NASA TECHNICAL MEMORANDUM

NASA TM X-1496

NASA TM X-1496

CALCULATIONS TO SUPPORT OPERATION OF THE PLUM BROOK REACTOR

by Harry J. Reilly, John H. Lynch, and Steven R. Borbash

Lewis Research Center

Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1968

NASA TM X-1496

CALCULATIONS TO SUPPORT OPERATION OF THE PLUM BROOK REACTOR

By Harry J. Reilly, John H. Lynch, and Steven R. Borbash

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

CALCULATIONS TO SUPPORT OPERATION OF THE PLUM BROOK REACTOR

by Harry J. Reilly, John H. Lynch, and Steven R. Borbash

Lewis Research Center

SUMMARY

Several calculations that have aided operation of the Plum Brook Reactor are described. First, a simple mathematical model and the resulting calculations which were done to study fuel utilization are described. A decision to buy 240-gram uranium 235 (U^{235}) fuel elements and to use them three times each was made on the basis of the study. The power distribution in the mixed loading is discussed and is observed to be more uniform than that in a uniform fuel loading. Charge life weight factors are then derived; the derivation differs in approach and in results from previous derivations by others. Finally, a "xenon clock" on-line computer program in use at the Reactor is discussed, and an approximation useful for hand calculations of complex histories is derived.

INTRODUCTION

In regular operation of a high power test reactor like the Plum Brook Reactor, it is necessary to have efficient techniques of predicting the reactor charge life and power distribution, the xenon transients after shutdown from high power, and the effects of projected changes in fuel-element loadings or arrangements. Because of the short cycle time of the core (about 2 weeks) and the small amount of manpower available to do calculations for the reactor, the techniques must not be time consuming. It is helpful if the techniques can be set up for calculation on a small digital computer.

Some of the methods devised at Plum Brook for predicting reactor performance are different in some respects from those used elsewhere. This report describes several of the methods: a model for an analysis of reactor fuel utilization, a derivation of charge life weight factors, and a "xenon clock" program for an on-line digital computer. Some of the results are presented and their significance is discussed.

CORE DESCRIPTION

The core consists of a 3 by 9 array of MTR-type fuel elements, cooled and moderated by water, with beryllium reflectors. The core volume (metal and water) is about 100 liters and the core power is 60 megawatts (thermal). The core has 22 fixed elements with 18 fuel plates each and 5 shim control rods with fueled followers having 14 fuel plates each. Control is with the shim rods in a bank. The normal method of operation is with new fuel elements placed at the core ends at startup and moved toward the center at the end of each reactor operating cycle. Thus operation is with a "mixed loading" of new and used elements at the start of each cycle.

FUEL UTILIZATION STUDY

A study was made to determine what gains in reactor charge life and fuel utilization could be made by using fuel elements of different loadings than the 200-gram uranium 235 (U^{235}) elements being used at the time. The quantities of interest are the following:

(1) Reactor charge life, which is the maximum amount of time, in megawatt-days (MWD), that the reactor will operate with a given loading. Charge lives of the order of 900 megawatt-days or greater were wanted.

(2) Critical height, which is the indicated control rod position at startup. This must remain above 15 inches (about halfway inserted) to avoid excessive flux peaking below the rods and to provide sufficient shutdown margin at startup.

(3) Fuel utilization, which is the average amount of U^{235} fuel in grams used from each element before it is discarded. (The cost of an element is nearly independent of its new fuel loading, so that the average gram burnup per element is a good measure of the fuel utilization.) The value had been approximately 80 grams per element with the existing mixed loadings of new and used 200-gram elements.

(4) The maximum burnup which can safely be achieved. On this type of element, it was about 144 grams of U^{235} (private communication with Mr. T. P. Hamrick of ORNL). This value, which was taken as a limit for this study, established an upper limit on the fuel utilization. Also, the maximum burnup per element differs the least from the average when elements are used the same number of times each. Therefore, the average number of times that elements are used should be an integer.

(5) Flux perturbation on existing fluxes in experiment facilities was to be minimized and a flat power distribution maintained. This would be true if the relative fuel distribution at startup remained nearly the same.

To study the variation of all these quantities as functions of fuel loading, the following equations were used:

With rods out, not including xenon or samarium,

$$K_{eff} = K_{ex} = K_{\infty}^{RO} P_{NL} \tag{1}$$

With rods inserted (critical) including xenon and samarium,

$$K_{eff} = 1 = K_{ex} - \left|\Delta K_{rods} + \Delta K_{xe+sm}\right| \tag{2}$$

Substituting equation (1) into (2) yields

$$\left|\Delta K_{rods} + \Delta K_{xe+sm}\right| = K_{\infty}^{RO} P_{NL} - 1 \tag{3}$$

where $K_{\infty}^{RO} = \eta f \epsilon P$. For PBR, $\epsilon P \doteq 1.0$ and is not sensitive to fuel loading. Also, P_{NL} is nearly constant for the fuel loadings of interest. From reference 1 K_{ex} was measured to have a value of 1.137 for a uniform loading of 168-gram elements. When values of $\eta = 2.07$, $\sigma^{25} = 600$ barns, and a total metal-water cross section of 1700 centimeters squared are assumed, equation (1) gives $P_{NL} = 0.6929$ for that core loading. This is assumed to vary only with experiment reactivity worth for the 3 by 9 core. Thus,

$$P_{NL} = 0.6929\ (1 + \rho_{exp}) \tag{4}$$

For this study the value of P_{NL} was 0.6896.

The shim rod worth varies with fuel loading. Most of the worth is due to the fast neutron thermalization inside the control rod water passages. According to reference 2, the total reactivity worth of the control rods in the PBR with 168-gram elements (4365 g total) is 0.362 $\Delta K/K$, of which 0.227 $\Delta K/K$ is due to fast neutrons and 0.135 $\Delta K/K$ is due to thermal neutrons from outside the rods. The thermal flux is inversely proportional to fuel loading. Thus, the relative rod worth R is

$$R = \frac{0.227 + 0.135\left(\frac{4365}{M}\right)}{0.227 + 0.135} \tag{5}$$

and

$$\Delta K_{rods} = \Delta K_{rods}^{ref} R \tag{6}$$

where ΔK_{rods}^{ref} was measured in the reference loading of 168-gram elements.

In general, with rods out, not including xenon or samarium,

$$f = \frac{\int_{core} \Sigma^{25}(r)\varphi(r)\,dr}{\int_{core} \Sigma^{25}(r)\varphi(r)\,dr + \int_{core} \Sigma^{MW}(r)\varphi(r)\,dr + \int_{core} \Sigma^{FP}(r)\varphi(r)\,dr} \tag{7}$$

Multiplying by the volume V, dividing by $\overline{\varphi}$, and rearranging yield

$$f = \frac{V\Sigma_0^{25} + \int_{core} \Delta\Sigma^{25}\,\frac{\varphi(r)}{\overline{\varphi}}\,dr}{V\Sigma_0^{25} + V\Sigma^{MW} + \int_{core} \Delta\Sigma^{25}(r)\,\frac{\varphi(r)}{\overline{\varphi}}\,dr + \int_{core} \Sigma^{FP}(r)\,\frac{\varphi(r)}{\overline{\varphi}}\,dr} \tag{8}$$

where the subscript zero refers to the value for unirradiated elements and $\Delta\Sigma$ is the change in cross section due to irradiation. Assuming that

$$\frac{\Delta\Sigma^{25}(r)}{\Delta\overline{\Sigma}^{25}} \doteq \frac{\varphi(r)}{\overline{\varphi}}$$

when rods are out and that

$$\Sigma^{FP}(r) = \sigma^{FP}\,\Delta M^{25}(r)$$

and noting that $V\Sigma_0^{25} = \sigma^{25}M_0^{25}$ give

$$K_\infty^{RO} = \frac{\eta\sigma^{25}\left[M_0^{25} - F\left(M_0^{25} - M^{25}\right)\right]}{\sigma^{25}\left[M_0^{25} - F(M_0^{25} - M^{25})\right] + V\Sigma^{MW} + \sigma^{FP}\left(M_0^{25} - M^{25}\right)} \tag{9}$$

where

$$F \equiv \frac{1}{V}\int_{core} \left[\frac{\varphi(r)}{\overline{\varphi}}\right]^2 dr \tag{10}$$

The value of F was determined to be 1.192 by a criticality measurement on a depleted core with rods out. This value of F is assumed to be constant, and $\sigma^{FP} = 50$ barns per fission. If these values are used, K_∞^{RO} is computed and plotted in figure 1.

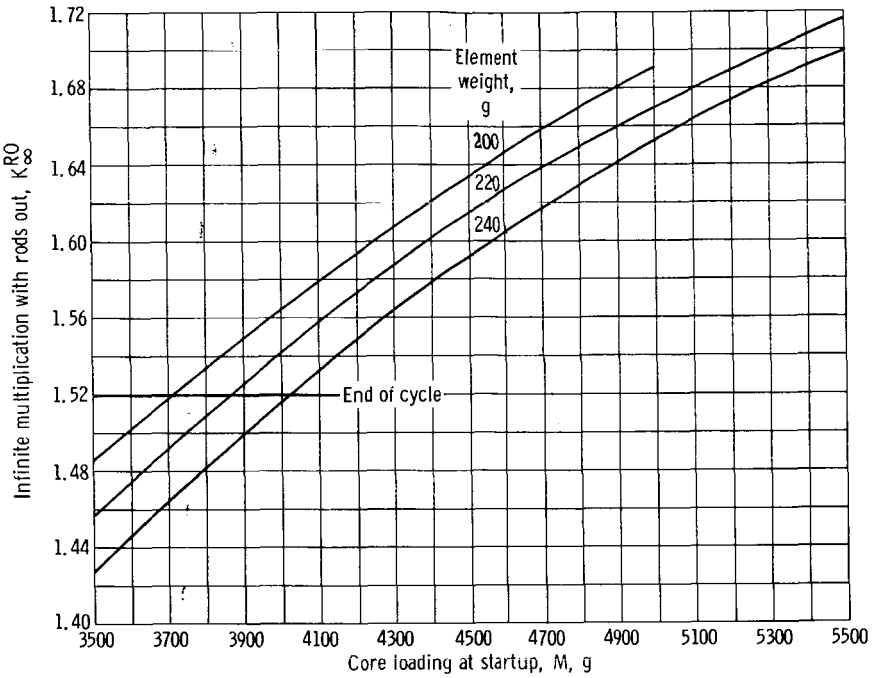

Figure 1. - Infinite multiplication with rods out against core loading for several element weights. No samarium or xenon.

For rods out (end of cycle),

$$|\Delta K_{Sm}| \doteq \frac{\Sigma_{Sm}^{eq}}{\Sigma_{core}} = \frac{\gamma_{Pm}\Sigma_f}{\Sigma_{core}} \doteq \frac{60}{V\Sigma_{core}} \tag{11}$$

$$|\Delta K_{Xe}| = \frac{\Sigma_{Xe}^{eq}}{\Sigma_{core}} \doteq \frac{\gamma_I\Sigma_f}{\Sigma_{core}} = \frac{\gamma_I}{\nu} K_\infty^{RO} = 0.0247\, K_\infty^{RO} \tag{12}$$

$$1 + 0.0247\, K_\infty^{RO} \frac{60}{V\Sigma_{core}} = 0.6896\, K_\infty^{RO} \tag{13}$$

$$K_\infty^{RO} = 1.505 + \frac{90}{V\Sigma_{core}} \doteq 1.52 \tag{14}$$

Operation will stop when K_∞^{RO} is reduced to 1.52. This establishes the end of cycle value of core loading M_f for a given fuel-element weight (see fig. 1). The charge life

is then $T = (M - M_f)/1.27$, where 1.27 is the number of grams of U^{235} depleted in 1 MWD of operation. In figure 2, T is plotted against core loading for several element weights.

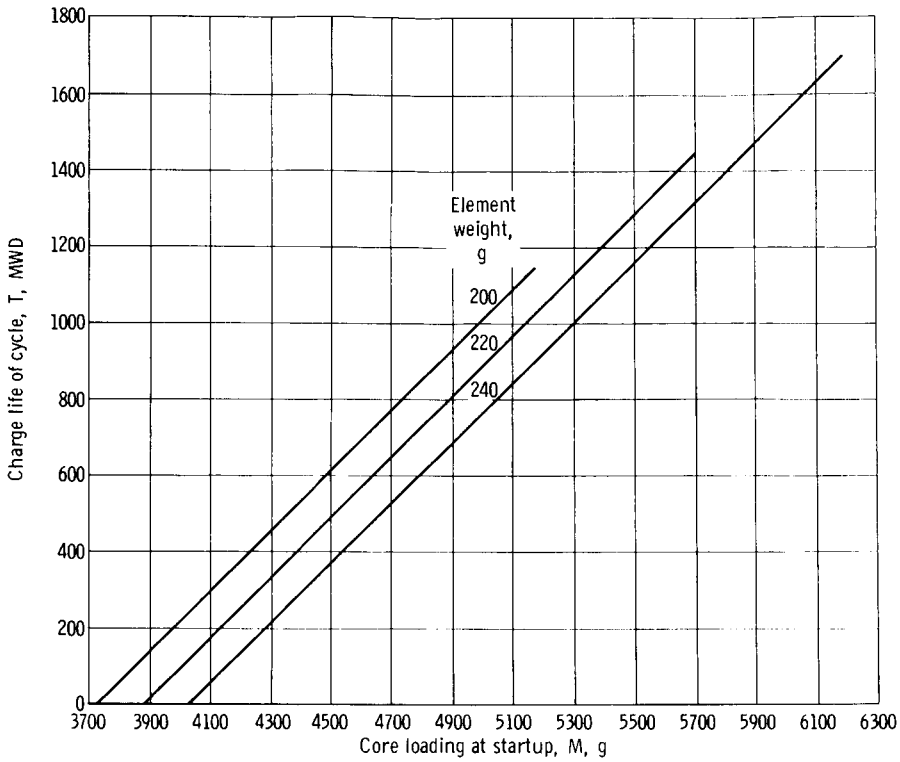

Figure 2. - Charge life as function of startup loading for several element weights.

With rods inserted (at startup),

$$\left|\Delta K_{rods}^{ref} R + \Delta K_{Sm}\right| = 0.6896\, K_{\infty}^{RO} - 1 \tag{15}$$

where

$$\left|\Delta K_{Sm}\right| = \frac{\hat{\Sigma}_{Sm}}{\Sigma_{core}} \tag{16}$$

and

$$\hat{\Sigma}_{Sm} = N_{el}^{used} \frac{\Sigma_{Sm}}{element} \tag{17}$$

The Σ_{Sm} per element equals about 16 centimeters squared for each used element at startup. The problem is to determine the number of used elements in the core at start-

up. It is noted that $M - M_f = 1.27\ T$. There are 22 fixed elements with 18 fuel plates each and 5 control rod fueled followers with 14 fuel plates each. This makes $22+(14/18)\ 5 = 25.89$ equivalent fixed elements in the core. Thus, the depletion per element in a cycle is

$$\Delta m = \frac{1.27T}{25.89} = \frac{(M - M_f)}{25.89} \tag{18}$$

If it is assumed that each cycle is the same as the previous cycle, then

$$M = N_{el}^{new}\, m_0 + N_{el}^{new}(m_0 - \Delta m) + N_{el}^{new}(m_0 - 2\Delta m) + \ldots \tag{19}$$

$$M = nN_{el}^{new}\, m_0 - N_{el}^{new}\, \frac{M - M_f}{25.89} \sum_{\xi=0}^{n-1} \xi \qquad n = \text{number of uses} \tag{20}$$

$$M = M_0 - (M - M_f)\left(\frac{1}{n} + \frac{2}{n} + \ldots + \frac{n-1}{n}\right) \tag{21}$$

If each element is used three times, $M = 1/2\ (M_0 + M_f)$, and so on. Resulting values of N_{el}^{new} are shown in figure 3. Since $N_{el}^{used} = 25.89 - N_{el}^{new}$ equations (15) to (17) can now be solved for ΔK_{rods}^{ref} and from figure 4 the starting critical height can be obtained

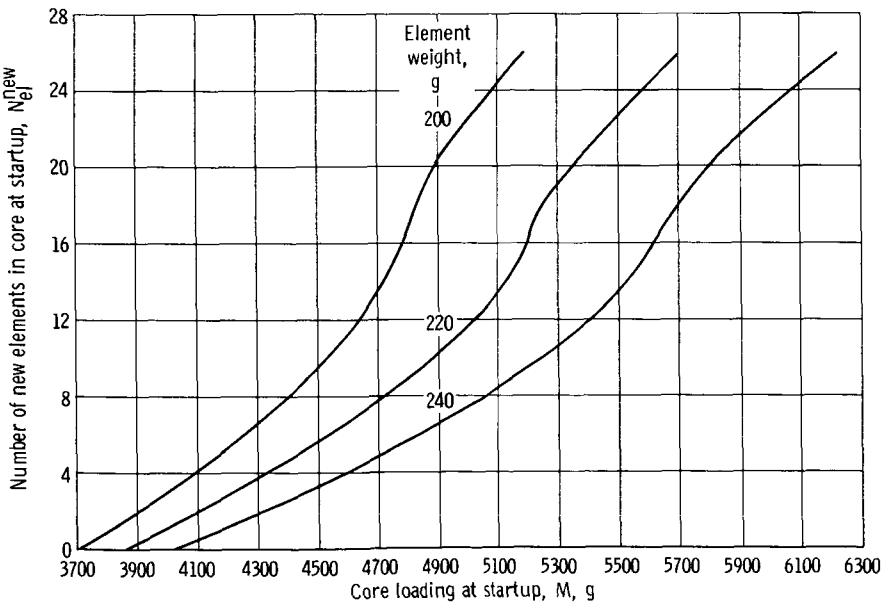

Figure 3. - Number of new elements at startup as function of core loading for several element weights.

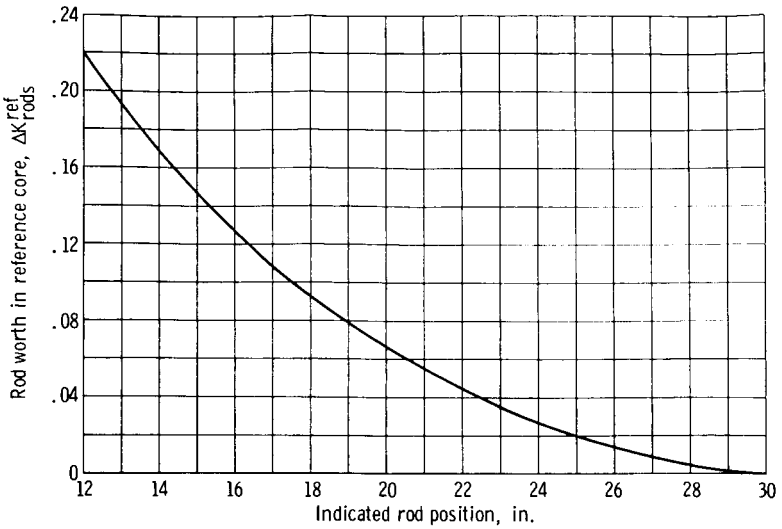

Figure 4. - Control rod calibration in clean core with 4365-gram loading.

for each case. This condition is plotted in figure 5. Figures 2 and 3 are cross-plotted to obtain the average utilization in grams per element in figure 6. Then figures 1 to 6 were cross-plotted to obtain estimates of core performance as a function of new element weight and number of elements added per cycle. An example is given in figure 7 for core performance with elements used three times each.

Many observations can be made about the results. For example, figure 2 shows that the total core loading at end of cycle will always be about the same for given element weight and experiment reactivity worth. This has been observed experimentally. Also some of the variables behave in a nonobvious fashion, such as the average utilization in figure 6. The results indicated that operation with 240-gram elements used three times each would provide improved fuel-element utilization while also giving longer charge life

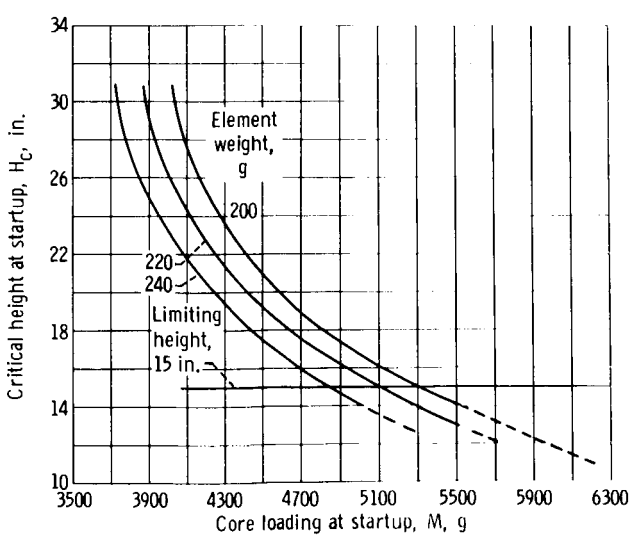

Figure 5. - Critical height at startup as function of fuel loading for several element weights.

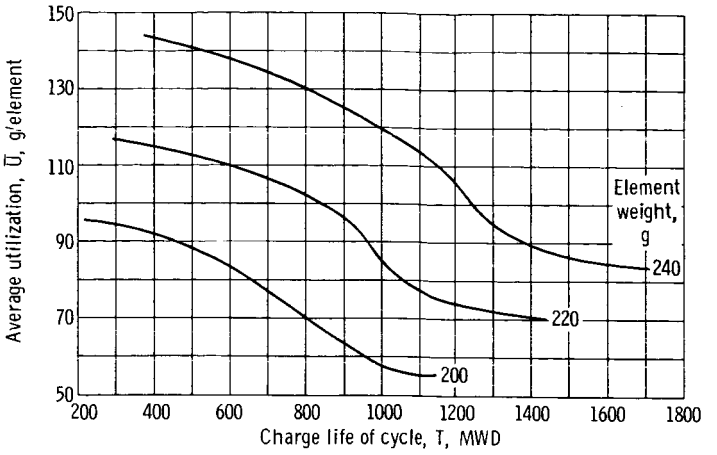

Figure 6. - Average utilization as function of charge life for several element weights.

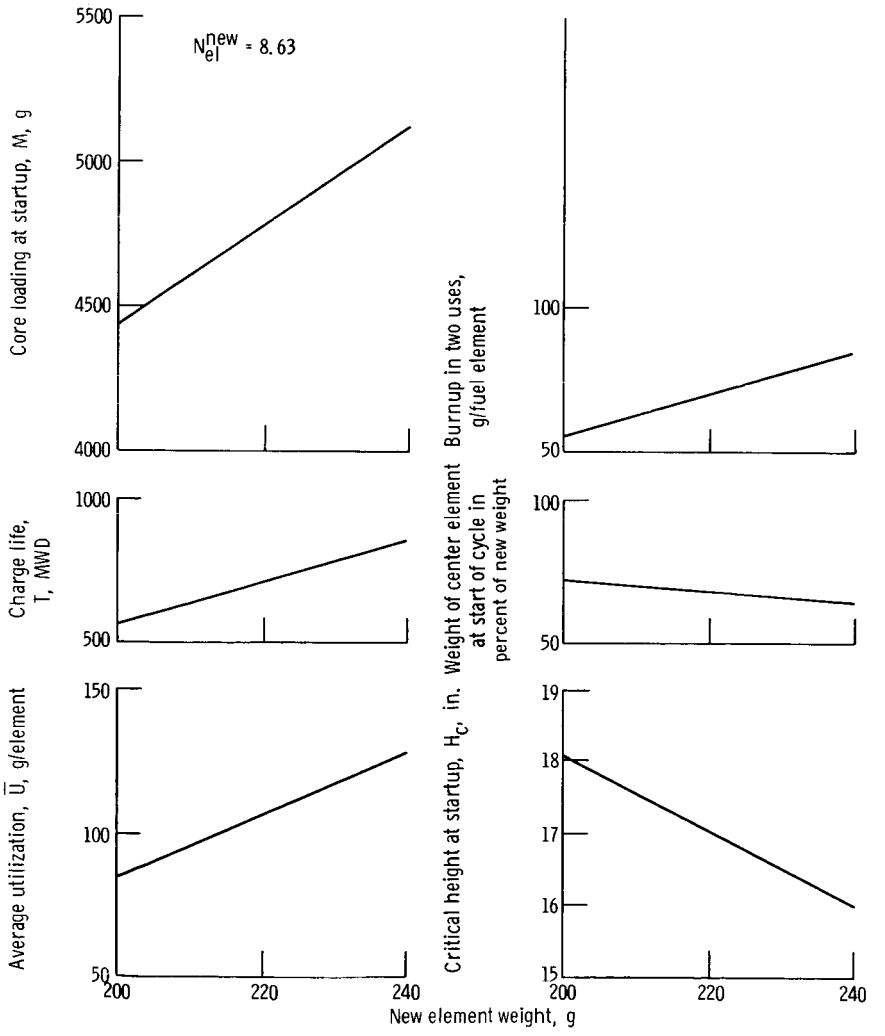

Figure 7. - Plum Brook Reactor operation with elements used three times each.

without any significant penalities. This was verified by calculations using computer codes for two-dimensional diffusion theory. Therefore, the decision was made to use this type of operation.

POWER DISTRIBUTIONS

In addition to the greater fuel utilization, the mixed loadings have the advantage of a more uniform power distribution in the core. This effect is shown in figure 8, which

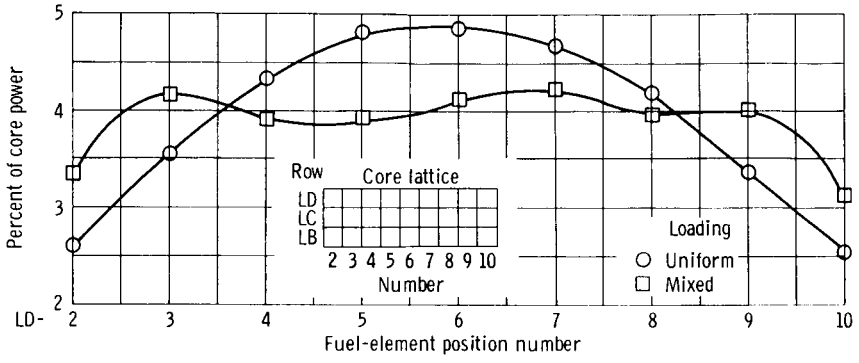

Figure 8. - Comparison of power distributions in uniform and mixed loadings.

compares the power distribution among elements in the mixed loading of 200-gram elements with that for a uniform loading. The maximum power density is reduced about 10 percent; this gives a greater heat-transfer margin so that the reactor can run at 60 megawatts with control rods at deeper insertions than was the case with uniform loadings.

A property of these loadings is that the distribution of thermal flux among elements is not sensitive to the loading or even to the control rod position. Evidently the increased fission power at the ends of the core in the mixed loading is compensated for by the larger absorption cross section in the end elements. As a result, it is possible to calculate the power distribution (and the burnup distribution) among elements at the start of each cycle using

$$P_i = \frac{F_i M_i}{\sum_i F_i M_i} \tag{22}$$

The use of this expression makes fuel accountability easy and aids in loading the reactor for a flat power distribution.

CHARGE LIFE WEIGHT FACTOR DERIVATION

Before the start of each cycle, the charge life of the PBR is predicted using a method originally developed at MTR (ref. 3). The core to be used is compared to some previous "reference" core by the equation

$$T = T_{ref} + \sum_i W_i(M_i - M_{ref,i}) \tag{23}$$

The weight factors were derived differently than those of reference 3. The units of the weight factors were written as

$$\frac{\text{MWD}}{\text{Gram}} = \frac{\text{MWD}}{\text{Unit reactitity}} \times \frac{\text{Reactivity}}{\text{Gram}} \tag{24}$$

The perturbation expression for the latter quantity is

$$\frac{\text{Reactivity}}{\text{Gram}} = \frac{\int_i \left[\frac{\delta\nu\Sigma_f}{\text{Gram}}\varphi_f^+(r)\varphi_s(r)\right] dV - \int_i \left[\frac{\delta\Sigma_a}{\text{Gram}}\varphi_s^+(r)\varphi_s(r)\right] dV}{\nu \int_{core} \left[\Sigma_f\varphi_f^+(r)\varphi_s(r)\right] dV} \tag{25}$$

A similar expression relating MWD to reactivity is

$$\frac{\text{Reactivity}}{\text{MWD}} = \frac{\int_{core} \left[\frac{\delta\nu\Sigma_f}{dt}\varphi_f^+(r)\varphi_s(r)\right] dV - \int_{core} \left[\frac{\delta\Sigma_a}{dt}\varphi_s^+(r)\varphi_s(r)\right] dV}{\nu \int_{core} \left[\Sigma_f\varphi_f^+(r)\varphi_s(r)\right] dV} \tag{26}$$

Dividing equation (25) by equation (26) gives

$$W_i = \frac{\dfrac{\delta\nu\Sigma_f}{\text{Gram}} \displaystyle\int_i \varphi_f^+(r)\varphi_s(r)\,dV - \dfrac{\delta\Sigma_a}{\text{Gram}} \int_i \varphi_s^+(r)\varphi_s(r)\,dV}{\displaystyle\int_{\text{core}} \left[\frac{\delta\nu\Sigma_f}{dt}\,\varphi_f^+(r)\varphi_s(r)\right] dV - \int_{\text{core}} \left[\frac{\delta\Sigma_a}{dt}\,\varphi_s^+(r)\varphi_s(r)\right] dV} \tag{27}$$

The change in the fission cross section due to a 1-gram change in the fuel density is

$$\frac{\delta\nu\Sigma_f}{\text{Gram}} = \frac{\nu}{V_e}\,\frac{N_0\sigma_f}{A} \tag{28}$$

where V_e is the volume of a fuel element. The change in absorption cross section per gram is

$$\frac{\delta\Sigma_a}{\text{Gram U}^{235}} = -\frac{1+\alpha}{\nu}\,\frac{\nu\delta\Sigma_f}{\text{Gram}} + \frac{\delta\Sigma_a^{FP}}{\text{Gram}} + \frac{\delta\Sigma_a^{26}}{\text{Gram}} \tag{29}$$

This fission cross section at any time t is given by

$$\nu\Sigma_{f_i}(f) = \nu\Sigma_{f_i}(o) - \delta\nu\Sigma_{f_i}(t)$$

$$= \nu\Sigma_{f_i}(o) - \left(\frac{\delta\nu\Sigma_f}{\text{Gram U}^{235}}\,\frac{\overline{\Delta M}_i^{25}}{T}\,T\,\frac{\Delta N_i^{25}}{\Delta N_{\text{core}}^{25}}\right) \tag{30}$$

where $\overline{\Delta M}_i^{25}$ is the average number of grams depleted at i, $\Delta N_i^{25}/\Delta N_{\text{core}}^{25}$ is the i^{th} fuel-element burnout relative to the core average burnout, and T is the charge life of cycle in megawatt-days. Differentiating equation (30) yields

$$\frac{d\nu\Sigma_{f_i}}{dt} = \frac{-\delta\nu\Sigma_f}{\text{Gram U}^{235}}\left(\frac{\Delta M_i^{25}}{T}\,\frac{\Delta N_i^{25}}{\Delta N_{\text{core}}^{25}}\right) \tag{31}$$

In equation (31), the quantity $(\Delta N_i^{25}/\Delta N_{core}^{25})$ equals P_i and the quantity $\overline{\Delta M_i^{235}}/T$ equals the average number of grams depleted at i per total megawatt-days, which is (1.27/27). The total cross section at i at any time after equilibrium xenon has been attained is

$$\Sigma a_i(t) = \Sigma a_i^{25}(t) + \Sigma a_i^{MW}(t) + \Sigma a_i^{Xe+Sm} + \Sigma a_i^{FP} + \Sigma a_i^{26} \tag{32}$$

where

$$\left.\begin{aligned} \Sigma a_i^{25}(t) &= \Sigma a_i^{25}(o) - \frac{1+\alpha}{\nu}\,\delta\nu\Sigma_{f_i}(t) \\ \Sigma a_i^{Xe+Sm}(t) &= \left[\gamma_{Xe} + \gamma_I + \gamma_{Pm}\right]\Sigma_{f_i}(t) \\ \Sigma_{a_i}^{FP}(t) &= \sigma^{FP}\,\frac{\delta\Sigma_{f_i}}{\sigma_{f_i}} \\ \Sigma a_i^{26}(t) &= \frac{\delta\Sigma_{a_i}^{25}}{\sigma_a^{25}}\,\frac{\sigma_a^{26}\,\alpha}{1+\alpha} \end{aligned}\right\} \tag{33}$$

Equation (32) is then differentiated to give $d\Sigma a_i(t)/dt$. Equations (28 to (33) were evaluated with the parameter values shown as follows: ′s:

Neutrons per fission in U^{235}, ν	2.47
Avagadro's number, N_0 atoms/g-mole	0.6023×10^{24}
Fission cross section, σ_f	$\sigma_a/(1+\alpha)$
Atomic weight, A, g/g-mole	235
Ratio of total capture to fission captures for U^{235}, $1+\alpha$	1.19
Fuel-element volume, V_e, cm^3	$10^5/27$
Rate of depletion of U^{235}, g/MWD	1.27
Yield of xenon, γ_{Xe}	0.003
Yield of iodine, γ_I	0.061
Yield of promethium, γ_{Pm}	0.0113
Microscopic fission product cross section, σ^{FP}, b/fission	50
Microscopic absorption cross section for U^{235}, σ_a^{235}, b/atom	$694\times0.8862\times0.975$ corrected to Maxwellian with plate disadvantage factor included
Microscopic absorption cross section for U^{236}, σ_a^{236}, b/atom	2.71

Then, they were substituted into equation (27) to give

$$W_i = \frac{2.07 \int_i \varphi_f^+ \varphi_s \, dV - 0.935 \int_i \varphi_s^+ \varphi_s \, dV}{1.27 \sum_{i=1}^{27} \left(2.07 \int_i \varphi_f^+ \varphi_s \, dV - \int_i \varphi_s^+ \varphi_s \, dV \right) P_i} \tag{34}$$

Two-dimensional multigroup diffusion theory calculations were run to obtain values of φ_f^+, φ_s^+, and φ_s for use in equation (34). The core loading is shown in figure 9. The

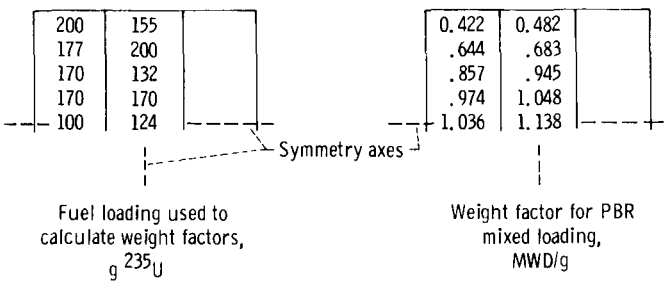

Figure 9. - Fuel-element configuration and weight factor for mixed loading of new and used 200-gram fuel elements.

calculations were done with all rods out and the core poisoned out to critical. The resulting weight factors are given in figure 9. These values differed significantly from those obtained by the method of reference 3, both in magnitude and distribution. The average of these values should be about 1.27^{-1} megawatt-day per gram as shown in the FUEL UTILIZATION STUDY section; this is true for the values shown in figure 9. Therefore, these values were adopted for routine prediction of charge life of the PBR.

POISON EQUATIONS

"Xenon Clock" Computer Program

The program calculates the poison transients occurring after shutdown in the PBR due to iodine-xenon and promethium-samarium buildup in the reactor core during before-shutdown operation. The program is divided into two parts. The first part samples the reactor flux and rod position every 10 minutes and updates the calculated iodine, xenon, promethium, and samarium concentrations while the reactor is operating. The second part, done on demand, predicts what the transient would be if the reactor were to scram.

In the first part, the equations are solved assuming that the flux is constant over the previous 10-minute interval. The new values of the concentration replace the previous values and the computer reverts to other scan-log functions until the next computation 10 minutes later. The equations used are

$$I(t_2) = e^{-\lambda_I(t_2-t_1)} I(t_1) + \frac{\gamma_I \Sigma_f}{\lambda_I} \left[1 - e^{-\lambda_I(t_2-t_1)}\right] \varphi$$

$$Xe(t_2) = e^{-\alpha_{Xe}(t_2-t_1)} \left[Xe(t_1) - \frac{I(t_1)\lambda_1 - \gamma_I \Sigma_f \varphi}{\alpha_{Xe} - \lambda_I} - \frac{\gamma_I + \gamma_{Xe}}{\alpha_{Xe}} \Sigma_f \varphi\right]$$

$$+ e^{-\lambda_I(t_2-t_1)} \left[\frac{I(t_1)\lambda_I - \gamma_I \Sigma_f \varphi}{\alpha_{Xe} - \lambda_I} + \frac{(\gamma_I + \gamma_{Xe})\Sigma_f \varphi}{\alpha_{Xe}}\right] \quad (35)$$

where

$$\alpha_{Xe} \equiv \lambda_{Xe} + \sigma_{Xe} \varphi$$

A similar set of equations describes the promethium-samarium chain.

The second part of the program calculates the times after a scram at which the core would be just critical with rods out. The reactor operator need just press a button on the console to have these times displayed for him. Figure 10 is a plot of the total poison

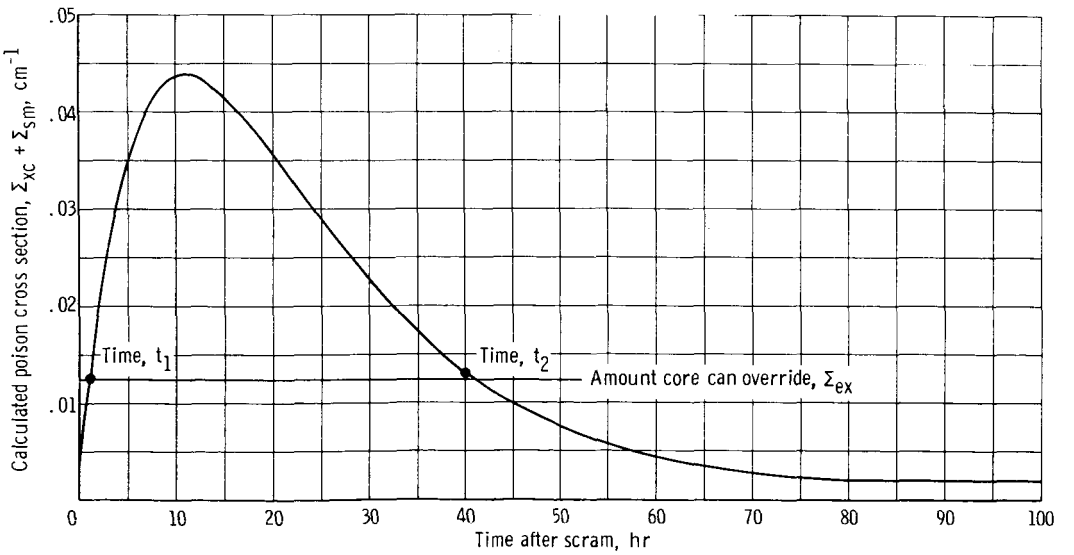

Figure 10. - Calculated poison cross section as function of time after reactor scram for the Plum Brook Reactor. Operation, 3 days at 60 megawatts.

cross section as a function of time after shutdown from high power operation. The notation Σ_{ex} represents the maximum poison cross section which the rods can override if they are pulled clear out of the core. The problem is to calculate the times t_1 and t_2 shown in figure 10. Time t_1 is the amount of time after scram which the reactor operator has to restart the reactor before the poison transient builds beyond the control rod capabilities; t_2 is the waiting time before the reactor can be restarted if the operator fails to restart the reactor within t_1.

After scram the equations governing poison dynamics are

$$I(t) = I_0 e^{-\lambda_I t}$$

$$Xe(t) = Xe_0 e^{-\lambda_{Xe} t} + \frac{\lambda_I I_0}{\lambda_I - \lambda_{Xe}} \left(e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right) \tag{36}$$

$$Sm(t) = Sm_0 + Pm_0 \left(1 - e^{-\lambda_{Pm} t} \right)$$

The xenon equation can be written

$$Xe(t) = Xe_0 f_1(t) + I_0 f_2(t) \tag{37}$$

and the samarium equation can be written

$$Sm(t) = Sm_0 + Pm_0 f_3(t) \tag{38}$$

where

$$f_1(t) = e^{-\lambda_{Xe} t}$$

$$f_2(t) = \frac{\lambda_I}{\lambda_I - \lambda_{Xe}} \left(e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right) \tag{39}$$

$$f_3(t) = \left(1 - e^{-\lambda_{Pm} t} \right)$$

Thus the after-scram poison transients are functions of the Xe, I, Pm and Sm concentrations at scram and of the three time functions f_1, f_2, and f_3.

The total poison transient after scram is made up of the xenon and samarium tran-

sients. It is convenient to work in terms of the total poison macroscopic cross section

$$\Sigma_T(t) = \Sigma_{Xe}(t) + \Sigma_{Sm}(t) = \left[\sigma_{Xe}Xe(t) + \sigma_{Sm}Sm(t)\right] \tag{40}$$

The magnitude of the change in total poison macroscopic cross section after scram is $\Delta\Sigma_T = \Sigma_T(t) - \Sigma_T(o)$. However, the reactivity worth of the xenon is greater than if it were uniformly distributed in the core because the iodine concentration is greater in regions of greater-than-average worth. Measurements in the PBR have shown that the "effective" xenon-samarium cross section after a shutdown is more nearly

$$\Delta\Sigma_T(t) = K_1\Sigma_T(t) - K_2\Sigma_T(o) \tag{41}$$

where

$$K_1 = 1.28$$

$$K_2 = 1.02$$

Expanding $\Delta\Sigma_T(t)$ gives

$$\Delta\Sigma_T(t) = K_1\Sigma_T(t) - K_2\Sigma_T(0) = K_1\left[\sigma_{Xe}Xe(t) + \sigma_{Sm}Sm(t)\right] - K_2(\sigma_{Xe}Xe_0 + \sigma_{Sm}Sm_0)$$

Let

$$Xe(t) = Xe_0f_1(t) + I_0f_2(t) \tag{42}$$

$$Sm(t) = Sm_0 + Pm_0f_3(t) \tag{43}$$

$$\Delta\Sigma_T(t) = K_1(\sigma_{Xe}Xe_0)f_1(t) + K_1(\sigma_{Xe}I_0)f_2(t) + K_1(\sigma_{Sm}Pm_0)f_3(t) + K_1(\sigma_{Sm}Sm_0)$$

$$- K_2(\sigma_{Xe}Xe_0 + \sigma_{Sm}Sm_0) \tag{44}$$

The method of calculation is as follows:

(1) The amount of poison the rods can override when drawn full out Σ_{ex} is computed from figure 11.

(2) The value of $\Delta\Sigma_T(t)$ is computed for time after scram increments from 0 to

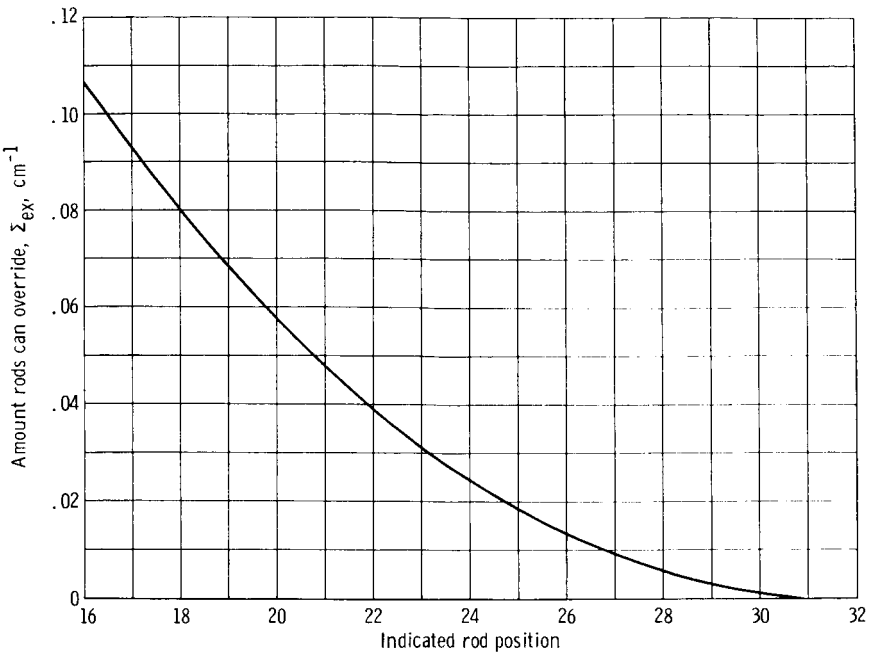

Figure 11. - Amount poison rods can override as function of rod position.

12 hours. Time t_1 is calculated by locating the two values of $\Delta\Sigma_T(t)$ which bracket Σ_{ex} and by linearly interpolating between the time points which bracket t_1.

(3) Time t_2 is calculated as time t_1 except that $\Delta\Sigma_T(t)$ is computed in increments from t_{max} to 100 hours. Values of f_1, f_2, and f_3 are included in table I.

Approximation of Xenon Transient for Complex Histories

An approximate hand-calculation method was devised for predicting the xenon poison transient after scram for complex power histories. The xenon equation after a scram is

$$Xe(t) = Xe_0 f_1(t) + I_0 f_2(t) \tag{45}$$

where f_1 and f_2 are given in equation (39)

$$\frac{\Delta Xe(t)}{Xe(t)} = \frac{f_1(t)\,\Delta Xe_0 + f_2(t)\,\Delta I_0}{f_1 Xe_0 + f_2 I_0} = \frac{\Delta I_0}{I_0} + \frac{f_1 \dfrac{Xe_0}{I_0}\left(\dfrac{\Delta Xe_0}{Xe_0} - \dfrac{\Delta I_0}{I_0}\right)}{f_1 \dfrac{Xe_0}{I_0} + f_2} \tag{46}$$

for 60-megawatt operation $(I_0 \geq 10\ Xe_0)$, and

TABLE I. - TIME FUNCTIONS

[Iodine decay probability, λ_I, 0.1037/hr; xenon decay probability, λ_{Xe}, 0.07596/hr; promethium decay probability, λ_{Pm}, 0.01285/hr.]

Time, t, hr	$f_1(t) = e^{-\lambda_{Xe}t}$	$f_2(t) = \frac{\lambda_I}{\lambda_I - \lambda_{Xe}}\left(e^{-\lambda_{Xe}t} - e^{-\lambda_I t}\right)$	$f_3(t) = 1 - e^{-\lambda_{Pm}t}$
0	1.0000	0.0	0.0
1	.9268	.09479	.01276
2	.8590	.1733	.02537
3	.7962	.2376	.03781
4	.7379	.2897	.05010
5	.6839	.3311	.06222
6	.6339	.3633	.07420
7	.5875	.3876	.08602
8	.5446	.4051	.09769
9	.5047	.4169	.1092
10	.4678	.4236	.1205
11	.4336	.4263	.1318
12	.4019	.4254	.1428
13	.3725	.4216	.1538
14	.3452	.4153	.1646
15	.3200	.4072	.1753
20	.2188	.3484	.2266
25	.1497	.2799	.2747
30	.1024	.2162	.3198
35	.07004	.1626	.3622
40	.04791	.1200	.4019
45	.03277	.08734	.4391
50	.02241	.06286	.4740
55	.01533	.04485	.5067
60	.01048	.03178	.5374
65	.007173	.02239	.5662
70	.004906	.01571	.5932
75	.003356	.01097	.6185
80	.002295	.007648	.6422
85	.001570	.005314	.6645
90	.001073	.003684	.6854
95	.0007345	.002549	.7049
100	.0005024	.001761	.7233

$$\frac{\Delta Xe(t)}{Xe(t)} = \frac{\Delta I_0}{I_0} + 0.091\left(\frac{\Delta Xe_0}{Xe_0} - \frac{\Delta I_0}{I_0}\right) \doteq \frac{\Delta I_0}{I_0} \tag{47}$$

Thus, the relative error in the xenon concentration after scram is almost entirely dependent on the relative error in the equilibrium iodine concentration at scram.

The iodine equation is

$$\frac{dI(t)}{dI} = \gamma_I \Sigma_f \varphi(t) - \lambda_I I \tag{48}$$

A solution is

$$I(t) = \gamma_I \Sigma_f \int_0^{t_s} e^{-\lambda_I (t_s - t')} \varphi(t')\, dt' \tag{49}$$

where t_s is the time of the scram. Approximating the flux over the time before shutdown in intervals of constant flux yields

$$0 \rightarrow t_1 \quad \varphi_1$$

$$t_1 \rightarrow t_2 \quad \varphi_2$$

$$t_{n-1} \rightarrow t_n \quad \varphi_n$$

$$t_n \rightarrow t_s \quad \varphi_s$$

$$I(t) = \gamma_I \Sigma_f \left[\varphi_1 \int_0^{t_1} e^{-\lambda_I (t_s - t')} dt' + \ldots + \varphi_n \int_{t_{n-1}}^{t_n} e^{-\lambda_I (t_s - t')} dt' + \varphi_s \int_{t_n}^{t_s} e^{-\lambda_I (t_s - t')} dt' \right] \tag{50}$$

Evaluating the integrals yields

$$I(t_s) = \frac{\gamma_I \Sigma_f}{\lambda_I} (\varphi_1 B_1 + \ldots + \varphi_s B_s) \tag{51}$$

where the constants B_n are dependent only on the time intervals. To find an equivalent constant flux $\hat{\varphi}_1$ which will produce the same $I(t_s)$ over the time interval $0 \rightarrow t$

$$I(t_s) = \hat{\varphi} \gamma_1 \Sigma_f \int_0^{t_s} e^{-\lambda_I (t_s - t')} dt' = \frac{\hat{\varphi} \gamma_1 \Sigma_f}{\lambda_I} \left(1 - e^{-\lambda_I t_s}\right) \tag{52}$$

Equating the two values of $I(t_s)$ and then solving and assuming $\hat{\varphi}$ is proportional to core power P yields

$$\hat{P} = P_1 B'_1 + \ldots + P_n B'_n + P_s B'_s \tag{53}$$

where

$$B'_n = \frac{B_n}{1 - e^{-\lambda_I t_s}} = \frac{e^{-\lambda_I (t_s - t_n)} - e^{-\lambda_I (t_s - t_{n-1})}}{1 - e^{-\lambda_I t_s}} \tag{54}$$

The coefficient B'_n will be different for each power history. However, if the time intervals are fixed beforehand and an average power over each interval is chosen, then the values of B'_n are fixed and the equivalent average power over the 48 hours preceding the scram is

$$\hat{P} = \overline{P}_1 B'_1 + \ldots + \overline{P}_s B'_s \tag{55}$$

The length of the intervals was chosen to give

(1) Equal weight to each $\overline{P}_n$

(2) Total 48 hours before scram

(3) No fractions of hours

The intervals and their weights are as follows:

Interval	Weight, percent
0–1	9.92
1–2	8.91
2–4	15.31
4–6	12.49
6–8	10.16
8–12	14.91
12–16	9.87
16–24	10.78
24–48	7.65

Given these values and a set of calculations of $\Delta\Sigma_{Xe}$ against t after scram from equilibrium xenon for several power levels, the xenon transient for any power history before scram can be calculated with an accuracy of 5 percent of the poison concentration.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 29, 1967,
120-27-04-54-22.

APPENDIX - SYMBOLS

A	atomic weight
B_n	coefficients in xenon transient approximation
F_i	thermal flux at element i, N/cm^2-sec
f	thermal utilization, not including xenon and samarium
H_c	critical height, in.
I	atom density of iodine
i	subscript referring to core position
K_{eff}	effective multiplication factor
K_{ex}	K_{eff} with rods out
K_{∞}^{RO}	infinite multiplication with rods out, no xenon or samarium
ΔK_{rods}	control rod worth
ΔK_{rods}^{ref}	rod worth in reference core
ΔK_{Xe+Sm}	worth of xenon and samarium
M	core loading at startup, g
M_f	core loading at end of cycle, g
M_i	U^{235} in element i, g
M_0^{25}	core loading of all new elements, g
m_0	weight of a new element, g U^{235}
Δm	average depletion of U^{235} in one fuel element in one cycle, g
N_0	Avogadro's number
N_{el}^{new}	number of new elements in core at startup
N_{el}^{used}	number of used elements in core at startup
N_i^{25}	atom density of U^{235} in fuel element i
n	number of times each element is to be used
P	probability of escape from capture while slowing down
$\hat{P}$	effective core power over period 0 to 48 hr before scram
Pm	atom density of promethium
P_i	fraction of core power produced by element i
P_{NL}	nonleakage probability
R	relative rod worth
Sm	atom density of samarium
T	charge life of cycle, MWD
t	time, MWD or sec
t_{max}	time after scram at which poison concentration reaches a maximum, hr
t_s	time of scram

$\overline{U}$ average utilization, g/element

V core volume, cm^3

V_e fuel-element volume, cm^3

W_i charge life weight factor at position i, MWD/g U^{235}

Xe atom density of xenon

α ratio of captures to fissions in U^{235}

γ yield , nuclei/fission

ϵ fast fission factor

η neutrons/capture in U^{235}

λ decay probability, sec^{-1}

ν neutrons/fission in U^{235}

ξ dummy variable

ρ reactivity

ρ_{exp} reactivity worth of experiments

Σ_a absorption cross section, cm^{-1}

Σ_{core} core cross section, cm^{-1}

Σ_{ex} amount core can override

Σ_f core fission cross section, cm^{-1}

Σ_s peak samarium after scram, cm^{-1}

Σ^{FP} macroscopic absorption cross section of long lived low cross section fission products, cm^{-1}

Σ^{MW} macroscopic absorption cross section of metal and water, cm^{-1}

Σ^{25} macroscopic absorption cross section of U^{235}, cm^{-1}

Σ_{Sm}^{eq} equilibrium cross section of samarium, cm^{-1}

Σ_{Xe}^{eq} equilibrium cross section of xenon, cm^{-1}

$\Delta\Sigma_T$ change in total poison cross section after scram, cm^{-1}

σ^{FP} microscopic fission product cross section, cm^2/g U^{235}

σ_f microscopic fission cross section, b

σ^{25} microscopic absorption cross section for U^{235}

φ_f^+ above-thermal adjoint flux

φ_s^+ thermal adjoint flux

$\varphi(r)$ thermal flux at r, n/cm^2-sec

φ_s thermal flux

$\overline{\varphi}$ average thermal flux, n/cm^2-sec

REFERENCES

1. Giesler, Harold W.; Reilly, Harry J.; and Poley, William A.: Low-Power Tests of the Plum Brook Reactor. NASA TN D-1560, 1963.

2. Thompson, T. J.; and Beckerley, J. G.: Reactor Physics and Control. Vol. 1 of the Technology of Nuclear Reactor Safety. MIT Press, 1964, pp. 48-57.

3. McMurry, H. L.; Cazier, G. A.; and Goin, R. W.: Calculation of MTR Fuel Loadings. Nucl. Sci. Eng., vol. 6, no. 1, July 1959, pp. 44-48.

NASA-Langley, 1968 —— 22 E-4017