TE-18

STUDY OF AIRCRAFT POSITION FIXING USING THE NAVY NAVIGATIONAL SATELLITE SYSTEM

by

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B. S., Mathematics, University of Kentucky (1962)

B. S., Electrical Engineering, University of Kentucky (1963)

Submitted in partial fulfillment
of the requirements for the degree
of Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
May, 1967

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ABSTRACT

The possibility of using the Navy Navigational Satellite System for position fixing of supersonic aircraft (speeds up to Mach 2.7) is examined. The effects of errors in required input data on the computed position of the aircraft are determined for various pass angles of the satellite with respect to the aircraft. Results indicate this system could be a valuable aid to supersonic aircraft navigation.

Thesis Supervisor: Winston R. Markey

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ACKNOWLEDGMENTS

I wish to thank Professor Winston Markey for his supervision of this thesis. I would like to express my thanks to Ken Britting for his assistance and advice. In addition, I wish to thank those members of the Computation Center who gave help and advice in debugging the computer program. The computational work found in this thesis was done at the Computation Center of the Massachusetts Institute of Technology, Cambridge, Massachusetts.

This thesis was prepared under DSR Project 70343 sponsored by the National Aeronautics and Space Administration Electronic Research Center, Cambridge, Mass., through NASA Grant number NGR 22-009-229.

The publication of this thesis does not constitute approval by National Aeronautics and Space Administration or by the MIT Experimental Astronomy Laboratory of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas..

FOREWORD

As air traffic along the major air routes of the world increases, separation standards for commercial aircraft will be reduced further and further. This reduction will require greatly increased accuracy in aircraft navigation equipment. If the aircraft concerned are supersonic, with speeds up to about Mach 2.7, the navigation problem becomes more acute. This thesis is concerned with the study of the Navy Navigational Satellite System as a possible aid in the solution of this problem. This system, heretofore, has been used primarily for fixed or slow-moving earth-bound vehicles and has yielded excellent results with these type craft. If used with high-speed aircraft, additional and more accurate data are required. This study determines the effects of errors in this data on the accuracy of the computed position of a supersonic aircraft for several flight paths.

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LIST OF SYMBOLS

a	semi-major axis of the satellite orbit
a_e	radius of the earth at the Equator
b	radius of the earth at the Poles
β	geocentric latitude
β_n	geocentric latitude of the aircraft at $t_n + \delta t_n$
$\Delta \beta_n$	defined as $\beta_n - \beta_1$
c	speed of light
$d(P_n, p_n)$	distance between P_n and p_n
e	eccentricity of satellite orbit
f	true anomaly of satellite position
f_G	frequency of reference oscillator in receiving station receiver
f_R	Doppler-shifted frequency received by the receiving station
f_t	frequency of the transmitter aboard the satellite (400. 0 mc.)
G	gravitational constant
γ_n	planar angle between the $\bar{r}_n$ and $\bar{\rho}_n$ vectors
h_A	altitude of the aircraft above the reference ellipsoid of the earth
i	inclination angle of satellite orbital plane
k	defined as $1 - b^2/a_e^2$
λ_e	longitude of the aircraft measured eastward from the Greenwich meridian
λ_{e_n}	longitude of the aircraft at time $t_n + \delta t_n$ measured eastward from Greenwich

λ_{I_n}	longitude of the aircraft at time $t_n + \delta t_n$ with respect to the geocentric inertial frame
p_n	position of aircraft at time of reception of t_n timing marker
ϕ_n	angle between the $\bar{r}_n$ position vector and the equatorial plane (a latitude angle)
r_e	radius of the earth at β
r_n	distance from the center of the earth to the satellite at time t_n
r_o	distance from the center of the earth to the satellite
ρ_n	distance from the center of the earth to the aircraft at time $t_n + \delta t_n$
t_a	time at which the e-frame and i-frame are coincident
t_n	time of transmission of the n-th timing marker by the satellite transmitter
$\Delta\lambda_{e_n}$	defined as $\lambda_{e_n} - \lambda_{e_1}$
M	mean anomaly of satellite position
m_e	mass of the earth
$N_{n, n+1}$	number of cycles of the $f_G - f_R$ signal counted between times of reception of the t_n and t_{n+1} timing markers
n_a	actual mean angular motion of the satellite
n_c	computed mean angular motion of the satellite
Ω	right ascension of the ascending node of the satellite orbital plane
ω	argument of perigee in satellite orbit
ω_{ie}	angular velocity of the earth with respect to inertial space
P_n	position of satellite at time of transmission of the t_n timing marker
t_p	time of perigee passage
ΔT_n	defined as $t_n - t_a$

Δt_n	defined as $t_n - t_1$
δt_n	time required for the signal carrying the t_n timing marker to travel from P_n to p_n
θ_n	longitude of the satellite at time t_n
V_E	eastward component of aircraft velocity
V_N	northward component of aircraft velocity

CHAPTER I

INTRODUCTION

With the advent of commercial aircraft capable of flying at supersonic speeds, the problem of accurate navigation becomes increasingly difficult, particularly in the heavily-traveled regions over the North Atlantic. In an effort to meet this increasing demand for airspace in the North Atlantic region, separation standards (Reference 7) have recently been reduced to 90 n. miles lateral (cross-track) separation, 1,000 ft. vertical separation and 15 minutes longitudinal (along track) separation. To meet these demands for increased navigation accuracy, particularly for supersonic aircraft, many types of navigation systems and combinations of navigation systems are being examined in order to determine the system or systems most suitable for use with commercial supersonic aircraft. Factors that must be considered in the evaluation of these systems, in addition to accuracy, are cost, present availability, reliability, coverage area, etc. One particular navigation system being examined for these purposes is the Navy Navigational Satellite System. This system has been used successfully for several years to enable fixed or slow-moving earth-bound stations to determine their position accurately. Recent tests have shown that this system could also be used by aircraft to determine position if the aircraft velocity and altitude are known to a high degree of accuracy.

The purpose of this thesis is to determine the accuracy with which a supersonic aircraft can determine it's position using the Navy Navigational Satellite System with given errors in the required input data which define the motion of the satellite and the aircraft. These input data consist of the orbital elements defining the satellite's position and motion and the aircraft speed, heading, and altitude.

The operation of the system is simulated by a digital computer program. The general approach is to define a particular satellite orbit and a particular flight path for a supersonic aircraft. The measurements the aircraft would make in the actual flight (assuming no imperfections) are determined first. Then, the position of the aircraft is computed, based on input data which is

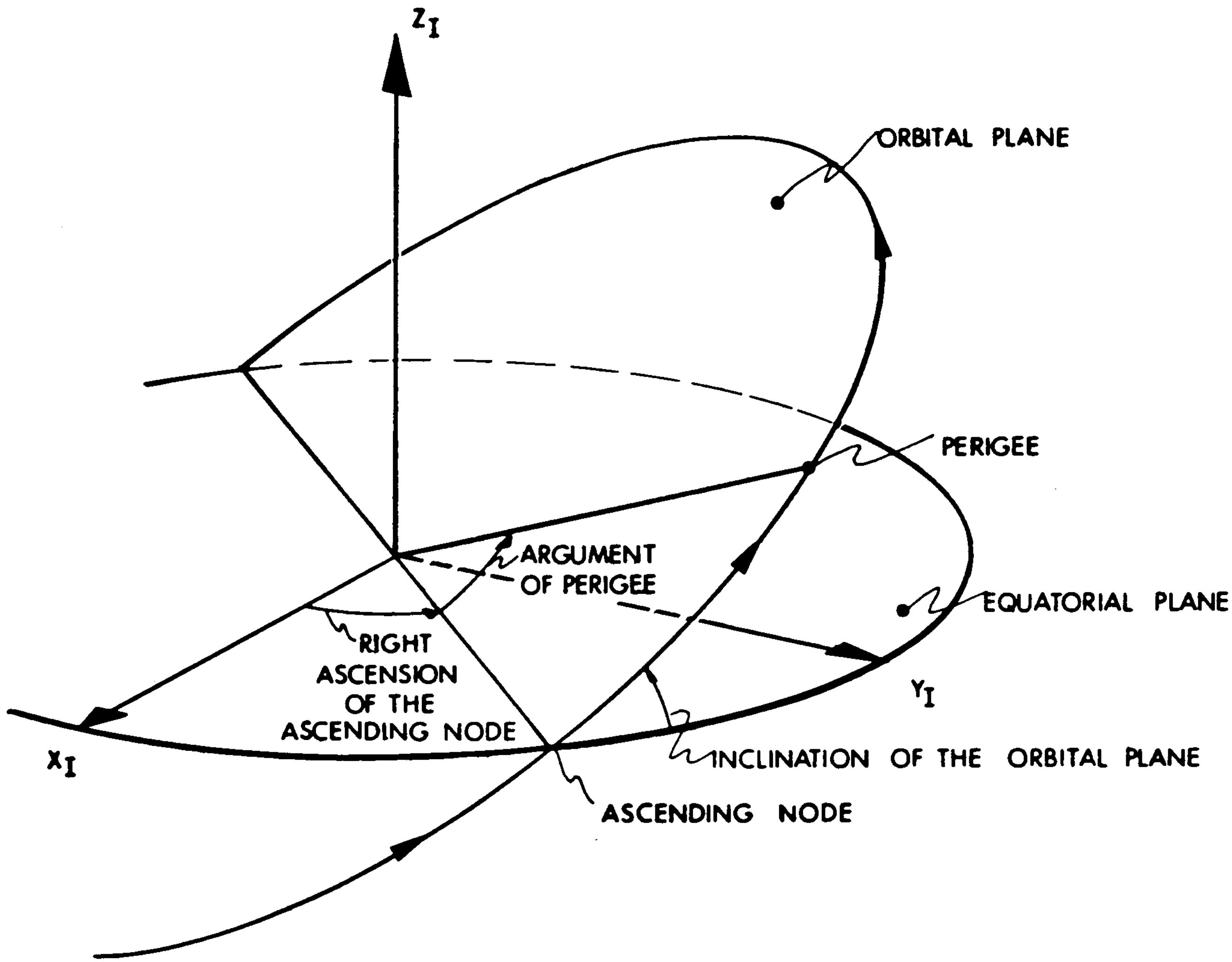

Figure 1. Orbital Elements Defining Orientation of Orbital Plane

correct, except for one parameter that is perturbed by a given error. In this way, the sensitivity of the accuracy of the position determination to the various errors is determined. This procedure is carried out for three separate aircraft flight paths--one for which the satellite is just above the aircraft's horizon (low satellite pass angle), a second for intermediate pass angle, and a third for which the satellite passes almost directly over the aircraft (pass angle near 90^{o}).

Because the actual satellite orbits are essentially circular, polar orbits, the digital computer simulation considers the satellite orbit to be a perfect circular, polar one. The satellite position and motion are determined from the six orbital elements used in the simulation; these are right ascension of the ascending node, argument of perigee, inclination of the orbital plane, eccentricity, semi-major axis, and time of perigee passage (See Figure 1). The effects of inaccuracies in the knowledge of these elements are considered by adding, one at a time, a given error to a particular orbital parameter and then, the aircraft position is computed using correct input data except for the perturbed value of that particular orbital element. The error in computed position resulting from this particular error gives an indication as to how accurately this parameter should be known. For a perfect circular, polar orbit, the inclination is 90^{o}, the eccentricity is zero, the argument of perigee is arbitrary, and the semi-major axis equals the radius of the orbit. For this simulation, only one satellite is considered and it's orbital plane is somewhat arbitrarily chosen to coincide with the Greenwich meridian plane near the time it is to be in sight of the (hypothetical) aircraft.

The aircraft in the simulation is constrained to fly at a constant speed of 1,800 statute miles per hour, constant heading, and constant 70,000 feet altitude. It is felt that this is not an unreasonable restriction because, once the aircraft has reached its planned altitude, its velocity and altitude are likely to remain essentially constant during as short a period as the satellite will be in view of the aircraft. The three separate flight paths considered take into account the heading the actual aircraft would have at the three corresponding stages of it's flight across the North Atlantic from New York to London. It is noted again that only one satellite orbit is considered in the simulation and the (hypothetical) aircraft's position,p, is moved to correspond to the starting position of each of the three flight paths. The input data required to define the aircraft's motion are speed, heading, and altitude; these are expressed in the simulation as the east component of velocity, north component of velocity, and altitude. The procedure for determining the effects of errors in these quantities on the computed position of the aircraft is exactly the same as that followed for errors in the knowledge of the orbital elements. The values chosen for the errors in vehicle motion are intended to be in line with the accuracy to which this input data is likely to be known in an operational situation.

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CHAPTER II

OPERATION OF THE NAVY NAVIGATIONAL SATELLITE SYSTEM

In this chapter, the operation of the Navy Navigational Satellite System will be discussed.

The Navy Navigational Satellite System uses the phenomenon known as the Doppler effect to enable a receiving station to determine its position based on its measurements of a radio frequency signal transmitted by an orbiting artificial satellite.

The Doppler effect results from the relative motion of the transmitter of a signal with respect to the receiver (or vice versa) which detects the signal and it is manifested by a shift in the frequency received even though the transmitter frequency may remain fixed. The equation describing the Doppler effect can be written

$$f_R = f_t (1 + \dot{r}/c)$$

where f_R is the frequency of the signal detected by the receiver, f_t is the transmitter frequency (constant for this system), $\dot{r}$ is the velocity of the transmitter with respect to the receiver (or vice versa), and c is the speed of light. This equation does not consider the constant phase lag due to propagation time; this is equal to $\frac{2\pi f_t d_o}{c}$, where d_o is the distance between the satellite and aircraft at $t = t_o$. If there is no relative motion of the transmitter with respect to the receiver, then the Doppler effect (shift) is not in evidence.

The Navy Navigational Satellite System is set up so that each of the satellites (there are four in evenly spaced orbital planes) transmits a stable radio frequency signal of frequency f_t = 400 mc. which has superimposed on it timing markers that are transmitted at precise two minute intervals. The receiving station (aboard the aircraft) receives the Doppler-shifted signal f_R from the particular satellite in view and mixes this signal with a reference oscillator signal of frequency f_G to obtain a beat frequency $f_G - f_R$, where f_G is chosen to be greater than any expected value of f_R. The actual measurements used by the receiving station in determining its position are the number of cycles of the $f_G - f_R$ signal received between the times of reception of consecutive time markers. The number of cycles counted between the times of reception of the t_n and t_{n+1} timing markers can be expressed as

$$N_{n,n+1} = \int_{t_n+\delta t_n}^{t_{n+1}+\delta t_{n+1}} (f_G - f_R)\, dt$$

where δt_n represents the time required for the signal containing the t_n timing marker to travel from the satellite to the aircraft; a similar definition holds for δt_{n+1}. Expansion of this equation yields

$$N_{n,n+1} = \int_{t_n+\delta t_n}^{t_{n+1}+\delta t_{n+1}} f_G\, dt - \int_{t_n+\delta t_n}^{t_{n+1}+\delta t_{n+1}} f_R\, dt$$

$$= f_G (t_{n+1} - t_n) + f_G (\delta t_{n+1} - \delta t_n) - \int_{t_n+\delta t_n}^{t_{n+1}+\delta t_{n+1}} f_R\, dt$$

Because the number of cycles <u>received</u> by the receiver between the <u>times of reception</u> of the t_n and t_{n+1} timing markers is equal to the number of cycles transmitted between the t_n and t_{n+1} timing markers, the integral

$$\int_{t_n+\delta t_n}^{t_{n+1}+\delta t_{n+1}} f_R\, dt = \int_{t_n}^{t_{n+1}} f_t\, dt = \text{constant}$$

Therefore,

$$N_{n,n+1} = (f_G - f_t)(t_{n+1} - t_n) + f_G(\delta t_{n+1} - \delta t_n)$$

where f_G and f_t are constants and $(t_{n+1} - t_n)$ is equal to 2 minutes. The final expression takes the form

$$N_{n,n+1} = (f_G - f_t)(t_{n+1} - t_n) + f_G \left\{ \frac{1}{c} \left[\left| \bar{r}_{n+1} - \bar{\rho}_{n+1} \right| - \left| \bar{r}_n - \bar{\rho}_n \right| \right] \right\}$$

where $\bar{r}_n$ is the position vector of the satellite with respect to the center of the earth at the time of <u>transmission</u> of the t_n timing marker and $\bar{\rho}_n$ is the position vector of the receiving station (aircraft) with respect to the center of the earth at the time of <u>reception</u> of the t_n timing marker. An equation of this form may be written for each two-minute interval over which Doppler measurements (counts of cycles) are taken.

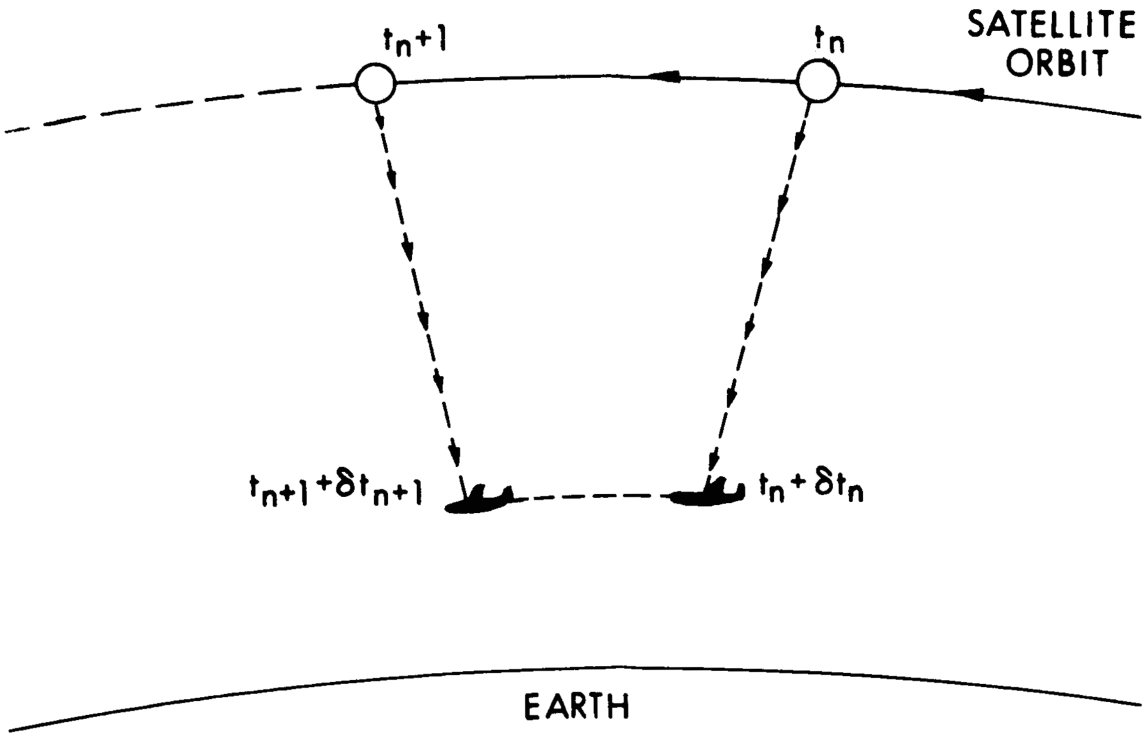

Figure 2. Operation of Navy Navigational Satellite System.

Writing $|\bar{r}_n - \bar{\rho}_n|$ as d (P_n, p_n), where P_n is the (satellite) position corresponding to $\bar{r}_n$, p_n is the (aircraft) position corresponding to $\bar{\rho}_n$, and d (P_n, p_n) is the distance between P_n and p_n, it is noted, for each two-minute interval, that $N_{n,\, n+1}$ is the measured quantity and f_G, f_t, and c are constants; this leaves as unknowns the quantities d (P_n, p_n) and d (P_{n+1}, p_{n+1}).

$$N_{n,\, n+1} = (f_G - f_t)(t_{n+1} - t_n) + \frac{f_G}{C}\left[d\,(P_{n+1}, p_{n+1}) - d\,(P_n, p_n)\right]$$

The satellite positions P_n and P_{n+1}, corresponding to the times of transmission of the t_n and t_{n+1} timing markers, can be determined by the received station from data transmitted by the satellite in a manner which is described later. This, then, leaves only the aircraft positions p_n and p_{n+1} as unknowns. If the altitude of the aircraft is known, p_n and p_{n+1} each represent two unknowns, which may be expressed as latitude and longitude with respect to the equator and the Greenwich meridian, respectively. However, if the motion of the aircraft (speed and heading) is accurately known during the two minute interval, it is possible to express p_{n+1} in terms of p_n, leaving only two unknowns in the equation, i.e. the aircraft latitude and longitude corresponding to p_n, the position at the time it receives the t_n timing marker.

Even though both f_G and f_t are very stable during as short a time as the satellite is in view of the receiving station, it is difficult to determine the essentially constant difference $f_G - f_t$ to the accuracy desired. This adds a third unknown to the above equation. By taking Doppler measurements over three intervals, however, (n = 1, 2, 3) and expressing the aircraft positions p_2, p_3, and p_4 in terms of p_1 and aircraft motion after t_1, the three resulting equations in three unknowns can be used to eliminate the term $(f_G - f_t)$ from these equations. This procedure yields the following two basic equations by which the aircraft position p_1 (hence p_2, p_3, and p_4) may be determined:

$$- \left[d\,(P_1,\ p_1) \right] + 2 \left[d\,(P_2,\ p_2) \right] - \left[d\,(P_3,\ p_3) \right] = \frac{c}{f_G}\,(N_{1,2} - N_{2,3})$$

$$- \left[d\,(P_2,\ p_2) \right] + 2 \left[d\,(P_3,\ p_3) \right] - \left[d\,(P_4,\ p_4) \right] = \frac{c}{f_G}\,(N_{2,3} - N_{3,4})$$

where the subscripts relate to the relevant time markers t_1, t_2, t_3, and t_4 ($t_2 = t_1 + 2$ minutes, etc.). The aircraft latitude and longitude corresponding to positions p_2, p_3, and p_4 are easily determined since the motion of the aircraft for the time after p_1 is known.

In order that the receiving station can determine the satellites' positions, the satellites' transmitted signals are phase-modulated with the information necessary to perform these computations. This information consists of a set of orbital elements, plus corrections, which serve to define a specific orbit corresponding to each timing marker. This is necessary because the actual orbit of the satellite is not perfectly circular or elliptical due to the ellipticity of the earth and to the nonuniformity of the earth's gravitational field (gravity anomalies).

The remaining input data required are aircraft speed, heading, and altitude, which describe the motion of the aircraft and enable the positions p_2, p_3, and p_4 to be expressed in terms of the position p_1.

The equipment necessary for the receiving station requires a base area of approximately 4 by 6 feet and the maximum component height is about 3 feet. The cost of the complete receiving station is about $60,000. However, if there is already a digital computer aboard that could be used for the computations required, the cost (exclusive of the computer) would be only about $5,000. The antenna required at the receiving station can be fairly small due to the short wavelength (0.75 m) of the transmitted signals. Even though this antenna is small, it may, however, present a troublesome drag problem for supersonic aircraft. Study and experimentation could determine more fully the magnitude of this problem.

This summarizes the principles, computations, and equipment which the Navy Navigational Satellite System employs in its operation.

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CHAPTER III

DIGITAL COMPUTER SIMULATION

In this chapter, the details of the digital computer simulation will be discussed.

The digital computer program can be logically divided into four major parts. Part 1 is concerned with the computation of the $N_{1,2}$, $N_{2,3}$, and $N_{3,4}$ that the aircraft would actually measure if it were flying the defined flight path, given the defined satellite orbit. These computed quantities are not changed in any subsequent part of the program. Part 2 computes the satellite positions P_1, P_2, P_3, and P_4 based on input parameters of which one may be perturbed from its correct value for a particular run. Part 3 determines the aircraft positions p_1, p_2, p_3, and p_4 based on an estimated value of p_1 and on the given input data, which may contain one parameter that has been perturbed from its correct value for a particular run. Part 4 then computes the aircraft position based on the values of $N_{n, n+1}$, P_n, and p_n computed in Parts 1, 2, and 3. As stated before, only one input parameter is perturbed from it's correct value for each performance of the program, so the error in the computed position of the aircraft is dependent on only one input error.

A. COMPUTATION OF DOPPLER MEASUREMENTS

The operations carried out in Part 1 will now be discussed. Before the computation of $N_{n,\ n+1}$ (n = 1, 2, 3), the data defining the exact positions and motions of the aircraft and satellite are read into the computer. These data include the time corresponding to satellite position P_1 ($t_0 = t_1$), the time at which the satellite passes through perigee, the remaining orbital elements, the aircraft geocentric latitude and longitude at time t_1, aircraft east and north velocity components, aircraft altitude, and the time, t_a, at which the geocentric inertial frame (i - frame) coincides with the geocentric earth frame (e - frame). Next, values of needed constants are set into the computer. As given previously, the value of $N_{n,\ n+1}$ is

$$\left(f_G - f_t\right)\left(t_{n+1} - t_n\right) + \frac{f_G}{C}\left[d\left(P_{n+1},\ p_{n+1}\right) - d\left(P_n,\ p_n\right)\right]$$

where n = 1, 2, 3 for a six minute interval.

In the simulation f_G, f_t, C, and $\left(t_{n+1} - t_n\right)$ are known. Therefore, the values of d $\left(P_n,\ p_n\right)$ for n = 1, 2, 3, 4 must be computed based on the defined positions and motion of the aircraft and satellite. Before this is done, the coordinates of the position of each craft are referred to the i-frame. This frame is chosen because the orbital elements defining the satellite motion are given with respect to this frame, and if the vehicle position is given in spherical coordinates with respect to the e-frame, it is a very simple matter to compute its coordinates in the i-frame, also.

Because the satellite orbit (in the simulation) is a circular, polar orbit, the computation of the satellite position coordinates in the i-frame is very straightforward. The longitude angle (θ_n) coordinate is a constant for each P_i and is equal to the right ascension of the ascending node; the length of the radius vector, r_n, is also a constant and is equal to the radius of the orbital path. The angle corresponding to latitude varies as the satellite moves along its path. It is assumed in the simulation that the satellite is traveling from south to north when it is in view of the aircraft. The angular velocity of the satellite is given by the equation

$$N_a = \text{Mean angular motion} = \sqrt{\frac{Gm_e}{a^3}} \quad \frac{\text{rad}}{\text{sec}}$$

where G is the gravitational constant, m_e is the mass of the earth (mass of the satellite is ignored), and a is the semimajor axis of the orbit. For a circular orbit, a is equal to r. In order to place the satellite at a particular point in its orbit at a given time, the time it passes through perigee (t_p) is specified as one of the orbital elements. Because this is a circular orbit, the argument of perigee is arbitrary and was set equal to zero degrees in the simulation. Therefore, the latitude angle of the satellite can be expressed as $\theta_n = N_a (t_n - t_p)$ rad. The position coordinates of the satellite are, therefore, r_n, θ_n, and ϕ_n referred to the geocentric inertial frame.

The computation of the positions p_i of the aircraft presents more difficulty because, though the aircraft altitude is maintained constant, the radius of the earth changes with latitude and this must be taken into account. Both latitude and longitude rate are affected by this variation even though aircraft speed, heading, and altitude remain fixed. The expression for the radius of the accepted analytic figure of the earth (Hayford ellipsoid) may be written as

$$\frac{r_e^2 \cos^2 \beta}{a_e^2} + \frac{r_e^2 \sin^2 \beta}{b^2} = 1$$

where β is the geocentric latitude, r_e is the radius of the earth at β, a_e is the radius of the earth at the equator, and b is the radius of the earth at the poles. This equation may be rearranged to yield

$$r_e^2 = \mathbf{b} \left(1 - k \cos^2 \beta\right)^{-1/2}$$

where k is defined to be $\frac{a_e^2 - b^2}{a_e^2}$. The square root term may be expanded into a power series of the form $(1 - x)^{-n} = 1 + n + \frac{n(n+1)}{2} x^2 + \ldots$.

By eliminating all terms of the power series which yield magnitudes less than 1 ft. the expression for r_e may be written as

$$r_e = b\left[1 + \frac{k}{4}(1 + \cos 2\beta) + \frac{3k^2}{32}(1 + \cos 2\beta)^2\right] .$$

In the simulation, b and r_e are expressed in kilometers. In order to compute positions p_1, p_2, p_3, and p_4 of the aircraft given its position at t_1, the values of latitude rate and longitude rate are computed and integrated piecewise starting at t_1 to yield, eventually, p_1, p_2, p_3, and p_4. It is noted that the position of the aircraft at t_1, the time of <u>transmission</u> of the t_1 timing marker is the position read into the computer. First, the positions of the aircraft corresponding to t_2, t_3, and t_4 are determined; then, these positions are adjusted to account for the very short time required for the radio frequency signal to travel from the satellite to the aircraft, δt_1, δt_2, etc.

These adjusted positions are the ones corresponding to p_1, p_2, p_3, and p_4.

In order to compute the geocentric latitude rate (also the longitude rate), it is necessary to know the distance from the center of the earth to the aircraft, r_A. It has been shown that this distance can be expressed, with an error less than 1 ft, as the sum of the altitude of the aircraft (measured perpendicularly from the surface of the earth), h_A, and the radius of the earth at the point directly below the aircraft, r_e. Because the earth is, essentially, an ellipsoid and not a sphere, these two displacement vectors are not coincident (Fig. 3). In the simulation, the latitude on which computations are based is the geocentric latitude of the aircraft, not the geocentric latitude of the point directly below the aircraft on the surface of the earth. Calculations show that, if the latitude of the aircraft is used to compute the theoretical radius of the earth at the point beneath the aircraft, a maximum error of less than 1 ft. is experienced; therefore, this approximation is a very good one.

The geocentric latitude rate of the aircraft can be expressed as $\frac{d\beta}{dt} = \frac{V_n(\cos \delta_0)}{r_e + h_A}$ where V_n is the north velocity component of the aircraft velocity, r_e is the radius of the earth, h_A is the altitude of the aircraft, and δ_0 is the angle shown in Fig. 3. In the simulation, the $\cos \delta_0$ term was considered to be 1.0. This term differs from 1.0 by, at most, 5.7×10^{-6} and therefore causes an error

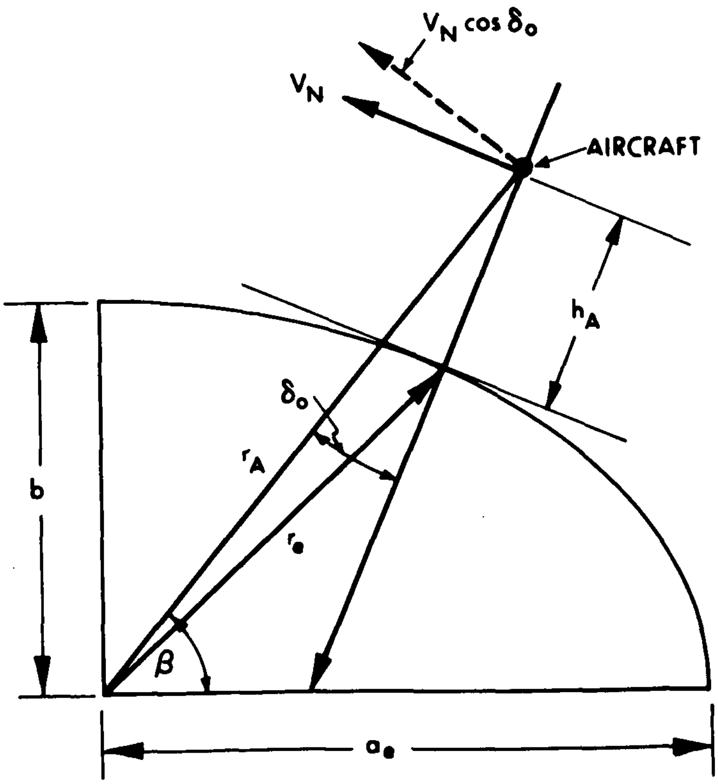

Figure 3. Geometry of Aircraft Latitude Motion

in north velocity less than 0.01 mph. Even though both V_N and h_A are considered to be constant during the flight, latitude rate changes because the radius of the earth varies with latitude. In order to determine the change in latitude between t_1 and t_2, t_2 and t_3, and t_3 and t_4, this equation is integrated piecewise for small intervals of time, i.e., the value of r_e is considered to be a constant for a small Δt. The change in latitude is, thus, determined for the time from t to t + Δt, then a new value of r_e is computed and this procedure is continued until the aircraft latitude is determined for times t_2, t_3, and t_4.

Along with each computation for latitude change, the change in longitude is determined in a similar manner. The expression for longitude rate is

$$\frac{d\lambda_e}{dt} = \frac{V_E}{\left(r_e + h_A\right) \cos \beta}$$

where V_E is the (constant) east component of aircraft velocity. Because β, thus r_e, also, is changing if $V_N \neq 0$, the change in longitude must be determined by integrating this equation in a piecewise fashion, as for latitude. For each increment Δt, the change in longitude is computed first, then the change in latitude is determined. These changes are added to the previous values, and this procedure is repeated until the aircraft latitude and longitude are determined for times t_2, t_3, and t_4. In this simulation Δt was set to 1 second because V_N, thus $\frac{d\beta}{dt}$, is great in magnitude for two of the three flight paths considered.

Because the aircraft positions p_1, p_2, p_3, and p_4 correspond to the times at which the aircraft <u>receives</u> the timing markers ($t_1 + \delta t_1$, $t_2 + \delta t_2$, etc.) and the positions just computed correspond to the time of <u>transmission</u> of the timing markers (t_1, t_2, etc.), these positions must be adjusted by computing the time required for the radio signal to travel from the satellite to the aircraft. This requires that the distance between the satellite (at t_n) and the aircraft (at $t_n + \delta t_n$) be known. Because, at the present stage, only the aircraft and satellite positions at t_n are known, these positions are used to calculate this distance. The maximum δt_n will occur for the low satellite pass angle case (pass angle $\approx \sin^{-1} \frac{600}{2600} \cong 13^o$) and will be on the order of 16 milliseconds. Because the aircraft is traveling at 1,800 mph, the aircraft will travel, at most, about 42 ft. in the time δt_n; therefore, the above computed δt_n may be in error equal to the time it takes a radio wave to travel 42 ft. (for the worst case). This approximation leads to an error in the computed position p_n on the order of

$$\frac{\left(\frac{V_N}{3,600}\right)\left(\frac{42}{5,280}\right) \times 5,280}{186,000} = 0.11 \times 10^{-3} \text{ ft}$$

which is completely negligible. Therefore, the values of δt_n may be computed very accurately by using the positions of the aircraft and satellite at the times t_n for the computations. The change in latitude and longitude between times t_n and $t_n + \delta t_n$ are computed in the same manner as mentioned above.

In this simulation, all distances are computed using the latitude, longitude, and radius vector length of each position with respect to the i-frame. The distance $d(P_n, p_n)$ may be expressed as

$$d\left(P_n, p_n\right) = \left[r_n^{\ 2} + \rho_n^{\ 2} - 2\, r_n \rho_n \cos \gamma_n\right]^{1/2}$$

where r_n is the length of the $\bar{r}_n$ vector, ρ_n is the length of the $\bar{\rho}_n$ vector, and γ_n is the planar angle between these two vectors. This equation follows directly from the cosine law of plane trigonometry. The term $\cos \gamma_n$ may be written as

$$\cos \gamma_n = \sin \phi_n \sin \beta_n + \cos \phi_n \cos \beta_n \cos \left(\theta_n - \lambda_{I_n}\right)$$

where ϕ_n is the angle between the equatorial plane and the vector $\bar{r}_n$ (a latitude angle), β_n is the geocentric latitude of the aircraft, θ_n is the angle from the i-frame X-axis to the meridian plane containing the satellite radius vector $\bar{\rho}_n$, and λ_{I_n} is the corresponding (longitude) angle of the aircraft measured from the X_I axis. This expression is derived from the cosine law of spherical trigonometry. The angle λ_{I_n} of the aircraft is equal to its longitude with respect to the earth, λ_{e_n}, plus the angle $\omega_{ie}(t - t_a)$ where ω_{ie} is the constant angular velocity of the earth with respect to the i-frame.

At this point, the actual number of cycles counted by the aircraft equipment in each interval between consecutive timing markers can be calculated. The number of cycles counted between times of reception of the t_n and t_{n+1} timing markers, as given above, is expressed as

$$N_{n,\, n+1} = \left(f_G - f_t\right)\left(t_{n+1} - t_n\right) + \frac{f_G}{C}\left[d\left(P_{n+1}, p_{n+1}\right) - d\left(P_n, p_n\right)\right]$$

The value chosen for f_G in the simulation was $f_G = 400.01$ mc. with $f_t = 400.0$ mc. It was desired to make f_G as close to f_t in frequency as possible in order to reduce the number of counts; this was done to reduce computational error (not wanting to lose significant digits). The value of $(t_{n+1} - t_n)$ for all n is two minutes (the error in time between $t_{n+1} - t_n$ is held to less than 10 microseconds in the actual system). Since P_n and p_n (for n = 1, 2, 3, 4) have been determined, the computation of number of cycles counted, $N_{n,\, n+1}$, takes place. Note that, up to this point, all input data is, essentially, error-free. From this point on, one of the input parameters used may be in error, as explained previously.

B. GENERAL COMPUTATION OF SATELLITE POSITIONS

Part 2 of the simulation will be discussed here. This section computes the satellite positions P_1, P_2, P_3, and P_4 in a manner that will allow the input data (the orbital elements) to contain errors. The previous (error-free) computations of satellite positions allowed the use of greatly simplified equations because the (hypothetical) orbit is circular and polar. Now, with possible error terms added to the correct values of orbital elements, the orbit defined by this (perturbed) data may not be a polar, circular orbit. Therefore, general equations must be used to determine the satellite positions. First, the error terms are read into the computer and added to the correct values of orbital elements. Next, it is necessary to compute the value of mean angular motion,

$$N_c = \sqrt{\frac{G m_e}{a^3}} \ \frac{\text{rad}}{\text{sec}}$$

Then, the mean anomaly

$$M = N_c \, (t - t_p) \text{ rad}$$

is computed for the time corresponding to the transmission of the time marker t_n. Because the mean anomaly normally does not equal the actual angle that the radius vector to the satellite makes with the line of apsides (through perigee), it is necessary to compute this angle f, called the true anomaly, using the value of mean anomaly. For small values of eccentricity, Reference 1 shows that the relationship between f and M may be expressed as

$$f = M + \left(2e - \frac{e^3}{4}\right) \sin M + \left(\frac{5e^2}{4}\right) \sin 2M + \left(\frac{13e^3}{12}\right) \sin 3M$$

where e is the eccentricity and is near or equal to zero. Let a right-handed rectangular coordinate system with origin at the center of the earth be designated with axes X_{op}, Y_{op}, Z_{op} such that X_{op} passes through perigee, Y_{op} is also in the orbital plane and 90° from X_{op}, and Z_{op} is perpendicular to the orbital plane. The position of the satellite in this frame can be expressed as

$$X_{op} = r_o \cos f$$

$$Y_{op} = r_o \sin f$$

$$Z_{op} = 0$$

The value of r_o is determined by the equation

$$r_o = \frac{a\,(1 - e^2)}{1 + e \cos f}$$

where a is the semi-major axis of the orbit (See Figure 4).

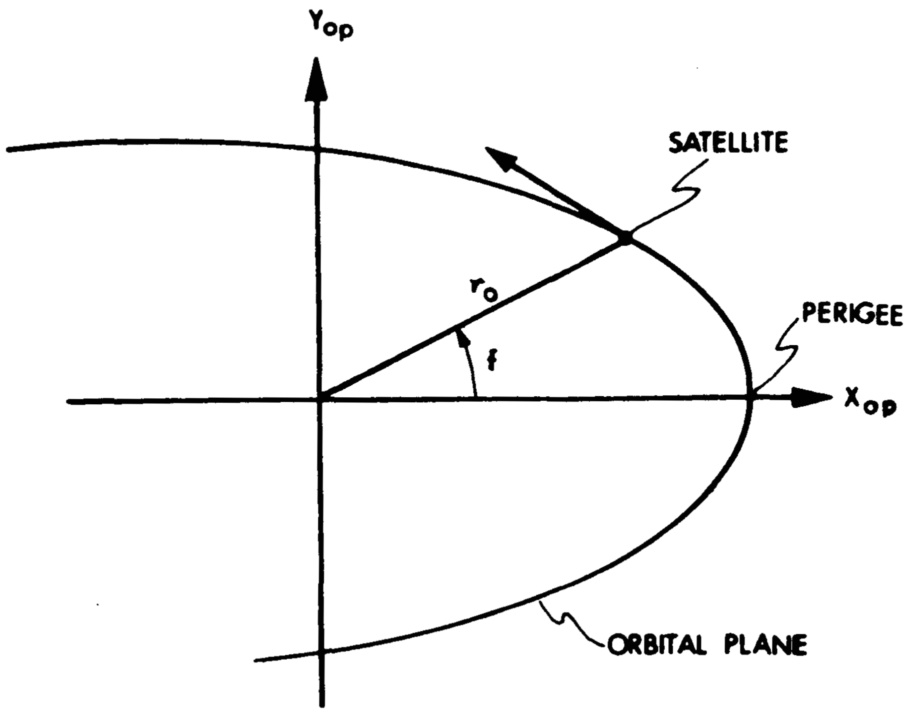

Figure 4. Geometry of Satellite Motion

Because the X_{op}, Y_{op}, Z_{op} frame (OP-frame) may not be oriented in a simple fashion with respect to the i-frame, it is necessary to perform a coordinate transformation between the OP-frame and i-frame by means of the rotation matrix

$$\underline{R} = \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$

where

$$\begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix} = \underline{R} \begin{pmatrix} X_{op} \\ Y_{op} \\ Z_{op} \end{pmatrix}$$

Because the satellite is always in the X_{op}, Y_{op} plane, the Z_{op} coordinate is zero and there is no need to compute l_3, m_3, and n_3. The expressions for the remaining terms in the matrix are:

$$l_1 = \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i$$
$$l_2 = - \cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i$$

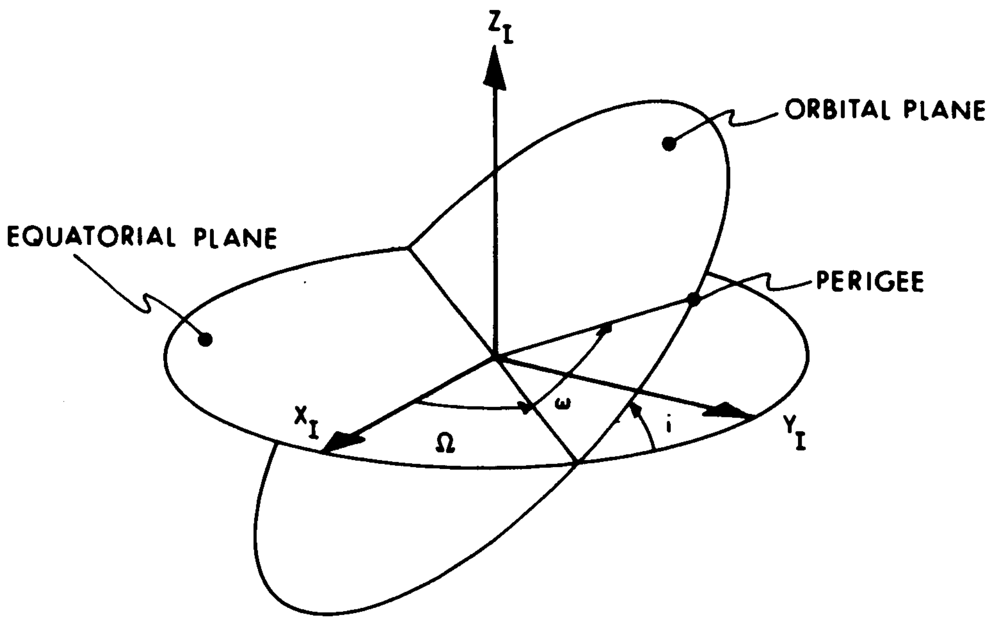

Figure 5. Orbital Elements Ω, ω, i

$$m_1 = \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i$$
$$m_2 = -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i$$
$$n_1 = \sin\omega\sin i$$
$$n_2 = \cos\omega\sin i$$

where Ω is the right ascension of the ascending node, ω is the argument of perigee, and i is the angle of inclination (See Figure 5). This coordinate transformation yields the rectangular coordinates of the satellite position with respect to the i-frame. Because the range difference computations use coordinates of longitude, radius vector length, and latitude, it is now necessary to convert from X_I, Y_I, Z_I coordinates to latitude, longitude, and radius vector length. From Figure 6, it is seen that the following relationships hold:

$$\tan\theta = \frac{Y_I}{X_I}, \quad \tan\phi = \frac{Z_I}{\sqrt{X_I^2 + Y_I^2}} .$$

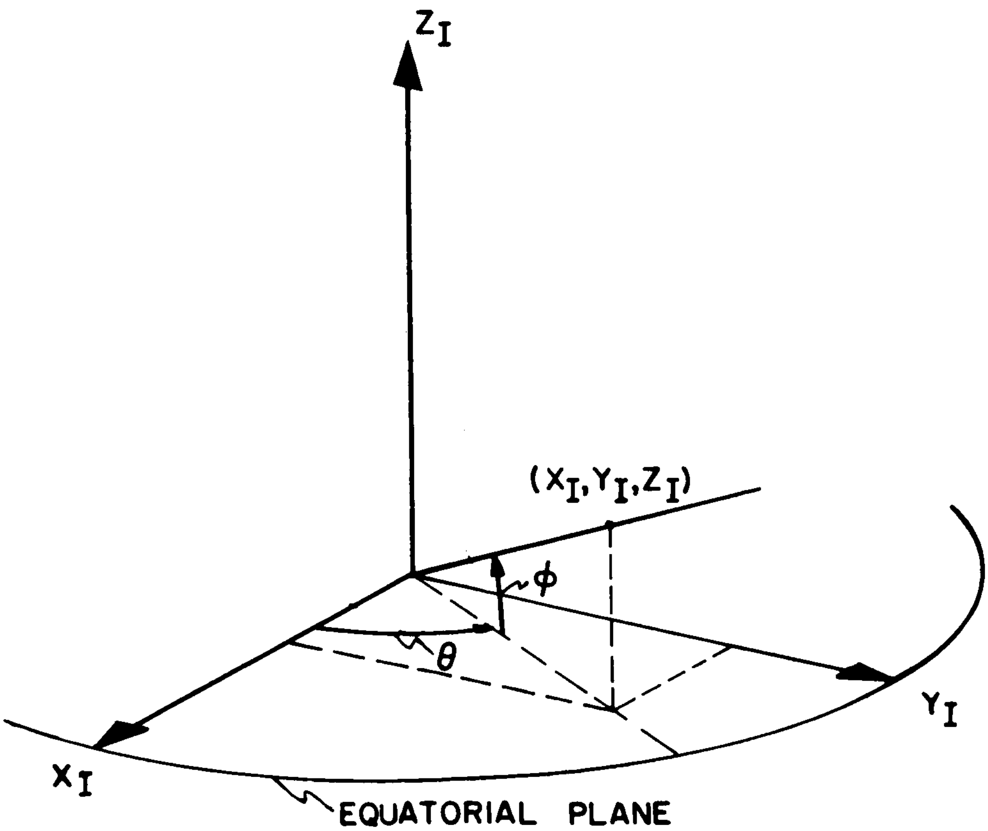

Figure 6. Relationship of Coordinates

The value of r is the same as r_o, computed previously.

This summarizes the procedure carried out to determine the positions P_1, P_2, P_3, and P_4 of the satellite for all cases including those for which the orbital elements define an orbit other than a perfect circular, polar orbit; this will be the case for errors in either i or e.

C. GENERAL COMPUTATION OF AIRCRAFT POSITIONS

The computation of aircraft positions p_2, p_3, and p_4 based, initially, on an estimated position for p_1, is the concern of Part 3 of the simulation. The perturbations in the input data defining the aircraft motion are read into the computer at the start of this procedure and are added to the known correct values before computation of p_1, p_2, p_3, and p_4 is begun. Since the actual value of p_1 is not known to the navigator, an estimated value must be used to start. In this simulation, as a mere convenience, the estimated position read into the computer was made to correspond to the estimated aircraft position at t_1-not $t_1 + \delta t_1$ which corresponds to p_1. Starting with this initial position, the changes in latitude and longitude are determined between t_1 and t_2, t_2 and t_3, and t_3 and t_4 in exactly the same manner as was done in Part 1. Since these positions do not correspond exactly to p_1, p_2, etc., they must be adjusted in the same manner as in Part 1; this time, however, initially, the estimated positions at t_1, t_2, t_3, and t_4 are used to compute the δt_n's. Because the estimated positions are expected to be within 20 or 30 miles of the actual positions, this causes no great difficulty. Once this set of p_1, p_2, p_3, and p_4 is determined, the latitude and longitude of p_1, β_1 and λ_{e_1}, respectively, are considered as the two unknowns in the two basic equations given in Chapter II and the positions p_2, p_3, and p_4 are now expressed in terms of β_1 and λ_{e_1}. This is done by considering p_n to be expressed as $\beta_n = \beta_1 + \Delta\beta_n$ and $\lambda_{e_n} = \lambda_{e_1} + \Delta\lambda_{e_n}$. This will be discussed more fully in the discussion of Part 4 of the simulation. Because the accuracy of the computation of $\Delta\beta_n$ and $\Delta\lambda_{e_n}$ depends on the accuracy of the initial estimate, there will be some error in the determination of the $\Delta\beta_n$'s and $\Delta\lambda_{e_n}$'s due only to this error. Since the error in the estimated position will be less than 30 miles, the resulting errors in $\Delta\beta_n$ and $\Delta\lambda_{e_n}$ are small. To eliminate this source of error completely, however, after each correction to β_1 and λ_{e_1} is computed, the values of $\Delta\beta_n$ and $\Delta\lambda_{e_n}$ are computed again, this time being based on the improved values of β_1 and λ_{e_1}.

D. SOLUTION OF THE NAVIGATION EQUATIONS

In Part 4, the procedure by which the two basic navigation equations are solved for β_1 and λ_{e_1} is discussed in detail. As has been mentioned, the solution of these two equations starts with an estimated value for β_1 and λ_{e_1}. The procedure followed is one of iteration - starting with the estimated values of β_1 and λ_{e_1}, a set of corrections are computed for β_1 and λ_{e_1} which, when added to the estimated values, yield new values which are closer to the solution. This procedure continues until the corrections come sufficiently close to zero, at which time the values of β_1 and λ_{e_1} are considered to be the solution of the equations. The particular method used is Newton's Method for two equations in two unknowns. A brief description of this method will now be given. Let two equations be given,

$$f(x, y) = 0$$

$$g(x, y) = 0,$$

from which the unknowns x and y are to be determined. Starting with an estimate for x and y, the corrections Δx and Δy to the estimated values are:

$$\Delta x = \frac{g\frac{\partial f}{\partial y} - f\frac{\partial g}{\partial y}}{\frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}}, \quad \Delta y = \frac{f\frac{\partial g}{\partial x} - g\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}}$$

where the quantities in the equations are evaluated, initially, by using the estimated values of x and y. Substitution into these equations yields values of Δx and Δy which are added to the estimated values. This results in an improved pair of values for the roots. Now, a new set of corrections Δx and Δy are computed using the improved values of x and y to evaluate the above expressions. This process continues until the corrections become negligable, at which time the values obtained for x and y are the solutions to the equations. Following this procedure, the two basic navigation equations can be written as

$$f(\beta_1, \lambda_{e_1}) = -\left[d(P_1, p_1)\right] + 2\left[d(P_2, p_2)\right] - \left[d(P_3, p_3)\right] + \frac{c}{f_G}\left(N_{2,3} - N_{1,2}\right)$$

$$g(\beta_1, \lambda_{e_1}) = -\left[d(P_2, p_2)\right] + 2\left[d(P_3, p_3)\right] - \left[d(P_4, p_4)\right] + \frac{c}{f_G}\left(N_{3,4} - N_{2,3}\right)$$

The computation of the corrections $\Delta\beta_1$ and $\Delta\lambda_{e_1}$ require the evaluation of the following expressions:

$$\frac{\partial f}{\partial \beta_1} = -\frac{\partial\left[d\left(P_1, p_1\right)\right]}{\partial \beta_1} + 2\frac{\partial\left[d\left(P_2, p_2\right)\right]}{\partial \beta_1} - \frac{\partial\left[d\left(P_3, p_3\right)\right]}{\partial \beta_1}$$

$$\frac{\partial g}{\partial \beta_1} = -\frac{\partial\left[d\left(P_2, p_2\right)\right]}{\partial \beta_1} + 2\frac{\partial\left[d\left(P_3, p_3\right)\right]}{\partial \beta_1} - \frac{\partial\left[d\left(P_4, p_4\right)\right]}{\partial \beta_1}$$

$$\frac{\partial f}{\partial \lambda_{e_1}} = -\frac{\partial\left[d\left(P_1, p_1\right)\right]}{\partial \lambda_{e_1}} + 2\frac{\partial\left[d\left(P_2, p_2\right)\right]}{\partial \lambda_{e_1}} - \frac{\partial\left[d\left(P_3, p_3\right)\right]}{\partial \lambda_{e_1}}$$

$$\frac{\partial g}{\partial \lambda_{e_1}} = -\frac{\partial\left[d\left(P_2, p_2\right)\right]}{\partial \lambda_{e_1}} + 2\frac{\partial\left[d\left(P_3, p_3\right)\right]}{\partial \lambda_{e_1}} - \frac{\partial\left[d\left(P_4, p_4\right)\right]}{\partial \lambda_{e_1}}$$

It is noted that, in order to evaluate these partial derivatives, it is necessary to develop expressions for $\frac{\partial\left[d\left(P_n, p_n\right)\right]}{\partial \beta_1}$ and $\frac{\partial\left[d\left(P_n, p_n\right)\right]}{\partial \lambda_{e_1}}$ for n = 1, 2, 3, 4. It is simplest to develop the general equation with subscript n and the value of n concerned can be sustituted into the general equation to obtain the expression needed. First, the expression for $d\left(P_n, p_n\right)$ is written as

$$d\left(P_n, p_n\right) = \left[r_n^2 + \rho_n^2 - 2r_n\rho_n\left(\sin\phi_n \sin\beta_n + \cos\phi_n \cos\beta_n \cos\left[\theta_n - \lambda_{e_n} - \omega_{ie}\left(\Delta T_n\right)\right]\right)\right]^{1/2}$$

where ω_{ie} (ΔT) represents the angle between the e-frame and i-frame and the other terms are as defined previously. Taking the required partial derivatives of this expression yields

$$\begin{aligned}\frac{\partial\left[d\left(P_n, p_n\right)\right]}{\partial \beta_1} = \frac{1}{\left[d\left(P_n, p_n\right)\right]}\Bigg\{&\left[\frac{\partial \rho_n}{\partial \beta_1}\right]\Big[\rho_n - r_n\Big\{\sin\theta_n \sin\beta_n + \\ &\cos\theta_n \cos\beta_n \cos\left[\theta_n - \lambda_{e_n} - \omega_{ie}\left(\Delta T_n\right)\right]\Big\}\Big] + \\ &-r_n\rho_n\left[\left(\frac{\partial\beta_n}{\partial\beta_1}\right)\left(\sin\phi_n \cos\beta_n - \cos\phi_n \sin\beta_n \cos\left[\theta_n - \lambda_{e_n} - \omega_{ie}\left(\Delta T_n\right)\right]\right) + \right. \\ &\left. + \left(\frac{\partial\lambda_{e_n}}{\partial\beta_1}\right)\left(\cos\phi_n \cos\beta_n \sin\left[\theta_n - \lambda_{e_n} - \omega_{ie}\left(\Delta T_n\right)\right]\right)\right]\Bigg\}\end{aligned}$$

$$\frac{\partial\left[d\left(P_n, p_n\right)\right]}{\partial \lambda_{e_1}} = \frac{-r_n\rho_n}{d\left(P_n, p_n\right)}\left[\cos\phi_n \cos\beta_n \sin\left\{\theta_n - \lambda_{e_n} - \omega_{ie}\left(\Delta T_n\right)\right\}\right]$$

It is seen that, in order to evaluate $\frac{\partial[d(P_n, p_n)]}{\partial\beta_1}$, expressions for $\frac{\partial\rho_n}{\partial\beta_1}$, $\frac{\partial\beta_n}{\partial\beta_1}$, and $\frac{\partial\lambda_{e_n}}{\partial\beta_1}$ must be developed. Because ρ_n is equal to $r_e + h_A$, it can be expressed as

$$\rho_n = b\left[1 + \frac{k}{4}\left(1 + \cos 2\beta_n\right) + \frac{3k^2}{32}\left(1 + \cos 2\beta_n\right)^2\right] + h_A$$

where $\beta_n = \beta_1 + \Delta\beta_n$ (note that when $n = 1$, $\Delta\beta_n = 0$). Taking the partial derivative of this expression with respect to β_1 yields

$$\frac{\partial\rho_n}{\partial\beta_1} = -\frac{(bk)}{2}\left(\sin 2\beta_n\right)\left(\frac{\partial\beta_n}{\partial\beta_1}\right)\left[1 + \frac{3k}{4}\left(1 + \cos 2\beta_n\right)\right].$$

This requires the expression for $\frac{\partial\beta_n}{\partial\beta_1}$, which may be written as $\frac{\partial(\beta_n)}{\partial\beta_1} = \frac{\partial(\beta_1)}{\partial\beta_1} + \frac{\partial(\Delta\beta_n)}{\partial\beta_1} = 1 + \frac{\partial(\Delta\beta_n)}{\partial\beta_1}$. In developing this expression, the approximation

$$\Delta\beta_n \approx \frac{V_N\left(\Delta t_n\right)}{\rho_1 + \frac{\Delta\rho_n}{2}} = \frac{2V_N\left(\Delta t_n\right)}{\rho_1 + \rho_n}$$

is used where $\Delta t_n \triangleq t_n - t_1$. Substituting this expression into the equation for $\frac{\partial\rho_n}{\partial\beta_1}$ yields

$$\frac{\partial\rho_n}{\partial\beta_1} = \frac{-\frac{(bk)}{2}\left(\sin 2\beta_n\right)\left[1 + \frac{3k}{4}\left(1 + \cos 2\beta_n\right)\right]\left\{1 - \frac{2V\left(\Delta t_n\right)\frac{\partial\rho_1}{\partial\beta_1}}{\left(\rho_1 + \rho_n\right)^2}\right\}}{1 - \frac{(bk)\left(\sin 2\beta_n\right)\left(V_N\right)\left(\Delta t_n\right)\left[1 + \frac{3k}{4}\left(1 + \cos 2\beta_n\right)\right]}{\left(\rho_1 + \rho_n\right)^2}}$$ where

$$\frac{\partial\rho_1}{\partial\beta_1} = -\frac{(bk)}{2}\left(\sin 2\beta_1\right)\left[1 + \frac{3k}{4}\left(1 + \cos 2\beta_1\right)\right]$$

Writing $\lambda_{e_n} = \lambda_{e_1} + \Delta\lambda_{e_n}$, the expression for $\frac{\partial\lambda_{e_n}}{\partial\beta_1}$ can be written as $\frac{\partial\lambda_{e_1}}{\partial\beta_1} + \frac{\partial\left(\Delta\lambda_{e_n}\right)}{\partial\beta_1}$ where $\frac{\partial\lambda_{e_1}}{\partial\beta_1}$ is zero. Therefore, $\frac{\partial\lambda_{e_n}}{\partial\beta_1} = \frac{\partial\left(\Delta\lambda_{e_n}\right)}{\partial\beta_1}$. This expression is developed using the approximation

$$\Delta\lambda_{e_n} \approx \frac{V_E\,(\Delta t_n)}{\left(\rho_1 + \frac{\Delta\rho_n}{2}\right)\left[\cos\left(\beta + \frac{\Delta\beta_n}{2}\right)\right]} = \frac{2V_E\,(\Delta t_n)}{\left(\rho_1 + \rho_n\right)\left[\cos\left(\frac{\beta_1 + \beta_n}{2}\right)\right]}$$

Therefore, the quantity $\frac{\partial\left(\Delta\lambda_{e_n}\right)}{\partial\beta_1} = \frac{\partial\lambda_{e_n}}{\partial\beta_1}$ can be expressed as

$$\frac{\partial\lambda_{e_n}}{\partial\beta_1} = \frac{-2V_E\,(\Delta t_n)\left\{-\frac{(\rho_1+\rho_n)}{2}\left[\sin\left(\frac{\beta_1+\beta_n}{2}\right)\right]\left[1+\frac{\partial\beta_n}{\partial\beta_1}\right]+\left[\cos\left(\frac{\beta_1+\beta_n}{2}\right)\right]\left[\frac{\partial\rho_1}{\partial\beta_1}+\frac{\partial\rho_n}{\partial\beta_1}\right]\right\}}{\left(\rho_1+\rho_n\right)^2\left[\cos\left(\frac{\beta_1+\beta_n}{2}\right)\right]^2}$$

Because the δt_n's are not all equal, the values for the Δt_n's are not all exactly equal. However, because the magnitudes of the δt_n's are on the order of milliseconds and the differences in the δt_n's will be even less, the Δt_n's were set equal to the relevant multiples of 2 minutes in the evaluation of the above expressions. It is pointed out again that only β_1 and λ_{e_1} are the unknowns in the expressions; the $\Delta\beta_n$'s, $\Delta\lambda_{e_n}$'s, Δt_n's, etc. are known, within the limitations mentioned previously. This, then, summarizes the procedures by which the two basic navigation equations are solved for β_1 and λ_{e_1}.

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CHAPTER IV

RESULTS OF THE SIMULATION

As stated before, three separate flight paths were simulated by this computer program. For each flight path, the same satellite and satellite motion were simulated, i.e. for the start of each particular flight path (starting time was the same for each path), t_1, the satellite was at the same point in it's orbit and it's (error-free) orbital elements were the same. Flight path #1 begins at $\beta = 45^{o}N$ and $\lambda e = 65^{o}W$ at time $t_1 = 7200$ sec GMT. The total velocity is 1,800 mph, as it is for all three flight paths. The heading is a constant 55^{o} measuring clockwise from true north. This value of heading is expressed by setting the north velocity component, $V_N = 1,032.44$ mph and the east velocity component, $V_E = 1,474.47$ mph. As for all three cases, the altitude above the reference ellipsoid is 70,000 ft. The second flight path, Flight path #2, starts at $\beta = 50^{o}N$ and $\lambda e_1 = 45^{o}W$ at time $t_1 = 7200$ sec GMT. The heading in this case is 75^{o} expressed as $V_N = 465.876$ mph and $V_E = 1,738.656$ mph. Flight path #3 commences at $\beta = 53^{o}N$ and $\lambda_e = 20^{o}W$ at time $t_1 = 7,200$ sec GMT. In this case the heading is 90^{o} from true north; therefore, $V_N = 0.$ and $V_E = 1,800.00$ mph. In all three cases the time at which the e-frame and i-frame are coincident is set to 0 sec GMT.

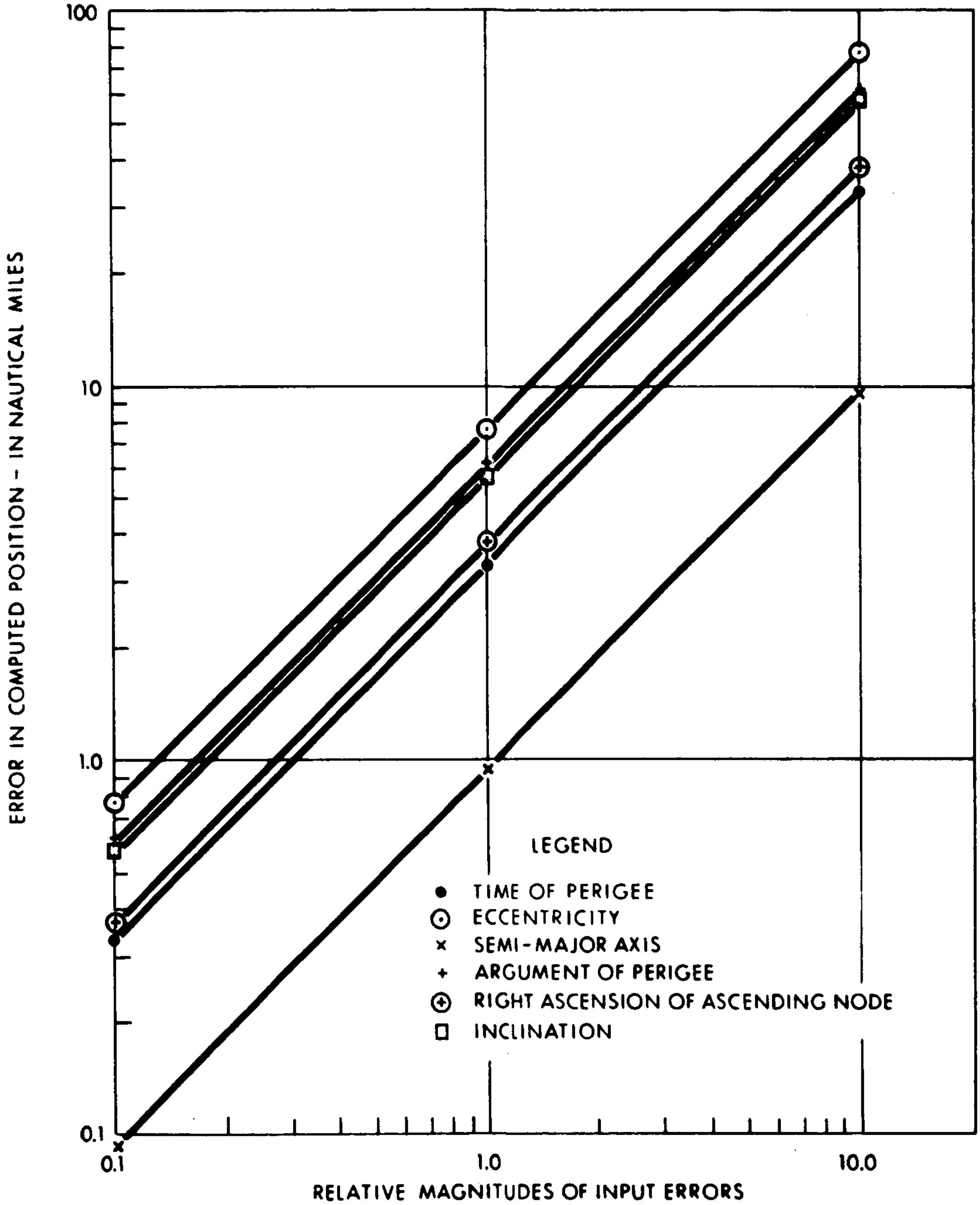
The effect on the computed position of errors in the orbital elements and errors in the knowledge of the aircraft motion are determined for each of the three flight paths. As stated previously, only one type of input error is considered at a time. The reason three separate flight paths are considered instead of just one, is that it is expected that the magnitude of error in computed position is a function of the satellite pass angle, the angle the position vector from the aircraft to the satellite makes with the horizon. Flight path #1 corresponds to a satellite pass angle of about 13^{o}, flight path #2 corresponds to a 27^{o} pass angle, and the pass angle is near 90^{o} for flight path #3.

The values of errors used for the orbital elements are as follows:

Δ time of perigee = 1.0 sec
Δ eccentricity = 0.001
Δ semi-major axis = 1.0 km.
Δ argument of perigee = 0.1 degree

Error Source (magnitude)	Flight Path #1	Flight Path #2	Flight Path #3
No errors	0.019 n. m.	0.018 n. m.	0.007 n. m.
Time of perigee (1.0 sec)	3.28	3.39	3.32
Eccentricity (0.001)	8.68	7.73	23.91
Semi-major axis (1.0 km)	1.03	0.95	6.13
Argument of perigee (0.1 deg)	5.86	6.05	5.96
Rt. Ascension of Ascending node (0.1 deg)	4.13	3.80	3.64
Inclination (0.1 deg)	6.32	5.77	1.50
East Velocity (3.5 mph)	0.67	0.28	0.16
North Velocity (3.5 mph)	0.72	0.50	1.73
Heading (4 $\widehat{\text{min}}$)	0.54	0.33	1.09
Altitude (1,000 ft)	0.10	0.13	1.60

Table 1. Effect of Errors on Computed Position



Graph 1. Variation of Output Error with Errors in Orbital Elements (Flight Path #2)

Δ right ascension of the ascending node = 0.1 degree
Δ inclination = 0.1 degree

The values of errors used for the parameters describing the aircraft motion are based on magnitudes of errors likely to be experienced if an inertial navigation system with a 1.5 n.m./hr CEP performance figure is used to supply velocity and heading information. A system of this quality was chosen because, recently, the navigation systems chosen for the Boeing 747 commercial aircraft are inertial navigation systems with this performance figure. It is unlikely that, because of the greatly increased cost, inertial navigation systems of higher quality than this would be chosen for commercial aircraft unless the cost of inertial navigation systems decreases significantly in the next several years; this is a definite possibility, however. Reference 8 gives several rules of thumb for determining the velocity and heading accuracy of an inertial navigation system when the only performance figure given is the n.m./hour CEP figure; this is the only performance figure the author has obtained for the Boeing 747 system. Using these rules of thumb, the following values of errors are those used in the simulation:

North velocity error = 3.5 mph
East velocity error = 3.5 mph
Heading error (initial) = 4.0 $\widehat{\text{min}}$

The effects of north velocity error and east velocity error are considered separately. The heading error is expressed as velocity errors, i.e. considering the heading to be in error and the total velocity to be correct, new values of east and north velocity are computed based on the incorrect heading angle. The changes resulting in the north and east velocity values, therefore, are considered to be the errors in velocity corresponding to the heading error. These velocity changes are then used to determine the effect of heading error on the accuracy of the computed position. The value used for altitude error is 1,000 ft.

The results of the digital computer simulation are summarized in Table 1. The numbers shown in the table are the errors in the computed position of p_4 in nautical miles due to the error source on the left side of the table. The position error in p_4 was chosen instead of p_1 because p_4 is the position at the end of the six minute interval during which the Doppler measurements are taken and therefore, is the most recent computed position. It should be pointed out again that the position of p_1 is considered to be the unknown in the basic navigation equations and that p_2, p_3, and p_4 are determined in terms of p_1 and the motion of the aircraft during the six minute interval.

For each flight path, the program was executed once with zero error in in the input data and using an estimated initial position that was in error by about 20 or 30 miles. This was done to check the accuracy of the program. The maximum error in computed position with zero error input, as seen in Table 1, is only 0.019 n.m. or about 115 ft. This error probably results from the use of approximations, round-off errors, etc. It is felt that this magnitude of error is completely tolerable and, thus, verifies the validity of the simulation.

It is noted that the magnitude of error in the computed position differs little with pass angle for errors in time of perigee, argument of perigee and right ascension of ascending node.

Surprisingly, the errors in computed position due to errors in inclination and east velocity are less for high satellite pass angle (Flight Path #3) than for lower pass angles.

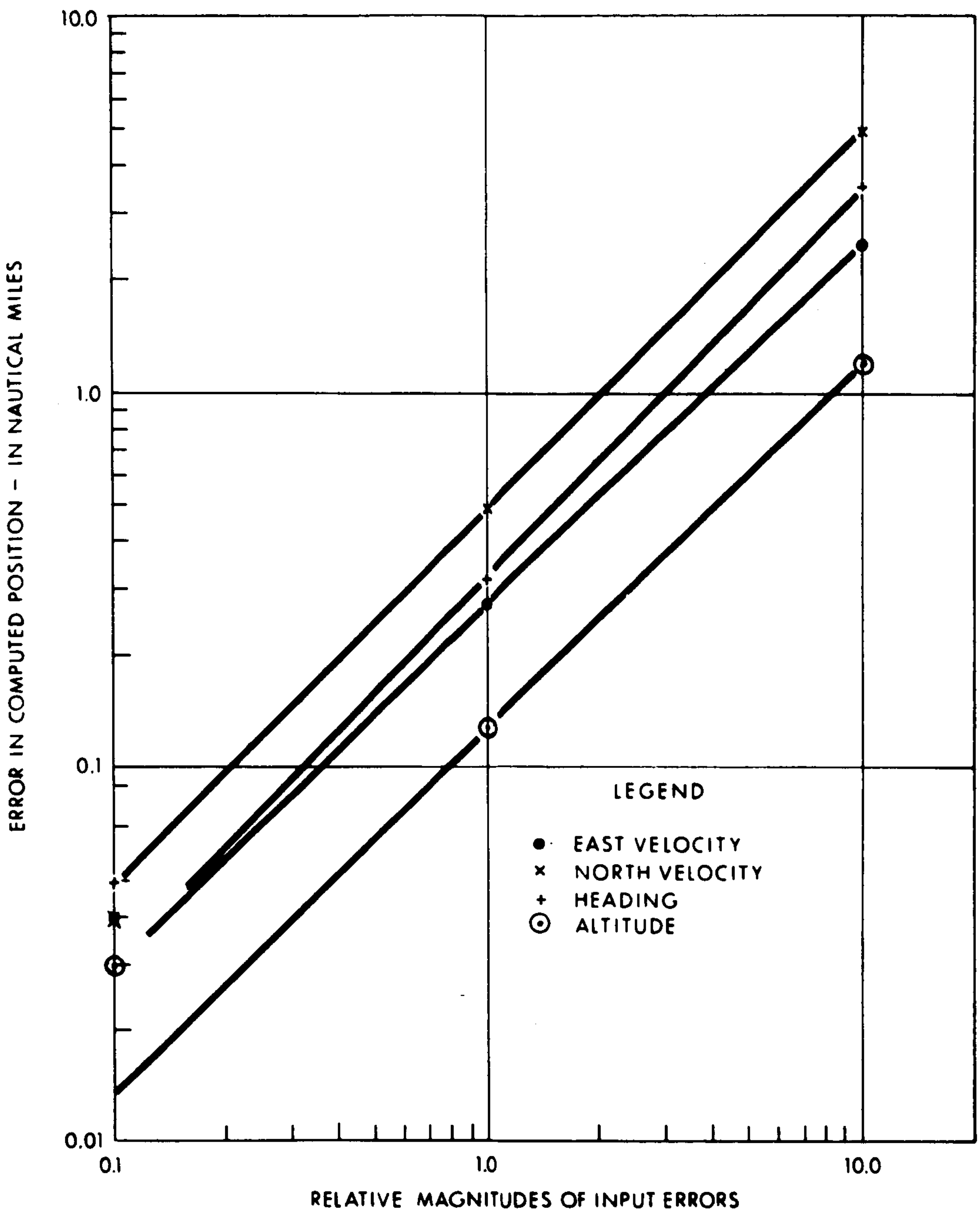
However, for high satellite pass angle, the errors in computed position due to errors in eccentricity, semi-major axis, north velocity, heading and altitude are much greater than for low or medium pass angles. It appears that the system is particularly sensitive, at high pass angles, to errors affecting the altitude determination of both the aircraft and satellite as manifested by errors in eccentricity, semi-major axis, and aircraft altitude. For example, the error in computed position resulting from eccentricity error jumped from around 8 n. miles for Flight Paths #1 and 2 to nearly 24 n. miles for Flight Path #3. The error in computed position due to aircraft altitude error at high satellite pass angle (Flight Path 3) was over 10 times the value for Flight Paths 1 and 2.

The breakdown of position error into latitude and longitude error reveals that, generally, the latitude and longitude errors are of the same order of magnitude, with some exceptions. For example, at low satellite pass angle, the position error resulting from the error in the right ascension of the ascending node is equal to the error in longitude; this is due to the fact that the input error "displaces" the satellite longitudinally only. It should be noted that latitude error yields error in longitude because longitude rate is a function of latitude; thus, input errors affecting only the latitude of the satellite position cause errors in longitude of the aircraft, also.

To determine if the error in the computed position is related to the input error in a linear fashion, Flight Path #2 was run for input errors $\frac{1}{10}$ the magnitude of those listed in Table 1 and again for input errors 10 times the magnitude of those in Table 1. In other words it was desired to determine whether the output error would increase K times if the input error was increased K times. The

Error Source (Magnitude of Nominal input error)	0.1 x Nominal Input Errors	Nominal Input Errors	10 x Nominal Input Errors
Time of perigee (1 sec)	0.34 n.m.	3.39 n.m.	33.79 n.m.
Eccentricity (0.001)	0.77	7.73	77.74
Semi-major axis (1 km)	0.09	0.95	9.59
Argument of perigee (0.1 deg)	0.60	6.05	60.74
Rt Ascension of Ascending node (0.1 deg)	0.37	3.80	38.11
Inclination (0.1 deg)	0.56	5.77	57.98
East Velocity (3.5 mph)	0.04	0.28	2.55
North Velocity (3.5 mph)	0.04	0.50	4.95
Heading (4 $\widehat{\text{min}}$)	0.05	0.33	3.54
Altitude (1,000 ft)	0.03	0.13	1.21

Table 2. Variation of Output Error with Input Error for Flight Path #2



Graph 2. Variation of Output Error with Errors in Knowledge of Aircraft Motion (Flight Path #2)

results of these runs for Flight Path #2 are given in Table 2. The plots of these results are shown in Graphs 1 and 2. Graph 1 contains the plots for errors only in the orbital elements in Graph 2 shows the results for errors only in the knowledge of the aircraft motion.

In Table 2 and Graphs 1 and 2 it is noted that the output errors vary very nearly linearly with the input errors. The only exceptions are for errors in the knowledge of aircraft motion that are $\frac{1}{10}$ the magnitude of the nominal input errors. It is believed this apparent non-linearity can be explained by noting the very small magnitudes of output errors for this set of input errors. The maximum output error for this set is 0.05 nautical miles and when one considers that the error in computed position with no input errors is 0.018 n.m., it is seen that there is an uncertainty in the figure for output error of about 0.02 n.m. This means the values of output error obtained in this set are not reliable due to the fact the computational error is so great relative to the values computed. With these exceptions, however, the output errors generally increase linearly with the increase in input errors for medium satellite pass angle.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

Results of using the Navy Navigational Satellite System for a fixed earth-bound station, according to Reference 6, are errors in computed position averaging 0.13 nautical miles. Comparing this figure with the results shown in Table 1, it appears that the errors in the knowledge of the orbital elements in the actual operational system are, generally, much less than the magnitudes used for the runs whose results appear in Table 1. It would seem that, in general, no serious problem exists with the accuracy of this (orbital element) input data for high-speed aircraft, except that, for high satellite pass angles, errors in eccentricity and semi-major axis become particularly troublesome.

Judging from the results obtained with errors in the knowledge of the aircraft's motion, which are the magnitudes of errors likely to result from using a good inertial navigation system for this information, the Navy Navigational Satellite System could be of definite benefit in giving accurate position fixes of supersonic aircraft, although there is some degradation at high satellite pass angles.

There are, however, some problems which may arise for high-speed aircraft use of this system which should be mentioned. One of these difficulties is locking onto the satellite's transmitted signal; this may be difficult in a high-speed aircraft, particularly, if it is in maneuvers during the satellite pass. Another problem is that, at present, there are only four satellites in orbit and this may require a wait of up to an hour and a half to obtain a position fix; this would make this system of little value for some flights of less than an hour or two in duration. It is, however, entirely possible that the number of satellites may be increased; estimates are that, if eight evenly-spaced satellites were used instead of four, the maximum wait would be only about 15 minutes, which would make this system a very useful one for high-speed aircraft.

No attempt has been made in this thesis to determine the benefits of using an optimum filtering technique for combining the output of the Navy Navigational Satellite System with the aircraft's inertial navigational system. Such a study would help in more fully evaluating the benefits of the Navy Navigational Satellite System for use with high-speed aircraft.

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APPENDIX

COMPUTER PROGRAM AND FORMAT OF INPUT DATA

The format of the input data cards to the program and the description and units of the variables will now be given.

Card 1:

TO,	time satellite is at position P_1, in seconds from 0 hr. GMT
ATP,	time of perigee passage of satellite, in seconds from 0 hr. GMT
AAØP,	argument of perigee, in degrees
AECC,	eccentricity of orbit, set equal to 0.0
ASALT,	satellite altitude at equator, in nautical miles, set equal to 600.0
ARAAN,	right ascension of the ascending node, in degrees
AINCL,	inclination of the orbit, in degrees, set equal to 90.0

Card 2:

BETA(1),	actual geocentric latitude of aircraft at TO, in degrees
LAM(1),	actual longitude of aircraft at TO measured eastward from Greenwich, in degrees
AVE,	actual east velocity of aircraft with respect to the earth, in st. miles per hour
AVN,	actual north velocity of aircraft with respect to the earth, in st. miles per hour
AHA,	actual altitude of aircraft, in ft.
TT,	time at which e-frame coincides with i-frame, in seconds from 0 hr. GMT

Card 3:

DTMP,	error in time of perigee, in seconds
DECC,	error in eccentricity of orbit

DSMA,	error in semi-major axis, in km.
DAØP,	error in argument of perigee, in degrees
DØMN,	error in rt. ascension of the ascending node, in degrees
DINC,	error in orbit inclination, in degrees

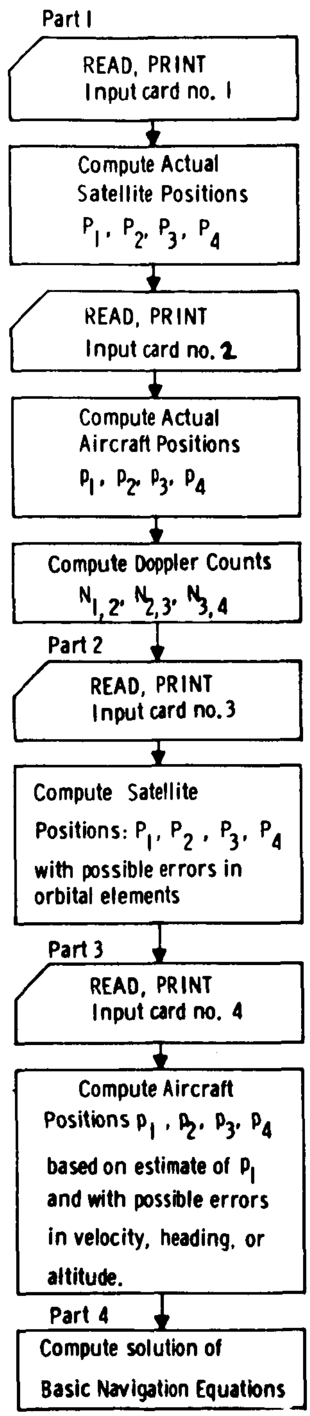
Card 4:

DVE,	error in knowledge of aircraft east velocity, in st. miles per hour
DVN,	error in knowledge of aircraft north velocity, in st. miles per hour
DHA,	error in knowledge of aircraft altitude, in ft.
BIE,	estimated value of BETA(1) at TO, in degrees
LIES,	estimated value of LAM(1) at TO measured eastward from Greenwich, in degrees

Next, an outline of the computer program is given.

Also included in this appendix is the complete computer program listing. The language of this program is MAD.

OUTLINE OF COMPUTER PROGRAM



```
 DIMENSION THETA(5),R(5),PHI(5),BETA(5),LAM(5),DT(5),
1ALAM(5),AILG(5),ALAT(5),D(5),NC(5),TA(5),CSR(5),CSTHT(5),
2CSPH(5),CBETA(5),CLAM(5),CRHO(5)
 DIMENSION DB(5),DL(5),DR(5),DXC(5),DLDB(5),DDDB(5),DDDL(5),
1RLAT(5),RLAM(5),FBETA(5),FLAM(5),ELAT(5),ELAM(5),
2TOTERR(5),RHO(5),DCT(5)
 DIMENSION PRB(5),PBB(5),PLB(5)
 INTEGER I,J,K
 R'A
 GME=1.407639E16
 KMTFT=3281.
 FTKM=.3048E-3
 MPHTFS=1.467
 DTR=.17453293E-1
 RTD=57.29578
 REP=6356.912
 REE=6378.388
 MPHKH=1.609
 NMTFT=6080.
 WIE=7.29E-5
 C=3.37267E-3
 CL=299793.0
 SMA=REE*KMTFT+ASALT*NMTFT
 A3=SMA.P.3
 M2M=GME/A3
 AMM=SQRT.(M2M)
 IANG=ARAAN*DTR
 THETA(1)=IANG
 THETA(2)=IANG
 THETA(3)=IANG
 THETA(4)=IANG
 IDTN=SMA*FTKM
 R(1)=IDTN
 R(2)=IDTN
 R(3)=IDTN
 R(4)=IDTN
 EA=AAOP*DTR
 PHI(1)=AMM*(TO-ATP)+EA
 PHI(2)=AMM*(TO+120.-ATP)+EA
 PHI(3)=AMM*(TO+240.-ATP)+EA
 PHI(4)=AMM*(TO+360.-ATP)+EA
 R'A
 KKK=1.-(REP/REE)*(REP/REE)
 K1=KKK/4.
 K2=(3.*KKK*KKK)/32.
 AVE=AVE*MPHTFS
 AVN=AVN*MPHTFS
 BNUM=AVN*FTKM
 LNUM=AVE*FTKM
 K=0
```

```
          BETA(1)=BETA(1)*DTR
          LAM(1)=LAM(1)*DTR
          B=BETA(1)
          L=LAM(1)
AAA       T'H ST1, FOR I=0, 1, I.GE.120
          BDNM=ERAD.(B)+AHA*FTKM
          LDNM=BDNM*COS.(B)
          L=L+(LNUM/LDNM)
          B=B+(BNUM/BDNM)
ST1       K=K+1
          W'R K.E.120
          BETA(2)=B
          LAM(2)=L
          T'O AAA
          O'R K.E.240
          BETA(3)=B
          LAM(3)=L
          T'O AAA
          O'E
          BETA(4)=B
          LAM(4)=L
          E'L
BBB       T'H ST2, FOR J=1, 1, J.GE.5
          RB=BETA(J)
          RDS=ERAD.(RB)+AHA*FTKM
          IFEF=WIE*(T0+(J-1.)*120.-TT)
          LIF=LAM(J)+IFEF
          SP=PHI(J)
          SR=R(J)
          AB=BETA(J)
          LD=THETA(J)-LIF
          CGT=(SIN.(SP)*SIN.(AB))+(COS.(SP)*COS.(AB)*COS.(LD))
          DSK=(SR-RDS)*(SR-RDS)+2.*SR*(1.-CGT)*RDS
          DT(J)=SQRT.(DSK)/CL
          ALAM(J)=LAM(J)+(AVE*DT(J)*FTKM)/(RDS*COS.(AB))
          AILG(J)=ALAM(J)+IFEF+WIE*DT(J)
          ALAT(J)=BETA(J)+(AVN*DT(J)*FTKM)/RDS
          ALA=ALAT(J)
          LK=THETA(J)-AILG(J)
          CGT=SIN.(SP)*SIN.(ALA)+COS.(SP)*COS.(ALA)*COS.(LK)
          ADSK=(SR-RDS)*(SR-RDS)+2.*SR*(1.-CGT)*RDS
          RHO(J)=RDS
          D(J)=SQRT.(ADSK)
          P'S R(J),PHI(J),THETA(J),RHO(J),ALAT(J)
ST2       P'S AILG(J),ALAM(J),DT(J),D(J)
          FG=400.01E+6
          FT=4.0E+8
          DF=1.0E+4
CCC       T'H ST3, FOR J=1, 1, J.GE.4
          NC(J)=DF*120.+(FG/CL)*(D(J+1)-D(J))
ST3       P'S NC(J)
DDD       R'A
          TP=ATP+DTMP
          ECC=AECC+DECC
          SMJA=SMA+DSMA*KMTFT
```

```
            AOP=(AAOP+DAOP)*DTR
            RAAN=(ARAAN+DOMN)*DTR
            INC=(AINCL-DINC)*DTR
            A3=SMJA.P.3
            M2M=GME/A3
            MM=SQRT.(M2M)
            E1=2.*ECC-((ECC.P.3)/4.)
            E2=(5.*ECC*ECC)/4.
            E3=(13.*(ECC.P.3))/12.
            E4=SMJA*FTKM*(1.-ECC.P.2)
            CMG=COS.(RAAN)
            SMG=SIN.(RAAN)
            CW=COS.(AOP)
            SW=SIN.(AOP)
            CI=COS.(INC)
            SI=SIN.(INC)
            L1=CMG*CW-SMG*SW*CI
            L2=-(CMG*SW)-(SMG*CW*CI)
            M1=SMG*CW+CMG*SW*CI
            M2=-(SMG*SW)+CMG*CW*CI
            N1=SW*SI
            N2=CW*SI
EEE         T'H ST4, FOR J=1, 1, J.GE.5
            MAM=MM*(T0+(J-1.)*120.-TP)
            M2AM=2.*MAM
            M3AM=3.*MAM
            TA(J)=MAM+E1*SIN.(MAM)+E2*SIN.(M2AM)+E3*SIN.(M3AM)
            TAN=TA(J)
            RORB=E4/(1.+ECC*COS.(TAN))
            XPSI=RORB*COS.(TAN)
            XETA=RORB*SIN.(TAN)
            XI=L1*XPSI+L2*XETA
            YI=M1*XPSI+M2*XETA
            ZI=N1*XPSI+N2*XETA
            RPSK=XI*XI+YI*YI
            RPL=SQRT.(RPSK)
            CSR(J)=RORB
            IR=YI/XI
            CSTHT(J)=ATAN.(IR)
            IR=ZI/RPL
            CSPH(J)=ATAN.(IR)
ST4         P'S TA(J),CSR(J),CSTHT(J),CSPH(J)
            R'A
            VE=AVE+DVE*MPHTFS
            VN=AVN+DVN*MPHTFS
            HA=AHA+DHA
            BIE=BIE*DTR
            LIES=LIES*DTR
            CBETA(1)=BIE
            CLAM(1)=LIES
            B=BIE
            L=LIES
            BNUM=VN*FTKM
            LNUM=VE*FTKM
            K=0
```

```
FFF       T'H ST5, FOR I=0, 1, I.GE.120
          BDNM=ERAD.(B)+HA*FTKM
          LDNM=BDNM*COS.(B)
          L=L+(LNUM/LDNM)
          B=B+(BNUM/BDNM)
ST5       K=K+1
          W'R K.E.120
          CBETA(2)=B
          CLAM(2)=L
          T'O FFF
          O'R K.E.240
          CBETA(3)=B
          CLAM(3)=L
          T'O FFF
          O'E
          CBETA(4)=B
          CLAM(4)=L
          E'L
GGG       T'H ST6, FOR J=1, 1, J.GE.5
          AGCBT=CBETA(J)
          BDNM=ERAD.(AGCBT)+HA*FTKM
          LDNM=BDNM*COS.(AGCBT)
          IP=CSPH(J)
          IT=CSTHT(J)
          IVL=CLAM(J)+WIE*(T0+(J-1.)*120.-TT)
          IRK=IT-IVL
          CGT=SIN.(IP)*SIN.(AGCBT)+COS.(IP)*COS.(AGCBT)*COS.(IRK)
          IVR=(CSR(J)-BDNM)*(CSR(J)-BDNM)+2.*CSR(J)*(1.-CGT)*BDNM*
          DCT(J)=SQRT.(IVR)/CL
          BADD=(VN*DCT(J)*FTKM)/BDNM
          CBETA(J)=CBETA(J)+BADD
          CLAM(J)=CLAM(J)+(VE*DCT(J)*FTKM)/LDNM
          CRHO(J)=BDNM
ST6       P'S CRHO(J),CBETA(J),CLAM(J),DCT(J)
          DB(1)=0.
          DB(2)=CBETA(2)-CBETA(1)
          DB(3)=CBETA(3)-CBETA(1)
          DB(4)=CBETA(4)-CBETA(1)
          DL(1)=0.
          DL(2)=CLAM(2)-CLAM(1)
          DL(3)=CLAM(3)-CLAM(1)
          DL(4)=CLAM(4)-CLAM(1)
          DR(1)=0.
          DR(2)=CRHO(2)-CRHO(1)
          DR(3)=CRHO(3)-CRHO(1)
          DR(4)=CRHO(4)-CRHO(1)
          P'S DB(1),DB(2),DB(3),DB(4),DL(1),DL(2),DL(3),DL(4),DR(1),
         1 DR(2),DR(3),DR(4)
          RBD=2.*CBETA(1)
          K1C=1.+COS.(RBD)
          SB=SIN.(RBD)
          IID=1.+(3.*KKK*K1C)/4.
          DRDB=-(REP*KKK*SB*IID)/2.
HHH       T'H ST7, FOR J=1, 1, J.GE.5
          IP=CSPH(J)
```

```
      IB=CBETA(J)
      IT=CSTHT(J)
      ILI=CLAM(J)+WIE*(TO+(J-1.)*120.+DCT(J)-TT)
      LK=IT-ILI
      CGT=SIN.(IP)*SIN.(IB)+COS.(IP)*COS.(IB)*COS.(LK)
      DCSS=(CSR(J)-CRHO(J))*(CSR(J)-CRHO(J))
      DCST=2.*CSR(J)*(1.-CGT)*CRHO(J)
      DCS=DCSS+DCST
      DXC(J)=SQRT.(DCS)
      DELTM=(J-1.)*120.
      A2B=2.*IB
      SA2B=SIN.(A2B)
      CA2B=COS.(A2B)
      CBTT=1.+(3.*KKK*(1.+CA2B))/4.
      AB1=CBETA(1)
      RTT=ERAD.(AB1)+ERAD.(IB)+2.*HA*FTKM
      NMT=1.-((2.*FTKM*VN*DELTM)/RTT)*(DRDB/RTT)
      DMT1=((REP*KKK*SA2B)/RTT)*((VN*FTKM*DELTM*CBTT)/RTT)
      NTM=REP*KKK*SA2B*CBTT*NMT
      PRB(J)=-(.5*NTM)/(1.-DMT1)
      SCTM=((2.*VN*FTKM*DELTM)/RTT)*((DRDB+PRB(J))/RTT)
      PBB(J)=1.-SCTM
      ABL=CBETA(1)+DB(J)/2.
      CABL=COS.(ABL)
      SABL=SIN.(ABL)
      TRMA=((VE*FTKM*DELTM)/(RTT*CABL))*((SABL*(1.+PBB(J)))/CABL)
      TRMB=((2.*VE*FTKM*DELTM)/RTT)*((DRDB+PRB(J))/(RTT*CABL))
      PLB(J)=TRMA-TRMB
      PN=ERAD.(IB)+HA*FTKM
      BXT1=-(CGT*CSR(J)-PN)*PRB(J)
      BXIA=(SIN.(IP)*COS.(IB)-COS.(IP)*SIN.(IB)*COS.(LK))*PBB(J)
      BXIB=COS.(IP)*COS.(IB)*SIN.(LK)*PLB(J)
      TMA=BXT1/DXC(J)
      TMB=(PN/DXC(J))*BXIA*CSR(J)
      TMC=(PN/DXC(J))*BXIB*CSR(J)
      DDDB(J)=TMA-TMB-TMC
      LLIT=COS.(IP)*COS.(IB)*SIN.(LK)
ST7   DDDL(J)=-(CSR(J)/DXC(J))*LLIT*CRHO(J)
      DFDB=-DDDB(1)+2.*DDDB(2)-DDDB(3)
      DGDB=-DDDB(2)+2.*DDDB(3)-DDDB(4)
      DFDL=-DDDL(1)+2.*DDDL(2)-DDDL(3)
      DGDL=-DDDL(2)+2.*DDDL(3)-DDDL(4)
      F=-DXC(1)+2.*DXC(2)-DXC(3)+(CL/FG)*(NC(2)-NC(1))
      G=-DXC(2)+2.*DXC(3)-DXC(4)+(CL/FG)*(NC(3)-NC(2))
      CRDN=DFDB*DGDL-DFDL*DGDB
      DELBT=(G*DFDL)/CRDN-(F*DGDL)/CRDN
      DELLM=(F*DGDB)/CRDN-(G*DFDB)/CRDN
      P'S DELBT,DELLM,F,G,DFDB,DGDL,DFDL,DGDB
      CBETA(1)=CBETA(1)+DELBT
      CLAM(1)=CLAM(1)+DELLM
      RB=CBETA(1)
      BDNM=ERAD.(RB)+HA*FTKM
      LDNM=BDNM*COS.(RB)
      BSUB=(VN*DCT(1)*FTKM)/BDNM
      CBETA(1)=CBETA(1)-BSUB
```

```
          CLAM(1)=CLAM(1)-(VE*DCT(1)*FTKM)/LDNM
          B=CBETA(1)
          L=CLAM(1)
          K=0
          W'R .ABS.(DELBT).GE. 1.0E-6, T'O FFF
          EPS=1.0E-6/COS.(B)
          W'R .ABS.(DELLM).GE.EPS, T'O FFF
JJJ       T'H ST8, FOR I=0, 1, I.GE.120
          BDNM=ERAD.(B)+HA*FTKM
          LDNM=BDNM*COS.(B)
          L=L+(LNUM/LDNM)
          B=B+(BNUM/BDNM)
ST8       K=K+1
          W'R K.E.120
          CBETA(2)=B
          CLAM(2)=L
          T'O JJJ
          O'R K.E.240
          CBETA(3)=B
          CLAM(3)=L
          T'O JJJ
          O'E
          CBETA(4)=B
          CLAM(4)=L
          E'L
LLL       T'H ST9, FOR J=1, 1, J.GE.5
          RBB=CBETA(J)
          BDNM=ERAD.(RBB)+HA*FTKM
          LDNM=BDNM*COS.(RBB)
          CBETA(J)=CBETA(J)+(VN*DCT(J)*FTKM)/BDNM
          CLAM(J)=CLAM(J)+(VE*DCT(J)*FTKM)/LDNM
          FBETA(J)=CBETA(J)*RTD
          FLAM(J)=CLAM(J)*RTD
          RLAT(J)=ALAT(J)*RTD
          RLAM(J)=ALAM(J)*RTD
ST9       P'S FBETA(J),FLAM(J),RLAT(J),RLAM(J)
MMM       T'H SST1, FOR J=1, 1, J.GE.5
          BKI=ALAT(J)
          BDNM=ERAD.(BKI)+AHA*FTKM
          ELAT(J)=((CBETA(J)-ALAT(J))*BDNM*KMTFT)/6080.
          ELAM(J)=((CLAM(J)-ALAM(J))*BDNM*COS.(BKI)*KMTFT)/6080.
          P'S ELAT(J),ELAM(J)
          XX2=ELAM(J)*ELAM(J)+ELAT(J)*ELAT(J)
          TOTERR(J)=SQRT.(XX2)
SST1      P'S TOTERR(J)
          T'O DDD
          INTERNAL FUNCTION (X)
          ENTRY TO ERAD.
          AUX=2.*X
          K1C=1.+COS.(AUX)
          K2C=K1C*K1C
          FUNCTION RETURN REP*(1.+(K1C*K1)+(K2*K2C))
          END OF FUNCTION
          END OF PROGRAM
```

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