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IMPEDANCE CHARACTERISTICS OF CONICAL SHELLS UNDER AXIAL EXCITATIONS

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FOREWORD

This report is prepared as a self-contained technical document and submitted as Part I of Final Report in lieu of the Second Quarterly Report. It contains all technical results obtained prior to May 28, 1967, and represents a complete description of Phase I of the contract work that deals with the impedance characteristics of thin conical shells under axial excitations. A part of the contents of the first quarterly report, with necessary revision and corrections, has been included herein for completeness, and the first quarterly report should be superseded by the present report.

A digital computer program in CDC 3600 Fortran Compiler Language is submitted accompanying this report as a part of the technical results obtained in Phase I of the contract.

ABSTRACT

A combined analytical and experimental study is presented to demonstrate that the transfer matrix or four-pole parameters of a truncated, thin conical shell, under axial excitations, may be accurately obtained by applying membrane shell theory. A general calculation procedure is described, and the numerical results are compared with test data for three shell models with semivertex angles 0°, 15° and 30° over a frequency range from 20 to 600 cps. The excellent agreement indicates that the four-pole parameters calculated through the present analysis are adequate for vibration analyses if the input excitation frequency is appreciably lower than a theoretical singularity on the frequency spectrum inherent to the membrane shell theory. If the excitation frequency is near or above this singularity (around 6000 cps for the models considered), a more accurate bending theory of shell must be used.

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NOMENCLATURE

a	radius of the major base of the conical shell, or radius of the cylindrical shell
b	radius of the minor base of the conical shell
C	capacitance
E	Young's modulus
F	total axial force transmitted through a shell cross section
F_1, F_2	axial forces at the input and output ends, respectively
h	shell thickness
L	inductance
ℓ	length of cylindrical shell
M	mass attached to output terminal
m	total mass of the shell
N_s, N_θ	meridional and circumferential stress resultants in shell, respectively
s	meridional coordinate of conical shell, distance measured from the apex
s_1	meridional distance from apex to major base of conical shell
t	time
U	axial displacement of a shell cross section
U_1, U_2	axial displacements of input and output ends, respectively
u, w	displacements of shell wall in meridional and outward normal directions, respectively

V_1, V_2	axial velocity of input and output ends, respectively
$\bar{x}$	meridional coordinate of cylindrical shell, distance measured from output end
$x = \bar{x}/a$	dimensionless coordinate of cylindrical shell
Z_{11}, Z_{12}	driving point impedance and transfer impedance, respectively
Z_1, Z_2, Z_3	equivalent impedance parameters in mobility circuit analog
α_{ij}	four-pole parameters
β_{ij}	transfer matrix
$\gamma = b/a$	completeness parameter
ν	Poisson's ratio
$\xi = s/s_1$	dimensionless coordinate of conical shell
ρ	mass density
$\Omega = \omega/\omega_o$	dimensionless frequency
ω	circular frequency in rad/sec
$\omega_o = (E/\rho)^{1/2}/a^2$	

INTRODUCTION

In recent years, a growing interest has been directed toward the application of the mechanical impedance approach to analyze vibrations of complex structures (e.g., Refs. 1-10). Although derived from a rather old concept in electrical engineering, the impedance technique and mechanical circuit analysis offer a much needed, complementary alternative to the normal-mode analysis of structural vibration problems. The former is especially suitable and superior in obtaining structural response information when the main excitation source may be definitely identified, such as: massive, rotating engines in a factory building; the rocket engine of a launch vehicle or missile structure; the earthquake waves felt by foundations; or the power plant in a ship hull structure. These excitation sources usually exert oscillatory forces at some definite "singular points" of the structures and thus may excite a large number of normal modes within a given frequency range. This multiplicity of modes will obviously further amplify the analytical difficulties associated with structural complexity, and often renders an accurate normal mode analysis of the response impractical, if not infeasible. In this class of engineering problems, the impedance method not only makes available the systematic techniques of circuit analysis through electric analogies, but also allows a direct correlation with test data or actual vibration record of the structure during operation.

In order to develop the full advantage of the mechanical circuit analysis, preliminary studies must be made to provide complete characteristic information for the basic structural elements, such as: lumped spring-mass units, beams, plates, and shells of commonly used configurations. These basic impedance characteristics (transfer matrix or four-pole parameters) may be used subsequently in analyzing any complex structures containing such elements, which are then replaced by the so-called "black box" in the mechanical circuit model for the entire structure. The present study is intended to provide the impedance characteristics of the truncated, thin conical shell (including the cylindrical shell as a special case) with respect to axisymmetric longitudinal excitations. This information will be useful, for example, in handling longitudinal vibration problems of complex launch vehicle structures which include a number of cylindrical and conical shells in tandem arrangements.

FOUR-POLE PARAMETER VIBRATION ANALYSIS OF STRUCTURES

Consider a linear, elastic, structural element which has a single input terminal, with oscillatory input force F_1 and velocity V_1, and a single output terminal, with oscillatory output force F_2 and velocity V_2. Under the restriction that no dynamic instability occurs, the relation between these four quantities can be uniquely described by a linear transformation:

$$F_1 = \alpha_{11}F_2 + \alpha_{12}V_2$$
$$V_1 = \alpha_{21}F_2 + \alpha_{22}V_2 \tag{1}$$

where the four coefficients α_{ij} are termed four-pole parameters[1] which are, in general, frequency-dependent complex quantities. When damping is considered in the system, α_{ij} will generally comprise both real and imaginary parts; but, if damping is neglected, α_{11} and α_{22} reduce to real, dimensionless numbers, whereas α_{12} and α_{21} become pure imaginary, the former having the dimension of mechanical impedance (force/velocity, or mechanical ohm) and the latter having the dimension of mobility or mechanical admittance (velocity/force).

For those elastic systems which consist of a finite number of simple, lumped elements (massless springs and rigid masses), the four-pole parameters can always be written in simple algebraic forms. However, when the elastic system contains elements with distributed mass and stiffness, the four-pole parameters become complicated transcendental functions of the input frequency and, in most cases, cannot be obtained in closed forms. It will be shown later that, for a continuous element such as a nonuniform column or the conical shell under consideration, for which the force and velocity variables are governed by a pair of coupled, first-order differential equations, an efficient numerical integration procedure may be used to calculate the required four-pole parameters.

Moreover, since such an elastic element always possesses an infinite number of natural frequencies (i.e., infinite degrees of freedom), regardless of the imposed boundary conditions at the two terminals, the electrical circuit analogies for such a mechanical four-pole element, [Fig. 1(a)] cannot be exactly represented by a finite number of simple capacitors and inductors.* Using mobility circuit analogy (force-current, velocity-voltage analogy), one can construct an exact circuit representation either by incorporating an infinite number of capacitors $C_1, C_2, \ldots, C_n$, and inductors L_1, L_2, ..., L_{n+1}, arranged as shown in the left of Figure 1(b), or by using equivalent impedances of transcendental expressions Z_1, Z_2 and Z_3, as shown in the right of Figure 1(b). Due to the lack of symmetry between the two terminals "1" and "2" of a conical shell, the two equivalent impedances Z_1 and Z_2 will not be equal as in the case of the uniform bar or cylindrical shell. The relations between these equivalent impedances and the four-pole parameters can be obtained from Reference 1, as follows:

$$\alpha_{11} = \frac{Z_2 + Z_3}{Z_3} \qquad \alpha_{12} = \frac{1}{Z_3}$$
$$\alpha_{21} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} \qquad \alpha_{22} = \frac{Z_1 + Z_3}{Z_3} \tag{2}$$

It may be noted that the four-pole parameters α_{ij} can be expressed in

*Resistors are not used because damping effects are not considered in the following analysis.

terms of only three independent impedance parameters; this is because the system obeys the reciprocity principle so that α_{ij} must satisfy the restraint condition

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix} = 1 \tag{3}$$

This condition may be used as an accuracy criterion to check the numerical results.

For small frequencies which are well below the lowest frequency at which any of α_{ij} vanishes, the equivalent impedance parameters reduce to the approximate form as shown in Figure 1(c). In this case, the parameter Z_3 reduces to a simple capacitor, with the magnitude equal to the total mass of the cone, while the other two parameters, Z_1 and Z_2, reduce to simple inductors, which represent the elastic property of the cone.

TRANSFER MATRIX OF A TRUNCATED CONICAL SHELL IN AXISYMMETRIC VIBRATIONS

In the following, we shall consider the analytical calculation of the four-pole parameters α_{ij} for a truncated conical shell in axisymmetric vibrations. No boundary conditions are prescribed except the restriction that all the boundary quantities are harmonic with the excitation frequency ω. It will be assumed that, for thin conical shells with α no larger than, say, 45°, the axisymmetric vibrations may be satisfactorily governed by membrane theory. Thus, there are two equations of motion,

$$N_s' + \frac{1}{s}(N_s - N_\theta) = \rho h \ddot{u}$$
$$- \frac{1}{s} N_\theta \cot \alpha = \rho h \ddot{w} \tag{4}$$

and two stress-displacement relations,

$$N_s = \frac{Eh}{1 - \nu^2}\left(u' + \nu \frac{u + w \cot \alpha}{s}\right)$$
$$N_\theta = \frac{Eh}{1 - \nu^2}\left(\frac{u + w \cot \alpha}{s} + \nu u'\right) \tag{5}$$

where s is the meridional distance measured from the vertex, prime denotes differentiation with respect to s, dot denotes time differentiation, and the other notations are given in the Nomenclature. Since only the steady-state harmonic motions of the shell are under consideration, the time dependence of all the stress and displacement variables may be put in the usual form $e^{i\omega t}$; therefore, in the following analysis, the factor $e^{i\omega t}$ will be ignored, and the time differentiation may be replaced by the operator $i\omega$; e.g.,

$$\dot{u} = i\omega u, \qquad \ddot{u} = (i\omega)^2 u = -\omega^2 u$$

For convenience, we shall introduce two new variables defined by:

$$F = -2\pi s N_s \sin\alpha \cos\alpha$$
$$U = -u \cos\alpha + w \sin\alpha \tag{6}$$

which represent the resultant axial force transmitted through the cone and the axial displacement, respectively. It may be noted that, at the input terminal (major base), $s = s_1$, the force and velocity are

$$F_1 = F(s_1), \qquad V_1 = i\omega U(s_1) \tag{7}$$

and, at the output terminal (minor base), $s = \gamma s_1$, the force and velocity are

$$F_2 = F(\gamma s_1), \qquad V_2 = i\omega U(\gamma s_1) \tag{8}$$

Using Eqs. (6) to eliminate N_s and w from Eqs. (4) and (5) and introducing the dimensionless spatial coordinates $\xi = s/s_1$, we can write the governing equations in the following form:

$$U = \cos\alpha(\cos^2\alpha - \Omega^2\xi^2)^{-1}\left[\frac{\nu \tan\alpha}{2\pi Eh} F - (1 - \Omega^2\xi^2)u\right]$$
$$N_\theta = \frac{Eh}{a} \frac{\Omega^2\xi}{\sin\alpha \cos\alpha} [u \cos\alpha + U]$$
$$\frac{dF}{d\xi} = -2\pi Eh\Omega^2\xi U \csc\alpha \tag{9}$$
$$\frac{du}{d\xi} = -\frac{1}{Eh \sin\alpha}\left[\frac{F}{2\pi\xi \cos\alpha} + \nu a N_\theta\right]$$

in which the dimensionless frequency parameter:

$$\Omega = \omega/\omega_o \tag{10}$$

with

$$\omega_o^2 = E/\rho a^2 \tag{10a}$$

The set of Eqs. (9) is of second order and in a convenient form for numerical integration. It may be noted that a singularity exists at the frequency

$$\Omega = \cos \alpha \tag{11}$$

at or above which the coefficient of the first equation of Eq. (9) tends to infinity.

Independent numerical integrations of Eqs. (9) for two sets of initial values, $(F, U) = (1, 0)$ and $(0, 1)$ at $\xi = \gamma$, will yield two sets of influence coefficients, $(F, U) = (\beta_{11}, \beta_{21})$ and (β_{12}, β_{22}) at $\xi = 1$, respectively. This provides the transfer matrix $[\beta_{ij}]$ that relates the boundary values of (F, U) at $\xi = 1$ and γ.

$$\begin{Bmatrix} F_1 \\ U_1 \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{Bmatrix} F_2 \\ U_2 \end{Bmatrix} \tag{12}$$

where the subscript 1 refers to the input end $\xi = 1$, and the subscript 2 refers to the output end $\xi = \gamma$. Since the axial velocities at the two terminals are given by:

$$V_1 = i\omega U_1, \qquad V_2 = i\omega U_2$$

therefore, there follows from Eq. (12),

$$\begin{aligned} F_1 &= \beta_{11} F_2 + (\beta_{12}/i\omega) V_2 \\ V_1 &= i\omega\beta_{21} F_2 + \beta_{22} V_2 \end{aligned} \tag{13}$$

Comparing Eqs. (13) to Eqs. (1), we find the four-pole parameters

$$\begin{aligned} \alpha_{11} &= \beta_{11}, \qquad \alpha_{12} = (-\beta_{12}/\omega)i \\ \alpha_{21} &= i\omega\beta_{21}, \qquad \alpha_{22} = \beta_{22} \end{aligned} \tag{14}$$

In the calculation of β_{ij}, a standard subroutine for numerical integration may be used to integrate Eqs. (9).

SPECIAL CASE OF THE CYLINDRICAL SHELL ($\alpha = 0$)

The set of Eqs. (9) is not in a suitable form for the special case of the cylindrical shell ($\alpha = 0$). However, since the governing differential equations for cylindrical shells have constant coefficients, it is possible to obtain closed form solutions.

Referring to the dimensionless coordinate $x = \bar{x}/a$, where $\bar{x}$ is the meridional distance measured from the output terminal "2," it can be shown that the governing Eqs. (4) and (5) may be reduced to a single second-order differential equation for u,

$$\frac{d^2u}{dx^2} + \lambda^2 u = 0 \tag{15}$$

where

$$\lambda^2 = \frac{\Omega^2[1 - (1 - \nu^2)\Omega^2]}{(1 - \Omega^2)} \tag{16}$$

with the frequency parameter Ω defined in Eqs. (10). It is obvious that Eq. (15) has a singularity at $\Omega = 1$ [cf. Eq. (11)], or $\omega = \omega_o$, which limits the applicability of membrane shell theory. We shall restrict our attention to the frequency range $\Omega < 1$, then λ has a real value, and the general solution of Eq. (15) may be written in the form

$$u = A \sin \lambda x + B \cos \lambda x \tag{17}$$

From the first equation of Eqs. (4), we have

$$N_x = \frac{\rho h a \omega^2}{\lambda} (A \cos \lambda x - B \sin \lambda x)$$

Introducing the axial force and axial displacement variables with equivalent definitions as Eqs. (6), we obtain

$$\begin{aligned} F &= - \frac{2\pi \rho h a^2 \omega^2}{\lambda} (A \cos \lambda x - B \sin \lambda x) \\ U &= - (A \sin \lambda x + B \cos \lambda x) \end{aligned} \tag{18}$$

Therefore, at the input terminal, $x = \ell/a$, where ℓ is the length of the cylinder:

$$F_1 = -\frac{2\pi\rho h a^2\omega^2}{\lambda}\left(A\cos\frac{\lambda\ell}{a} - B\sin\frac{\lambda\ell}{a}\right)$$
$$U = -\left(A\sin\frac{\lambda\ell}{a} + B\cos\frac{\lambda\ell}{a}\right) \tag{19}$$

and, at the output terminal, $x = 0$,

$$F_2 = -\frac{2\pi\rho h a^2\omega^2}{\lambda} A$$
$$U_2 = -B \tag{20}$$

Elimination of the integration constants A and B from Eqs. (19) and (20) yields

$$\begin{Bmatrix} F_1 \\ U_1 \end{Bmatrix} = \begin{bmatrix} \cos\frac{\lambda\ell}{a} & -\frac{2\pi\rho h a^2\omega^2}{\lambda}\sin\frac{\lambda\ell}{a} \\ \frac{\lambda\sin\frac{\lambda\ell}{a}}{2\pi\rho h a^2\omega^2} & \cos\frac{\lambda\ell}{a} \end{bmatrix} \begin{Bmatrix} F_2 \\ U_2 \end{Bmatrix} \tag{21}$$

This gives the desired transfer matrix $[\beta_{ij}]$ for cylindrical shells. Using the relations, Eqs. (13), the four-pole parameters may be readily obtained in closed form valid for $\omega < \omega_o$:

$$\alpha_{11} = \alpha_{22} = \cos\Lambda$$
$$\alpha_{12} = i\omega m\Lambda^{-1}\sin\Lambda \tag{22}$$
$$\alpha_{21} = i\omega^{-1}m^{-1}\Lambda\sin\Lambda$$

in which, $\Lambda = \lambda\ell/a$, and $m = 2\pi\rho h a\ell$ is the total mass of the cylindrical shell. It may be noted that the parameters α_{ij} given by Eqs. (22) satisfy the condition (3).

Substitution of Eqs. (22) into the relations (2) gives the equivalent impedance parameters for the cylindrical shell:

$$\left.\begin{aligned} Z_1 = Z_2 &= \frac{(1 - \cos\Lambda)\Lambda i}{m\omega \sin\Lambda} \\ Z_3 &= \frac{\Lambda}{m\omega i \sin\Lambda} \end{aligned}\right\} \quad (\omega < \omega_o) \tag{23}$$

which define the mobility circuit analog in Figure 1(b). If ω is small (i.e., $\omega << \omega_o$) and the shell is not too long, then $\Lambda << 1$, and Eqs. (23) reduce to the following approximate expressions:

$$\left.\begin{aligned} Z_1 = Z_2 &\approx \frac{\Lambda^2 i}{2m\omega} \equiv \frac{\ell\omega i}{4\pi Eha} \\ Z_3 &\approx \frac{1}{m\omega i} \end{aligned}\right\} \quad (\omega << \omega_o) \tag{24}$$

which have the expected form defining the approximate circuit analog in Figure 1(c), with the inductors $L_1 = L_2 = \dfrac{(\ell/2)}{Eh(2\pi a)}$, and the capacitor $C = m$.

It should be pointed out that, as ω approaches ω_o, the parameter λ increases without bound; therefore, the four-pole parameters become infinitely oscillatory, i.e., have infinite number of maxima and minima within an arbitrarily small frequency interval enclosing ω_o. Therefore, when the excitation frequency is near or above this singularity, a more accurate bending theory of shell is needed to correct the membrane solution.

The calculated four-pole parameters, α_{ij}, for the three shell models described below, are shown in Figures 2-4, in which the frequency singularities are indicated by vertical dashed lines near 6000 cps.

IMPEDANCE EXPERIMENTS OF CONICAL SHELLS SUPPORTING AN ARBITRARY MASS

It is evident that Eqs. (1) provide two equations for four unknowns, the two terminal forces and the two terminal velocities, and therefore represent an indeterminate set. This is, in fact, an advantageous feature of the method which permits versatile applications of the characteristic four-pole parameters of structural elements. When the two terminals are connected to other elements in any complex mechanical circuit system, such as various stages of a launch-vehicle structure or its payload assembly, two additional conditions are provided by the connecting joints.

In the present study, the calculated four-pole parameters will be experimentally tested through the following arrangements: the major base of the conical shell will be excited by an electrodynamic shaker with prescribed input level and sweep frequency control, while the minor base will be attached to a rigid mass M (Fig. 5). Since the impedance of the mass is known, the boundary condition at the output terminal can be readily obtained:

$$F_2 = M\omega i V_2 \tag{25}$$

Substitution of the above into Eqs. (1) yields

$$\begin{aligned} F_1 &= (\alpha_{11} M\omega i + \alpha_{12}) V_2 \\ V_1 &= (\alpha_{21} M\omega i + \alpha_{22}) V_2 \end{aligned} \tag{26}$$

From these, one can easily calculate the input impedance

$$Z_{11} = \frac{F_1}{V_1} = \frac{\alpha_{11} M\omega i + \alpha_{12}}{\alpha_{21} M\omega i + \alpha_{22}} \tag{27}$$

and the transfer impedance

$$Z_{12} = \frac{F_1}{V_2} = \alpha_{11} M\omega i + \alpha_{12} \tag{28}$$

Since, as mentioned before, α_{12} and α_{21} are pure imaginary, while α_{11} and α_{22} are real, both Z_{11} and Z_{12} are pure imaginary quantities, indicating the usual 90° phase-shift between the force and velocity variables. In the correlation between calculated and measured results, only their absolute values $|Z_{11}|$ and $|Z_{12}|$ need be considered.

It may be noted that two special cases of Eq. (25) are of particular interest in view of their relation to the numerical calculation of β_{ij} as well as for their own physical significance. The first case is $M = 0$ (free end), which implies $F_2 = 0$. If we set V_2 equal to unit velocity, Eqs. (26) immediately give $F_1 = \alpha_{12}$ and $V_1 = \alpha_{22}$. The second case is $M = \infty$

(blocked end) which implies $V_2 = 0$. If we now set F_2 equal to unit force, Eqs. (1) yield $F_1 = \alpha_{11}$ and $V_1 = \alpha_{21}$. Therefore, $(\alpha_{11})^{-1}$ represents the force transmissibility for the case of blocked end, and $(\alpha_{22})^{-1}$ represents the velocity transmissibility for the case of free end.

APPARATUS AND EXPERIMENTAL PROCEDURE

Figure 5 shows a schematic diagram of the overall apparatus used for impedance measurements. Two conical and one cylindrical shell models have been investigated in the experiments; their detailed dimensions are given in Table 1.

TABLE 1. DIMENSIONS OF THE SHELL MODELS

Model No.	Cone Angle, α	Radius a (in.)	Radius b (in.)	Thickness h (in.)	Net Weight of Shell (lb)
1	0°	5.0	5.0	0.005	0.670
2	15°	5.0	2.5	0.005	0.323
3	30°	5.0	2.5	0.005	0.167

All the three models were made of tempered mild-steel sheet-stock which was rolled and butt-welded along a generatrix, with negligible discontinuity at the seam. Each specimen was made to support a rigid mass through a steel, upper end-plate which was spot-welded to the upper (or smaller) edge of the shell. Two rows of 0.020-in. diameter spots, spaced at 1/8-in. center-to-center, were used in the welding to secure firm connections. The total weight of the upper end-plate and supported mass was

32.8 lb which was much heavier than those of the shells. It was selected such that the resonance of the first longitudinal mode of the system would occur below 400 cps, the estimate frequency limit for accurate experimental investigations (see discussion below).

The major base of the specimen was similarly spot-welded to the lower end-ring (a steel annular plate weighing about 15 lb), which was mounted on a thick, steel base-plate through four piezoelectric force transducers. The entire arrangement was then bolted to the armature of an electrodynamic shaker.

Throughout the design, special emphasis has been placed on maintaining high rigidity in all parts relative to that of the shell models so that the usual mass-cancellation procedure may be used to eliminate the inertia force of the lower end-ring. Thus, a portion of the resultant acceleration signal from this end-ring was properly scaled and inverted, then fed into the force signal to produce a vector cancellation (Fig. 5). Preliminary checkout of this procedure by exciting the end-ring alone indicated that it was rigid enough to produce only inertia-force signal up to about 500 cps, and started to show appreciable elastic deformation at about 600 cps. Therefore, no experimental data were taken beyond this frequency.

The experiments were designed to measure both the driving-point impedance at the base of the shell as well as the transfer impedance through the specimen. The input force F_1 was measured as described in the preceding paragraph, and the accelerations at the two terminals were measured by piezoelectric accelerometers. Integrations of the acceleration signals (to give direct velocity readings) as well as the mass cancellation procedure were performed electronically through operational amplifiers. Narrow-band frequency filters were also used to filter out high frequency noise from various sources. The output force and velocity signals were then displayed on an oscilloscope for examination and recorded through a digital voltmeter. The high sensitivity of the transducers allowed excitations at low input levels and maintaining good accuracy throughout the experiments.

RESULTS AND DISCUSSIONS

The numerical results presented here were calculated on a CDC-3600 computer. In the calculation of the transfer matrix, β_{ij}, for conical shells, a numerical integration subroutine using fifth-order Adams method* was incorporated in the program for integrating Eqs. (9). The calculated four-pole parameters, α_{ij}, of the three shell models are shown graphically in Figures 2-4. If we use the reciprocity condition (3) as an accuracy criterion, the results indicate that the numerical integration procedure introduces higher error as the frequency approaches the singularity (6212 cps for the 15° cone, and 5569 cps for the 30° cone). For example, for the 15° cone, the error is about 4% at 4000 cps and increases

*The subroutine, written by R. H. Hudson, is a self-starting variant of the Adams method incorporating automatic step-size control.

to 12% at 5000 cps; for the 30° cone, the error is 3% at 2000 cps and increases to 8% at 3000 cps. As mentioned before, the four-pole parameters become infinitely oscillatory as the frequency approaches the singularity; therefore, the error is inherent to the ill-behaved Eqs. (9) near this frequency and cannot be eliminated by the step-size control in the integration process.

The correlation of the calculated and measured impedances, Z_{11} and Z_{12}, are shown in Figures 6-8. It may be seen that the agreement is in general very good. In Figure 7, some unusual scattering of the input impedance Z_{11} may be seen near 250 cps. Careful examination showed that this was a result of dynamic instability of geometric imperfections along the butt-joint seam, which showed excessive lateral vibration at this frequency. In Figure 8, the so-called "split-resonance" was observed in the experimental data of the input impedance Z_{11}, which exhibited two sharp peaks with close frequencies (the first peak at about 380 cps and the second at about 410 cps, whereas the calculated resonance is 392 cps). This may also be associated with some model imperfections, but no definite explanation can be found.

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PLAN FOR RESEARCH DURING THE NEXT QUARTER

During the next quarter, it is planned to continue on both the analytical and experimental work to determine the (4×4) transfer matrix, driving-point impedance and transfer impedance of the conical shell under lateral excitations. Since the beam-type bending and transverse shearing modes are always coupled, it is equivalent to coupled electric circuits with eight poles. In the experiments, we have restricted our attention to pure translational vibrations at the input end, but the responses of the attached mass at the output end have both a translational and rotational component. Both the analytical and experimental work on this phase are presently well under way and are expected to produce satisfactory progress during the next quarter.

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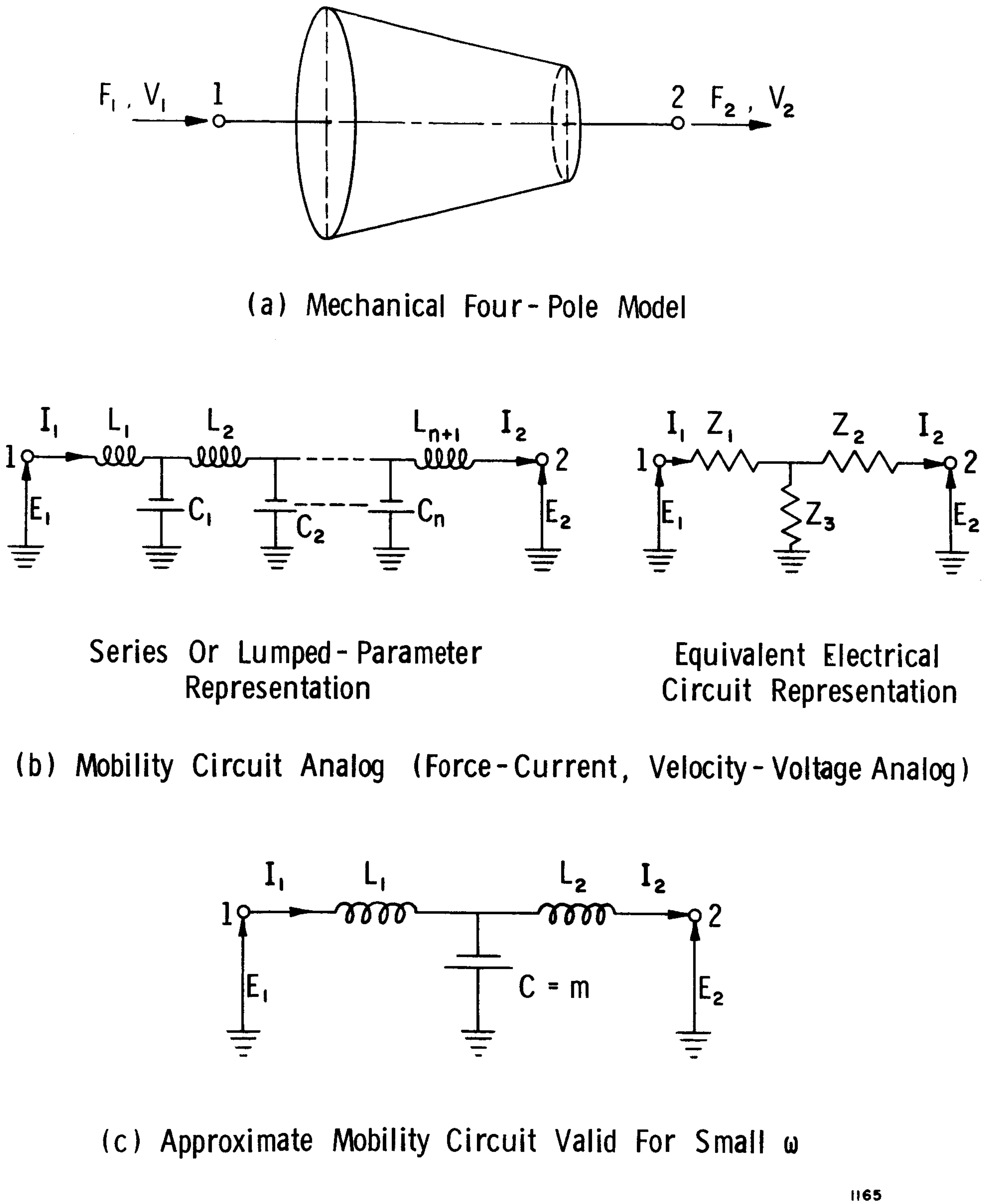

(a) Mechanical Four-Pole Model

(b) Mobility Circuit Analog (Force-Current, Velocity-Voltage Analog)

(c) Approximate Mobility Circuit Valid For Small ω

FIGURE 1. MECHANICAL FOUR-POLE AND ELECTRICAL ANALOGIES FOR CONICAL SHELL WITH DISTRIBUTED MASS

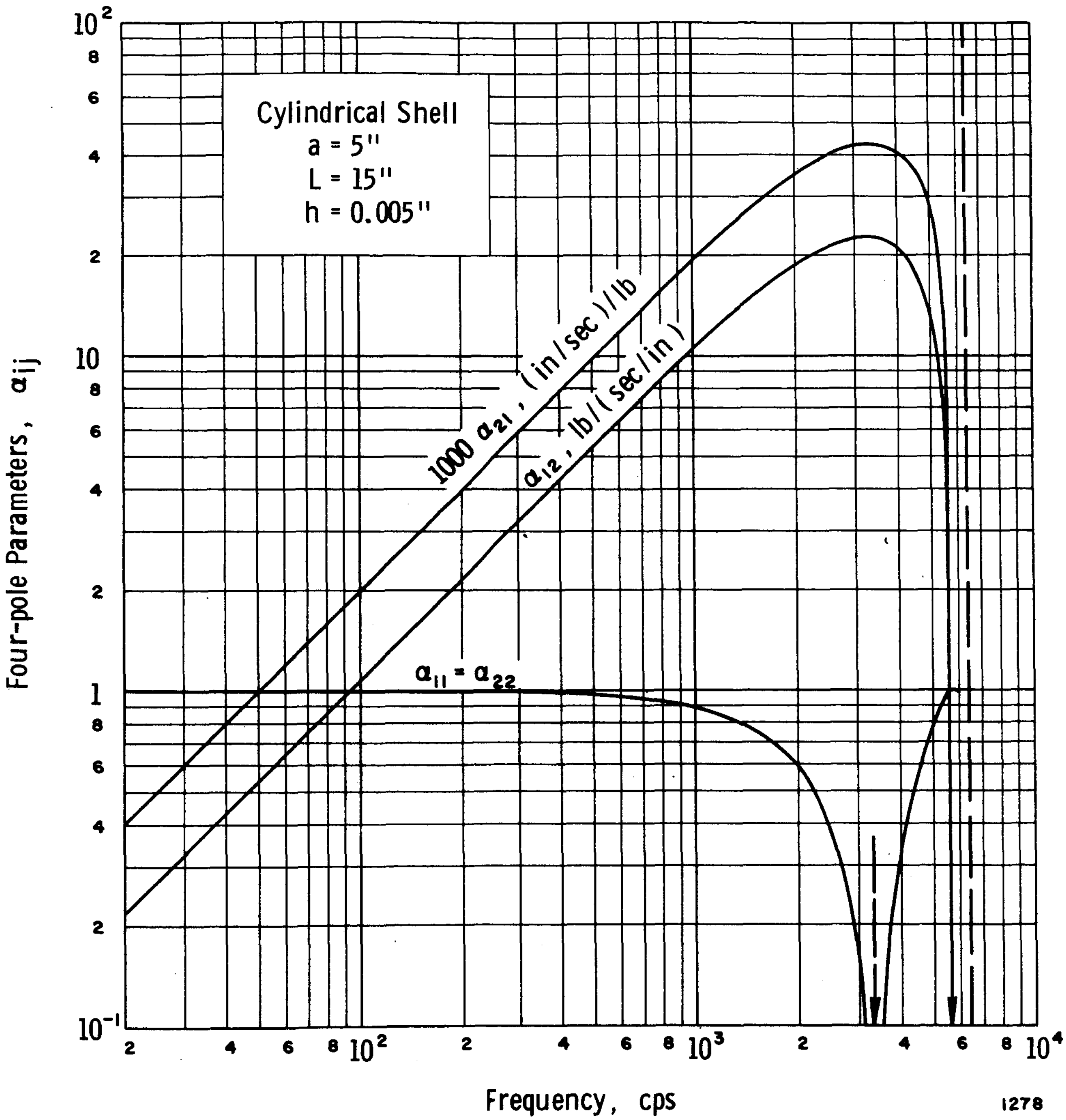

FIGURE 2. FOUR-POLE PARAMETERS FOR THE CYLINDRICAL SHELL MODEL

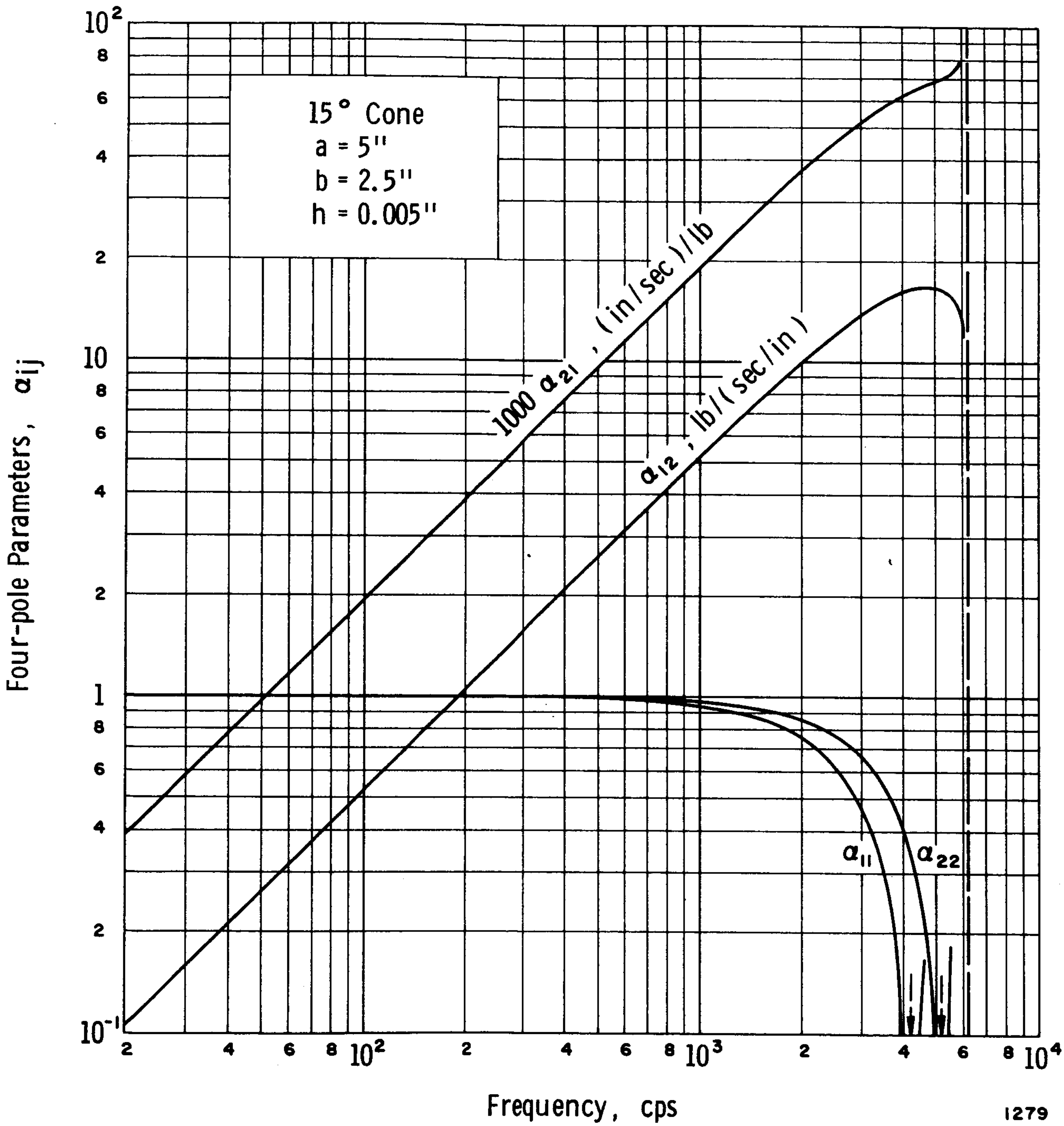

FIGURE 3. FOUR-POLE PARAMETERS FOR THE 15° CONICAL SHELL MODEL

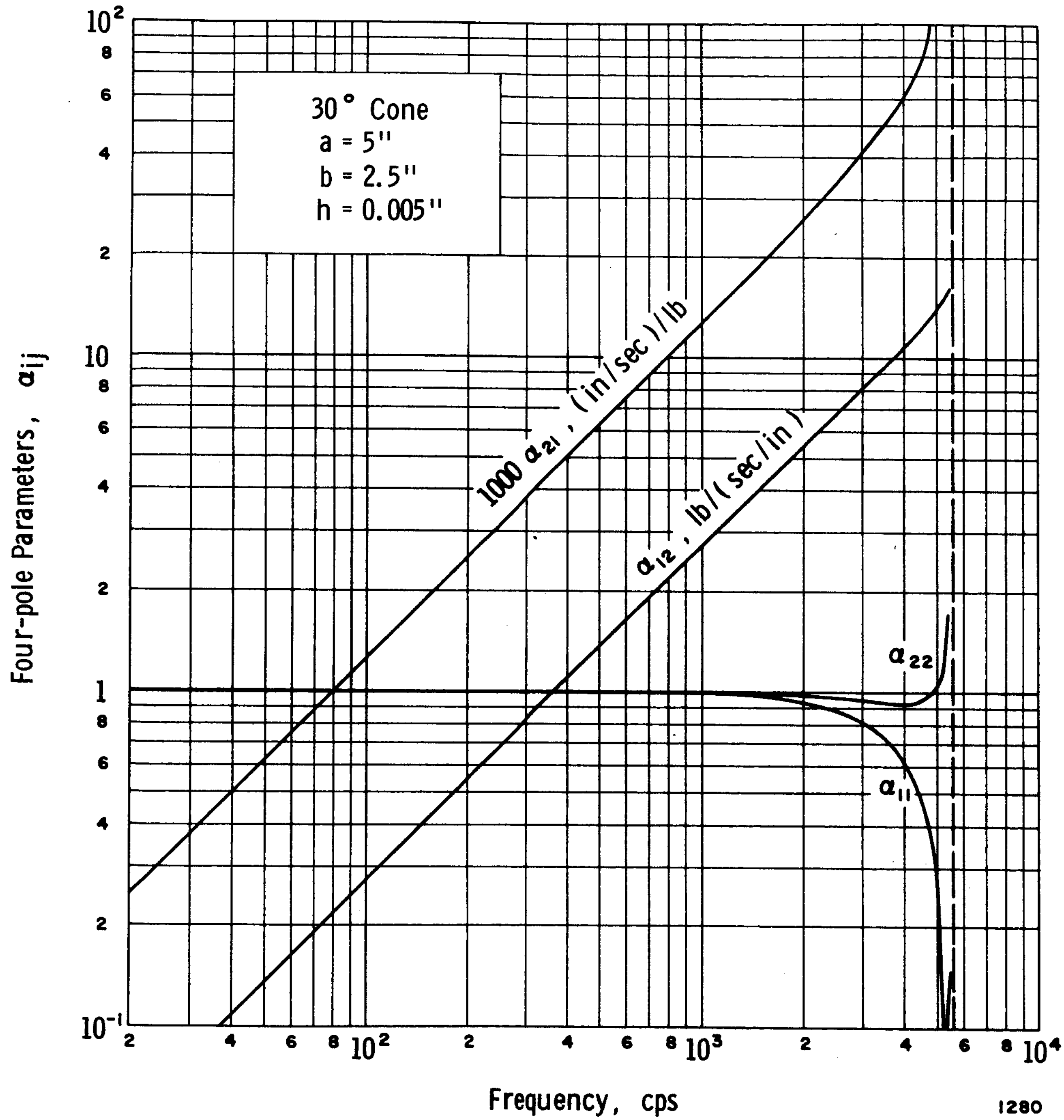

FIGURE 4. FOUR-POLE PARAMETERS FOR THE 30° CONICAL SHELL MODEL

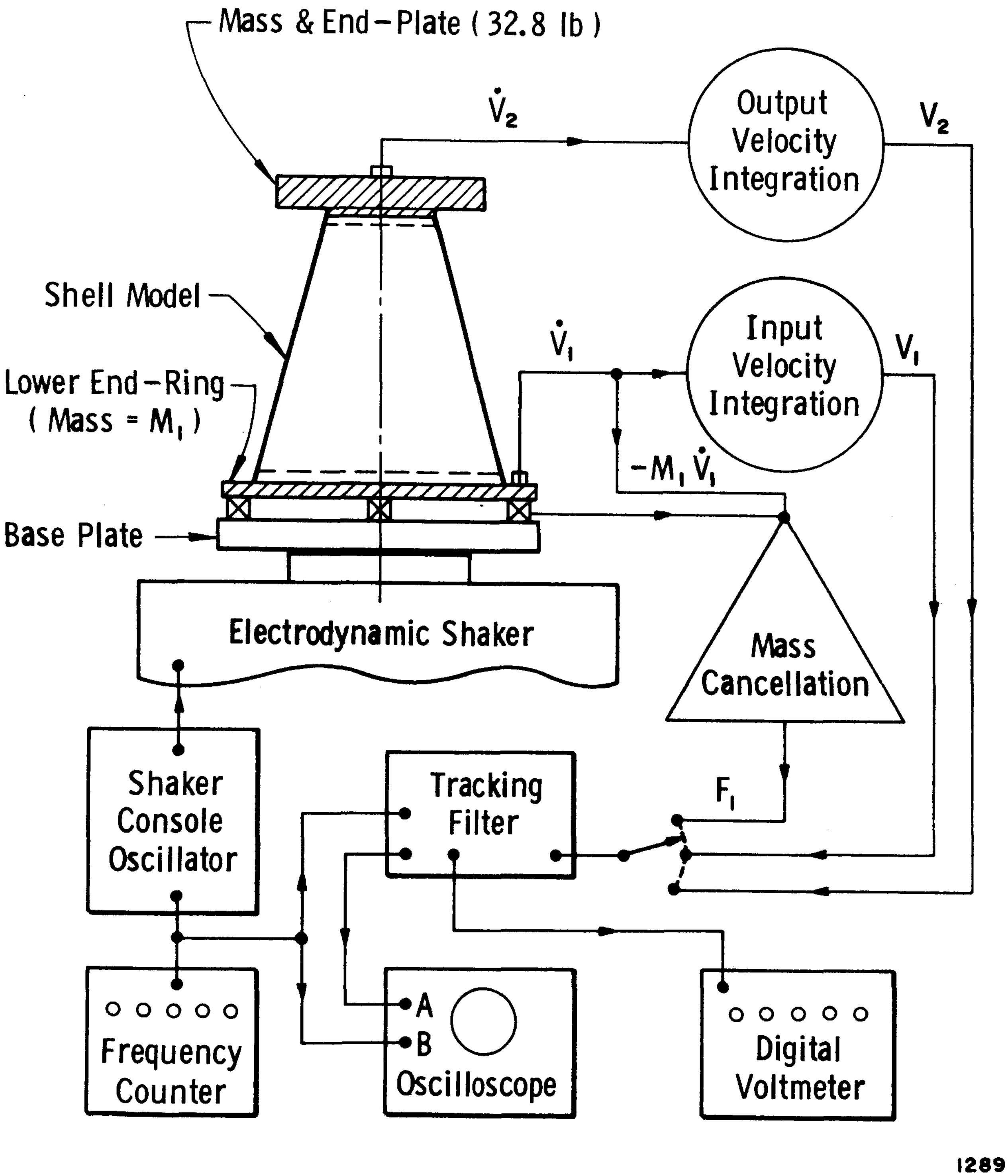

FIGURE 5. SCHEMATIC DIAGRAM OF EXPERIMENTAL APPARATUS

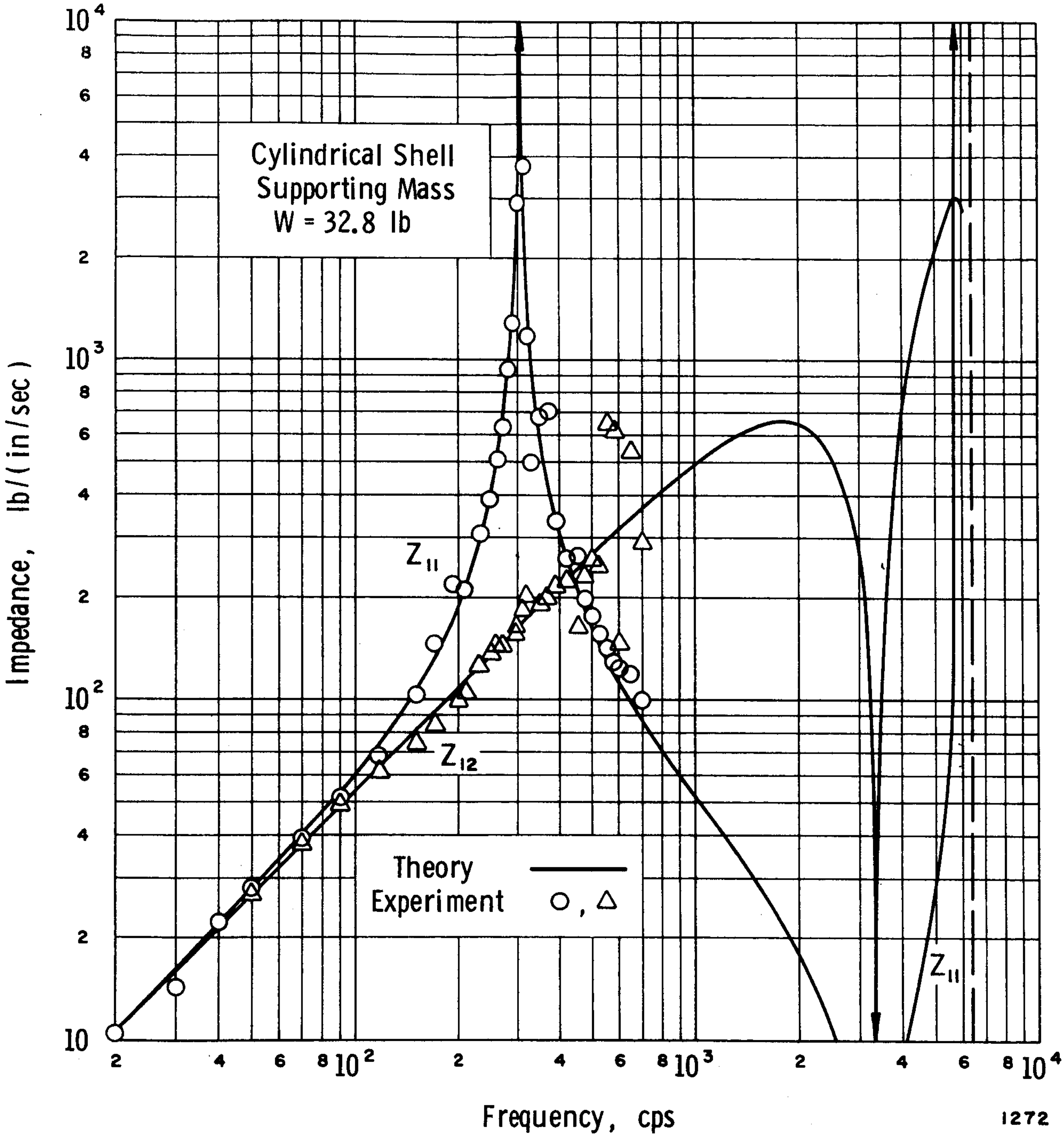

FIGURE 6. INPUT AND TRANSFER IMPEDANCE FOR THE CYLINDRICAL SHELL SUPPORTING MASS

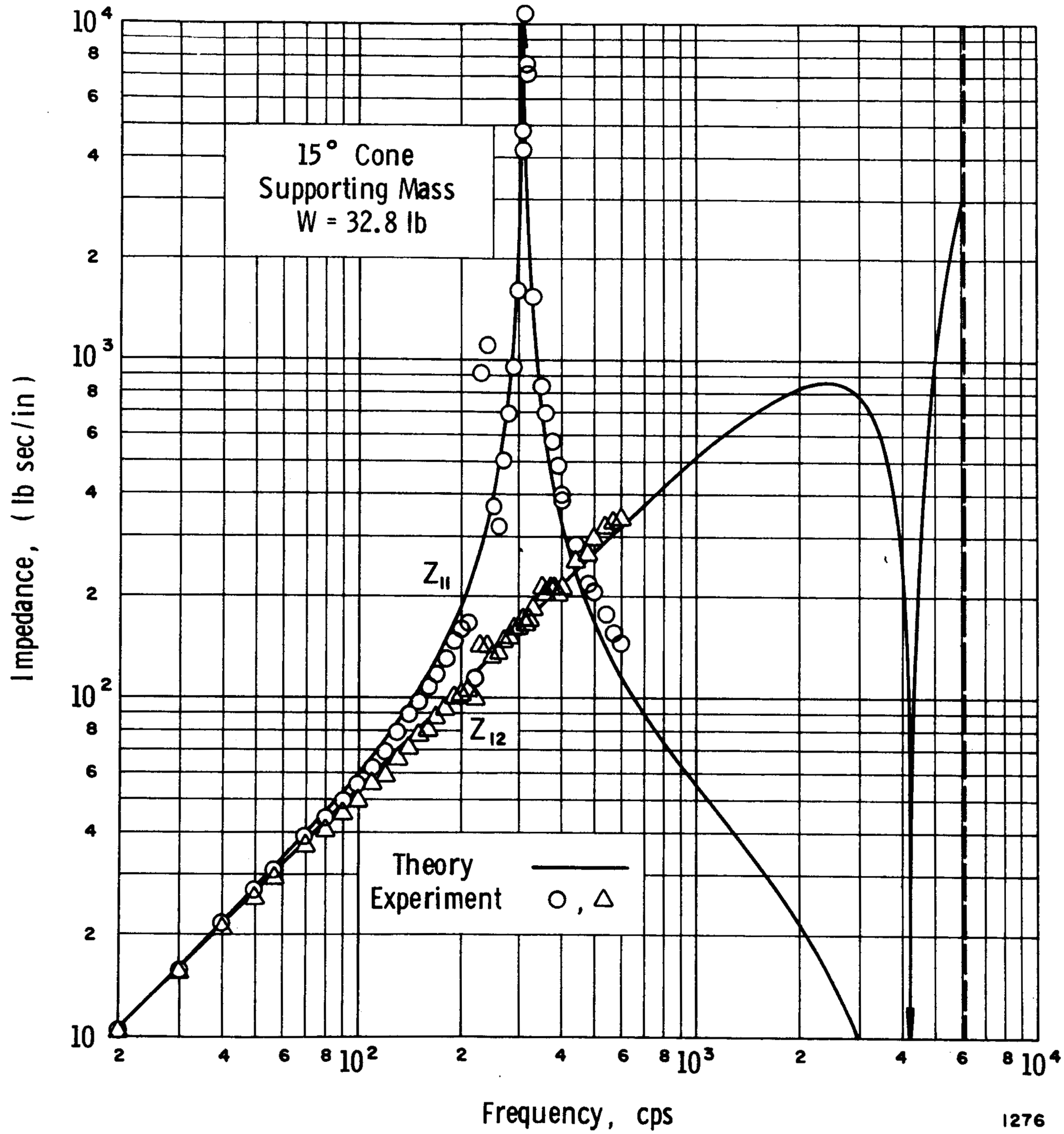

FIGURE 7. INPUT AND TRANSFER IMPEDANCE FOR THE 15° CONICAL SHELL SUPPORTING MASS

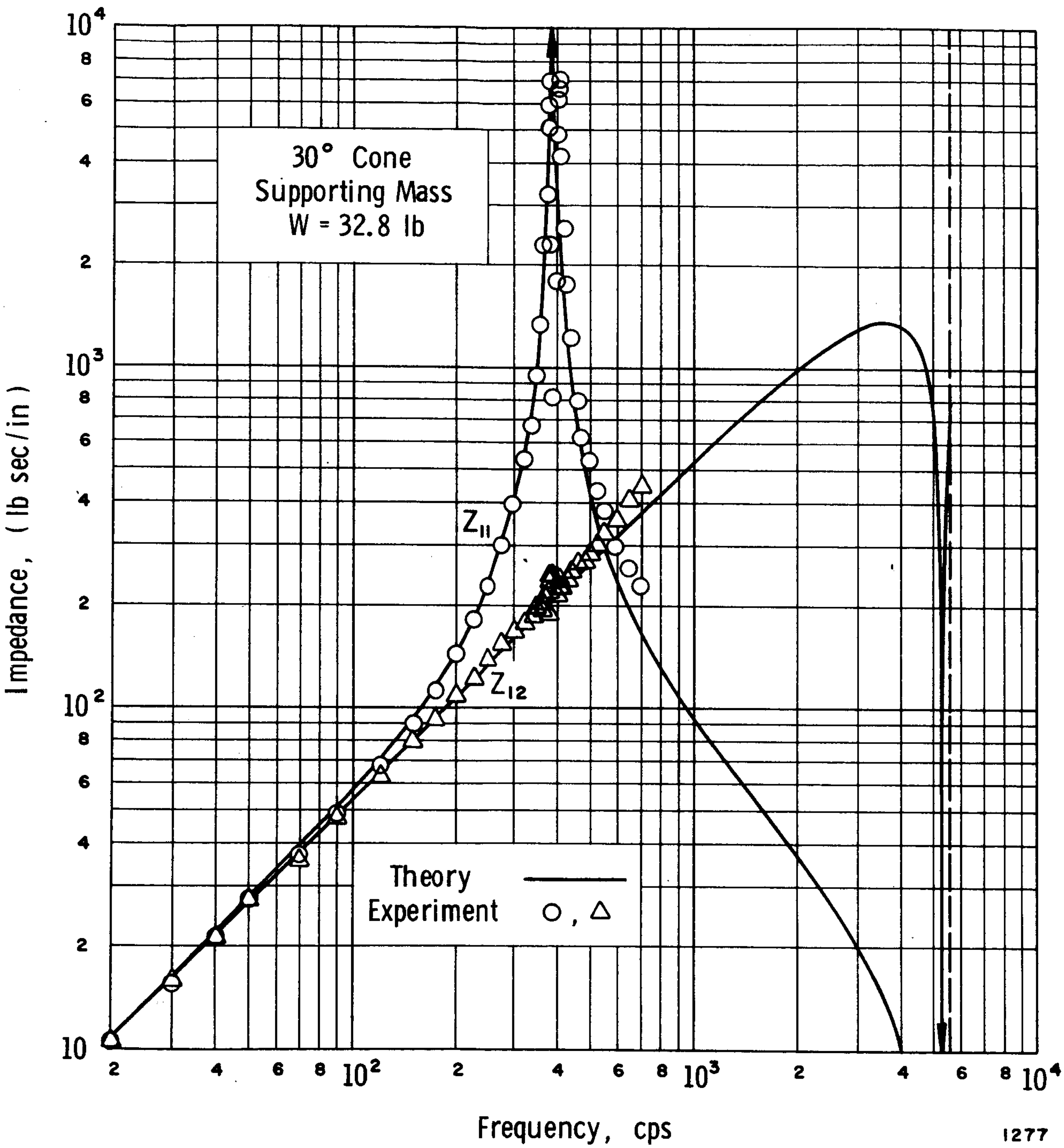

FIGURE 8. INPUT AND TRANSFER IMPEDANCE FOR THE 30° CONICAL SHELL SUPPORTING MASS

APPENDIX

LISTING OF COMPUTER PROGRAM AND FORMAT OF INPUT DATA CARDS

```
C      PROGRAM CONEIMP
C      PROJECT  0 2 - 2 0 3 4                                            SWR00010
C      CDC 3600 FORTRAN                                                  SWR00020
       DIMENSION  Y(2),F(2),B(20),TL(2)                                  SWR00030
       DIMENSION  FRQ(20),FRQX(20),FRQN(20),FI(2),UI(2)                  SWR00040
       DATA (PI=3.14159265),(C1=386.0),(C2=1.74532925E-02),(ERR=1.E-5)   SWR00050
 2000 READ 200, N,N1OPT                                                  SWR00060
  200 FORMAT ( 2I5 )                                                     SWR00070
      IF (EOF,60)80,85                                                   SWR00080
   80 STOP                                                               SWR00090
C *** GEOMETRIC PARAMETERS                                               SWR00100
   85 READ 205, A,SB,ALPHA,H                                             SWR00110
  205 FORMAT ( 4F10.0 )                                                  SWR00120
C *** MATERIAL PARAMETERS                                                SWR00130
      READ 210,ENU,E,RHO                                                 SWR00140
  210 FORMAT ( F10.0,2E10.2)                                             SWR00150
C *** RIGID MASS                                                         SWR00160
      READ 215, WT                                                       SWR00170
  215 FORMAT ( F10.0 )                                                   SWR00180
      BM = WT/C1                                                         SWR00190
      ALFR = C2*ALPHA                                                    SWR00191
      DCN = COSF(ALFR)                                                   SWR00192
      DSN = SINF(ALFR)                                                   SWR00193
      W0SQ = E/(RHO*A*A)                                                 SWR00194
      W0 = SQRTF(W0SQ)                                                   SWR00195
      WS = W0*DCN                                                        SWR00196
      FS = WS/(2.0*PI)                                                   SWR00197
C *** FREQUENCY RANGE                                                    SWR00200
      READ 220, (FRQ(I),FRQX(I),FRQN(I),I=1,N)                           SWR00210
  220 FORMAT ( 3F10.0)                                                   SWR00220
      PRINT 300                                                          SWR00230
  300 FORMAT(1H1,30X,60H    CONICAL SHELL LONGITUDINAL IMPEDANCE PROGRAMSWR00240
     1 - W.C.L. HU//68H COMMENT - THIS PROGRAM CALCULATES TRANSFER MATRISWR00250
     2X BETAIJ(OMEGA) AND/10X,58HFOUR-POLE PARAMETERS ALPHAIJ(OMEGA) FORSWR00260
     3     CONICAL SHELLS/10X,61HUNDER LONGITUDINAL EXCITATION,ALSO CALCSWR00270
     4ULATES INPUT IMPEDANCE/10X,62HZ11(OMEGA) AND TRANSFER IMPEDANCE Z1SWR00280
     52(OMEGA) WHEN AN ARBITRARY/10X,44HMASS M IS ATTACHED TO THE OUTPUTSWR00290
     6 TERMINAL 2.)                                                      SWR00300
      PRINT 305, A,SB,ALPHA,H                                            SWR00310
  305 FORMAT (1H0,28HINPUT - GEOMETRIC PARAMETERS/8X,22HMAJOR BASE RADIUSWR00320
     1S A = ,F4.1, 8H INCHES,,4X,22HMINOR BASE RADIUS B = ,F4.1, 8H INCHSWR00330
     2ES,,3X,25HSEMIVERTEX ANGLE ALPHA = ,F5.1, 9H DEGREES,,/8X,14HTHICKSWR00340
     3NESS H = ,F6.3, 8H INCHES,)                                        SWR00350
      PRINT 310, E,ENU,RHO                                               SWR00360
  310 FORMAT (1H0,27HINPUT - MATERIAL PARAMETERS/8X,19HYOUNGS MODULUS E SWR00370
     1= ,E8.1, 5H PSI,,4X,20HPOISSONS RATIO NU = ,F4.1,1H,,4X,19HMASS DESWR00380
     2NSITY RHO = ,E10.3,19H LB(SEC)**2/(IN)**4)                         SWR00390
      PRINT 315, WT,BM                                                   SWR00400
  315 FORMAT (1H0,61HCALCULATE IMPEDANCE Z11(OMEGA) AND Z12(OMEGA) FOR WSWR00410
     1EIGHT W = ,F5.1, 3H LB,4X, 11H( MASS M = ,F6.3,21H LB(SEC)**2/(IN)SWR00420
     2     )/////)                                                       SWR00430
      PRINT 320                                                          SWR00440
  320 FORMAT (1H0,32HCALCULATED FOR FREQUENCY ( CPS ))                   SWR00450
      PRINT 325, (FRQ(I),FRQX(I),FRQN(I),I=1,N)                          SWR00460
  325 FORMAT (8X,3(F8.1,2H (,F7.1,2H ),F8.1,4X))                         SWR00470
      PRINT 340, FS,WS                                                   SWR00471
  340 FORMAT (1H0,29HFREQUENCY SINGULARITY - FS = ,F10.1, 5H CPS,,4X,    SWR00472
     1  9HOMEGAS = ,F10.1, 8H RAD/SEC)                                   SWR00473
      PRINT 330                                                          SWR00480
  330 FORMAT (1H0,4X,4HFREQ,6X, 5HOMEGA,5X, 6HBETA11,6X, 6HBETA22,6X,    SWR00490
     1 6HBETA12,6X, 6HBETA21,6X, 7HALPHA12,5X, 7HALPHA21,5X,3HZ11,       SWR00500
     2 9X,3HZ12/24X, 8H=ALPHA11,4X, 8H=ALPHA22)                          SWR00510


      IF (N1OPT)95,90,95                                                 SWR00511
   95 PRINT 355                                                          SWR00512
  355 FORMAT (1H0,20HINTERMEDIATE RESULTS,10X, 1HX,13X, 4HCAPU,12X,      SWR00513
     1    6HNTHETA,15X, 1HF,14X, 1HU)                                    SWR00514
   90 DO 40 I=1,N                                                        SWR00520
      FREQ = FRQ(I)                                                      SWR00530
```

```
1000 W = 2.0*PI*FREQ                                                         SWR00540
     WSQ = W*W                                                               SWR00550
     TM2 = WSQ/W0SQ                                                          SWR00560
     TM3 = WSQ/(WS*WS)                                                       SWR00570
     DX = (1.0-SB/A)/64.0                                                    SWR00620
     FI(1) = 1.0                                                             SWR00630
     UI(1) = 0.                                                              SWR00640
     FI(2) = 0.                                                              SWR00650
     UI(2) = 1.0                                                             SWR00660
     IF (TM3.GT.0.80)15,10                                                   SWR00670
  15 PRINT 350, FREQ,W                                                       SWR00680
 350 FORMAT (1H ,2F10.1,4X,25HNEAR OR ABOVE SINGULARITY)                     SWR00690
     GO TO 55                                                                SWR00700
  10 CONTINUE                                                                SWR00710
     DO 45 J=1,2                                                             SWR00720
     X = SB/A                                                                SWR00730
     Y(1) = FI(J)                                                            SWR00740
     UC = UI(J)                                                              SWR00750
     Y(2) = (1.0/(1.0-TM2*X*X))*((ENU*DSN*Y(1))/(2.0*PI*E*H*DCN)-            SWR00760
    1  (1.0-TM3*X*X)*UC*DCN)                                                 SWR00770
     BNT = ((E*H*TM2*X)/(A*DSN*DCN))*(Y(2)*DCN+UC)                           SWR00780
     TL(1) = X                                                               SWR00800
     TL(2) = 1.0                                                             SWR00810
     CALL NORDSET (K,X,DX,2  ,Y,F,ERR,B,2,TL,0,0)                            SWR00820
     K = 0                                                                   SWR00830
   1 CALL NORDINT                                                            SWR00840
     GO TO (20,1,25,30)K                                                     SWR00850
  20 DFDX = 2.0*PI*E*H*(TM2*X*Y(2)*(DCN/DSN)-(BNT*A*DCN)/(E*H))              SWR00860
     DUDX = -(1.0/(E*H*DSN))*(Y(1)/(2.0*PI*X*DCN)+ENU*BNT*A)                 SWR00870
     F(1) = DFDX                                                             SWR00930
     F(2) = DUDX                                                             SWR00940
     GO TO 1                                                                 SWR00950
  25 TL = TL+DX                                                              SWR00960
     UC = (1.0/(DCN-(TM2*X*X)/DCN))*((ENU*DSN*Y(1))/(2.0*PI*E*H*DCN)-        SWR00961
    1  (1.0-TM2*X*X)*Y(2))                                                   SWR00962
     BNT = ((E*H*TM2*X)/(A*DSN*DCN))*(Y(2)*DCN+UC)                           SWR00963
     IF (N1OPT)60, 1,60                                                      SWR00970
  60 PRINT 345, X,UC,BNT,Y(1),Y(2)                                           SWR00980
 345 FORMAT (1H0,20X,5E17.8)                                                 SWR00990
     GO TO 1                                                                 SWR01000
  30 UC = (1.0/(DCN-(TM2*X*X)/DCN))*((ENU*DSN*Y(1))/(2.0*PI*E*H*DCN)-        SWR01001
    1  (1.0-TM2*X*X)*Y(2))                                                   SWR01002
     BNT = ((E*H*TM2*X)/(A*DSN*DCN))*(Y(2)*DCN+UC)                           SWR01003
     IF (N1OPT)50,65,50                                                      SWR01004
  50 PRINT 345, X,UC,BNT,Y(1),Y(2)                                           SWR01005
  65 CONTINUE                                                                SWR01006
     IF (J-1)75,70,75                                                        SWR01010
  70 B11 = Y(1)                                                              SWR01020
     B21 = UC                                                                SWR01030
     GO TO 45                                                                SWR01040
  75 B12 = Y(1)                                                              SWR01050
     B22 = UC                                                                SWR01060
  45 CONTINUE                                                                SWR01070
     A12 = -B12/W                                                            SWR01080
     A21 = B21*W                                                             SWR01090
     Z12 = B11*BM*W-B12/W                                                    SWR01100


     Z11 = Z12/(-B21*BM*WSQ+B22)                                             SWR01110
     PRINT 335, FREQ,W,B11,B22,B12,B21,A12,A21,Z11,Z12                       SWR01120
 335 FORMAT (1H ,2F10.1,8(2X,E10.3))                                         SWR01130
  55 IF (FREQ-FRQN(I))35,40,40                                               SWR01140
  35 FREQ = FREQ+FRQX(I)                                                     SWR01150
     GO TO 1000                                                              SWR01160
  40 CONTINUE                                                                SWR01170
     GO TO 2000                                                              SWR01180
     END                                                                     SWR01190
```

```
      SUBROUTINE NORDSET (K,T,H,N,Y,F,DELTAY,B,NTL,TL,NPL,PL)
C
C     K           CONTROL INTEGER FOR USER  STATEMENTS          INTEGER
C     T           INDEPENDENT VARIABLE                          REAL
C     H           INTEGRATION STEP SIZE                         REAL
C     N           NUMBER OF FIRST ORDER EQUATIONS               INTEGER
C     Y           DEPENDENT VARIABLES                           REAL
C     F           DERIVATIVES                                   REAL
C     DELTAY      ERROR CONTROL VECTOR                          REAL
C     B           TEMPORARY STORAGE, DIMENSION 10*N             REAL
C     NTL         NUMBER OF ENTRIES IN TL                       INTEGER
C     TL          LIST OF INTERUPT TIMES                        REAL
C     NPL         NUMBER OF ENTRIES IN PL                       INTEGER
C     PL          LIST OF INTERUPT FUNCTIONS                    REAL
C
C     B(1,I)      EQUIVALENT
C     B(2,I)          OF
C     B(3,I)              ADAMS
C     B(4,I)                  DIFFERENCES
C     B(5,I)      PREDICTED DERIVATIVES
C     B(6,I)      Y AT START OF INTEGRATION STEP
C     B(7,I)      SECOND PRECISION PART OF Y ABOVE
C     B(8,I)      F AT START OF INTEGRATION STEP
C     B(9,I)      HOLE FOR INITIAL Y  WHILE STARTING
C     B(10,I)     SECOND PRECISION PART OF Y
C
      DIMENSION Y(1),F(1),B(10,N),TL(1),PL(1),DPTA(2),TEST(2),
     1FIND(10),PLEFT(10),PRITE(10)
      EQUIVALENCE (DPTA,DPTEMA)
      TYPE INTEGER STEP
      TYPE DOUBLE DPTEMA
      TYPE LOGICAL FIND,HALVE,DOUBLE
      COMMON/NORDCOM/IDER,IEOS,ITL,IPL,STEP,HMAX,HMIN,HBIG,HL
      DATA (HBIG=0),(HL=0)
C
      DELY(I)=H*(B(8,I)+(B(1,I)+(B(2,I)+(B(3,I)+(B(4,I)+Z)))))
      RETURN
      ENTRY NORDINT
      IF (K) GO TO KFLIP
C
C     TEST FOR CALLING SEQUENCE ERROR
C
      IF (H.LE.0.OR.N.LE.0.OR.N.GT.2000.OR.NTL.LT.0.OR.NTL.GT.500.OR.NPL
     X.LT.0.OR.NPL.GT.500.OR.DELTAY.LE.0.OR.T.LT.0)
     XCALL Q8QERROR (0,34HERROR IN NORDSET CALLING SEQUENCE.)
C
C     SET SUBROUTINE COUNTERS AND STEP SIZE DATA
C
      HMAX=HMIN=IDER=IEOS=ITL=IPL=STEP=0
C
C     CONTROL SECTION FOR STARTING INTEGRATION
C
      ASSIGN 3000 TO KFLIP
      GO TO 1001
 3000 H=H.AND.3777400000000000B
      DO 3002 J=1,NTL
      IF (T.EQ.TL(J)) 3001,3002
 3001 ASSIGN 3002 TO KFLIP
      GO TO 1002
 3002 CONTINUE
      T LEFT=T
      DO 3004 J=1,NPL
```

```
      PLEFT(J)=PL(J)
      IF(PL(J).EQ.0.)3003,3004
 3003 ASSIGN 3004 TO KFLIP
      GO TO 1003
 3004 CONTINUE
      DO 3010 I=1,N
 3010 B(9,I)=Y(I)
      D1=-1.
      ASSIGN 3100 TO ISFOUR
      GO TO 1400
 3020 I=STEP.AND.3
      IF (I) GO TO 2000
      I=STEP/4
      GO TO (3030,3050,3030,3080,3030,3040) I
 3030 D1=-1.
      ASSIGN 2000 TO ISFOUR
      GO TO 1400
 3040 D1=2.
      HMAX=HMIN=-H
      ASSIGN 3050 TO ISFOUR
      GO TO 1400
 3050 DO 3060 I=1,N
      Y(I)=B(9,I)
 3060 B(10,I)=0.0
 3070 ASSIGN 3030 TO KFLIP
      GO TO 1000
 3080 D1=.5
      ASSIGN 3090 TO ISFOUR
      GO TO 1400
 3090 IF(HALVE)3100,3050
 3100 STEP=0
      DO 3110 I=1,N
 3110 B(1,I)=B(2,I)=B(3,I)=B(4,I)=0.0
      GO TO 3050
C
C     CONTROL SECTION FOR TIME INTERUPTS DURING NORMAL INTEGRATION
C                   STATEMENT 1700 INTEGRATES FORWARD,RETURNING TO 1701
C
 1700 GO TO 1600
 1701 DO 1702 I=1,N
      B(6,I)=Y(I)
 1702 B(8,I)=F(I)
      TSAVE=T
 1703 Z=2.*TSAVE
      DO 1705 I=1,NTL
      IF (TL(I).LT.Z) 1704,1705
 1704 Z=TL(I)
      J=I
 1705 CONTINUE
      IF (Z.GE.TSAVE) GO TO 1707
      ASSIGN 1706 TOKFLIP
      RTEST=TSAVE/Z
      RTEST=RTEST.AND..NOT.3
      IF (RTEST.EQ.1.0) 17051,17053
17051 DO 17052 I=1,N
17052 Y(I)=B(6,I)
      T=TSAVE
      GO TO 1001
17053 HP=Z-TSAVE
      ASSIGN 1001 TO ISTWO
      GO TO 1200
 1706 ASSIGN 1703 TO KFLIP
```

```
      ASSIGN 1002 TO ISTHREE
      GO TO 1300
 1707 DO 1708 I=1,N
      F(I)=B(8,I)
 1708 Y(I)=B(6,I)
      T=TSAVE
      ASSIGN 1300 TO KFLIP
      ASSIGN 1709 TO ISTHREE
      GO TO 1001
 1709 RTEST=Z/T
      RTEST=RTEST.AND..NOT.3
      IF (RTEST.EQ.1.0) 1710,1711
 1710 ASSIGN 1711 TO KFLIP
      GO TO 1002
 1711 DO 1712 I=1,NPL
 1712 FIND(I)=.FALSE.
      GO TO 1700
C
C     INTEGRATE ONE STEP
C
C                     SAVE CONDITIONS AT START OF STEP
C
 2000 DO 2010 I=1,N
      B(6,I)=Y(I)
      B(7,I)=B(10,I)
 2010 B(8,I)=F(I)
      TSTART=T
C
C                     ENTRY FOR HALVED STEP
C
 2020 T=T+H
      DO 2030 I=1,N
      Z=0
      Y(I)=B(6,I)+DELY(I)
 2030 B(5,I)=F(I)+(2.*B(1,I)+(3.*B(2,I)+(4.*B(3,I)+5.*B(4,I))))
C
C                     ITERATE TWICE,DEVELOP TEST PARAMETERS
C
      HALVE=.FALSE.
      DOUBLE=.TRUE.
      TEST(1)=TEST(2)=0.
      DO 2070 J=1,2
      ASSIGN 2040 TO KFLIP
      GO TO 1000
 2040 DO 2070 I=1,N
      Z=F(I)-B(5,I)
      IF(J.EQ.2) 2050,2060
 2050 ZZ=ABSF(Z*H)
      RTEST=DELTAY*ABSF(Y(I))
      IF (ZZ.GT.RTEST) HALVE=.TRUE.
      IF (ZZ.GT.RTEST*.015625) DOUBLE=.FALSE.
2060  DPTA(1)=B(6,I)
      DPTA(2)=B( 7,I)
      Z=Z*.329861111111
      DPTEMA=DPTEMA+DELY(I)
      ZZ=ABSF(DPTA(1)-Y(I))
      IF (ZZ.GT.TEST(J))  TEST(J)=ZZ
      Y(I)=DPTA(1)
      B(10,I)=DPTA(2)
2070  CONTINUE
C
C     CHECK TEST PARAMETERS,BUMP COUNT OF INTEGRATION STEPS
```

```
C
      STEP=STEP+1
      IF (STEP.GT.1.AND.STEP.LT.25) GO TO 1100
      IF (8.*TEST(2).GT.TEST(1).AND..NOT.DOUBLE) GO TO 1500
      IF (8.*TEST(2).GT.TEST(1)) DOUBLE=.FALSE.
      IF (STEP.EQ.1) GO TO 1100
      IF (HALVE) GO TO 1500,1100
C
C     UPDATE ROUTINE,RETURNS TO 3020 IF STARTING - 1701 OTHERWISE
C
 1100 DO1101 I=1,N
      Z=F(I)-B(5,I)
      B(1,I)=B(1,I)+(3.*B(2,I)+(6.*B(3,I)+(10.*B(4,I)+Z/.96)))
      B(2,I)=B(2,I)+(4.*B(3,I)+(10.*B(4,I)+Z*0.4861111111))
      B(3,I)=B(3,I)+(5.*B(4,I)+Z/9.6)
 1101 B(4,I)=B(4,I)+Z/120.
      IF (STEP.LE.24) GO TO 3020
      IF (H.GT.HMAX) HMAX=H
      IF (H.LT.HMIN) HMIN=H
      GO TO 1701
C
C     ROUTINE TESTPHI,FALSE EXIT IS S1300,TRUE EXIT IS 1800
C
 1300 DO 1301 I=1,NPL
      IF (FIND(I)) GO TO 1301
      IF (PL(I)*PLEFT(I).LT.0) GO TO 1303
 1301 CONTINUE
      DO 1302 I=1,NPL
 1302 PLEFT(I)=PL(I)
      TLEFT=T
      GO TO ISTHREE
 1303 DO 1304 I=1,NPL
 1304 PRITE(I)=PL(I)
      TRITE=T
      GO TO 1800
C
C     DEPENDENT VARIABLE SEARCH PROCEDURE,ENTERED IF PL(I) CHANGES SIGN
C
 1800 Z=0.0
      DO 1802 I=1,NPL
      IF (FIND(I)) GO TO 1802
      IF (PRITE(I).EQ.0) PLEFT(I)=0
      ZZ=PLEFT(I)/PRITE(I)
      IF (ZZ.LE.Z) 1801,1802
 1801 Z=ZZ
      J=I
 1802 CONTINUE
      HP=(TRITE-TSAVE)-(TRITE-TLEFT)/(1.-Z)
      IF ((TSAVE+HP).EQ.T.OR.Z.EQ.0) 1803,1804
 1803 ASSIGN 1703 TO KFLIP
      FIND(J)=.TRUE.
      GO TO 1003
 1804 ASSIGN 1001 TO ISTWO
      ASSIGN 1300 TO KFLIP
      ASSIGN 1800 TO IS THREE
      GO TO 1200
C
C     CHECK FOR DOUBLE OF STEP SIZE
C
 1600 IF (DOUBLE.AND.(.NOT.HBIG.OR.(H+H).LE.HBIG)) 1601,2000
 1601 D1=2.
      ASSIGN 2000 TO ISFOUR
```

```
      GO TO 1400
C
C     SUBROUTINE CALLS, ASSUMES KFLIP SET PRIOR TO ENTRY
C
 1000 K=1
      IDER=IDER+1
      RETURN
 1001 K=2
      IEOS=IEOS+1
      RETURN
 1002 K=J+2
      ITL=ITL+1
      RETURN
 1003 K=J+NTL+2
      IPL=IPL+1
      RETURN
C
C     SUBROUTINE TO CHANGE STEP SIZE
C
 1400 H=H*D1
      D2=D1*D1
      D3=D2*D1
      D4=D3*D1
      DO 1401 I=1,N
      B(1,I)=B(1,I)*D1
      B(2,I)=B(2,I)*D2
      B(3,I)=B(3,I)*D3
 1401 B(4,I)=B(4,I)*D4
      GO TO ISFOUR
C
C     ROUTINE TO PREDICT INTERMEDIATE VALUES OF Y(I)
C
 1200 T=TSAVE+HP
      D1=HP/H
      D2=D1*D1
      D3=D2*D1
      D4=D3*D1
      DO 1201 I=1,N
 1201 Y(I)=B(6,I)+HP*(B(8,I)+(D1*B(1,I)+(D2*B(2,I)+(D3*B(3,I)+D4*B(4,I
     1)))))
      GO TO ISTWO
C
C     RESTORE T,Y,F.  HALVE STEP SIZE, TRY STEP AGAIN
C
 1500 RTEST=H/2
      IF (HL.AND.RTEST.LT.HL) GO TO 2020
      IF (T.EQ.(T+RTEST)) CALL Q8QERROR (0,23HH LESS THAN 2**(-36)*T.)
      STEP=STEP-1
      T=TSTART
      DO 1501 I=1,N
      Y(I)=B(6,I)
      B(10,I)=B(7,I)
 1501 F(I)=B(8,I)
      D1=.5
      ASSIGN 2020 TO ISFOUR
      GO TO 1400
      END
```

Format of Input Data for Conical Shell

The user provides the following data cards in the order as listed:

(1) The first card gives two integers each in a field of five columns. The first integer is the number of frequency sets to be calculated (see below). The second is a boolean integer indicating whether the intermediate results of numerical integrations are needed or not, (1 for printout and 0 for not printout).

(2) The second card provides the geometric parameters of the cone: four floating-point numbers each in a field of ten columns, giving the major base radius a, minor base radius b, semivertex angle α (in degrees) and shell thickness h (in inches), respectively.

(3) The third card provides the material properties of the cone: one floating-point number in a field of ten columns, giving the Poisson's ratio, and two real numbers in exponential form, each in a field of ten columns (2E10.2), giving Young's modulus E (in psi) and mass density ρ (in lb-sec^2/in^4), respectively.

(4) The fourth card gives the weight of the attached mass (in lb), a floating-point number in a field of ten columns.

(5) Each of the remaining data cards gives a set of input frequencies in cps. For example, the fifth card should provide three floating-point numbers: the first frequency of the first set, the increment of the set, and the final frequency of the set, each in a field of ten columns. The

sixth card should provide similar numbers for the second set of frequencies, etc. The total number of sets should be the same as the first integer in the first data card.

```
C      PROGRAM CYLINDER
C      SWRI PROJECT 02-2034
C      CDC 3600 FORTRAN
       DIMENSION F(12),FDX(12),FN(12)
       DATA (PI=3.14159265),(C1=386.0)
 2000  READ 200, N
  200  FORMAT ( I5 )
       IF (EOF,60)20,25
   20  STOP
C ***  GEOMETRIC PARAMETERS
   25  READ 205, A,SL,H
  205  FORMAT ( 3F10.0 )
C ***  MATERIAL PARAMETERS
       READ 210, ENU,E,RHO
  210  FORMAT (F10.0,2E10.2)
       TM1 = 1.0-ENU*ENU
       W0SQ = E/(TM1*RHO*A*A)
C ***  RIGID MASS
       READ 215, WT
  215  FORMAT ( F10.0 )
       BM = WT/C1
C ***  FREQUENCY RANGE
       READ 205, (F(I),FDX(I),FN(I),I=1,N)
       PRINT 300
  300  FORMAT(1H1,30X,60HCYLINDRICAL SHELL LONGITUDINAL IMPEDANCE PROGRAM
      1 - W.C.L. HU//68H COMMENT - THIS PROGRAM CALCULATES TRANSFER MATRI
      2X BETAIJ(OMEGA) AND/10X,58HFOUR-POLE PARAMETERS ALPHAIJ(OMEGA) FOR
      3 CYLINDRICAL SHELLS/10X,61HUNDER LONGITUDINAL EXCITATION,ALSO CALC
      4ULATES INPUT IMPEDANCE/10X,62HZ11(OMEGA) AND TRANSFER IMPEDANCE Z1
      52(OMEGA) WHEN AN ARBITRARY/10X,44HMASS M IS ATTACHED TO THE OUTPUT
      6 TERMINAL 2.)
       PRINT 305, A,SL,H
  305  FORMAT (1H0,28HINPUT - GEOMETRIC PARAMETERS/8X,11HRADIUS A = ,
      1F4.1, 8H INCHES,,4X,11HLENGTH L = ,F5.1, 8H INCHES,,4X,14HTHICKNES
      2S H = ,F6.3, 8H INCHES.)
       PRINT 310, E,ENU,RHO
  310  FORMAT (1H0,27HINPUT - MATERIAL PARAMETERS/8X,19HYOUNGS MODULUS E
      1= ,E8.1, 5H PSI,,4X,20HPOISSONS RATIO NU = ,F4.1,1H,,4X,19HMASS DE
      2NSITY RHO = ,E10.3,19H LB(SEC)**2/(IN)**4)
       PRINT 315, WT,BM
  315  FORMAT (1H0,61HCALCULATE IMPEDANCE Z11(OMEGA) AND Z12(OMEGA) FOR W
      1EIGHT W = ,F5.1, 3H LB,4X, 11H( MASS M = ,F6.3,17H LB(SEC)**2/(IN)
      2))
       PRINT 320
  320  FORMAT (1H0,32HCALCULATED FOR FREQUENCY ( CPS ))
       PRINT 325, (F(I),FDX(I),FN(I),I=1,N)
  325  FORMAT (8X,3(F8.1,2H (,F7.1,2H ),F8.1,4X))
       PRINT 330
  330  FORMAT (////4X,4HFREQ,6X, 5HOMEGA,5X, 6HBETA11,6X, 6HBETA22,6X,
      1 6HBETA12,6X, 6HBETA21,6X, 7HALPHA12,5X, 7HALPHA21,5X,3HZ11,
      2 9X,3HZ12/24X, 8H=ALPHA11,4X, 8H=ALPHA22)
       DO 40 I=1,N
       FREQ = F(I)
 1000  W = 2.0*PI*FREQ
       WSQ = W*W
       TM2 = WSQ/W0SQ
       TM3 =  TM2*(1.0-TM2)*(1.0/(TM1-TM2))
       TM4 = 2.0*PI*RHO*H*A*A*WSQ
       IF (TM2.GT.(TM1*0.99))15,10
   10  ALDA = SQRTF(TM3)
       ARG = (ALDA*SL)/A
       B11 = COSF(ARG)


       SN = SINF(ARG)
       B12 = -(TM4/ALDA)*SN
       B21 = (ALDA/TM4)*SN
       B22 = B11
       A12 = -B12/W
       A21 = B21*W
       Z12 = B11*BM*W-B12/W
       Z11 = Z12/(-B21*BM*WSQ+B22)
       PRINT 335, FREQ,W,B11,B22,B12,B21,A12,A21,Z11,Z12
  335  FORMAT (1H ,2F10.1,8(2X,E10.3))
   45  IF (FREQ-FN(I))35,40,40
   35  FREQ = FREQ+FDX(I)
       GO TO 1000
   40  CONTINUE
       GO TO 2000
   15  PRINT 340, W
  340  FORMAT (1H ,3X,E10.3,4X,25HNEAR OR ABOVE SINGULARITY)
       GO TO 45
       END
```

<u>Format of Input Data for Cylindrical Shell</u>

The user provides the following data cards in the order as listed:

(1) The first card gives the number of frequency sets, an integer in a field of five columns.

(2) The second card provides the geometric parameters of the cylinder: three floating-point numbers each in a field of ten columns, giving the radius a, length ℓ, and wall thickness h of the cylindrical shell (all in inches).

(3) The third card provides the material properties of the cylindrical shell, same as the third data card for the previous case of conical shell. The fourth card provides the attached weight.

(4) Each of the remaining data cards gives a set of input frequencies in cps, same as the previous case of conical shell. The total number of sets should equal the integer in the first data card.