NASA TECHNICAL NOTE

NASA TN D-4354

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NASA TN D-4354

AN UNCERTAINTY ANALYSIS FOR SATELLITE CALORIMETRIC MEASUREMENTS

by John P. Millard

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Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1968

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SUMMARY

The use of an uncertainty analysis for analyzing current data and for designing future satellite calorimetric experiments is described. Calorimetric experiments considered are those for measuring the radiative properties of solar absorptance and infrared emittance of test surfaces, and those for measuring the emissive and reflective properties of the sun and planets. The paper describes the uncertainty-analysis technique, lists all pertinent equations for each measurement, presents several illustrative examples, and includes a section on design guides for future experiments.

INTRODUCTION

Several calorimetric flight experiments have contributed data on the thermal-radiation properties of materials and planetary thermal environments (refs. 1-10). These data have added to knowledge of the long-term ultraviolet stability of thermal-control surfaces, the magnitude of infrared energy emitted by the earth, and the albedo, or portion of direct sunlight reflected by the earth. Additional measurements are required both to complement existing ones and to provide data on other planets, the sun, and new materials in the various radiation environments of space.

A valuable tool for analyzing current data and for designing future experiments is an uncertainty analysis. It can identify major sources of uncertainties, the effect of each on overall measurement, and values of experiment and orbit design variables that will minimize those effects. The use of the analysis for satellite calorimetric measurements is demonstrated herein; examples are included. Pertinent equations and design guides for the following measurements are tabulated:

(1) Solar absorptance of a test surface
(2) Infrared emittance of a test surface
(3) Solar absorptance to infrared emittance ratio of a test surface
(4) Solar constant
(5) Planetary albedo
(6) Planetary infrared emission

The content of the report was derived from a study of uncertainty associated with the Ames OSO-III Thermal Control Coatings Experiment.

NOMENCLATURE

a	albedo
A	area of the sensor, m^2
c	specific heat of the coating, J/gm $^{\circ}K$
F_a	view factor for albedo, defined by $H_a = F_a aS$
F_P	view factor for planetary infrared radiation, defined by $H_P = F_P P$
F_S	view factor for solar radiation defined by $H_S = F_S S$
H_a	energy incident on the coating due to albedo, W/m^2
H_P	energy incident on the coating due to planetary infrared radiation, W/m^2
H_S	energy incident on the coating due to the sun, W/m^2
K	radiation heat loss coefficient, $W/^{\circ}K^4$
P	planetary infrared radiation, W/m^2
Q_L	net heat loss due to imperfect thermal isolation of sensor on back side, W
S	solar constant, 1360 W/m^2
T	temperature of the coating, $^{\circ}K$
T_b	temperature of the base plate, $^{\circ}K$
V	function of n independent variables
w	mass of sensor disk, g
x_i	independent variable
δx_i	$\frac{\partial V}{\partial x_i} \Delta x_i$
Y	conduction heat loss coefficient, $W/^{\circ}K$
α_P	planetary-radiation absorptance of sensor surface
α_S	solar-radiation absorptance of sensor surface

α_a albedo-radiation absorptance of sensor surface

δ $\alpha_S - \alpha_a$

ϵ emittance of sensor surface

θ time, sec

ν $\epsilon - \alpha_P$

σ Stefan-Boltzmann constant, 5.67×10^{-8} W/m^2 $^{\circ}$K^4

CALORIMETRIC TECHNIQUE

A brief description of the calorimetric technique is presented to establish basic equations, terminology, and nomenclature.

The calorimeter considered is a thin wafer of material, called a sensor, which is thermally isolated on one side and which views the environment of space on the other. See figure 1 for a typical sensor. The temperature of the sensor is measured in orbit. From that measurement, one can deduce the amount of radiant flux emitted or reflected by a planet or the sun, the absorptance of the sensor for that radiation, or the infrared emittance of the sensor. The technique of deducing one of these unknowns utilizes an energy balance. An unknown is solved in terms of other variables, such as sensor temperature, heat capacity, surface radiative properties, solar constant, and view factors. Two or more sensors may be used simultaneously if more than one unknown exists. The simultaneous technique exploits differences in optical properties of sensors. Thus two sensors, one sensitive to long-wavelength and the other sensitive to short-wavelength radiation, could be used to measure the radiant flux emitted by a planet and also that reflected by the planet.

The basic energy equation from which an unknown may be determined is:

$$\underset{\text{(Solar)}}{F_S S A \alpha_S} + \underset{\text{(Albedo)}}{F_a a S A \alpha_a} + \underset{\substack{\text{(Planetary}\\\text{emission)}}}{F_P P A \alpha_P} = \underset{\substack{\text{(Sensor}\\\text{emission)}}}{A \epsilon \sigma T^4} + \underset{\substack{\text{(Heat}\\\text{storage)}}}{wc \frac{dT}{d\theta}} + \underset{\substack{\text{(Heat}\\\text{leak)}}}{Q_L} \qquad (1)$$

The terms on the left are radiation inputs to the sensor. These are defined by the flux at the source, the view factors from source to sensor, and the absorptance of the sensor for that radiation. The terms on the right represent radiant emittance of the sensor, changes in heat storage of the sensor, and heat lost from the back because of imperfect thermal isolation. See Nomenclature for definition of symbols.

Certain modifications of equation (1) are desirable. First, albedo absorptance α_a can usually be related to solar absorptance α_S by the relation:

$$\alpha_S - \alpha_a = \delta \tag{2}$$

where δ represents a small deviation. These absorptances may be nearly equal because of close spectral match of direct and reflected sunlight. Second, the absorptance of a sensor for planetary infrared radiation, α_P, can be related to infrared emittance, ϵ, of the sensor by

$$\epsilon - \alpha_P = \nu \tag{3}$$

where ν represents a small deviation. The equivalence of these values can result from either similar temperatures of sensor and planet, or flat sensor spectral characteristics. Third, the heat-leak term Q_L can be expressed as

$$Q_L = K(T^4 - T_b{}^4) + Y(T - T_b) \tag{4}$$

where K and Y are proportionality factors, T is temperature of sensor, and T_b is a characteristic temperature of the structure behind the sensor. When these three substitutions are made in equation (1), the energy equation becomes

$$F_S SA\alpha_S + F_a aSA(\alpha_S - \delta) + F_P PA(\epsilon - \nu) = A\epsilon\sigma T^4 + wc\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b) \tag{5}$$

Equation (5) is the form of the energy equation used in this report. Solutions of it for various variables selected to be the dependent variable are listed in appendix A. Simultaneous solutions of two such equations are listed in appendix B.

UNCERTAINTY[1] ANALYSIS

The calorimetric technique defines an unknown in terms of other variables. The effect of uncertainties of these variables on accuracy of measurement of the unknown is discussed in this section.

A general method of analyzing uncertainty will first be described; it is that of Kline and McClintock (ref. 11). Consider V to be defined by the variables $x_1, x_2, \ldots, x_i, \ldots, x_n$. An uncertainty Δx_i in the value of x_i, will produce an uncertainty $(\partial V/\partial x_i)\Delta x_i$ in the value of V, to a first-order approximation. The overall effect of uncertainty in more than one variable is not necessarily the sum of the individual effects. If the uncertainties are independent and equally probable, then the uncertainty in V can be described as:

---

[1]Uncertainty is defined as a possible value within which an error of measurement is estimated to fall with a given degree of certainty.

$$\Delta V = \left[\left(\frac{\partial V}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial V}{\partial x_2}\Delta x_2\right)^2 + \ldots + \left(\frac{\partial V}{\partial x_i}\Delta x_i\right)^2 + \ldots + \left(\frac{\partial V}{\partial x_n}\Delta x_n\right)^2\right]^{1/2} \tag{6}$$

where ΔV has the same assurance associated with it as that used for selecting Δx_i.

Overall uncertainty of calorimetric results may be computed from equation (6). The computation requires two inputs: (1) the partial derivatives of the unknown with respect to each variable, and (2) the uncertainty of each variable. Appendixes C and D are tabulations of the partial derivatives of α_S, α_S/ϵ, ϵ, S, a, and P with respect to each of their variables.[2] Appendix C pertains to single energy-equation solutions, and appendix D to simultaneous solutions. The uncertainty of each variable must be known or estimated.

Selecting Values of Experiment and Orbit Design Variables

At design, one must specify the uncertainty with which each variable is to be measured, plus the magnitudes of the variables relating to orbit, orientation, and sensor characteristics. The magnitudes of the variables are important because the values of the partial derivatives, used to compute overall uncertainty, are functions of them.

The uncertainty-analysis equation, equation (6), and the partial derivatives in appendix C provide means for judiciously selecting the foregoing values. A closed solution, however, does not exist; an iterative procedure must be used. The procedure is as follows:

(1) Examine the partial derivatives; select conditions that appear to minimize each.

(2) Select reasonable values of uncertainty for each variable.

(3) Calculate and compare overall uncertainty for each set of conditions; select those conditions yielding least uncertainty for refinement.

(4) Compare individual-effect terms, $(\partial V/\partial x_i)\Delta x_i$, to locate major sources of uncertainty.

(5) Iterate as required.

All values must be consistent with any constraints, such as orbit of a specific spacecraft or accuracy of its telemetry system.

[2]All variables employed herein are independent, except the view factors F_P and F_a which are mutually dependent. Therefore to use equation (6), replace the terms $\left(\frac{\partial V}{\partial F_P}\Delta F_P\right)^2 + \left(\frac{\partial V}{\partial F_a}\Delta F_a\right)^2$ with $\left(\frac{\partial V}{\partial F_P}\Delta F_P + \frac{\partial V}{\partial F_a}\Delta F_a\right)^2$.

Examples

Three examples will illustrate some of the uses of an uncertainty analysis. The first two relate to the Ames experiment on OSO-III; the last is hypothetical.

The Ames experiment is composed of several flat-disc sensors, figure 1, each mounted in a special cup to minimize heat leak. In orbit, the sensors scan both the earth and the sun once very 2 seconds because the spacecraft is spin-stabilized; the spin axis is maintained perpendicular to the satellite sun line. The orbit of the spacecraft is nearly circular at 550 km altitude and inclined 33° relative to the equator.

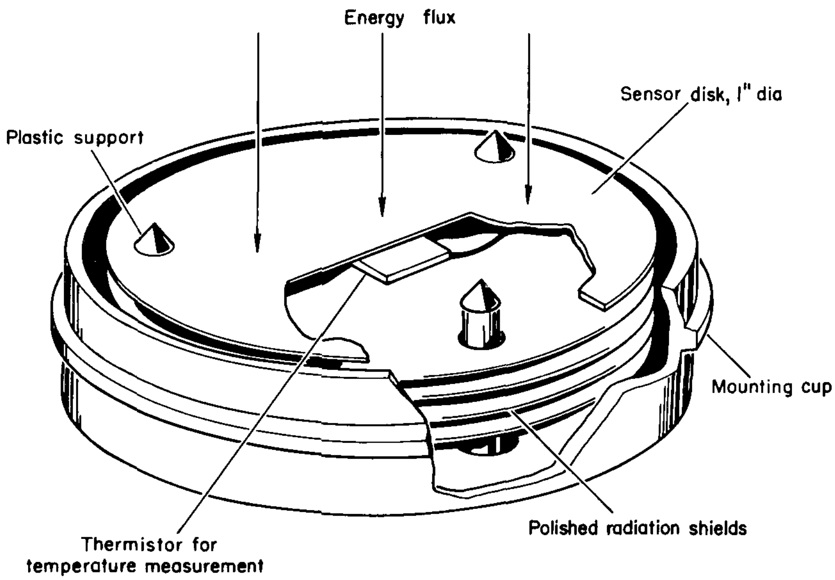


Figure 1.- Design of sensors for OSO-III experiment.

The first example illustrates the selection of best location in the OSO-III orbit for measuring α_S/ϵ of a white coating. The magnitude of the variables and the uncertainty associated with each have been selected (table I). Figure 2 is a theoretical time-temperature profile of the sensor for a typical orbit. The reasons for the shape of the profile are as follows: (1) the satellite entered sunlight at time zero, hence the rapid rise in temperature, (2) the albedo view factor from satellite to earth reaches a maximum when the satellite is at the subsolar point, hence the maximum temperature at midday, and (3) the satellite entered darkness, hence the rapid fall in temperature. Figure 3 is a plot of uncertainty of measurement as a function of time in orbit. Both individual effects $(\partial V/\partial x_i)\Delta x_i$, represented as δx_i, and overall uncertainty, computed by equation (6), are plotted. Waviness of some of the curves is due to the continuous change of view factors and temperature of the sensor. It can be seen that the major individual effects result from uncertainties in the variables albedo, earth radiation, heat capacity, and rate of change of temperature with time. The effect of uncertainty in albedo is greatest near the subsolar point because albedo input to a sensor is greatest there. Heat capacity and temperature rate effects are maximum at

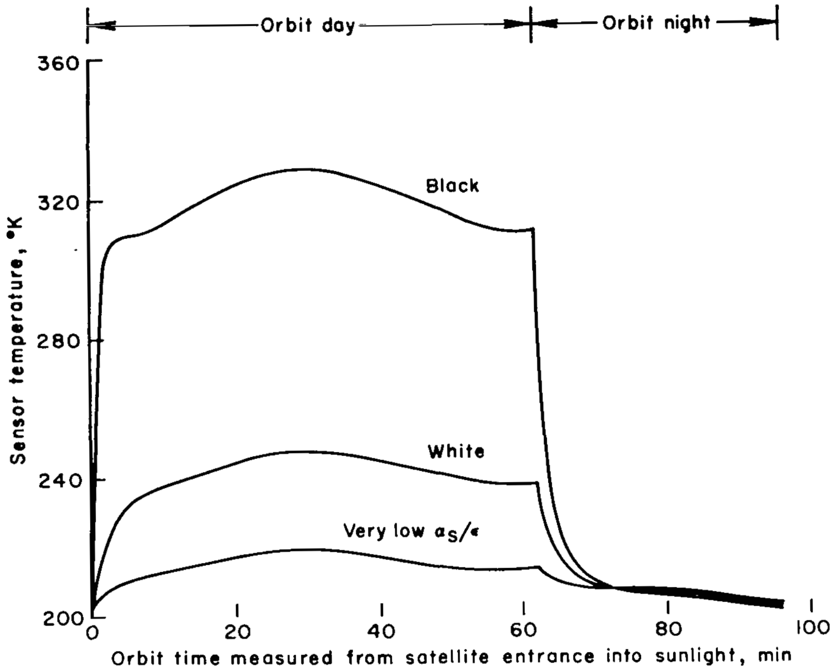


Figure 2.- Temperature of illustrative-example sensors vs. orbit time.

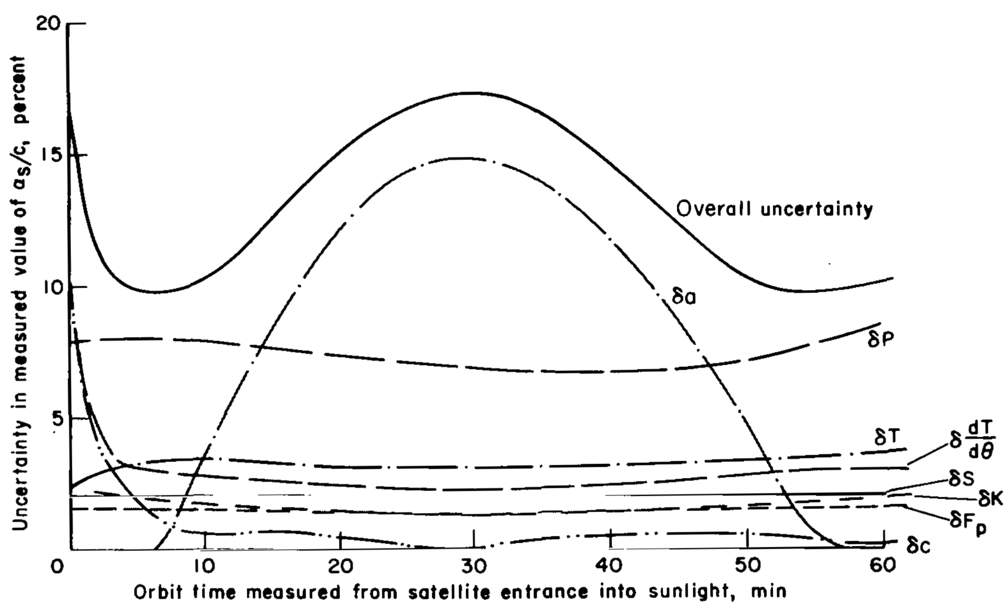
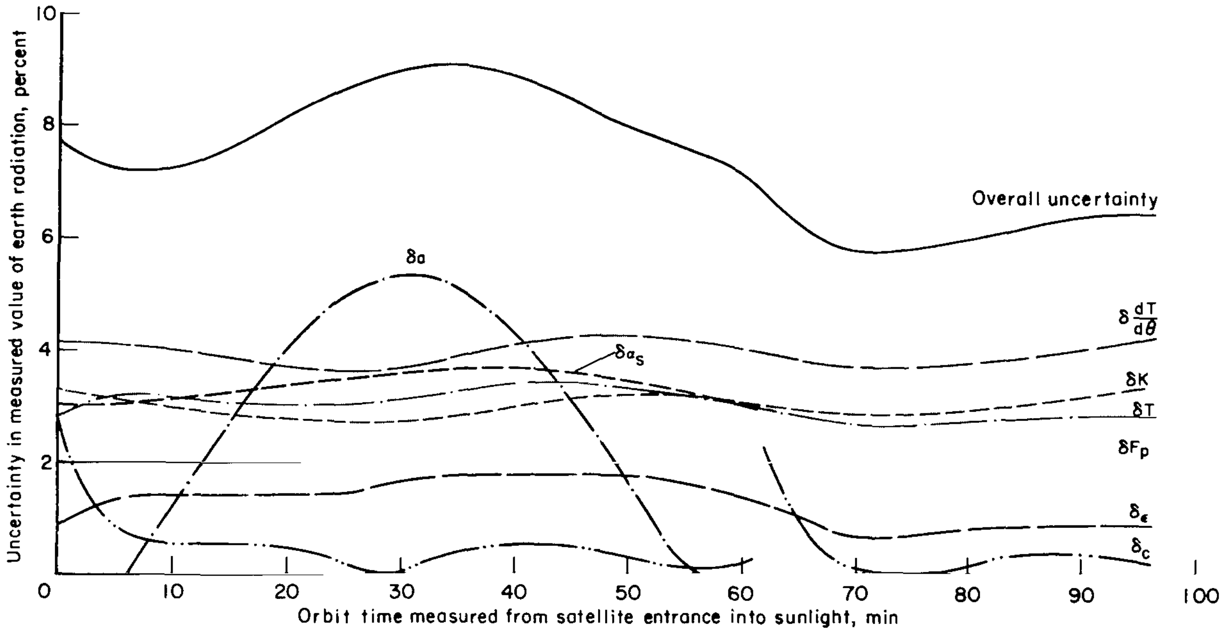


Figure 3.- Uncertainty in measured α_S/ϵ of a white coating vs. orbit time.

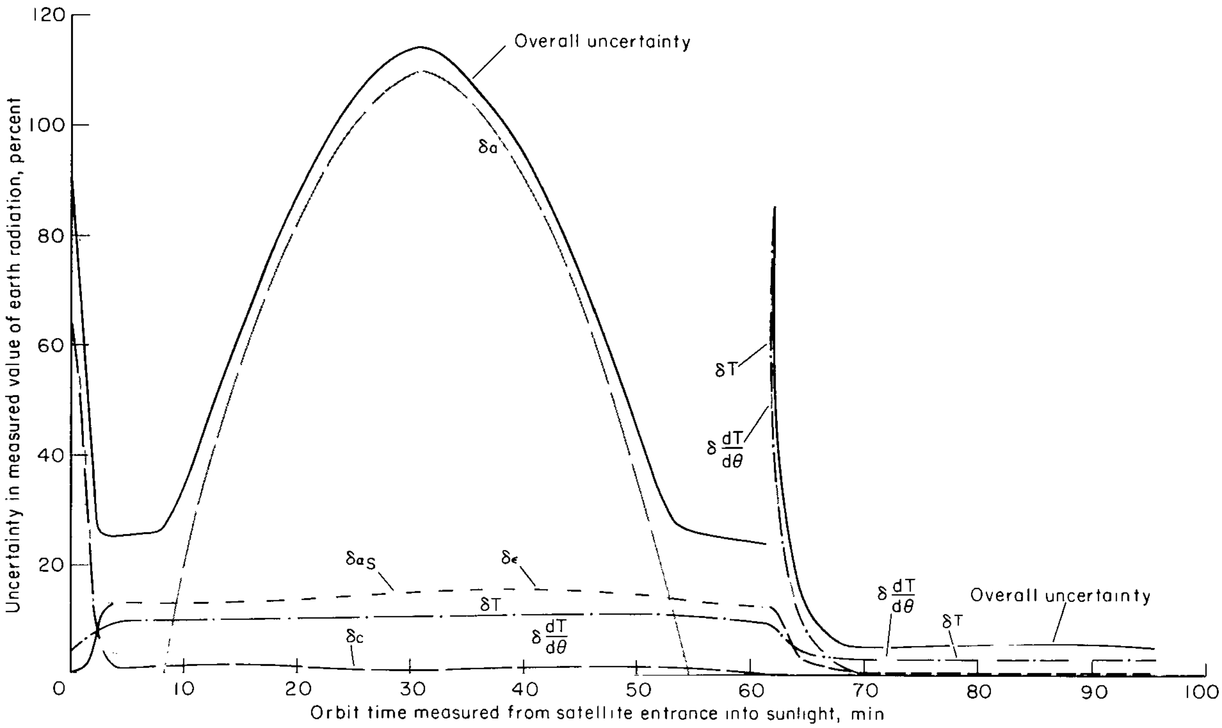
satellite entrance to sunlight because the temperature of the sensor changes so rapidly. As a result, overall uncertainty is a minimum at about 6 minutes after the satellite enters sunlight, and just before it enters night. These are the best locations for making the measurements.



(a) Very low α_S/ϵ coating vs. orbit time.

Figure 4.- Measured uncertainty in earth radiation.

The second example illustrates the selection of a very low α_S/ϵ coating in preference to an optically black one for measuring earth radiation. The magnitudes of the variables and the uncertainty to be associated with each are included in table I. The value of albedo assigned to the example is 0.30, and uncertainty of the value, ±0.20. This value of uncertainty may be high if a second sensor were employed to measure albedo. The uncertainty selected, however, will emphasize the advantage gained from using the low α_S/ϵ coating. Theoretical time-temperature profiles of the two sensors are plotted in figure 2. Uncertainty of measurement made with the very low α_S/ϵ sensor is plotted in figure 4(a); uncertainty associated with the optically black one is plotted in figure 4(b). The major difference between the two is the large effect of uncertainty of albedo on the optically-black sensor. The effect of nearly all uncertainties, however, seems to be larger. Overall uncertainty associated with the black sensor is more than an order of magnitude higher than that for the low α_S/ϵ sensor at the subsolar point.



(b) Optically black coating vs. orbit time.

Figure 4.- Concluded.

The third example illustrates the procedure for selecting magnitudes of variables for measuring the solar constant. First, an examination of the equations in appendix C (pp. 23-25) shows that the magnitudes of the partial derivatives will be minimized if (1) the sensor is sun-oriented, that is, $F_S = 1$, $F_P = 0$, $F_a = 0$, (2) α_S is large, (3) thermal mass is small, and (4) heat leak is small. What is not obvious is the best magnitude for ϵ. To determine that magnitude, values of magnitude and uncertainty were assigned to all other variables. Uncertainty values were assigned to ϵ, and then individual effects and overall uncertainty were computed as a function of ϵ. The magnitudes of the variables are tabulated in table 1. The resulting uncertainty as a function of ϵ is plotted in figure 5. It is seen that minimum overall uncertainty is obtained when ϵ approaches unity, hence ϵ should be assigned a value near unity. It is interesting that the effect of heat leak is nearly negligible. Note, however, that heat leak effects for other measurements are not usually negligible.

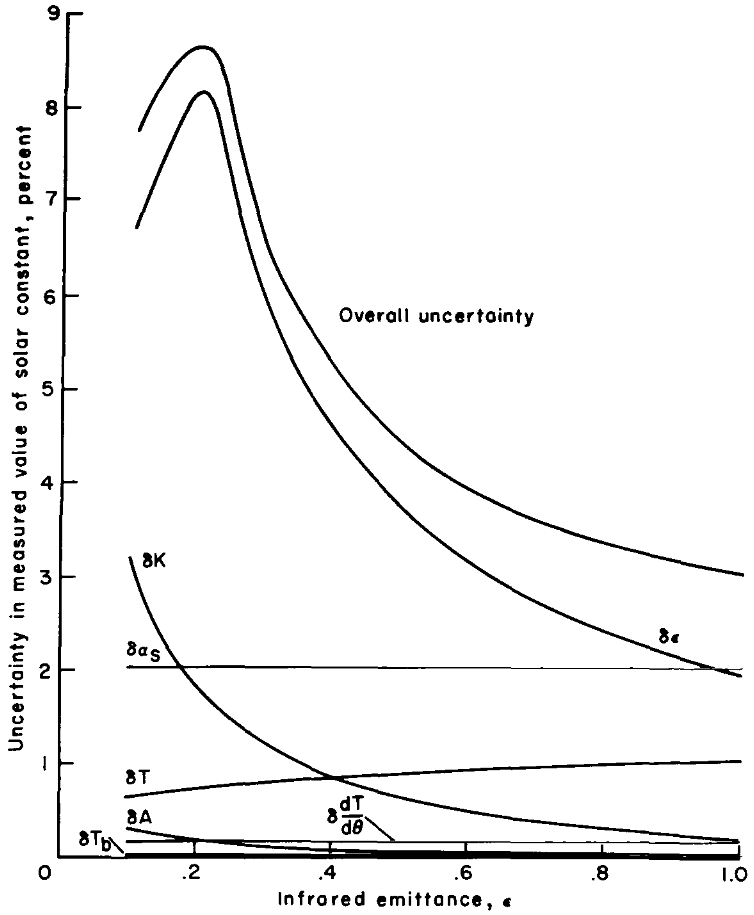


Figure 5.- Uncertainty in measured value of solar constant vs. ϵ of sensor.

Design Guides

Several important factors that will guide both the design of experiments and the selection of conditions at which to reduce data were obtained from a general analysis of the energy-equation partial derivatives. Those factors are as follows:

I. Material Optical Properties: Solar Absorptance and Solar Absorptance to Infrared Emittance Ratio

A. Under steady-state and negligible heat-leak conditions, the solution for α_s/ϵ tends to become independent of knowledge of ϵ. Determination of α_s, however, is dependent upon knowledge of ϵ.

B. In general, the solution for α_s and α_s/ϵ of low ϵ coatings is critically dependent upon accurate knowledge of ϵ and heat leak.

C. Temperature sensitivity, $\partial T/\partial \alpha_S$ or $\partial T/\partial(\alpha_S/\epsilon)$ decreases with increasing temperature; therefore, this effect should be fully evaluated.

D. Effects of uncertainty in albedo input can be minimized by making measurements near the day-night terminator, where albedo input to a sensor is nearly zero.

II. Material Optical Property: Infrared Emittance

A. The quantity $\sigma T^4 - F_P P$, which represents net infrared heat exchange must be maximized; when it is zero, ϵ cannot be determined.

B. To avoid confusion between infrared emittance and absorptance, sensor should be oriented so as not to view a planet.

III. Solar Constant and Albedo

A. Sensor should have a high value of α_S.

B. The value of ϵ for the sensor should satisfy two conflicting requirements; a high value to achieve operation at cool temperatures where both dependence on ϵ and heat leak from back side are small, and a low value to minimize planetary infrared input.

C. Two adjacent sensors with the same field of view <u>cannot</u> distinguish between direct and albedo sunlight.

IV. Planetary Infrared Radiation

A. Sensor should have a high value of ϵ.

B. Sensor should have a low value of α_S for both operation at cool temperature where temperature sensitivity, $\partial T/\partial P$, is greatest, and for minimizing effects of uncertainties in solar and albedo terms.

C. Measurements are most accurate on the dark side of an orbit where uncertainties in solar and albedo inputs can be eliminated.

CONCLUDING REMARKS

An uncertainty analysis was shown to be valuable for the design of satellite calorimetric instruments, and for selecting most appropriate orbit conditions for making measurements. Mechanics of the analysis were described and examples presented to demonstrate its use. Equations tabulated in the appendix plus design guides listed in the text should aid design of future experiments.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, Sept. 27, 1967
124-09-05-03-00-21

APPENDIX A

SINGLE ENERGY-EQUATION SOLUTIONS

Solar absorptance, α_S

$$\alpha_S = \frac{1}{S(F_S + F_a a)} \left[\epsilon(\sigma T^4 - F_P P) + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P + \delta F_a a S \right]$$

Solar absorptance to infrared emittance ratio, α_S/ϵ

$$\frac{\alpha_S}{\epsilon} = \frac{1}{S(F_S + F_a a)} \left[\sigma T^4 - F_P P + \frac{wc}{\epsilon A}\frac{dT}{d\theta} + \frac{K}{\epsilon A}(T^4 - T_b{}^4) + \frac{Y}{\epsilon A}(T - T_b) + \frac{\nu}{\epsilon} F_P P + \frac{\delta}{\epsilon} F_a a S \right]$$

Infrared emittance, ϵ

$$\epsilon = \frac{1}{\sigma T^4 - F_P P} \left[S\alpha_S(F_S + F_a a) - \frac{wc}{A}\frac{dT}{d\theta} - \frac{K}{A}(T^4 - T_b{}^4) - \frac{Y}{A}(T - T_b) - \nu F_P P - \delta F_a a S \right]$$

Solar constant, S

$$S = \frac{1}{\alpha_S(F_S + F_a a) - \delta F_a a} \left[\epsilon(\sigma T^4 - F_P P) + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P \right]$$

Albedo, a

$$a = \frac{1}{F_a S(\alpha_S - \delta)} \left[\epsilon(\sigma T^4 - F_P P) - \alpha_S F_S S + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P \right]$$

Planetary infrared radiation, P

$$P = \frac{1}{F_P(\epsilon - \nu)} \left[\epsilon\sigma T^4 - (F_S + F_a a)S\alpha_S + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \delta F_a a S \right]$$

APPENDIX B

SIMULTANEOUS ENERGY EQUATION SOLUTIONS[1]

$\underline{\alpha_{S_1}}$ $\underline{F_aA \text{ eliminated}}$

$$\alpha_{S_1} = \frac{\alpha_{S_2} - \delta_2}{\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \nu_2 F_P P - \delta_2 F_S S}$$

$$\left\{\epsilon_1\sigma T_1{}^4 - \epsilon_1 F_P P + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) + \nu_1 F_P P\right.$$

$$+ \frac{\delta_1}{\alpha_{S_2} - \delta_2}\left[\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P - \alpha_{S_2} F_S S + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \nu_2 F_P P\right]\right\}$$

$\underline{\alpha_{S_1}}$ $\underline{F_PP \text{ eliminated}}$

$$\alpha_{S_1} = \frac{1}{(F_S + F_a a)S}\left\{\epsilon_1\sigma T_1{}^4 + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) + \delta_1 F_a a S\right.$$

$$+ \frac{\nu_1 - \epsilon_1}{\epsilon_2 - \nu_2}\left[\epsilon_2\sigma T_2{}^4 - (F_S + F_a a)S\alpha_{S_2} + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \delta_2 F_a a S\right]\right\}$$

[1]Subscripts 1 and 2 denote sensors one and two.

$\left.\frac{\alpha_S}{\epsilon}\right|_1$ $F_a a$ eliminated

$$\left.\frac{\alpha_S}{\epsilon}\right|_1 = \frac{\alpha_{S_2} - \delta_2}{\epsilon_1\left[\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \nu_2 F_P P - \delta_2 F_S S\right]}$$

$$\left\{\epsilon_1\sigma T_1^4 - \epsilon_1 F_P P + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) + \nu_1 F_P P\right.$$

$$+ \frac{\delta_1}{\alpha_{S_2} - \delta_2}\left[\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P - \alpha_{S_2} F_S S + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \nu_2 F_P P\right]\right\}$$

$\left.\frac{\alpha_S}{\epsilon}\right|_1$ $F_P P$ eliminated

$$\left.\frac{\alpha_S}{\epsilon}\right|_1 = \frac{1}{(F_S + F_a a)\epsilon_1 S}\left\{\epsilon_1\sigma T_1^4 + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) + \delta_1 F_a aS\right.$$

$$+ \frac{\nu_1 - \epsilon_1}{\epsilon_2 - \nu_2}\left[\epsilon_2\sigma T_2{}^4 - (F_S + F_a a)S\alpha_{S_2} + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \delta_2 F_a aS\right]\right\}$$

ϵ_1 $F_a a$ eliminated

$$\epsilon_1 = \frac{1}{\sigma T_1{}^4 - F_P P}\left\{\alpha_{S_1} F_S S - \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 - \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) - \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) - \nu_1 F_P P\right.$$

$$+ \frac{\alpha_{S_1} - \delta_1}{\alpha_{S_2} - \delta_2}\left[\epsilon_2(\sigma T_2{}^4 - F_P P) - \alpha_{S_2} F_S S + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \nu_2 F_P P\right]\right\}$$

ϵ_1 F_pP eliminated

$$\epsilon_1 = \frac{\epsilon_2 - \nu_2}{(\epsilon_2 - \nu_2)\sigma T_1^4 - \left[\epsilon_2\sigma T_2^4 - (F_S + F_a a)S\alpha_{S_2} + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \delta_2 F_a aS\right]}$$

$$\left\{(F_S + F_a a)S\alpha_{S_1} - \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 - \left.\frac{K}{A}\right|_1 (T_1^4 - T_{b_1}^4) - \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) - \delta_1 F_a aS - \frac{\nu_1}{\epsilon_2 - \nu_2}\left[\epsilon_2\sigma T_2^4\right.\right.$$

$$\left.\left. - (F_S + F_a a)S\alpha_{S_2} + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \delta_2 F_a aS\right]\right\}$$

Solar constant F_pP eliminated

$$S = \frac{1}{\left(\left.\frac{\alpha_S}{\epsilon - \nu}\right|_1 - \left.\frac{\alpha_S}{\epsilon - \nu}\right|_2\right)(F_S + F_a a) - \frac{\delta_1 F_a a}{\epsilon_1 - \nu_1} + \frac{\delta_2 F_a a}{\epsilon_2 - \nu_2}} \left\{\left(\frac{1}{\epsilon_1 - \nu_1}\right)\left[\epsilon_1\sigma T_1^4 + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1\right.\right.$$

$$\left. + \left.\frac{K}{A}\right|_1 (T_1^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1})\right] - \left(\frac{1}{\epsilon_2 - \nu_2}\right)\left[\epsilon_2\sigma T_2^4 + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2\right.$$

$$\left.\left. + \left.\frac{K}{A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2})\right]\right\}$$

Albedo F_pP eliminated

$$a = \frac{1}{F_a S\left(\left.\frac{\alpha_S - \delta}{\epsilon}\right|_1 + \left.\frac{\epsilon - \nu}{\epsilon}\right|_1 \left.\frac{\delta - \alpha_S}{\epsilon - \nu}\right|_2\right)} \left\{\sigma T_1^4 - \left.\frac{\alpha_S}{\epsilon}\right|_1 F_S S + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1^4 - T_{b_1}^4)\right.$$

$$+ \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_b) + \left(\frac{\epsilon_2}{\epsilon_2 - \nu_2}\right)\left(\left.\frac{\nu}{\epsilon}\right|_1 - 1\right)\left[\sigma T_2^4 - \left.\frac{\alpha_S}{\epsilon}\right|_2 F_S S + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2\right.$$

$$\left.\left. + \left.\frac{K}{\epsilon A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]\right\}$$

Planetary infrared radiation $\quad F_a a$ eliminated

$$P = \frac{-1}{F_P\left(\left.\frac{\alpha_S - \delta}{\epsilon}\right|_1 - \left.\frac{\nu}{\epsilon}\right|_2 \left.\frac{\alpha_S - \delta}{\epsilon}\right|_1 - \left.\frac{\alpha_S - \delta}{\epsilon}\right|_2 \left.\frac{\epsilon - \nu}{\epsilon}\right|_1\right)} \left\{\left(\left.\frac{\alpha_S}{\epsilon}\right|_2 - \left.\frac{\delta}{\epsilon}\right|_2\right)\left[\sigma T_1{}^4 - \left.\frac{\alpha_S}{\epsilon}\right|_1 F_S S\right.\right.$$

$$\left. + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})\right] - \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\delta}{\epsilon}\right|_1\right)\left[\sigma T_2{}^4 - \left.\frac{\alpha_S}{\epsilon}\right|_2 F_S S\right.$$

$$\left.\left. + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]\right\}$$

APPENDIX C

PARTIAL DERIVATIVES OF SINGLE ENERGY EQUATION SOLUTIONS

α_S - single

$$\alpha_S = \frac{1}{S(F_S + F_a a)} \left[\epsilon(\sigma T^4 - F_P P) + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P + \delta F_a a S \right]$$

$$\frac{\partial \alpha_S}{\partial \epsilon} = \frac{\sigma T^4 - F_P P}{S(F_S + F_a a)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial \alpha_S}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{wc}{A}\frac{\partial}{dT}\frac{dT}{d\theta} + \frac{4KT^3}{A} + \frac{Y}{A}}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial \theta} = \frac{\frac{wc}{A}\frac{\partial}{\partial \theta}\frac{dT}{d\theta}}{S(F_S + F_a a)}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial \alpha_S}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{4KT^3}{A} + \frac{Y}{A}}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial \frac{dT}{d\theta}} = \frac{\frac{wc}{A}}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial F_S} = \frac{-\alpha_S}{F_S + F_a a}$$

$$\frac{\partial \alpha_S}{\partial F_a} = \frac{-a(\alpha_S - \delta)}{F_S + F_a a}$$

$$\frac{\partial \alpha_S}{\partial F_P} = \frac{-\epsilon P\left(1 - \frac{\nu}{\epsilon}\right)}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial a} = \frac{-F_a(\alpha_S - \delta)}{F_S + F_a a}$$

$$\frac{\partial \alpha_S}{\partial S} = \frac{-\alpha_S(F_S + F_a a) + \delta F_a a}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial P} = \frac{-F_P \epsilon \left(1 - \frac{\nu}{\epsilon}\right)}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial w} = \frac{\frac{c}{A} \frac{dT}{d\theta}}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial c} = \frac{\frac{w}{A} \frac{dT}{d\theta}}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial A} = \frac{-\left[wc \frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{SA^2(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial T_b} = \frac{-(4KT_b{}^3 + Y)}{SA(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial K} = \frac{T^4 - T_b{}^4}{SA(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial Y} = \frac{T - T_b}{SA(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial \nu} = \frac{F_P P}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_S}{\partial \delta} = \frac{F_a a}{F_S + F_a a}$$

α_S/ϵ - single

$$\frac{\alpha_S}{\epsilon} = \frac{1}{S(F_S + F_a a)} \left[\sigma T^4 - F_P P + \frac{wc}{\epsilon A} \frac{dT}{d\theta} + \frac{K}{\epsilon A} (T^4 - T_b{}^4) + \frac{Y}{\epsilon A} (T - T_b) + \frac{\nu}{\epsilon} F_P P + \frac{\delta}{\epsilon} F_a a S\right]$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial \epsilon} = \frac{-\left[\frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P + \delta F_a a S\right]}{S\epsilon^2(F_S + F_a a)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial T} = \frac{4\sigma T^3 + \frac{wc}{\epsilon A}\frac{\partial}{\partial T}\frac{dT}{d\theta} + \frac{4KT^3}{\epsilon A} + \frac{Y}{\epsilon A}}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial \theta} = \frac{\frac{wc}{\epsilon A}\frac{\partial}{\partial \theta}\frac{dT}{d\theta}}{S(F_S + F_a a)}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial T} = \frac{4\sigma T^3 + \frac{4KT^3}{\epsilon A} + \frac{Y}{\epsilon A}}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial \frac{dT}{d\theta}} = \frac{\frac{wc}{\epsilon A}}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial F_S} = \frac{-\frac{\alpha_S}{\epsilon}}{F_S + F_a a}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial F_a} = \frac{-a\left(\frac{\alpha_S}{\epsilon} - \frac{\delta}{\epsilon}\right)}{F_S + F_a a}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial F_P} = \frac{-P\left(1 - \frac{\nu}{\epsilon}\right)}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial a} = \frac{-F_a\left(\frac{\alpha_S}{\epsilon} - \frac{\delta}{\epsilon}\right)}{F_S + F_a a}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial S} = \frac{-\frac{\alpha_S}{\epsilon}(F_S + F_a a) + \frac{\delta}{\epsilon} F_a a}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial P} = \frac{-F_P\left(1 - \frac{\nu}{\epsilon}\right)}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial w} = \frac{\frac{c}{\epsilon A}\frac{dT}{d\theta}}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial c} = \frac{\frac{w}{\epsilon A}\frac{dT}{d\theta}}{S(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial A} = \frac{-\left[wc\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{S\epsilon A^2(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial T_b} = \frac{-(4KT_b{}^3 + Y)}{S\epsilon A(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial K} = \frac{T^4 - T_b{}^4}{S\epsilon A(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial Y} = \frac{T - T_b}{S\epsilon A(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial \nu} = \frac{F_P P}{S\epsilon(F_S + F_a a)}$$

$$\frac{\partial \frac{\alpha_S}{\epsilon}}{\partial \delta} = \frac{F_a a}{\epsilon(F_S + F_a a)}$$

<u>ϵ - single</u>

$$\epsilon = \frac{1}{\sigma T^4 - F_P P}\left[S\alpha_S(F_S + F_a a) - \frac{wc}{A}\frac{dT}{d\theta} - \frac{K}{A}(T^4 - T_b{}^4) - \frac{Y}{A}(T - T_b) - \nu F_P P - \delta F_a a S\right]$$

$$\frac{\partial \epsilon}{\partial \alpha_S} = \frac{S(F_S + F_a a)}{\sigma T^4 - F_P P}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial \epsilon}{\partial T} = \frac{-\left[4\epsilon\sigma T^3 + \frac{wc}{A}\frac{\partial}{\partial T}\left(\frac{dT}{d\theta}\right) + \frac{4KT^3}{A} + \frac{Y}{A}\right]}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial \theta} = \frac{-\frac{wc}{A}\frac{\partial}{\partial \theta}\left(\frac{dT}{d\theta}\right)}{\sigma T^4 - F_P P}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial \epsilon}{\partial T} = \frac{-\left(4\epsilon\sigma T^3 + \frac{4KT^3}{A} + \frac{Y}{A}\right)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial \frac{dT}{d\theta}} = \frac{-\frac{wc}{A}}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial F_S} = \frac{S\alpha_S}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial F_a} = \frac{aS(\alpha_S - \delta)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial F_P} = \frac{P(\epsilon - \nu)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial a} = \frac{F_a S(\alpha_S - \delta)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial S} = \frac{\alpha_S(F_S + F_a a) - \delta F_a a}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial P} = \frac{F_P(\epsilon - \nu)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial w} = \frac{-\frac{c}{A}\frac{dT}{d\theta}}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial c} = \frac{-\frac{w}{A}\frac{dT}{d\theta}}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial A} = \frac{\frac{1}{A^2}\left[wc\,\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial T_b} = \frac{\frac{1}{A}\,(4KT_b{}^3 + Y)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial K} = \frac{-\frac{1}{A}\,(T^4 - T_b{}^4)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial Y} = \frac{-\frac{1}{A}\,(T - T_b)}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial \nu} = \frac{-F_P P}{\sigma T^4 - F_P P}$$

$$\frac{\partial \epsilon}{\partial \delta} = \frac{-F_a a S}{\sigma T^4 - F_P P}$$

Solar constant - single

$$S = \frac{1}{\alpha_S(F_S + F_a a) - \delta F_a a}\left[\epsilon(\sigma T^4 - F_P P) + \frac{wc}{A}\,\frac{dT}{d\theta} + \frac{K}{A}\,(T^4 - T_b{}^4) + \frac{Y}{A}\,(T - T_b) + \nu F_P P\right]$$

$$\frac{\partial S}{\partial \alpha_S} = \frac{-S(F_S + F_a a)}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial \epsilon} = \frac{\sigma T^4 - F_P P}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial S}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{wc}{A}\frac{\partial}{\partial T}\left(\frac{dT}{d\theta}\right) + \frac{4KT^3}{A} + \frac{Y}{A}}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial \theta} = \frac{\frac{wc}{A}\frac{\partial}{\partial \theta}\left(\frac{dT}{d\theta}\right)}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial S}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{4KT^3}{A} + \frac{Y}{A}}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial \frac{dT}{d\theta}} = \frac{\frac{wc}{A}}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial F_S} = \frac{-\alpha_S S}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial F_a} = \frac{-(\alpha_S - \delta)aS}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial F_P} = \frac{-(\epsilon - \nu)P}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial P} = \frac{-(\epsilon - \nu)F_P}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial a} = \frac{-(\alpha_S - \delta)F_a S}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial w} = \frac{\frac{c}{A}\frac{dT}{d\theta}}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial c} = \frac{\frac{w}{A}\frac{dT}{d\theta}}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial A} = \frac{-\left[wc\,\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{A^2[\alpha_S(F_S + F_a a) - \delta F_a a]}$$

$$\frac{\partial S}{\partial T_b} = \frac{-(4KT_b{}^3 + Y)}{A[\alpha_S(F_S + F_a a) - \delta F_a a]}$$

$$\frac{\partial S}{\partial K} = \frac{T^4 - T_b{}^4}{A[\alpha_S(F_S + F_a a) - \delta F_a a]}$$

$$\frac{\partial S}{\partial Y} = \frac{T - T_b}{A[\alpha_S(F_S + F_a a) - \delta F_a a]}$$

$$\frac{\partial S}{\partial \nu} = \frac{F_P P}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

$$\frac{\partial S}{\partial \delta} = \frac{F_a a S}{\alpha_S(F_S + F_a a) - \delta F_a a}$$

Albedo - single

$$a = \frac{1}{F_a S(\alpha_S - \delta)}\left[\epsilon(\sigma T^4 - F_P P) - \alpha_S F_S S + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \nu F_P P\right]$$

$$\frac{\partial a}{\partial \alpha_S} = \frac{-(F_S + F_a a)}{F_a(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial \epsilon} = \frac{\sigma T^4 - F_P P}{F_a S(\alpha_S - \delta)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial a}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{wc}{A}\frac{\partial}{\partial T}\left(\frac{dT}{d\theta}\right) + \frac{4KT^3}{A} + \frac{Y}{A}}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial \theta} = \frac{\frac{wc}{A}\frac{\partial}{\partial \theta}\left(\frac{dT}{d\theta}\right)}{F_a S(\alpha_S - \delta)}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial a}{\partial T} = \frac{4\epsilon\sigma T^3 + \frac{4KT^3}{A} + \frac{Y}{A}}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial \frac{dT}{d\theta}} = \frac{\frac{wc}{A}}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial F_S} = \frac{-\alpha_S}{F_a(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial F_a} = \frac{-a}{F_a}$$

$$\frac{\partial a}{\partial F_P} = \frac{-P(\epsilon - \nu)}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial P} = \frac{-F_P(\epsilon - \nu)}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial S} = \frac{-\left(F_a a + \frac{\alpha_S F_S}{\alpha_S - \delta}\right)}{F_a S}$$

$$\frac{\partial a}{\partial w} = \frac{\frac{c}{A}\frac{dT}{d\theta}}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial c} = \frac{\frac{w}{A}\frac{dT}{d\theta}}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial A} = \frac{-\left[wc\,\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{F_a S A^2(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial T_b} = \frac{-(4KT_b{}^3 + Y)}{F_a SA(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial K} = \frac{T^4 - T_b{}^4}{F_a SA(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial Y} = \frac{T - T_b}{F_a SA(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial \nu} = \frac{F_P P}{F_a S(\alpha_S - \delta)}$$

$$\frac{\partial a}{\partial \delta} = \frac{a}{\alpha_S - \delta}$$

Planetary infrared radiation - single

$$P = \frac{1}{F_P(\epsilon - \nu)}\left[\epsilon\sigma T^4 - (F_S + F_a a)S\alpha_S + \frac{wc}{A}\frac{dT}{d\theta} + \frac{K}{A}(T^4 - T_b{}^4) + \frac{Y}{A}(T - T_b) + \delta F_a a S\right]$$

$$\frac{\partial P}{\partial \alpha_S} = \frac{-S(F_S + F_a a)}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial \epsilon} = \frac{\sigma T^4 - F_P P}{F_P(\epsilon - \nu)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T and θ.

$$\frac{\partial P}{\partial T} = \frac{1}{F_P(\epsilon - \nu)} \left(4\epsilon\sigma T^3 + \frac{wc}{A} \frac{\partial}{\partial T} \frac{dT}{d\theta} + \frac{4KT^3}{A} + \frac{Y}{A} \right)$$

$$\frac{\partial P}{\partial \theta} = \frac{\frac{wc}{A} \frac{\partial}{\partial \theta} \frac{dT}{d\theta}}{F_P(\epsilon - \nu)}$$

The second uses T and $dT/d\theta$.

$$\frac{\partial P}{\partial T} = \frac{1}{F_P(\epsilon - \nu)} \left(4\epsilon\sigma T^3 + \frac{4KT^3}{A} + \frac{Y}{A} \right)$$

$$\frac{\partial P}{\partial \frac{dT}{d\theta}} = \frac{\frac{wc}{A}}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial F_S} = \frac{-S\alpha_S}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial F_a} = \frac{-aS(\alpha_S - \delta)}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial F_P} = \frac{-P}{F_P}$$

$$\frac{\partial P}{\partial a} = \frac{-F_a S(\alpha_S - \delta)}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial S} = \frac{-[\alpha_S(F_S + F_a a) - \delta F_a a]}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial w} = \frac{\frac{c}{A} \frac{dT}{d\theta}}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial c} = \frac{\frac{w}{A}\,\frac{dT}{d\theta}}{F_P(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial A} = \frac{-\left[wc\,\frac{dT}{d\theta} + K(T^4 - T_b{}^4) + Y(T - T_b)\right]}{F_P A^2(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial T_b} = \frac{-(4KT_b{}^3 + Y)}{F_P A(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial K} = \frac{T^4 - T_b{}^4}{F_P A(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial Y} = \frac{T - T_b}{F_P A(\epsilon - \nu)}$$

$$\frac{\partial P}{\partial \nu} = \frac{P}{\epsilon - \nu}$$

$$\frac{\partial P}{\partial \delta} = \frac{F_a a S}{F_P(\epsilon - \nu)}$$

APPENDIX D

PARTIAL DERIVATIVES OF SIMULTANEOUS SOLUTION

Conditions for solution of α_S:

(1) $(F_S + F_a a)S$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $\big|_1$ = View factors $\big|_2$

Substitutions:

(1) Let D_1 represent $\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})$.

(2) Where possible, expressions have been simplified by writing them in terms of α_{S_1}.

<u>Equation for α_S:</u>

$$\alpha_{S_1} = \frac{\alpha_{S_2}\left[\epsilon_1\sigma T_1{}^4 - \epsilon_1 F_P P + \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{A}\Big|_1 (T_1{}^4 - T_{b_1}^4) + \frac{Y}{A}\Big|_1 (T_1 - T_{b_1})\right]}{\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})}$$

<u>Partial Derivatives:</u>

$$\frac{\partial\alpha_{S_1}}{\partial\alpha_{S_2}} = \frac{\alpha_{S_1}}{\alpha_{S_2}}$$

$$\frac{\partial\alpha_{S_1}}{\partial\epsilon_1} = \frac{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}{\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})}$$

$$= \frac{\sigma T_1{}^4 - F_P P}{S(F_S + F_a a)}$$

$$\frac{\partial\alpha_{S_1}}{\partial\epsilon_2} = \frac{-\alpha_{S_1}(\sigma T_2{}^4 - F_P P)}{D_1}$$

$$= \frac{-\alpha_{S_1}(\sigma T_2{}^4 - F_P P)}{\alpha_{S_2} S(F_S + F_a a)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial \alpha_{S_1}}{\partial T_1} = \frac{\alpha_{S_2}\left[4\epsilon_1\sigma T_1^3 + \frac{wc}{A}\Big|_1 \frac{\partial}{\partial T_1}\left(\frac{dT}{d\theta}\Big|_1\right) + \frac{K}{A}\Big|_1 4T_1^3 + \frac{Y}{A}\Big|_1\right]}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_2} = \frac{-\alpha_{S_1}\left[4\epsilon_2\sigma T_2^3 + \frac{wc}{A}\Big|_2 \frac{\partial}{\partial T_2}\left(\frac{dT}{d\theta}\Big|_2\right) + \frac{K}{A}\Big|_2 4T_2^3 + \frac{Y}{A}\Big|_2\right]}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial \theta_1} = \frac{\alpha_{S_2}\frac{wc}{A}\Big|_1 \frac{\partial}{\partial \theta_1}\left(\frac{dT}{d\theta}\Big|_1\right)}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial \theta_2} = \frac{-\alpha_{S_1}\frac{wc}{A}\Big|_2 \frac{\partial}{\partial \theta_2}\left(\frac{dT}{d\theta}\Big|_2\right)}{D_1}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial \alpha_{S_1}}{\partial T_1} = \frac{\alpha_{S_2}\left(4\epsilon_1\sigma T_1^3 + \frac{K}{A}\Big|_1 4T_1^3 + \frac{Y}{A}\Big|_1\right)}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_2} = \frac{-\alpha_{S_1}\left(4\epsilon_2\sigma T_2^3 + \frac{K}{A}\Big|_2 4T_2^3 + \frac{Y}{A}\Big|_2\right)}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial \frac{dT}{d\theta}\Big|_1} = \frac{\alpha_{S_2}\frac{wc}{A}\Big|_1}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial \frac{dT}{d\theta}\Big|_2} = \frac{-\alpha_{S_1}\frac{wc}{A}\Big|_2}{D_1}$$

$$\frac{\partial \alpha_{S_1}}{\partial F_P} = \frac{P(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial F_a} = 0 \qquad \frac{\partial\alpha_{S_1}}{\partial F_S} = 0 \qquad \frac{\partial\alpha_{S_1}}{\partial S} = 0$$

$$\frac{\partial\alpha_{S_1}}{\partial P} = \frac{F_P(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial w_1} = \frac{\alpha_{S_2} \left.\frac{c}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial w_2} = \frac{-\alpha_{S_1} \left.\frac{c}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial c_1} = \frac{\alpha_{S_2} \left.\frac{w}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial c_2} = \frac{-\alpha_{S_1} \left.\frac{w}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial A_1} = \frac{-\alpha_{S_2}\left[wc|_1 \left.\frac{dT}{d\theta}\right|_1 + K_1(T_1{}^4 - T_{b_1}^4) + Y_1(T_1 - T_{b_1})\right]}{A_1{}^2(D_1)}$$

$$\frac{\partial\alpha_{S_1}}{\partial A_2} = \frac{\alpha_{S_1}\left[wc|_2 \left.\frac{dT}{d\theta}\right|_2 + K_2(T_2{}^4 - T_{b_2}^4) + Y_2(T_2 - T_{b_2})\right]}{A_2{}^2(D_1)}$$

$$\frac{\partial\alpha_{S_1}}{\partial T_{b_1}} = \frac{-\alpha_{S_2}\left(\left.\frac{K}{A}\right|_1 4T_{b_1}^3 + \left.\frac{Y}{A}\right|_1\right)}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial T_{b_2}} = \frac{\alpha_{S_1}\left(\left.\frac{K}{A}\right|_2 4T_{b_2}^3 + \left.\frac{Y}{A}\right|_2\right)}{D_1}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial\alpha_{S_1}}{\partial T_b} = \frac{-\alpha_{S_2}\left(\left.\frac{K}{A}\right|_1 4T_{b_1}^3 + \left.\frac{Y}{A}\right|_1\right) + \alpha_{S_1}\left(\left.\frac{K}{A}\right|_2 4T_{b_2}^3 + \left.\frac{Y}{A}\right|_2\right)}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial K_1} = \frac{\alpha_{S_2}(T_1{}^4 - T_{b_1}^4)}{A_1(D_1)}$$

$$\frac{\partial\alpha_{S_1}}{\partial K_2} = \frac{-\alpha_{S_1}(T_2{}^4 - T_{b_2}^4)}{A_2(D_1)}$$

$$\frac{\partial\alpha_{S_1}}{\partial Y_1} = \frac{\alpha_{S_2}(T_1 - T_{b_1})}{A_1(D_1)}$$

$$\frac{\partial\alpha_{S_1}}{\partial Y_2} = \frac{-\alpha_{S_1}(T_2 - T_{b_2})}{A_2(D_1)}$$

[1] $$\frac{\partial\alpha_{S_1}}{\partial\delta_1} = \frac{\epsilon_2\sigma T_2{}^4 - \epsilon_2 F_P P + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) + \alpha_S F_S S}{D_1}$$

$$= \frac{\alpha_{S_2} F_a aS}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial\delta_2} = \frac{-\left[\epsilon_1\sigma T_1{}^4 + \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) - \epsilon_1 F_P P - \alpha_{S_1} F_S S\right]}{D_1}$$

$$= \frac{-\alpha_{S_1} F_a aS}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial\nu_1} = \frac{\alpha_{S_2} F_P P}{D_1}$$

$$\frac{\partial\alpha_{S_1}}{\partial\nu_2} = \frac{-\alpha_{S_1} F_P P}{D_1}$$

[1]Although the stipulated conditions are that ν and δ are zero, the effect of that stipulation can be tested via the partial derivatives of the unknown with respect to those variables. The partial derivatives were derived from the simultaneous solutions in appendix B.

Conditions for solution of α_S:

(1) $F_P P$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $\Big|_1$ = View factors $\Big|_2$

Substitutions:

Where possible, expressions have been simplified by writing them in terms of α_{S_1}.

Equation for α_S:

$$\alpha_{S_1} = \frac{1}{S(F_S + F_a a)} \left\{ \epsilon_1 \sigma T_1^4 + \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{A}\Big|_1 (T_1^4 - T_{b_1}^4) + \frac{Y}{A}\Big|_1 (T_1 - T_{b_1}) - \epsilon_1 \left[\sigma T_2^4 \right.\right.$$

$$\left.\left. -(F_S + F_a a) S \frac{\alpha_S}{\epsilon}\Big|_2 + \frac{wc}{\epsilon A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{\epsilon A}\Big|_2 (T_2^4 - T_{b_2}^4) + \frac{Y}{\epsilon A}\Big|_2 (T_2 - T_{b_2}) \right] \right\} .$$

Partial derivatives:

$$\frac{\partial \alpha_{S_1}}{\partial \alpha_{S_2}} = \frac{\epsilon_1}{\epsilon_2}$$

$$\frac{\partial \alpha_{S_1}}{\partial \epsilon_1} = \frac{\sigma T_1^4 - \left[\sigma T_2^4 - (F_S + F_a a) S \frac{\alpha_S}{\epsilon}\Big|_2 + \frac{wc}{\epsilon A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{\epsilon A}\Big|_2 (T_2^4 - T_{b_2}^4) + \frac{Y}{\epsilon A}\Big|_2 (T_2 - T_{b_2}) \right]}{S(F_S + F_a a)}$$

$$= \frac{\sigma T_1^4 - F_P P}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \epsilon_2} = \frac{\epsilon_1 \left[-(F_S + F_a a) S \frac{\alpha_S}{\epsilon}\Big|_2 + \frac{wc}{\epsilon A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{\epsilon A}\Big|_2 (T_2^4 - T_{b_2}^4) + \frac{Y}{\epsilon A}\Big|_2 (T_2 - T_{b_2}) \right]}{\epsilon_2 S(F_S + F_a a)}$$

$$= \frac{-\epsilon_1(\sigma T_2^4 - F_P P)}{\epsilon_2 S(F_S + F_a a)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial \alpha_{S_1}}{\partial T_1} = \frac{4\epsilon_1 \sigma T_1^3 + \frac{wc}{A}\Big|_1 \frac{\partial}{\partial T_1}\left(\frac{dT}{d\theta}\Big|_1\right) + \frac{K}{A}\Big|_1 4T_1^3 + \frac{Y}{A}\Big|_1}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_2} = \frac{-\epsilon_1 \left[4\epsilon_2 \sigma T_2^3 + \frac{wc}{A}\Big|_2 \frac{\partial}{\partial T_2}\left(\frac{dT}{d\theta}\Big|_2\right) + \frac{K}{A}\Big|_2 4T_2^3 + \frac{Y}{A}\Big|_2\right]}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \theta_1} = \frac{\frac{wc}{A}\Big|_1 \frac{\partial}{\partial \theta_1}\left(\frac{dT}{d\theta}\Big|_1\right)}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \theta_2} = \frac{-\epsilon_1 \frac{wc}{A}\Big|_2 \frac{\partial}{\partial \theta_2}\left(\frac{dT}{d\theta}\Big|_2\right)}{\epsilon_2 S(F_S + F_a a)}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial \alpha_{S_1}}{\partial T_1} = \frac{4\epsilon_1 \sigma T_1^3 + \frac{K}{A}\Big|_1 4T_1^3 + \frac{Y}{A}\Big|_1}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_2} = \frac{-\epsilon_1 \left(4\epsilon_2 \sigma T_2^3 + \frac{K}{A}\Big|_2 4T_2^3 + \frac{Y}{A}\Big|_2\right)}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \frac{dT}{d\theta}\Big|_1} = \frac{\frac{wc}{A}\Big|_1}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \frac{dT}{d\theta}\Big|_2} = \frac{-\epsilon_1 \left(\frac{wc}{A}\Big|_2\right)}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial F_S} = \frac{\epsilon_1 \alpha_{S_2} - \epsilon_2 \alpha_{S_1}}{\epsilon_2 (F_S + F_a a)} \qquad \frac{\partial \alpha_{S_1}}{\partial F_P} = 0$$

$$\frac{\partial \alpha_{S_1}}{\partial F_a} = \frac{a(\epsilon_1 \alpha_{S_2} - \epsilon_2 \alpha_{S_1})}{\epsilon_2 (F_S + F_a a)} \qquad \frac{\partial \alpha_{S_1}}{\partial P} = 0$$

$$\frac{\partial \alpha_{S_1}}{\partial S} = \frac{\epsilon_1 \alpha_{S_2} - \alpha_{S_1}\epsilon_2}{\epsilon_2 S}$$

$$\frac{\partial \alpha_{S_1}}{\partial a} = \frac{F_a(\epsilon_1 \alpha_{S_2} - \epsilon_2 \alpha_{S_1})}{\epsilon_2 (F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial w_1} = \frac{\left.\frac{c}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial w_2} = \frac{-\epsilon_1 \left.\frac{c}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial c_1} = \frac{\left.\frac{w}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial c_2} = \frac{-\epsilon_1 \left.\frac{w}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial A_1} = \frac{-\left[wc|_1 \left.\frac{dT}{d\theta}\right|_1 + K_1(T_1^{\,4} - T_{b_1}^4) + Y_1(T_1 - T_{b_1}) \right]}{A_1^{\,2} S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial A_2} = \frac{\epsilon_1 \left[wc|_2 \left.\frac{dT}{d\theta}\right|_2 + K_2(T_2^{\,4} - T_{b_2}^4) + Y_2(T_2 - T_{b_2}) \right]}{\epsilon_2 A_2^{\,2} S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_{b_1}} = \frac{-\left(\left.\frac{K}{A}\right|_1 4T_{b_1}^3 + \left.\frac{Y}{A}\right|_1 \right)}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial T_{b_2}} = \frac{\epsilon_1 \left(\left.\frac{K}{A}\right|_2 4T_{b_2}^3 + \left.\frac{Y}{A}\right|_2 \right)}{\epsilon_2 S(F_S + F_a a)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial \alpha_{S_1}}{\partial T_b} = \frac{-\left(\frac{K}{A}\Big|_1 4T_{b_1}^3 + \frac{Y}{A}\Big|_1\right)}{S(F_S + F_a a)} + \frac{\epsilon_1 \left(\frac{K}{A}\Big|_2 4T_{b_2}^3 + \frac{Y}{A}\Big|_2\right)}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial K_1} = \frac{T_1{}^4 - T_{b_1}^4}{A_1 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial K_2} = \frac{-\epsilon_1 (T_2{}^4 - T_{b_2}^4)}{\epsilon_2 A_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial Y_1} = \frac{T_1 - T_{b_1}}{A_1 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial Y_2} = \frac{-\epsilon_1 (T_2 - T_{b_2})}{\epsilon_2 A_2 S(F_S + F_a a)}$$

[2] $$\frac{\partial \alpha_{S_1}}{\partial \delta_1} = \frac{F_a a S}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \delta_2} = \frac{-\epsilon_1 F_a a S}{\epsilon_2 S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \nu_1} = \frac{\epsilon_2 \sigma T_2{}^4 - (F_S + F_a a) S \alpha_{S_2} + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})}{\epsilon_2 S(F_S + F_a a)}$$

$$= \frac{F_P P}{S(F_S + F_a a)}$$

$$\frac{\partial \alpha_{S_1}}{\partial \nu_2} = \frac{-\left[\epsilon_1 \sigma T_1{}^4 - (F_S + F_a a) S \alpha_{S_1} + \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{A}\Big|_1 (T_1{}^4 - T_{b_1}^4) + \frac{Y}{A}\Big|_1 (T_1 - T_{b_1})\right]}{\epsilon_2 S(F_S + F_a a)}$$

$$= \frac{-\epsilon_1 F_P P}{\epsilon_2 S(F_S + F_a a)}$$

[2]See footnote 1, page 33.

Conditions for solution of α_S/ϵ:

(1) $(F_S + F_a a)S$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $|_1$ = View factors $|_2$

Equation for α_S/ϵ:

$$\frac{\alpha_S}{\epsilon}\bigg|_1 = \frac{\alpha_{S_2}\left[\epsilon_1\sigma T_1^{\,4} - \epsilon_1 F_P P + \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{A}\Big|_1 (T_1^{\,4} - T_{b_1}^4) + \frac{Y}{A}\Big|_1 (T_1 - T_{b_1})\right]}{\epsilon_1\left[\epsilon_2\sigma T_2^{\,4} - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2^{\,4} - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})\right]}$$

Partial derivatives:

The above equation is for α_{S_1} divided by ϵ_1. Therefore, the partial derivatives of $\alpha_S/\epsilon|_1$ with respect to all variables except ϵ_1 are those for α_{S_1} divided by ϵ_1. The partial derivative with respect to ϵ_1 is as follows:

$$\frac{\partial \frac{\alpha_S}{\epsilon}\Big|_1}{\partial\epsilon_1} = \frac{\alpha_{S_2}(\sigma T_1^{\,4} - F_P P) - \frac{\alpha_S}{\epsilon}\Big|_1\left[\epsilon_2\sigma T_2^{\,4} - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2^{\,4} - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})\right]}{\epsilon_1\left[\epsilon_2\sigma T_2^{\,4} - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2^{\,4} - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})\right]}$$

Conditions for solution of α_S/ϵ:

(1) $F_P P$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $|_1$ = View factors $|_2$

Equation for α_S/ϵ:

$$\frac{\alpha_S}{\epsilon}\bigg|_1 = \frac{1}{S(F_S + F_a a)}\left\{\sigma T_1^4 + \frac{wc}{\epsilon A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{\epsilon A}\Big|_1 (T_1^{\,4} - T_{b_1}^4) + \frac{Y}{\epsilon A}\Big|_1 (T_1 - T_{b_1}) - \left[\sigma T_2^{\,4}\right.\right.$$

$$\left.\left. - (F_S + F_a a)S\,\frac{\alpha_S}{\epsilon}\Big|_2 + \frac{wc}{\epsilon A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{\epsilon A}\Big|_2 (T_2^{\,4} - T_{b_2}^4) + \frac{Y}{\epsilon A}\Big|_2 (T_2 - T_{b_2})\right]\right\}$$

Partial derivatives:

The above equation is for α_{S_1} divided by ϵ_1. Therefore, the partial derivatives of $\alpha_S/\epsilon|_1$ with respect to all variables except ϵ_1 are those for α_{S_1} divided by ϵ_1. The partial derivative with respect to ϵ_1 is as follows:

$$\frac{\partial \left.\frac{\alpha_S}{\epsilon}\right|_1}{\partial \epsilon_1} = \frac{-\left[wc|_1 \left.\frac{dT}{d\theta}\right|_1 + K_1(T_1{}^4 - T_{b_1}^4) + Y_1(T_1 - T_{b_1}) \right]}{\epsilon_1{}^2 A_1 S(F_S + F_a a)}$$

Conditions for solution of ϵ:

(1) $(F_S + F_a a)S$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $|_1$ = View factors $|_2$

Substitutions:

Where possible, expressions have been simplified by writing them in terms of ϵ_1.

Equation for ϵ:

$$\epsilon_1 = \frac{1}{\sigma T_1{}^4 - F_P P}\left\{ -\left[\left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1}) \right] + \frac{\alpha_{S_1}}{\alpha_{S_2}} \left[\epsilon_2(\sigma T_2{}^4 - F_P P) \right.\right.$$

$$\left.\left. + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2}) \right] \right\}$$

Partial derivatives:

$$\frac{\partial \epsilon_1}{\partial \alpha_{S_1}} = \frac{\epsilon_2(\sigma T_2{}^4 - F_P P) + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2})}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$= \frac{S(F_S + F_a a)}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial \alpha_{S_2}} = \frac{-\alpha_{S_1}\left[\epsilon_2(\sigma T_2{}^4 - F_P P) + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2})\right]}{\alpha_{S_2}^2(\sigma T_1{}^4 - F_P P)}$$

$$= \frac{-\alpha_{S_1} S(F_S + F_a a)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial \epsilon_2} = \frac{\alpha_{S_1}(\sigma T_2{}^4 - F_P P)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

Partial derivatives were derived with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial \epsilon_1}{\partial T_1} = \frac{-\left[4\epsilon_1 \sigma T_1{}^3 + \left.\frac{wc}{A}\right|_1 \left.\frac{\partial}{\partial T}\right|_1 \left(\left.\frac{dT}{d\theta}\right|_1\right) + \left.\frac{K}{A}\right|_1 4T_1{}^3 + \left.\frac{Y}{A}\right|_1\right]}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial T_2} = \frac{\alpha_{S_1}\left[4\epsilon_2 \sigma T_2{}^3 + \left.\frac{wc}{A}\right|_2 \frac{\partial}{\partial T_2}\left(\left.\frac{dT}{d\theta}\right|_2\right) + \left.\frac{K}{A}\right|_2 4T_2{}^3 + \left.\frac{Y}{A}\right|_2\right]}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial \theta_1} = \frac{-\left.\frac{wc}{A}\right|_1 \frac{\partial}{\partial \theta_1}\left(\left.\frac{dT}{d\theta}\right|_1\right)}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial \theta_2} = \frac{\alpha_{S_1} \left.\frac{wc}{A}\right|_2 \frac{\partial}{\partial \theta_2}\left(\left.\frac{dT}{d\theta}\right|_2\right)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial \epsilon_1}{\partial T_1} = \frac{-\left(4\epsilon_1 \sigma T_1{}^3 + \left.\frac{K}{A}\right|_1 4T_1{}^3 + \left.\frac{Y}{A}\right|_1\right)}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial T_2} = \frac{\alpha_{S_1}\left(4\epsilon_2 \sigma T_2{}^3 + \left.\frac{K}{A}\right|_2 4T_2{}^3 + \left.\frac{Y}{A}\right|_2\right)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial \left.\frac{dT}{d\theta}\right|_1} = \frac{-\left.\frac{wc}{A}\right|_1}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial \left.\frac{dT}{d\theta}\right|_2} = \frac{\alpha_{S_1} \left.\frac{wc}{A}\right|_2}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial F_P} = \frac{P(\epsilon_1 \alpha_{S_2} - \epsilon_2 \alpha_{S_1})}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)} \qquad \frac{\partial \epsilon_1}{\partial F_S} = 0$$

$$\frac{\partial \epsilon_1}{\partial P} = \frac{F_P(\epsilon_1 \alpha_{S_2} - \epsilon_2 \alpha_{S_1})}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)} \qquad \frac{\partial \epsilon_1}{\partial S} = 0$$

$$\frac{\partial \epsilon_1}{\partial F_a} = 0 \qquad \frac{\partial \epsilon_1}{\partial a} = 0$$

$$\frac{\partial \epsilon_1}{\partial w_1} = \frac{-\left.\frac{c}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial w_2} = \frac{\alpha_{S_1} \left.\frac{c}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial c_1} = \frac{-\left.\frac{w}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial c_2} = \frac{\alpha_{S_1} \left.\frac{w}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial A_1} = \frac{wc|_1 \left.\frac{dT}{d\theta}\right|_1 + K_1(T_1{}^4 - T_{b_1}^4) + Y_1(T_1 - T_{b_1})}{A_1{}^2(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial A_2} = \frac{-\alpha_{S_1}\left[wc\big|_2 \frac{dT}{d\theta}\Big|_2 + K_2(T_2{}^4 - T_{b_2}^4) + Y_2(T_2 - T_{b_2})\right]}{\alpha_{S_2}A_2{}^2(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial T_{b_1}} = \frac{\frac{K}{A}\Big|_1 4T_{b_1}^3 + \frac{Y}{A}\Big|_1}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial T_{b_2}} = \frac{-\alpha_{S_1}\left(\frac{K}{A}\Big|_2 4T_{b_2}^3 + \frac{Y}{A}\Big|_2\right)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial \epsilon_1}{\partial T_b} = \frac{\frac{K}{A}\Big|_1 4T_{b_1}^3 + \frac{Y}{A}\Big|_1}{\sigma T_1{}^4 - F_P P} - \frac{\alpha_{S_1}\left(\frac{K}{A}\Big|_2 4T_{b_2}^3 + \frac{Y}{A}\Big|_2\right)}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial K_1} = \frac{-(T_1{}^4 - T_{b_1}^4)}{A_1(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial K_2} = \frac{\alpha_{S_1}(T_2{}^4 - T_{b_2}^4)}{\alpha_{S_2}A_2(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial Y_1} = \frac{-(T_1 - T_{b_1})}{A_1(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial Y_2} = \frac{\alpha_{S_1}(T_2 - T_{b_2})}{\alpha_{S_2}A_2(\sigma T_1{}^4 - F_P P)}$$

$$^3\frac{\partial \epsilon_1}{\partial \delta_1} = \frac{-\left[\epsilon_2(\sigma T_2{}^4 - F_P P) - \alpha_{S_2}F_S S + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})\right]}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$= \frac{-F_a aS}{\sigma T_1{}^4 - F_P P}$$

[3]See footnote 1, page 33.

$$\frac{\partial \epsilon_1}{\partial \delta_2} = \frac{\alpha_{S_1}\left[\epsilon_2(\sigma T_2{}^4 - F_P P) - \alpha_{S_2} F_S S + \left.\frac{wc}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{A}\right|_2 (T_2 - T_{b_2})\right]}{\alpha_{S_2}^2(\sigma T_1{}^4 - F_P P)}$$

$$= \frac{\alpha_{S_1} F_a a S}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

$$\frac{\partial \epsilon_1}{\partial \nu_1} = \frac{-F_P P}{\sigma T_1{}^4 - F_P P}$$

$$\frac{\partial \epsilon_1}{\partial \nu_2} = \frac{\alpha_{S_1} F_P P}{\alpha_{S_2}(\sigma T_1{}^4 - F_P P)}$$

Conditions for solution of ϵ:

(1) $F_P P$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $|_1$ = View factors $|_2$

Substitutions:

(1) Let D_6 represent

$$\sigma T_1{}^4 - \left[\sigma T_2{}^4 - (F_S + F_a a)S \left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]$$

(2) Where possible, expressions have been simplified by writing them in terms of ϵ_1.

<u>Equation for ϵ:</u>

$$\epsilon_1 = \frac{(F_S + F_a a)S\alpha_{S_1} - \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 - \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) - \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1})}{\sigma T_1{}^4 - \left[\sigma T_2{}^4 - (F_S + F_a a)S \left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{\epsilon A}{}_2 (T_2 - T_{b_2})\right]}$$

$$= \frac{(F_S + F_a a)S\alpha_{S_1} - \left.\frac{wc}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 - \left.\frac{K}{A}\right|_1 (T_1{}^4 - T_{b_1}^4) - \left.\frac{Y}{A}\right|_1 (T_1 - T_{b_1})}{\sigma T_1{}^4 - F_P P}$$

Partial derivatives:

$$\frac{\partial\epsilon_1}{\partial\alpha_{S_1}} = \frac{(F_S + F_a a)S}{\sigma T_1{}^4 - \left[\sigma T_2{}^4 - (F_S + F_a a)S\left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}$$

$$\frac{\partial\epsilon_1}{\partial\alpha_{S_2}} = \frac{-\epsilon_1(F_S + F_a a)S}{\epsilon_2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial\epsilon_2} = \frac{\epsilon_1\left[(F_S + F_a a)S\left.\frac{\alpha_S}{\epsilon}\right|_2 - \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 - \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) - \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}{\epsilon_2(D_6)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first involves T_1, T_2, θ_1, and θ_2.

$$\frac{\partial\epsilon_1}{\partial T_1} = \frac{-\left[4\epsilon_1\sigma T_1{}^3 + \left.\frac{wc}{A}\right|_1 \frac{\partial}{\partial T_1}\left(\left.\frac{dT}{d\theta}\right|_1\right) + \left.\frac{K}{A}\right|_1 4T_1{}^3 + \left.\frac{Y}{A}\right|_1\right]}{D_6}$$

$$\frac{\partial\epsilon_1}{\partial T_2} = \frac{\epsilon_1\left[4\epsilon_2\sigma T_2{}^3 + \left.\frac{wc}{A}\right|_2 \frac{\partial}{\partial T_2}\left(\left.\frac{dT}{d\theta}\right|_2\right) + \left.\frac{K}{A}\right|_2 4T_2{}^3 + \left.\frac{Y}{A}\right|_2\right]}{\epsilon_2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial\theta_1} = \frac{-\left.\frac{wc}{A}\right|_1 \frac{\partial}{\partial\theta_1}\left(\left.\frac{dT}{d\theta}\right|_1\right)}{D_6}$$

$$\frac{\partial\epsilon_1}{\partial\theta_2} = \frac{\epsilon_1\left.\frac{wc}{A}\right|_2 \frac{\partial}{\partial\theta_2}\left(\left.\frac{dT}{d\theta}\right|_2\right)}{\epsilon_2(D_6)}$$

The second involves T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial\epsilon_1}{\partial T_1} = \frac{-\left(4\epsilon_1\sigma T_1{}^3 + \left.\frac{K}{A}\right|_1 4T_1{}^3 + \left.\frac{Y}{A}\right|_1\right)}{D_6}$$

$$\frac{\partial\epsilon_1}{\partial T_2} = \frac{\epsilon_1\left(4\epsilon_2\sigma T_2{}^3 + \left.\frac{K}{A}\right|_2 4T_2{}^3 + \left.\frac{Y}{A}\right|_2\right)}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial \left.\frac{dT}{d\theta}\right|_1} = \frac{-\left.\frac{wc}{A}\right|_1}{D_6}$$

$$\frac{\partial \epsilon_1}{\partial \left.\frac{dT}{d\theta}\right|_2} = \frac{\epsilon_1 \left.\frac{wc}{A}\right|_2}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial F_S} = \frac{(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)S}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial F_a} = \frac{(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)aS}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial F_P} = 0 \qquad \frac{\partial \epsilon_1}{\partial E} = 0$$

$$\frac{\partial \epsilon_1}{\partial S} = \frac{(F_S + F_a a)(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial a} = \frac{(\alpha_{S_1}\epsilon_2 - \alpha_{S_2}\epsilon_1)F_a S}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial w_1} = \frac{-\left.\frac{c}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{D_6}$$

$$\frac{\partial \epsilon_1}{\partial w_2} = \frac{\epsilon_1 \left.\frac{c}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\epsilon_2(D_6)}$$

$$\frac{\partial \epsilon_1}{\partial c_1} = \frac{-\left.\frac{w}{A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{D_6}$$

$$\frac{\partial \epsilon_1}{\partial c_2} = \frac{\epsilon_1 \left.\frac{w}{A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\epsilon_2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial A_1} = \frac{wc\big|_1 \frac{dT}{d\theta}\Big|_1 + K_1(T_1{}^4 - T_{b_1}^4) + Y_1(T_1 - T_{b_1})}{A_1{}^2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial A_2} = \frac{-\epsilon_1 \left[wc\big|_2 \frac{dT}{d\theta}\Big|_2 + K_2(T_2{}^4 - T_{b_2}^4) + Y_2(T_2 - T_{b_2})\right]}{\epsilon_2 A_2{}^2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial T_{b_1}} = \frac{\frac{K}{A}\Big|_1 4T_{b_1}^3 + \frac{Y}{A}\Big|_1}{D_6}$$

$$\frac{\partial\epsilon_1}{\partial T_{b_2}} = \frac{-\epsilon_1 \left(\frac{K}{A}\Big|_2 4T_{b_2}^3 + \frac{Y}{A}\Big|_2\right)}{\epsilon_2(D_6)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial\epsilon_1}{\partial T_b} = \frac{\frac{K}{A}\Big|_1 4T_{b_1}^3 + \frac{Y}{A}\Big|_1}{D_6} - \frac{\epsilon_1 \left(\frac{K}{A}\Big|_2 4T_{b_2}^3 + \frac{Y}{A}\Big|_2\right)}{\epsilon_2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial K_1} = \frac{-(T_1{}^4 - T_{b_1}^4)}{A_1(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial K_2} = \frac{\epsilon_1(T_2{}^4 - T_{b_2}^4)}{\epsilon_2 A_2(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial Y_1} = \frac{-(T_1 - T_{b_1})}{A_1(D_6)}$$

$$\frac{\partial\epsilon_1}{\partial Y_2} = \frac{\epsilon_1(T_2 - T_{b_2})}{\epsilon_2 A_2(D_6)}$$

[4] $$\frac{\partial\epsilon_1}{\partial\delta_1} = \frac{-F_a aS}{D_6}$$

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[4]See footnote 1, page 33.

$$\frac{\partial \epsilon_1}{\partial \delta_2} = \frac{\epsilon_1 F_a a S}{\epsilon_2 (D_6)}$$

$$\frac{\partial \epsilon_1}{\partial \nu_1} = \frac{-\left[\sigma T_2^4 - (F_S + F_a a) S \left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}{D_6}$$

$$= \frac{-F_P P}{D_6}$$

$$\frac{\partial \epsilon_1}{\partial \nu_2} = \frac{\epsilon_1 \left[\sigma T_2^4 - (F_S + F_a a) S \left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}{\epsilon_2 (D_6)}$$

$$= \frac{\epsilon_1 F_P P}{\epsilon_2 (D_6)}$$

Conditions for solution of S:

(1) $F_P P$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $|_1$ = View factors $|_2$

Substitutions:

Where possible, expressions have been simplified by writing them in terms of S.

<u>Equation for S:</u>

$$S = \frac{1}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)} \left\{\sigma T_1^4 + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})\right.$$

$$\left. - \left[\sigma T_2^4 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]\right\}$$

Partial derivatives:

$$\frac{\partial S}{\partial \alpha_{S_1}} = \frac{-S}{\epsilon_1 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right)}$$

$$\frac{\partial S}{\partial \alpha_{S_2}} = \frac{S}{\epsilon_2 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right)}$$

$$\frac{\partial S}{\partial \epsilon_1} = \frac{\alpha_{S_1} S(F_S + F_a a) - \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 - \frac{K}{A}\Big|_1 (T_1^4 - T_{b_1}^4) - \frac{Y}{A}\Big|_1 (T_1 - T_{b_1})}{\epsilon_1^2 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

$$= \frac{\sigma T_1^4 - F_P P}{\epsilon_1 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

$$\frac{\partial S}{\partial \epsilon_2} = \frac{-\alpha_{S_2} S(F_S + F_a a) + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})}{\epsilon_2^2 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

$$= \frac{-(\sigma T_2^4 - F_P P)}{\epsilon_2 \left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial S}{\partial T_1} = \frac{4\sigma T_1^3 + \frac{wc}{\epsilon A}\Big|_1 \frac{\partial}{\partial T_1}\left(\frac{dT}{d\theta}\Big|_1 \right) + \frac{K}{\epsilon A}\Big|_1 4T_1^3 + \frac{Y}{\epsilon A}\Big|_1}{\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

$$\frac{\partial S}{\partial T_2} = \frac{-\left[4\sigma T_2^3 + \frac{wc}{\epsilon A}\Big|_2 \frac{\partial}{\partial T_2}\left(\frac{dT}{d\theta}\Big|_2 \right) + \frac{K}{\epsilon A}\Big|_2 4T_2^3 + \frac{Y}{\epsilon A}\Big|_2 \right]}{\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2 \right) (F_S + F_a a)}$$

$$\frac{\partial S}{\partial \theta_1} = \frac{\left.\frac{wc}{\epsilon A}\right|_1 \frac{\partial}{\partial \theta_1}\left(\left.\frac{dT}{d\theta}\right|_1\right)}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \theta_2} = \frac{-\left.\frac{wc}{\epsilon A}\right|_2 \frac{\partial}{\partial \theta_2}\left(\left.\frac{dT}{d\theta}\right|_2\right)}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial S}{\partial T_1} = \frac{4\sigma T_1^3 + \left.\frac{K}{\epsilon A}\right|_1 4T_1^3 + \left.\frac{Y}{\epsilon A}\right|_1}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial T_2} = \frac{-\left(4\sigma T_2^3 + \left.\frac{K}{\epsilon A}\right|_2 4T_2^3 + \left.\frac{Y}{\epsilon A}\right|_2\right)}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \left.\frac{dT}{d\theta}\right|_1} = \frac{\left.\frac{wc}{\epsilon A}\right|_1}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \left.\frac{dT}{d\theta}\right|_2} = \frac{-\left.\frac{wc}{\epsilon A}\right|_2}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial F_S} = \frac{-S}{F_S + F_a a} \qquad \frac{\partial S}{\partial F_P} = 0$$

$$\frac{\partial S}{\partial F_a} = \frac{-aS}{F_S + F_a a} \qquad \frac{\partial S}{\partial P} = 0$$

$$\frac{\partial S}{\partial a} = \frac{-F_a S}{F_S + F_a a}$$

$$\frac{\partial S}{\partial w_1} = \frac{\left.\frac{c}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial w_2} = \frac{-\left.\frac{c}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial c_1} = \frac{\left.\frac{w}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial c_2} = \frac{-\left.\frac{w}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial A_1} = \frac{-\left[\left.\frac{wc}{\epsilon}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon}\right|_1 (T_1 - T_{b_1})\right]}{A_1{}^2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial A_2} = \frac{\left.\frac{wc}{\epsilon}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon}\right|_2 (T_2 - T_{b_2})}{A_2{}^2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S - F_a a)}$$

$$\frac{\partial S}{\partial T_{b_1}} = \frac{-\left(\left.\frac{K}{\epsilon A}\right|_1 4T_{b_1}^3 + \left.\frac{Y}{\epsilon A}\right|_1\right)}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial T_{b_2}} = \frac{\left.\frac{K}{\epsilon A}\right|_2 4T_{b_2}^3 + \left.\frac{Y}{\epsilon A}\right|_2}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial S}{\partial T_b} = \frac{-\left.\frac{K}{\epsilon A}\right|_1 4T_{b_1}^3 - \left.\frac{Y}{\epsilon A}\right|_1 + \left.\frac{K}{\epsilon A}\right|_2 4T_{b_2}^3 + \left.\frac{Y}{\epsilon A}\right|_2}{\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial K_1} = \frac{T_1{}^4 - T_{b_1}^4}{\epsilon_1 A_1 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial K_2} = \frac{-(T_2{}^4 - T_{b_2}^4)}{\epsilon_2 A_2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial Y_1} = \frac{T_1 - T_{b_1}}{\epsilon_1 A_1 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial Y_2} = \frac{-(T_2 - T_{b_2})}{\epsilon_2 A_2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

[5] $$\frac{\partial S}{\partial \delta_1} = \frac{F_a a S}{\epsilon_1 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \delta_2} = \frac{-F_a a S}{\epsilon_2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \nu_1} = \frac{F_P P}{\epsilon_1 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

$$\frac{\partial S}{\partial \nu_2} = \frac{-F_P P}{\epsilon_2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)(F_S + F_a a)}$$

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[5]See footnote 1, page 33.

Conditions for solution of a:

(1) $F_P P$ eliminated by virtue of simultaneous solution

(2) $\nu = \delta = 0$

(3) View factors $\Big|_1$ = View factors $\Big|_2$

Substitutions:

Where possible, expressions have been simplified by writing them in terms of a.

Equation for a:

$$a = \frac{1}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)} \left\{\sigma T_1^4 - \left.\frac{\alpha_S}{\epsilon}\right|_1 F_S S + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})\right.$$

$$\left. - \left[\sigma T_2^4 - \left.\frac{\alpha_S}{\epsilon}\right|_2 F_S S + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]\right\}$$

Partial derivatives:

$$\frac{\partial a}{\partial \alpha_{S_1}} = \frac{-(F_S + F_a a)}{\epsilon_1 F_a\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial \alpha_{S_2}} = \frac{F_S + F_a a}{\epsilon_2 F_a\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial \epsilon_1} = \frac{\sigma T_1^4 - F_P P}{\epsilon_1 F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$= \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 S(F_S + F_a a) - \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 - \left.\frac{K}{\epsilon A}\right|_1 (T_1^4 - T_{b_1}^4) - \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})}{\epsilon_1 F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial \epsilon_2} = \frac{-(\sigma T_2^{\,4} - F_p P)}{\epsilon_2 F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

$$= \frac{-\left[\left.\frac{\alpha_S}{\epsilon}\right|_2 S(F_S + F_a a) - \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 - \left.\frac{K}{\epsilon A}\right|_2 (T_2^{\,4} - T_{b_2}^4) - \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2}) \right]}{\epsilon_2 F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial a}{\partial T_1} = \frac{4\sigma T_1^{\,3} + \left.\frac{wc}{\epsilon A}\right|_1 \frac{\partial}{\partial T_1}\left(\left.\frac{dT}{d\theta}\right|_1 \right) + \left.\frac{K}{\epsilon A}\right|_1 4T_1^{\,3} + \left.\frac{Y}{\epsilon A}\right|_1}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

$$\frac{\partial a}{\partial T_2} = \frac{-\left[4\sigma T_2^{\,3} + \left.\frac{wc}{\epsilon A}\right|_2 \frac{\partial}{\partial T_2}\left(\left.\frac{dT}{d\theta}\right|_2 \right) + \left.\frac{K}{\epsilon A}\right|_2 4T_2^{\,3} + \left.\frac{Y}{\epsilon A}\right|_2 \right]}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

$$\frac{\partial a}{\partial \theta_1} = \frac{\left.\frac{wc}{\epsilon A}\right|_1 \frac{\partial}{\partial \theta_1}\left(\left.\frac{dT}{d\theta}\right|_1 \right)}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

$$\frac{\partial a}{\partial \theta_2} = \frac{-\left.\frac{wc}{\epsilon A}\right|_2 \frac{\partial}{\partial \theta_2}\left(\left.\frac{dT}{d\theta}\right|_2 \right)}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial a}{\partial T_1} = \frac{4\sigma T_1^{\,3} + \left.\frac{K}{\epsilon A}\right|_1 4T_1^{\,3} + \left.\frac{Y}{\epsilon A}\right|_1}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2 \right)}$$

$$\frac{\partial a}{\partial T_2} = \frac{-\left(4\sigma T_2{}^3 + \left.\frac{K}{\epsilon A}\right|_2 4T_2{}^3 + \left.\frac{Y}{\epsilon A}\right|_2\right)}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial \left.\frac{dT}{d\theta}\right|_1} = \frac{\left.\frac{wc}{\epsilon A}\right|_1}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial \left.\frac{dT}{d\theta}\right|_2} = \frac{-\left.\frac{wc}{\epsilon A}\right|_2}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial F_S} = \frac{-1}{F_a} \qquad \frac{\partial a}{\partial F_P} = 0$$

$$\frac{\partial a}{\partial F_a} = \frac{-a}{F_a} \qquad \frac{\partial a}{\partial P} = 0$$

$$\frac{\partial a}{\partial S} = \frac{-(F_S + F_a a)}{F_a S}$$

$$\frac{\partial a}{\partial w_1} = \frac{\left.\frac{c}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial w_2} = \frac{-\left.\frac{c}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial c_1} = \frac{\left.\frac{w}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{F_a S\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial c_2} = \frac{-\left.\frac{w}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial A_1} = \frac{-1}{A_1{}^2 F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)} \left[\left.\frac{wc}{\epsilon}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon}\right|_1 (T_1 - T_{b_1})\right]$$

$$\frac{\partial a}{\partial A_2} = \frac{1}{A_2{}^2 F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)} \left[\left.\frac{wc}{\epsilon}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon}\right|_2 (T_2 - T_{b_2})\right]$$

$$\frac{\partial a}{\partial T_{b_1}} = \frac{-\left(\left.\frac{4K}{\epsilon A}\right|_1 T_{b_1}^3 + \left.\frac{Y}{\epsilon A}\right|_1\right)}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial T_{b_2}} = \frac{\left.\frac{4K}{\epsilon A}\right|_2 T_{b_2}^3 + \left.\frac{Y}{\epsilon A}\right|_2}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial a}{\partial T_b} = \frac{-\left.\frac{4K}{\epsilon A}\right|_1 T_{b_1}^3 - \left.\frac{Y}{\epsilon A}\right|_1 + \left.\frac{4K}{\epsilon A}\right|_2 T_{b_2}^3 + \left.\frac{Y}{\epsilon A}\right|_2}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial a}{\partial K_1} = \frac{1}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)} \left(\frac{T_1{}^4 - T_{b_1}^4}{\epsilon_1 A_1}\right)$$

$$\frac{\partial a}{\partial K_2} = \frac{-1}{F_a S \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)} \left(\frac{T_2{}^4 - T_{b_2}^4}{\epsilon_2 A_2}\right)$$

$$\frac{\partial a}{\partial Y_1} = \frac{1}{F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)} \left(\frac{T_1 - T_{b_1}}{\epsilon_1 A_1}\right)$$

$$\frac{\partial a}{\partial Y_2} = \frac{-1}{F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)} \left(\frac{T_2 - T_{b_2}}{\epsilon_2 A_2}\right)$$

[6] $$\frac{\partial a}{\partial \delta_1} = \frac{a}{\epsilon_1 \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

$$\frac{\partial a}{\partial \delta_2} = \frac{-a}{\epsilon_2 \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

$$\frac{\partial a}{\partial \nu_1} = \frac{F_P P}{\epsilon_1 F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

$$= \frac{\sigma T_1{}^4 - (F_S + F_a a) S \left.\frac{\alpha_S}{\epsilon}\right|_1 + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})}{\epsilon_1 F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

$$\frac{\partial a}{\partial \nu_2} = \frac{-F_P P}{\epsilon_2 F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

$$\frac{-\left[\sigma T_2{}^4 - (F_S + F_a a) S \left.\frac{\alpha_S}{\epsilon}\right|_2 + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}{\epsilon_2 F_a S \left(\frac{\alpha_S}{\epsilon}\bigg|_1 - \frac{\alpha_S}{\epsilon}\bigg|_2\right)}$$

[6]See footnote 1, page 33.

Conditions for solution of P:

(1) $F_a a$ eliminated by virtue of simultaneous solution.

(2) $\nu = \delta = 0$

(3) View factors $\big|_1$ = View factors $\big|_2$

Substitutions:

Where possible, expressions have been simplified by writing them in terms of P.

Equation for P:

$$P = \frac{-1}{F_P\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2\right)} \left\{ \frac{\alpha_S}{\epsilon}\Big|_2 \left[\sigma T_1{}^4 - \frac{\alpha_S}{\epsilon}\Big|_1 F_S S + \frac{wc}{\epsilon A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{\epsilon A}\Big|_1 (T_1{}^4 - T_{b_1}^4)\right.\right.$$

$$\left. + \frac{Y}{\epsilon A}\Big|_1 (T_1 - T_{b_1})\right] - \frac{\alpha_S}{\epsilon}\Big|_1 \left[\sigma T_2{}^4 - \frac{\alpha_S}{\epsilon}\Big|_2 F_S S + \frac{wc}{\epsilon A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{\epsilon A}\Big|_2 (T_2{}^4 - T_{b_2}^4)\right.$$

$$\left.\left. + \frac{Y}{\epsilon A}\Big|_2 (T_2 - T_{b_2})\right]\right\}$$

Partial derivatives:

$$\frac{\partial P}{\partial \alpha_{S_1}} = \frac{\alpha_{S_2} S(F_S + F_a a)}{\epsilon_1 \epsilon_2 F_P\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2\right)}$$

$$= \frac{\epsilon_2 \sigma T_2{}^4 - \epsilon_2 F_P P + \frac{wc}{A}\Big|_2 \frac{dT}{d\theta}\Big|_2 + \frac{K}{A}\Big|_2 (T_2{}^4 - T_{b_2}^4) + \frac{Y}{A}\Big|_2 (T_2 - T_{b_2})}{\epsilon_1 \epsilon_2 F_P\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2\right)}$$

$$\frac{\partial P}{\partial \alpha_{S_2}} = \frac{-\alpha_{S_1} S(F_S + F_a a)}{\epsilon_1 \epsilon_2 F_P\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2\right)}$$

$$= \frac{-\left[\epsilon_1 \sigma T_1{}^4 - \epsilon_1 F_P P + \frac{wc}{A}\Big|_1 \frac{dT}{d\theta}\Big|_1 + \frac{K}{A}\Big|_1 (T_1{}^4 - T_{b_1}^4) + \frac{Y}{A}\Big|_1 (T_1 - T_{b_1})\right]}{\epsilon_1 \epsilon_2 F_P\left(\frac{\alpha_S}{\epsilon}\Big|_1 - \frac{\alpha_S}{\epsilon}\Big|_2\right)}$$

$$\frac{\partial P}{\partial \epsilon_1} = \frac{-\alpha_{S_2}(\sigma T_1^4 - F_P P)}{\epsilon_1\epsilon_2 F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \epsilon_2} = \frac{\alpha_{S_1}(\sigma T_2^4 - F_P P)}{\epsilon_1\epsilon_2 F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

Partial derivatives were developed with respect to two sets of independent variables involving temperature and time; either set may be used. The first uses T_1, T_2, θ_1, and θ_2.

$$\frac{\partial P}{\partial T_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2\left[4\sigma T_1^3 + \left.\frac{wc}{\epsilon A}\right|_1 \frac{\partial}{\partial T_1}\left(\left.\frac{dT}{d\theta}\right|_1\right) + \left.\frac{4K}{\epsilon A}\right|_1 T_1^3 + \left.\frac{Y}{\epsilon A}\right|_1\right]}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial T_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1\left[4\sigma T_2^3 + \left.\frac{wc}{\epsilon A}\right|_2 \frac{\partial}{\partial T_2}\left(\left.\frac{dT}{d\theta}\right|_2\right) + \left.\frac{4K}{\epsilon A}\right|_2 T_2^3 + \left.\frac{Y}{\epsilon A}\right|_2\right]}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \theta_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 \left.\frac{wc}{\epsilon A}\right|_1 \frac{\partial}{\partial \theta_1}\left(\left.\frac{dT}{d\theta}\right|_1\right)}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \theta_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 \left.\frac{wc}{\epsilon A}\right|_2 \frac{\partial}{\partial \theta_2}\left(\left.\frac{dT}{d\theta}\right|_2\right)}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

The second uses T_1, T_2, $dT/d\theta|_1$, and $dT/d\theta|_2$.

$$\frac{\partial P}{\partial T_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2\left(4\sigma T_1^3 + \left.\frac{4K}{\epsilon A}\right|_1 T_1^3 + \left.\frac{Y}{\epsilon A}\right|_1\right)}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial T_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 \left(4\sigma T_2{}^3 + \left.\frac{4K}{\epsilon A}\right|_2 T_2{}^3 + \left.\frac{Y}{\epsilon A}\right|_2\right)}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \left.\frac{dT}{d\theta}\right|_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 \left.\frac{wc}{\epsilon A}\right|_1}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \left.\frac{dT}{d\theta}\right|_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 \left.\frac{wc}{\epsilon A}\right|_2}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial F_P} = \frac{-P}{F_P} \qquad \frac{\partial P}{\partial F_a} = \frac{\partial P}{\partial F_S} = \frac{\partial P}{\partial S} = 0$$

$$\frac{\partial P}{\partial w_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 \left.\frac{c}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial w_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 \left.\frac{c}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial c_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 \left.\frac{w}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial c_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 \left.\frac{w}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2}{F_P\left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial A_1} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_2 \left[\left.\frac{wc}{\epsilon}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon}\right|_1 (T_1{}^4 - T_b{}^4) + \left.\frac{Y}{\epsilon}\right|_1 (T_1 - T_{b_1})\right]}{F_P A_1{}^2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial A_2} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_1 \left[\left.\frac{wc}{\epsilon}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon}\right|_2 (T_2{}^4 - T_b{}^4) + \left.\frac{Y}{\epsilon}\right|_2 (T_2 - T_{b_2})\right]}{F_P A_2{}^2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial T_{b_1}} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_2 (4K_1 T_{b_1}^3 + Y_1)}{\epsilon_1 A_1 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial T_{b_2}} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_1 (4K_2 T_{b_2}^3 + Y_2)}{\epsilon_2 A_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

When a common base plate is used for both sensors, and a single temperature measurement is made, the partial derivative is the sum of the previous two.

$$\frac{\partial P}{\partial T_b} = \frac{\epsilon_2 A_2 \left.\frac{\alpha_S}{\epsilon}\right|_2 (4K_1 T_{b_1}^3 + Y_1) - \epsilon_1 A_1 \left.\frac{\alpha_S}{\epsilon}\right|_1 (4K_2 T_{b_2}^3 + Y_2)}{\epsilon_1 \epsilon_2 A_1 A_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial K_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 (T_1{}^4 - T_{b_1}^4)}{\epsilon_1 A_1 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial K_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 (T_2{}^4 - T_{b_2}^4)}{\epsilon_2 A_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial Y_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 (T_1 - T_{b_1})}{\epsilon_1 A_1 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial Y_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 (T_2 - T_{b_2})}{\epsilon_2 A_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

[7] $$\frac{\partial P}{\partial \delta_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 F_a a S}{\epsilon_1 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$= \frac{-\left[\sigma T_2{}^4 - F_P P - \left.\frac{\alpha_S}{\epsilon}\right|_2 F_S S + \left.\frac{wc}{\epsilon A}\right|_2 \left.\frac{dT}{d\theta}\right|_2 + \left.\frac{K}{\epsilon A}\right|_2 (T_2{}^4 - T_{b_2}^4) + \left.\frac{Y}{\epsilon A}\right|_2 (T_2 - T_{b_2})\right]}{\epsilon_1 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \delta_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 F_a a S}{\epsilon_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$= \frac{\sigma T_1{}^4 - F_P P - \left.\frac{\alpha_S}{\epsilon}\right|_1 F_S S + \left.\frac{wc}{\epsilon A}\right|_1 \left.\frac{dT}{d\theta}\right|_1 + \left.\frac{K}{\epsilon A}\right|_1 (T_1{}^4 - T_{b_1}^4) + \left.\frac{Y}{\epsilon A}\right|_1 (T_1 - T_{b_1})}{\epsilon_2 F_P \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

$$\frac{\partial P}{\partial \nu_1} = \frac{-\left.\frac{\alpha_S}{\epsilon}\right|_2 P}{\epsilon_1 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_1\right)}$$

$$\frac{\partial P}{\partial \nu_2} = \frac{\left.\frac{\alpha_S}{\epsilon}\right|_1 P}{\epsilon_2 \left(\left.\frac{\alpha_S}{\epsilon}\right|_1 - \left.\frac{\alpha_S}{\epsilon}\right|_2\right)}$$

[7]See footnote 1, page 33.

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TABLE I.- MAGNITUDES AND UNCERTAINTIES OF VARIABLES EMPLOYED IN ILLUSTRATIVE EXAMPLES

Variable		White coating		Very low α_S/ϵ		Optical black		Solar constant	
		Magnitude	Uncertainty	Magnitude	Uncertainty	Magnitude	Uncertainty	Magnitude	Uncertainty
α_S,	Dimensionless	0.18	(1)	0.04	0.004	0.98	0.02	0.97	0.02
ϵ,	Dimensionless	0.88	0.02	0.80	0.02	0.95	0.02	(1)	$\begin{cases} 0.02 \text{ for } 0.2 \leq \epsilon \leq 1 \\ 10\% \ \epsilon \text{ for } 0.1 \leq \epsilon \leq 0.2 \end{cases}$
S,	W/m^2	1360	0.02 S	(2)	(2)	(2)	(2)	(1)	(1)
P,	W/m^2	250	0.10 P	(1)	(1)	(1)	(1)	(2)	(2)
a,	Dimensionless	0.30	0.20	(2)	(2)	(2)	(2)	(2)	(2)
F_S,	Dimensionless	0.32	0.01 F_S	(2)	(2)	(2)	(2)	1	(2)
F_P,	Dimensionless	(3)	0.02 F_P	(2)	(2)	(2)	(2)	0	(2)
F_a,	Dimensionless	(3)	0.05 F_a	(2)	(2)	(2)	(2)	0	(2)
w,	g	0.30	0.01 w	(2)	(2)	(2)	(2)	(2)	(2)
c,	J/g °K	0.80	0.10 c	(2)	(2)	(2)	(2)	(2)	(2)
T,	°K	(3)	1	(2)	(2)	(2)	(2)	(2)	(2)
$dT/d\theta$,	°K/sec	(3)	$\begin{cases} 0.10 \ dt/d\theta \\ \text{min value } 0.005 \ ^\circ K/sec \end{cases}$	(2)	(2)	(2)	(2)	0	0.005
T_b,	°K	300	1	(2)	(2)	(2)	(2)	350	(2)
A,	m^2	0.0005	0.01 A	(2)	(2)	(2)	(2)	(2)	(2)
K,	$W/^\circ K^4$	1.5×10^{-12}	0.10 K	(2)	(2)	(2)	(2)	(2)	(2)
Y,	W/°K	0	0	(2)	(2)	(2)	(2)	(2)	(2)
δ,	Dimensionless	0	0	(2)	(2)	(2)	(2)	(2)	(2)
ν,	Dimensionless	0	0	(2)	(2)	(2)	(2)	(2)	(2)

[1]Values not required for illustrative example.
[2]Values are the same as those for the white coating.
[3]Values are functions of position in orbit; these values determined by computer.

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