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Technical Memorandum 33-346

JPL Lunar Ephemeris Number 4

J. Derral Mulholland
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CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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Abstract

This memorandum provides a detailed discussion of the revised Lunar ephemeris contained in Development Ephemeris 19 (DE 19), the second version of the JPL Ephemeris System to be issued for general distribution. The revision is based on improvements in the Lunar Theory, improvements in computational processes, and improvements in the fundamental astronomical constants. This latter category represents the practical application of scientific data provided by the space research programs. The new version of the Lunar ephemeris is compatible with the system of astronomical constants currently adopted by the International Astronomical Union and consists of a corrected form of the Improved Brown Lunar Theory.

JPL Lunar Ephemeris Number 4

I. Introduction

Space vehicle operations in the vicinity of the Moon require highly accurate Lunar ephemerides. At the same time, such operations provide experimental information that can be used to improve the existing ephemerides. Any significant improvement in an ephemeris, whether from observational or analytical sources, causes a direct increase in the utility of tracking data for other purposes.

From 1964 to the present, the standard ephemerides for spacecraft trajectory work have been those distributed on the JPL Ephemeris Tapes (Refs. 1 and 2).[1] During this period, however, work has continued on improvement of the ephemerides. Specifically, the Lunar ephemeris has received several corrections that greatly extend its basic accuracy, and the revised ephemeris has been incorporated into a new set of ephemeris tapes which are intended to supplant those described in Ref. 1. This memorandum provides complete details of the corrections applied to the Lunar ephemeris, and the information necessary for proper evaluation and use of the new ephemeris.

[1]The tapes described in Ref. 1 contain the rectangular coordinates of Moon, Mercury, Venus, Earth-Moon barycenter, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto, as well as the nutations in longitude and obliquity. JPL tape identifications and associated time intervals follow:

E-9510 JPL Ephemeris, 1949 Dec. 30.0–1969 Dec. 29.0

E-9511 JPL Ephemeris, 1967 Jan. 10.0–1987 Jan. 1.0

E-9512 JPL Ephemeris, 1984 Jan. 9.0–2000 Jan. 5.0

II. The Basic Lunar Theory Program

For computational efficiency, a complete recomputation of the Lunar Theory was avoided, and modifications to the Lunar ephemeris were applied as additive corrections to the previous evaluation of the Lunar Theory. To provide background, the computation represented by Lunar Ephemeris No. 2 (LE 2), the Lunar data contained in the ephemeris (DE 3) of Ref. 1, will be described briefly.

These data were computed in 1963 by means of a double precision FORTRAN II program written to recompute the Improved Lunar Ephemeris (ILE) according to the precepts of Ref. 3, and using the algebraic expressions and numerical values given in Refs. 3 and 4.

The ILE is a modification of the Brown Lunar Theory (Refs. 5 and 6), and corrects deficiencies in the Lunar tables as recommended by the International Astronomical Union (IAU) at its 1952 meeting (Ref. 7). It has been the basis of the Lunar positions published in the various national ephemerides since 1960. Reference 3 provides explicit instructions for execution of the computation; these directions were followed as rigorously as possible in the JPL Brown Lunar Theory program. Concurrently, Woolard's expressions (Ref. 8) for the mean obliquity and the nutations in longitude and obliquity were evaluated. The principal result of this computation was a set of magnetic tapes giving the apparent geocentric Lunar coordinates, the mean obliquity of date, and the nutations in longitude and obliquity, as computed for each half-day for the years 1850 to 2000.[2] More specifically, each tape record contains the following quantities in double precision:

JED = Julian Ephemeris Date (0^h or 12^h)

λ = apparent geocentric longitude of the Moon, referred to the mean equinox and ecliptic of date, including a partial aberration correction

β = apparent geocentric latitude of the Moon, referred to the mean equinox and ecliptic of date

Π^* = a quantity functionally related to Π, the equatorial horizontal parallax of the Moon

S^* = a quantity functionally related to S, the semidiameter of the Moon

$\delta\theta$ = nutation in longitude

$\delta\psi$ = nutation in obliquity

$\bar{\epsilon}$ = mean obliquity of the ecliptic date

[2]The magnetic tapes, called the Lunar Source Ephemeris, recorded at 556 bpi. The following data are recorded at half-day intervals: Julian Ephemeris Date, apparent geocentric Lunar coordinates, mean obliquity of date, nutation in longitude and obliquity. JPL tape identifications and associated time intervals follow:

N-6070	Lunar Source Ephemeris, 1850–1875
N-6071	Lunar Source Ephemeris, 1875–1900
N-6072	Lunar Source Ephemeris, 1900–1925
N-6073	Lunar Source Ephemeris, 1925–1950
N-6074	Lunar Source Ephemeris, 1950–1975
N-6075	Lunar Source Ephemeris, 1975-2000

The data for the period 1875-1900 are not complete. A gap of 319.5 days exists, covering the interval from JD 241 1526.0 to ID 241 1845.5.

The quantity Π^* is a two-term approximation to arc sin σ, where $\sigma = \sin \Pi$; S^* is computed from the approximate relation (Ref. 9, p. 109)

$$S^* = \frac{0.0796}{1296000} + 0.272446\, \Pi^*$$

All of the angular data are expressed in revolutions.

Subsequently, these data were used to compute the apparent Cartesian rectangular coordinates referred to the mean equinox and ecliptic of date; the coordinates were then transformed into apparent rectangular coordinates by means of the standard formulae (Ref. 9, pp. 29–31). Finally, the aberration was removed by an iterative numerical procedure, so that the resulting ephemeris was geometric rather than apparent. These data were recorded on magnetic tape in binary mode.[3]

The three components of the Lunar position vector and the two nutations were then used to obtain velocity components by numerical differentiation, using second and fourth modified differences. The ten quantities, with their second and fourth modified differences, were then merged into the complete JPL Ephemeris Tapes (Ref. 1). These tapes were the distributed version of the ephemeris in use until the present.

III. Discussion of the Corrections

The modifications contained in the present revision of the Lunar ephemeris may be separated into four categories: (1) transformation corrections, (2) corrections based on a new solution of the problem, (3) an improved treatment of aberration, and (4) improved astronomical constants. The Appendix contains the step-by-step program specifications followed in the computation, and the data in Tables A-1 through A-14 are arranged in the sequence of their use. In the following paragraphs, the principles underlying each of these modifications are described.

The basic data required for evaluating either the Lunar Theory or corrections to it are the collection of

[3]Lunar Geometric Ephemeris, derived from Lunar Source Ephemeris tapes and recorded on magnetic tapes at 556 bpi; Julian Ephemeris Dates and geocentric rectangular Lunar coordinates, referred to the mean equinox and equator of 1950.0, are recorded at half-day intervals. JPL tape identifications and associated time intervals follow:

N-6619	Lunar Geometric Ephemeris, 1950–1975
N-6620	Lunar Geometric Ephemeris, 1975–2000

angular quantities (L, Ω, ℓ, ℓ', F, D, $\Delta\gamma_c$, ...) which describe the instantaneous configuration of the system, and which affect the geocentric ecliptic coordinates of the Moon. Each of these quantities consists of a "mean value," which is given as a power series in time (Table A-1), to which must be added, in some cases, a periodic part (Table A-2).[4] These periodic functions consist of Fourier series whose arguments are linear functions of the mean values of the fundamental arguments, and thus may be represented as power series in time. This representation was chosen as the most convenient for the present application.

A. Transformation Corrections

The most significant improvement in the ephemeris deals directly with E. W. Brown's (Ref. 5) solution of the "main problem" of the Lunar Theory in rectangular coordinates, and its reduction to the tables (Ref. 6) on which the ILE is founded. The solution was constructed in terms of rotating rectangular coordinates, and then transformed into the spherical coordinates commonly used in the Lunar problem: longitude, latitude and sine parallax. Because of the observational limitations of that time, inaccuracies were allowed in the coordinate transformations that were notably greater than those inherent to the theory. In particular, Brown accepted a greatly reduced precision in sine parallax (effectively the radial distance) because it lacked any practical use in observational astronomy. This justification disappeared, of course, with the rise of radar astronomy. Direct line-of-sight measurements of both distance and radial velocity are now possible with high precision, and an ephemeris in which the radial components of position and velocity are of a precision commensurate with the angular coordinates is necessary. Improvements in observational capabilities, and the execution of spacecraft missions in near-Lunar space also require improvement in the absolute precision of *all* coordinates.

Eckert, Walker, and Eckert (Ref. 10) recomputed the coordinate transformations with higher precision than did Brown. The corrections to the ILE coordinates extend the spherical coordinates to include terms as small as 0.″0001 in longitude and latitude, and 0.″000002 in sine parallax, each of which correspond to linear distances of about 20 cm. The largest terms in each of the three coordinate corrections are

longitude: $-0.''0090 \sin(\ell - \ell' + 2F + 2D)$

latitude: $+0.''0154 \sin(-\ell + 3F)$

sine parallax: $+0.''005507 \cos(\ell - 2F)$

The coefficients correspond to linear distances of about 18, 31, and 617 m, respectively.

The computations correspond to Steps 4 and 5 of the Appendix.

B. Correction from the New Solution

Eckert and Smith (Ref. 11) have constructed a new solution of the Lunar problem that is independent of the Brown Theory. It is a numerical general theory said to contain all terms of size 0.″00001 and larger, and many terms of even greater precision. On the basis of this work, Dr. Eckert recommends that the ILE be augmented by the longitude correction

$$+0.''072 \sin(2F - 2\ell)$$

which corresponds to a maximum value of about 144 m in-track change.

C. Aberration Correction

Clemence, Porter, and Sadler (Ref. 12) noted that Brown apparently made *no* allowance for aberration in correcting the elements of the Lunar orbit from Greenwich meridian observations. Thus the elements deduced by Brown are affected by aberration, although the coordinates themselves contain the major part of the individual quantities required for a strictly apparent theory, again because of Brown's procedure. The data of Ref. 12 are reproduced here as a part of Table A-14 (Appendix); a description of these data and their use follow:

- column (a): the terms which are required to be subtracted from the Lunar Theory to fully correct it from a strictly geometric to a strictly apparent theory
- column (b): the terms to be subtracted from the Brown Lunar Theory to remove the effects of aberration in the lunar elements
- column (c): the sum of the two preceding columns, which represent the total correction necessary to make the Brown Lunar Theory strictly apparent
- column (d): a two-term correction to the longitude, as recommended by Clemence, et al., for inclusion in the Lunar ephemeris (this was, in fact, included in the ILE and the JPL Ephemeris based on it)

[4]To avoid possible confusion with radians, the superscript c (circles) denotes units of rev.

Unlike most astronomical applications, the uses of the JPL Ephemeris require a geometric rather than an apparent ephemeris of the Moon. Thus, it seems desirable to reverse the process indicated in Ref. 12.

To obtain the utmost precision in computing a rigorously geometric ephemeris, the following steps must be performed:

(1) Subtract column (d) from the JPL Ephemeris; this removes the correction of Ref. 12, and essentially restores the situation created by Brown.

(2) Subtract column (c), thus rendering the ephemeris strictly apparent.

(3) Add column (a), which corrects the strictly apparent theory to a strictly geometric theory.

These corrections, which are, in effect, subtracting the sum of columns (b) and (d), are combined in column (e) of Table A-14.

Thus, it is possible to replace the iterative numerical procedure used previously to remove the aberration from the JPL Ephemeris by a rigorous set of mathematical expressions applied directly to the spherical coordinates, as represented by Step 8 in the Appendix. The actual quantitative effect that was expected on the ephemeris is quite small.

D. Corrections due to Improved Constants

The ILE and the modifications to it are based on a set of astronomical constants that are no longer adequate. Largely because of *Mariner II* flight data, satellite data, and radar-bounce data, new values for the Earth-Moon mass ratio, the astronomical unit in meters, the Earth's gravitational constant, and the equatorial Earth radius were adopted by the International Astronomical Union in 1964 (Ref. 13). These changes affect the Lunar Theory strongly, and enter into the numerical data in two ways.

1. The parameter α_1. The ILE is compatible with an Earth-Moon mass ratio

$$E/M = \mu^{-1} = 81.53$$

and a ratio of mean geocentric distances of Moon and Sun

$$\alpha = 0.0025\ 7279$$

The IAU 1964 value for μ^{-1} is 81.30, that for the astronomical unit (AU) is 1496×10^5 km, and that for the Earth's gravitational constant (GE) is 398603 km^3/sec^2. These figures, when used in conjunction with Kepler's third law, yield a perturbed mean Lunar distance of 384400 km, and

$$\alpha = 0.0025\ 69518$$

These data enter into the Lunar Theory through the parameter

$$\alpha_1 = [(E - M)/(E + M)]\ \alpha$$

$$= [(1 - \mu)/(1 + \mu)]\ \alpha$$

The ILE value of α_1 is 0.0025 12730, while the new value is 0.0025 09351. To understand the way in which the correction must be applied, reference must be made to the way in which the disturbing function is treated in the Lunar Theory. Because the Lunar Theory is geocentric rather than barycentric, the disturbing function, if expanded in Legendre polynomials, is of the form

$$R = \alpha^2\ [f_2(r,r')\ P_2(S) + \alpha_1 f_3(r,r') P_3(S)$$

$$+ \alpha_2 f_4(r,r') P_4(S)]$$

where α is the ratio of the mean geocentric distances of Moon and Sun, S is the angular separation of Moon and Sun, and the f_i are functions of the position vectors of Moon and Sun. Also,

$$\alpha_2 = [(1 - \mu + \mu^2)/(1 + \mu)^2]\ \alpha^2$$

Because M/E is sufficiently small in the derivations of the Lunar Theory, the quantity α was substituted for both α_1 and α_2, and corrections (or adjustments) were made to the affected terms at the end of the computation. If a subsequent change is made in the value of either E/M or AU, then the first term of this expression is unaffected, but the other two are affected. In particular, if the value of α_1 is changed, the terms that arise from the P_3 term of the disturbing function must be identified. This is possible because the expression for the angular separation S is linear in D. Thus, all terms in the Lunar Theory whose arguments have an odd factor for D have coefficients that are multiplied by α_1.

The appropriate corrections to longitude and sine parallax are simple and straightforward. The relative change of α_1 is $(2509351/2512730 - 1) = -0.001344752$;

it is necessary to apply this factor to only the terms of λ and sin Π that contain odd multiples of D. The Solar terms required for the correction of λ and sin Π are given in Tables A-6 and A-7. Because the planetary terms are quite small, they will be sufficiently accurate if the arguments are computed from expressions linear in time, as given in Table A-8; the corresponding terms from L have been included in Table A-8 as a higher order correction. Sine parallax contains no planetary terms that require correction. The correction terms themselves are given in Table A-11.

The expressions for the corrections to the latitude are more complicated. For this discussion, the following two conventions will be adopted: (1) the symbol δ denotes the variation produced by a change in the terms that contain odd multiples of D, and (2) the notation [code i] indicates the terms having that same designation in Ref. 3. Tables A-9 and A-10 contain those parts of [code 1] and [code 2] that contain odd multiples of D.

The latitude β is given in Ref. 3 by

$$\beta = A\,\Delta_3 \sin S + B\,\Delta_3^3 \sin 3S + C\,\Delta_3^5 \sin 5S + D\,[\text{code } 3] + [\text{code } 4]$$

where

$$A = 18519''.700 + [\text{code } 2]$$

$$B = -3.36992 \times 10^{-4} A$$

$$C = +2.16 \times 10^{-7} A$$

$$D = +5.3996 \times 10^{-5} A$$

$$\Delta_3 = +1.00000\ 2708 + 139.978\,\Delta\gamma_c$$

$$S = F + [(\Delta L - \Delta\Omega) \text{ from Table A-2}] + [\text{code } 1]$$

evidently,

$$\delta A = \delta\,[\text{code } 2],\ \delta B = (B/A)\delta A,\ \delta C = (C/A)\delta A,$$

$$\delta D = (D/A)\delta A$$

Also, one can verify from Ref. 3 that

$$\delta\,[\text{code } 3] = 0 \qquad \delta\,[\text{code } 4] = 0$$

$$\delta\,(\Delta\gamma_c) = 0 \qquad \delta\Delta_3 = 0$$

Using these relations, one may derive the expression

$$\delta\beta = \left(\Delta_3 \cos S + 3\frac{B}{A}\Delta_3^3 \cos 3S + 5\frac{C}{A}\Delta_3^5 \cos 5S\right) A\,\delta S + \left(\Delta_3 \sin S + \frac{B}{A}\Delta_3^3 \sin 3S + \frac{C}{A}\Delta_3^5 \sin 5S + \frac{D}{A}[\text{code } 3]\right)\delta A$$

This expression is rigorous, but its application would not be easy. It is desirable, at this point, to make whatever approximations are possible, consistent with the desired accuracy. If the present corrections are required to meet the accuracy of the transformation corrections (0''.0001), all terms that contain the factors B/A, C/A and D/A may be safely neglected, which reduces the above equation to

$$\delta\beta = \Delta_3\,(A\,\delta S \cos S + \delta A \sin S)$$

To the same order of approximation,

$$\Delta_3 = 1.0$$

$$A = 18519''.7 = 0.089786 \text{ rad}$$

$$\delta S = \delta\,[\text{code } 1]$$

However, to maintain the desired accuracy, it is necessary to retain all terms in S whose coefficients are greater than about 1400''.

Thus, the approximate value is

$$S = F + 2373''.36 \sin 2D + 22609''.07 \sin \ell - 4578''.13 \sin(\ell - 2D)$$

The correction must be computed from

$$\delta\beta = \eta_1 \cos S + \eta_2 \sin S$$

where

$$\eta_1 = -\,0.00134\,4752\,A \text{ (Table A-9)}$$

$$\eta_2 = -\,0.00134\,4752 \text{ (Table A-10)}$$

The two quantities η_1 and η_2 may be obtained directly from Table A-11, while the angle S must be evaluated separately from Table A-12, where it is given in rev.

So far, only the corrections arising from the P_3 term of the disturbing function have been treated. The P_4 term is also factored by a function of the mass ratio E/M

and the AU, and corresponding corrections must be applied to the appropriate terms. Eckert, Walker, and Eckert (Ref. 10) present two-term corrections, one to the longitude and one to the sine parallax. These, presumably, represent the only non-negligible terms arising from this cause. They are given in Table A-13.

The largest terms in each of the coordinate corrections were

longitude: $+ 0''.1683 \sin D$

latitude: $+ 0''.0136 \sin D \cos S$

sine parallax: $+ 0''.001315 \cos D$

The coefficients correspond to linear distances of about 340, 27, and 150 m, respectively.

2. ***Constant term in sine parallax.*** In earlier times, the constant in sine parallax of the Moon was measured directly or derived from direct application of observations, just as the Solar parallax has always been related to observations until recently. In fact, the value of a used by Brown was calculated from the relation

$$a = \frac{\sin \Pi_\odot}{\sin \Pi_{☽}}$$

In recent years, radar astronomy has provided a tool for relating the AU directly to observations of very high precision, reversing the functional relationship between the AU and the Solar parallax. Similarly, radar has been used to measure the Lunar mean distance, but this quantity is now most accurately provided by its dependence on GE and μ, the values of which have been determined from observations of unmanned space vehicles. Accordingly, we may rearrange the above equation to give

$$\sin \Pi_{☽} = \frac{\sin \Pi_\odot}{a} = \frac{R_e}{a_{☽}}$$

where $R_e = 6378.160$ km is the adopted equatorial Earth radius and $a_{☽} = 384400$ km, as given earlier. Thus

$$\sin \Pi_{☽} = 3422''.451$$

is the new value of the constant of sine parallax cited in Ref. 13.

There is a slight difference between the *constant of sine parallax* and the constant term in the algebraic expression for sine parallax. Judging from the discussion in Ref. 10, pp. 327–329, the fourth decimal is quite uncertain. Accordingly, the spirit of the IAU Working Group on the System of Astronomical Constants has been followed in the sense of adopting the above value of $\sin \Pi_{☽}$, and observing that rounded to three decimals, the constant term is identical to the value of $\sin \Pi_{☽}$ adopted. Eckert, Walker, and Eckert recommended a value of $3422''.452$, based in part on the work of Eckert and Smith (Ref. 11).

This quantity linearly affects all of the Solar terms in sine parallax. A rigorous correction, then, would consist of multiplying each of these terms by $(3422.451/3422.540 - 1)$, and using the resultant values as additive corrections to the previous value of sine parallax; in practice, this is not necessary. The correction is of the order of 3 parts in 10^5, and, because the greatest of the planetary terms has a coefficient of $0''.0095$, no error is introduced by simply multiplying the value of sine parallax by the factor 3422451/3422540. This should be done, however, after the application of all corrections that are based on previous values of this constant; in the present calculations, then, it was the last of the corrections to be applied.

This correction would have the effect of increasing the predicted distance of the Moon by nearly 10 km, if the adopted equatorial Earth radius is retained. A discussion of the real effect, or lack of it, on space vehicle trajectory calculations is given in Section V.

IV. Reduction to Inertial Coordinates

The calculations in Section III resulted in an ephemeris of the Moon that was stated in terms of corrected longitude, latitude and sine parallax referred to the mean equinox and ecliptic of date. These coordinates were subsequently converted into rectangular coordinates in the same reference frame, by means of the standard formulae (e.g., Ref. 9, p. 27). Finally, the conversion to rectangular coordinates referred to the mean equinox and equator 1950.0 was accomplished by means of the combined precession and obliquity transformation matrix whose elements are given as power series in time in Ref. 14. The velocities were obtained by numerical differentiation after all of the coordinate computations had been completed.

V. Evaluation of the Corrected Ephemeris

At this point, the new ephemeris must be discussed in terms of its use, how it compares with the previous ephemeris, and its accuracy.

A. Application of the New Ephemeris

Because the Lunar ephemeris is recorded in units of "equatorial Earth radii," the proper scale factor R_{em} is necessary for converting these data into kilometers. Although $R_e = 6378.160$ km was used in constructing the ephemeris, this value is the appropriate scaling factor *only* so long as the IAU system of astronomical constants is adopted for uniform use. Inherent in the concept of "adopted constants" is the reservation that these constants may not necessarily represent the best available values at some subsequent epoch. This is certainly true of the IAU system. In particular, recent analysis of spacecraft trajectory data indicate that

$$GE = 398601.3 \text{ km}^3/\text{sec}^2 \qquad (\pm 0.4)$$

$$\mu^{-1} = 81.302 \qquad (\pm 0.002)$$

If these more accurate values are to be used in trajectory calculations, it is not permissible to identify R_{em} with R_e. R_{em} must be regarded as a fictitious Earth radius to be used *only* for scaling the Lunar ephemeris (cf Ref. 15). It is that value required to satisfy certain constraint equations; it need not be the same as the radius used for near-Earth space vehicle or geodetic work. The primary constraint is Kepler's third law as it applies to the intermediary orbit in the Lunar Theory[5]:

$$n^2 a^3 = F_2^3 \, GE \, (1 + \mu)$$

The mean distance a is related to the scaling factor through the relation

$$R_{em} = a \sin \Pi_{☽}$$

Using the values of GE and μ given above, in conjunction with the IAU values of sine parallax and the Lunar mean motion n, one finds

$$a = 384\,399.30 \text{ km}$$

$$R_{em} = 6378.1495 \text{ km}$$

which is the Lunar scale factor recommended for use with the new ephemeris (LE 4).

[5] F_2 is a constant arising in the derivation of the Lunar Theory. Its value is 0.99909 31419 75298.

B. Comparison of Data of LE 4 and LE 2

Figures 1–9 are several graphs that indicate differences between the Lunar data on LE 4 and those on DE 3, the ephemeris tapes of Ref. 1. Figures 1–6 show the fine structure of the corrections to the polar coordinates for two arbitrarily selected periods.[6] They are to be regarded as representative, but not necessarily typical. From Figs. 7 through 9, a perspective can be obtained on the long period effects of the corrections, as well as an indication of the extreme magnitudes of the changes. Figures 7 through 9 give the differences in spherical ecliptic coordinates, rectangular equatorial coordinates, and rectangular equatorial velocity components for a six-year interval. The rectangular data are in units of Earth radii (R_{em}) and Earth radii per day; the sin $\Pi_{☽}$ scale change has been eliminated by plotting

$$(3422451/3422540) \text{ LE } 4 - \text{LE } 2$$

rather than the actual difference. Thus, the differences shown are indicative of the differences *in meters* when each ephemeris is scaled with its own appropriate value for R_{em}.

The first practical application of the new Lunar ephemeris was reported by Mulholland and Sjogren (Ref. 16),[7] who published the effect of the corrections of Section III on ranging data from *Lunar Orbiters I* and *II*. Figure 10 is reproduced from that paper.

C. Estimated Accuracy

The question of accuracy has several ramifications and, as yet, no really firm answers. Taking the simplest and most favorable possible view of the subject, *if* the new solution of Ref. 11 is as exhaustive as it is believed to be, it is implied that the theory underlying DE 19 does not suffer from the omission of significantly large terms in any coordinate. If this implication be true, the estimated error in the coordinates would be on the order of 10^{-6} Earth radii. With the proper R_{em}, this is equivalent to about 6 m.

In fact, the R_{em} given above is not likely to be the best one because of the uncertainty in the value of sin $\Pi_{☽}$. The resulting uncertainty in Lunar position is not likely

[6] Rough estimates of metric equivalents may be obtained by noting that, in longitude and latitude, $0\overset{''}{.}1 \approx 186$ m; in sine parallax, $0\overset{''}{.}001 \approx 112$ m.

[7] The cited work actually used DE 15, an interim ephemeris whose Lunar data differed from that of DE 19 by an amount negligible in the context of Ref. 16.

to exceed 60 m. This source of potential error, however, should manifest itself as a nearly constant bias in ranging residuals from *Lunar Orbiter*-type vehicles. The implication is that the *Lunar Orbiter* data, if continued over a sufficiently long interval, can lead rather directly to an improved value of sin $\Pi_{☽}$. Thus, this uncertainty need not be regarded as inherent to the ephemeris.

There is some indication that a sizable error may yet exist in the way in which observations were fitted to the Lunar Theory; this kind of error would affect the disparity between the center of mass and the center of figure. We have no way of evaluating these indications at the present time, although the question is under intensive study elsewhere.

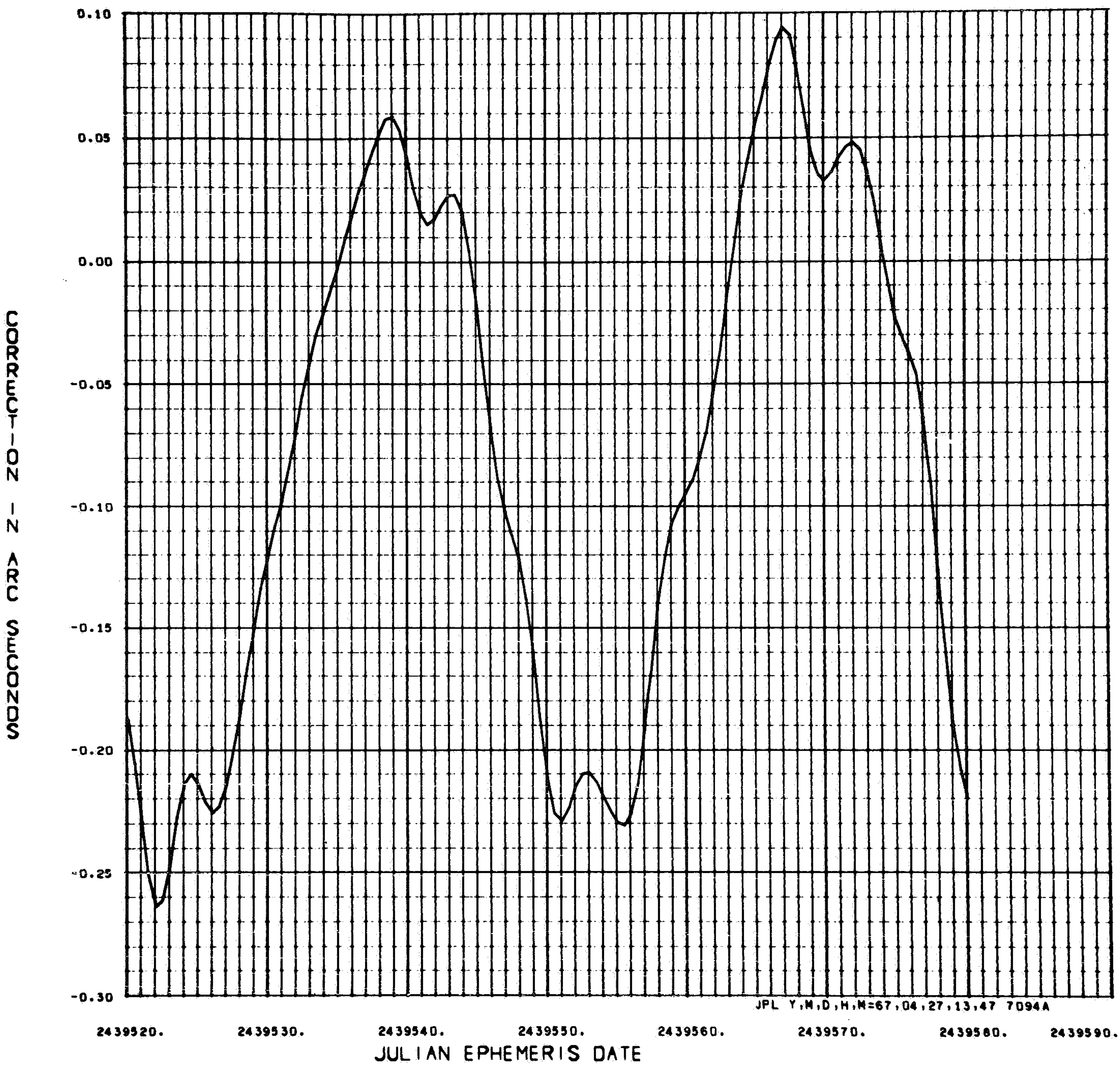

Fig. 1. Longitude corrections to the Lunar ephemeris, period 1 (LE 4—LE 2)

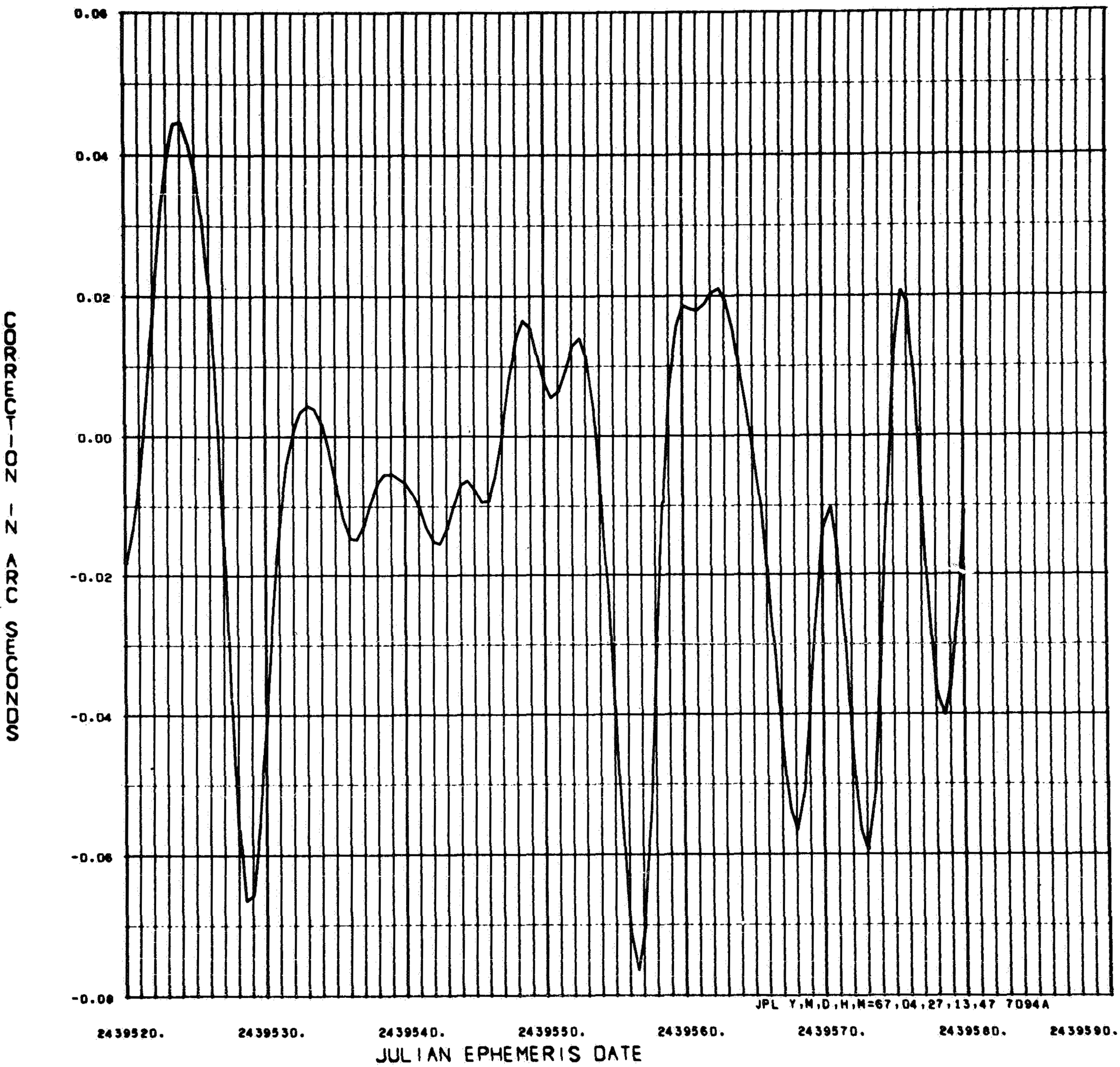

Fig. 2. Latitude corrections to the Lunar ephemeris, period 1 (LE 4—LE 2)

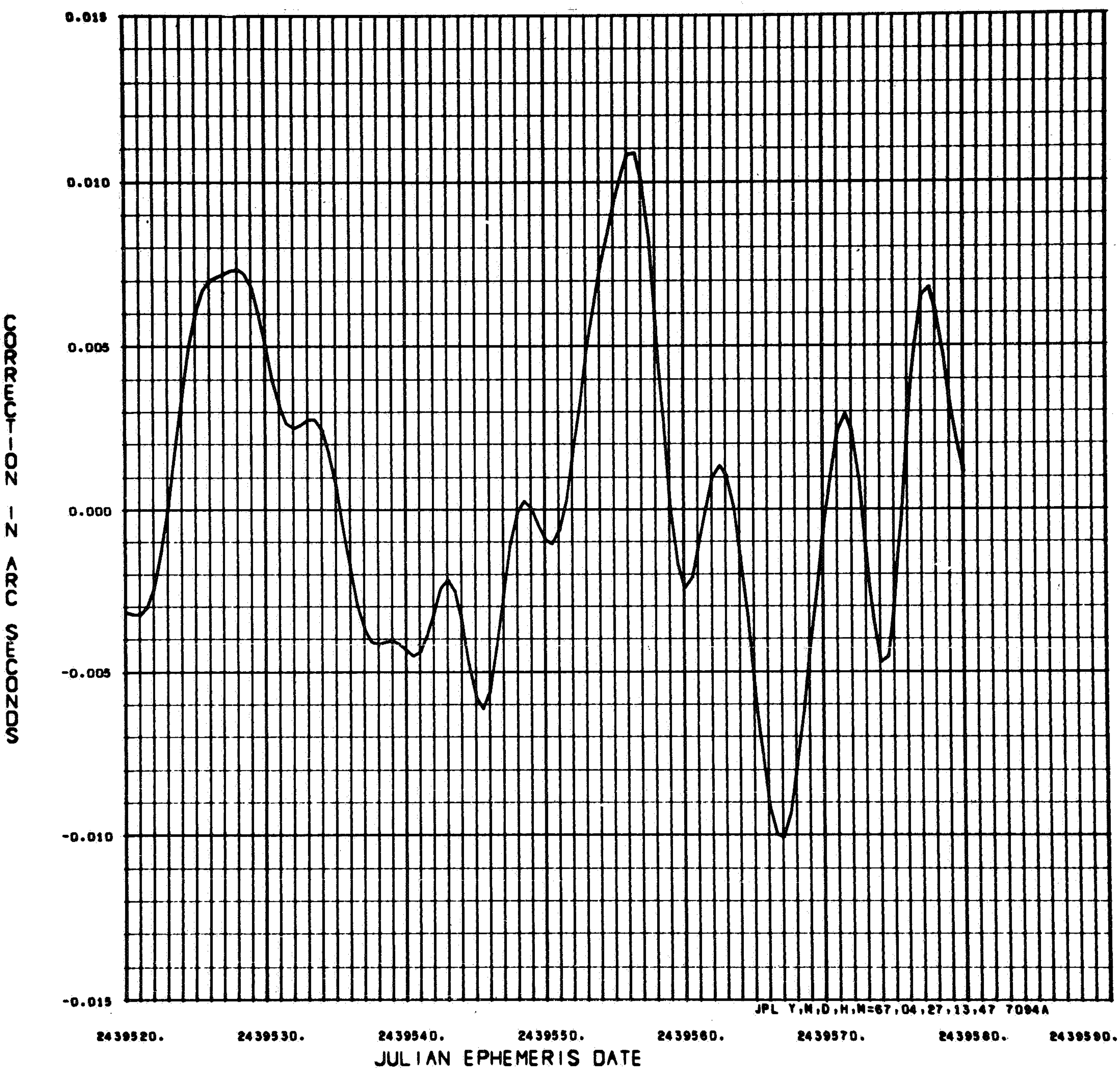

Fig. 3. Sine parallax corrections to the Lunar ephemeris, period 1 (LE 4—LE 2)

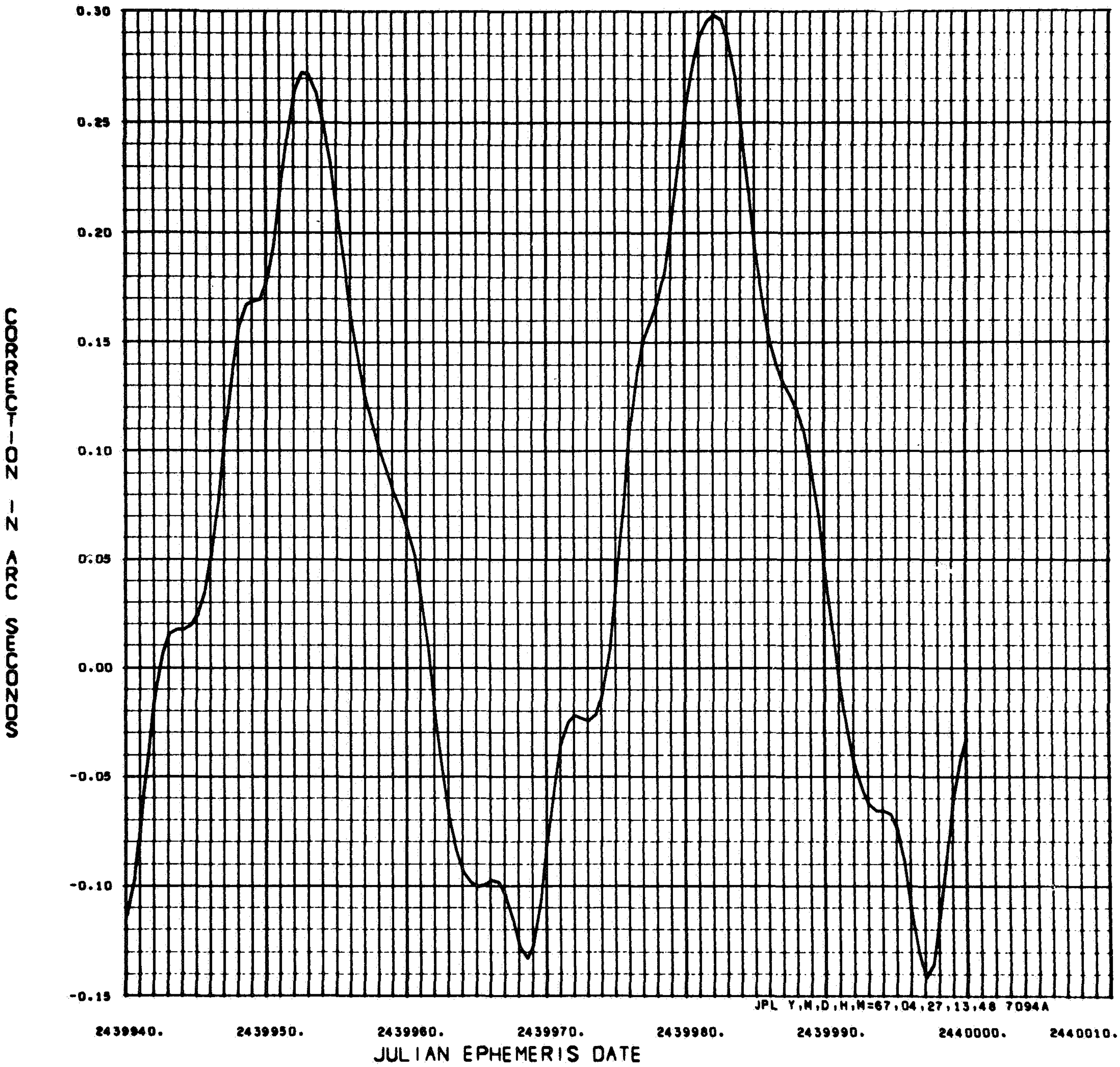

Fig. 4. Longitude corrections to the Lunar ephemeris, period 2 (LE 4—LE 2)

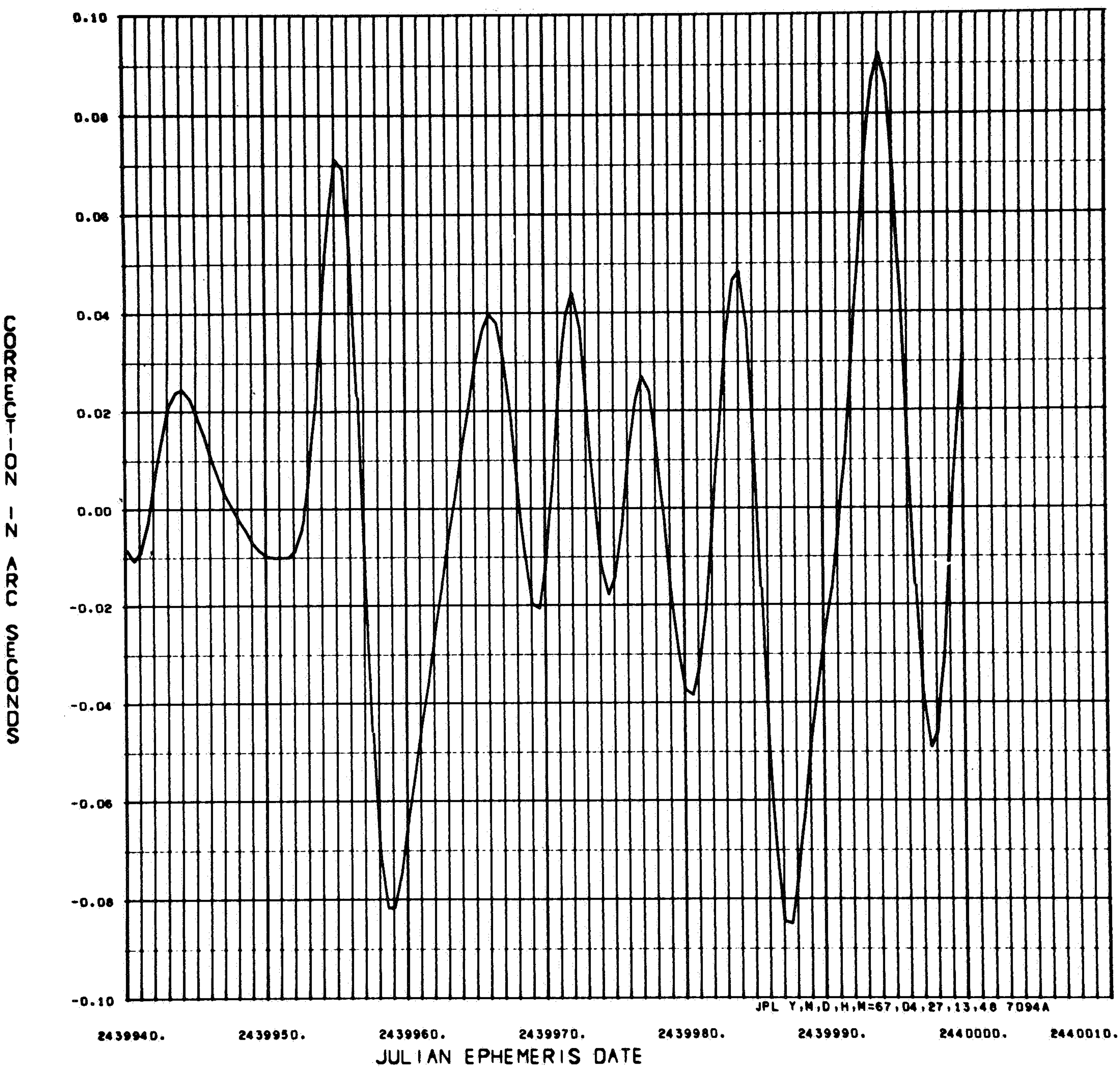

Fig. 5. Latitude corrections to the Lunar ephemeris, period 2 (LE 4—LE 2)

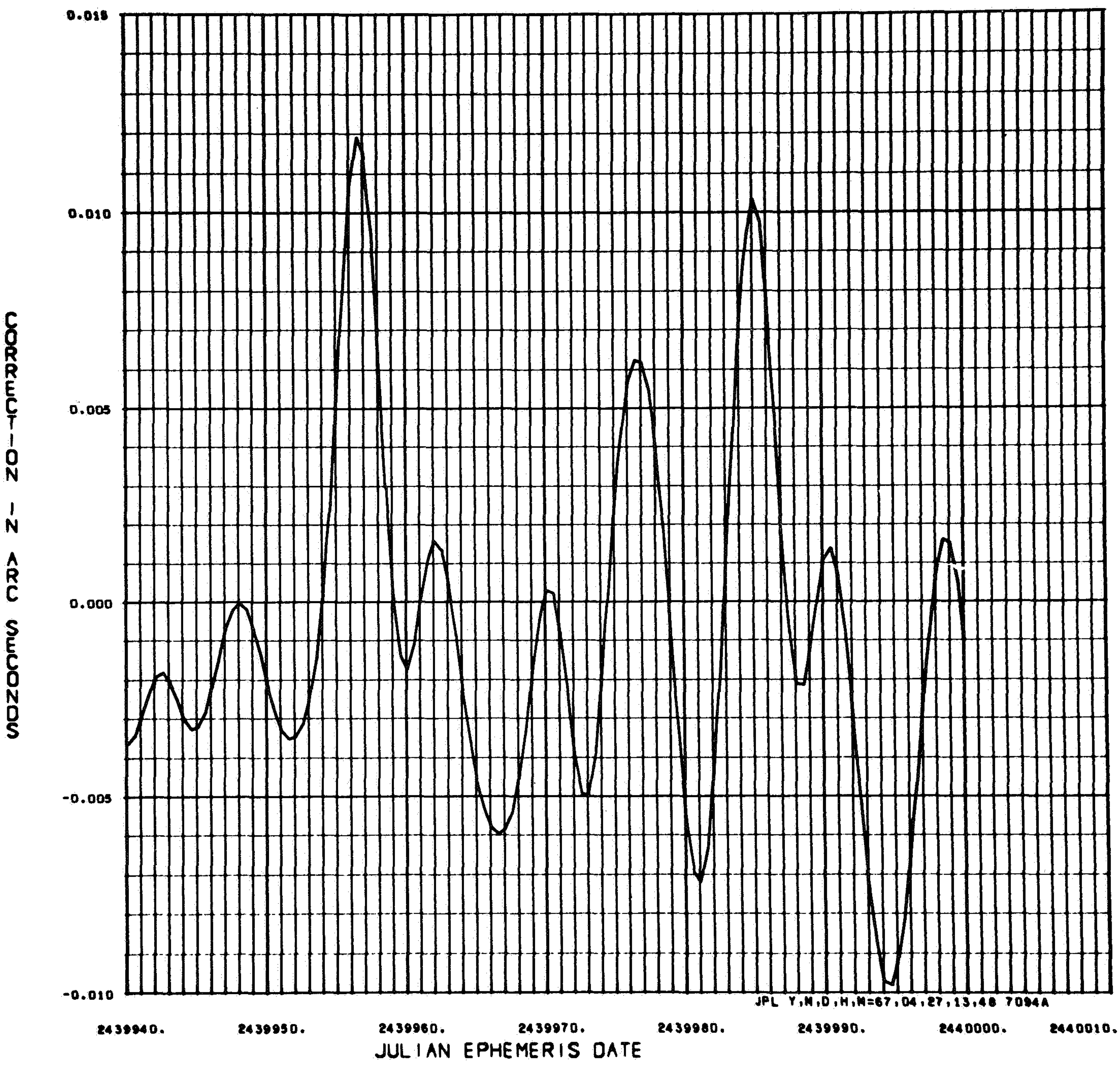

Fig. 6. Sine parallax corrections to the Lunar ephemeris, period 2 (LE 4—LE 2)

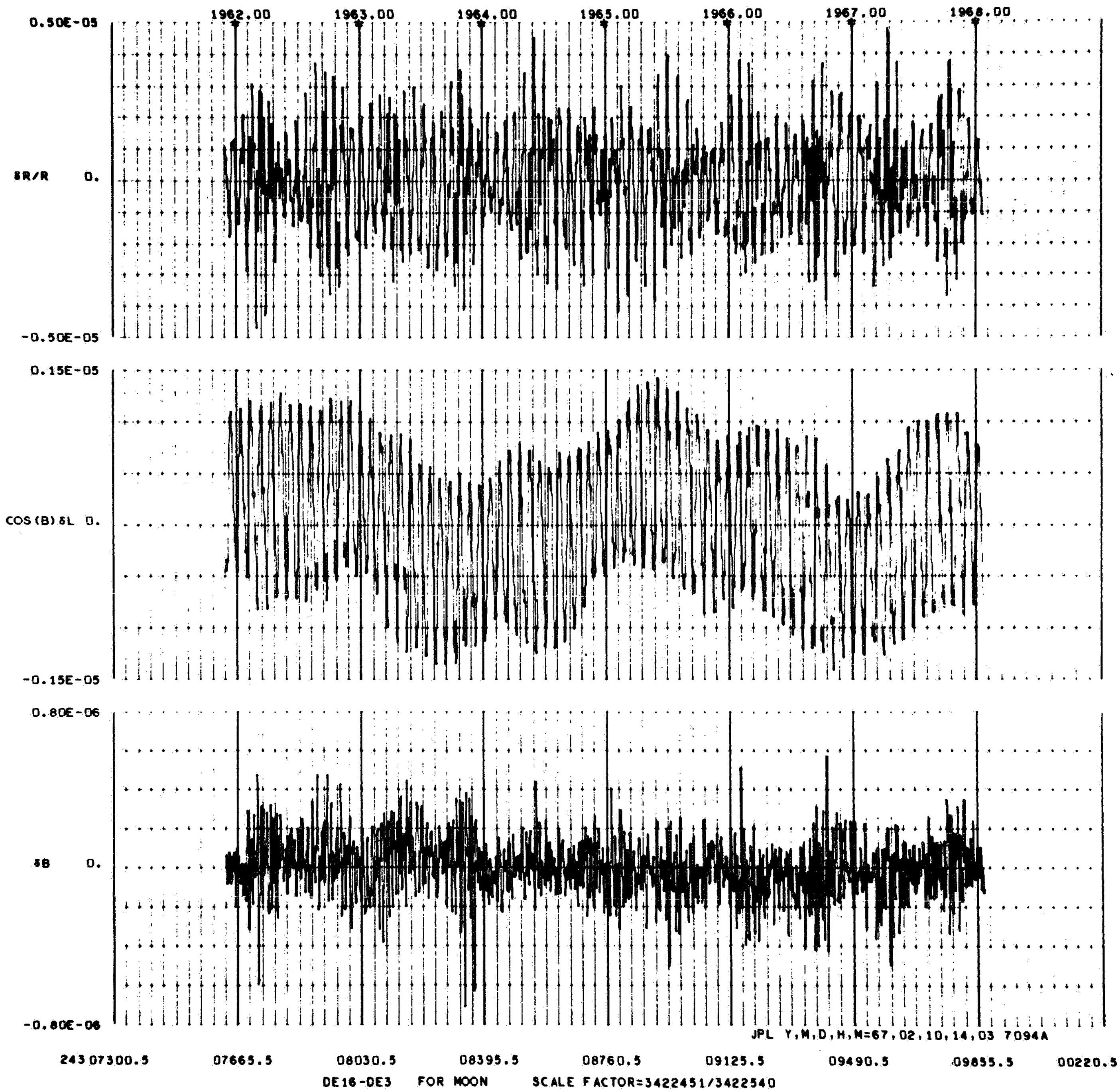

Fig. 7. Corrections to the ecliptic Lunar coordinates (1962–1968)

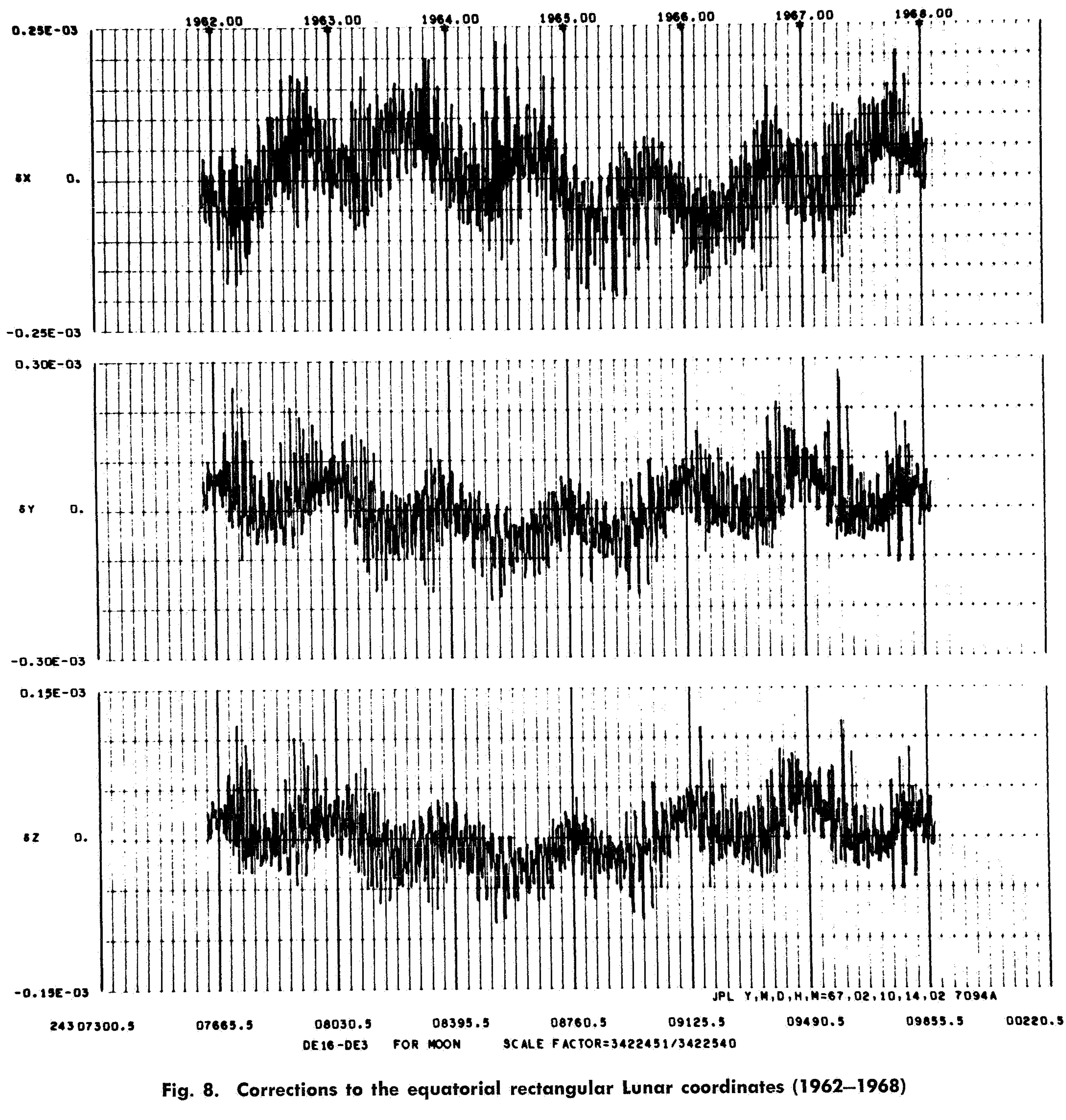

Fig. 8. Corrections to the equatorial rectangular Lunar coordinates (1962–1968)

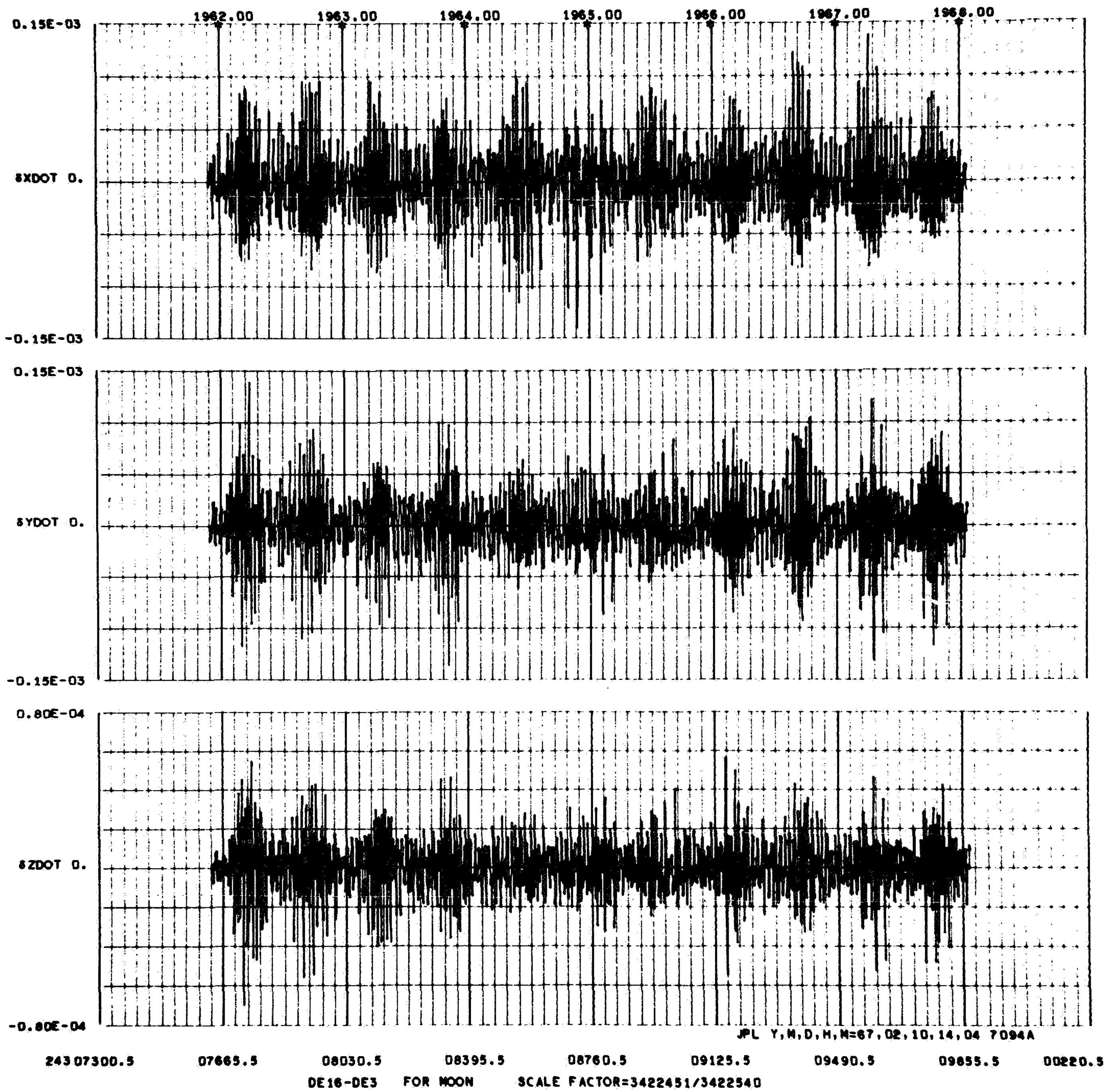

Fig. 9. Corrections to the equatorial rectangular Lunar velocities (1962–1968)

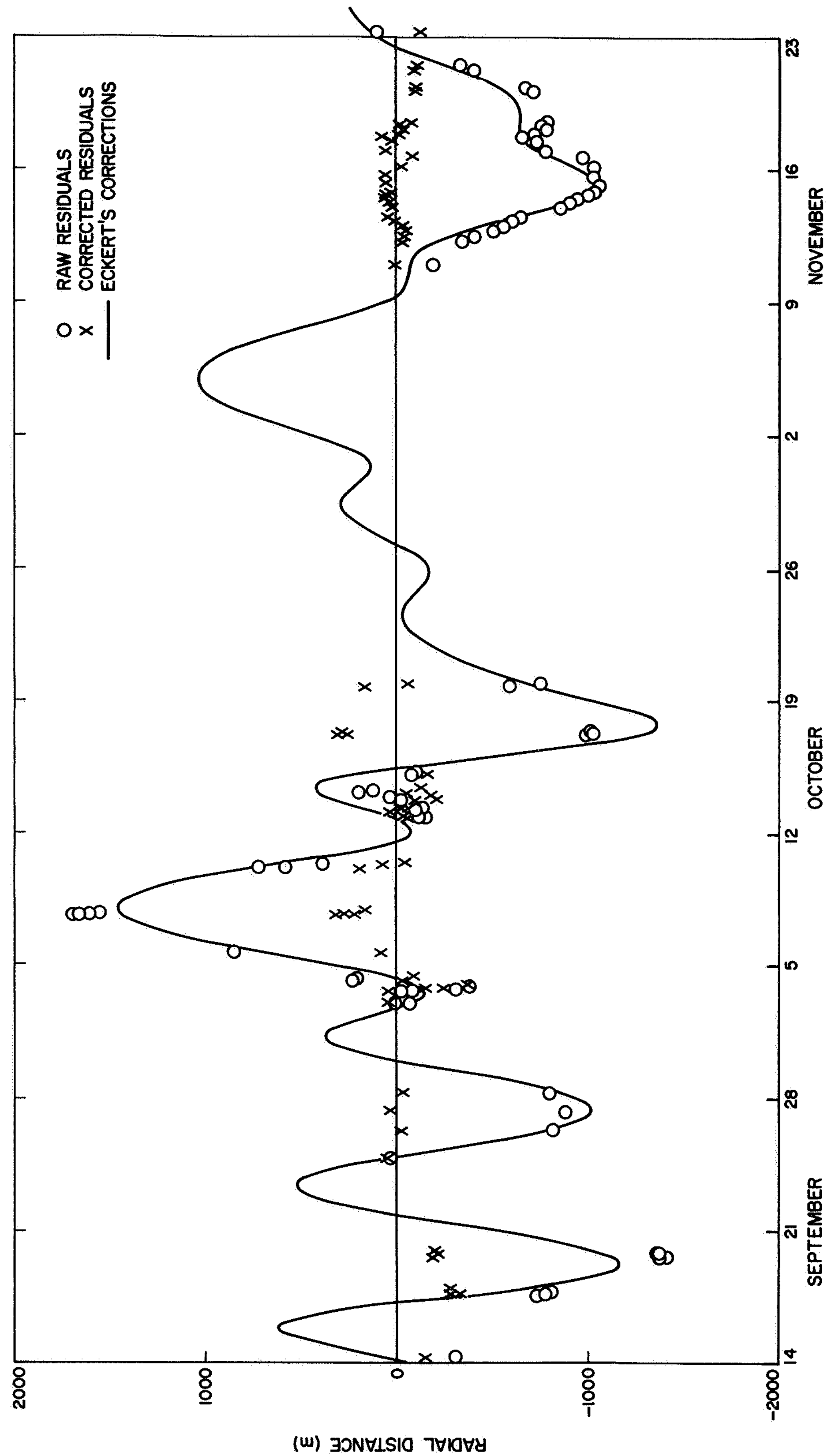

Fig. 10. ***Lunar Orbiter*** **residuals and Eckert's transformation correction to the Lunar radial distance. The interval 14 September–20 October corresponds to** ***Lunar Orbiter I*****, and 11–23 November to** ***Lunar Orbiter II***

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Appendix

Program Specifications

This appendix specifies the steps followed in applying the corrections to LE 2. The procedure chosen avoids the need to evaluate many of the "Fundamental Arguments" of Ref. 3, having, in addition, the advantage of only one time variable,

$$t = JED - 2415020.0, \text{ the number of ephemeris days elapsed since 1900 January 0.5 ET,}$$

rather than the three used in that work.[8]

The variable t, and all explicit functions of it, were computed in double precision, although the explicit functions of time were subsequently truncated to single precision with no detriment to the effective accuracy of the corrections. With the exception of the aberration correction (Table A-14), all corrections may be expected to have an accuracy of 0.″0001 in longitude and latitude, 0.″000001 in sine parallax. The corresponding numbers for the aberration correction are 0.″001 and 0.″0001, respectively.

Each of the tabular arguments in all tables but Tables A-1, A-2, and A-8 are to be regarded as four pairs of integers that specify a linear combination of ℓ, ℓ', F, and D. The actual coefficients of this linear combination have been augmented by +50, to make all of the numbers positive. If the actual combination used in the computation is

$$H_{j1n}\ell + H_{j2n}\ell' + H_{j3n}F + H_{j4n}D$$

then the tabular argument is

$$H_{j1n} + 50, H_{j2n} + 50, H_{j3n} + 50, H_{j4n} + 50$$

As an example, the last entry in Table A-11 is 49515051, which represents

$$-\ell + \ell' + D$$

[8] Although irrelevant to the present discussion, it may be worth noting that one of the three time variables in Ref. 3 carries the unfortunate notation $(t_c - 18.5)$, where t_c has previously been defined as the number of Julian centuries elapsed since 1900 Jan. 0.5 ET. This notation must not be taken literally. The intent, as clearly illustrated by the example on p. 360 of Ref. 3, is to specify 1850.0 as the epoch from which time should be counted for the corresponding variations.

The following are the program specifications as they were given to the programmer:

(1) Read and store the following data from punched cards:

Table A-1: A_n, B_n, C_n, D_n (double precision)

Table A-2: $Z_{ni}, A_{ni}, B_{ni}, C_{ni}$ (double precision)

Table A-3: $K_{11n}, K_{12n}, K_{13n}, K_{14n}, a_{1n}, b_{1n}$

Table A-4: $H_{11n}, H_{12n}, H_{13n}, H_{14n}, c_{1n}$

Table A-5: $K_{21n}, K_{22n}, K_{23n}, K_{24n}, u_{1n}, v_{1n}, w_{1n}, u_{2n}, v_{2n}, w_{2n}, u_{3n}, v_{3n}, w_{3n}$

Table A-11: $K_{31n}, K_{32n}, K_{33n}, K_{34n}, a_{3n}, b_{3n}, c_{3n}, d_{3n}$

Table A-14: $K_{51n}, K_{52n}, K_{53n}, K_{54n}, a_{5n}, b_{5n}, c_{5n}$

Because of their brevity, it was desirable to omit Tables A-12 and A-13 and program the calculations directly.

(2) Read a control card that specifies the first ($JEDZ$) and last ($JEDF$) Julian Ephemeris Dates to be processed and step size (DELT). Rewind the ephemeris tape and the output tape, preparatory to entering the major computation loop. Set $JED = JEDZ$.

(3) Compute t. Using the numerical data from Table A-1, evaluate the mean values of the seven required fundamental arguments L, Ω, ℓ, ℓ', F, D and $\Delta\gamma_c$, each of which is of the form

$$(A_n + B_n t + D_n t^2 + D_n t^3) \text{ rev}$$

To the values for L, Ω and ℓ', add the contributions from Table A-2

$$\sum_i Z_{ni} \sin [2\pi (A_{ni} + B_{ni} t + C_{ni} t^2)]$$

For ℓ, F and D, one uses the relations

$$\Delta\ell = \Delta L - \Delta\omega$$

$$\Delta F = \Delta L - \Delta\Omega$$

$$\Delta D = \Delta L - \Delta\ell'$$

where $\Delta\omega$ is computed in the same way as ΔL, $\Delta\Omega$ and $\Delta\ell'$.

Each of these six arguments was reduced modulo 1 rev (superscript c, cf footnote 4, p. 3) and adjusted to be non-negative.

In the case of $\Delta\gamma_c$, the addition is

$$\sum_i Z_{ni} \cos [2\pi (A_{ni} + B_{ni}\, t + C_{ni}\, t^2)]$$

(4) Evaluate the primary transformation corrections by means of the formulae

$$\theta_n = 2\pi\,(K_{11n}\ell + K_{12n}\ell' + K_{13n}\,F + K_{14n}\,D)$$

$$\theta_n' = 2\pi\,(H_{11n}\ell + H_{12n}\ell' + H_{13n}\,F + H_{14n}\,D)$$

$$\Delta_1\lambda = \sum_n a_{1n} \sin \theta_n$$

$$\Delta_1(\sin \Pi) = \sum_n b_{1n} \cos \theta_n$$

$$\Delta_1\beta = \sum_n c_{1n} \sin \theta_n'$$

Table IV of Ref. 6 provides check values of these corrections for 124 dates.

(5) Evaluate the higher-order transformation corrections by means of the formulae

$$\theta_n = 2\pi\,(K_{21n}\ell + K_{22n}\ell' + K_{23n}\,F + K_{24n}\,D)$$

$$\Delta_2\lambda = \sum_n (u_{1n}t + v_{1n}\,\Delta\gamma_c + w_{1n}) \sin \theta_n$$

$$\Delta_2(\sin \Pi) = \sum_n (u_{2n}t + v_{2n}\,\Delta\gamma_c + w_{2n}) \cos \theta_n$$

$$\Delta_2\beta = \sum_n (u_{3n}t + v_{3n}\,\Delta\gamma_c + w_{3n}) \sin \theta_n$$

using the data from Table A-5.

(6) Using the data of Tables A-11 and A-12, evaluate the primary corrections due to the adoption of a new value for the parameter a_1. The formulae are

$$\theta_n = 2\pi\,(K_{31n}\ell + K_{32n}\ell' + K_{33n}\,F + K_{34n}\,D)$$

$$S = 2\pi \left\{ \sum_n (f_n \sin [2\pi(H_{31n}\ell + H_{32n}\ell' + H_{33n}\,F + H_{34n}\,D)]) + F \right\}$$

$$\Delta_3\lambda = \sum_n a_{3n} \sin \theta_n$$

$$\Delta_3(\sin \Pi) = \sum_n b_{3n} \cos \theta_n$$

$$\Delta_3\beta = \sum_n (c_{3n} \sin \theta_n \cos S + d_{3n} \cos \theta_n \sin S)$$

Note that the terms of Table A-8 contribute no significant corrections.

(7) Evaluate the higher order corrections of Table A-13, combined with the correction to longitude suggested by the new Eckert-Smith solution to the main problem, by means of the formulae

$$\theta_1 = 2\pi\,(2D),\ \theta_2 = 2\pi\,(\ell - 2D)$$

$$\theta_3 = 2\pi\,(2F - 2\ell)$$

$$\Delta_4\lambda = -\,0''.0001 \sin \theta_1 + 0''.0001 \sin \theta_2 + 0''.072 \sin \theta_3$$

$$\Delta_4(\sin \Pi) = -\,0''.000001 \cos \theta_1 - 0''.000001 \cos \theta_2$$

$$\Delta_4\beta = 0.0$$

(8) Using the data of Table A-14, evaluate the aberration corrections by means of the formulae

$$\theta_n = 2\pi\,(K_{51n}\ell + K_{52n}\ell' + K_{53n}\,F + K_{54n}\,D)$$

$$\Delta_5\lambda = \sum_n a_{5n} \cos \theta_n$$

$$\Delta_5(\sin \Pi) = \sum_n b_{5n} \sin \theta_n$$

$$\Delta_5\beta = \sum_n c_{5n} \cos \theta_n$$

where the a, b and c coefficients are the values given in column (e) of the table.

(9) Compute the total correction to be applied to each of the three coordinates by summing the results of Steps 4 through 8; convert to units of rev:

$$\Delta\lambda = \sum_1^5 \Delta_i \lambda/1296000$$

$$\Delta(\sin \Pi) = \sum_1^5 \Delta_i (\sin \Pi)/1296000$$

$$\Delta\beta = \sum_1^5 \Delta_i\beta/1296000$$

(10) Read one logical record from the "old" Lunar Ephemeris tape, obtaining the double precision quantities

$$TJED, \lambda_i, \beta_i, \Pi_i^*, S^*, \bar{\epsilon}, \delta\psi, \delta\epsilon$$

(11) Test if $TJED = JED$. If not, search tape for correct $TJED$.

(12) Reconstruct the ephemeris value of sine parallax. If one lets

$$\sigma = \sin \Pi \text{ expressed in rev}$$

$$R = 2\pi = 6.283185\ldots$$

$$Q = (206\,264.806\,2471/206\,265)$$

then

$$R\Pi^* = R\,\sigma + R^3(\sigma^3/6)\,Q^2$$

represents the first two terms of the expansion of arc sin $(R\,\sigma)$.

The quantity Q represents the ratio of the number of arc seconds per radian to the approximate value used in the construction of Π^*.

To calculate σ, rewrite the previous equation as

$$\Pi^* = \sigma + \sigma^3/a, \qquad a = 6/R^2\,Q^2$$

Using a Newton-Raphson iteration technique,

$$\sigma_0 = \Pi^*$$

$$\sigma_{n+1} = \sigma_n - [\sigma_n^3 + a(\sigma_n - \Pi^*)]/(3\sigma_n^2 + a)$$

$$n = 0, 1, 2, \ldots$$

Then

$$\sin \Pi_i = \sigma = \lim_{n \to \infty} \sigma_n$$

(13) Compute the new values of the lunar coordinates by means of the formulae

$$\lambda_f = (\lambda_i + \Delta\lambda) \text{ modulo 1 rev}$$

$$\beta_f = \beta_i + \Delta\beta$$

$$\sin \Pi_f = (\sin \Pi_i + \Delta \sin \Pi)\,(3422451/3422540)$$

where the factor applied to sine parallax is that discussed in Section III-D-2. The subscripts i and f denote initial and final values.

(14) Convert to geocentric ecliptic rectangular coordinates by means of the formulae

$$\rho = 1/(2\pi \sin \Pi_f)$$

$$x = \rho \cos 2\pi\,\lambda_f \cos 2\pi\,\beta_f$$

$$y = \rho \sin 2\pi\lambda_f \cos 2\pi\beta_f$$

$$z = \rho \sin 2\pi\beta_f$$

These data are in units of Earth radii.

(15) Rotate the x, y, z from the mean ecliptic system of date to the mean equatorial system 1950.0.

(16) Write one logical tape record on the output tape; it should contain the quantities JED, λ_i, β_i, $\sin \Pi_i$, $\bar{\epsilon}$, $\delta\psi$, $\delta\epsilon$, $\Delta\lambda$, $\Delta\beta$, $\Delta \sin \Pi$, λ_f, β_f, $\sin \Pi_f$, x, y, z, all in double precision.

(17) Test if $JED = JEDF$. If not, set $JED = JED +$ DELT and repeat from Step 3. If the equality is satisfied, write end-of-file on the output tape, rewind both tapes and terminate execution.

Table A-1. Fundamental arguments, mean values

n	Argument	$A_n \times 10^{11}$	$B_n \times 10^{15}$	$C_n \times 10^{20}$	$D_n \times 10^{25}$
1	L	7 51206 01080^c	+3660 11014 63356^c	− 2 35980^c	+ 1077^c
2	Ω	7 19953 54167	− 14 70942 28332	+ 4 32630	+ 1266
3	ℓ	8 22512 80093	+3629 16456 84716	+19 13865	+ 8203
4	ℓ'	9 95766 20370	+ 273 77785 19279	− 31233	− 1900
5	F	31252 46914	+3674 81956 91688	− 6 68609	− 190
6	D	9 74270 79475	+3386 31921 98393	− 2 99023	+ 1077
7	$\Delta\gamma_c$	0	0	0	0

Notes: This table was extracted from Table I of Ref. 3.
The superscript c denotes units of rev.

Table A-2. Fundamental arguments, additive terms

n	Argument	Serial no.	$Z_{in} \times 10^{12}$	$A_{in} \times 10^{8}$	$B_{in} \times 10^{12}$	$C_{in} \times 10^{16}$
1	L					
		1628	+ 6 48148^c	+142 22222^c	+ 15 36238^c	
		1629	+ 2 39197	233 63774	+ 12 32723	+ 191^c
		1630	+ 30864	641 40745	− 4 62723	
		1636	+110 10802	537 33431	− 101 04982	+ 191
		1638	+ 56 02623	719 95354	−1470 94228	+ 43
		1639	+ 2 17592	483 98132	−1472 69147	+ 43
		1640	+ 30864	587 50000	+ 90 50118	
		1641	+ 2315	56 90595	+4163 95407	
		1642	+ 57870	213 68238	+ 697 18458	
		1643	+ 2315	279 08004	− 522 76150	
		1644	+ 2315	842 88755	−3466 76949	
		1645	+ 1 82870	845 36324	− 114 59387	
		1646	+ 83333	403 53088	− 214 88317	
		1647	+ 23148	643 40759	+3278 99186	
		1648	+ 97222	655 44893	− 786 45335	
		1649	+ 25463	765 36348	− 671 85949	
		1650	+ 41667	762 98245	− 549 86705	
		1651	+ 7716	314 30515	+ 471 28088	
		1652	+ 10031	749 12667	− 386 07439	
		1653	+ 10031	522 54546	− 500 66826	

Notes: This table was extracted from Table II of Ref. 3.
The superscript c denotes units of rev.

Table A-2 (contd)

n	Argument	Serial no.	$Z_{in} \times 10^{12}$	$A_{in} \times 10^{8}$	$B_{in} \times 10^{12}$	$C_{in} \times 10^{16}$
1	L (contd)	1654	+ 2315[c]	+ 568 41949[c]	+ 971 33573[c]	
		1655	+ 6173	545 34917	−1572 29330	
		1656	+ 2315	273 33997	+6939 64168	
		1657	+ 6173	798 48142	−5881 91858	
		1658	+ 4630	234 87197	−4147 00816	
		1659	+ 4630	710 15141	−2412 09774	
		1660	+ 20062	129 87529	− 677 18733	
		1661	+ 3086	538 48806	+1057 72309	
		1662	+ 13117	29 40541	+ 262 90643	
2	Ω					
		1632	+ 4 86111	142 22222	+ 15 36238	
		1669	+ 1 31172	537 33431	− 101 04982	+ 191[c]
		1670	+740 43210	719 95354	−1470 94228	+ 43
		1671	+120 21605	483 98132	−1472 69147	+ 43
		1672	+ 14 35185	524 53688	−1471 62675	+ 43
4	ℓ'					
		1633	− 49 38271	142 22222	+ 15 36238	
		1673	− 2 08333	587 50000	+ 90 50118	
		1674	− 14 58333	845 36324	− 114 59387	
		1675	+ 1 54321	610 43085	− 677 18733	
7	$\Delta\gamma_C$					
		1676	− 33 31790	719 95354	−1470 94228	+ 43
		1677	− 5 38580	483 98132	−1472 69147	+ 43
		1678	− 64043	524 53688	−1471 62675	+ 43
8	$\Delta\omega$					
		1631	+ 16 20370	142 22222	+ 15 36238	
		1663	+ 91049	537 33431	− 101 04982	+ 191
		1664	+ 16 01851	719 95354	−1470 94228	+ 43
		1665	+ 6 48148	483 98132	−1472 69147	+ 43
		1666	+ 77160	587 50000	+ 90 50118	
		1667	+ 4 57562	845 36324	− 114 59387	
		1668	+ 50154	129 87529	− 677 18733	

Table A-3. Transformation corrections to longitude and sine parallax

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
50505058	4	41	51504842	1	
50505056	-11	-116	50515254	-2	
50505054	-18	268	50515252	-6	37
50505052	88	569	50515250	-6	87
51505058	3	9	50515248	-1	162
51505056	-3	-76	50515246	3	-136
51505054	-12	266	50515244	3	8
51505052	20	-210	50514856	1	-3
51505050		-504	50514854	5	-154
51505048	-11	-131	50514852	-10	-35
51505046	-15	-90	50514850	-7	-241
51505044	-11	-46	50514848	-11	-65
51505042	-5	-76	50514846	1	-4
51505040		2	50514844	2	
50515058	-1	-2	52505053	1	-127
50515056		-99	52505051	8	75
50515054	-7	-271	52505049	16	-379
50515052	-5	-334	52505047	-9	157
50515050	-4	-122	52505045	-16	-90
50515048	-20	-1065	52505043	-3	-16
50515046	16	771	51515055	4	9
50515044	-11	8	51515053	-5	157
50515042	-4	11	51515051	-2	385
50505057	1		51515049	-2	129
50505055	4	43	51515047	-4	-161
50505053	12	32	51515045	5	-39
50505051	-1	282	51495055	1	2
52505058		2	51495053	3	22
52505056	-9	97	51495051	7	-98
52505054	47	91	51495049	-9	59
52505052	-37	-501	51495047	11	-131
52505050	29	233	51495045	-17	94
52505048	-8	-141	51495043	-1	2
52505046	2	137	50525053	2	-35
52505044	-19	-98	50525051	-5	-62
52505042	4	11	50525049	1	
52505040	-1	4	50525047	4	54
51515056	-8	-23	50525045	-1	
51515054	5	50	50505255	1	
51515052	-6	283	50505253	-2	4
51515050	15	-147	50505251	-4	265
51515048	-3	-83	50505249		11
51515046	48	-17	50505247		-4
51515044	7	40	50505245	4	-30
51515042	-1	31	54505054	-1	63
51495058	1	3	54505052	56	249
51495056	-3	130	54505050	-9	-226
51495054	-9	181	54505048	22	88

Note: The data in this table were obtained from W. J. Eckert and are identical to those in Table 2 of Ref. 10.

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
51495052	-73	- 265	54505046	-12	12
51495050	57	52	54505044	3	- 40
51495048	-14	- 121	54505042	6	62
51495046	7		54505040	- 1	4
51495044	- 4	263	53515054	- 7	- 23
51495042	1	- 5	53515052	-10	- 41
50525054	10	- 45	53515050	38	143
50525052	19	257	53515048	25	- 20
50525050	-15	- 39	53515046	- 4	- 28
50525048	32	- 154	53515044	- 1	107
50525046	-13	7	53515042	- 6	38
50525044	- 8	70	53495056	1	3
50525042		2	53495054	2	97
50505256	- 3		53495052	34	453
50505254	3	- 30	53495050	-71	- 140
50505252	9	- 285	53495048	37	244
50505250	40	297	53495046	16	- 18
50505248	15	- 91	53495044	- 2	- 79
50505246	- 6	180	53495042	2	- 5
50505244	9	47	52525052	- 4	- 19
51505055	5	3	52525050	17	- 4
51505053	23	- 139	52525048	9	23
50524848	18	3	52494852	4	- 339
50524846	9		52494850	1	32
50505454	3		52494848	1	41
50505452	- 3		52494846	- 1	6
50505450	23		52494844	- 1	
50505448	11	92	51525252	1	
50505446	1		51525250	- 3	2
50505444	1		51525248	3	- 95
53505053	- 5	- 21	51525246	- 2	4
53505051	9	68	51525244		2
53505049	-14	15	51524852	- 1	- 17
53505047	- 9	100	51524850	- 3	22
53505045	9		51524848	4	- 34
53505043		- 18	51524846	6	- 2
52515055		2	51524844	1	
52515053	- 5	- 137	51485254	- 2	
52515051	1	102	51485252	- 7	
52515049	13	258	51485250	2	4
52515047	- 5	- 46	51485248	- 4	94
52515045	9	92	51485246	1	6
52515043	1	- 3	51484854	6	- 5
52495053	- 1	- 8	51484852	- 2	- 105
52495051	- 9	253	51484850	- 1	- 37
52495049	- 1	- 101	51484848	- 1	
52495047	11	177	51484846	- 1	
52495045	- 9	- 136	50535250	1	

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
52495043	- 1	2	50535248	3	- 10
51525053	- 4	- 8	50535246	- 1	2
51525051	- 5	- 118	50534852	1	
51525049	- 1	- 11	50534850	- 2	
51525047	- 3	- 130	50534848	- 3	
51525045	1	- 3	50534846	1	
51485053	4	3	51505454	1	
51485051	- 4	- 5	51505452	4	
51485049	2	- 7	51505450	7	- 4
51485047	2	- 17	51505448	- 6	13
51485045	1	- 2	51505446	- 1	10
50535051	5	- 5	51504654		- 11
50535049	- 5	16	51504652	- 5	- 10
50535047	- 3	4	51504650	12	11
51505253	8		51504648	- 7	- 3
51505251	- 6	- 157	51504646	- 7	
51505249		- 218	50515452	- 2	
51505247	2	- 123	50515450	2	
51505245	13	159	50515448	- 1	4
51504855	- 2	- 6	50514654	1	
51504853	- 8	61	50514652		- 2
51504851	6	43	50514650	- 2	
51504849	- 3	- 97	50514648	1	
51504847	-10	- 390	54505053	- 1	3
51504845	- 2		54505051	1	- 68
50515253	1		54505049	- 4	166
50515251	- 7	- 5	54505047	- 2	16
50515249	- 2	120	54505045	- 4	- 2
50515247		- 126	54505043	2	- 4
50515245	2	- 4	53515053	2	7
50514855	1	2	53515051	- 2	142
50514853	4	68	53515049	3	- 10
50514851	- 4	12	53515047		9
50514849	4	8	53515045	- 1	- 22
50514847	- 2		53515043	2	- 3
55505054	1	6	53495051	5	- 29
55505052	8	- 35	53495049	- 1	- 308
55505050	-21	38	53495047	- 7	9
55505048	7	2	53495045	5	- 6
55505046	1	46	52525051	- 7	- 25
55505044	- 1	2	52525049	- 2	- 13
55505042	- 1	4	52525047	1	- 7
54515054	- 1	- 3	52525045	3	- 10
54515052	- 3	- 63	52485051	- 3	- 7
54505250	2	16	54495049	-11	- 36
54505248	11	- 99	54495047	2	12
54505246	1	21	53525051	- 1	- 3
54504854	- 1	- 2	53525049		- 2

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
54504852	- 8	- 55	53525045		- 2
54504850	48	- 51	53485049	- 1	- 4
54504848	-19	12	53505251	5	
54504846	1	- 2	53505249	- 7	10
54504844	1	- 5	53505247	- 1	2
53515252	3		53504851		- 2
53515250	0	- 4	53504847		4
53515248	36	- 35	53504845	- 1	
53515246	- 5	12	52515251	- 8	
53514852	6	16	52515249	1	
53514850	6	24	52515247		3
53514848	3	6	52514847	- 1	
53514846	3	- 28	52495251	1	
53514844	3		52495249	6	- 4
53495254	- 1		52495247	- 1	
53495252	-11		52494851	- 2	- 5
53495250	0	6	52494849	2	12
53495248	12	- 10	52494847	- 1	- 3
53494854	- 1	- 3	51505451	- 1	
53494852	-16	- 47	51505449	- 1	
53494850	- 4	- 12	51504651		- 3
53494848		- 3	51504649	1	
53494846	1	3	50515451	1	
52525250	3		57505050	3	14
52525248	10	- 15	57505048	- 3	- 10
52525246	- 2	4	57505046	1	
52524852	- 1	- 3	56515050	- 2	- 6
52524850	- 1		56515048	- 1	- 3
52524848	3	- 29	56495050	2	7
52524846	7	- 2	56495048	- 1	- 4
52524844	1		55485050	1	3
52485252	- 5		55505252	- 1	
52485250	- 9	2	55505250	-19	3
52485248	- 1	17	55505248	10	- 9
52485246	1	- 3	55505246	- 1	2
52484854	- 1		55504852	- 2	- 5
52484852	- 9	- 23	55504850	- 2	- 3
52484850	1	5	55504848	1	
51505051	12	- 156	52525046	20	- 194
51505049	13	- 85	52525044	- 2	159
51505047	- 9	342	52525042	- 2	6
51505045	- 5	- 37	52485056	1	3
51505043	1	- 3	52485054	- 2	74
50515055	- 2	43	52485052	15	- 36
50515053	4	- 173	52485050	-22	31
50515051	3	44	52485048	-10	- 46
50515049	- 2	- 56	52485046	- 7	51
50515047	7	- 48	52485044	- 4	- 13

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
50515045		13	51535050	- 2	27
53505056	3	13	51535048	4	91
53505054	5	- 103	51535046	-12	68
53505052	- 9	- 50	51535044	- 6	13
53505050	- 2	46	51475054	5	36
53505048	- 7	- 14	51475052	8	122
53505046		75	51475050	15	- 38
53505044	3	29	51475048	-10	- 217
53505042	5	- 15	51475046	- 2	4
53505040	- 1	5	50545050	- 1	
52515056	- 1	- 4	50545048	1	142
52515054	2	- 172	50545046	- 5	10
52515052	- 9	- 736	52505256	- 1	
52515050	-10	463	52505254	3	
52515048	- 2	89	52505252	3	- 11
52515046	11	- 198	52505250	5	- 564
52515044	5	- 13	52505248	9	201
52515042	11	48	52505246	- 4	- 163
52495056	7	21	52505244	- 6	38
52495054	- 2	116	52505242		3
52495052	-11	1579	52504856	- 2	- 2
52495050	10	-1387	52504854	18	- 114
52495048	- 9	117	52504852	30	- 71
52495046	-14	- 181	52504850	- 9	- 117
52495044	-12	- 49	52504848	-14	- 41
52495042	3	- 7	52504846	-37	
51525054	- 1	- 2	52504844	1	- 8
51525052	5	- 242	52504842	1	
51525050		54	51515254	4	
51525048	- 8	- 198	51515252	38	2
51525046	6	281	51515250	1	134
51525044	6	- 41	51515248	- 9	90
51525042	- 1	4	51515246	12	83
51485056	6	15	51515244	8	15
51485054	3	22	51514854	- 2	
51485052	-14	538	51514852		64
51485050	-12	- 646	51514850	12	- 119
51485048	- 1	105	51514848	-11	- 157
51485046	- 3	- 54	51514846	34	- 21
51485044	2	- 5	51514844	6	
50535052	4	6	51495256	- 1	
50535050	8	35	51495254	- 4	
50535048	13	190	51495252	-90	4
50535046	2	- 23	51495250	28	- 244
50535044	- 3	6	51495248	- 3	95
51505256	- 3		51495246	9	- 117
51505254	3	- 3	51495244	1	- 7
51505252	24	- 118	51494856	- 1	

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
51505250	3	-17	51494854		-64
51505248	-4	72	51494852	2	73
51505246	4	42	51494850	-14	87
51505244		-92	51494848	-18	110
51505242		3	51494846	-14	3
51504856	1		51494844	-1	
51504854	11	63	50525252	8	-2
51504852		247	50525250	11	29
51504850	-25	5507	50525248	8	148
51504848	-1	-283	50525246	10	22
51504846	-9	-180	50524852	16	
51504844	5	-3	50524850	-17	42
54515050	5	-828	52485049	-2	-14
54515048	6	-417	52485047	-1	4
54515046	1	2	51535051	-1	-2
54515044	2	20	51535049		-4
54515042	4	11	51535047	1	
54495054	3	10	51475051	1	3
54495052	2	191	51475047	1	-3
54495050	-8	966	52505253	1	
54495048	9	-243	52505251	-6	5
54495046	3	-2	52505249	1	77
54495044	-2	3	52505247	2	21
54495042	1		52505245	3	-2
53525050	-4	-61	52504853	5	-10
53525048	5	-154	52504851	1	-6
53525046	3	29	52504849	2	-34
53525044	-1	49	52504847	1	23
53525042	-1	5	52504845	-3	
53485054	3	8	51515253	-2	
53485052	3	124	51515251	-6	
53485050	-8	254	51515249		3
53485048	-4	88	51515247	-1	7
53485046	-5	-26	51515245	4	-5
53485044	-5	-10	51514853	2	3
52535050		-14	51514851	3	
52535048	3	-37	51514849	-1	
52535046	5	84	51514847	3	
52535044	4	11	51495251	6	
52475054	2	5	51495249	2	-25
52475052	-1	58	51495247	-2	-2
52475050	2	52	51495245	-2	2
52475048	-1		51494853	1	-12
52475046	-2	3	51494851	-1	4
51545048	59	6	51494849		-60
51545046	-6	15	51494847	4	-2
51465054	1	2	50525251	1	
51465052	10	25	50525249		-3

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
51465050	1	3	50524853	- 1	
53505254	27		50505451	5	
53505252	-22	2	50505449	- 2	
53505250	22	- 30	50505447	- 2	
53505248	- 9	102	56505052	3	15
53505246	- 2	60	56505050	0	210
53505244	- 3		56505048	0	- 138
53505242	- 1		56505046	1	17
53504854		- 17	55515052	- 2	- 7
53504852	- 2	- 8	55515050	0	- 84
53504850	23	- 136	55515048	0	- 45
53504848	7	34	55515046	- 1	11
53504846	4	548	55495052	4	18
53504844	- 2	- 11	55495050	0	97
53504842	1		55495048	- 9	- 44
52515254	1		55495046	- 1	2
52515252	6		54525050	- 1	- 4
52515250	4	11	54525048	- 5	- 20
52515248		- 284	54525046	- 2	5
52515246	- 4	6	54525044	- 1	3
52515244	- 5	4	54525042		2
52514854	3	2	54485052	3	13
52514852	2	103	54485050	8	32
52514850	- 2	12	54485048	3	12
52514848	- 7	- 563	54485046	1	- 2
52514846	- 7	- 45	53535048	- 3	- 10
52514844	- 2	- 2	53535046	- 2	5
52495254	- 4		53535044	- 1	4
52495252	- 9		53475052	2	7
52495250	12	- 2	53475050	2	8
52495248	- 5	- 6	53475048		2
52495246	4		52545046	- 1	5
52495244	3	- 3	52465052	1	3
52494854	- 2	- 16	54505252	- 3	
52484848	- 3	10	54515250	5	
52484846	- 2		54515248	3	- 2
51535248	2	- 6	54515246	- 1	
51535246	- 1	2	54514852	1	2
51534848	3		54514850	1	2
51534846	1		54495252	- 1	
51475252	- 1		54495250	- 6	
51475250	- 1		54495248	2	
51474852	1	2	54494852	- 2	- 4
52505452	- 4		53525248	1	
52505450	19		53524846		- 2
52505448	- 8	- 6	53485252	- 1	
52505446	- 2	5	53485250	- 1	
52504652	- 7	- 17	53484852	- 1	- 2

Table A-3 (contd)

Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$	Argument	$a_{1n} \times 10^4$	$b_{1n} \times 10^6$
52504650	33	202	53505452	1	
52504648	9	− 7	53505450	15	
52504646	4		53505448	− 3	
51515452	− 1		53504652	− 1	− 2
51515450	− 6		53504650	2	
51515448		− 3	53504648	1	6
51515446	− 1	2	52515450	− 2	
51514652	− 1	− 8	52515448	− 1	
51514650	− 1	3	52514648	− 1	
51514648	− 9		52514646	− 1	
51514646	− 1		52495450	2	
51495452	2		52494650	1	
51495450	6		51505650	− 2	
51495448	− 1		51504450	2	
51494654	− 1	− 2	56505049		2
51494652	4	18	55495049	− 1	− 3
51494650	1	− 4	55495047	1	
51494648	2		54505251	1	
50505650	− 6		54505249	− 1	
50505648	− 1		53515251	− 1	
55505051	− 1	− 6	53495249	1	
55505049	5	19	56505250	− 1	
55505047	1		56505248	1	
54515051	4	15	55495250	− 1	
54515049	− 1	− 4	54505450	2	
54515047	1	2	54505448	− 1	
54515045	1				
54495051	− 1	− 4			

Table A-4. Transformation corrections to latitude

Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$
50505158	1	48505154	18	50515145	16
50505156	− 1	48505152	− 10	50495153	4
50505154	13	48505150	31	50495151	1
50505152	5	48505148	36	50495149	− 2
50505150	4	48505146		50495147	− 5
50505148	1	48505144	− 13	50495145	− 6
50505146	46	48505142	− 1	51505354	3
50505144	4	51515156		51505352	9
50505142	− 8	51515154	− 6	51505350	8
51505156	− 13	51515152	17	51505348	5
51505154	− 10	51515150	− 19	51505346	− 19
51505152	24	51515148	− 12	51505344	− 3
51505150	− 27	51515146	14	49505356	
51505148	− 38	51515144	− 3	49505354	7
51505146	16	51515142	− 12	49505352	− 1
51505144	32	49495158	1	49505350	154
51505142	− 10	49495156	3	49505348	2
51505140	− 1	49495154	− 12	49505346	− 2
49505158	2	49495152	5	49505344	− 2
49505156	− 2	49495150	7	50515354	− 1
49505154		49495148	− 8	50515352	9
49505152	51	49495146	− 13	50515350	11
49505150	− 97	49495144	9	50515348	7
49505148	49	51495158	1	50515346	6
49505146	12	51495156	1	50515344	− 3
49505144	15	51495154	− 2	50495354	2
49505142	− 7	51495152	7	50495352	− 18
50515156		51495150	− 14	50495350	− 2
50515154	− 31	51495148	− 55	50495348	− 11
50515152	15	51495146	18	50495346	1
50515150	55	51495144	8	53505156	1
50515148	21	51495142	3	53505154	1
50515146	9	49515158	1	53505152	− 14
50515144	− 9	49515156	6	53505150	− 2
50515142	− 7	49515154	2	53505148	23
50495156	− 9	49515152	25	53505146	14
50495154	15	49515150	38	53505144	− 3
50495152	−132	49515148	− 2	53505142	− 11
50495150	2	49515146	45	53505140	− 1
50495148	2	49515144	− 6	47505160	1
50495146		49515142	− 2	47505158	5
50495144	− 1	50525154	− 3	47505156	
50495142	1	50525152	16	47505154	− 14
50505157	− 1	50525150	− 14	47505152	− 20
50505155	1	50525148	4	47505150	61
50505153	10	50525146	10	47505148	9
50505151	− 46	50525144	− 25	47505146	− 1
50505149	45	50485156	1	47505144	− 7

Note: The data in this table were obtained from W. J. Eckert and are identical to those in Table 2 of Ref. 10.

Table A-4 (contd)

Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$
50505147	12	50485154	- 3	52515156	1
50505145	3	50485152	- 21	52515154	- 2
50505143	3	50485150	- 3	52515152	- 52
50505356	1	50485148	- 6	52515150	22
50505354	8	50485146	- 4	52515148	40
50505352	- 1	51505153	- 9	52515146	10
50505350	62	51505151	- 8	52515144	9
50505348	- 2	51505149	17	52515142	- 13
50505346	12	51505147	- 5	48495158	
50505344	2	51505145	- 18	48495156	- 4
52505156	2	51505143	8	48495154	- 24
52505154	10	49505155	7	48495152	11
52505152	- 42	49505153	31	48495150	- 29
52505150	- 17	49505151	- 10	48495148	- 18
52505148	1	49505149	- 20	48495146	16
52505146	22	49505147	5	48495144	3
52505144	- 17	49505145	21	52495156	
52505142	7	50515155	1	52495154	5
52505140	- 1	50515153	- 11	52495152	76
48505160	1	50515151	18	52495150	-121
48505158	- 8	50515149	10	52495148	- 4
48505156	- 1	50515147	- 5	52495146	- 16
52495144	- 1	49495155	- 1	50485348	4
52495142	2	49495153	- 2	54505154	2
48515158	1	49495151	1	54505152	13
48515156	7	49495149	10	54505150	- 36
48515154	- 7	49495147	- 8	54505148	24
48515152	28	49495145	- 3	54505146	- 11
48515150	35	51495155	1	54505144	17
48515148	- 14	51495153	1	54505142	- 2
48515146	- 9	51495151	13	46505158	1
48515144	- 12	51495149	1	46505156	- 4
51525152	- 13	51495147		46505154	- 11
51525150		51495145	- 8	46505152	- 5
51525148	6	51495143	- 1	46505150	26
51525146	- 3	49515155	2	46505148	- 15
51525144	- 16	49515153	- 16	46505146	- 18
51525142	- 1	49515151	- 5	53515154	
49485156	- 5	49515149	- 7	53515152	7
49485154		49515147	19	53515150	18
49485152	- 18	49515145	2	53515148	12
49485150	9	50525153	- 1	53515146	7
49485148	4	50525151	- 1	53515144	13
49485146	4	50525149	- 3	53515142	- 5
51485154	1	50525147	6	47495158	
51485152	33	50525145	1	47495156	- 4
51485150	- 19	50485153	2	47495154	- 6
51485148	- 9	50485151	5	47495152	5

Table A-4 (contd)

Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$
51485146	− 1	50485149	1	47495150	7
51485144	1	50485147	5	47495148	10
49525156	− 1	50505552	1	47495146	11
49525154	− 6	50505550	16	53495156	1
49525152	14	50505548	− 1	53495154	2
49525150	4	50505546		53495152	− 5
49525148	28	52505354		53495150	− 11
49525146	− 21	52505352		53495148	19
49525144	− 6	52505350	5	53495146	− 11
50535150	2	52505348	22	53495144	1
50535148	5	52505346	− 7	47515156	7
50535146	− 23	52505344	− 3	47515154	13
50535144	− 1	48505356		47515152	− 11
50475156	1	48505354	3	47515150	6
50475154	3	48505352	1	47515148	− 4
50475152	16	48505350	11	47515146	− 18
50475150	− 8	48505348	− 7	47515144	− 1
50475148	− 4	48505346		52525152	
50475146	− 1	51515352	− 2	52525150	1
50505353	− 1	51515350	− 6	52525148	8
50505351	8	51515348	12	52525146	6
50505349	− 9	51515346	− 2	52525144	− 7
50505347	3	51515344	− 4	52525142	− 1
50505345	4	49495354		48485156	− 6
52505153	9	49495352	6	48485154	− 12
52505151	5	49495350	− 8	48485152	− 8
52505149	− 12	49495348	8	48485150	− 1
52505147	− 8	49495346	3	48485148	7
52505145	− 12	51495354		52485154	1
52505143	6	51495352	− 5	52485152	− 9
48505157	− 1	51495350	− 12	52485150	− 14
48505155	− 5	51495348	− 10	52485148	3
48505153	10	51495346	5	52485146	2
48505151	− 28	49515354		52485144	1
48505149	− 1	49515352	3	48525156	− 1
48505147	10	49515350	1	48525154	4
48505145	3	49515348	− 3	48525152	
51515155		49515346	5	48525150	− 5
51515153	7	50525352	− 1	48525148	− 6
51515151	− 8	50525350	2	48525146	− 18
51515149	3	50525348	3	51535150	1
51515147	− 3	50525346	3	51535148	4
51515145	14	50485352	− 7	51535146	− 15
51515143	1	50485350	2	51535144	− 2
49475156	1	49485153	− 4	53485154	1
49475154	− 2	49485151	2	53485152	
49475152	− 8	49485149	5	53485150	2
49475150		49485147	1	53485148	1

Table A-4 (contd)

Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$
49475148	-1	51485153	1	53485146	1
51475154	1	51485149	1	47525152	-5
51475152	3	49525153	-1	47525150	-9
51475150	4	49525151		47525148	-15
51475148	1	49525149	-1	47525146	-1
51475146	-1	49525147	2	54515154	-1
49535150	2	51505552	1	54515152	
49535148	-7	51505550	4	54515150	-37
49535146	-6	51505548	-1	54515148	-40
50545148	4	51505546	1	54515146	-1
50545146	-1	49505552		54515144	
50465154	1	49505550	-6	46495158	1
50465152	4	49505548	9	46495156	-1
51505353	-1	49505546	1	46495154	-2
51505351		50515548	1	46495152	1
51505349	-5	53505352	9	46495150	15
51505347	-3	53505350	23	46495148	15
51505345	12	53505348	8	46495146	1
49505353	-9	53505346	-2	54495152	4
49505351	-11	47505356		54495150	55
49505349	12	47505354	28	54495148	-20
49505347	13	47505352	-1	54495146	-2
50515351		47505350	13	46515152	5
50515349	7	47505348	-1	46515150	-18
50515347	-1	47505346	1	46515148	-9
50515345	1	52515352		46515146	-2
50495351	1	52515350	7	55505154	1
50495349	11	52515348	-3	55505152	-6
50495347	-5	52515346	-2	55505150	-4
53505153	-1	48495354	-1	55505148	6
53505151	-8	48495352	-15	55505146	4
53505149	-4	48495350	1	45505154	3
53505147	3	48495348	1	45505152	9
53505145	-5	52495352	1	45505150	11
53505143	2	52495350	-9	45505148	-1
47505157		52495348	4	45505146	-1
47505155	12	48515354	1	51515351	
47505153	14	48515350	1	49495349	-3
47505151	-13	48515348	-1	49515351	-1
47505149	5	51525350	1	52505351	
47505147	10	51525348	-2	52505349	5
52515153	2	51525346	-2	52505345	2
52515151	-8	49485352	-1	48505353	1
52515149	-6	51485352		48505351	-1
52515147	-3	51485350	1	48505349	2
52515145	9	51485348	2	52525151	-1
48495155	-3	49525352		52525149	-1
48495153	4	49525350	-2	53515153	1

Table A-4 (contd)

Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$	Argument	$c_{1n} \times 10^4$
48495151	6	49525348	−7	53515151	11
48495149	−2	51465152	1	53515149	
48495147	−11	52535148	1	53515147	−1
52495153		52535146	−4	47495155	−1
52495151	11	48475154	1	47495153	−1
52495149	5	48475152	4	47495149	−7
52495147	5	52475152	1	47495147	−1
52495145	−3	52475150		53495151	
48515155	1	48535150	−2	53495149	−15
48515153		48535148	−4	53495147	−1
48515151	−15	53525150	−1	47515153	−2
48515149	16	53525148	1	47515151	3
48515147	7	53525146	−2	47515149	3
51525153	−1	53525144	−2	54505153	−1
51525151	−2	47485156		54505151	
51525149	1	47485154		54505149	1
51525147	−2	47485152	5	54505147	1
51525145	1	47485150	3	46505153	−1
46505151	−10	46505350	3	56505148	
46505149	7	46505348	2	56505146	1
52505550	1	54525148	−1	44505152	8
48505552	−1	54525146	1	44505150	−14
48505550	5	54485152	−1	44505148	−3
48505548	−1	54485150	1	55505151	−1
53515350		55515152	−1	55505149	1
53515348	−1	55515150	−4	54515151	1
47495354	−1	55515148	−2	46495149	−1
47495352	3	55515146	2	54495149	−2
47495350	3	45495152	2	54495147	1
53495352	1	45495150	3	46515151	1
53495350		55495152	1	57505150	
47515350	−2	55495150	4	57505148	−1
54505352	1	55495148	−2	56515150	−1
54505350	−6	45515152	1	56495150	1
54505348	−7	45515150	−3	55505350	−1
46505354	−1	45515148	−1		
46505352	2	56505150	2		

Table A-5. Transformation corrections to higher order terms

Argument	$u_{1n} \times 10^{9}$	$v_{1n} \times 10^{-1}$	$w_{1n} \times 10^{4}$	$u_{2n} \times 10^{11}$	$v_{2n} \times 10$	$w_{2n} \times 10^{6}$
50505052	−1″	−4″	−2″	0	−310″	+16″
51505052	−1	0	+1	+53″	−34	+1
51505048	0	0	+1	0	+2	−1
51505046	−11	0	0	+15	−15	0
50515052	0	0	0	0	+5	0
50515050	+4	0	0	−2	+13	0
50515048	0	0	−1	+19	−20	+1
50505051	+3	−1	0	+2	+68	0
52505050	+8	−1	0	+12	0	0
52505048	+9	0	0	0	+3	0
51515050	+11			+9	+27	
51515048	−12			+7	−17	
51495050	−2			−9	−35	
50505250	0			0	−1	
51504850	+3			0	−28	

Argument	$u_{3n} \times 10^{9}$	$v_{3n} \times 10^{-1}$	$w_{3n} \times 10^{4}$
50505152	0	0	+1″
50505148	+1″	+1″	0
51505148	−7	0	0
49505152	+10	0	0
49505150	0	−1	0

Table A-6. [Code 0] Solar terms containing odd multiples of *D*

Serial no.	Argument	Coefficient	Serial no.	Argument	Coefficient
19	50505055	+0″004	227	53505049	+0″130
20	50505053	+0.403	228	53505047	+0.045
21	50505051	−125.154	229	53505045	+0.016
57	51505053	−0.002	230	53505043	+0.001
58	51505051	−8.466	231	52515053	+0.003
59	51505049	+18.609	232	52515051	+0.092
60	51505047	+3.215	233	52515049	+0.006
61	51505045	+0.014	234	52515047	+0.084
62	50515055	+0.002	235	52515045	+0.006
63	50515053	+0.150	236	52495051	−0.014
64	50515051	+18.023	237	52495049	−0.352
65	50515049	+0.560	238	52495047	+0.042
66	50515047	−0.066	239	52495045	−0.003
67	50515045	−0.001	240	51525051	−0.008
122	52505053	−0.004	241	51525049	−0.002

Note: This table was extracted from Table III of Ref. 3.

Table A-6 (contd)

Serial no.	Argument	Coefficent	Serial no.	Argument	Coefficent
123	52505051	− 0.″586	242	51525047	+ 0.″012
124	52505049	+ 1.750	243	51485049	+ 0.003
125	52505047	+ 1.225	244	51485047	+ 0.001
126	52505045	+ 0.059	245	50535051	− 0.001
127	52505043	+ 0.001	246	50535049	− 0.002
128	51515053	+ 0.023	247	51505251	+ 0.045
129	51515051	+ 1.267	248	51505249	+ 0.024
130	51515049	+ 0.137	249	51505247	+ 0.030
131	51515047	+ 0.233	250	51505245	+ 0.002
132	51515045	+ 0.001	251	51504853	− 0.010
133	51495053	+ 0.003	252	51504851	− 0.041
134	51495051	− 0.122	253	51504849	− 0.016
135	51495049	− 1.089	254	51504847	− 0.011
136	51495047	− 0.276	255	50515253	− 0.001
137	51495045	− 0.003	256	50515251	− 0.035
138	50525053	− 0.002	257	50515249	+ 0.013
139	50525051	− 0.039	258	50515247	+ 0.020
140	50525049	− 0.042	259	50514853	+ 0.009
141	50525047	− 0.006	261	50514851	− 0.001
142	50505253	+ 0.004	262	50514849	− 0.002
143	50505251	+ 0.255	348	54505051	− 0.003
144	50505249	+ 0.584	349	54505049	+ 0.010
145	50505247	+ 0.254	350	54505047	+ 0.002
146	50505245	+ 0.001	351	54505045	+ 0.001
226	53505051	− 0.042	352	53515051	+ 0.007
353	53515049	− 0.001	365	51515251	− 0.006
354	53515047	+ 0.003	366	51515249	+ 0.001
355	53515045	+ 0.002	367	51515247	+ 0.002
356	53495051	− 0.002	368	51514851	− 0.002
357	53495049	− 0.023	370	51514847	− 0.001
358	53495047	+ 0.007	371	51495247	− 0.001
359	52505251	+ 0.006	372	51494853	− 0.001
360	52505249	− 0.003	373	51494851	− 0.004
361	52504853	− 0.001	375	51494847	+ 0.001
362	52504851	− 0.001	376	50505451	− 0.001
363	52504849	+ 0.001	377	50505449	− 0.001
364	52504847	− 0.003			

Table A-7. [Code 5] Solar terms containing odd multiples of *D*

Serial no.	Argument	Coefficient	Serial no.	Argument	Coefficient
819	50505053	+ 0.″0023	911	50505249	+ 0.″0071
820	50505051	− 0.9781	912	50505247	− 0.0017
848	51505053	− 0.0003	958	53505051	− 0.0009
849	51505051	− 0.1093	959	53505049	+ 0.0017
850	51505049	+ 0.0118	960	53505045	− 0.0002
851	51505047	− 0.0386	961	52515053	+ 0.0002
852	51505045	− 0.0003	962	52515051	+ 0.0015
853	50515053	+ 0.0027	963	52515049	− 0.0002
854	50515051	+ 0.1494	964	52515047	− 0.0005
855	50515049	− 0.0037	965	52515045	− 0.0002
856	50515047	+ 0.0007	966	52495051	− 0.0005
900	52505051	− 0.0100	967	52495049	− 0.0028
901	52505049	+ 0.0155	968	52495047	− 0.0005
902	52505047	− 0.0088	969	52495045	+ 0.0002
903	52505045	− 0.0008	970	51505251	+ 0.0002
904	51515053	+ 0.0003	971	51505249	+ 0.0010
905	51515051	+ 0.0164	972	51505247	+ 0.0002
906	51515047	− 0.0025	973	51505245	− 0.0002
907	51495051	− 0.0014	974	51504853	− 0.0002
908	51495047	+ 0.0036	975	51504849	+ 0.0006
909	50525051	− 0.0003	976	51504847	+ 0.0004
910	50525049	+ 0.0003	977	50514851	−0.0003

Note: This table was extracted from Table III of Ref. 3.

Table A-8. [Code 0] Planetary terms containing odd multiples of *D*

Serial no.	H_{1n}	H_{2n}	Coefficient
1161	0^c03347	-0^c03626 30672	$+0''005$
1162	0.85599	+0.00002 85785	+0.011
1652	0.74913	−0.00003 86074	+0.013
1653	0.52255	−0.00005 00668	+0.013
1655	0.54535	−0.00015 72293	+0.008
1662	0.02941	+0.00002 62906	+0.017

Notes: This table was extracted from Tables II and III of Ref. 3; it is included for completeness, although it contributes no significant corrections, because of the smallness of the coefficients.
The superscript c denotes units of rev.

Table A-9. [Code 1] Solar terms containing odd multiples of *D*

Serial no.	Argument	Coefficient	Serial no.	Argument	Coefficient
397	50505051	$-$ 112''79	459	50515049	$+$ 0''32
399	50505053	− 4.01	461	50515047	+ 0.29
401	50505055	− 0.13	466	50525051	− 0.04
405	51505055	− 0.01	468	50525049	− 0.04
407	51505053	− 0.74	470	50525047	+ 0.01
409	51505051	− 13.51	476	51515053	+ 0.08
411	51505049	+ 3.59	478	51515051	+ 1.52
413	51505047	+ 5.44	480	51515049	− 0.12
415	51505045	+ 0.25	482	51515047	+ 0.36
420	52505053	− 0.10	484	51515045	+ 0.01
422	52505051	− 1.20	491	49515055	+ 0.01
424	52505049	+ 2.01	493	49515053	+ 0.38
426	52505047	+ 0.91	495	49515051	− 0.55
428	52505045	+ 0.12	497	49515049	+ 0.33
433	53505051	− 0.09	499	49515047	+ 0.04
435	53505049	+ 0.19	508	52515051	+ 0.14
437	53505047	+ 0.05	510	52515049	+ 0.02
439	53505045	+ 0.03	512	52515047	+ 0.07
453	50515055	+ 0.01	517	52495051	− 0.04
455	50515053	+ 0.53	519	52495049	− 0.37
457	50515051	+ 17.93	521	52495047	+ 0.04

Note: This table was extracted from Table III of Ref. 3.

Table A-9 (contd)

Serial no.	Argument	Coefficient	Serial no.	Argument	Coefficient
525	53515051	− 0.″01	578	49505251	+ 0.″06
562	50505249	+ 0.84	580	49505249	+ 0.04
564	50505247	+ 0.25	582	49505247	− 0.01
569	51505249	+ 0.07	584	52505249	− 0.01
571	51505247	+ 0.04	605	50515247	+ 0.02
573	51505245	− 0.02	609	50495249	− 0.06
576	49505253	+ 0.02			

Table A-10. [Code 2] Solar terms containing odd multiples of *D*

Serial no.	Argument	Coefficient	Serial no.	Argument	Coefficient
641	50505051	− 0.″725	699	50515051	+ 0.″007
643	50505053	+ 0.394	701	50515049	− 0.001
645	50505055	+ 0.012	703	50515047	+ 0.031
649	51505055	+ 0.001	710	50525047	+ 0.001
651	51505053	+ 0.068	716	51515053	− 0.007
653	51505051	+ 0.455	718	51515051	− 0.022
655	51505049	− 0.094	720	51515049	+ 0.005
657	51505047	+ 0.192	722	51515047	+ 0.012
659	51505045	+ 0.020	724	51515045	+ 0.001
664	52505053	+ 0.007	730	49515055	− 0.001
666	52505051	+ 0.054	732	49515053	− 0.006
668	52505049	− 0.018	734	49515051	+ 0.021
670	52505047	− 0.030	736	49515049	+ 0.016
672	52505045	+ 0.007	738	49515047	+ 0.004
677	53505051	+ 0.006	745	52515051	− 0.003
679	53505049	− 0.005	748	52515047	− 0.002
681	53505047	− 0.002	754	52495049	+ 0.001
683	53505045	− 0.001	759	53515051	− 0.001
695	50515055	− 0.001	767	53495049	+ 0.002
697	50515053	− 0.032			

Note: This table was extracted from Table III of Ref 3.

Table A-11. Correction terms due to modification of the value of a_1

Argument	$a_{3n} \times 10^4$	$b_{3n} \times 10^6$	$c_{3n} \times 10^4$	$d_{3n} \times 10^4$
50505053	− 5″	− 3″	+ 5″	− 5″
50505051	+1683	+1315	+ 136	+ 10
51505051	+ 114	+ 147	+ 16	− 6
51505049	− 250	− 16	− 4	+ 1
51505047	− 43	+ 52	− 7	− 3
50515053	− 2	− 4	− 1	0
50515051	− 242	− 201	− 22	0
50515049	− 8	+ 5	0	0
50515047	+ 1	− 1	0	0
52505051	+ 8	+ 13	+ 1	− 1
52505049	− 24	− 21	− 2	
52505047	− 16	+ 12	− 1	
52505045	− 1	+ 1	0	
51515051	− 17	− 22	− 2	
51515049	− 2	0	0	
51515047	− 3	+ 3		
51495051	+ 2	+ 2		
51495049	+ 15	0		
51495047	+ 4	− 5		
50525051	+ 1	0		
50525049	+ 1	0	0	
50505251	− 3	0	0	
50505249	− 8	− 10	− 1	
50505247	− 3	+ 2	0	
53505051	+ 1	+ 1	0	
53505049	− 2	− 2		
53505047	− 1	0		
52515051	− 1	− 2		
52515047	− 1	+ 1		
52495049	+ 5	+ 4		
52495047	− 1	+ 1		
51505251	− 1	0		
51504851	+ 1	0		
52495051	0	+ 1		
51505249	0	− 1		
51504849		− 1	0	0
51504847		− 1	0	0
51505053		0	+ 1	− 1
49515051		0	+ 1	0

Table A-12. Additive terms for S

Argument	Coefficient f_n
50505052	$+0^c00183\ 130$
51505050	+0.01744 527
51505048	−0.00353 251

Note: The superscript c denotes units of rev.

Table A-13. Higher order correction terms due to change in α_1

Argument	a_{4n}	b_{4n}
50505052	−0″.0001	−0″.000001
51505048	+0.0001	−0.000001

Table A-14. Aberration corrections

Coordinate argument	(a)	(b)	(c)	(d)	(e)
Longitude:					a_{5n}
50505050	+0″.703	−0″.703	0	0	+0″.703
50505052	+0.009	−0.016	−0″.007	+0″.007	+0.009
51505052	+0.001	−0.002	−0.001	0	+0.002
51505050	+0.038	−0.038	0	0	+0.038
51505048	+0.006	−0.024	−0.018	+0.018	+0.006
50515048	+0.001	−0.001	0		+0.001
52505050	+0.002	−0.003	−0.001		+0.003
52505048	−0.001	0	−0.001		0
51515048	0	−0.001	−0.001		+0.001
50505250	−0.003	+0.003	0		+0.003
Sine parallax:					b_{5n}
51505050	−0″.0006	+0″.0003	−0″.0003		−0″.0003
51505048	+0.0001	−0.0002	−0.0001		+0.0002
50505052	−0.0002	+0.0002	0		−0.0002
Latitude:					c_{5n}
50505150	+0″.063	−0″.063	0		+0″.063
50505152	+0.001	−0.001	0		+0.001
50505148	+0.002	−0.002	0		+0.002
51505150	+0.005	−0.005	0		+0.005
49505150	−0.002	+0.002	0		−0.002
49505152	+0.001	−0.002	−0″.001		+0.002
52505150	+0.001	0	+0.001		0

Notes: This table was derived from data of Ref. 12.
Definitions of the columns (a) through (e) are given in Section III-C.