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Lewis Research Center
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SUMMARY

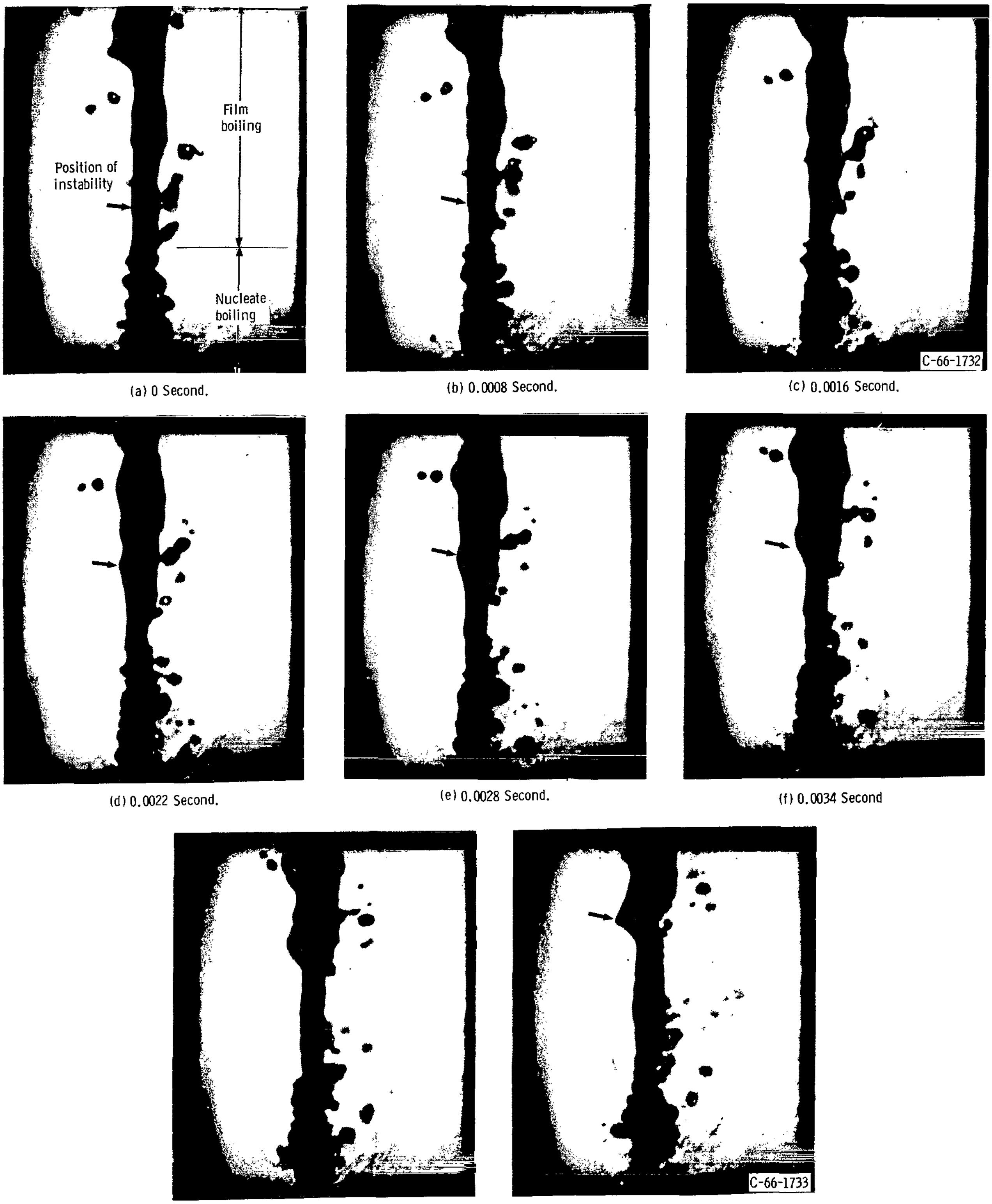
A study was made of the minimum film-boiling heat flux of liquid nitrogen flowing in a vertical, electrically heated tube. Inlet liquid velocities up to 3 meters per second were studied for reduced pressures P/P_c of 0.071, 0.20, and 0.40. An analysis of the wall conduction was used to predict the wall temperatures occurring at the minimum heat flux for an electrically heated system. The predicted wall temperatures were then used in conjunction with a heat-transfer analysis for predicting the minimum heat flux. The analytical results are in good agreement with the experimental data.

INTRODUCTION

Little attention has been given to the stability of film boiling in forced-convection flow. Theoretical and experimental work on the stability of film boiling in a nonflow system was done in an effort to determine the minimum heat flux required to maintain film boiling (refs. 1 to 3). Although no theories exist in the literature for the minimum heat flux of a flow system, some experimental evidence is available.

Critical heat-flux studies for liquid nitrogen and hydrogen flowing in a vertical, electrically heated tube (ref. 4) demonstrate that the heat flux may be lowered below the critical heat flux and still permit stable film boiling. Dougall (ref. 5) noted that, when the heat flux was too low to maintain film boiling, nucleate boiling would appear at the tube inlet and then spread until the entire tube was in nucleate boiling. Dougall assumed that this effect was governed by axial conduction.

If an axial temperature gradient exists in the heating surface, then it is possible to have nucleate and film boiling coexisting along the surface. This situation can occur when the heat flux is being controlled such as in the case of electrical heating. The co-



(a) 0 Second. (b) 0.0008 Second. (c) 0.0016 Second.

(d) 0.0022 Second. (e) 0.0028 Second. (f) 0.0034 Second

(g) 0.0042 Second. (h) 0.0054 Second

Figure 1. - Coexistence of nucleate and film boiling in region of minimum heat flux. Velocity, 0.76 meter per second; heat flux, 6.52×10^4 watts per square meter.

existence of nucleate and film boiling was noted in pool boiling by Farber (ref. 6) and studied by Nishikawa (ref. 7). Dougall qualitatively observed that film boiling was less stable with increased flow rate. This effect is shown graphically by Kutateladze and Borishanski (ref. 8) for the film boiling of water and isopropyl alcohol. It is demonstrated in reference 8 that, to maintain film boiling at its lower limit (minimum heat flux), a higher heat flux is required at larger liquid velocities.

Many investigators have used classic hydrodynamic stability theory (refs. 1 to 3) to determine theoretically the minimum heat flux in a nonflow system. The theoretical models have considered film boiling on a horizontal cylinder and a horizontal flat plate. Film boiling of a liquid with the accelerating field directed toward the heating surface produces an instability (the Raleigh-Taylor instability) of the vapor-liquid interface. The analysis for Raleigh-Taylor instability, may be found in reference 9. The assumption is made that, as long as the instability can be maintained with sufficient vapor generation, film boiling will persist. No analysis has been reported for the minimum heat flux of a vertical heater in a pool or with the added complication of liquid flow.

A visual observation was made (ref. 10) of the hydrodynamic conditions existing at the minimum heat flux of liquid nitrogen. The visual facility of reference 10 consisted of a flow channel with windows for viewing the edge view of an electrically heated vertical strip, which was located in the main stream of flowing liquid nitrogen. High-speed (5000 frames per sec), close-up motion pictures were taken in the vicinity where film boiling and nucleate boiling coexist. A sequence of photographs from reference 10 is shown in figure 1 that shows the region of film boiling and the region of nucleate boiling which is gradually replacing film boiling.

Semeria and Martinet (ref. 11) analyzed the motion of the interface between nucleate and film boiling for an electrically heated system. Their analysis is based on the wall conduction between the film boiling and nucleate boiling regions. In the study presented herein, use is made of Semeria and Martinet's analysis for predicting the wall temperatures associated with the minimum heat flux in an electrically heated system. A heat-transfer analysis is made, which, together with the analysis of Semeria and Martinet and the visual results of reference 10, provides an analytical prediction of the minimum heat flux as a function of the liquid velocity.

Also of interest in the photographs of figure 1 is a wave instability which begins to become evident at a certain distance from the position where nucleate and film boiling coexist. This instability is discussed in more detail in reference 10, and its behavior appears to be a symptom associated with the minimum heat flux.

Tests were conducted in a vertical electrically heated tube for the experimental determination of the minimum heat flux. Results are reported for reduced pressures P/P_c of 0.071, 0.20, and 0.40 and inlet liquid velocities up to 3 meters per second.

SYMBOLS

A	constant, dimensionless
b	$\overline{k}/\overline{T}$, W/(m)(^{o}K)
C_o	$\overline{\rho k}$, (kg)(W)/(m^4)(^{o}K)
C_p	specific heat, J/(kg)(^{o}K)
D	tube diameter, cm
g	acceleration of gravity, 9.8 m/sec^2
h	heat-transfer coefficient, q/θ, W/(m^2)(^{o}K)
k	thermal conductivity, W/(m)(^{o}K)
Le	effective length, cm
N	superheat constant, dimensionless
Nu	Nusselt number, $h_l D/k_l$, dimensionless
P	pressure, N/m^2
Pr	Prandtl number, $C_p\mu/k$, dimensionless
q	heat flux, W/m^2
Re	Reynolds number, $\overline{U}_L D_l \rho_l/\mu_l$, dimensionless
T	temperature, ^{o}K
t	wall thickness, cm
U	x component of velocity, m/sec
V	y component of velocity, m/sec
v	volume, m^3
W	mass flow reat, kg/(m)(sec)
x	distance along heating surface, cm
y	distance away from heating surface, cm
α	thermal diffusivity, $k/\rho C_p$, m^2/sec
β	intercept
Γ	heat of vaporization, J/kg
Γ'	effective heat of vaporization, J/kg
δ	film thickness, cm

θ temperature difference, $(T_w - T_{sat})$, ^{o}K

μ viscosity, $(N)(sec)/m^2$

ρ density; k/m^3

σ surface tension, N/m

Subscripts

C convection

c critical

$\bar{h}$ variable is evaluated at conditions corresponding to average heat-transfer coefficient

l liquid

m metal

min minimum

PB pool nucleate boiling, $U_l = 0$

s saturation

v vapor

w wall

wet wetting

1 nucleate boiling side

2 film boiling side

Superscripts

— average quantity

* minimum film boiling condition

EXPERIMENTAL

Apparatus

Flow system. - The cryogenic fluid flow system used for the determination of the minimum heat flux is shown in figure 2. The basic components are a gaseous helium high-pressure storage tank, a 0.15 cubic meter liquid-nitrogen Dewar, and a vacuum test section tank.

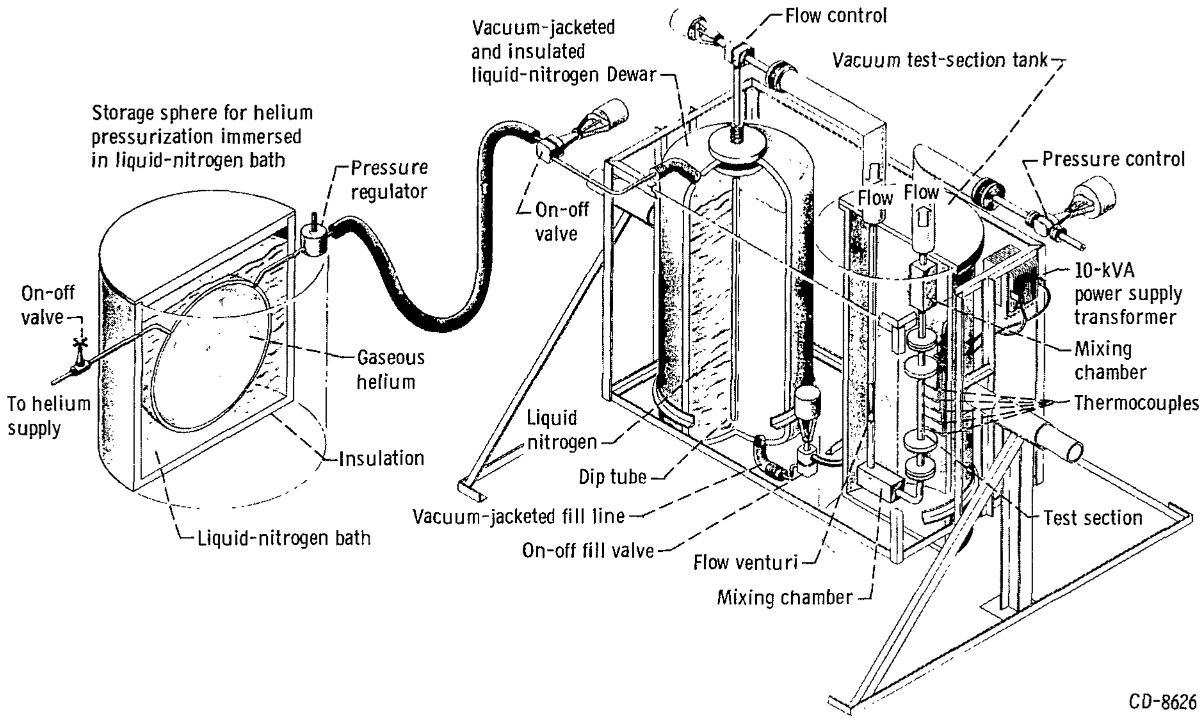


Figure 2. - Liquid-nitrogen heat-transfer apparatus.

The helium gas was used to pressurize the liquid-nitrogen Dewar. A liquid-nitrogen bath surrounding the helium tank precooled the gas to essentially liquid-nitrogem temperatures in order to minimize heat transfer between the pressurizing gas and the bulk liquid. Pressurization of the Dewar forced the liquid nitrogen to flow through an instrumented, resistance-heated test section. Flow control was achieved by a throttling valve located before the tube inlet, and the pressure was controlled by a throttling valve at the exit side of the tube. The entire flow system was vacuum jacketed to minimize the heat leak to the cryogenic fluid.

Test section and instrumentation. - The test section was made from a nickel alloy tubing with an inside diameter of 1.28 centimeters and a wall thickness of 0.025 centimeter. Figure 3 is a schematic drawing of the test section showing thermocouple locations and positions for voltage drop measurements. The heated length of the test section is 30.5 centimeters. Power to the test section was supplied by a 10 000-watt, 400-hertz generator. The power input was controlled by a system of transformers to permit delivery of 16 volts and 600 amperes.

Outside surface temperatures were recorded by using copper-constantan thermocouples. Bulk temperatures were measured with platinum resistance thermometers at the mixing chambers located near the inlet and exit of the test section. Liquid-nitrogen flow rate was determined with a Venturi meter. Pressure, flow rate, temperature, and electrical power were recorded by means of digital potentiometer.

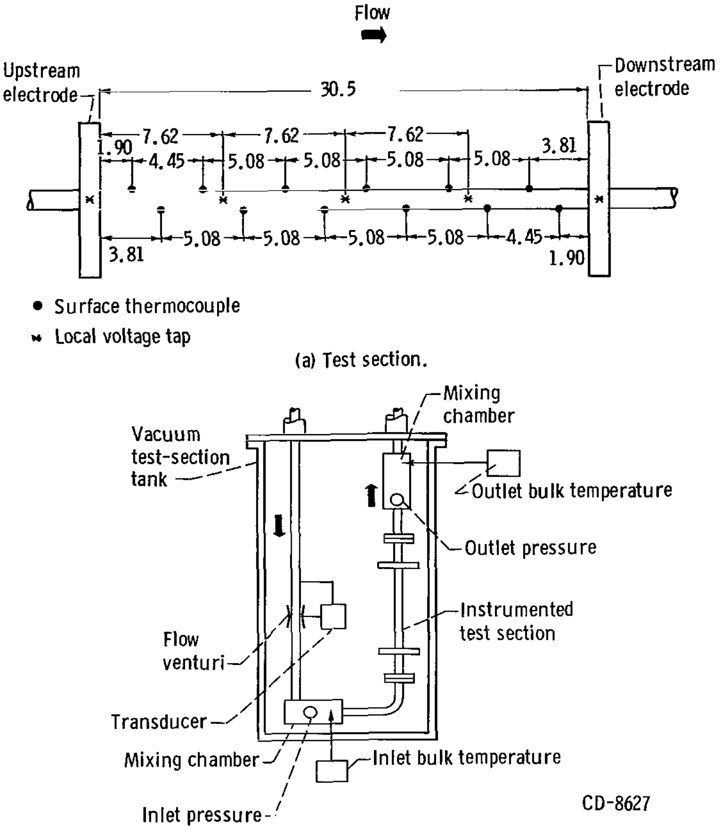


Figure 3. - Instrumentation. (All dimensions are in cm.)

Procedure

Initially, the test facility was cooled down with liquid nitrogen from a large supply Dewar. The helium tank bath and the 0.15-cubic-meter Dewar were filled, and the Dewar was vented through the test section until the thermocouples on the electrodes were at the liquid-nitrogen temperature. Since the liquid nitrogen was initially subcooled, it was necessary to bubble nitrogen gas through the liquid nitrogen while maintaining pressure in the Dewar until the liquid nitrogen was at saturated conditions. Before application of power to the test section, system pressure and flow rate were set. A step input in power was applied to the test section to permit film boiling over the entire test section, because a gradual increase in power requires higher power levels for the entire heating surface to be in film boiling. Once film boiling was obtained, the power level was reduced in small increments while at the same time flow and pressure were held at a constant value. At each power level, sufficient time was permitted (approximately $1\frac{1}{2}$ min) to note whether nucleate boiling would appear at the first thermocouple station. Nucleate

TABLE I. - EXPERIMENTAL RESULTS

Nominal reduced pressure, P/P_c	Reduced pressure, P/P_c	Temperature difference[a], θ, ^{o}K	Average liquid velocity, U_l, m/sec	Minimum heat flux, q_{min}, W/m^2
0.071	0.079	365	0.49	6.2×10^4
	.071	429	.82	7.0
	.071	413	.95	7.4
	.071	398	1.1	7.0
	.075	373	1.6	7.9
	.069	411	1.7	8.0
	.071	391	1.9	8.5
	.069	400	1.9	8.5
	.075	420	2.2	9.3
	.077	340	2.2	9.5
	.075	390	2.5	9.3
	.079	408	2.7	10.3
	.073	395	2.8	10.8
	.073	390	3.1	11.8
0.20	0.20	402	0.32	8.3
		426	.49	8.7
		367	.52	8.0
		391	.76	8.3
		380	1.1	9.2
		369	2.0	10.0
		378	2.1	11.4
		386	2.4	12.9
		360	3.1	13.7
0.40	0.41	275	0.29	6.7×10^4
	.40	254	.34	6.7
	.41	266	.49	7.0
	.40	248	.58	6.9
	.40	256	.61	6.9
	.41	253	.73	7.4
	.40	264	.82	6.9
	.41	293	1.4	8.8
	.40	302	1.7	10.3
	.41	299	2.2	12.2
	.40	282	2.2	11.6
	.41	334	2.5	15.2

[a]Temperature difference measured at x = 1.9 cm.

boiling would begin to occur at an increment of heat flux below the minimum heat flux. The minimum heat flux was taken as the lowest power level that allowed film boiling to exist over the entire heating surface.

Data

In the experimental phase of this study, the variables of interest are the inlet pressure, the minimum heat flux, the average liquid velocity at the inlet, and the wall temperature at the first thermocouple location. The measured values of these variables are listed in table I. Saturation temperatures were, in general, attained within a few degrees. It is interesting to note that, for a given pressure, the wall temperature differences appear independent of liquid velocity for the range of liquid velocities in this study.

It was observed that once nucleate boiling began to replace film boiling at heat fluxes below the minimum heat flux, the time required for nucleate boiling to completely replace film boiling decreased with decreasing heat flux (fig. 4). Figure 4 is a plot of the length of tube in nucleate boiling (wetted length) against the time in which this was achieved. The three curves shown are for three heat fluxes below the minimum heat flux at liquid velocities which do not differ significantly from each other.

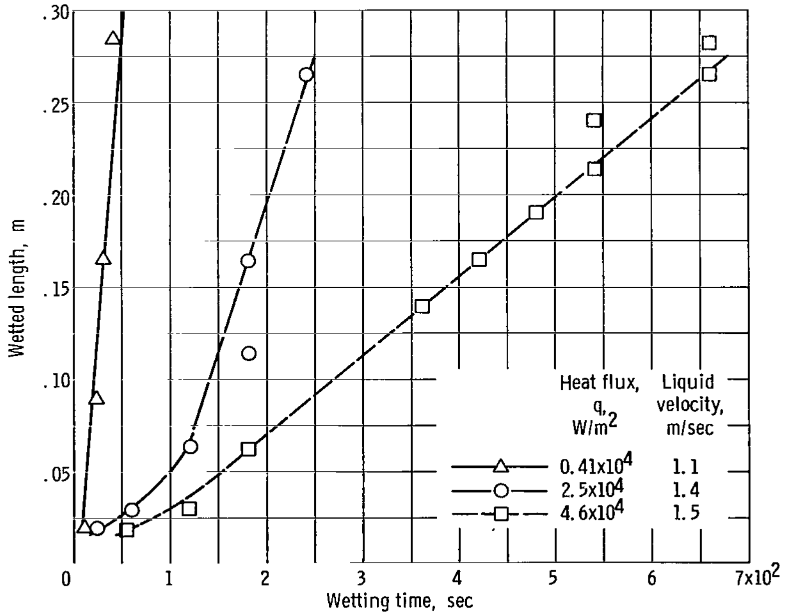


Figure 4. - Wetted length as function of wetting time for heat fluxes below the minimum for nitrogen.

ANALYTICAL

Boiling Curve

A boiling curve for a given pressure and liquid velocity may be obtained by plotting heat flux against the temperature difference between the wall and the liquid saturation temperatures. Figure 5 represents the general shape of such a plot and indicates the regions of nucleate boiling, transition boiling, and film boiling. In figure 5 constant heat flux refers to the boundary condition of uniform surface heat flux which can be controlled at various levels. This boundary condition is readily obtained with electrical heating. In a constant-temperature system the boundary condition is uniform wall temperature. In cryogenic systems this is often accomplished by using condensing steam as the heat source.

Examination of the functional relation between heat flux q and temperature difference θ, which are represented in figure 5, reveals that, in boiling, if the wall temperature is held constant, the heat flux can be uniquely determined by use of figure 5. In the range of heat flux between B and E, however, if the heat flux is held constant, figure 5 will not uniquely yield the wall temperature, and three separate cases are possible:

(1) The entire surface is in nucleate boiling at some temperature difference θ_1.[†]

(2) The entire surface is in film boiling at some temperature difference θ_2.[†]

(3) The surface is partially in nucleate and partially in film boiling with an axial gradient in the temperature along the surface.

These three conditions are all possible and have been obtained experimentally; in fact, the normal operation of an electrically heated system passes through each of these conditions successively. Examination of the operating procedure of reference 4 indicates that conditions (1), (2), and (3) were encountered. It is probable that condition (3) always occurs in these systems but might not always be observed since it is not very stable. In starting up an electrically heated system, the heat flux is increased along path A-B (fig. 5), and all nucleate boiling (condition 1) occurs. It will not jump over to film boiling path C-E because a higher temperature difference is required. At point B, the vapor accumulation blankets the surface and drives the mechanism into film boiling, B → C. The heat flux is then lowered along path C-E. Because the temperature difference required to maintain nucleate boiling is lower than that for film boiling, one might expect that anywhere along path C-E the mechanism could jump back to nucleate. However, investigators (refs. 4 and 5) have reported that it was necessary to lower the heat flux well below C before the system would transition to nucleate. During this lowering, the system is in

[†]It should be noted that in vertical systems, free or forced flow convection effects will impose a gradient on θ_1 or θ_2. In these preliminary remarks this effect can be omitted. It will be included in the analysis.

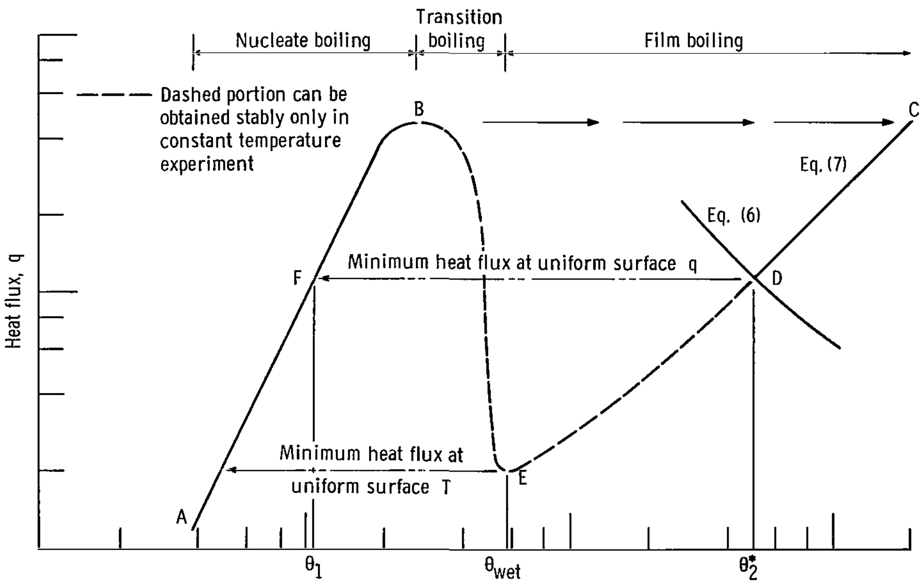


Figure 5. - Boiling curve.

condition 2. It has been possible to lower a constant heat flux system all the way to point E. This will be discussed in RESULTS AND DISCUSSION. Point E is the minimum heat flux associated with a constant-temperature system, and the wall temperature associated with this point is the wetting temperature. Lowering q below point E would lower θ below the wetting value, and the surface would automatically rewet and nucleate boil. The present experiments indicate that in constant heat flux systems it is more normal for the transition back to nucleate to occur at some point D between C and E. At, or just slightly above, point D it is possible to maintain stablely a condition of both nucleate and film boiling on the surface (condition 3) (ref. 10 and fig. 1 herein). It is the determination of this point D for vertical flow systems which is the subject of the following analysis.

Wall Conduction Model

Point D has been defined as the point where this relatively delicate condition of combined nucleate and film boiling can be held stable. There must be, for this condition, an interface or transition position which marks the separation of nucleate from film boiling (fig. 1). There will be, of course, an axial gradient in wall temperature across this transition position.

If the heat flux is lowered a finite amount below point D, axial conduction in the heated wall becomes more dominating and gradually lowers the film-boiling temperature difference to a value θ, which permits liquid rewetting of the metal surface. This axial conduction effect will cause the transition position to move upstream. The transition position (fig. 1) moves at a given velocity depending on the heat flux (fig. 4).

Similarly, a finite increase in wall heat flux above point D will drive the transition position toward the nucleate region thus increasing the amount of surface under film boiling. This motion has been analyzed by Semeria and Martinet (ref. 11). For the condition when conduction is beginning to affect the motion of the leading edge of film boiling (leading edge has no velocity or is stable, point D), Semeria and Martinet relate the temperature difference on the film boiling side (θ_2^* in fig. 5) and the temperature difference on the nucleate boiling side (θ_1 in fig. 5) to what they call the "calefaction" temperature difference. In figure 5 the calefaction temperature difference is expressed as the wetting temperature difference θ_{wet}. The explicit form of the relation is as follows:

$$\theta_{wet}^2 = \theta_1 \theta_2^* \tag{1}$$

Semeria and Martinet simplified their calculations by assuming constant heat-transfer coefficients for nucleate and film boiling.

In the present study the variation of heat-transfer coefficient is considered, and the temperature difference corresponding to an average heat -transfer coefficient is used in the analysis of Semeria and Martinet. The result is a modified version of equation (1).

$$\theta_{wet}^2 = \left[\theta_{1,\bar{h}}\right]\left[\theta_{2,\bar{h}}\right]^* \tag{2}$$

Equation (2) may be rearranged as follows:

$$\left[\theta_{2,\bar{h}}\right]^* = \frac{\theta_{wet}^2}{\left[\theta_{1,h}\right]} \tag{3}$$

In general, the temperature difference $\theta_{1,\bar{h}}$ is a function of pressure, heat flux, and liquid velocity.

$$\theta_{1,\bar{h}} = f_1(P, q, \overline{U}_l) \tag{4}$$

The wetting temperature T_{wet} is assumed to be a property of the fluid (ref. 12), thus the wetting temperature difference can be expressed as

$$\theta_{wet} = f_2(P, P_c, T_c) \tag{5}$$

Use of equations (4) and (5) in equation (3) results in the following functional relation for the application of Semeria and Martinet's analysis to the present study.

$$(\theta_{2,\bar{h}})^* = g(P, P_c, T_c, q, U_l) \tag{6}$$

Equation (6) is shown graphically in figure 5 for a given fluid, pressure, and liquid velocity. For values of temperature difference $\theta_{2,h}$ less than expressed by equation (6), transition from film boiling to nucleate boiling will occur. The film-boiling temperature difference $\theta_{2,\bar{h}}$ is a function of pressure, heat flux, and liquid velocity (C-D, fig. 5).

$$\theta_{2,\bar{h}} = f_3(P, q, \overline{U}_l) \tag{7}$$

Equation (6) is only valid at that one heat flux where the interface has no axial motion. Since above this flux there will be all film boiling and below it all nucleate boiling, this flux is defined as q_{min}, the minimum heat flux for a vertical flowing, constant heat flux system.

The solution of equations (6) and (7) for the minimum heat flux q_{min} may be seen graphically in figure 5. This may be expressed as follows

$$(\theta_{2,\bar{h}})^* = \theta_{2,\bar{h}}; \; q = q_{min} \tag{8}$$

For a given fluid, pressure and liquid velocity equations (6), (7), and (8) may be solved for the minimum heat flux.

In deriving equation (1) Semeria and Martinet assumed that θ_1 and θ_2 extended for an infinite distance away from the transition position. This is done because the axial heat conduction at infinity can be taken as zero, and the boundary condition $d\theta/dx \to 0$ as $x \to \pm\infty$ can be used. On the nucleate boiling side this presents no problem in the present analysis. On the film boiling side the model used herein is only valid for a finite length and the average heat transfer must be evaluated over some finite length designated Le, which is that length beyond which the conduction gradient $d\theta/dx$ is negligible. This will be discussed, in the film boiling section.

The boiling model associated with the minimum heat flux is shown in figure 6. The average temperature differences ($\theta_{1,\bar{h}}$ and $\theta_{2,\bar{h}}$) are evaluated based on this model. The property of the fluid θ_{wet} can be evaluated from an equation of state. Thus, it only remains to determine the functions f_1, f_2, and f_3 of equations (4), (5), and (7).

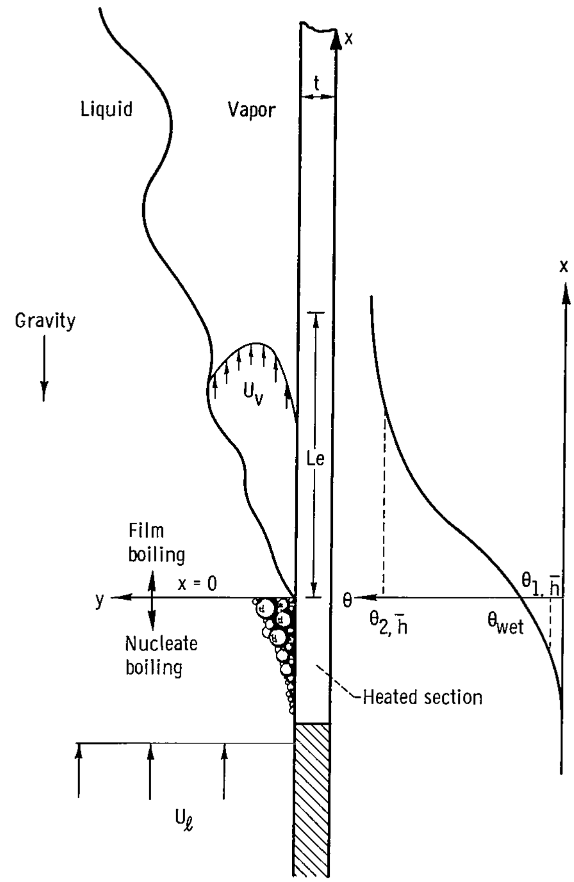


Figure 6. - Film boiling model.

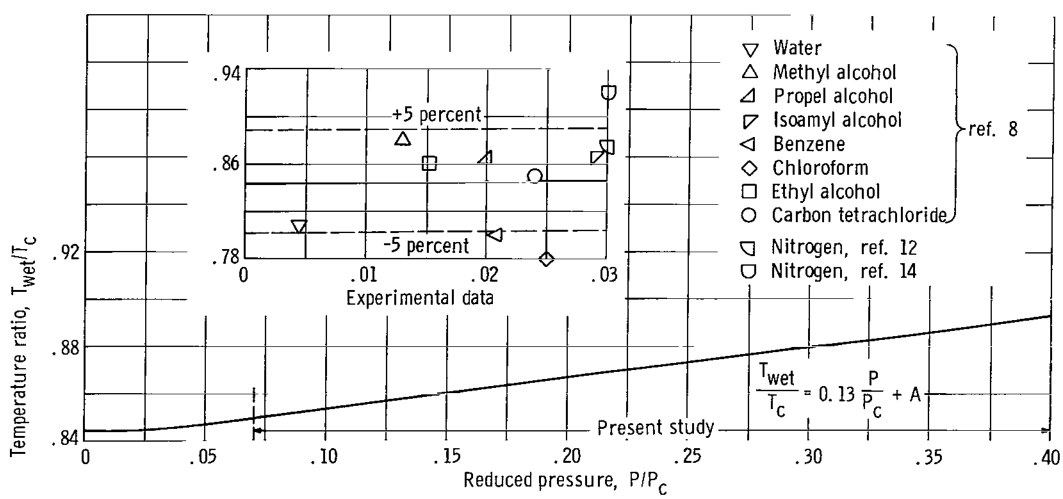


Figure 7. - Theoretical determination of wetting temperature from reference 13.

Wetting temperature difference $f_3(P, T)$. - The temperature difference θ_{wet} at which wetting occurs permitting transition from film boiling to nucleate boiling has been measured in temperature controlled systems. The data curve of Merte and Clark (ref. 13) for the pool boiling of liquid nitrogen is one source which gives an experimental value for the wetting temperature difference at atmospheric pressure. However, it is important in this analysis to determine the effect of pressure on the wetting temperature difference. It was demonstrated by Spiegler, et al. (ref. 12), that θ_{wet} is a state property of the fluid. It was assumed in reference 12 that the wetting fluid temperature T_{wet} corresponds to the maximum superheat temperature of a liquid. Spiegler, et al., used Van der Waals equation of state for a liquid to determine the maximum superheat. The authors of reference 12 concluded that the analysis based on maximum superheat agreed with existing data. The results of their analysis in terms of the critical pressure P_c and the critical temperature T_c is plotted in figure 7 and are compared with the results for drops given on page 214 of reference 8 and the pool boiling data points of Merte and Clark (ref. 13) and Ruzicka (ref. 14). The wetting temperature difference for drops is normally known as the Leidenfrost point. The experimental points on figure 7 from reference 8 appear to justify the analysis of Spiegler. The theoretical equation for the range of reduced pressures P/P_c in this study (fig. 7) is linear and is as follows:

$$\frac{T_{wet}}{T_c} = 0.13\frac{P}{P_c} + A \tag{9}$$

The projected intercept of the linear portion of the theoretical line of figure 7 yields a value $A = 0.840$; however, this value may be approximate considering the simplicty of Van der Waals equation of state. A small correction of this constant, which will shift the level of the curve, may be made by the use of known wetting temperature difference data at 1 atmosphere for saturated nitrogen. Merte and Clark (ref. 13) report a value of 33.4° K for the wetting temperature difference. Ruzicka (ref. 14) reports a value of $38.9\ ^{\circ}$K. Based on this data the intercept becomes either $A = 0.872$, assuming a value of $\theta_{wet} = 33.4^{\circ}$ K at 1 atmosphere (1×10^5 N/m^2) or $A = 0.916$, assuming $\theta_{wet} = 38.9\ ^{\circ}$K at 1 atmosphere. As a means of establishing the importance of the wetting temperature difference both constants were used in the calculation of θ_{wet} at the reduced pressures P/P_c of 0.071, 0.20, and 0.40. (See RESULTS AND DISCUSION.)

Film boiling temperature difference $f_2(P, q, U_l)$. - The analysis for film boiling heat transfer with liquid flow is given in appendix A for a flat plate and assumes that the vapor velocity at the vapor-liquid interface is equal to the liquid velocity. Inertial terms are neglected, and vapor property variations are accounted for in an approximate manner. The relation between position x and the wall temperature difference is given by the equation

$$x = a_1\left(\theta^5 - \theta_{wet}^5\right) + a_2\left(\theta^4 - \theta_{wet}^4\right) + a_3\left(\theta^3 - \theta_{wet}^3\right) + a_4\left(\theta^2 - \theta_{wet}^2\right) + a_5\left(\theta - \theta_{wet}\right) \qquad \text{(A17)}$$

where

$$a_1 = \frac{2}{5} \frac{A_o N C_p}{q_w^4}$$

$$a_2 = \frac{A_o}{q_w^4}\left(\frac{\Gamma}{2} + \frac{3T_s N C_p}{4}\right)$$

$$a_3 = \frac{A_o \Gamma T_s}{q_w^4}$$

$$a_4 = \frac{C_o \overline{U}_l N C_{pv}}{4q_w^2}$$

$$a_5 = \frac{C_o \overline{U}_l \Gamma}{2q_w^2}$$

$$A_o = \frac{bg\rho_l C_o C_{pv}}{12 \text{ Pr}}$$

and

$$C_o = \overline{\rho}_v \overline{k}_v$$

The superheat constant was assumed to be $N = 0.5$. This assumption is discussed in appendix A and was satisfactory. The temperature difference based on an average film-boiling heat-transfer coefficient (not to be confused with the average temperature difference) is shown in appendix A to be at any x.

$$\theta_{2,\bar{h}}(x) = \frac{x}{\left[\frac{5}{4} a_1\left(\theta^4 - \theta^4_{wet}\right) + \frac{4}{3} a_2\left(\theta^3 - \theta^3_{wet}\right) + \frac{3}{2} a_3\left(\theta^2 - \theta^2_{wet}\right) + 2a_4\left(\theta - \theta_{wet}\right) + a_5 \ln \frac{\theta}{\theta_{wet}}\right]} \tag{A21}$$

The value of x to be used is the distance from the leading edge of the film boiling (the nucleate-film boiling transition position), where the axial conduction is at its greatest, to the point where axial conduction is no longer effective in the wetting process. This is called (fig. 6) the effective conduction length Le. Therefore, equation (A21) becomes

$$\theta_{2,\bar{h}} = \frac{Le}{\left[\frac{5}{4} a_1\left(\theta^4_{Le} - \theta^4_{wet}\right) + \frac{4}{3} a_2\left(\theta^3_{Le} - \theta^3_{wet}\right) + \frac{3}{2} a_3\left(\theta^2_{Le} - \theta^2_{wet}\right) + 2a_4\left(\theta_{Le} - \theta_{wet}\right) + a_5 \ln \frac{\theta_{Le}}{\theta_{wet}}\right]} \tag{10}$$

The effective conduction length is calculated in appendix B by the use of Semeria's analysis and the experimental information from reference 10. The analysis in appendix B results in the following equation for the effective conduction length:

$$Le = 1.86\sqrt{\frac{k_m t \theta_{2,\bar{h}}}{q_w}} \tag{B4}$$

Nucleate boiling temperature difference $f_1(\rho, q, \bar{U}_l)$. - At the entrance of the heated tube used in the experimental phase very little void is expected to exist, thus a method proposed by Rohsenow (ref. 15), which applies to low-void, forced-convection, nucleate boiling, is used. This method assumes that the total heat flux may be considered a summation of the heat flux due to boiling in the absence of flow and the heat flux due to convection in the absence of boiling. This is stated in equation form as follows:

$$q_{w,1} = q_C + q_{PB} \tag{11}$$

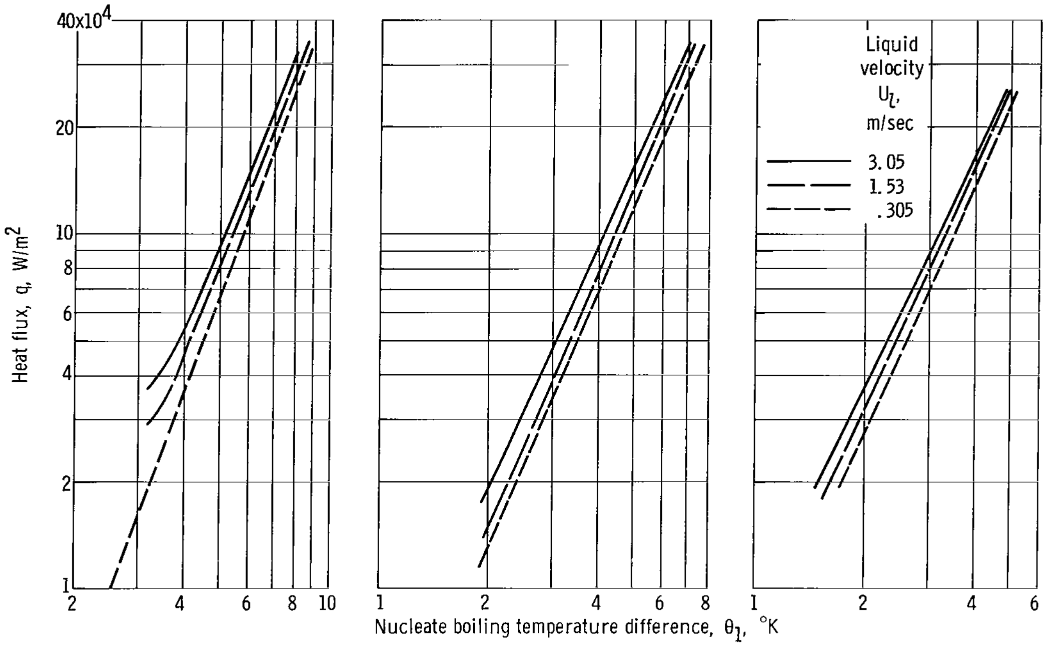
Brentari and Smith (ref. 16) adapted Kutateladzes' pool-boiling equation for the pool boiling of liquid nitrogen. The equation is

$$q_{PB} = 4.87\times10^{-7}\left(\frac{C_{p,l}}{\Gamma\rho_v}\right)^{1.50}\left(\frac{k_l\rho_l^{1.282}P^{1.75}}{\sigma^{0.906}\mu_l^{0.626}}\right)\theta_1^{2.50} \tag{12}$$

The convection heat flux is obtained from the standard Dittus-Boelter equation

$$Nu_l = 0.024\ Re_l^{0.8}Pr_l^{0.4} \tag{13}$$

The computed results of equation (11) using equations (12) and (13) are shown in figure 8.



(a) Reduced pressure, 0.071. (b) Reduced pressure, 0.20. (c) Reduced pressure, 0.40.

Figure 8. - Nucleate boiling heat flux as function of temperature difference.

The average heat-transfer coefficient for nucleate boiling is evaluated over the entire region of nucleate boiling. For liquid nitrogen the incipience of nucleate boiling occurs at very low θ. Thus, the lowest limit for averaging is $\theta = 0$, and the upper limit is the nucleate boiling temperature difference corresponding to the prescribed heat flux at the wall $\theta_{q,w}$.

Examination of figure 8 reveals that the slope of the q against θ curves are relatively constant

$$2.20 \leq \text{slope} \leq 2.80$$

with most of them being about 2.5. The intercepts are functions of both pressure and liquid velocity. Thus for simplicity, the following approximation is made to equation (11) for the ranges of this investigation:

$$q \approx \beta\left(\overline{U}_l, \frac{P}{P_c}\right)\theta_1^{2.5} \tag{14}$$

where the intercept β is a function liquid velocity and pressure ratio. The heat-transfer coefficient then is

$$h = \beta\theta_1^{1.5} \tag{15}$$

The average h over the nucleate region ($\theta = 0$ to $\theta = \theta_{q,w}$) is expressed as

$$\bar{h} = \frac{\beta}{\theta_{q,w}}\int_0^{\theta_{q,w}} \theta_1^{1.5}\, d\theta = \frac{\beta\theta_{q,w}^{1.5}}{2.5} \tag{16}$$

From equation (15), the average h can be used to define $\theta_{1,\bar{h}}$.

$$\bar{h} = \beta\theta_{1,\bar{h}}^{1.5} \tag{17}$$

Combining equations (16) and (17) yields for the ranges of liquid velocity and pressure of this investigation

$$\theta_{1,h} = \frac{\theta_{q,w}}{1.84} \tag{18}$$

where $\theta_{q,w}$ is a function of liquid velocity and pressure ratio and can be obtained from figure 8.

Analytical Prediction of Minimum Film Boiling Heat Flux

Now that the functions f_1, f_2, and f_3 of equations (4), (5), and (7) are determined, one can proceed to evaluate q_{min}. These functions are expressed in equations (18), (10), and (9), respectively. These equations along with the constraining equation (3) will be solved parametrically for fixed values of liquid velocity and pressure. For a fixed pressure, T_{wet} is a constant independent of q. Thus eliminating $\theta_{1,\bar{h}}$ and θ_{wet} from equations (3), (9), (18) yields

$$\left(\theta_{2,\bar{h}}\right)^{*} = \frac{1.84}{\theta_{q,w}}\left[T_c\left(0.13\frac{P}{P_c} + A\right) - T_{sat}\right]^2 \tag{19}$$

where $A = 0.872$ or 0.916 and $\theta_{q,w}$ is function of pressure, liquid velocity, and heat flux to be obtained from figure 8. Equation (19) was solved simultaneously, graphically with equation (10) to obtain the minimum heat flux for the pressures and liquid velocities of this investigation as shown in figure 9.

$$\theta_{2,\bar{h}} = \frac{Le}{\left[\frac{5}{4}a_1\left(\theta_{Le}^4 - \theta_{wet}^4\right) + \frac{4}{3}a_2\left(\theta_{Le}^3 - \theta_{wet}^3\right) + \frac{3}{2}a_3\left(\theta_{Le}^2 - \theta_{wet}^2\right) + 2a_4\left(\theta_{Le} - \theta_{wet}\right) + a_5 \ln\frac{\theta_{Le}}{\theta_{wet}}\right]} \tag{10}$$

where

$$Le = 1.86\sqrt{\frac{k_m t\theta_{2,\bar{h}}}{q_w}} \tag{B4}$$

and a_1, a_2, a_3, a_4, and a_5 are functions of q_w.

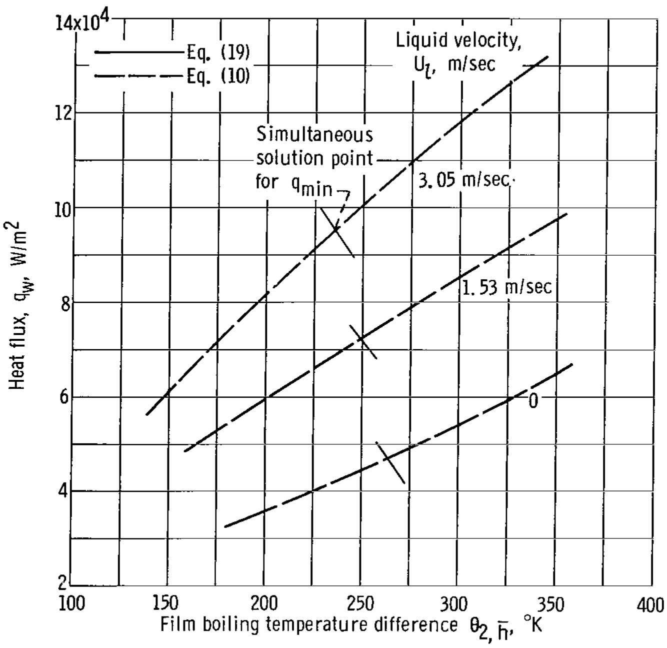


Figure 9. - Typical graphical solution of minimum heat flux. Reduced pressure, 0.071; dimensionless constant, 0.872 (eq. (9)).

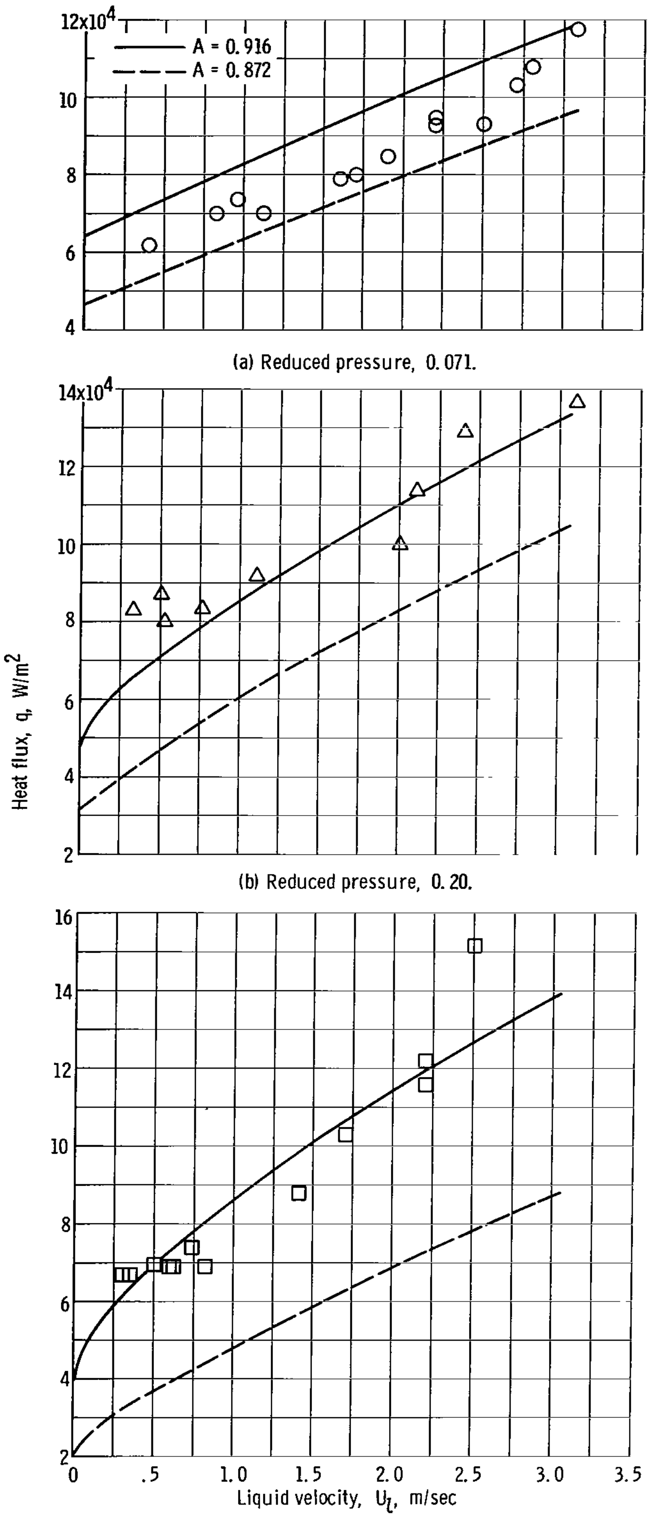
RESULTS AND DISCUSSION

Comparison of Analysis and Experiments

The graphical solutions of equations (10) and (19) demonstrated by figure 9 are compared with the experimental data in figure 10. The comparisons are made for two values of the constant A in equation (19). Calculating for both values of A demonstrated the importance of accurately determining the wetting temperature difference because its value is raised to the second power in equation (19). When the sensitivity of the analysis to the wetting temperature difference and the simplicity of the heat-transfer analysis, are considered, it is encouraging to find good agreement between analysis and experiment. This is especially true in the analytical prediction of the slope of the experimental data. In addition, it was found from the solutions of equations (10) and (19) (see fig. 9) that the average film-boiling temperature difference $\theta_{2,\bar{h}}$ associated with the analytically determined minimum heat flux was fairly constant. This effect was noted in the experimental phase (table I). The analytical average temperature difference decreases with pressure, which is also in agreement with the experimental findings.

The model proposed in this study for a uniform heat-flux situation considers conduction at the interface separating nucleate boiling and film boiling to govern the minimum heat flux. The model evaluates the conduction gradient due solely to the coexistence of nucleate boiling and film boiling and does not consider additional heat sinks which could alter the basic model presented herein.

Since forced-convection heat-transfer passages often have some kind of entrance region where there is no heating, it seemed appropriate to gain some additional information concerning the influence of a heat sink upstream of the heated section. The test section of this experiment has such a heat sink in the form of the electrode flanges and an unheated entrance length (fig. 3). The procedure was to precool the system so the flanges would be at approximately 80° K ($\theta_{flanges} \leq 0$) initially, and their large heat capacity would keep them near this temperature throughout the short run. There are two consequences of these cold flanges: (1) they ensure that there will always be a point along the wall at the nucleate boiling temperature difference which will promote at least a short length of nucleate boiling which is inherent to the analysis (any unheated length will do this) and (2) the large sink could permit increased axial conduction and consequently promote transition to nucleate boiling at a higher heat flux. It was important therefore to determine whether this heat sink had an effect large enough to invalidate these results in other configurations. We performed bench type experiments on the film boiling of liquid nitrogen in a pool exposed to the atmosphere, using several electrically heated configurations with no unheated entrance length. The values of the minimum heat fluxes obtained were all in agreement with each other and with the very low-velocity and



(c) Reduced pressure, 0.40.

Figure 10. - Minimum heat flux as function of average liquid velocity (see eq. (6)).

low-pressure data reported in this study. This is also true for the value of the minimum heat flux reported in reference 10. This agreement of the values of the minimum heat flux, independent of the physical configuration, would appear to discount excessive influence of the unheated entrance length and the electrode flanges.

Apparently, the very efficient heat transfer to the nucleate zone dominates the value of the conduction gradient existing at the interface of nucleate boiling and film boiling, thus, permitting application of the conduction model in those situations where additional heat sinks exist, such as the heated tube experiments reported herein.

The first consequence of the heat sink, the ensurance of a nucleate region, is a little more basic to the analysis. The bench experiments performed in a pool used heaters with sharp leading edges and thus there was no point that was always in nucleate boiling. Nevertheless, for these heaters the minimum heat flux occurred in accordance with the analytic model above. It would seem that the hydrodynamic force of the liquid on the thin film at the leading edge was sufficient to break the film and cause intermittent wetting. If the heat flux was above the minimum, the film would reestablish, but, if the heat flux was at the minimum, the axial conduction gradient would be sufficient to promote motion of the nucleate-film boiling transition position, and the surface would remain wetted and would transition to nucleate boiling. If the surface is kept above the wetting temperature, then there is no point for nucleate boiling, and the establishment of a conduction gradient for the model is prevented. To demonstrate this point in the present experiment, for some data runs the electrode flanges (fig. 3) were not precooled, and they stayed at a temperature difference of $\theta \approx 50^\circ$ K, ($\theta_{wet} \approx 35^\circ$ K). In a typical run it was possible to lower the heat flux to $q_{min} = 1.48\times10^4$ watts per square meter with a $\theta_{2,\bar{h}} = 78^\circ$ K. This is very near the constant temperature minimum. This excessively low minimum for a heat-flux controlled system occurs because the flange temperature difference was greater than θ_{wet} and wetting was retarded.

These results are valid primarily for low-heat capacity ($\rho C_p v$) heaters. The temperature in high-heat capacity ($\rho C_p v$) heaters (particularly in high diffusivity metallic surfaces) will tend to redistribute itself, and the axial gradient cannot be maintained.

There is some discrepancy between the results and analysis in figure 10. Since the basic analysis of an axial conduction model (eq. (2)) predicts the proper trends and predicts a wide range of data quite satisfactorily, the deviations should probably be assigned to the simplicity of the heat-transfer models used to determine $\theta_{1,\bar{h}}$ and $\theta_{2,\bar{h}}$. While the bench experiments indicated that the heat sink does not dominate the problem, it could account for some of the deviation between analysis and experiment.

Comparison of the Minimum Heat Flux for Constant Heat Flux and Constant Wall Temperature

The experimental evidence of this study indicates the minimum heat flux in a heat flux controlled system is one order of magnitude greater than in a temperature controlled system. In a temperature controlled system the minimum heat flux is controlled by the lowest possible wall temperature difference without transition to nucleate boiling. This lowest possible temperature difference is the wetting temperature difference θ_{wet}. In an electrically heated system, nucleate boiling and film boiling can coexist with a temperature gradient existing between the film boiling and nucleate boiling regions. This situation becomes unstable if there is insufficient heat to maintain the temperature on the film boiling side above the temperature required by the fluid to wet the wall.

Hydrodynamic Instabilities at the Minimum Heat Flux

Zuber (ref. 1) predicated an analysis of the minimum heat flux in horizontal pool systems on the existence of a Raleigh-Taylor instability.

The present authors took high-speed pictures (2000 frames/sec) of the transition from film boiling to nucleate for the pool boiling of liquid nitrogen on a horizontal electrically heated strip. The wave instability used by Zuber for the prediction of the minimum heat flux is clearly evident in these pictures. The pictures indicate that, at the minimum heat flux, the wave instability becomes less evident as required by Zuber's analysis; however, instead of a vapor collapse, the transition back to nucleate boiling occurs by fluid rapidly wetting the surface, a motion that goes from the sides of the heater toward the middle. Its appears that, while Zuber's wave instability approach is correct hydrodynamically for a horizontal system, it does not offer a complete explanation of the transition from film boiling to nucleate boiling. It is more likely that the conditions required for the wetting of the surface more exactly determine the transition and that the hydrodynamic evidence of the gradual disappearance of wave instability is a sympton occurring at the minimum heat flux. The same stability symptom appears in the vertical forced flow case (fig. 1); however, this instability is likely to be the Kelvin-Helmholtz type (ref. 10). From the visual observations of reference 10 in vertical film boiling, a condition of increasing hydrodynamic stability appears at the minimum heat flux, which is analogous to the previously mentioned observations for horizontal film boiling. Of course, since the symptom appears to be always present, an accurate analysis of the instability such as by Zuber (ref. 1) for the horizontal case should yield the minimum heat flux.

CONCLUSIONS

The results of the study of minimum film-boiling heat flux yielded the following conclusions:

1. Wall conduction governs the transition of film boiling to nucleate boiling in an electrically heated system. A method of analytically predicting the transition heat flux (minimum heat flux) for vertical systems was derived. Use was made of the analysis of Semeria and Mertinet for obtaining the wall temperature differences associated with the minimum heat flux. Good agreement was found between experiment and analysis, especially in predicting the positive rate of change of the minimum heat flux with liquid velocity.

2. The analysis is also consistent with the experimental findings which show that the film boiling wall temperature differences at the minimum heat flux are approximately independent of the liquid velocity for the range of liquid velocities studied (0 to 3 m/sec). The analytical values of the average film boiling wall temperature differences at the minimum heat flux decrease with pressure which is also consistent with the experiments.

3. In vertical pool boiling of liquid nitrogen when the surface heat flux is a constant, the minimum heat flux is higher by an order of magnitude than when the wall temperature is a constant. This difference can be attributed to the wall conduction which exists for constant heat flux and which is not present when the heating surface is at constant temperature.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 1, 1967,
129-01-11-02-22.

APPENDIX A

FILM BOILING ANALYSIS

The film boiling analysis is made assuming the following conditions applicable.

(1) Laminar, two dimensional vapor flow

(2) Uniform wall heat flux

(3) Linear temperature profile across the vapor film

(4) Inertia terms neglected

(5) Greater velocity gradient in the y direction than the velocity gradient in the x direction, $\partial U/\partial y >> \partial U/\partial x$

(6) Slug flow velocity profile in the liquid stream

(7) Liquid at the saturation temperature

(8) Evaluation of vapor properties across the vapor film at the average temperature of the wall and the saturated liquid, $\overline{T}(x) = (T_w(x) + T_s)/2$

(9) Constant heat capacity and Prandtl number

A complete solution of this problem should consider the inertia terms and the property variations in the x and y direction. The exact analytical results of McFadden and Grosh (ref. 17) for film boiling with a constant temperature boundary condition justifies assumptions (4) and (8). In this simplified analysis, the property variations in the vertical x direction will be considered in an approximate manner.

The basic equation relating the heat transfer at the wall and the vapor generation at the vapor-liquid interface is

$$q_w = \Gamma' \frac{dW}{dx} \tag{A1}$$

where

$$\Gamma' = \Gamma + NC_{p,v}\theta(x) \tag{A2}$$

and

$$W = \overline{U}_v\overline{\rho}_v\delta_v \tag{A3}$$

(Average conditions are for a fixed position x.) The superheat constant N represents the fraction of the heat content which goes into superheating the vapor above the saturation temperature. A wide variety of values have been reported in the literature, some greater and some less than 0.5 (ref. 18 to 20). None of these references used the same heating configuration as discussed herein. In this analysis, the superheat constant is taken as $N = 0.5$. Calculations were performed which show that a 20 percent change in N yields a 2 percent change in the final result q_{min}. Thus, $N = 0.5$ was considered satisfactory.

Using equations (A2) and (A3), equation (A1) can be expressed as follows.

$$q_w = \left(\Gamma + NC_{p,v}\theta(x)\right) \frac{d}{dx} (\overline{U}_v \rho_v \delta_v) \tag{A4}$$

Based on assumptions (4), (5), and (8), the momentum equation for vertical film boiling may be given as

$$0 = -\frac{\partial P}{\partial x} - \overline{\rho}_v g + \overline{\mu}_v \frac{\partial^2 U_v}{\partial y^2} \tag{A5}$$

and

$$\frac{\partial P}{dx} = -\rho_l g \tag{A6}$$

The applicable boundary conditions are

$$U_v = \overline{U}_l; \; y = \delta_v$$

$$U_v = 0; \; y = 0$$

Solution of equation (A5) for the boundary conditions given results in the following equation for the vapor velocity

$$U_v = \frac{g(\rho_l - \overline{\rho}_v)}{2\overline{\mu}_v} (\delta_v y - y^2) + \frac{\overline{U}_l y}{\delta_v} \tag{A7}$$

The average velocity at any station x is defined as

$$\overline{U}_v = \frac{1}{\delta_v}\int_0^{\delta_v} U_v \, dy \tag{A8}$$

The average velocity becomes

$$\overline{U}_v = \frac{g(\rho_l - \overline{\rho}_v)}{12\overline{\mu}_v}\delta_v^2 + \frac{\overline{U}_l}{2} \tag{A9}$$

Since $\rho_l >> \overline{\rho}_v$, the average velocity is simply

$$\overline{U}_v = \frac{g\rho_l \delta_v^2}{12\overline{\mu}_v} + \frac{\overline{U}_l}{2} \tag{A10}$$

Combining equations (A10) and (A4) yields

$$q_w = \left(\Gamma + NC_{p,v}\theta(x)\right)\frac{d}{dx}\left(\frac{g\rho_l \delta_v^3 \overline{\rho}_v}{12\overline{\mu}_v} + \frac{\overline{U}_l \delta_v^3 \overline{\rho}_v}{2}\right) \tag{A11}$$

From assumption (4) the film thickness δ_v may be expressed in terms of the temperature difference $\theta(x)$ as

$$\delta_v = \frac{\overline{k}_v \theta}{q_w} \tag{A12}$$

When it is assumed that the thermal conductivity varies directly with temperature, the product of the vapor density ρ_v and the thermal conductivity k_v, for an ideal gas is constant $(\rho_v k_v = C_o)$. With this assumption and equation (A12), equation (A11) is written as

$$q_w = (\Gamma + NC_{p,v}\theta)\frac{d}{dx}\left(\frac{g\rho_l C_o C_{p,v}}{12q_w^3 Pr_v}\bar{k}_v\theta^3 + \frac{\bar{U}_l C_o}{2q_w}\theta\right) \tag{A13}$$

Everything in the brackets to be differentiated with respect to x can be considered constant except the thermal conductivity $\bar{k}_v$ and the temperature difference θ.

Expressing the thermal conductivity $\bar{k}_v$ as

$$\bar{k} = b\bar{T} \tag{A14}$$

and carrying out the differentiation of equation (A13) with respect to x results in the following equation

$$q_w = (\Gamma + NC_{p,v}\theta)\left[\frac{g\rho_l C_o}{12q_w^3}\frac{C_{p,v}}{Pr_v}b\left(2\theta^3 + 3\theta^2 T_s\right) + \frac{\bar{U}_l C_o}{2q_w}\right]\frac{d\theta}{dx} \tag{A15}$$

Variables are separated to permit integration.

$$q_w\int_{\theta}^{x} dx = \Gamma\int_{\theta_{wet}}^{\theta}\left[\frac{g\rho_l C_o C_{p,v}}{12q_w^3 Pr_v}b\left(2\theta^3 + 3\theta^2 T_s\right) + \frac{\bar{U}_l C_o}{2q_w}\right]d\theta$$

$$+ \frac{g\rho_l C_o C_{p,v}^2}{12q_w^3 Pr_v}bN\int_{\theta_{wet}}^{\theta}\left(2\theta^4 + 3\theta^3 T_s\right)d\theta + \frac{NC_{p,v}\bar{U}_l C_o}{2q_w}\int_{\theta_{wet}}^{\theta}\theta\, d\theta \tag{A16}$$

The resulting integrated equation may be expressed as

$$x = a_1\left(\theta^5 - \theta_{wet}^5\right) + a_2\left(\theta^4 - \theta_{wet}^4\right) + a_3\left(\theta^3 - \theta_{wet}^3\right) + a_4\left(\theta^2 - \theta_{wet}^2\right) + a_5\left(\theta - \theta_{wet}\right) \tag{A17}$$

where

$$a_1 = \frac{2}{5}\frac{A_o NC_{p,v}}{q_w^4}$$

$$a_2 = \frac{A_o}{q_w^4}\left(\frac{\Gamma}{2} + \frac{3T_s N}{4} C_{p,v}\right)$$

$$a_3 = \frac{A_o \Gamma T_s}{q_w^4}$$

$$a_4 = \frac{C_o \overline{U}_l N C_{p,v}}{4q_w^2}$$

$$a_5 = \frac{C_o \overline{U}_l \Gamma}{2q_w^2}$$

$$A_o = \frac{g\rho_l C_o C_{p,v} b_v}{12 Pr_v}$$

and

$$C_o = \overline{\rho}_v \overline{k}_v = \rho_v k_v$$

An average film-boiling heat-transfer coefficient at any point x is defined as

$$\overline{h} = \frac{\int_0^x h\,dx}{\int_0^x dx} = \frac{q}{x}\int_0^x \frac{1}{\theta}\,dx \tag{A18}$$

The average heat-transfer coefficient is also used to define $\theta_{2,\overline{h}}$

$$\overline{h} = \frac{q_w}{\theta_{2,\overline{h}}} \tag{A19}$$

Therefore, from equations (A18) and (A19), the equation for the film boiling temperature difference based on the average heat-transfer coefficient $\theta_{2,\bar{h}}$ is

$$\theta_{2,\bar{h}} = \frac{x}{\int_0^x \frac{dx}{\theta}} \tag{A20}$$

Use of equation (A17) permits evaluation of the integral in equation (A20); the result is

$$\theta_{2,\bar{h}} = \frac{x}{\left[\frac{5}{4}a_1\left(\theta^4 - \theta_{wet}^4\right) + \frac{4}{3}a_2\left(\theta^3 - \theta_{wet}^3\right) + \frac{3}{2}a_3\left(\theta^2 - \theta_{wet}^2\right) + 2a_4\left(\theta - \theta_{wet}\right) + a_5 \ln\frac{\theta}{\theta_{wet}}\right]} \tag{A21}$$

APPENDIX B

EFFECTIVE CONDUCTION LENGTH

The heat-transfer analysis presented herein is for laminar film boiling and yields an increasing wall temperature with distance x, or a heat-transfer coefficient which decreases with distance. In reality this laminar film will eventually transition to a turbulent film. Once transition to turbulent film boiling occurs, the analytical results of Hsu and Westwater (ref. 21) show an increase in the heat-transfer coefficient. Based on this information once the vapor film becomes turbulent, the boiling mechanism is more efficient and axial conduction gradient no longer has a significant effect on the wetting mechanism. It was observed during the experimental phase of this report that before the minimum was reached (as described in procedure) the wall temperatures were nearly constant along the tube axis. This would imply $d\theta/dx \approx 0$, at least for all $x \geq 1.90$ centimeters which is the first observation station. This would further imply that the entire region of analytic concern (both nucleate and film boiling) is confined within the first 1.90 centimeters. This is consistent with the results of reference 10, which indicate that a transition from laminar to turbulent film boiling may be associated with the instability (fig. 1) and that this transition occurred at a very short distance (0.38 cm).

The conduction gradient for film boiling existing in the region where nucleate and film boiling coexist can be obtained from the analysis of Semeria and Martinet by differentiating equation (3.8) of reference 11.

$$\frac{d\theta}{dx} = \left(\theta_{2,\bar{h}} - \theta_{wet}\right)\left(\frac{\bar{h}_2}{k_m t}\right)^{1/2} \exp\left[-\left(\frac{\bar{h}_2}{k_m t}\right)^{1/2} x\right] \tag{B1}$$

The film temperature difference $\theta_{2,\bar{h}}$ is based on the average heat-transfer coefficient evaluated over the effective length Le. This evaluation makes the resulting expression implicit and requires an iterative solution.

The effective conduction length Le is determined when the conduction gradient becomes effectively too small to influence the region where nucleate boiling is gradually replacing film boiling. This may be expressed as a fractional change:

$$\frac{\left(\frac{d\theta}{dx}\right)_{x=Le}}{\left(\frac{d\theta}{dx}\right)_{x=0}} = Z \tag{B2}$$

Solving equation (B1) for Z yields

$$Z = \exp\left[-\left(\frac{\bar{h}_2}{k_m t}\right)^{1/2} Le\right] \tag{B3}$$

From the visual information of reference 10, a value of Z may be obtained. Visual information at the minimum flux is shown in figure 1 for a heat flux of 6.52×10^4 watts per square meter and a liquid velocity of 0.76 meter per second. For these conditions the transition point from laminar to turbulent film boiling of 0.38 centimeters can be taken as the effective conduction length. From the derived analytical expression for $\theta_{2,\bar{h}}(x)$ (eq. (A21)), a value of $\theta_{2,\bar{h}} = 282^\circ$ K was calculated for the condition of figure 1. The authors of reference 10 measured θ_2 values of 220° K in the region of the minimum heat flux. To find the fraction conduction gradient Z for the conditions of figure 1, the following values were used in equation (B3)

$$\bar{h}_2 = \frac{6.52\times10^4 \text{ W/m}^2}{282^\circ \text{ K}} = 231 \text{ W/(m}^2)(^\circ\text{K})$$

$$Le = 3.8\times10^{-3} \text{ m}$$

$$k_m = 9.7 \text{ W/(m)}(^\circ\text{K})$$

$$t = 1.02\times10^{-4} \text{ m}$$

The value obtained for Z is Z = 0.16, indicating, according to equation (B3), that the conduction gradient has diminished to 16 percent of its initial value at x = 0. For the purpose of determining the effective conduction length at other conditions, a constant value of Z is assumed which results in the following equation for the effective conduction length:

$$Le = 1.86\sqrt{\frac{k_m t \theta_{2,\bar{h}}}{q_w}} \tag{B4}$$

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