X-721-67-625

PREPRINT

NASA TM X-63097

A PHASE LOCK SERVO SYSTEM

RAYMOND J. STATTEL

JOHN W. FENNEL, JR.

DECEMBER 1967

GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

A PHASE LOCK SERVO SYSTEM

Raymond J. Stattel
John W. Fennel, Jr.

December 1967

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

PRECEDING PAGE BLANK NOT FILMED.

A PHASE LOCK SERVO SYSTEM

Raymond J. Stattel
John W. Fennel, Jr.

SUMMARY

This report presents the concept and basic considerations for the design of a phase-lock servo system. The servo system is for use in ground stations which decode pulse-position modulation (PPM) signals from sounding rockets. The theoretical concept for a system, capable of producing clock pulses which remain in phase-lock with a signal transmitted from the rocket-borne telemeter, is presented and analyzed (Section I) on the basis of the three definitive modes of deviation of the airborne clock. A mathematical model for the concept is then presented and the constants for a physical system are applied to the model (Section II). Verification of the physical system by means of laboratory tests, first in terms of transient performance (Section III), and then in terms of steady-state performance (Section IV) is next presented. Comparative conformity of performance between the theoretical concept, calculated values from the mathematical model, and laboratory tests gives weight to the feasibility of the design.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A PHASE LOCK SERVO SYSTEM*

INTRODUCTION

In a pulse-position modulation (PPM) system the information is contained in the separation resulting from the varying position in time of a data pulse with respect to an associated reference pulse fixed in time. The reference pulse is fixed in time by virtue of its recurrence in regular intervals. When the PPM system is a multi-channel one employing time sharing to define discrete channels, the reference pulses recur in a fixed interval and, within a single frame of data, in a fixed number. As such the reference pulses serve as markers of the boundaries of the channels within the frame.

In the sixteen-channel PPM telemetry system used by the Sounding Rocket Branch of Goddard Space Flight Center the signal transmitted from the rocket consists of a unique pulse train. Three closely-spaced pulses, called the triple pulse, serving to indicate the beginning of the sixteen-channel frame, are followed by sixteen data pulses containing sixteen separate kinds of information from the rocket. It is to be noted that the reference pulses are not transmitted. These must be derived in ground-station equipment from the triple pulse. The purpose of the Phase-Lock Servo System, or more simply, the Servo Clock, is to detect the received triple pulse, provide fifteen reference pulses between each triple pulse, and to space them so as to compensate, as well as possible, for any variations in the frequency or phase of the airborne clock. Since there must be sixteen reference pulses, the first reference pulse from the Servo Clock is made coincident with the first pulse of the received triple pulse. The Servo Clock also produces a frame pulse coincident with and identifying the first reference pulse.

Reference pulses in the airborne or transmitting portion of the system are generated by an oscillator called the clock. The block diagram and waveforms of Figure 1 are descriptive of the airborne transmitting system. A reference pulse is generated, but not transmitted, by every cycle of the airborne clock. Binary division of the clock frequency by sixteen yields a square wave of the frame frequency. The fast negative-going edge of the frame waveform triggers the triple-pulse generator to produce the triple pulse, which is transmitted. The frame pulse or first reference pulse is not transmitted, but it is identified by its coincidence with the first pulse of the transmitted triple pulse as shown in the expanded view in Figure 1.

*This report was submitted by John W. Fennel, Jr. to the University of Maryland as partial fulfillment of the requirements for the degree of Master of Science.

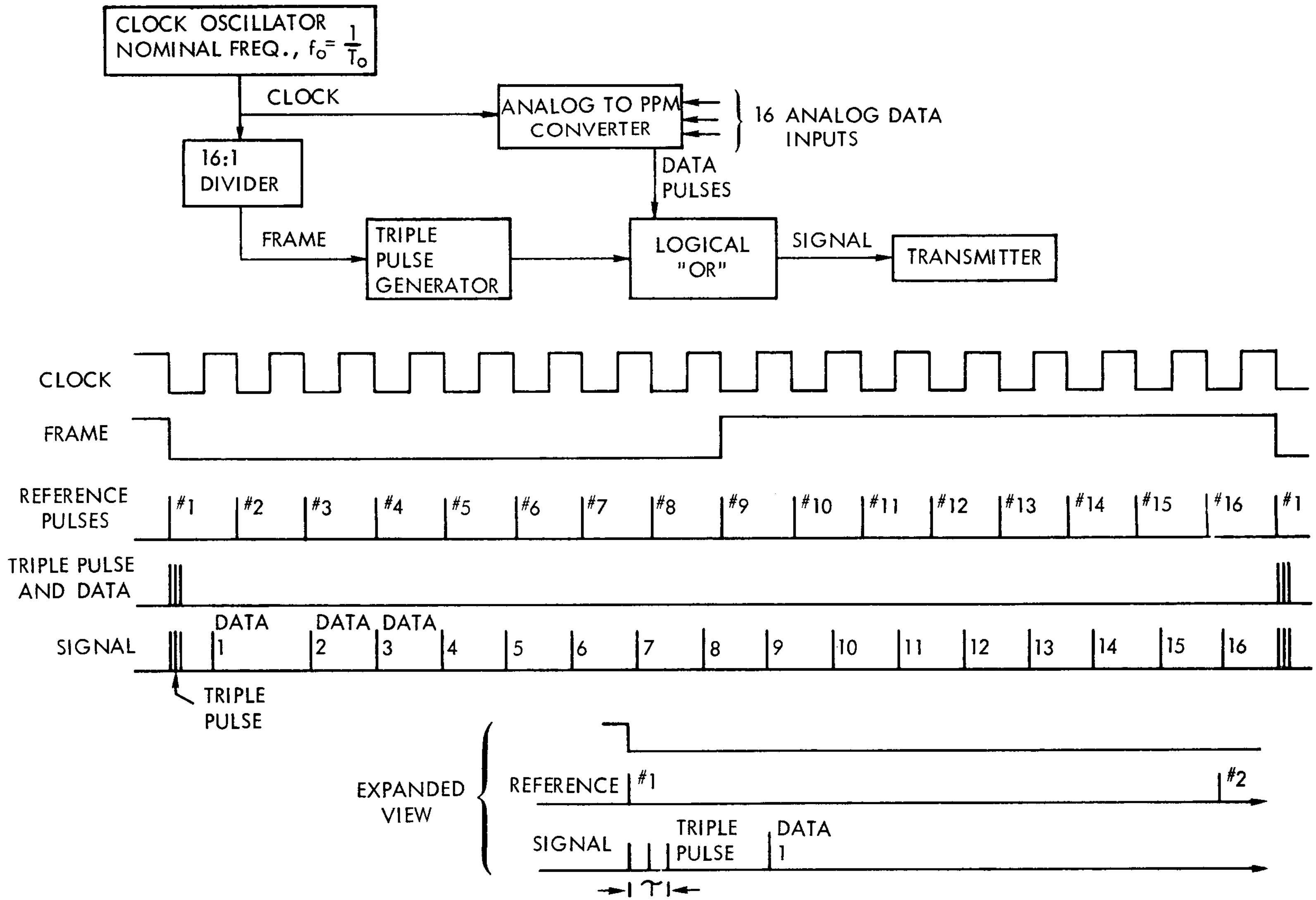


Figure 1. Airborne Telemetry System

The spacing between the three pulses of the received triple pulse is constant. The third pulse of the triple pulse is designated the sample pulse and always occurs a fixed time, T microseconds, after the first pulse which is coincident with the frame pulse. Since direct frequency division of the airborne clock yields the frame pulse, any shift in the repetition rate of the received triple pulse indicates a shift in the frequency of the airborne clock. A primary function of the Servo Clock is to detect any changes in the repetition rate of the triple pulse and make adjustments so that reference pulses, generated in the Servo Clock, coincide with those of the airborne system. Frequency and phase lock between the Servo Clock and the airborne system is accomplished by means of the sample pulse.

A block diagram of the system of the Servo Clock is given in Figure 2. The sample pulse is generated in a triple pulse detector in coincidence with the third pulse of the triple pulse. The sample pulse is applied from the detector to an error-sensing network the purpose of which is to produce, when necessary, a voltage which adjusts the frequency of a voltage-controlled oscillator (VCO) serving as the servo system clock. The output of the VCO is passed through two frequency-dividing networks, which convert it to the frame frequency, and back to the error-sensing network. This is the servo loop. The frame waveform and next-incoming sample pulse are compared in the error-sensing network, and, if a pulse difference is detected, a correcting voltage is automatically applied to the VCO to bring the servo system clock back into step with the airborne system clock. Between samples the error voltage is constant.

The normal frequency of the VCO was chosen to be eight times that of the airborne clock. Binary division of the normal frequency of the VCO by eight yields a square waveform at the reference-pulse frequency of the servo system clock; binary division of this frequency by sixteen yields a square waveform at the frame-pulse frequency of the servo system clock. These two waveforms are applied to two separate monostable multivibrators which provide reference-outputs and frame-pulse outputs from the Servo Clock for use in the ground station processing and recording equipments.

Other systems, essentially phase-locked loops, have been designed to slave one clock to another, but all required initial manual adjustment of the system oscillator to achieve phase lock. In these systems, if the master clock frequency shifted abruptly, or if the transmitted signal dropped out, even for the shortest time, manual readjustment was necessary to relock the system, and, in the time required for readjustment, data losses resulted.

The present servo system achieves proper lock in a fraction of a second and is completely automatic. By the elimination of unnecessary data losses it provides a means for a more effective telemetry system.

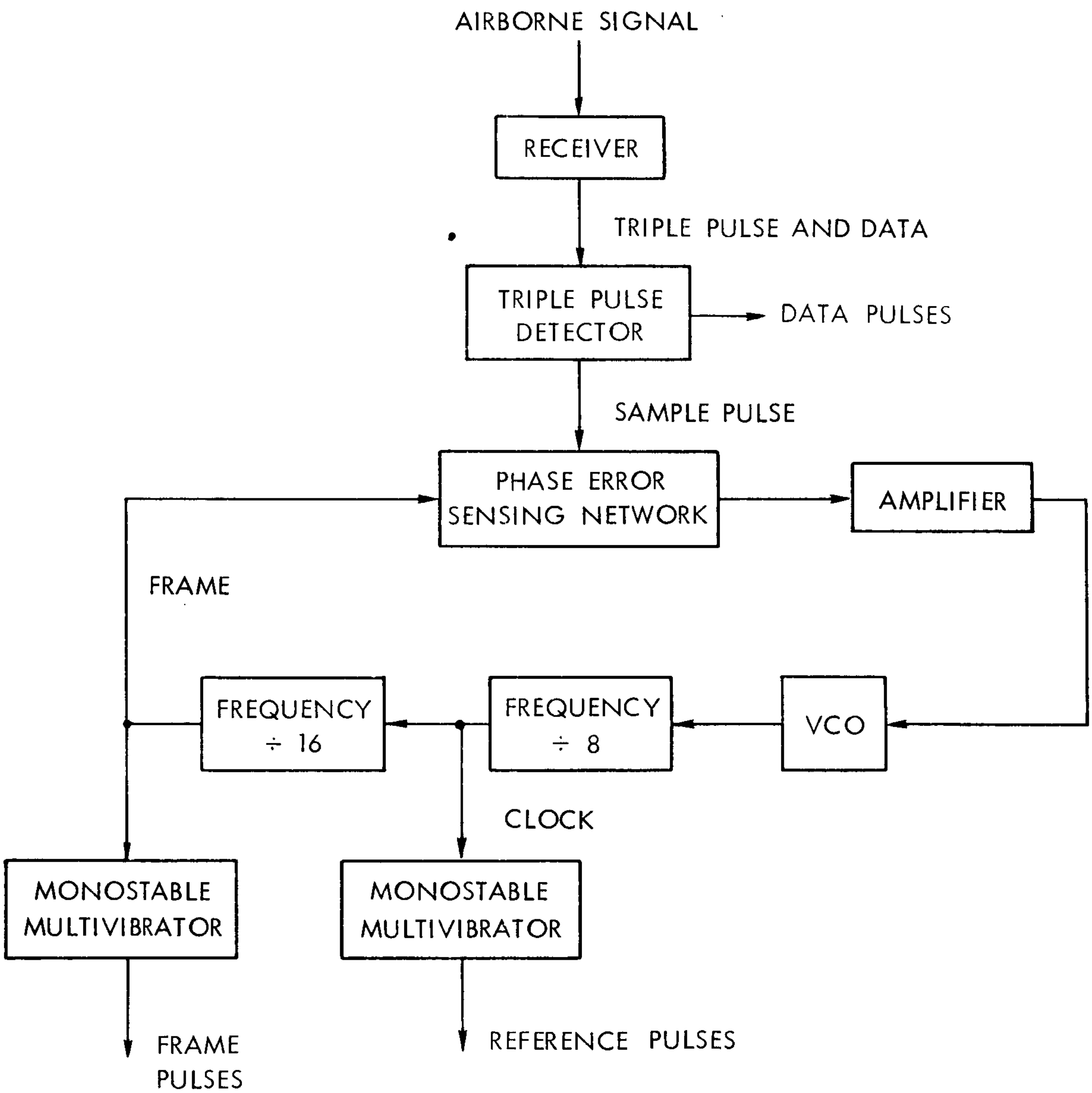


Figure 2. Basic Servo System

SECTION I

THEORY OF OPERATION

The following discussion develops theory indicating how the Servo Clock will apply self corrections. A list of symbols used is given on page 6.

Phase error at sample pulse n, θ_n, is defined as θ_{n-1} plus the time difference between the previous sixteen cycles of the airborne clock and the previous sixteen cycles of the servo system's clock. More specifically,

$$\theta_n = \theta_{n-1} + \left[\sum_{i=1}^{16} \frac{1}{(f_i)_a} - \sum_{i=1}^{16} \frac{1}{(f_i)_{ss}} \right]_{n-1} . \tag{1}$$

The second term sums the time required for sixteen cycles of the airborne clock; $1/(f_i)_a$ is the period of the i'th cycle. The third term does likewise for the servo system's clock. Since input information is available only at sampling instants, the system VCO will be controlled so that it changes frequency only at the sampling instant, that is, the voltage to the VCO will change only during sampling. This restriction is reflected in Equation 1 as,

$$\theta_n = \theta_{n-1} + \left[\sum_{i=1}^{16} \frac{1}{(f_i)_a} - \frac{16}{f_{ss}} \right]_{n-1} . \tag{2}$$

PHASE ERROR

A hypothetical situation will now be assumed. The servo system is properly locked to an airborne system which has a clock frequency f_0. When properly locked, the phase error, θ_n, is zero. Arbitrarily call a particular sample pulse (derived from the received triple pulse) sample zero. With no changes in either the airborne or the servo system, the phase error will remain zero. Thus, $\theta_0 = 0$, $\theta_1 = 0$.

At sample one the airborne clock suddenly decreases frequency to $f_0 - \Delta f$, which will result in a phase error at sample two (since the Servo Clock has no way to sense this frequency shift until sample two, its clock frequency continues

SYMBOLS

$(f_i)_a$	frequency of i'th cycle of airborne clock
$(f)_{ss}$	frequency of servo system clock
$(f_0)_a$	normal frequency of airborne clock, 5 kHz
G	transfer function of voltage controlled oscillator (VCO), 9.3 kHz/V
K	constant, .0071, Equal to Rt_0/C_2 and C_1/C_2
M	slope of phase error detector, 32/6 V/μs
S_n	magnitude of correction voltage applied to VCO between samples n and n + 1
t_0	duration of sample pulse, 2 microseconds
T_{ss}	period of servo system clock
$(T_n)_a$	period of airborne clock cycles between samples n and n + 1
$(T_n)_{ss}$	period of servo system clock cycles between samples n and n + 1
$(T_0)_a$	normal period of airborne clock, 200 microseconds
V_{C^n}	output voltage of $C_1 - C_2$ sample and hold network between samples n and n + 1
V_{R^n}	output voltage of R - C sample and hold network between samples n and n + 1
Δf	change in clock frequency
ΔT	change in clock period
$\Delta\theta$	magnitude of assumed phase error
θ_n	phase error at sample n
θ_{ss}	steady state phase error
T	time between first pulse of triple pulse and sample pulse, 16μs.

at f_0 between samples one and two). It is reasonable to assume, considering specifications of the airborne system to be used, that Δf will always be less than 1/10 f_0. If $T_0 = 1/f_0$ what period is associated with $1/(f_0 - \Delta f)$? More specifically,

$$T_0 + \Delta T = \frac{f}{f_0 - \Delta f} = \frac{1}{f_0}\left[\frac{1}{1 - \frac{\Delta f}{f_0}}\right],$$

$$T_0 + \Delta T = \frac{1}{f_0}\left[\frac{1}{1 - \frac{\Delta f}{f_0}}\right]\left[\frac{1 + \frac{\Delta f}{f_0}}{1 + \frac{\Delta f}{f_0}}\right],$$

$$T_0 + \Delta T = \frac{1}{f_0}\left[\frac{1 + \frac{\Delta f}{f_0}}{1 - \left(\frac{\Delta f}{f_0}\right)^2}\right].$$

But,

$$\left(\frac{\Delta f}{f_0}\right)^2 << 1 .$$

Then,

$$T_0 + \Delta T = \frac{1}{f_0}\left[1 + \frac{\Delta f}{f_0}\right]$$

$$T_0 + \Delta T = T_0 + \frac{\Delta f}{f_0^{\,2}}$$

$$\Delta T = + \frac{\Delta f}{f_0^{\,2}} .$$

In other words, a frequency decrease of Δf from some f_0, if $\Delta f < 1/10\ f_0$, corresponds to a period increase of $\Delta f/f_0^2$.

At sample two, there will clearly be a phase error θ_2, where

$$\theta_2 = \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0\right)_{ss}\right]_1 .$$

The first term is 16 cycles of the airborne clock between samples one and two. The second term is 16 cycles of the servo system's clock between samples one and two. At sample two the VCO will shift frequency because of the phase error, θ_2. The question is, what frequency should it assume between samples two and three, so that at sample three the phase error will be zero? If this is achieved, then at sample three the VCO will shift frequency again, so that its clock period is $T_0 + \Delta T$, thus maintaining frequency and phase lock.

Phase error at sample three, θ_3, is

$$\theta_3 = \theta_2 + \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_x\right)_{ss}\right]_2 .$$

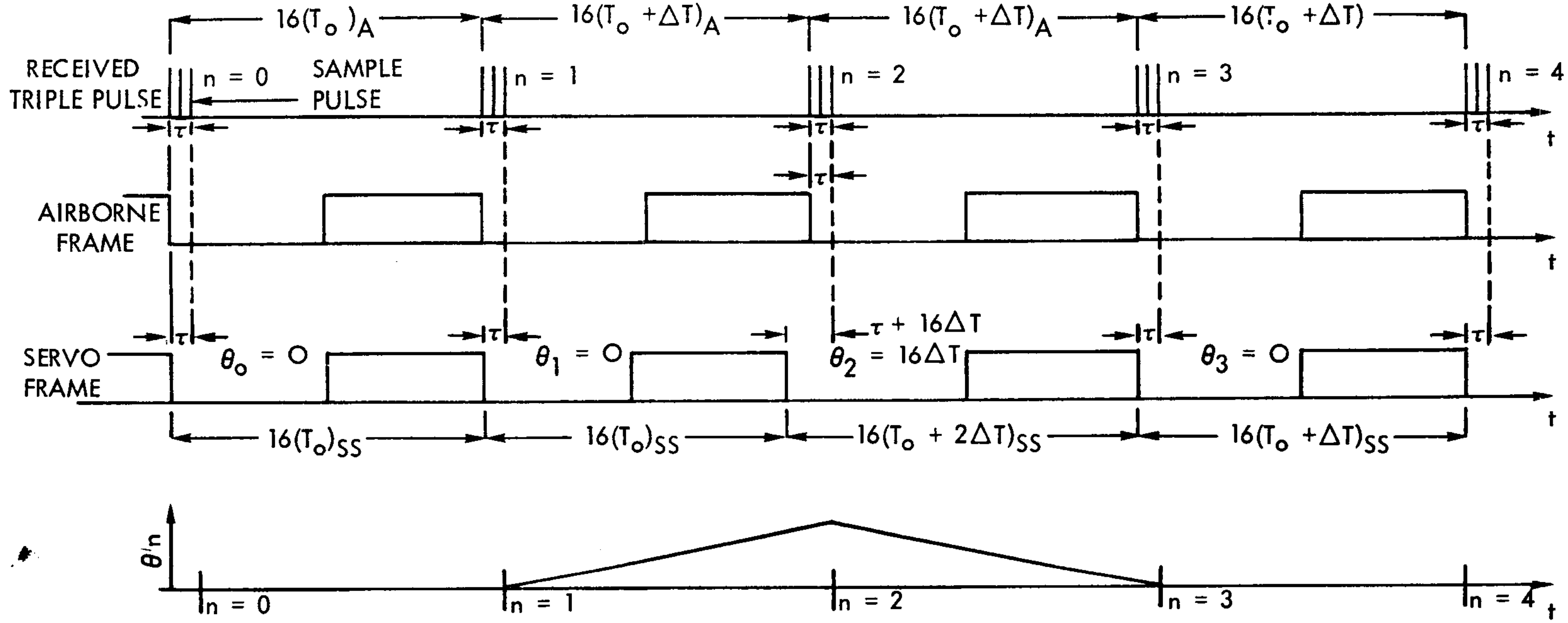
When

$$\theta_3 = 0 = \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0\right)_{ss}\right]_1 + \left[16\left(T_0 + \Delta T\right) - 16\left(T_x\right)_{ss}\right]_2 ,$$

and, if subscripts are dropped so that $\left(T_0\right)_{ss} = \left(T_0\right)_a = T_0$, then, $\left(T_x\right)_{ss} = T_0 + 2\Delta T$. This result, illustrated in Figure 3, means that to remove the phase error resulting from the frequency shift at sample one by sample three, the Servo must alter its clock period by twice the apparent input clock period shift between samples two and three. At sample three, the Servo will shift frequency again, to a clock period of $T_0 + \Delta T$, remaining in both phase and frequency lock.

Numerical Example

A numerical example will illustrate the usefulness of this result. Again, suppose $\theta_1 = 0$, with $T_0 = 200$ microseconds. At sample one, a frequency shift in the airborne clock causes the period to change by $\Delta T = 2$ microseconds. Since the system will always be operating at approximately a constant frequency, phase error will be expressed in microseconds and not radians. At sample two,



OPERATION:

(1) WHEN PROPERLY LOCKED, EDGE OF SERVO FRAME OCCURS τ μSEC BEFORE SAMPLE PULSE. UNDER THIS CONDITION, $\theta_n = 0$.
(2) THE AIRBORNE SYSTEM CLOCK SHIFTS FREQUENCY AT $n = 1$.
(3) SERVO DETECTS PHASE ERROR AT $n = 2$ AND RESPONDS.
(4) AT $n = 3$, $\theta_3 = 0$

Figure 3. System Operation

the resulting phase error, θ_2 is,

$$\theta_2 = \theta_1 + \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0\right)_{ss}\right]_1 ,$$

$$\theta_2 = 0 + \left[16(200 + 2)_a - 16(200)_{ss}\right]_1 = 3232 - 3200$$

$$\theta_2 = +\,32 \text{ microseconds.}$$

$$\theta_3 = \theta_2 + \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0 + 2\Delta T\right)_{ss}\right]_2$$

$$\theta_3 = 32 + 16(200 + 2)_a - 16(200 + 2 \times 2)_{ss} = 32 + 3232 - 3264$$

$$\theta_3 = 0 \text{ microseconds.}$$

$$\theta_4 = 16\left(T_0 + \Delta T\right)_a - 16\left(T_0 + \Delta T\right)_{ss} = 0 .$$

In the above ideal situation, the error begins to accumulate at sample one. At sample two it is detected by the servo system and the system reacts, altering its clock period by $2\Delta T$. At sample three the phase error is again zero, half the correction is removed so as to continue with a clock period of $T_0 + \Delta T$.

FREQUENCY ERROR

A similar result can be obtained on a frequency basis. Consider the following similar situation:

$$\theta_1 = 16\left(\frac{1}{f_0}\right)_a - 16\left(\frac{1}{f_0}\right)_{ss} = 0 .$$

Immediately after sample one is taken, the airborne clock frequency shifts by Δf.

$$\theta_2 = \left(\theta_1 = 0\right) + 16\left(\frac{1}{f_0 + \Delta f}\right)_a - 16\left(\frac{1}{f_0}\right)_{ss} .$$

Requiring that $\theta_3 = 0$,

$$\theta_3 = 0 = \theta_2 + 16\left(\frac{1}{f_0 + \Delta f}\right)_a - 16\left(\frac{1}{f_x}\right)_{ss}$$

$$0 = 16\left(\frac{1}{f_0 + \Delta f}\right)_a - 16\left(\frac{1}{f_0}\right)_{ss} + 16\left(\frac{1}{f_0 + \Delta f}\right)_a - 16\left(\frac{1}{f_x}\right)_{ss}$$

Dropping the airborne and servo system subscripts,

$$0 = f_0 f_x - f_x (f_0 + \Delta f) + f_x f_0 - f_0 (f_0 + \Delta f)$$

$$f_x = \frac{f_0 (f_0 + \Delta f)}{(f_0 - \Delta f)} = \frac{f_0\left[1 + \frac{\Delta f}{f_0}\right]}{\left[1 - \frac{\Delta f}{f_0}\right]} \cdot \frac{\left[1 + \frac{\Delta f}{f_0}\right]}{\left[1 + \frac{\Delta f}{f_0}\right]}$$

$$f_x = \frac{f_0\left[1 + \frac{2\Delta f}{f_0} + \frac{\Delta f^2}{f_0}\right]}{\left[1 - \frac{\Delta f^2}{f_0}\right]} .$$

Using the restriction that $\Delta f < 1/10 \; f_0$,

$$f_x = f_0\left[1 + \frac{2\Delta f}{f_0}\right] = f_0 + 2\Delta f$$

$$f_x = f_0 + 2\Delta f . \qquad (3)$$

This result is analogous to $T_x = T_0 + 2\Delta T$.

THEORETICAL SYSTEM

Figure 4 represents a system that will function as required. The frame waveform from the VCO (corresponding to sixteen cycles of the servo system's clock) is interrogated by the sample pulse in the phase error detector. The resulting output voltage, present only during the sampling instant, can be expressed as, $V_n = M\theta_n$, where M, to be expressed in volts per microsecond, is a constant.

Two independent sample and hold circuits sample V_n at the sampling instants and hold the result until the next sample. It is the holding characteristic of these two circuits which allows the VCO frequency to change only during sampling instants. A constant K appears in both circuits; it is a ratio of voltage division. Continuous output voltages $\frac{V_n}{R}$ and $\frac{V_n}{C}$ are added, inverted (multiplication by minus one) and the result, $-S_n$, drives the VCO. The VCO output is divided by 8, resulting in the system clock. The gain of the VCO is G hertz/volt. The VCO is externally biased so that with $S_n = 0$, the system clock period is T_0. To completely describe the feedback loop, it is necessary to describe the shift in system clock period accompanying a frequency shift of the VCO. The gain of the VCO referenced to the clock frequency is G. A shift in clock frequency referenced to the VCO frequency may be expressed as $\Delta f = -S_n\,(G/8)$ hertz. The shift in clock period accompanying a shift in clock frequency is ΔT. From an earlier result,

$$\Delta T = \frac{-\Delta f}{f_0^2},$$

$$\Delta T = \frac{-\left[\left(+\frac{G}{8}\right)\left(-S_n\right)\right]}{f_0^2} = \frac{GS_n}{8f_0^2}$$

or assuming

$$T_0 = \frac{1}{f_0},$$

$$\Delta T = \frac{GS_n T_0^2}{8}.$$

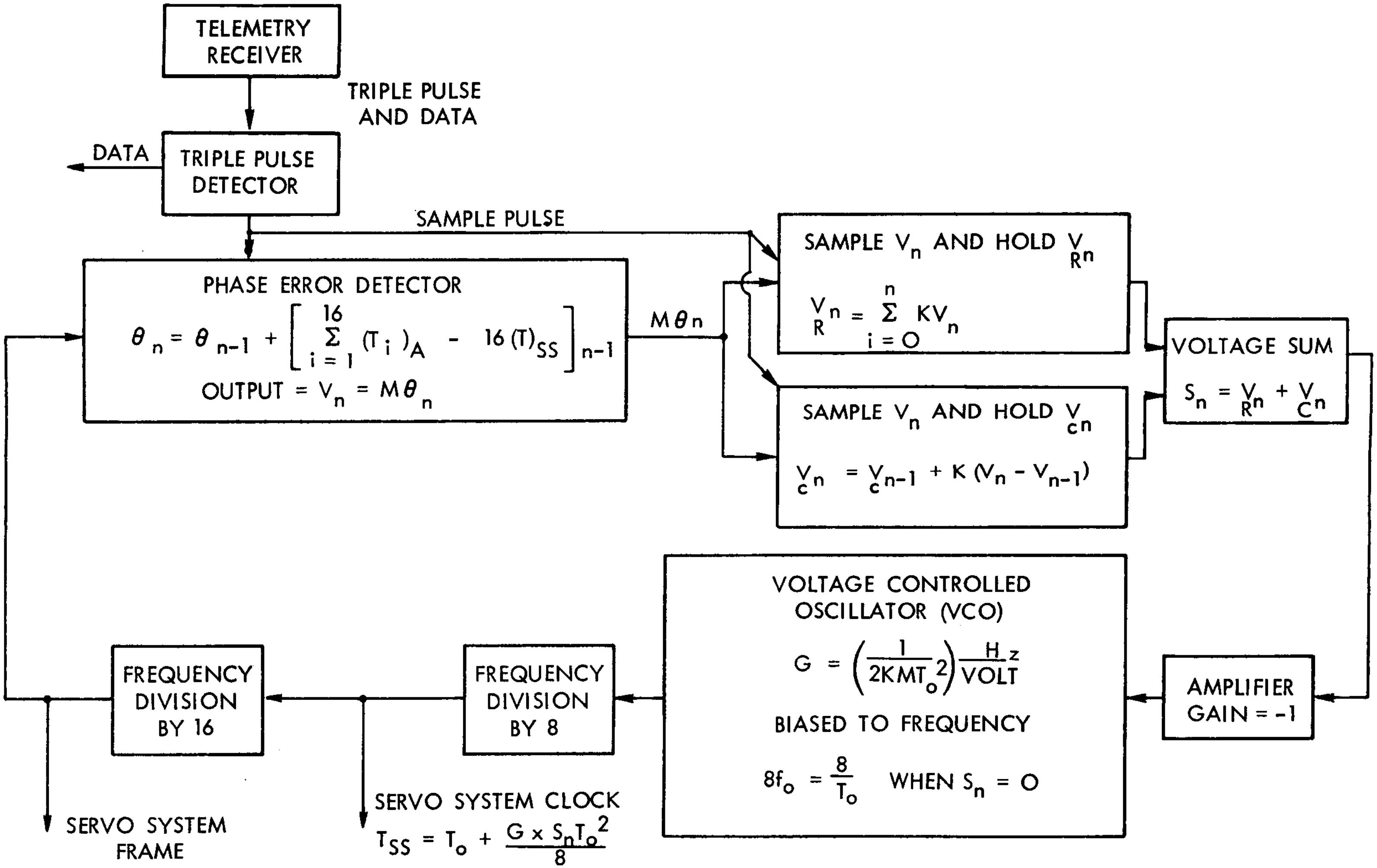


Figure 4. Servo System Block Diagram

Finally, the clock period of the servo system can be expressed as

$$\left(T_n\right)_{ss} = T_0 + \Delta T = T_0 + \frac{GS_n T_0^2}{8} \cdot \qquad (4)$$

Hypothetical Examples: Cases I, II, and III

It is useful to solve for G in terms of the other system parameters. This can be done by assuming again a hypothetical problem. Initially, assume $\theta_0 = 0$ and both clocks have periods of T_0. At sample one the airborne clock frequency shifts resulting in a new period $T_0 + \Delta T$.

$$\theta_0 = 0$$

$$\theta_1 = 0 + \left[16\left(T_0\right)_a - 16\left(T_0\right)_{ss}\right]_0 = 0$$

$$V_1 = 0, \qquad \underset{R}{V}{}_1^1 = 0, \qquad \underset{C}{V}{}_1^1 = 0, \qquad \left(T_1\right)_{ss} = T_0$$

$$\theta_2 = 0 + \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0\right)_{ss}\right]_1 = 16\Delta T$$

$$V_2 = 16\Delta TM$$

$$\underset{R}{V}{}_2^2 = 16\Delta TMK$$

$$\underset{C}{V}{}_2^2 = 16\Delta TMK$$

$$S_2 = 32\Delta TMK, \qquad -S_2 = -32\Delta TMK$$

$$\left(T_2\right)_{ss} = T_0 + \frac{GST_0^2}{8} \quad \text{(from Equation 4.)}$$

$$\theta_3 = \left(\theta_2 = 16\Delta T\right) + \left[16\left(T_0 + \Delta T\right) - 16\left(T_0 + \frac{32\Delta TMKGT_0^2}{8}\right)\right]_2 .$$

The requirement is $\theta_3 = 0$. The gain G can now be determined.

$$G = \frac{1}{2MKT_0^2} = \frac{f_0^2}{2MK} . \tag{5}$$

This example will be carried further to illustrate the final result of the problem.

$$\theta_3 = 0$$

$$V_3 = 0$$

$$\underset{R}{V_3} = \underset{R}{V_2} + 0 = 16\Delta TMK$$

$$\underset{C}{V_3} = \underset{C}{V_2} + MK\left(\theta_3 - \theta_2\right) = 16\Delta TMK + MK\; 0 - 16\Delta TM)$$

$$\underset{C}{V_3} = 0$$

$$S_3 = 16\Delta TMK$$

$$\left(T_3\right)_{ss} = T_0 + \frac{GST_0^2}{8}$$

$$= T_0 + \left[\frac{1}{2MKT_0^2} \cdot \frac{16\Delta TMK}{8} \cdot T_0^2\right]$$

$$\left(T_3\right)_{ss} = T_0 + \Delta T .$$

So, after sample three, both clocks have a period of $T_0 + \Delta T$ and $\theta_n = 0$. This example is summarized as Case I on page 16. Key values at each sample are presented in Table 1.

Table 1

Example 1: T_a Shifts at $n = 1$, T_{ss} Shifts at $n = 2$ and $n = 3$.

Sample n	θ_n	V_n	V_{R_n}	V_{C_n}	S_n	$(T_n)_a$	$(T_n)_{ss}$
0	0	0	0	0	0	T_0	T_0
1	0	0	0	0	0	$T_0 + \Delta T$	T_0
2	$16\Delta T$	$16M\Delta T$	$16KM\Delta T$	$16KM\Delta T$	$32K\Delta T$	$T_0 + \Delta T$	$T_0 + 2\Delta T$
3	0	0	$16KM\Delta T$	0	$16KM\Delta T$	$T_0 + \Delta T$	$T_0 + \Delta T$
4	0	0	$16KM\Delta T$	0	$16KM\Delta T$	$T_0 + \Delta T$	$T_0 + \Delta T$

Case I - The period of the airborne clock shifts from T_0 to $T_0 + \Delta T$ at sample one. The system equations are defined as follows:

$$\theta_n = \theta_{n-1} + \left[16(T)_a - 16(T)_{ss}\right]_{n-1}$$

$$V_n = M\theta_n$$

$$V_{R_n} = \sum^{n} KV_n$$

$$V_{C_n} = V_{C_{n-1}} + MK\left(\theta_n - \theta_{n-1}\right)$$

$$V_{C_n} = V_{C_{n-1}} + KV_n - KV_{n-1}$$

$$S_n = V_{R_n} + V_{R_n}$$

$$T_{ss} = T_0 + \frac{S}{16KM} .$$

By use of the system model of Figure 4 it is of interest to determine transient response in two other situations. Case II, summarized on pages 17 and 18 and in Table 2, shows the transient response when the airborne clock shifts period from T_0 to $T_0 + \Delta T$ eight cycles after sample one is taken; that is, the shift occurs midway in frame number one. Here, only half as much phase error is detected at sample two. Compared to Case 1, the system requires one extra sample to achieve phase lock.

In Case III, summarized on pages 19 and 20 and in Table 3, the frequency and period of the airborne clock remain constant. Assume a counting error occurs in the airborne system so that triple pulse number one (the sample command) occurs at the wrong time, resulting in a phase error of $\Delta\theta$. After sample one, the airborne clock continues with period T_0. The table shows that proper lock is obtained by sample three.

From the results of the three cases illustrated, it is concluded that proper lock will occur within a few cycles after a phase or frequency error. Furthermore, system operation is completely automatic, certainly an improvement over previous systems.

Table 2

Example 2: T_a Shifts from T_0 to $T_0 + \Delta T$ Eight Cycles after $n = 1$; T_{ss} Shifts at $n = 2, 3, 4$.

Sample n	θ_n	V_n	V_{R^n}	V_{C^n}	S_n	$(T_n)_a$	$(T_n)_{ss}$
0	0	0	0	0	0	T_0	T_0
1	0	0	0	0	0	$T_0 + \Delta T$	T_0
2	$8\Delta T$	$8M\Delta T$	$8KM\Delta T$	$8KM\Delta T$	$16KM\Delta T$	$T_0 + \Delta T$	$T_0 + \Delta T$
3	$8\Delta T$	$8M\Delta T$	$16KM\Delta T$	$8KM\Delta T$	$24KM\Delta T$	$T_0 + \Delta T$	$T_0 + \frac{3}{2}\Delta T$
4	0	0	$16KM\Delta T$	0	$16KM\Delta T$	$T_0 + \Delta T$	$T_0 + \Delta T$
5	0	0	$16KM\Delta T$	0	$16KM\Delta T$	$T_0 + \Delta T$	$T_0 + \Delta T$

Case II - The period of the airborne clock shifts from T_0 to $T_0 + \Delta T$ eight cycles after sample one is taken.

$$\theta_0 = 0$$

Case II (Continued)

$$\theta_1 = 0$$

$$\theta_2 = \left[8T_0 + 8\left(T_0 + \Delta T\right)_a - 16\left(T_0\right)_{ss}\right]_1 = 8\Delta T$$

$$V_2 = 8M\Delta T, \qquad \underset{R}{V}{}_2 = 0 + 8KM\Delta T, \qquad \underset{C}{V}{}_2 = 0 + 8KM\Delta T, \qquad S_2 = 16KM\Delta T$$

$$\left(T_2\right)_{ss} = T_0 + \frac{S_2}{16KM} = T_0 + \frac{16KM\Delta T}{16KM} = T_0 + \Delta T$$

$$\theta_3 = \left(\theta_2 = 8\Delta T\right) + \left[16\left(T_0 + \Delta T\right)_a - 16(T + \Delta T)_{ss}\right]_2$$

$$\theta_3 = 8\Delta T, \qquad V_3 = 8M\Delta T$$

$$\underset{R}{V}{}_3 = \left(\underset{R}{V}{}_2 = 8KM\Delta T\right) + 8KM\Delta T$$

$$\underset{C}{V}{}_3 = \left(\underset{C}{V}{}_2 = 8KM\Delta T\right) + MK\left(8\Delta T - 8\Delta T\right) = 8KM\Delta T$$

$$S_3 = 24KM\Delta T$$

$$\left(T_3\right)_{ss} = T_0 + \frac{24KM\Delta T}{16KM} = T_0 + \frac{3}{2}\Delta T$$

$$\theta_4 = \left(\theta_3 = 8\Delta T\right) + \left[16\left(T_0 + \Delta T\right)_a - 16\left(T_0 + \frac{3}{2}\Delta T\right)_{ss}\right]_3 = 0$$

$$V_4 = 0$$

$$\underset{R}{V}{}_4 = \left(\underset{R}{V}{}_3 = 16KM\Delta T\right) + 0$$

$$\underset{C}{V}{}_4 = \left(\underset{C}{V}{}_3 = 8KM\Delta T\right) + MK(0 - 8\Delta T) = 0$$

$$S_4 = 16KM\Delta T$$

Table 3

Example 3: Airborne Counting Error Results in Phase Error $\Delta\theta$;
Initially $T_a = T_{ss} = T_0$; T_{ss} Shifts at n = 1 and Shifts Back at n = 3.

Sample n	θ_n	V_n	V_{R_n}	V_{C_n}	S_n	$(T_n)_a$	$(T_n)_{ss}$
0	0	0	0	0	0	T_0	T_0
1	$\Delta\theta$	$M\Delta\theta$	$MK\Delta\theta$	$MK\Delta\theta$	$2MK\Delta\theta$	T_0	$T_0 + \frac{\Delta\theta}{8}$
2	$-\Delta\theta$	$-M\Delta\theta$	0	$-MK\Delta\theta$	$-MK\Delta\theta$	T_0	$T_0 - \frac{\Delta\theta}{16}$
3	0	0	0	0	0	T_0	T_0

Case III - An error occurs in the binary division of the airborne clock, resulting in a phase error at sample one of $\Delta\theta$. The airborne clock frequency does not change.

$$\theta_0 = 0$$

$$\theta_1 = \Delta\theta, \qquad V_1 = M\Delta\theta, \qquad V_{R_1} = V_{C_1} = KM\Delta\theta, \qquad S_2 = 2KM\Delta\theta$$

$$(T_1)_{ss} = T_0 + \frac{2KM\Delta\theta}{16KM} = T_0 + \frac{\Delta\theta}{8}$$

$$\theta_2 = (\theta_1 = \Delta\theta) + 16\left[(T_0)_a - 16\left(T_0 + \frac{\Delta\theta}{8}\right)_{ss}\right]_1 = -\Delta\theta$$

$$V_2 = -M\Delta\theta$$

$$V_{R_2} = \left(V_{R_1} = MK\Delta\theta\right) + K(-M\Delta\theta) = 0$$

$$V_{C_2} = \left(V_{C_1} = MK\Delta\theta\right) + MK(-\Delta\theta - \Delta\theta) = -MK\Delta\theta$$

Case III (Continued)

$$S_2 = -MK\Delta\theta$$

$$(T_2)_{ss} = T_0 - \frac{MK\Delta\theta}{16MK} = T_0 - \frac{\Delta\theta}{16}$$

$$\theta_3 = (\theta_2 = -\Delta\theta) + 16(T_0)_a - 16\left(T_0 - \frac{\Delta\theta}{16}\right) = 0$$

$$\theta_3 = 0$$

$$V_3 = 0$$

$$V_{R_3} = 0$$

$$V_{C_3} = (V_{C_2} = -MK\Delta\theta) + MK(0 - (-\Delta\theta)) = 0$$

$$S_3 = 0$$

$$(T_3)_{ss} = T_0 \, .$$

SECTION II

REALIZATION OF SYSTEM

Physical realization of circuits used to replace the ideal mathematical subsystems of the Servo Clock will now be considered. In many cases, in order to introduce simplifying approximations, numerical values will be used.

THE PHASE ERROR DETECTOR

The phase error detector will be considered first. As given in the system block diagram, Figure 4, the detector's equations are,

$$\theta_n = \theta_{n-1} + \left[\sum_{i=1}^{16} (T_i)_a - 16(T_{ss})_{n-1} \right]_{n-1} \tag{6}$$

(Equation 2, page 5) and,

$$V_n = M\theta_n . \tag{7}$$

(As defined, page 12.) The output of this error detector, V_n, is sampled by the two subsequent networks only at times n, as specified by the third pulse of the triple pulse. Therefore, it is necessary for V_n to be present only during sampling instants. As previously mentioned, the sample pulse occurs T microseconds after the airborne frame begins. Sixteen cycles of the servo system clock, $(16T)_{ss}$, are provided by binary division. Sixteen cycles of the airborne clock,

$$\sum_{i=1}^{16} (T_i)_a ,$$

are provided by the sample pulse.

The sample pulse occurs $T = 16$ microseconds after the beginning of an airborne frame. In consideration of this fact, it is convenient to start a linear sweep voltage from -3 volts to +3 volts of slope M, with the servo system frame. The voltage of the sweep, $v(t)$, can be described as follows:

$$v(t) = -3 + Mt \text{ volts.}$$

M is determined when it is required that $v(t) = 0$:

$$v(t) = 0 = -3 + Mt \text{ volts, or } Mt = 3, \text{ and when}$$

$$t = T = 16 \text{ microseconds,}$$

$$M = 3/16 \text{ volts/microsecond.} \tag{8}$$

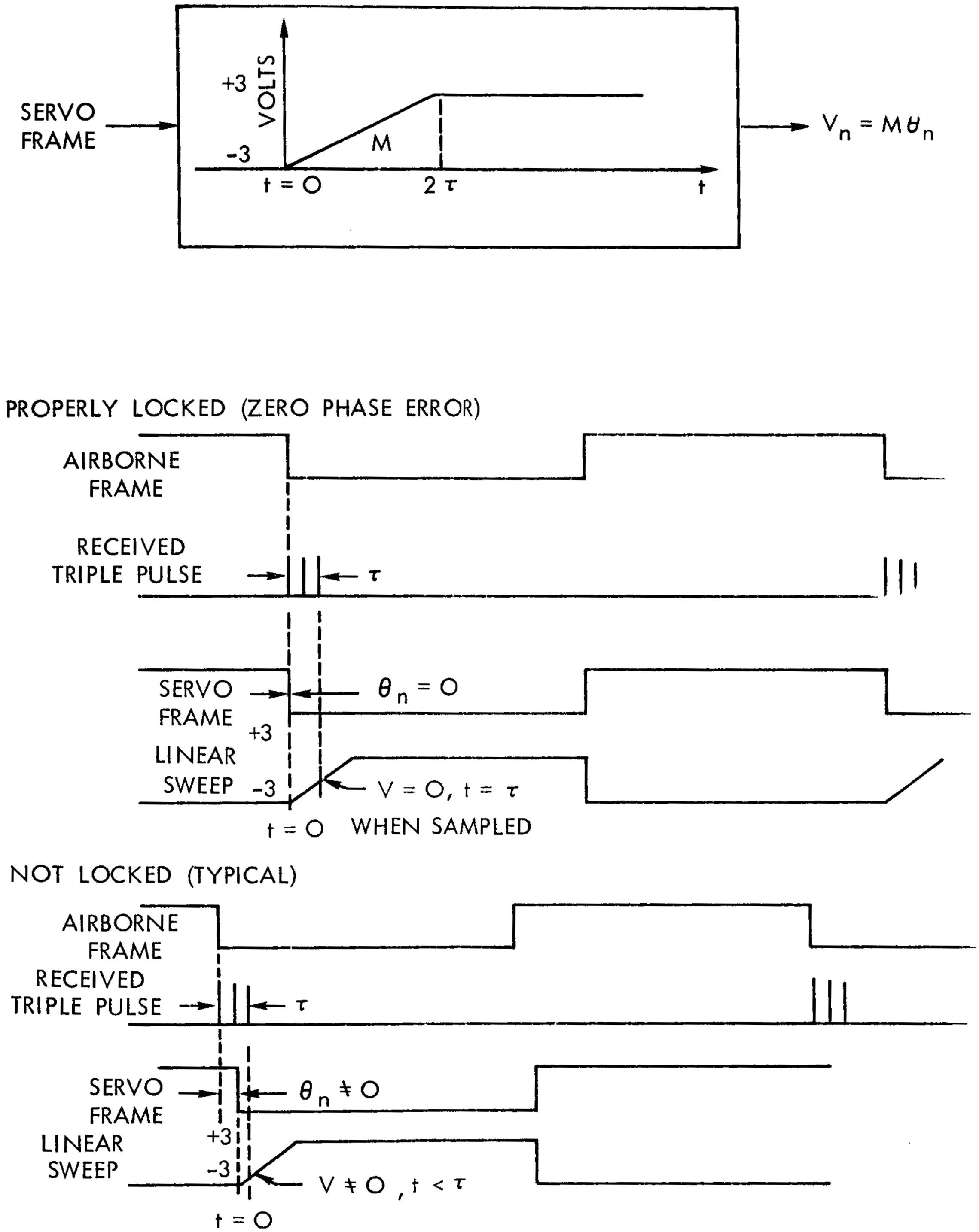
With this slope, if the sample pulse occurs 16 microseconds after the servo frame begins, zero volts are sampled, which means that no phase error occurred. This indicates that the servo frame and the airborne frame started at the same time. If, however, the phase error is greater than 16 microseconds, the output voltage magnitude is limited to 3 volts. This limitation will affect transient performance. The output at sampling instants n can be expressed as:

$$V_n = M\theta \text{ for } \theta < T$$

$$V_n = +3 \text{ volts for } \theta > T$$

$$V_n = -3 \text{ volts for } \theta < -T\ . \tag{9}$$

Figure 5 illustrates the linear sweep and associated waveforms in a locked and a typical unlocked condition.



$V_n = -3 + Mt$ VOLTS; IF $t = \tau$, $V_n = 0$.

Figure 5. Phase Error Detector

THE FIRST SAMPLE AND HOLD NETWORK

The first sample and hold network of the Servo Clock should satisfy the following equation:

$$V_{R^n} = \sum^{n} KV_n \quad . \tag{10}$$

(Where K is as defined on page 12.) Output V_{R^n} is held constant between sampling instants. To approximate Equation 10 a simple R - C circuit, as shown in Figure 6, is used. Assume C is initially charged to V_i. The voltage on C at time t due to a step input of magnitude E_1 at $t = 0$ can be represented as

$$v(t) = E_1 + (V_i - E_1)\, e^{-t/Rc} . \tag{11}$$

It is convenient to let t_0 be the width of the sample pulse. If the input voltage, E_1, is applied through a series switch closed only during the sampling time, a sample and hold circuit is created. Circuit operation is illustrated in Figure 6.

Equation 10 can be rewritten using infinite series expansion for exponential terms.

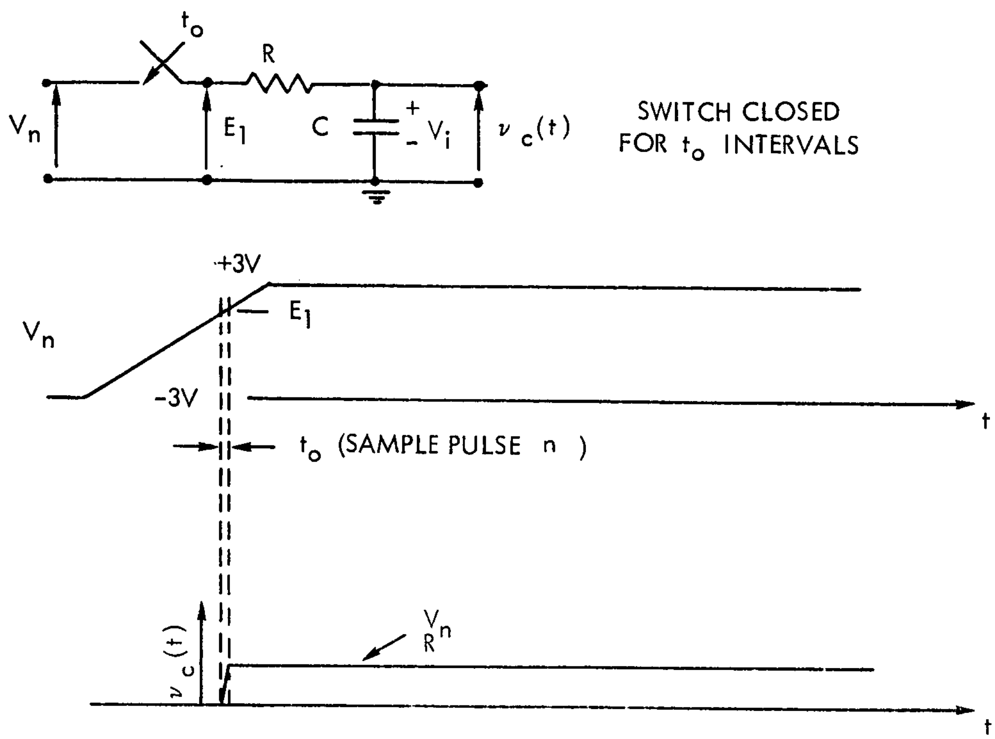
$$v(t) = E_1 + (V_i - E_1)\left[1 - \frac{t}{Rc} + \frac{1}{2}\frac{t^2}{Rc} - \cdots\right] .$$

By introducing the restriction that $K = t_0/Rc << 1$, the following simplifications can be made:

$$v(t_0) = E_1\left(\frac{t_0}{Rc}\right) + V_i\left(1 - \frac{t_0}{Rc}\right) \tag{12}$$

$$v(t_0) = KE_1 + V_i \quad . \tag{13}$$

Equation 13 is similar to Equation 10.



$$\nu_c(t) = E\,(1 - e^{-t/RC}) \doteq E\,\frac{t}{RC} \doteq EK$$

VALID ONLY FOR TRANSIENT CONDITION,
WHERE $\frac{nt_o}{RC} << 1$

Figure 6. Sample and Hold $V_{R^n} = \sum_n KV_n$

This approximation is valid only when considering transient conditions. After a certain number of samples, nt_0/Rc will not be less than one.

Recall the results of the transient analysis using the ideal model of the previous section. In all cases, the final V_n was zero. This desirable characteristic was due to the summation of correction voltage as specified by Equation 10. However, Equation 13 does not exhibit this property. For $v(t)$ to maintain a constant value, E_1 must be greater than zero. This result can be generalized for the entire system. When the Servo Clock corrects for an airborne clock frequency shift a phase error, to maintain some input voltage V_n, will result. The magnitude of the phase error will be determined later under steady state performance.

To illustrate the similarity of Equations 13 and 10, consider initially that $V_i = 0$ and that the input consists of two samples of magnitude E. Equation 10 will have as a result, $V_{R2} = 2KE$. Equation 13 has a result $v(2t_0) = KE + KE$. Rewritten in terms of samples, Equation 13 is:

$$V_{Rn} = V_n K + V_{R\,n-1} . \qquad (14)$$

Again, this is valid only during transient conditions.

THE SECOND SAMPLE AND HOLD NETWORK

A capacitor voltage divider network can be shown to approximate the sample and hold circuit producing V_{Cn} . Refer to Figure 7. The input voltage, V_n, is applied through a series switch similar to that of the R - C network. The switch allows the input voltage, E_1, always to be considered a step voltage. In addition opening the switch holds the output voltage, E_2. Consider the circuit with initial voltages E_{1i} and E_{2i} on C_1 and C_2 respectively. A loop equation for the circuit is

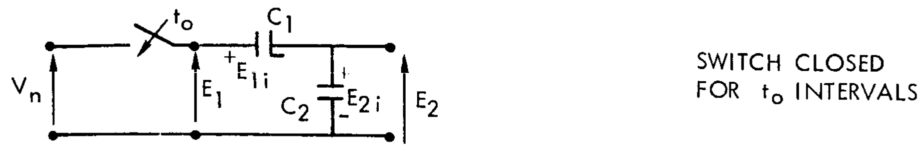
$$E_1(s) = I(s)\left[\frac{1}{C_1 s} + \frac{1}{C_2 s}\right] + \frac{1}{s}\left[E_{1i} + E_{2i}\right]$$

$$I(s) = \left[E_1(s) - \frac{E_{1i}}{s} - \frac{E_{2i}}{s}\right] \cdot \frac{C_1 C_2 s}{C_1 + C_2} .$$

The output, $E_2(s)$, can be written as,

$$E_2(s) = \frac{I(s)}{C_2(s)} + \frac{E_{2i}}{s}$$

$$E(s) = \frac{C_1}{C_1 + C_2}\left[E(s) - \frac{E_{1i}}{s} - \frac{E_{2i}}{s}\right] + \frac{E_{2i}}{s} .$$



$$\frac{E_2}{E_1} = \frac{C_1}{C_1 + C_2} = \frac{C_1}{C_2} = K, \text{ WHEN } C_2 >> C_1$$

$$E_2(s) = \frac{E_{2i}}{s} + K\left[\frac{E_1 - E_{2i}}{s}\right]$$

$$E_{2n} = E_{2(n-1)} + K\left[E_{1n} - E_{1(n-1)}\right]$$

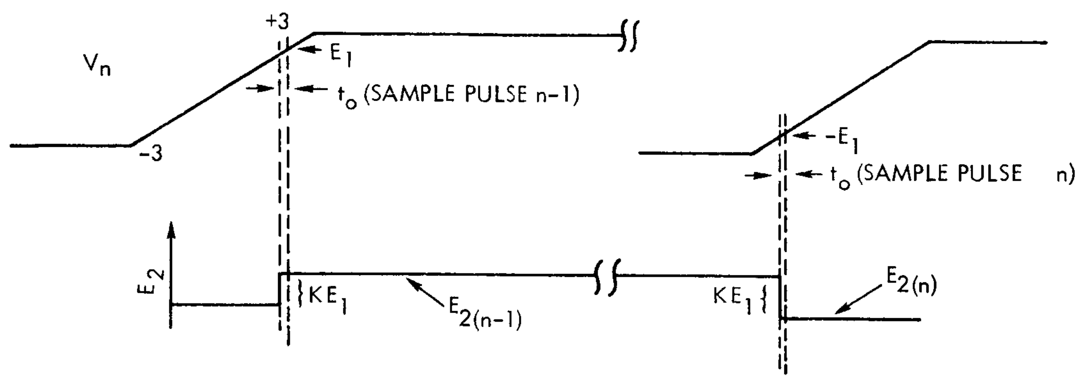


Figure 7. Sample and Hold $V_{C^n} = V_{C^{n-1}} + K(V_n - V_{n-1})$

Since $E_1(s)$ will always appear as a step voltage, $E_1(s) = E_1/s$. Finally,

$$E(s) = \frac{C_1}{C_1 + C_2}\left[\frac{E_1 - E_{1i} - E_{2i}}{s}\right] + \frac{E_{2i}}{s}.$$

Let $K = C_1/(C_1 + C_2)$, then,

$$E_2(s) = \frac{E_{2i}}{s} + K\left[\frac{E_1 - E_{1i} - E_{2i}}{s}\right]. \tag{15}$$

If, initially, before any samples are taken, $E_{1i} = E_{2i} = 0$, then in all subsequent samples, $E_{2i} = KE_{1i}$. Then

$$E_2(s) = \frac{K\left[E_1 - KE_{1i} - E_{1i}\right]}{s}. \tag{16}$$

Making the restriction that $C_2 >> C_1$, $K \doteq C_1/C_2$; $K << 1$. Introducing this approximation into Equation 16 yields,

$$E_2(s) = \frac{E_{2i}}{s} + K\left[\frac{E_1 - E_{2i}}{s}\right] . \tag{17}$$

Realizing that the output voltage will be a step function, the Laplace notation will be dropped. Initial voltage, E_{2i}, can be considered the result of the previous sample. If the input and output carry subscript n, then the initial condition carries subscript n - 1. Equation 17 can be rewritten as

$$E_{2n} = E_{2(n-1)} + K\left[E_{1n} - E_{1(n-1)}\right] . \tag{18}$$

This is similar to the sample and hold circuit,

$$\underset{C}{V}_n = \underset{C}{V}_{n-1} + K\left(V_n - V_{n-1}\right) . \tag{18a}$$

COMBINATION OF THE TWO SAMPLE AND HOLD NETWORKS

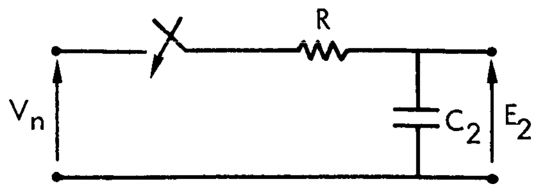
The next component of the block diagram to consider is addition of $\underset{C}{V}_n$ to $\underset{R}{V}_n$. Any one of many circuits could perform the desired operation. However, in the interest of overall system simplicity this circuit was omitted. Instead, the two sample and hold circuits were combined in a manner which approximates the required summation. Figure 8 shows the circuits previously described to produce $\underset{C}{V}_n$ and $\underset{R}{V}_n$. The combination circuit used to approximate S_n, where $S_n = \underset{C}{V}_n + \underset{R}{V}_n$ is also shown.

The transfer function of the combination circuit is

$$H(s) = \frac{1 + RC_1 s}{1 + R\left(C_1 + C_2\right) s} .$$

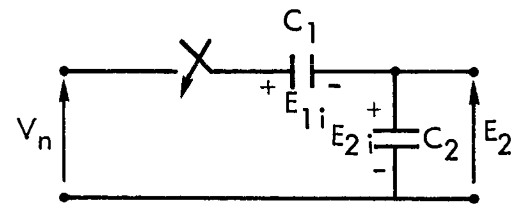
Since the input will always appear as a step function, $E_1(s) = E/s$,

$$E_2(s) = \frac{E}{s}\left[\frac{1 + RC_1 s}{1 + R\left(C_1 + C_2\right) s}\right]$$



$$E_2 = E_1 K = E_1 \frac{t_o}{RC}$$

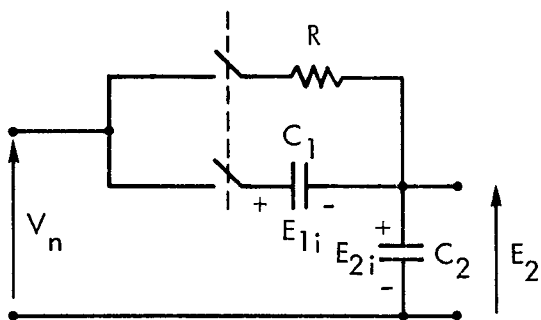
$$\frac{nt_o}{RC_2} << 1$$



$$E_2 = E_{2i} + K\left[E_1 - E_{1i}\right]$$

$$C_2 >> C_1$$

$$K = \frac{C_1}{C_1 + C_2} = \frac{C_1}{C_2} <<1$$



$$C_2 >> C_1$$

$$K = \frac{C_1}{C_1 + C_2} \doteq \frac{C_1}{C_2} = \frac{t_o}{RC_2} \; ; \; C_1 = \frac{t_o}{R}$$

$$E_2 \doteq E_1 \left[\frac{C_1}{C_2} + \frac{t_o}{RC_2} \right]$$

Figure 8. Combination Sample and Hold

$$E_2(s) = \frac{ERC_1\left(\frac{1}{RC_1} + s\right)}{sR\left(C_1 + C_2\right)\left(\frac{1}{R\left(C_1 + C_2\right)}\right) + s} .$$

Partial fraction expansion gives

$$E_2(s) = \frac{E}{s} - \frac{EC_2}{C_1 + C_2}\left[\frac{1}{s + \frac{1}{R(C_1 + C_2)}}\right]$$

$$E_2(t) = E - \frac{EC_2}{C_1 + C_2} e^{-t/R(C_1+C_2)} .$$

Evaluating E_2 after one sample (duration t_0) gives,

$$E_2(t_0) = E - \frac{EC_2}{C_1 + C_2} e^{-t_0/R(C_1+C_2)} .$$

Expanding the above into a series gives

$$E_2(t_0) = E\left[1 - \frac{C_1}{C_1 + C_2} + \frac{C_2}{C_1 + C_2} \cdot \frac{t_0}{R(C_1 + C_2)} \cdots\right] .$$

Making the assumption that $t_0/R(C_1 + C_2) << 1$, as previously made in the section on $\frac{V}{R}$", yields

$$E_2(t_0) = E\left[\frac{C_1}{C_1 + C_2} + \frac{C_2 t_0}{R(C_1 + C_2)^2}\right] .$$

Again requiring that $C_2 >> C_1$, the previous equation becomes

$$E_2(t_0) \doteq E\left[\frac{C_1}{C_2} + \frac{t_0}{RC_2}\right] . \tag{19}$$

The first term is that derived in Figure 7. The second term is the same as that of Figure 6 with no initial conditions. So, it is seen that the combination network of Figure 8 gives results similar to the sum of the R - C and C - C networks. The advantage to the combination network is the elimination of an active summation device.

The original equations of the sampling networks will now be combined to include initial conditions. Equation 14 is,

$$\underset{R}{V}_{n} = KV_{n} + \underset{R}{V}_{n-1} \quad . \tag{14}$$

Equation 18 is rewritten with the input voltage labeled V_n and $\underset{C}{V}_{n}$ is substituted for E_{2n}.

$$\underset{C}{V}_{n} = \underset{C}{V}_{n-1} + K\left[V_{n} - V_{n-1}\right] \quad . \tag{20}$$

Because of the summation implied by Equation 19, it is concluded that the output for transient conditions of the combination sampling network is Equation 14 plus Equation 20, and is called S_n.

$$S_{n} = \underset{R}{V}_{n} + \underset{C}{V}_{n} \quad .$$

THE AMPLIFIER, VOLTAGE-CONTROLLED OSCILLATOR, AND FREQUENCY-DIVIDING NETWORKS

The amplifier shown in the block diagram is a direct current amplifier with an adjustable output voltage level. This characteristic is necessary so that the VCO can be biased to the normal free-running frequency. Amplifier gain is determined by internal resistors. Since the amplifier is required to amplify step functions, its rise time must be much less than the normal clock period so that system performance is not hampered.

The VCO selected has a gain, G, of

$$G = 9.3 \times 10^{3} \text{ hertz/volt} = 9.3 \text{ kilohertz/volt.}$$

Binary frequency division of the VCO output is accomplished with bistable multivibrators. They are pre-set by each sample pulse so that the counting of 16 cycles of the servo system's clock begins with a zero count.

Numerical values for system parameters can now be selected. The VCO's relation to the system is expressed by Equation 5,

$$G = \frac{1}{2KM\,T_0^2} \,. \tag{5}$$

Equation 8 gives the value of M as $3V/16\mu s$. The value of T_0, the normal clock period of the airborne telemetry unit, is 200 microseconds, corresponding to a frequency of 5 kilohertz. Substitution into the above equation yields a value for K:

$$K = \frac{1}{2GM\,T_0^2} = \frac{1}{2(9.3\times 10^3)\,(3/16\times 10^6)\,(200\times 10^{-6})^2} = 0.0071\,.$$

From Figure 6,

$$K = \frac{C_1}{C_2} = \frac{t_0}{RC_2}\,.$$

Arbitrarily choose $C_2 = .01\mu F$, then $C_1 = 71pF$. Let the sample time, t, equal 2 microseconds. Now, a value for R can be determined:

$$R = \frac{t_0}{KC_2} = \frac{t_0}{C_1} = \frac{2\times 10^{-6}\ s}{71\times 10^{-12}\ F} = 28.1\ K\ ohms\,.$$

A system diagram is given in Figure 9.

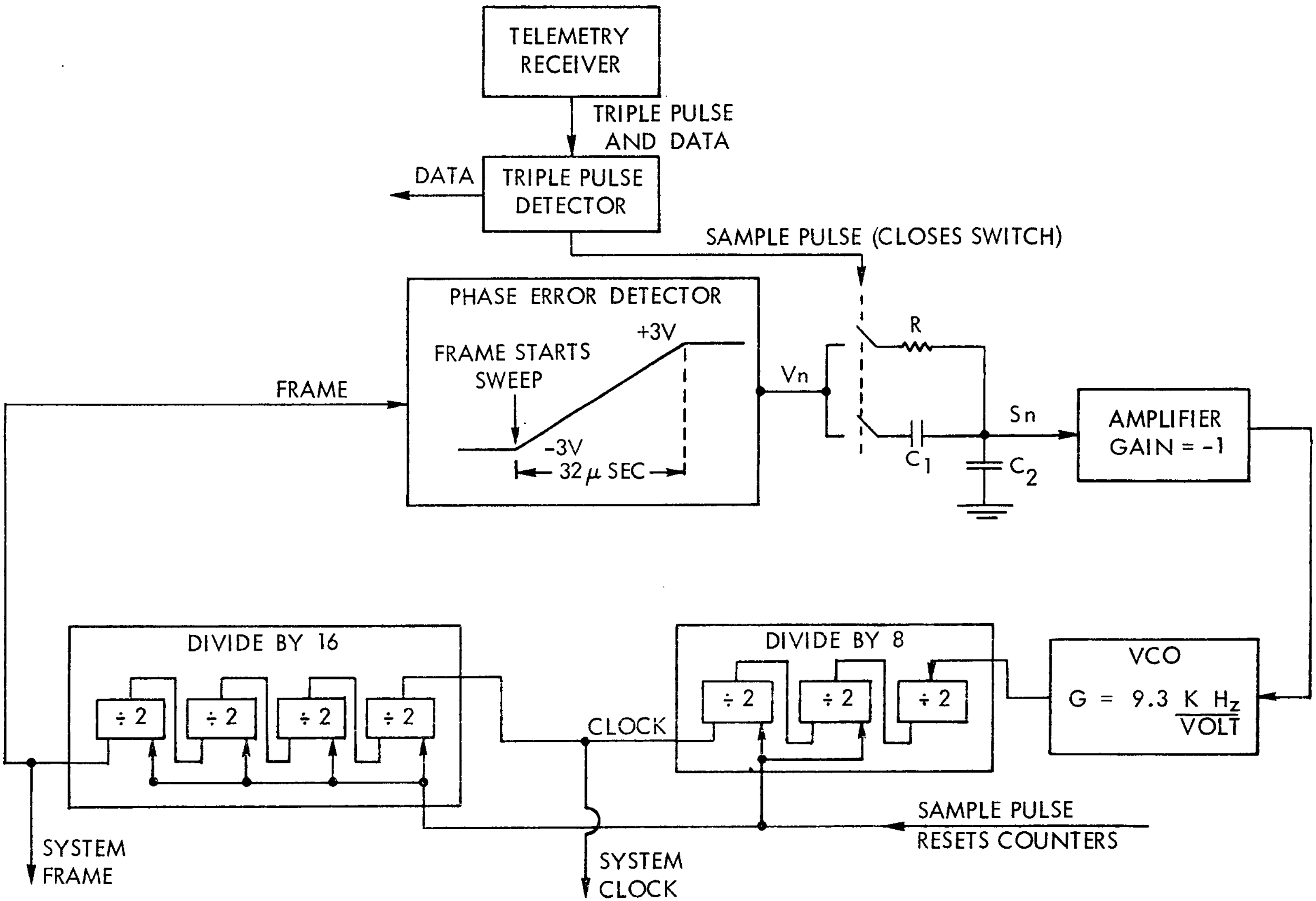


Figure 9. System Diagram

SECTION III

VERIFICATION OF TRANSIENT PERFORMANCE

In this section, transient performance characteristics of the Servo Clock will be verified. Calculations will be made to determine system response under two conditions, correcting for a frequency error and correcting for a phase error. Then measurements will be made and compared to the calculated results. The results will also be compared to Cases I and III. (See Tables 1 and 3.)

TRANSIENT PERFORMANCE WITH A FREQUENCY ERROR

Error Assumption and Calculation

The response of an ideal servo system to a frequency error of the airborne clock was examined in Case I. Response of the actual system to a frequency error will now be considered. First, a frequency error will be assumed, and the response will be calculated using the various parameters of the system; then, the same situation will be simulated with the physical system, and the results compared to those calculated.

Initially assume that $\theta_0 = 0$, $V_0 = 0$, $S_0 = 0$ and that the clock frequency of the servo is equal to that of the airborne unit, $T_{ss} = T_a = 1/5000$ second. At sample one assume that the airborne clock frequency increases by 20 hertz, from 5000 to 5020 hertz. At sample two a phase error will appear; it can be calculated from Equation 6.

$$\theta_2 = (\theta_1 = 0) + \left[16\left(\frac{1}{5020}\right) - 16\left(\frac{1}{5000}\right)\right]_1 \text{ seconds}$$

$$\theta_2 = -13 \text{ microseconds} .$$

From Equation 9,

$$V_2 = \frac{6}{32}\frac{V}{\mu s} \cdot (-13\mu s) = -2.44 \text{ volts} .$$

Equations 14 and 18a give V_{R^2} and V_{C^2} as,

$$V_{R^2} = V_2 K + V_{R^1} = KV_2 = (.0071) \times (-2.44)\,V = -17.3 \text{ millivolts}$$

$$V_{C^2} = V_{C^1} + \left(\frac{6}{32}\right) \cdot (.0071)\,(-13\mu s)\,\frac{V}{\mu s} = -17.3 \text{ millivolts} .$$

Then,

$$S_2 = V_{R^2} + V_{C^2} = -34.6 \text{ millivolts} .$$

From Equation 4, the new period of the servo is,

$$(T_2)_{ss} = T_0 + \frac{GS_n T_0^2}{8}$$

$$= \left[\frac{1}{5000} + \frac{9.3 \times 10^3 \times (-34.6) \times 4 \times 10^{-8}}{8}\right] \text{ seconds}$$

$$(T_2)_{ss} = 198.4 \text{ microseconds} .$$

At sample three the phase error is,

$$\theta_3 = -13 \times 10^{-6} + \left[16\left(\frac{1}{5020}\right) - 16\left(198.4 \times 10^{-6}\right)\right] \text{ seconds}$$

$$\theta_3 = (-13 + 13)\,\mu s = 0 \text{ microseconds} .$$

Then

$$V_3 = 0 \text{ volts} .$$

Equations 14 and 18a give V_{R3} and V_{C3} as,

$$V_{R3} = 0(K) + KV_{R2} ,$$

and

$$V_{C3} = V_{C2} + K(V_n - V_{n-1}) = V_{C2} - V_{C2} = 0 \text{ volts} .$$

The resultant voltage S_3 is,

$$S_3 = 17.3 \text{ millivolts} .$$

This value of S_3 is necessary to maintain frequency lock.

The value of $V_3 = 0$ is not valid under steady state conditions, as will be pointed out in the next section. The final value of V_n will equal S_3. As the system approaches steady state performance, the value of S_n will remain close to S_3 while V_n approaches S_3.

Physical Verification

Verification of the transient performance of the physical system in response to a frequency error was accomplished by the following test: The system was allowed to lock with a telemetry simulator with a clock frequency of 5000 hertz. At a particular sample, which may be designated sample one, the simulator clock frequency was changed to 5020 hertz. At sample two, the servo detected an error and reacted. To observe transient system performance, the voltage S_n was recorded on an oscillograph picture.

In the oscillograph picture in Figure 10, the voltage at sample two, S_2, goes negative approximately 35 millivolt or two units. At sample three, S_3 changes to -17 millivolt or one unit. At sample four, S_4 remains about one unit negative. When the above changes in the physical system are compared with those of the ideal system of Case I (Table 1), performance of the two systems is seen to be identical.

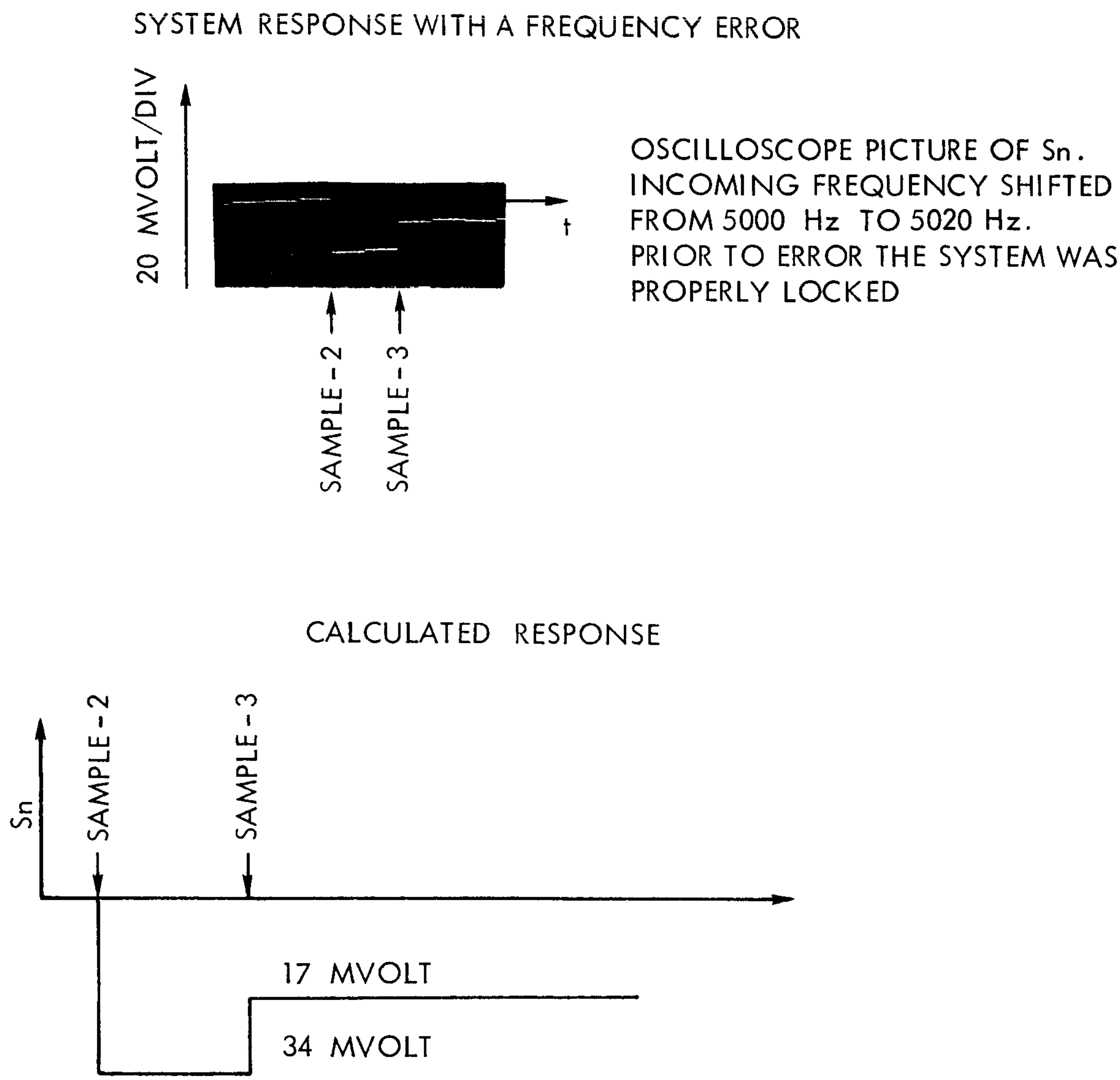


Figure 10. Transient Response with a Frequency Error

TRANSIENT PERFORMANCE WITH A PHASE ERROR

In contrast with the response of the ideal system, in which any phase error causes the same kind of correction, response of the actual system to a phase error must be considered in terms of the conditions of Equation 9. A small phase error, $\theta < T$, will be considered first, followed by the remaining two kinds of phase error both of magnitudes greater than T. In the first of these, the phase error, $\theta > T$, occurs later than the normal time of occurrence of the sampling interval, and, in the second, the error, $\theta < -T$, occurs in advance of it.

Assumption of a Phase Error, $\theta < T$

In this first example of phase error response a small error of 12 microseconds is assumed, the various values of circuit response are calculated, and the calculated values are then compared with values obtained from physical system tests.

Assume $\theta_0 = 0$, $V_0 = 0$, $S_0 = 0$ and the clock frequency of the servo is equal to that of the airborne unit,

$$T_{ss} = T_a = 200\mu s = \frac{1}{5kHz} .$$

Let $\theta_1 = 12$ microseconds. From Equation 9,

$$V_1 = M\theta = \frac{6V}{32\mu s} \times 12\mu s$$

$$V_1 = \frac{9}{4} \text{ volts} .$$

Equation 19 gives S_1,

$$S_1 = \frac{9}{4}\left[\frac{C_1}{C_2} + \frac{t_0}{RC_2}\right] = \frac{9}{4}\left[\frac{71 \times 10^{-12}}{1 \times 10^{-8}} + \frac{2 \times 10^{-6}}{2.8 \times 10^{4} \times 10^{-8}}\right] \text{ volts}$$

$$S_1 = 2.25(7.1 + 7.14) \times 10^{-3} = (2.25)(14.24) \times 10^{-3} \text{ volts}$$

$$S_1 = 32.2 \times 10^{-3} \text{ volts} .$$

Equation 4 gives the new period of the servo as

$$(T_1)_{ss} = T_0 + \frac{GS_n T_0^2}{8}$$

$$= \left[200 \times 10^{-6} + \frac{9.3 \times 10^3 \times 32.1 \times 10^{-3} \times 4 \times 10^{-8}}{8}\right] \text{ seconds}$$

$$= \left(200 \times 10^{-6} + 1.49 \times 10^{-6}\right) \text{ seconds}$$

$$(T_1)_{ss} = 201.5 \text{ microseconds} .$$

From Equation 6 the phase error at sample two is

$$\theta_2 = \left[\theta_1 + \left[16\left(200 \times 10^{-6}\right) - 16\left(201.5 \times 10^{-6}\right)\right]_1\right] \text{ seconds}$$

$$= (12 + 3200 - 3224) \text{ microseconds}$$

$$\theta_2 = -12 \text{ microseconds} .$$

$$V_2 = -9/4 \text{ volts} .$$

Equations 14 and 18a give $\underset{R^2}{V_2}$ and $\underset{C^2}{V_2}$ as,

$$\underset{R^2}{V_2} = V_2 K + \underset{R^1}{V_1}, \quad \text{where} \quad \underset{R^1}{V_1} = V_1 + 0 = (.0071)(2.25) = 16 \times 10^{-3} \text{ volts,}$$

$$\underset{R^2}{V_2} = -\frac{9}{4}(.0071)\,V + 16 \times 10^{-3} = (-16 + 16)\,\text{mV} = 0 \text{ volts} ,$$

$$V_{C^2} = V_{C^2} + MK[\theta_2 - \theta_1] = 16 \times 10^{-3} + MK[-12 - 12] \text{ volts}$$

$$= \left[16 \times 10^{-3} - 24 \frac{6}{32} (.0071)\right] = (16 - 32) \text{ millivolts}$$

$$V_{C^2} = -16 \text{ millivolts}$$

$$S_2 = -16 \text{ millivolts} .$$

$$(T_2)_{ss} = 200 \times 10^{-6} - \frac{9.3 \times 10^3 \times 16 \times 10^{-3} \times 4 \times 10^{-8}}{8} \text{ seconds}$$

$$(T_2)_{ss} = (199.3 \times 10^{-6}) \text{ seconds}$$

$$\theta_3 = (-12 + 3200 - 16(199.3)) \text{ microseconds}$$

$$= -12 + 3200 - 3188.2 \doteq 0 \text{ microseconds}$$

$$\theta_3 \doteq 0 \text{ microseconds}$$

$$V_3 = 0 \text{ volts}$$

$$S_3 = 0 \text{ volts}$$

$$(T_3)_{ss} = (200 \times 10^{-6}) \text{ seconds} .$$

Thus, at sample three the system has completely corrected for the phase error.

Verification of a Phase Error, $\theta < T$

Simulation of the above phase error was accomplished with the test setup shown in the block diagram at the top of Figure 11. A square-wave generator,

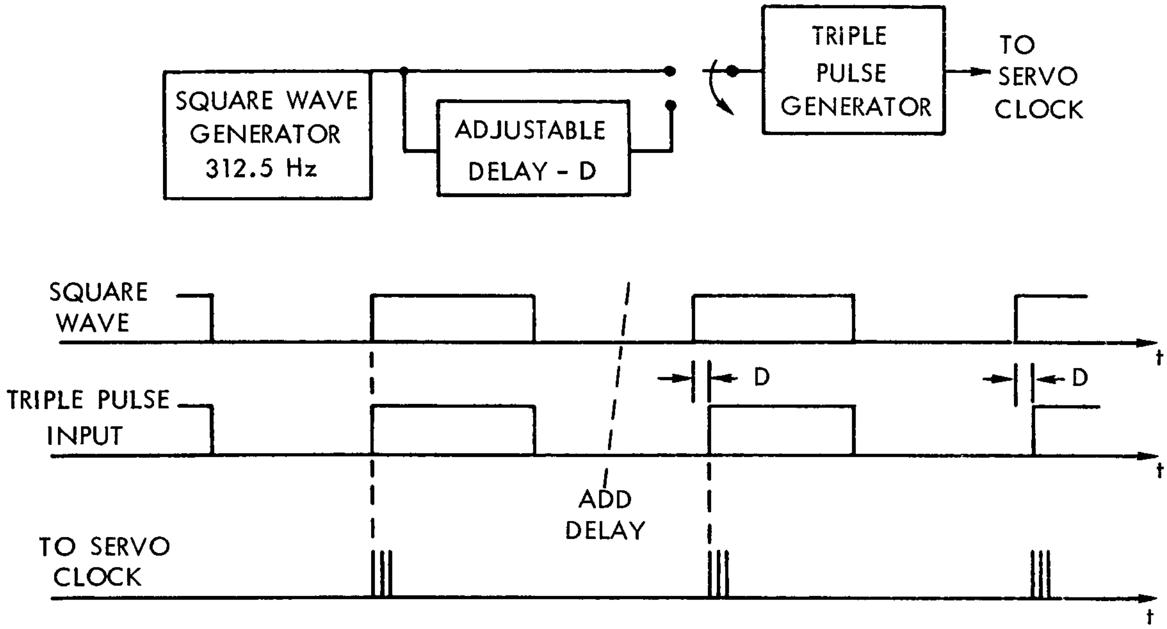


Figure 11. Phase Error Simulation

operating at the normal frame frequency of 312.5 hertz (5000 hertz/16 = 312.5 hertz), was used to drive a triple-pulse generator specially-constructed for the test to produce only the triple-pulse waveform (since the absence or presence of data pulses has no effect on the operation of the servo system). The output of the triple-pulse generator was applied directly to one position of a two-position switch, and through a 12-microsecond delay network, to the other position of the switch. The transfer connection of the switch was connected as the input to the Servo Clock. An oscillograph was then connected to record the voltage applied to the VCO. With this connection the oscillograph recorded variations of the voltage, S_n, before, during, and after adjustment to a phase error of θ = 12 microseconds, that is, before and after operation of the switch. The waveforms of Figure 11 illustrate these variations.

The resulting oscillograph waveform and waveforms depicting variations in the calculated values of S_n and V_n are presented in Figure 12. The oscillogram values of S_n are seen to be in close agreement with the calculated values. Similarity of performance between the physical system and the ideal system of Case III is evidenced by the fact that the recorded voltage S_n goes positive two units, then negative one unit, as does the ideal voltage S_n in Table 3.

Assumption and Verification of Phase Error Magnitudes, $\theta > T$

The second and third equations of Equation 9 deal with a phase error greater than T in magnitude. The second equation is for a phase error greater than T but

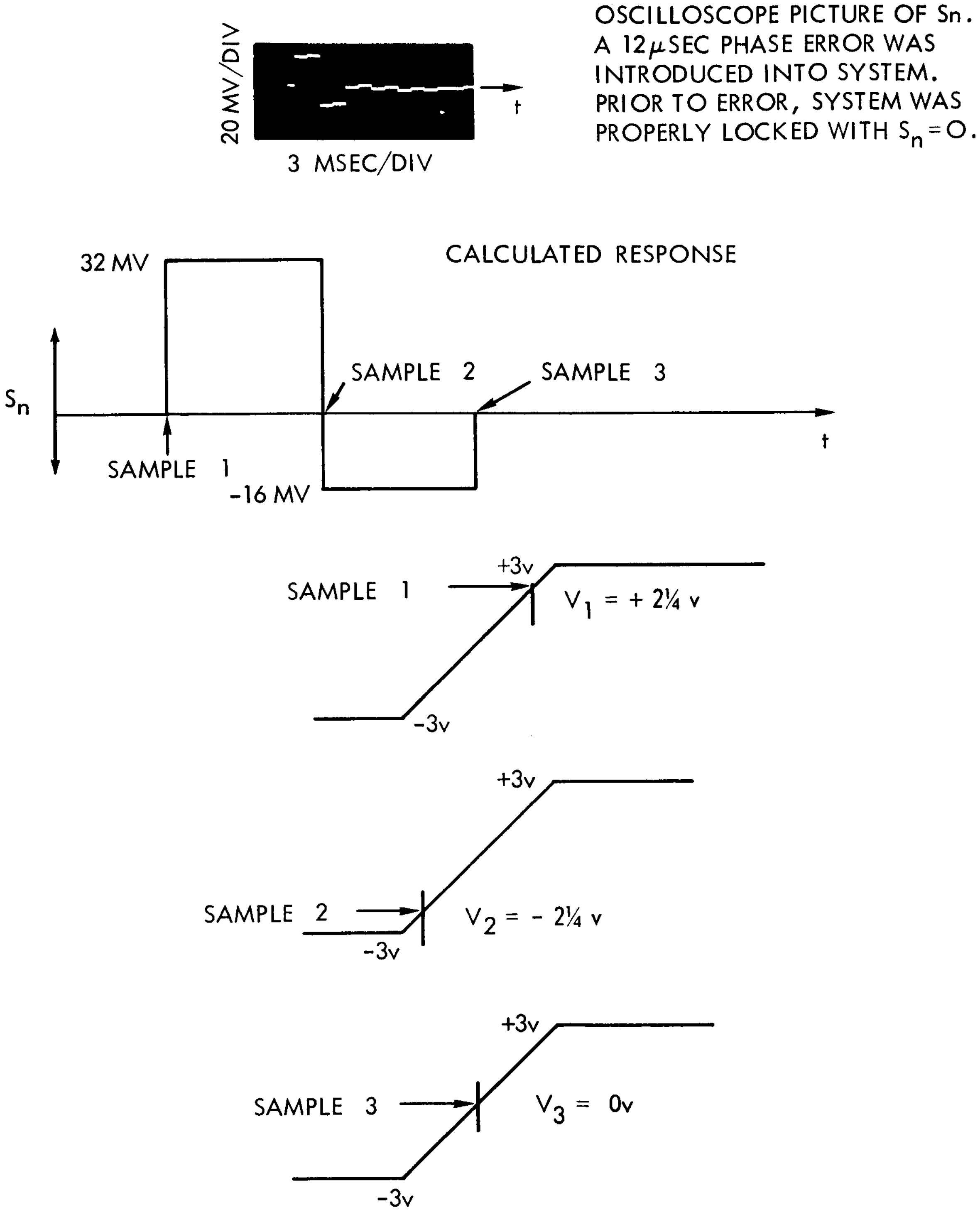


Figure 12. Response with 12 μSec Phase Error

less than half the frame period. The third equation is for a phase error less than minus T but greater than minus half of the frame period. The two conditions are illustrated in Figure 13.

First to be considered in detail is, $V_n = +3V$ for $\theta > T$, the first illustration in Figure 13. When sample two occurs, a value of +3 volts is read. This sample alone will have supplied a correction of only 32 microseconds when the next sample is taken. Note: In the previous example of a 12 microsecond phase error, the 9/4 volt sample resulted in a correction of 24 microseconds;

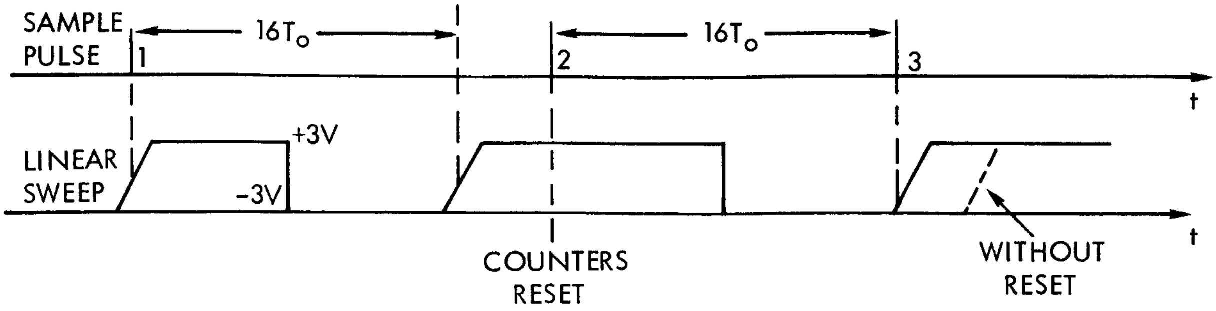
$$\frac{\frac{9}{4}V}{3V} = \frac{24\mu s}{x}, \qquad x = 32\mu s .$$

If the phase error is about one thousand microseconds, many samples would be needed before proper phase lock. However, when a sample is taken, all binary counters, except the first, are reset to the zero state, which reduces phase error to less than 50 microseconds, the normal period of the first counter. The phase shift due to the 3-volt sample will reduce phase error at the next sample by 32 microseconds, so that this sample occurs on the linear sweep. As verified by the previous example, only two more samples are required for complete phase lock. When the sampled value is +3 volt, a new sweep does not occur since the flip-flop driving the linear sweep generator is already in the reset condition.

The first photograph in Figure 14 is actual system operation when a +3 volt error is received. The top trace is that of the triple pulse. The bottom trace shows the linear sweep, going from -3 volts to +3 volts. In the frame before this picture was taken, the system was properly locked. A delay was introduced so the next triple pulse would be late, and the +3 volt sample was taken. At the next sample the system is again apparently in the locked condition (resolution is not great enough to show a phase error of up to 50 microseconds, which probably exists).

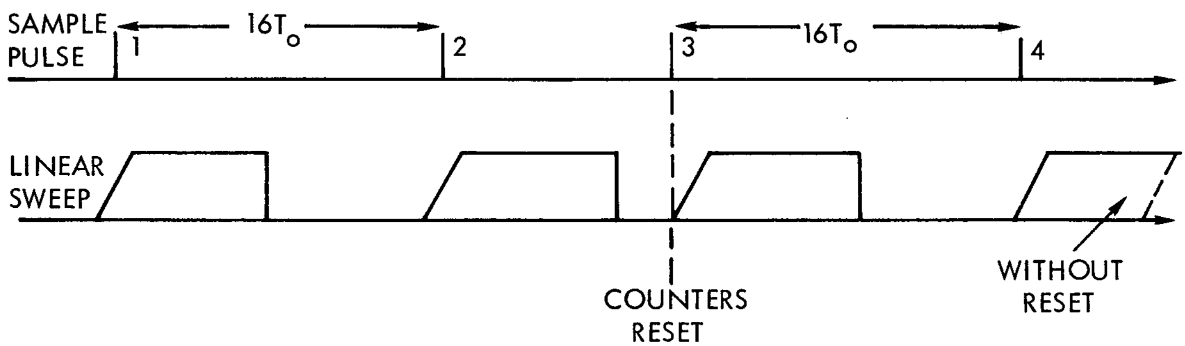
Next to be considered is a phase error resulting in a sample value of -3 volts, the last of Equation 9. This is the second illustration of Figure 13. Here, the sample pulse occurs early. All counters except the first are reset and the linear sweep occurs. When the next sample is taken, the previous reset plus the correction, due to the -3 volt sample, reduces the phase error to less than 18 microseconds. This sample occurs on the linear sweep. Only two more samples are required for complete phase lock.

PHASE ERROR $> \tau$ ($1600\ \mu s > \theta > \tau$)



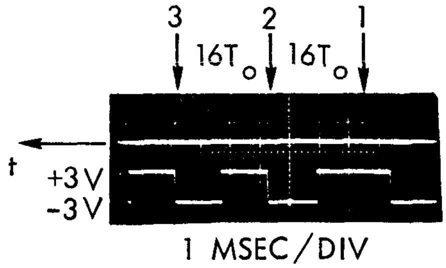
SAMPLE PULSE - 2 OCCURS TOO LATE, RESULTING IN AN ERROR VOLTAGE OF +3V. ALL COUNTERS EXCEPT FIRST ARE RESET; BY SAMPLE - 3 SYSTEM IS ALMOST IN SYNC. WITHOUT RESET, A LARGE PHASE ERROR WOULD STILL EXIST.

PHASE ERROR $< -\tau$ ($1600\ \mu s < \theta < -\tau$)

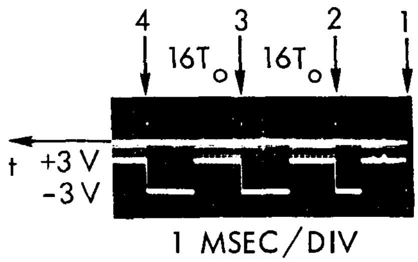


SAMPLE PULSE - 3 OCCURS TOO EARLY, RESULTING IN AN ERROR VOLTAGE OF -3V

Figure 13. Phase Error with Magnitude $> \tau$



SAMPLE - 1 OCCURS TOO LATE, RESULTING IN A +3V READING. BECAUSE OF RESET, BY SAMPLE - 2, SYSTEM HAS ALMOST ACHIEVED LOCK.



SAMPLE - 2 OCCURS TOO EARLY. A READING OF -3V IS TAKEN AND THE RESET OPERATION CAUSES SWEEP TO OCCUR. BY SAMPLE - 3 SYSTEM HAS ALMOST ACHIEVED LOCK.

Figure 14. Operation with a Phase Error of Magnitude $> \tau$

The second photograph of Figure 14 illustrates system operation. The top trace shows the triple pulses and the bottom trace is the linear sweep. At the first and preceding triple pulses, the system is properly locked. Triple pulse two occurs early, a sample of -3 volts is taken and the linear sweep occurs because of the reset action. At the third and fourth triple pulses, the system is approaching lock.

SECTION IV

STEADY STATE PERFORMANCE

The approximations used in the transient analysis portion of this paper were, in most cases, dependent upon sampling time being much less than the circuit time constants. Equation 12 uses, for example, the approximation that

$$\frac{t_0}{RC_2} = .0071 << 1 .$$

Consider this relationship after a number of samples. It is,

$$\frac{nt_0}{RC_2} << 1 .$$

After about fifteen samples this relationship is not valid. In order to study steady state performance, the original transfer function of the sampling network must be examined. It is,

$$H(s) = \frac{1 + RC_1 s}{1 + R(C_1 + C_2) s} .$$

The input is considered a step function, E_1/s. Solving for the output,

$$E(s) = \frac{E_1}{s} \left[\frac{1 + RC_1 s}{1 + R(C_1 + C_2) s}\right] .$$

Using the final value theorem,

$$E_2(t) = \lim_{s \to 0} sE_2(s) = E_1 .$$

In other words, if a non-zero value of S_n is required from the sampling network, a non-zero input, V_n, is required.

In terms of actual system operation, this means that under certain conditions there will be a steady state actuating error. In particular, when the ideal system model corrected for an error in frequency, the integrating network

$$\left(\sum KV_n\right)$$

provided the voltage necessary to achieve a permanent frequency change in the VCO. There was no steady state phase error since the required value of V_n was zero. In Table 1, note that at sample four V_4 is zero while S_4 is not zero. However, when the physical system corrects for a frequency error, a V_n other than zero will be required. In fact, the value of V_n required will be equal to the required value of S_n to achieve the necessary frequency change. To produce a non zero V_n, under steady state conditions, a phase error is required.

Case III is that of correcting for a phase shift between the airborne system and the servo system. Since no permanent frequency change resulted, it is entirely a transient problem. It has been shown that physical system operation is identical to ideal system operation in this case.

System performance in Case I under steady state conditions will now be considered in detail.

THEORETICAL STEADY STATE PERFORMANCE

In Case I of the ideal model, it was initially assumed that both the airborne and servo system had a clock frequency of $F_0 = 1/T_0$, and were in phase lock. The airborne clock changed frequency, and the servo followed, with the final result that no phase error resulted. If similar input conditions were applied to the actual servo system, the transient performance would be similar to the ideal. However, the steady state performance differs in a significant manner. In order for the physical system to correct for a clock frequency shift, a steady state phase actuating error results.

The ideal model had for an equation,

$$\frac{V_n}{R} = \sum^{n} KV_n \; .$$

This is essentially an integrating relationship which can produce a constant value for $\frac{V_n}{R}$ when the input, V_n, equals zero. If the value of its input samples are equal to V volts, the R - C network used to approximate this equation has a final value of V volts. This allows calculation of the steady state phase actuating error necessary to correct for a constant frequency error.

Suppose the airborne clock deviates from its normal value, f_0, by a value Δf. To produce steady state servo system clock frequency correction of Δf, a steady state correction voltage of S_{ss} must be applied to the VCO. The relation between Δf and the required input voltage is, $\Delta f = GS_{ss}/8$. Since, under steady state conditions, the output voltage of the R - C network equals the input, the voltage S_{ss} is equal to that sampled on the linear sweep; $S_{ss} = V_n$. The voltage sampled, V_n, is related to the phase error by Equation 9,

$$V_n = M\theta_n = \frac{3}{16}\,\theta \text{ volts/microsecond} \; .$$

Assume that the resulting phase error will be less than 16 microseconds (T = 16 μs). Solving for the resultant steady state phase error,

$$\Delta f = \frac{G}{8} \times M\theta_{ss}$$

$$\theta = \frac{8\Delta f}{GM} = \frac{8\Delta f}{9.3 \times 10^3} \times \left(\frac{16}{3}\right)$$

$$\theta = .0046 \text{ microseconds/hertz} \; .$$

For example, if under steady state conditions a 100 hertz difference exists between the free-running clock frequency of the servo system and the airborne clock, a steady state phase discrepancy of .46 microsecond will be present.

It is interesting to note that the transient and steady state performance are independent. That is, a VCO with a higher gain could be used in the system to improve steady state performance without influencing transient performance. The values for R, C_1, and C_2 would be changed to reflect the increased gain, thus maintaining transient performance.

VERIFICATION OF STEADY STATE PERFORMANCE

The following test was made to verify Case I steady state performance of the Servo Clock. A telemetry simulator with an adjustable clock frequency was used as the input for the servo system. With the Servo locked to the simulator, the simulator clock frequency was varied from 4800 hertz to 5112 hertz. Observation of the linear sweep in relation to the sample pulse gave the resulting system phase error, a total of 1.4 microseconds.

In more concise terms, for a Δf of 312 hertz, a θ_{ss} of 1.4 microseconds resulted. The θ_{ss} per cycle is,

$$\theta_{ss} = .0045 \text{ microseconds/hertz} .$$

On the basis of 100 hertz difference between the free-running clock frequency of the servo system and the airborne clock, the measured steady state phase error was .45 microseconds. This is comparable to the .46 microseconds value obtained through theoretical considerations.

SECTION V

CONCLUSIONS

In the Introduction it was stated that mathematical model of the Servo Clock would quickly regain lock after an input disturbance. Calculations and test results show that the physical system performs, under transient conditions, almost identically to the ideal system. Under steady state conditions, the phase actuating error is small, and could be reduced if high loop gain was introduced. With a higher loop gain, system parameters could be changed to maintain transient performance.

In conclusion, it can be said that the design objectives of an automatic system with fast response and low actuating phase error over a wide frequency have been met.

ACKNOWLEDGMENTS

The authors wish to express appreciation to H. W. Price of the University of Maryland for his assistance and guidance during the development of this paper.

SELECTED BIBLIOGRAPHY

1. M. Schwartz, Information Transmission, Modulation, and Noise, (McGraw-Hill, New York, 1959).

2. V. Del Toro and S. Parker, Principles of Control Systems Engineering, (McGraw-Hill, New York, 1960).

3. J. Millman and H. Taub, Pulse and Digital Circuits, (McGraw-Hill, New York, 1956).

4. D. Cheng, Analysis of Linear Systems, (Addison-Wesley, Reading, 1959).

5. F. Gardner, S. Kent and R. Dasenbrock, "Theory of Phase Lock Techniques," Contract No. NAS5-8225, Goddard Space Flight Center.