

CRITERIA FOR INVESTMENT SELECTION*

by

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ABSTRACT

A scarce resource is to be periodically allocated among several investment opportunities each of which produces a payout of the same resource in future periods. Future payouts may be reinvested, the object being to maximize some measure of accumulated wealth. The maximum internal rate of return criterion and the mathematical programming approach to this problem are analyzed and under certain stability conditions are shown to agree. Under these conditions the optimal prices in the dual program must initially lie near a discount vector derived from the maximum internal rate of return which is the systems von-Neumann price vector. In this sense the maximum internal rate of return may be used as an interest rate to "price out" the opportunities available on the planning horizon even though no external rate of interest is assumed.

INTRODUCTION

The problem of periodically allocating a scarce resource among a number of different productive opportunities has received a considerable amount of attention.

The more recent works of Dorfman [3], Weingartner [8], Baumol and Quandt [2] and Manne [5] largely agree that a mathematical programming formulation is appropriate for this problem. In this formulation the scarce resource may be invested periodically in one of a number of different investment opportunities, each of which produces a payout of the same resource in future periods. Future payouts may be reinvested in then currently available opportunities, the objective being to maximize some measure of accumulated wealth.

It is well established that the optimal solution to the dual programming problem is a set of prices which may be used to "price-out" current investment opportunities, and that those opportunities that are most valuable with respect to these prices are the ones to be chosen in an optimal program. The prices are, in general, dependent upon future opportunities for reinvestment and on the particular measure of wealth employed.

A number of pertinent questions remain unanswered. First, in what way is the mathematical programming approach related to the criterion of choosing that opportunity with maximum internal rate of return? This criterion is widely used in practice and has been discussed

by a number of authors in the literature of finance. The maximum rate of return available in a given period can be thought of as a discount factor used to "price-out" current opportunities. In what way is this discount factor related to the optimal prices obtained by the programming approach and why are these prices independent of future reinvestment opportunities? Second, what is an appropriate measure of wealth and are the results sensitive to the measure of wealth actually utilized in the programming approach? Third, is it possible to choose a time horizon such that opportunities available after that date will not affect the optimal choice among current investments?

The purpose of this work is to shed some light on the above questions. The model to be used is similar to the one proposed by Baumol and Quandt [2] and studied further by Manne [5], with the exception that wealth will be measured by the total amount of scarce resource generated by the program at some future time, instead of by the accumulated utility of dividend withdrawals. This departure insures that investments will be made during each period and, thus, forces assumption H1 of Manne [5] to be satisfied. It will be clear that the dividend model could have been employed together with this assumption to obtain the same results.

OPTIMALITY CONDITIONS

The situation described in the introduction can be formulated as a discrete time control problem. Let

$A(t) = (a_{ij}(t))$ be an $m \times n(t)$ matrix where $a_{ij}(t)$ is the net cash flow in period $t + i$ from a unit investment in the j^{th} project available at time t .

$x(t)$ = an $m \times 1$ dimensional state vector representing the firm's cash position for the next m periods as a result of previous investment.

$v(t)$ = an $n(t) \times 1$ dimensional control vector representing the amount to be invested in each of the currently available investment opportunities.

$L = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix}$ an $m \times m$ left shift operator.

$K = (1, \dots, 1)$ a row vector with dimension defined by context.

$U(\cdot)$ = a concave continuously differentiable function on $x(T + 1)$ giving the valuation of the final stock of capital.

$M = (1, 0, \dots, 0)$ a $1 \times m$ dimensional vector.

We wish to

Maximize $U(x(T+1))$

Subject to:

$$x(t+1) \leq Lx(t) + A(t)v(t) \quad t = 1, \dots, T$$

$$Kv(t) \leq x_1(t) \quad t = 1, \dots, T$$

$$v(t) \geq 0$$

$x(1)$ a given initial position.

Forming the Lagrangian,

$$U(x(T+1)) + \sum_{t=1}^T \Psi(t) (Lx(t) + A(t)v(t) - x(t+1)) + \sum_{t=1}^T \gamma(t) (x_1(t) - Kv(t)),$$

a straightforward application of the Kuhn-Tucker Theorem suffices to demonstrate that a program is optimal only if there are sequences of m dimensional row vectors $\Psi(t)$ and scalars $\gamma(t)$ such that

$$\nabla U(x(T+1)) = \Psi(T)$$

$$\Psi(t)L - \Psi(t-1) + \gamma(t)M = 0 \quad t = 2, 3, \dots, T$$

$$\Psi(t)A(t) - \gamma(t)K \leq 0 \quad t = 1, 2, \dots, T$$

$$(\Psi(t)A(t) - \gamma(t)K) v(t) = 0 \quad t = 1, 2, \dots, T$$

$$\Psi(t) \geq 0, \quad \gamma(t) \geq 0, \quad v(t) \geq 0 \quad t = 1, 2, \dots, T.$$

The second condition can be seen to be equivalent to

$$\Psi_1(t-1) = \gamma(t)$$

$$\Psi_i(t) = \Psi_{i+1}(t-1) \quad i = 1, 2, \dots, m-1.$$

Let

$$A_k(t) = \begin{pmatrix} a_{1k}(T-t+1) & 1 & 0 & \dots & 0 \\ a_{2k}(T-t+1) & 0 & 1 & \dots & 0 \\ \vdots & \vdots & & & \\ a_{m-1,k}(T-t+1) & 0 & & \dots & 0 & 1 \\ a_{mk}(T-t+1) & 0 & & \dots & 0 \end{pmatrix} \quad t=1, 2, \dots, T$$

$$\Psi(0) = (\gamma(1), \Psi_1(1), \Psi_2(1), \dots, \Psi_{m-1}(1))$$

$$p(t) = \Psi(T-t+1) \quad t=1, 2, \dots, T+1$$

A little calculation now yields the following necessary conditions which will be referred to as the optimality conditions:

$$(1) \quad \nabla U(x(T+1)) = p(1)$$

$$(2) \quad p(t)A_k(t) \leq p(t+1) \quad t=1, 2, \dots, T \quad k=1, 2, \dots, n(t)$$

with equality holding in equation $\hat{k}$ at time t if $v_{\hat{k}}(t) > 0$ where time is now being calculated from the end of the programming period.

$$(3) \quad v(t) \geq 0 \quad t=1, \dots, T$$

$$p(t) \geq 0 \quad t=1, \dots, T, T+1$$

The sequence $\langle p(t) \rangle_{t=1}^{T+1}$ of dual variables appearing in the optimality conditions is, of course, the usual sequence of shadow prices arising from the capital budgeting model and used to "price out" an investment opportunity in order to determine its profitability. The

investments not restricted to zero intensity are precisely those for which equality holds in (2) for the optimal price sequence. Postulating that wealth is a function only of the final state insures that at least one opportunity will be operated at a positive level during each period. Manne's assumption H1 would provide the same result for the dividend withdrawal model and all future results would remain valid. In the next section we shall use the optimality conditions to derive the asymptotic path of the prices thereby determining the investment to be made initially if the horizon is sufficiently far away.

PRICE CONVERGENCE

Our objective in this section is to establish a set of regularity conditions which, when placed on future opportunities, insure a useful form of price convergence. We first examine the eigenvalues of matrices of the form possessed by $A_k(t)$, dropping the t index for clarity. If λ is an eigenvalue of some A_k with associated left eigenvector p then $pA_k = \lambda p$ and thus $p_i = \lambda p_{i+1}$ $i=1, 2, \dots, m-1$ yielding $p_i = (\frac{1}{\lambda})^{i-1} p_1$ $i=1, 2, \dots, m$. Thus $\lambda p_1 = \sum_i p_i a_{ik} = p_1 \sum_i (\frac{1}{\lambda})^{i-1} a_{ik}$ or $1 = \frac{1}{\lambda} a_{1k} + \frac{1}{\lambda^2} a_{2k} + \dots + \frac{1}{\lambda^m} a_{mk}$ implying that the reciprocal of any eigenvalue of A_k is a root of the polynomial generated by the payment stream for investment opportunity k . Let $\lambda(k)$ be the largest positive eigenvalue of A_k where $\lambda(k) = 0$ if A_k has no positive eigenvalues. We shall define $\lambda(k) - 1$ to be the

rate of return of the k^{th} investment opportunity. Let $\lambda_t^* = \max \lambda(k)$, where the maximization is performed over all opportunities k available at time t , and let $A^*(t)$ and $p^*(t)$ be the associated A_k and left eigenvector where we take $p^*(t) = (\frac{1}{\lambda_t^*}, (\frac{1}{\lambda_t^*})^2, \dots, (\frac{1}{\lambda_t^*})^m)$.

Lemma 1:

$$p^*(t) A_k(t) \leq \lambda_t^* p^*(t) \quad t=1, \dots, T; \quad k=1, \dots, n(t)$$

with strict inequality holding in the first equation of group t if $\lambda(k) < \lambda_t^*$.

Proof:

Let $f_k(\beta) = \sum_{i=1}^m \beta^i a_{ik} - 1$. Then $f_k(0) = -1$ for all k and $\lambda(k) \leq \lambda_t^*$ implies that $f_k(\beta)$ has no zero crossing for $0 \leq \beta < \frac{1}{\lambda_t^*}$ and thus $f_k(\frac{1}{\lambda_t^*}) \leq 0$ with strict inequality in case $\lambda(k) < \lambda_t^*$.

This proves the inequality for the first equation of each group and the other equations hold with equality for all k as can be seen in a manner similar to the construction of the eigenvectors of A_k . QED

The following assumptions will be used:

A1: All investment opportunities are of the point input stream output type, i.e., $a_{ij}(t) \geq 0$ for all i, j , and $t = 1, 2, \dots, T$.

$$A2: \lambda_{t+1}^* = \lambda_t^* = \lambda^* \quad t = 1, 2, \dots, T-1$$

A3: There is a sequence of available opportunities $\langle A^*(t) \rangle_{t=1}^T$ each of which has maximum positive eigenvalue equal to λ^* , $a_k(t) > 0$, for some fixed $k \neq m$, $t=1, \dots, T$ and $a_m(t) > 0$ for $t=1, \dots, T$.

Assumption A2 assures future stationarity of the maximum rate of return, while A3 guards against the possibility of cyclic phenomena. Of course, A2 also implies that the maximum positive eigenvalue of each $A^*(t)$ in the sequence is also the eigenvalue of maximum modulus (Karlin [4]). Note that the set of available investment opportunities need not be stationary over time. Let $\hat{P}$ be a diagonal matrix with element $p_{ii} = (\frac{1}{\lambda^*})^i$ and observe that $p^*(t) = (\frac{1}{\lambda^*}, (\frac{1}{\lambda^*})^2, \dots, (\frac{1}{\lambda^*})^m) = p^*$ as a consequence of A2. The following lemma was established by Morishima [6] in a somewhat different setting.

Lemma 2:

Suppose $p(1) \geq 0$ is fixed and $\neq 0$. Then given any cone N containing the ray p^* , there exists an integer $t(p(1))$ such that every price path starting from $p(1)$ and satisfying optimality condition (2) enters, and remains within N , after at most $t(p(1))$ transitions.

Proof:

Following Morishima [6] we will show that $\lim_{t \rightarrow \infty} \left(\frac{1}{\lambda^*}\right)^t p(t) \hat{P}^{-1} = CK$ for some $C > 0$, if $\langle p(t) \rangle$ is a sequence satisfying the conditions of the lemma. Let

$$z(t) = \left(\frac{1}{\lambda^*}\right)^t p(t) \hat{P}^{-1}$$

and observe that optimality condition (2) and A3 imply

$$p(t+1) \geq p(t) A^*(t)$$

and, thus, multiplying on the right by $\hat{P}^{-1} \left(\frac{1}{\lambda^*}\right)^{t+1}$ we have

$$(4) \quad z(t+1) \geq z(t) \hat{P} A^*(t) \hat{P}^{-1} \left(\frac{1}{\lambda^*}\right)$$

as $\hat{P}^{-1}$ and λ^* are nonnegative. Since p^* is the left eigenvector of $A^*(t)$ for all t , $K\hat{P} = p^*$ implies

$$(5) \quad K = K\hat{P} A^*(t) \hat{P}^{-1} \frac{1}{\lambda^*} \geq K\hat{P} A_j(t) \hat{P}^{-1} \frac{1}{\lambda^*}$$

for all t and opportunities j available at time t where the inequality is a direct consequence of Lemma 1 and the nonnegativity of the $A_j(t)$.

Now form sequences $\langle c(t) \rangle$ and $\langle C(t) \rangle$ where $c(t) = \min_i z_i(t)$ and $C(t) = \max_i z_i(t)$.

Clearly

$$(6) \quad c(t)K \leq z(t) \leq C(t)K$$

and thus from (4) and (5)

$$c(t)K \leq z(t) \hat{P} A^*(t) \hat{P}^{-1} \frac{1}{\lambda^*} \leq z(t+1)$$

hence
$$c(t) \leq c(t+1) .$$

Now using (5) again

$$C(t)K \geq C(t)K \hat{P} A_j(t) \hat{P}^{-1} \frac{1}{\lambda^*}$$

for all opportunities j available at time t . Since $A_j(t) \geq 0$

$$\begin{aligned} C(t)K &\geq z(t) \hat{P} A_j(t) \hat{P}^{-1} \frac{1}{\lambda^*} \\ &\geq p(t) A_j(t) \hat{P}^{-1} \left(\frac{1}{\lambda^*}\right)^{t+1} \\ &\geq p(t+1) \hat{P}^{-1} \left(\frac{1}{\lambda^*}\right)^{t+1} \\ &\geq z(t+1) \end{aligned}$$

the penultimate line being obtained by assuming j to correspond to

the opportunity satisfying optimality condition (2) with equality at time t . Thus $C(t) \geq C(t+1)$ and hence both the sequences $\langle c(t) \rangle$ and $\langle C(t) \rangle$ converge monotonically. It remains to demonstrate that both sequences converge to the same limit and that the limit is positive. If A is any nonnegative matrix with elements $a_{i,i+1} > 0$ for $i=1, \dots, m-1$, $a_{m1} > 0$ and $a_{k1} > 0$ for some $k \neq m$, then there is some integer v such that A^v is a strictly positive matrix. Thus, using assumption A3

$$A^*(t) A^*(t+1) \dots A^*(t+v)$$

is a strictly positive matrix for each t and v is independent of t . Iterating on (4)

$$z(t+v) \geq z(t) \hat{P} A^*(t) A^*(t+1) \dots A^*(t+v) \hat{P}^{-1} \left(\frac{1}{\lambda^*} \right)^v$$

which implies that $z(t+v)$ is strictly positive. Letting

$$\lim_{t \rightarrow \infty} c(t) = c \quad \text{and} \quad \lim_{t \rightarrow \infty} C(t) = C$$

we have $0 < c \leq C$ since $\langle c(t) \rangle$ is monotonically increasing.

Let $C_1 = (C, c-\delta, c-\delta, \dots, c-\delta)$ and choose t^* large enough so that $c - c(t^*) < \delta$ for a fixed $\delta > 0$. Suppose $c < C$ and observe that $z_i(t) = z_i(t+1)$ from the definition of $z(t)$.

Then for some $t > t^*$ $C_1 \leq z(t)$ and thus

$$(7) \quad z(t+v) \geq C_1 \hat{P} A^*(t) \dots A^*(t+v) \hat{P}^{-1} \left(\frac{1}{\lambda^*} \right)^v.$$

Fixing the opportunities represented by $A^*(t), \dots, A^*(t+v)$ observe that the above results continue to hold as the time horizon is increased due to the monotonic convergence of $\langle c(t) \rangle$ and $\langle C(t) \rangle$. Thus, we may assume that an optimal price path has been followed long enough to insure a δ so small that each component of $z(t+v)$ is strictly larger than c from (5), (7) and the strict positivity of $\hat{P} A^*(t) \dots A^*(t+v) \hat{P}^{-1} \left(\frac{1}{\lambda^*} \right)^v$. This contradicts the fact that $\langle c(t) \rangle$ is monotone. QED

If we further postulate that the maximum rate of return is attained by only one investment opportunity during each period we can establish uniform convergence to the cone N for all prices in a sufficiently small neighborhood of $p(1)$. This result is needed if $U(\cdot)$ is not linear but will not be proven here since a similar demonstration was given by Morishima^{*}[6].

MYOPIA AND THE MAXIMUM RATE OF RETURN CRITERION

The eigenvector p^* and eigenvalue λ^* can be thought of as the von Neumann price ray and growth rate of the system (see

^{*}Lemma 4, page 167.

Seelenfreund [7]). Consequently, lemma 2 may be interpreted as asserting that if the time horizon is long enough, the first period prices obtained from the optimality conditions must be close to the system's von Neumann price vector and that initially the prices must grow at approximately the von Neumann growth rate. This is, in a sense, the reverse of the usual turnpike theorem since it implies that the optimal prices must start near the turnpike and can only move off the turnpike for a small number of periods at the end of the program. This last phenomena being the result of nonstationary ending conditions. We shall now see how this price condition establishes the important myopic property of the system.

Theorem:

If the horizon is sufficiently far away then the optimal first period investment must be one of the investments associated with the maximum ROR, i.e., with maximum eigenvalue equal to λ^* .

Proof:

If the objective function $U(x_{T+1})$ is linear, i.e., $U(x_{T+1}) = u \cdot x_{T+1}$ then no matter what value is chosen for T the gradient of $U(\cdot)$ is constant and independent of x_{T+1} . This implies that $p(1) = u$ for any horizon value T and, thus, the subsequent application of lemma 2 can be made directly. On the other hand, if $U(\cdot)$ is a more general concave function $p(1)$ will depend on x_{T+1} and,

therefore, on T necessitating recourse to the uniform convergence result mentioned in the discussion following lemma 2.

Let A^* be the matrix corresponding to an opportunity with maximal ROR in the first period. Then $\lambda^* p^* = p^* A^*$ and, thus, applying lemma 1,

$$p^* A_k \leq p^* A^*$$

for all opportunities k available in the first period, with strict inequality in the first equation whenever $\lambda(k) > \lambda^*$. Consequently, we may find a cone N containing p^* in its interior such that $p A_k \leq p A^*$ for all $p \in N$ and opportunities k such that $\lambda(k) < \lambda^*$ with strict inequality in the first equation and equality in all others.

Applying lemma 2 or its uniform generalization, we know that the first period optimal price vector must be in N if the horizon is far enough away and thus any opportunity K for which $\lambda(k) < \lambda^*$ is worth less when priced out using the optimal initial prices than an opportunity with maximal ROR. Thus, no opportunity k with $\lambda(k) < \lambda^*$ can be used in an optimal program since optimality condition 2 would be contradicted.

QED

The essence of the theorem is, of course, that under the assumptions A1-A3 the entity faced with capital rationing need only consider those investment opportunities with maximal internal ROR

and if there is only one opportunity with a ROR of λ^*-1 then the choice is unique. Consequently, the firm may optimally follow a policy that is myopic in the sense of Arrow [1] since the form of future investment opportunities does not affect the decision to be currently made.

It should now be possible to examine the ROR criterion when assumption A2 is relaxed. We note that A2 is an unappealing assumption since in effect it states that no technological progress will occur in the future. Suppose that A1-A3 hold for a sufficiently long period of time commencing with period t_1 so that the theorem can be invoked to establish that $p(t_1)$ is near to p^* . Now suppose that the investment with maximum ROR during period t_1-1 has maximum eigenvalue $\lambda_1 < \lambda^*$. (Technological progress has occurred between periods t_1-1 and t_1 .) Since p^* is approximately equal to the price vector appropriate to period t_1-1 it is possible to construct an investment opportunity with maximum eigenvalue $\lambda_2 < \lambda_1$ that prices out more profitably for prices near p^* than the opportunity associated with λ_1 . Of course, the reason for this is that if the returns for opportunity 2 occur earlier than those for opportunity 1 they may be sooner invested in the opportunity associated with λ^* , the combined return being greater than λ_1 . Thus, under certain conditions of technological progress the ROR criterion is not optimal and optimal policies are not myopic.

CONCLUSION

The investment selected by the maximum rate of return criterion has been shown to be the first period optimal investment of the mathematical programming approach under assumptions of a stable future technology and a point input-stream output type of investment profile. This result being independent of the measure of wealth employed so long as positive investments in some opportunity are required in every optimal program. However, in the case wherein technological progress occurs from one period to the next it has been demonstrated that the two approaches to investment selection are not equivalent. Thus, if the maximum ROR attainable varies from one period to the next, the ROR criterion may not be optimal in the programming sense. It is likely that the actual sequence of opportunities being evaluated will display this nonstable characteristic for at least a finite number of periods, after which it becomes difficult to precisely specify the profiles of the available opportunities. If the assumptions presented here are applicable beyond these initial periods, a reasonable approach would be to use the von Neumann price vector p^* to price out the opportunities in the last specified period and then to work backward using the Dorfman-Manne method to calculate prices and choose optimal investments.

REFERENCES

1. Arrow, K. J., "Optimal Capital Policy, the Cost of Capital, and Myopic Decision Rules," Annals of the Institute of Statistical Mathematics, Vol. XVI, Nos. 1-2, Tokyo, 1964.
2. Baumol, W. J., and R. E. Quandt, "Investment and Discount Rates under Capital Rationing - A Programming Approach," The Economic Journal, June 1965.
3. Dorfman, R., "Regional Allocation of Investment: Comment," Quarterly Journal of Economics, February 1963.
4. Karlin, S., Mathematical Methods and Theory in Games, Programming, and Economics, Vol. 1. Reading, Mass.: Addison-Wesley, 1959.
5. Manne, A. S., "Optimal Dividend and Investment Policies for a Self-Financing Business Enterprise," Working Paper No. 93R, Graduate School of Business, Stanford University (1966).
6. Morishima, M., Equilibrium Stability and Growth. Oxford at the Clarendon Press, 1964.
7. Seelenfreund, A. J., "Optimal Allocation in a Pure Capital Model with Withdrawals: Asymptotic Properties and Algorithms," Technical Report No. 149, Institute for Mathematical Studies in the Social Sciences, Stanford University (1967).
8. Weingartner, H. M., Mathematical Programming and the Analysis of Capital Budgeting Problems, Prentice-Hall, Englewood Cliffs, 1963.

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