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SUMMARY

The effects of cable and circuit parameters on the precision calibration of a charge amplifier for microsecond pulses, such as those obtained from semiconductor diode particle detectors, are evaluated. The precision calibration method is described. A mercury-wetted relay pulse generator driving a 1-picofarad capacitor was used as a charge generator. The accuracy of the calibration was ±0.4 percent.

INTRODUCTION

The effects of cable and circuit parameters on the calibration of a charge amplifier are important whenever precision measurements are made with charge amplifiers. Charge amplifiers are frequently used to reduce the effect of variations of charge source capacitance, cable capacitance, and input capacitance of associated electronics on the amplifier output voltage (ref. 1).

A charge amplifier is used with a piezoelectric transducer for applications such as pressure or acceleration measurements (ref. 2). Another application is made in nuclear radiation spectroscopy, where semiconductor diodes are used as detectors. One type of semiconductor detector consists of a reverse-biased p-n junction (ref. 1). Incident radiation causes charge generation in the depletion region and results in a current pulse with a duration of about 10 to 50 nanoseconds. The current pulse from the detector drives a charge amplifier. The amplifier output pulse drives a multichannel pulse height analyzer. The capacitance of the detector junction varies with the value of the detector reverse-bias voltage. Therefore, when a voltage pulse amplifier is used, the output is a function of the detector bias voltage and is not proportional to just the total charge q as desired. After pulse charge amplifiers were developed, they were used to measure q directly, since the detector capacitance has only a minor effect on the amplifier output voltage.

A fundamental quantity in nuclear radiation spectroscopy is the energy required to produce an electron-hole pair in a particular semiconductor detector material. The energy per electron-hole pair must be known accurately in order to calculate, from the charge collected, the energy deposited in the detector by an incident charged particle. The energy required to produce an electron-hole pair in a particular material can be calculated from the ratio of the independently measured energy deposited in the material by a test particle to the charge collected (ref. 3). The accuracy of this method depends on the accuracy to which the charge collected can be measured and, therefore, directly on the accuracy of the charge amplifier system calibration.

An accurate method for calibration of a charge amplifier - pulse height analyzer system is described in this report. The accuracy of the calibration is determined, and the effects of cable and circuit parameters on the calibration are evaluated.

DESCRIPTION OF METHOD

The calibration of a charge amplifier may be considered in terms of evaluating the two parameters in the following equation:

$$V_o = aq_i + b \tag{1}$$

where V_o is the output voltage, q_i is the input charge, and a and b are constants. (All symbols are defined in appendix A.) The quantity b has, of course, the same units as V_o, and a has the units of inverse capacitance. In the case of nuclear particle detectors, where the input charge q_i is in the form of a fast pulse, the output V_o is usually measured not in volts but in terms of the pulse height in units of channel number (proportional to V_o) in a multichannel analyzer.

The quantities a and b are determined by feeding known amounts of charge into the amplifier, measuring the average channel number as a function of q_i, and then using a least-squares fit to find a and b. After a and b have been determined, it is possible to calculate the amount of charge q_i corresponding to an output voltage (or channel number) obtained during an experimental measurement. The quantity a, however, is not always the same for the calibration as during the measurement, although for a carefully designed calibration only small corrections are required. One correction treated is the effect of cable capacitance, which may be different during calibration and measurement. It is often easier to make a minor correction for cable capacitance than to exactly match cable lengths or capacitances. The other correction arises from the resistance-capacitance (RC) time-constant decay of the charge on capacitor C_p through R_1 and R_2 in the calibrating circuit (see fig. 1). This decay has no counterpart in most measuring circuits. These corrections are normally a few percent or less. Also, the value of b is normally small compared with aq_i but must be included for accurate calibration and measurement.

A block diagram of the electronic system used in the calibration of the charge amplifier - pulse height analyzer system is presented in figure 1. This system consists of a precision mercury-wetted relay pulse generator, two charge amplifiers, a coincidence unit, and an 800-channel pulse height analyzer. The analyzer consists of an analog-to-digital converter which assigns a channel number to each input pulse proportional to its amplitude, a 10^6-counts-per-channel memory unit which records the number of counts falling into each channel number, and a cathode ray tube display or readout device (ref. 4). The coincidence unit receives its signals from the two charge amplifiers and delivers a gate signal to the analyzer if the two signals from the charge amplifiers are in time coincidence within about 6 microseconds. This gate signal enables the analyzer to accept the signals supplied to it from one of the charge amplifiers. The purpose of using a coincidence unit is to minimize the number of electronic noise signals counted by the pulse height analyzer. Although the calibration as reported is for a system with two charge amplifiers with coincidence, a system with only a single charge amplifier and with the coincidence unit removed would work equally well.

The pulse generator (ref. 5) has two outputs, as indicated in figure 1. However,

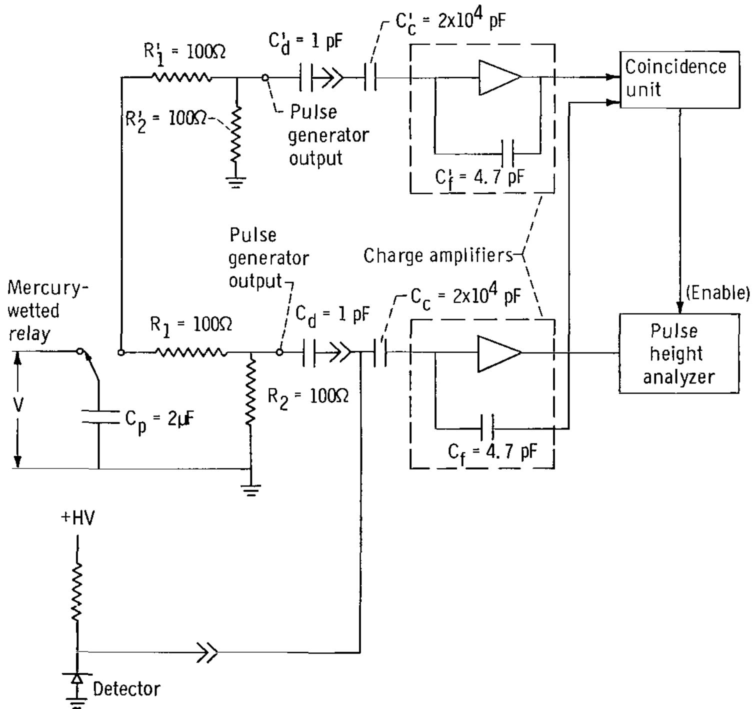

Figure 1. - System for charge calibration and measurement.

for simplicity, the pulse generator is described with only one of the outputs connected, since the two outputs of the generator and their loads were identical.

A mercury-wetted relay operating at 60 hertz, alternately connected the initial voltage V to the capacitor C_p and C_p to the resistor network R_1 and R_2, which were both 100 ohms to match the 50-ohm line normally used. The use of mercury-wetted relay contacts at low voltage prevents contact arcing and bounce and, hence, provides short-rise time and jitter-free pulses. Rise times less than 10 nanoseconds and fall times of 200 to 400 microseconds resulted. The pulse generator was charge terminated (i.e., a 1-pF capacitor (C_d) was connected between the middle point of the voltage divider (R_1 and R_2) and the charge amplifier input). The charge amplifier input was held near zero potential by the capacitor feedback network. The input circuit to the amplifier is shown in figure 2. Charge integration takes place on the amplifier feedback capacitor C_f. A 20 000-picofarad coupling capacitor (C_c) serves as a direct-current blocking capacitor when the amplifier system is operated with a reverse-biased semiconductor detector. The generator pulse amplitude was varied by adjusting the voltage V applied to C_p.

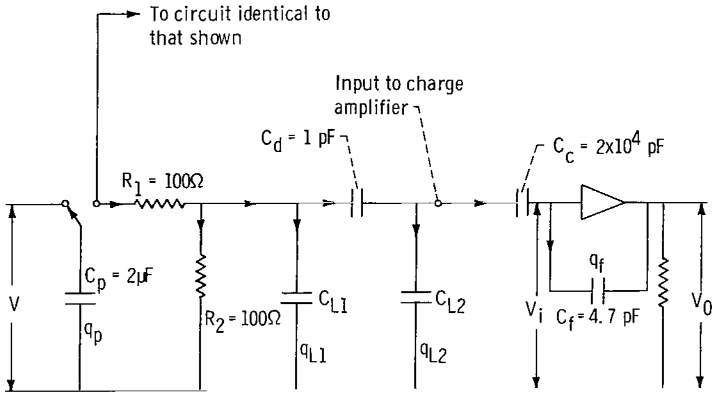

Figure 2. - Charges and currents in circuit analyzed to find charge on charge amplifier feedback capacitance.

CALIBRATION EQUATION

The output of the measuring system is proportional to the charge on the feedback capacitor C_f. Thus,

$$V_o - b \propto PH - PH' \equiv N = Kq_f \tag{2}$$

where the constant K is independent of the corrections given in this section and is in units of channels per coulomb, and where PH' is a small zero correction in the pulse height analyzer and is the pulse height corresponding to zero input. The feedback capacitor charge q_f may be written in terms of the input charge q_i as follows:

$$q_f = F_C F_{RC} q_i \tag{3}$$

and

$$N = K F_C F_{RC} q_i \tag{4}$$

where F_C and F_{RC} are the cable-capacitance and time-constant corrections, respectively.

The expressions for F_C, F_{RC}, and q_i are derived in appendix B. The results are

$$F_C = \left(1 + \frac{C_{L2} + C_d}{C_i}\right)^{-1} \tag{5}$$

$$F_{RC} = \exp\left(- \frac{2T}{(R_1 + R_2)C_p}\right) \tag{6}$$

and

$$q_i = \frac{VR_2C_d}{R_1 + R_2} \tag{7}$$

Combining these equations gives

$$K = N\left(1 + \frac{C_{L2} + C_d}{C_i}\right)\left\{\exp\left[\frac{2T}{(R_1 + R_2)C_p}\right]\right\}\left(\frac{VR_2C_d}{R_1 + R_2}\right)^{-1} \tag{8}$$

The quantity K, as discussed earlier, is the proportionality constant relating the measured pulse height and the charge on the feedback capacitance. The first factor N on the right side of the equation is the corrected pulse height. The second factor corrects for the loading effects of cable capacitance, while the third factor corrects for RC decay on the pulse generator capacitor C_p during integration. The last factor (in V, R_1, R_2, and C_d) is simply the charge placed on the capacitor C_d when a voltage $VR_2/(R_1 + R_2)$ is impressed upon it. In equation (8), V is the initial voltage on C_p; R_1, R_2, and C_d are defined by the circuit of figure 2; C_{L2} is the capacitance of the calibration cable connecting C_d and the charge amplifier input; and C_i is the experimentally measured value of the effective amplifier input capacitance (eq. (B2)). In the second factor, C_d can be neglected since $C_d \ll C_{L2}$. The quantity $1/C_i$ was evaluated from the slope of the experimental curve presented in figure 3 and is explained in the section CABLE-CAPACITANCE CORRECTION.

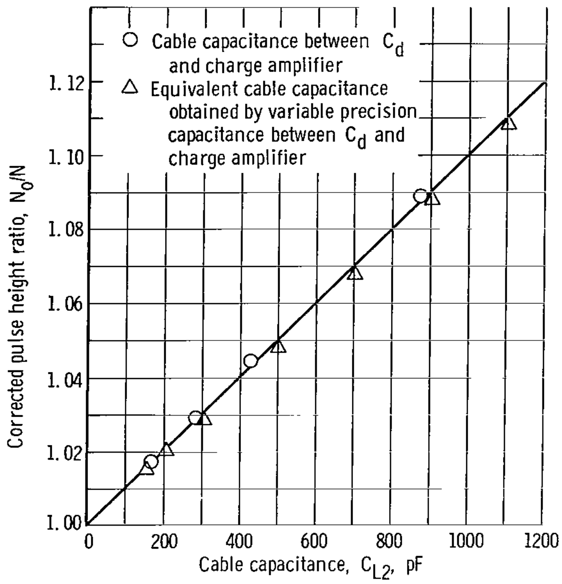

Figure 3. - Relation between pulse height and cable capacitance.

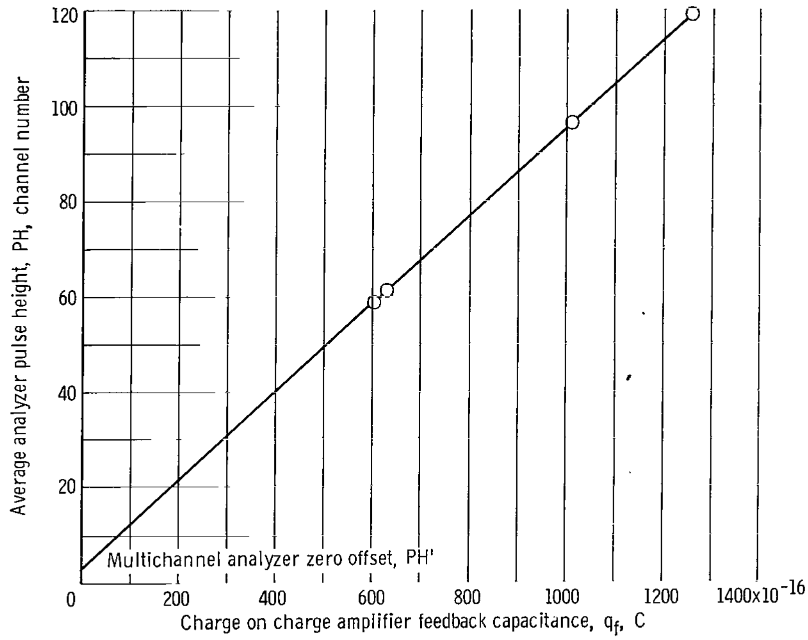

Figure 4. - Average analyzer pulse height as function of calibration charge.

The effect of the cable capacitance C_{L1} between the junction of the two voltage divider resistors (R_1 and R_2) and C_d is shown to be negligible in appendix B (the inequality following eq. (B36)). This result is in agreement with those of Emery and Rabson (ref. 6).

The analyzer pulse heights (channel numbers) corresponding to the charge q_f, calculated according to equation (B42), were weighted with respect to their frequency of occurrence. A least-squares fit of the average analyzer pulse height as a function of the calibration charge q_f is given in figure 4. The curve in figure 4 enables the determination of both K and PH' and, therefore, constitutes the charge amplifier - pulse height calibration curve. The correction factors given in equation (8) were used to obtain q_f from q_i.

After the calibration is complete, the value of K as determined from equation (8) can be used in the determination of an unknown charge. For the case of a semiconductor detector of capacitance C_{det}

$$q_x = \frac{N_x}{K}\left(1 + \frac{C_{LX} + C_{det}}{C_i}\right) \tag{9}$$

where N_x is the experimentally measured value of corrected pulse height and q_x is the unknown charge to be measured. In an actual measurement, there is, of course, no factor for exponential decay of charge on the pulse generator capacitor C_p. However, the detector capacitance C_{det}, which is now in parallel with the cable capacitance C_{LX}, must be included as an additive term in the cable-capacitance correction factor. Also, any other capacitance in the input must be included.

SYSTEM-RESPONSE-TIME CORRECTION

The factor $\exp\left[-2T/(R_1 + R_2)(C_p)\right]$ in equation (6) is caused by decay of the voltage on the 2-microfarad capacitor C_p while the charge amplifier is integrating the current pulse signal. The value of the system response time T in this term is 1.3 microseconds. This value was obtained from the slope of the experimental curve presented in figure 5, which is a plot of N against $1/C_p$. The magnitude of this factor is 0.9936,

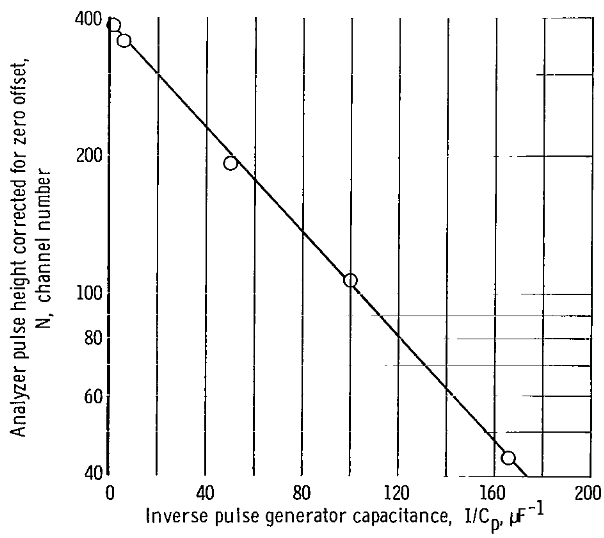

Figure 5. - Effect of pulse generation capacitance on output pulse height.

and has only a small effect on q_f and, hence, on the charge calibration. The reason for the small effect on this factor on q_f is that the time constant for the discharge of C_p is large compared with the system response time.

CABLE-CAPACITANCE CORRECTION

In the derivation of the equation for q_f, the cable between C_d and the input to the charge amplifier was assumed to be a lumped capacitor located at the input to the charge amplifier. When the equation for q_f and the transformation relation $N = Kq_f$ are used, the following expression can be obtained:

$$\frac{N_o}{N} = 1 + \frac{C_{L2} + C_d}{C_i} \tag{10}$$

where N is the corrected analyzer pulse height, N_o is the corrected analyzer pulse height if the capacitance loading is negligible (i.e., if $C_{L2} << C_i$). This linear equation (eq. (10)), based on the lumped-capacitor assumption, is in agreement with the experimental curve presented in figure 3. The quantity C_i is the inverse slope of this curve. The value of C_i determined from the slope of the experimental curve is substantially larger than that calculated from equation (B2), probably because stray capacitance was not taken into account in the value of C_f. Equation (10) can also be written as follows:

$$N = \frac{N_o}{1 + \frac{C_{L2} + C_d}{C_i}} \tag{11}$$

which gives the explicit dependence of N on the cable capacitance C_{L2}. The quantity C_d is included also even though for this particular calibration $C_d << C_{L2}$ and can be ignored, as mentioned earlier. A plot of N against cable capacitance is given in figure 6. Since the correction term is small, the plot of N against C_{L2} appears as a straight line, which means that the approximation

$$N = N_o\left(1 + \frac{C_{L2} + C_d}{C_i}\right)^{-1} \approx N_o\left(1 - \frac{C_{L2} + C_d}{C_i}\right) \tag{12}$$

is valid.

CABLE EFFECTS OTHER THAN CAPACITANCE LOADING ON CHARGE AMPLIFIER - PULSE HEIGHT ANALYZER CALIBRATION

The decrease in pulse height with increasing cable capacitance, shown in figure 6, could be due to either cable capacitance loading or other cable effects, such as reflection, attenuation, or inductance. In an attempt to distinguish between these effects, a precision variable capacitor was inserted (fig. 2) between C_d and the input to the charge amplifier, parallel to the amplifier input. This capacitance to ground was varied, for a fixed-pulse generator output, and the corresponding pulse heights were registered by the pulse height analyzer.

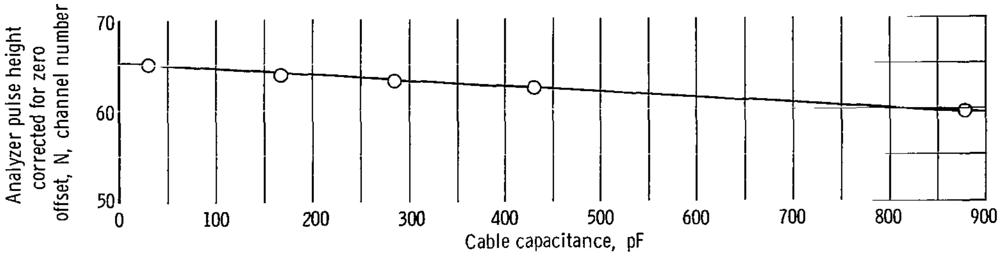

Figure 6. - Analyzer pulse height as function of cable capacitance. Cable capacitance, 94 picofarads per meter.

Next, the variable capacitor was removed from the circuit, and cables of various lengths, and hence, various capacitances were inserted between C_d and the input of the charge amplifier. The corresponding pulse heights were recorded. A curve of the pulse height ratio N_0/N as a function of capacitance for the variable capacitance-to-ground case and for the variable cable-length case (cable capacitance, 94 pF/m) is presented in figure 3. The curves coincide which indicates that cable effects other than capacitance have a negligible effect on the calibration.

As an additional check on reflection losses due to mismatch of the cable either at the sending or receiving end, two cables differing in characteristic impedance by a factor of two (50- and 100-Ω cables) but of equal total capacitance were used. The average analyzer pulse height ratio for these two cases was 1.0002 ± 0.0008. Hence, the factor-of-two difference in cable characteristic impedance has no appreciable effects on the average analyzer pulse height and, hence, on q_f; therefore, it is appropriate to replace the cable by its lumped capacitance in making corrections for cable effects.

This conclusion is further substantiated by the agreement of equation (10) with experimental results as shown in figure 3 and also by the fact that cable capacitance and additional capacitance in the form of a variable capacitance produced the same effect on the analyzer pulse height.

ERROR ANALYSES

The total error in the calibration was taken as the rms error of the systematic and statistical errors. The systematic errors are caused by the uncertainties in the parameters on the right side of equation (8). The statistical error, based on a 95-percent confidence level, was computed from the least-squares fit of the charge calibration curve. The total rms error in the calibration constant K was ±0.4 percent and was based on the data presented in table I. Table I consists of a list of parameters,

TABLE I. - RMS CONTRIBUTION TO ERROR IN K

[Σ (Systematic errors)^2, = (0.10 percent)^2;
statistical error, = 0.24 percent;
total rms error, = 0.4 percent.]

Parameter	Error in parameter, percent	rms contribution to error in K, percent
C_i	15	0.21
C_{L2}	10	.16
C_d	.1	.1
C_p	15	.1
R_1	.1	.05
R_2	.1	.05
V	.05	.05
T	1.2	.008

their percent errors, and their percent rms contribution to the total error in K. Data in this table indicate that the systematic error contributes more than the statistical error to the total rms error in K and also that C_i was the primary contributor to the systematic error.

For the purposes of error analysis, the error in C_i was taken to be about ±15 percent. This estimate includes a maximum 10 percent error caused by cable capacitance and errors in the quantity N_0/N. This error in C_i contributes 0.21 percent to the total error in K (table 1). The total rms error in the calibration was ±0.4 percent.

In an actual measurement, the determination of N_x and the cable-capacitance correction factor also contribute errors (eq. (9)). If the cable correction factors are nearly the same in the calibration and measurement cases, the combined error due to both the cable-capacitance corrections decreases. For a reasonably well-designed experiment, the total error, including both calibration and measurement, is probably about twice the calibration error, or about 1 percent.

SUMMARY OF RESULTS

In this evaluation of the effects of cable and circuit parameters on the precision calibration of a charge amplifier for microsecond pulses such as those obtained from semiconductor detector diodes, the following results were obtained:

1. The accuracy of calibration was ±0.4 percent.

2. The effect of cable capacitance on charge calibration was small but significant; however, calibration can be corrected for this effect.

3. Cable effects on charge calibration other than capacitance, such as reflection, attenuation, or inductance, were negligible.

4. The effects of circuit parameters on charge calibration were evaluated.

5. Effects of varying cable length on charge calibration were accurately approximated by treating the cable as a lumped capacitance. A derivation was made of the charge on the amplifier feedback capacitor (and therefore the amplifier output voltage) in terms of electrical parameters and input voltage of the calibration circuit.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 14, 1967,
120-27-01-03-22.

APPENDIX A

SYMBOLS

A	constant defined by eq. (B35)
a	constant defined by eq. (1), F^{-1}
b	constant defined by eq. (1), volts
C_c, C'_c	charge amplifier coupling capacitors (capacitance, 2×10^4 pF) (see fig. 1)
C_d, C'_d	charge termination capacitors (capacitance, 1 pF) (see fig. 1)
C_{det}	capacitance of detector, pF
C_f, C'_f	charge amplifier feedback capacitors (nominal capacitance, 4.7 pF) (see fig. 1) (Actual value may be larger because of stray capacitance.)
C_i	experimentally determined value of equivalent amplifier input capacitance (1×10^4 pF)
C_{L1}	cable capacitance to ground at junction of C_d and R_2 (approx. 100 pF) (see fig. 2)
C_{L2}	cable capacitance between C_d and charge amplifier input (approx. 170 pF) (see fig. 2)
C_{LX}	detector cable capacitance between detector and charge amplifier input, pF
C_p	capacitance (2 μF) (see fig. 2)
F_C	cable-capacitance correction
F_{RC}	time-constant correction
G	amplifier open-loop voltage gain (-1750)
K	constant defined by eq. (2), channels/C
K_1	constant defined by eq. (B10), 1×10^{10}/sec
K_2	constant defined by eq. (B11), 101
K_3	constant defined by eq. (B12), 1/pF
K_4	constant defined by eq. (B19), 2×10^8/sec
K_5	constant defined by eq. (20), $1\times10^{12}/\mathrm{sec}^2$
N	analyzer pulse height corrected for zero offset, PH - PH', analyzer channel number

N_o	limiting value of N for $C_{L2} + C_d << C_i$
N_x	pulse height analyzer channel number corrected for analyzer offset, experimental case
PH	average analyzer pulse height, analyzer channel number
PH'	multichannel analyzer zero offset, analyzer channel number
q	total charge
q_d	charge on C_d, C (see fig. 2)
q_f	charge on C_f, C
q_i	input charge, C
q_{L1}	charge on C_{L1}, C (see fig. 2)
q_{L2}	charge on C_{L2}, C (see fig. 2)
q_p	charge on C_p, C (see fig. 2)
q_x	charge to be measured from detector or other noncalibration source, C
R_1, R_1'	resistors (100 Ω) (see fig. 1)
R_2, R_2'	resistors (100 Ω) (see fig. 1)
T	system response time, 1.3 μsec
t	time, sec
V	initial voltage across C_p. volts
V_i	input voltage of charge amplifier, volts (see fig. 2)
V_o	output voltage of charge amplifier, volts (see fig. 2)

APPENDIX B

DERIVATION OF CALIBRATION EQUATION

A derivation of the expression used to determine the charge on the amplifier feedback capacitor C_f (and, therefore, the amplifier output voltage) is given below. As a starting point in this derivation, consider the circuit diagram presented in figure 2. From the application of Kirchhoff's circuit laws, the differential equation for the charge on the amplifier feedback capacitor may be obtained. The solution to this differential equation yields the charge on C_f.

The following equation was obtained from Kirchhoff's circuit laws:

$$q_d = q_f\left\{1 + C_{L2}\left[\frac{1}{C_c} + \frac{1}{C_f(1 - G)}\right]\right\} \tag{B1}$$

Define

$$\frac{1}{C_i} = \frac{1}{C_c} + \frac{1}{C_f(1 - G)} \tag{B2}$$

Substituting the constant $1/C_i$ into equation (B1) yields

$$q_d = q_f\left(1 + \frac{C_{L2}}{C_i}\right) \tag{B3}$$

Kirchhoff's circuit laws also yield

$$-\frac{\dot{q}_p}{2} = \frac{q_d}{R_2C_d} + \frac{q_f}{R_2C_i} + \left(1 + \frac{C_{L1}}{C_d}\right)\dot{q}_d + \frac{C_{L1}\dot{q}_f}{C_i} \tag{B4}$$

and

$$\frac{q_p}{C_p} = -\frac{R_1\dot{q}_p}{2} + \frac{q_d}{C_d} + \frac{q_f}{C_i} \tag{B5}$$

Eliminating q_d from equations (B4) and (B5) by using equation (B3) gives

$$-\frac{\dot{q}_p}{2} = \left[\frac{1}{R_2C_d}\left(1 + \frac{C_{L2}}{C_i}\right) + \frac{1}{R_2C_i}\right]q_f + \left(\frac{C_{L1} + C_d}{C_d}\right)\left(1 + \frac{C_{L2}}{C_i}\right)\dot{q}_f + \frac{C_{L1}}{C_i}\dot{q}_f \tag{B6}$$

$$-\frac{\dot{q}_p}{2} = \frac{q_f}{R_2C_d}\left(1 + \frac{C_{L2} + C_d}{C_i}\right) + \dot{q}_f\left\{\frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i}\left[C_{L1} + C_{L2}\left(\frac{C_{L1} + C_d}{C_d}\right)\right]\right\} \tag{B7}$$

and

$$\frac{q_p}{C_p} = -R_1\frac{\dot{q}_p}{2} + \frac{q_f}{C_d}\left(1 + \frac{C_{L2}}{C_i}\right) + \frac{q_f}{C_i} \tag{B8}$$

$$\frac{q_p}{C_p} = \frac{-R_1\dot{q}_p}{2} + \frac{q_f}{C_d}\left(1 + \frac{C_{L2} + C_d}{C_i}\right) \tag{B9}$$

The constants K_1, K_2, and K_3 are defined by the following equations:

$$K_1 = \frac{1}{R_2C_d}\left(1 + \frac{C_{L2} + C_d}{C_i}\right) \tag{B10}$$

$$K_2 = \frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i}\left[C_{L1} + \frac{C_{L2}}{C_d}\left(C_{L1} + C_d\right)\right] \tag{B11}$$

$$K_3 = \frac{1}{C_d}\left(1 + \frac{C_{L2} + C_d}{C_i}\right) \tag{B12}$$

Substituting these constants into equations (B7) and (B9) yields

$$\frac{-\dot{q}_p}{2} = K_1q_f + K_2\dot{q}_f \tag{B13}$$

and

$$\frac{q_p}{C_p} = -R_1\frac{\dot{q}_p}{2} + K_3q_f \tag{B14}$$

Taking the derivatives of equations (B13) and (B14) yields

$$\frac{-\ddot{q}_p}{2} = K_1\dot{q}_f + K_2\ddot{q}_f \tag{B15}$$

and

$$\frac{\dot{q}_p}{C_p} = -R_1\frac{\ddot{q}_p}{2} + K_3\dot{q}_f \tag{B16}$$

Substituting the right side of equation (B13) and the right side of equation (B15) into the left and right sides of equation (B16), respectively, yields

$$-\frac{2}{C_p}\left(K_1q_f + K_2\dot{q}_f\right) = R_1\left(K_1\dot{q}_f + K_2\ddot{q}_f\right) + K_3\dot{q}_f \tag{B17}$$

or

$$R_1K_2\ddot{q}_f + \left(K_3 + R_1K_1 + \frac{2K_2}{C_p}\right)\dot{q}_f + \frac{2}{C_p}K_1q_f = 0 \tag{B18}$$

The constants K_4 and K_5 are defined as

$$K_4 = \frac{K_3 + R_1K_1 + \frac{2K_2}{C_p}}{R_1K_2} \tag{B19}$$

$$K_5 = \frac{2K_1}{C_pR_1K_2} \tag{B20}$$

Dividing equation (B18) by R_1K_2 and substituting K_4 and K_5 into equation (B18) yields

$$\ddot{q}_f + K_4\dot{q}_f + K_5q_f = 0 \tag{B21}$$

Applying the initial condition $q_f(0) = 0$ yields the following solution:

$$q_f(t) = A\left\{\exp\left[-\frac{K_4 t}{2} + \frac{\sqrt{K_4^2 - 4K_5}\,t}{2}\right] - \exp\left[-\frac{K_4 t}{2} - \frac{\sqrt{K_4^2 - 4K_5}\,t}{2}\right]\right\} \tag{B22}$$

where

$$K_4 = \frac{\frac{1}{C_d}\left(1+\frac{C_{L2}+C_d}{C_i}\right)+\frac{R_1}{R_2 C_d}\left(1+\frac{C_{L2}+C_d}{C_i}\right)+\frac{2}{C_p}\left\{\frac{C_{L1}+C_d}{C_d}+\frac{1}{C_i}\left[C_{L1}+\frac{C_{L2}}{C_d}\left(C_{L1}+C_d\right)\right]\right\}}{R_1\left\{\frac{C_{L1}+C_d}{C_d}+\frac{1}{C_i}\left[C_{L1}+\frac{C_{L2}}{C_d}\left(C_{L1}+C_d\right)\right]\right\}} \tag{B23}$$

$$K_4 = \frac{(R_1 + R_2)(C_i + C_{L2} + C_d)}{R_1 R_2 C_d C_i\left\{\frac{C_{L1}+C_d}{C_d}+\frac{1}{C_i}\left[C_{L1}+\frac{C_{L2}}{C_d}\left(C_{L1}+C_d\right)\right]\right\}} + \frac{2}{R_1 C_p} \tag{B24}$$

$$K_5 = \frac{\frac{2}{R_2 C_d}\left(1+\frac{C_{L2}+C_d}{C_i}\right)}{R_1 C_p\left\{\frac{C_{L1}+C_d}{C_d}+\frac{1}{C_i}\left[C_{L1}+\frac{C_{L2}}{C_d}\left(C_{L1}+C_d\right)\right]\right\}} \tag{B25}$$

Comparing the numerical value of $4K_5$ and K_4^2 results in $4K_5 \ll K_4^2$, where K_4 and K_5 are defined by equations (B24) and (B25), respectively; and C_p = 2 microfarads, C_d = 1 picofarad, $C_{L1} \cong 100$ picofarads, $C_{L2} \cong 170$ picofarads, $C_i = 1\times10^4$ picofarads, R_1 = 100 ohms, R_2 = 100 ohms, $K_4 = 2\times10^8$ per second, and $K_5 = 1\times10^{12}$ per second^2. Therefore, the radical term in equation (B22) can be approximated by the first two terms of its Taylor series expansion as follows:

$$\sqrt{K_4^2 - 4K_5} = K_4\sqrt{1 - \frac{4K_5}{K_4^2}} \tag{B26}$$

$$\sqrt{K_4^2 - 4K_5} = K_4\left[1 - \frac{2K_5}{K_4^2} - \left(\frac{K_5}{K_4^2}\right)^2 \frac{1}{8} - \ldots\right] \tag{B27}$$

and

$$-\frac{K_4}{2} + \frac{\sqrt{K_4^2 - 4K_5}}{2} = -\frac{K_5}{K_4} \tag{B28}$$

$$-\frac{K_4}{2} - \frac{\sqrt{K_4^2 - 4K_5}}{2} = -K_4 + \frac{K_5}{K_4} \tag{B29}$$

Substituting equation (B29) into equation (B22) yields

$$q_f = A\left\{\exp\left[-\frac{K_5}{K_4}t\right] - \exp\left[\left(-K_4 + \frac{K_4}{K_5}\right)t\right]\right\} \tag{B30}$$

$$q_f = A\exp\left[-\frac{K_5}{K_4}t\right]\left\{1 - \exp\left[-K_4t + 2\frac{K_5}{K_4}t\right]\right\} \tag{B31}$$

$$q_f = A\exp\left[-\frac{K_5}{K_4}t\right]\left\{1 - \exp\left[-K_4t\left(1 + 2\frac{K_5}{K_4^2}\right)\right]\right\} \tag{B32}$$

However, because $K_5 << K_4^2$, to a good approximation

$$q_f = A\exp\left[-\frac{K_5}{K_4}t\right]\left(1 - \exp\left[-K_4t\right]\right) \tag{B33}$$

Evaluate A from the remaining initial condition which is $-\dot{q}_p/2 = V/R_1$ at the time $t = 0$. Taking the derivative of equation (B33) evaluated at time $t = 0$ and substituting it into the right side of equation (B13), using this condition, give

$$\frac{V}{R_1} = K_2K_4A \tag{B34}$$

Hence,

$$A = \frac{V}{R_1K_2K_4} \tag{B35}$$

The denominator of equation (B35)

$$R_1K_2K_4 = \frac{\left(R_1 + R_2\right)}{R_2C_d}\left(1 + \frac{C_{L2} + C_d}{C_i} + \frac{R_2C_d}{C_p(R_1 + R_2)}\left\{\frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i}\left[C_{L1} + \frac{C_{L2}}{C_d}\left(C_{L1} + C_d\right)\right]\right\}\right) \tag{B36}$$

and

$$\left(1 + \frac{C_{L2} + C_d}{C_i}\right) >> \left(\frac{R_2C_d}{C_p(R_1 + R_2)}\left\{\frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i}\left[C_{L1} + \frac{C_{L2}}{C_d}\left(C_{L1} + C_d\right)\right]\right\}\right)$$

(approximately six orders of magnitude greater).
Therefore,

$$R_1K_2K_4 \cong \frac{\left(R_1 + R_2\right)}{R_2C_d}\left(1 + \frac{C_{L2} + C_d}{C_i}\right) \tag{B37}$$

Substituting the right side of equation (B37) into the denominator of equation (B35) yields

$$A = \frac{VR_2C_d}{(R_1 + R_2)\left(1 + \frac{C_{L2} + C_d}{C_i}\right)} \tag{B38}$$

In equation (B24),

$$\frac{(R_1 + R_2)(C_i + C_{L2} + C_d)}{R_1R_2C_dC_i\left\{\frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i}\left[C_{L1} + \frac{C_{L2}}{C_d}\left(C_{L1} + C_d\right)\right]\right\}} >> \frac{2}{R_1C_p}$$

by about four orders of magnitude. Hence, to a good approximation

$$K_4 = \frac{(R_1 + R_2)(C_i + C_{L2} + C_d)}{R_1 R_2 C_d C_i \left\{ \frac{C_{L1} + C_d}{C_d} + \frac{1}{C_i} \left[C_{L1} + \frac{C_{L2}}{C_d} \left(C_{L1} + C_d \right) \right] \right\}} \tag{B39}$$

Taking the ratio K_5/K_4 by using this expression for K_4 gives

$$\frac{K_5}{K_4} = \frac{2}{(R_1 + R_2)C_p} \tag{B40}$$

Comparing the magnitude of the terms exp $-K_4 t$ and exp $-(K_5/K_4)t$ in equation (B33) reveals that $K_4 = 2\times10^8$ per second and $K_5/K_4 = 5\times10^3$ per second. Therefore, exp $-(K_5/K_4)t >>$ exp $-K_4 t$ and, for times of the order of a microsecond, q_f can be approximated as follows:

$$q_f = A \exp\left[-\frac{K_5}{K_4} t \right] \tag{B41}$$

Substituting equations (B38) and (B40) into the right side of equation (B41) yields

$$q_f = \left(1 + \frac{C_{L2} + C_d}{C_i}\right)^{-1} \left\{ \exp\left[-\frac{2T}{(R_1 + R_2)C_p} \right] \right\} \frac{VR_2 C_d}{R_1 + R_2} \tag{B42}$$

where t has been replaced by the system response time T. Equation (B42) may be written as

$$q_f = F_c F_{RC} q_i \tag{B43}$$

Each factor is discussed individually in the main text.

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