (1) and continue iterating until convergence is obtained, that is, until two successive solutions differ by less than some arbitrarily preassigned value.

The outlined computation scheme, which was programmed for a digital computer, gives rapidly and accurately the complete stress and strain fields both in the inclusion and the matrix. Both the elastic properties and stress-strain curves will, of course, be different in the two media.

RESULTS AND DISCUSSION

The technique presented will now be illustrated for three different cases: The two extreme cases of a plate with a void and a plate with a rigid inclusion, and the case of a relatively weak matrix with a relatively strong inclusion. In all cases, the condition of plane strain with $e_z = 0$ is assumed.

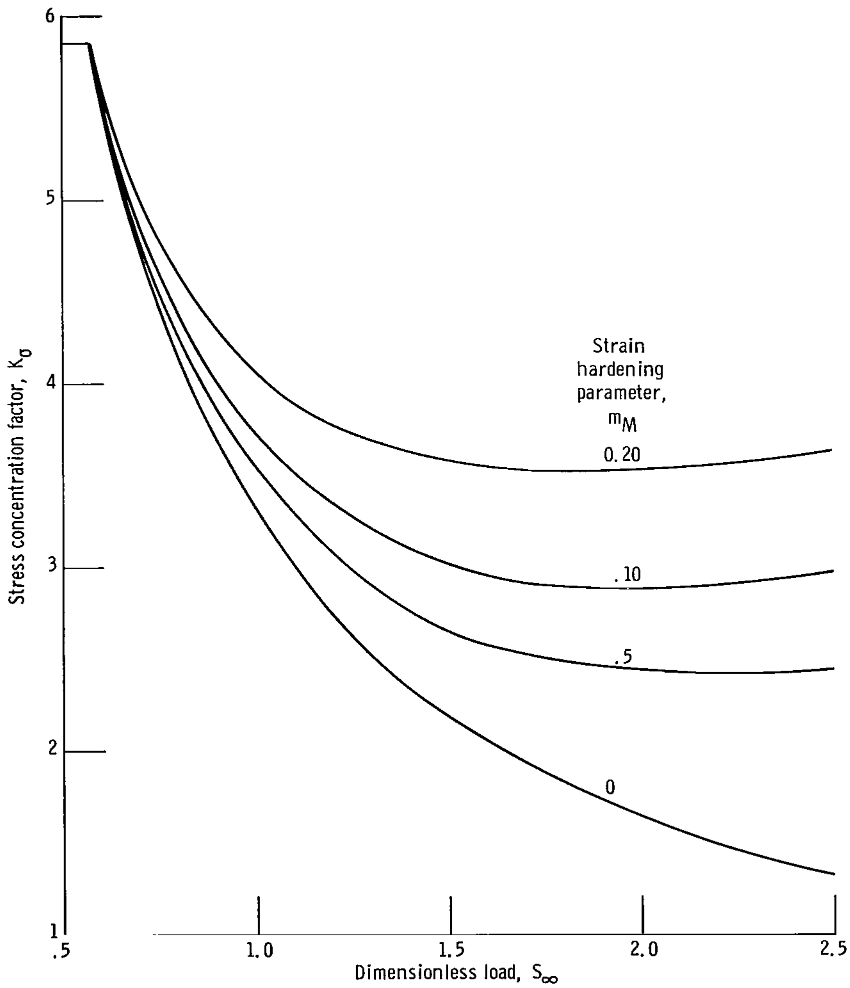
Plate With Hole or Void

A plate with a hole represents the limiting case

$$\alpha \to \infty \qquad \beta \to \infty \tag{19}$$

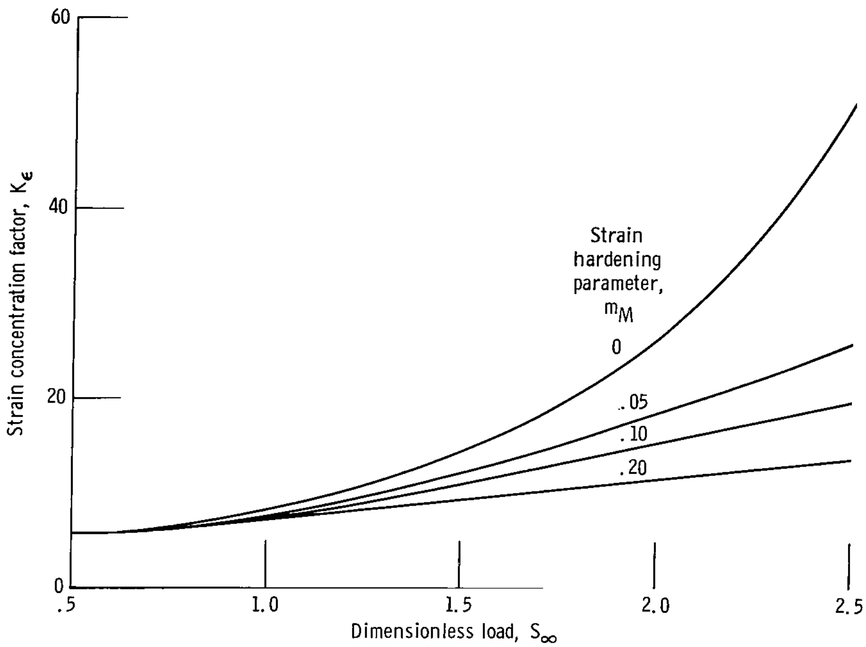
The calculation proceeds as described except that A_I need not be computed. The stress and strain fields are obtained only in the matrix. Some results are shown in figure 3.

In figures 3(a) and (b) the effective stress and strain concentration factors are plotted against the applied stress at infinity for various values of the strain hardening parameter m_M. The effective stress concentration factor is defined as the ratio of effective or equivalent stress at the edge of the hole to the equivalent stress at infinity;



(a) Effective stress concentration factor.

Figure 3. - Effect of strain-hardening parameter on plane-strain case of infinite plate with hole. Ratio of matrix yield stress to inclusion yield stress, ∞; ratio of matrix elastic modulus to inclusion elastic modulus, ∞.



(b) Effective strain concentration factor.

Figure 3. - Continued.

that is,

$$K_\sigma = \frac{S_e(1)}{S_e(\infty)} = \frac{S_e(1)}{(1 - 2\mu_M)S_\infty} \tag{20}$$

where

$$S_e = \left[\frac{(S_r - S_\theta)^2 + (S_\theta - S_z)^2 + (S_z - S_r)^2}{2}\right]^{1/2}$$

The elastic stress concentration factor as defined is readily shown as

$$K_\sigma(\text{elastic}) = \sqrt{1 + \frac{3}{(1 - 2\mu_M)^2}}$$

For a value of μ_M of 0.35, which is the value used in these calculations, the elastic stress concentration factor is equal to 5.86, which is the common starting point of all the curves in figure 3(a).

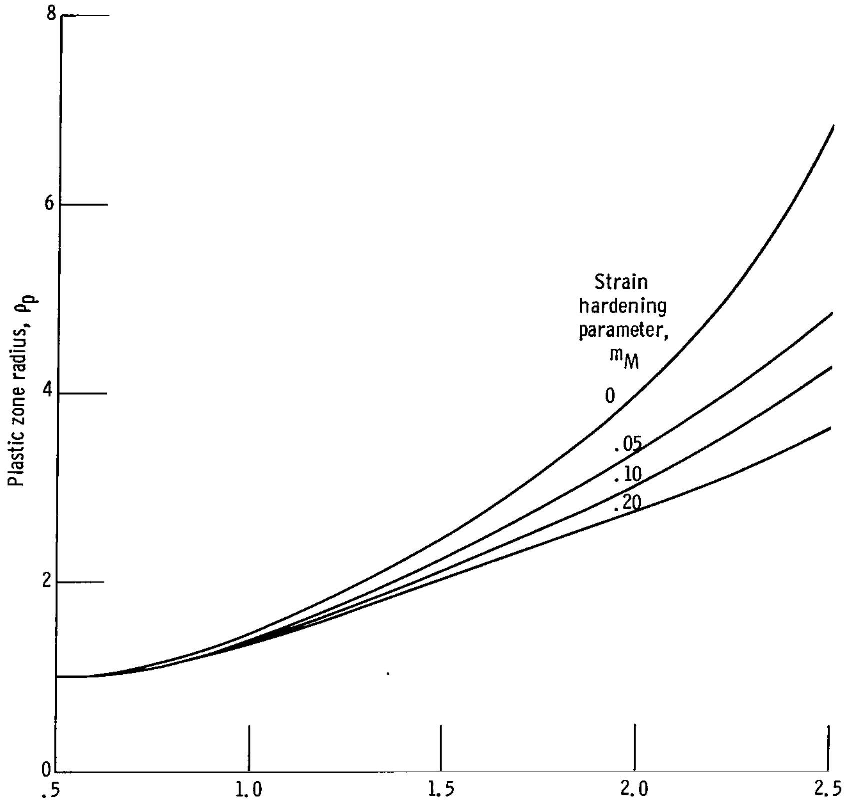
Note that, for the case of plane stress, the stress concentration factor is independent of Poisson's ratio and is equal to 2 for the elastic case. The large initial value of the stress concentration factor for the case of plane strain does not mean, however, that plastic flow will start at a lower load than for plane stress. On the contrary, it can readily be shown that the equivalent stress S_e will equal 1 at the hole for a value of $S_\infty = 0.569$ compared with $S_\infty = 0.5$ for plane stress. For a plate without a hole, S_∞ must equal 3.33 (for $\mu = 0.35$) in order that $S_e = 1$, whereas, for the plane stress case, $S_e = 1$ when $S_\infty = 1$. Thus, it requires more than 3 times the load for yielding to occur in the plane-strain case compared with the plane stress case. However, the effect of a hole is felt much more for plane strain than plane stress, so that the hole weakens the plate by a factor of almost 6 for plane strain compared with a factor of 2 for plane stress. The result is that the load to produce yielding at the hole is equal to 0.569 for plane strain compared to 0.5 for plane stress as indicated previously. The plane-strain case still requires a larger load for yielding to occur than the plane-stress case, but the difference is no longer as great when there is a hole present.

Whether definition (20) is appropriate depends on how the stress and strain concentration factors are to be used. The concentration factors, by definition, give the ratios of the stress or strain with the hole present to the corresponding values without the hole. They therefore represent the weakening effect of the hole. The equivalent stress and equivalent strain are used herein to represent the concentration factors because they are the determining quantities in plastic flow. If some other quantity is more useful in determing failure, then that quantity should be used rather than the equivalent stress or equivalent strain in defining the concentration factors.

Figures 3(a) and (b) show how the stress concentration factor decreases with load and the strain concentration factor increases with load because the material is flowing plastically. The importance of strain hardening is readily evident. It is interesting to note that, although without strain hardening the stress concentration factor decreases monotonically with load, the presence of strain hardening introduces a minimum point in the stress concentration as function of load curve. For a high enough load, the strain hardening of the material is sufficient to overcome the stress relaxation due to plastic flow, and the stress concentration factor starts rising. This effect does not occur for the plane stress case (see ref. 6), probably, because the additional axial constraint inherent in the plane-strain case is missing.

The strain concentration factor used in figure 3(b) is defined by

$$K_\epsilon = \frac{e_e(1)}{e_e(\infty)} = \frac{e_e(1)}{\frac{2}{3}(1 + \mu_M)(1 - 2\mu_M)S_\infty} \tag{21}$$



(c) Plastic zone radius.

Figure 3. - Concluded.

For the elastic case,

$$e_e(1) = \frac{2}{3}(1 + \mu_M)\left[3 + (1 - 2\mu_M)^2\right]^{1/2} S_\infty$$

so that, for $\mu_M = 0.35$,

$$K_\epsilon = 5.86 = K_\sigma$$

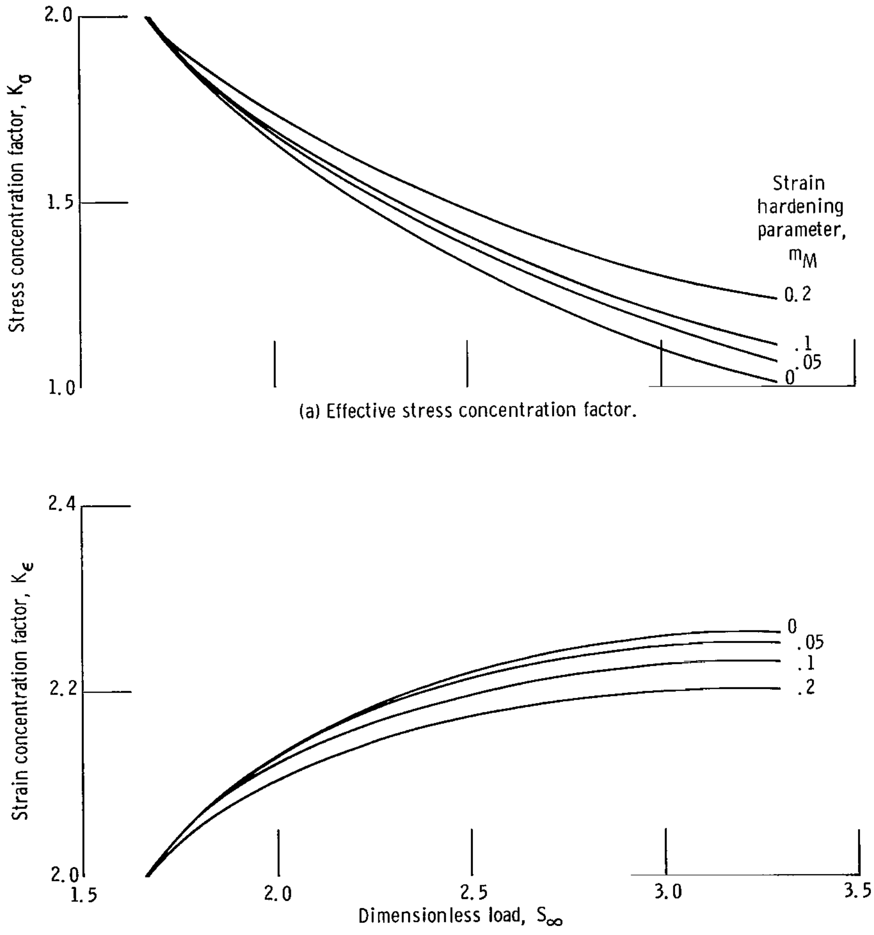
Figure 3(c) shows the growth of the plastic zone with load. Again the effect of strain hardening is very evident. The plate becomes completely plastic when S_∞ equals $3\frac{1}{3}$.

Plate With Rigid Inclusion

For a rigid inclusion

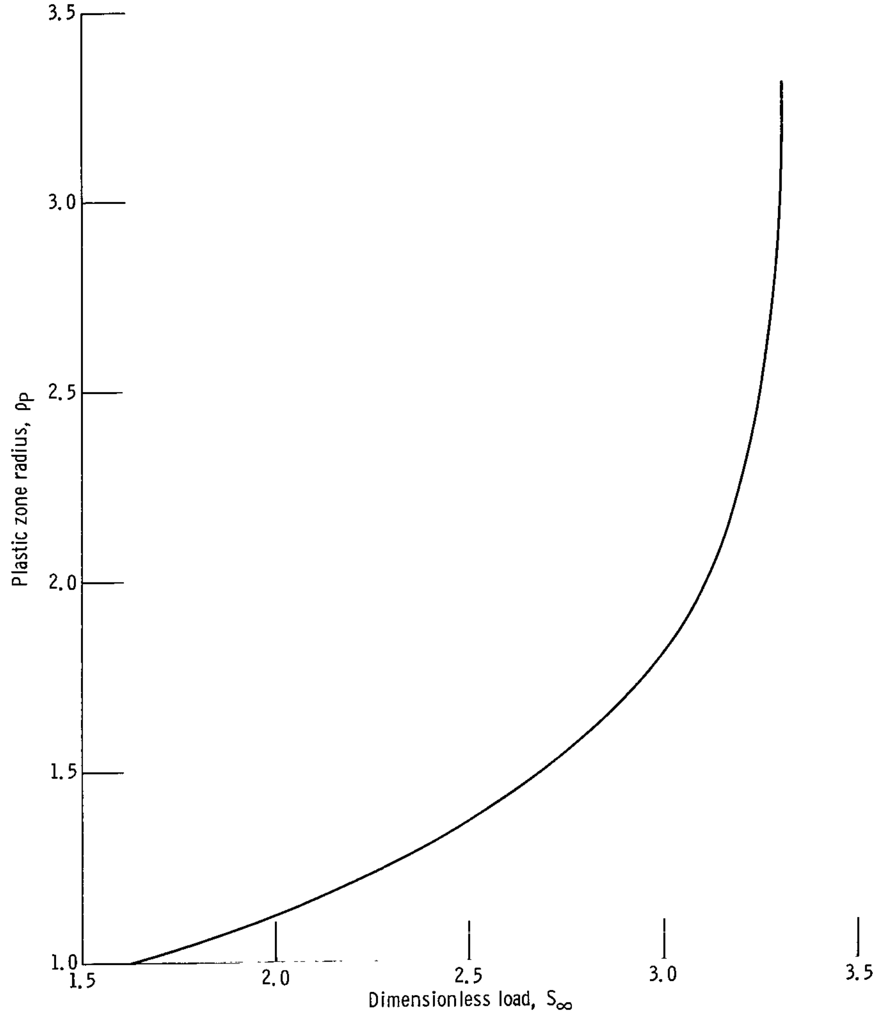
$$\alpha = \beta = 0 \tag{22}$$

The results using these values are shown in figure 4. Note the very large difference between these results and those for the plate with the hole. The restraints produced by the inclusion greatly lower both the stress and strain concentration factors. Furthermore, the effect of strain hardening is no longer as great. The plastic zone size is greatly reduced as compared with the hole (figs. 3(c) and 4(c)).



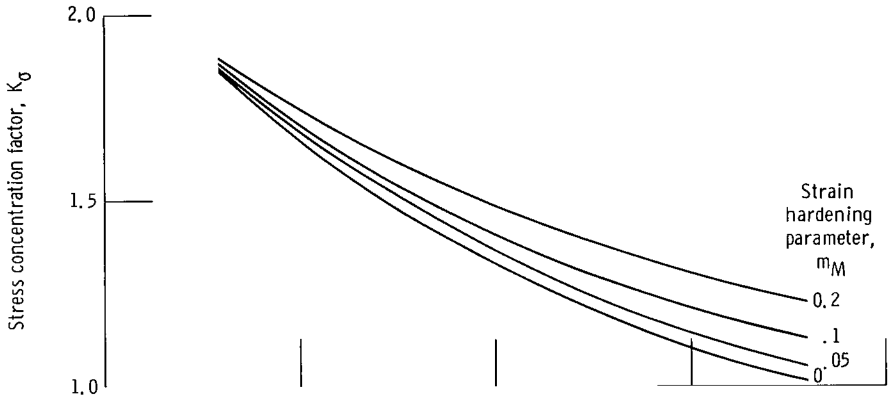
(b) Effective strain concentration factor.

Figure 4. - Effect of strain-hardening parameter on inclusion. Ratio of matrix yield stress to inclusion yield stress, 0; ratio of matrix elastic modulus to inclusion elastic modulus, 0.

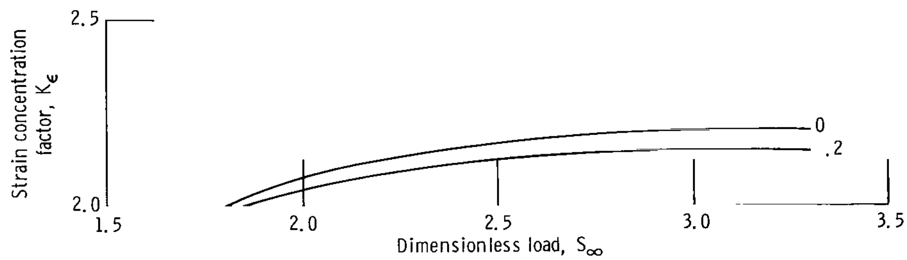


(c) Plastic zone radius.

Figure 4. - Concluded.

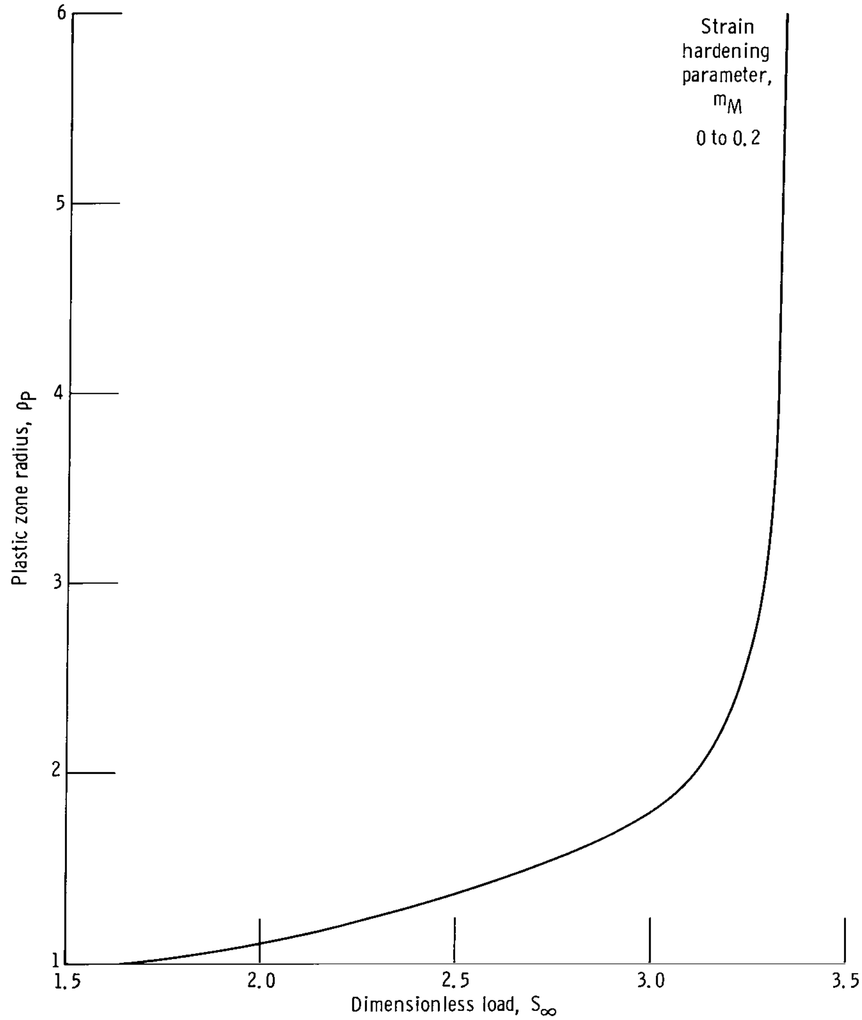


(a) Effective stress concentration factor.



(b) Effective strain concentration factor.

Figure 5. - Effect of strain-hardening parameter on plane-strain case of infinite plate with fiber inclusion. Ratio of matrix yield stress to inclusion yield stress, 0.1; ratio of matrix elastic modulus to inclusion elastic modulus, 0.02.



(c) Plastic zone radius.

Figure 5. - Concluded.

Inclusion With $\alpha = 0.1$, $\beta = 0.02$

As a final example, a matrix inclusion combination with $\alpha = 0.1$ and $\beta = 0.02$ was considered. These properties correspond roughly to a graphite fiber in a resin matrix. The results are shown in figure 5. Note the similarity of these results to those of figure 4 for the rigid inclusion. Of particular interest is the fact that the strain-hardening properties of the matrix are relatively unimportant. Only one curve was drawn in figure 5(c) because the difference between the different strain-hardening parameters is very small.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 28, 1967,
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