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NASA TN D-4302

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NASA TN D-4302

REDUCTION OF APPARENT-POWER REQUIREMENT OF PHASE-CONTROLLED PARASITICALLY LOADED TURBOALTERNATOR BY MULTIPLE PARASITIC LOADS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1968

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SUMMARY

Alternator output is improved when a phase-controlled parasitically loaded turboalternator has multiple sequentially actuated parasitic loads. Phase-controlled currents produce some unwanted effects. Included among these effects is an increase in the apparent-power (volt-ampere) requirement of the alternator. Phase control also produces strong harmonic components in the alternator current. The effects derive from the nonsinusoidal and reactive character of the parasitic load.

The results of the analysis give the alternator apparent power and the total harmonic content of the alternator current as a function of parasitic load.

One means of reducing the unwanted effects is to subdivide the parasitic load. The analysis shows this reduction as a function of the number of divisions of the parasitic load. For the model of the circuit analysis, an undivided parasitic load produces a total harmonic content of 32 percent in the alternator current when used with a basic 0.8 lagging power factor load. But when the parasitic load is divided into two loads, the total harmonic content is reduced to 18 percent. With the same basic 0.8 lagging power factor load, the apparent power with an undivided parasitic load is 7.6 percent greater than the apparent power without a parasitic load. However, when the parasitic load is divided, this difference is only 3 percent.

Additional division of the original parasitic load reduces the undesirable effects further. The amount of improvement at each division decreases appreciably with increasing number of loads, lessening the advantage obtained from load division.

INTRODUCTION

The use of static parasitic loading in order to maintain constant power loading of a turboalternator introduces undesirable effects into the electrical system. The chief undesirable effects are (1) the occurrence of nonsinusoidal currents, and (2) an increase in the alternator apparent-power (volt-ampere) requirement above that necessary to supply the maximum specified load. Despite these shortcomings, parasitic loading is being used for power control of advanced turboalternator systems (e.g., SNAP 2 and SNAP 8) for space auxiliary electric power application. Therefore, a better understanding of its performance is desirable. In these systems a variable electrical load (herein termed the "parasitic" load) is used in parallel with the vehicle electrical load (herein termed the "useful load"). As the demands of the vehicle useful load vary, sensing circuits provide for the adjustment of the parasitic load to complement the useful load, thereby keeping the total electrical load constant. (The plural term, parasitic loads, as used in this report, refers to the two or more loads which result from dividing the total parasitic load into multiple smaller parasitic loads.)

This report presents a steady-state analysis of one aspect of the circuitry involved in parasitic loading. In particular, it describes the manner in which the reactive loading of the alternator varies and the extent to which harmonics occur in the alternator current when parasitic load-compensation is phase-controlled.

An evaluation of the effects of phase-controlled parasitic loads is necessary for the proper design of turboalternator systems because apparent-power requirements determine alternator size and because nonsinusoidal currents contribute to the distortion of the generated voltage.

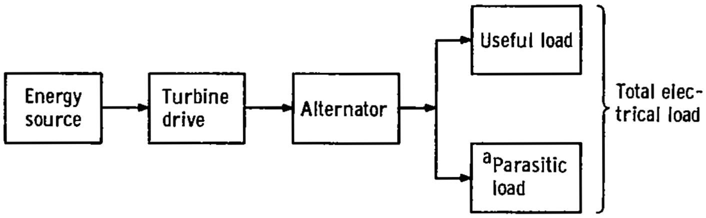
The analysis is presented for a single-phase circuit, but is also applicable to three-phase circuits if the electrical quantities are considered phase quantities.

MODEL OPERATION MODE

Purpose of Parasitic Loading

Figure 1 is a block diagram of a generating system typical of a parasitically loaded turboalternator system. The total load on the alternator has been divided into two electrical loads: a useful load which includes all the electrical equipments which are necessary for the performance of the mission to which the generating system is assigned, including components for the operation of the turboalternator; and a parasitic load which has been added only to control the total loading on the generating system.

When the turboalternator system is operating, the parasitic load must complement



[a]Includes only the load necessary to maintain constant power load.

Figure 1. - Turboalternator system with parasitic loading.

the useful load in such a manner that the combination of these loads is a constant power load on the generating system.

The loading system functions in the following manner: the amount of parasitic loading depends on the turbine speed (alternator electrical frequency). After the turboalternator and its load have attained a steady-state condition, any variation of the useful load changes the torque load on the turbine, which thereupon changes speed. This speed change is sensed by the parasitic-load control circuitry, and a compensating parasitic electrical load is applied to the alternator to produce an opposing speed change of the turbine. The sensitivity of the parasitic-load control circuitry is high enough so that the arrangement serves as a speed control.

With constant power load and constant speed, the driving torque of the turbine can be kept constant. As a consequence, the energy source and turbine controls are considerably simplified from what they would be if the turbine torque were varied to accommodate changing useful load.

Phase Control

The control of the parasitic loading described in this analysis is considered to be performed completely electrically (in contrast to the use of electromechanical components such as variable transformers or resistance potentiometers). The potential improvement in reliability provided by this mode of operation is especially valuable for space equipment. Electrical phase control of magnetic amplifier, controlled rectifier, or saturable reactor currents is the most convenient means of obtaining the necessary modulation of electrical power.

The term "phase control" describes the action whereby the current through a device such as a controlled rectifier can be started at any selected time (electrical angle) during a half cycle of the applied voltage. When phase-controlled devices are used back-to-back electrically, full-wave control is available. The electronic and magnetic devices used to effect phase control of the current in parasitic-loading systems permit

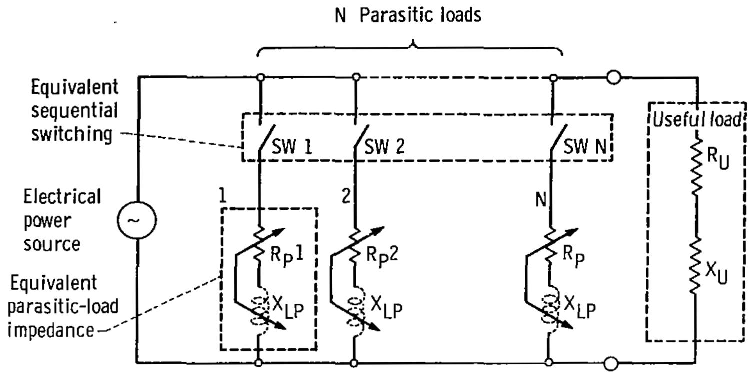


Figure 2. - Schematic diagram of ac voltage source with multiple parasitic loads.

totally electrical control of the parasitic-load currents, but in the process they generate nonsinusoidal periodic load currents.

Figure 2 is a schematic diagram of the model circuit which is analyzed in this report. The electrical power source is ideal and supplies power to two types of load: the useful load and the N parasitic loads. (Symbols are defined in appendix A.) The parasitic loads consist of N parallel, full-wave, phase-controlled loads, each with its own control circuitry. The control of the function performed by the indicated switches is in practice performed by frequency sensitive logic circuitry. The loading component is a resistor; however, it is shown as a resistance-inductance combination because (as is demonstrated in the analysis which follows) the effective impedance of the phase-controlled parasitic load has such a characteristic.

The current, and therefore the power, in each parasitic-load resistor varies continuously from fully off to fully on as the electrical frequency varies over the complete range for which that branch of the total parasitic load is sensitive. For this problem only one parasitic load (or branch) is phase-controlled at any given time. The frequency ranges of the N branches do not overlap. The loads are switched on and off sequentially. When, as a result of decreasing useful load, the frequency increases from the minimum value of its steady-state range, SW 1 closes, and parasitic load 1 turns on. When parasitic load 1 is fully on, switch SW 2 closes, and the power dissipated in parasitic load 2 increases from zero. When parasitic load 2 reaches the fully on condition, switch SW 3 closes, and the power in parasitic load 3 increases from zero. Although the useful and parasitic loads change, the total power of their combination remains constant. When all N parasitic loads are fully conducting, the power dissipated is equivalent to the rated load.

There is a similar action with an increasing useful load. When the power in parasitic load 2 decreases to zero as a result of decreasing frequency (increasing useful load), switch SW 2 opens, and the power in parasitic load 1, which has been fully on,

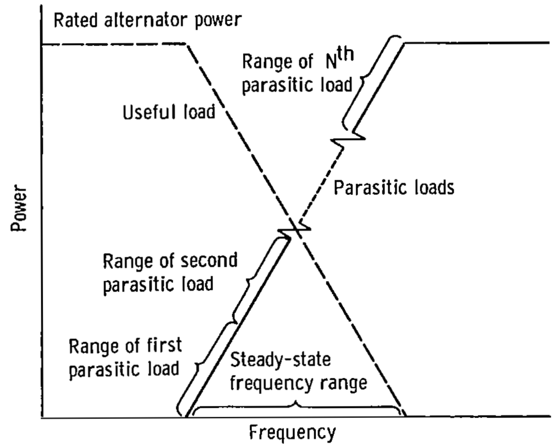


Figure 3. - Schematic diagram of ideal load distribution of parasitically loaded alternator for N parallel parasitic loads.

begins to decrease. The parasitic loads turn off sequentially as they are replaced by useful load.

The distribution of power between the useful load and the parasitic load is indicated diagramically in figure 3. This figure also illustrates how this parasitic load scheme serves as a frequency control. The frequency excursion from full useful load to zero useful load can be limited to any desired (and practical) range by design of the parasitic-load control.

Limitations of Analysis

The limiting conditions which were imposed on the model to simplify the analysis and the justification for their use are as follows:

(1) The power source is ideal: The internal impedance is zero, and the output is sinusoidal with a constant amplitude. Also, these characteristics do not change in the frequency range being considered.

The assumption that the internal impedance is zero is acceptable because the alternator apparent-power requirement is determined only by the external load. Consideration of internal impedance leads to a consideration of the harmonics generated in the alternator output voltage by the nonsinusoidal load currents. This condition would contradict the assumption of sinusoidal output voltage. The effect of harmonics in the alternator voltage is not a part of this study. The use of a constant voltage source is acceptable because virtually constant output voltage can be achieved with practical voltage regulators.

(2) It is assumed that the rise time of the phase-controlled current is zero.

This mode of operation is approached in practice with controlled rectifiers. The rise time of practical circuits is so fast that consideration of rise time makes no discernable change in the results reported herein.

(3) It is assumed that there is no transient current as the parasitic current pulses turn on and off.

The consideration of any ringing of the parasitic current complicates the analysis without significant results.

(4) The firing angle of the parasitic-load current is assumed to vary from 0° to 180°. There is no overlap or dead time between the frequency ranges of consecutively applied parasitic loads.

This much control of firing angle range of phase-controlled currents is not achieved in practice. Accounting for the circuit variations available to make up for the limitations of the firing angle is outside the scope of this analysis. Moreover, the significant results of this analysis are the calculated peak values of harmonic content and apparent power. These values are not affected by a shortened firing angle range.

(5) The useful load consists of linear resistances and reactances.

Such circuitry can be closely achieved in practice.

ANALYSIS AND DISCUSSION

Derivation of Analytical Expression for Total Harmonic Distortion

Analytical expression for total harmonic distortion. - The total harmonic distortion of the alternator current is defined as the ratio of the rms value of all the harmonic frequency components of the function to the rms value of the fundamental component of the function (ref. 1):

$$\text{Total harmonic distortion (THD)} = \frac{I_H}{I_{T1}} \tag{1}$$

The analysis evaluates equation (1) for a parasitically loaded alternator. The evaluation is in terms of two parameters: (1) the power factor of the useful load and (2) the number of parallel parasitic-load resistors.

In order to compute the harmonic content I_H, the rms (effective) value of the alternator current i_T must be determined. Since i_T is a nonsinusoidal periodic function, its effective value can be expressed in terms of the harmonic contents (ref. 2).

$$I_T = \left(I_{T0}^2 + I_{T1}^2 + \sum_{m=2}^{\infty} I_{Tm}^2 \right)^{1/2} \tag{2}$$

where I_{T0} represents the value of the dc component, I_{T1} the effective value of the fundamental, and I_{Tm} the effective value of the m^{th} harmonic.

The dc component (average value) of the alternator current is zero. Therefore,

$$I_T = \left(I_{T1}^2 + \sum_{m=2}^{\infty} I_{Tm}^2 \right)^{1/2} \tag{3}$$

The square root of the second term within the parentheses of equation (3) is I_H the root mean square of all the harmonic currents included in the function i_T.

$$I_H = \left(\sum_{m=2}^{\infty} I_{Tm}^2 \right)^{1/2} = \left(I_T^2 - I_{T1}^2 \right)^{1/2} \tag{4}$$

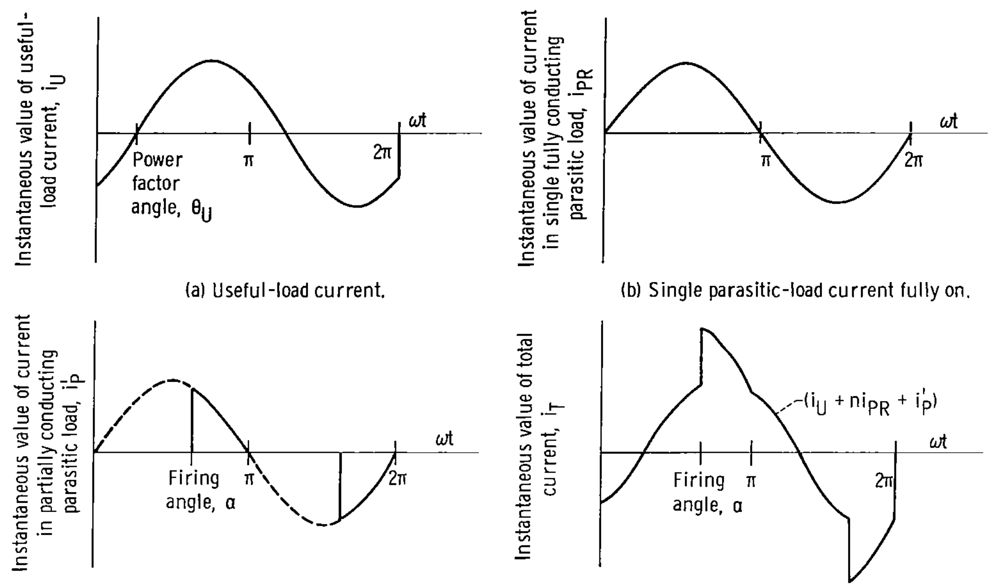
Substituting this expression for I_H into equation (1) gives, as the value of total harmonic distortion,

$$\mathrm{THD} = \frac{\left(I_T^2 - I_{T1}^2 \right)^{1/2}}{I_{T1}} \tag{5}$$

Effective value of total current. - To obtain analytical expressions for I_T and I_{T1}, the instantaneous values i_T and i_{T1} are needed.

The total alternator current i_T is the sum of the useful-load current i_U, and the total parasitic-load current i_P. The parasitic-load current is further divided into two components: ni_{PR}, the current through n parasitic-load resistances which are fully on and i_P', the current through the one parasitic-load resistance which is partially on. Thus

$$i_T = i_U + ni_{PR} + i_P' \tag{6}$$



(a) Useful-load current.

(b) Single parasitic-load current fully on.

(c) Single parasitic-load current partially on.

(d) Total load current.

Figure 4. - Load currents for alternator with phase controlled parasitic load (not to scale).

These currents are indicated in figure 4.

The first component of equation (6), the useful-load current i_U (fig. 4(a)), varies sinusoidally because the useful-load impedance is assumed to consist of linear resistances and reactances.

$$i_U = \sqrt{2}\, I_U \sin(\omega t - \theta_U) \tag{7}$$

where θ_U is the useful-load power factor angle.

The second component is proportional to the current through each of the n fully on parasitic-load resistors (fig. 4(b)). This current also is sinusoidal and is given as

$$i_{PR} = \sqrt{2}\, I_{PR} \sin \omega t \tag{8}$$

However, the third component, i'_P, the current through the partially turned on parasitic-load resistance, is nonsinusoidal (fig. 4(c)). It is described by the equation

$$
\left.\begin{aligned}
i'_P &= 0 \Bigg|_{\frac{\alpha}{\omega},\ \frac{\alpha+\pi}{\omega}} \Bigg|_{0,\ \frac{\pi}{\omega}} \\
&= \sqrt{2}\ I_{PR} \sin \omega t \Bigg|_{\frac{\pi}{\omega},\ \frac{2\pi}{\omega}} \Bigg|_{\frac{\alpha}{\omega},\ \frac{\alpha+\pi}{\omega}}
\end{aligned}\right\} \tag{9}
$$

where α is the firing angle of the phase-controlled current. All the harmonics in the total current are introduced by i'_P.

Effective value of total current fundamental. - The fundamental component i_{T1} of the total current is also composed of three signals:

$$
i_{T1} = i_U + ni_{PR} + i'_{P1} \tag{10}
$$

The first two signals, i_U and ni_{PR}, have been defined by equations (7) and (8). The third signal, i'_{P1}, is the fundamental sinusoidal component of the current in the partially turned on parasitic-load resistor.

Fourier analysis (appendix B) of the wave form described by equation (9) yields the following expression for i'_{P1}:

$$
i'_{P1} = \sqrt{2}\ I'_{P1} \sin\left(\omega t + \tan^{-1} \frac{\cos 2\alpha - 1}{2\pi - 2\alpha + \sin 2\alpha}\right) \tag{11}
$$

where

$$
I'_{P1} = I_{PR}\left[\left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right)^2 + \left(\frac{\cos 2\alpha - 1}{2\pi}\right)^2\right]^{1/2} \tag{12}
$$

Setting

$$
A = \frac{\cos 2\alpha - 1}{2\pi} \tag{13}
$$

$$
B = 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \tag{14}
$$

and substituting these variables in equations (11) and (12) makes

$$i'_{P1} = \sqrt{2}\ I_{PR}(A^2 + B^2)^{1/2} \sin\left(\omega t + \tan^{-1}\frac{A}{B}\right) \tag{15}$$

and

$$I'_{P1} = I_{PR}(A^2 + B^2)^{1/2} \tag{16}$$

The effective values of the currents i_T and i_{T1} of equations (6) and (10), required for evaluation of the total harmonic distortion, are obtained by integration.

$$I_T = \left[\frac{1}{2\pi}\int_0^{2\pi} i_T^2\ d(\omega t)\right]^{1/2}$$

$$= \left\{I_{PR}^2(n^2 + 2nB + B) + I_U^2 + 2I_{PR}I_U\left[(n + B)\cos\theta_U - A\sin\theta_U\right]\right\}^{1/2} \tag{17}$$

and

$$I_{T1} = \left[\frac{1}{2\pi}\int_0^{2\pi} i_{T1}^2\ d(\omega t)\right]^{1/2}$$

$$= \left\{I_{PR}^2\left[(n + B)^2 + A^2\right] + I_U^2 + 2I_{PR}I_U\left[(n + B)\cos\theta_U - A\sin\theta_U\right]\right\}^{1/2} \tag{18}$$

It is more meaningful to express I_T and I_{T1} as functions only of the alternator loading (proportion of parasitic power) and the parameters N and $\cos\theta_U$. To do this it is necessary to eliminate from the expressions for I_T and I_{T1} the variables A, B, and n. These variables can be expressed as functions of parasitic load; however, the functions are transcendental, and direct substitution of these functions for A, B, and n is not convenient. The approach which was used to obtain usable mathematical expressions for I_T and I_{T1} is described in the following sections.

Eliminating load currents. - The explicit variables I_U and I_{PR} can be eliminated from equations (17) and (18).

$$I_U = \frac{P_U}{E_G \cos \theta_U} \tag{19}$$

and

$$I_{PR} = \frac{E_G}{R_P} \tag{20}$$

where P_U is the useful-load power, E_G is the alternator terminal voltage, and R_P is the value of the individual parasitic-load resistor.

The total power provided by the alternator, P_G (the power quantity which is kept constant), is dissipated as a combination of useful-load power P_U and parasitic-load power P_P.

$$P_G = P_U + P_P \tag{21}$$

Substituting for P_U in equation (19) yields

$$I_U = \frac{P_G - P_P}{E_G \cos \theta_U} \tag{22}$$

When the useful load is zero, so that the total generator power is dissipated in parasitic load, all parasitic-load resistors are in parallel, and the equivalent load resistance is R_P/N. Then

$$P_G = P_P = \frac{E_G^2}{\frac{R_P}{N}} \tag{23}$$

From equations (20) and (23),

$$I_{PR} = \frac{P_G}{NE_G} \tag{24}$$

Equations (22) and (24), when substituted in equations (17) and (18), give

$$
I_T = \left\{ \left(\frac{P_G}{NE_G} \right)^2 (n^2 + 2nB + B) + \left(\frac{P_G - P_P}{E_G \cos \theta_U} \right)^2 \right.
$$

$$
\left. + \frac{2P_G(P_G - P_P)}{NE_G^2 \cos \theta_U} \left[(n + B) \cos \theta_U - A \sin \theta_U \right] \right\}^{1/2} \tag{25}
$$

$$
I_{T1} = \left\{ \left(\frac{P_G}{NE_G} \right)^2 \left[(n + B)^2 + A^2 \right] + \left(\frac{P_G - P_P}{E_G \cos \theta_U} \right)^2 \right.
$$

$$
\left. + \frac{2P_G(P_G - P_P)}{NE_G^2 \cos \theta_U} \left[(n + B) \cos \theta_U - A \sin \theta_U \right] \right\}^{1/2} \tag{26}
$$

Per unit quantities. - At this point the expressions for I_T and I_{T1} are converted to a per unit system. More general and convenient values are obtained when the electrical quantities are expressed in a per unit system (ref. 3). The following table presents base quantities used herein:

Actual quantity	Base quantity
Power	Rated alternator power, P_G
Voltage	Rated voltage source voltage, E_G
Apparent power	Rated alternator power, P_G

The subscript pu is used to indicate the per unit variables.

Because E_G and P_G are fixed quantities in this analysis, the corresponding per unit values are fixed at one.

The equations for I_T and I_{T1} in per unit form become

$$I_{Tpu} = \left\{ \frac{1}{N^2} (n^2 + 2nB + B) + \left(\frac{1 - P_{Ppu}}{\cos \theta_U} \right)^2 + \frac{2(1 - P_{Ppu})}{N \cos \theta_U} \left[(n + B) \cos \theta_U - A \sin \theta_U \right] \right\}^{1/2} \tag{27}$$

$$I_{T1pu} = \left\{ \frac{1}{N^2} \left[(n + B)^2 + A^2 \right] + \left(\frac{1 - P_{Ppu}}{\cos \theta_U} \right)^2 + \frac{2(1 - P_{Ppu})}{N \cos \theta_U} \left[(n + B) \cos \theta_U - A \sin \theta_U \right] \right\}^{1/2} \tag{28}$$

Elimination of n. - In order to eliminate n from equations (27) and (28) an expression for n as a function of P_{Ppu} is determined as follows.

Because average power results only from voltage and current components of the same frequency, the average power dissipated in the partially turned on parasitic-load resistor is given by the expression

$$P'_P = \frac{1}{2\pi} \int_0^{2\pi} e_G i'_{P1} \, d(\omega t) \tag{29}$$

where e_G is the terminal voltage of the alternator.

$$e_G = \sqrt{2} \, E_G \sin \omega t \tag{30}$$

Completion of the integration indicated in equation (29) yields

$$P'_P = E_G I_{PR} B \tag{31}$$

The power in the total parasitic load appears in n parasitic loads which are completely conducting plus one partially turned on load.

$$P_P = nE_G I_{PR} + E_G I_{PR} B$$

$$= (n + B)E_G I_{PR}$$

$$= \frac{(n + B)}{N} P_G \tag{32}$$

or

$$n = \frac{NP_P}{P_G} - B \tag{33}$$

In per unit terms

$$n = NP_{Ppu} - B \tag{34}$$

where n is an integer, and $0 \le B \le 1$. Examination of equation (34) shows that n must be the largest integer contained in the product NP_{Ppu}. This "largest included integer" is represented by the bracketed quantity $[NP_{Ppu}]$.

When n is eliminated from equations (27) and (28), these equations become

$$I_{Tpu} = \left\{ \frac{1}{N^2} \left([NP_{Ppu}]^2 + 2[NP_{Ppu}]B + B \right) + \left(\frac{1 - P_{Ppu}}{\cos \theta_U} \right)^2 + \frac{2(1 - P_{Ppu})}{N \cos \theta_U} (NP_{Ppu} \cos \theta_U - A \sin \theta_U) \right\}^{1/2} \tag{35}$$

$$I_{T1pu} = \left\{ \frac{1}{N^2} \left(N^2 P_{Ppu}^2 + A^2 \right) + \left(\frac{1 - P_{Ppu}}{\cos \theta_U} \right)^2 + \frac{2(1 - P_{Ppu})}{N \cos \theta_U} (NP_{Ppu} \cos \theta_U - A \sin \theta_U) \right\}^{1/2} \tag{36}$$

Calculation of total current and total current fundamental. - Values for A and B obtained from equations (37) and (38) were used to compute the magnitudes of I_{Tpu} and I_{T1pu}.

$$A = \frac{\cos 2\alpha - 1}{2\pi} \tag{37}$$

$$B = NP_{Ppu} - [NP_{Ppu}] = 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \tag{38}$$

Equation (38) was solved first for α for selected values of P_{Ppu}. The corresponding values of A and B then were calculated. This process permitted the elimination of A and B from equations (35) and (36). These equations were then evaluated at the selected values of P_{Ppu} with N and $\cos\theta_U$ as parameters.

The calculations for this report were performed by computer. As an aid to making these calculations without the help of an electronic computer, graphs of A and B as functions of α have been included as figure 5.

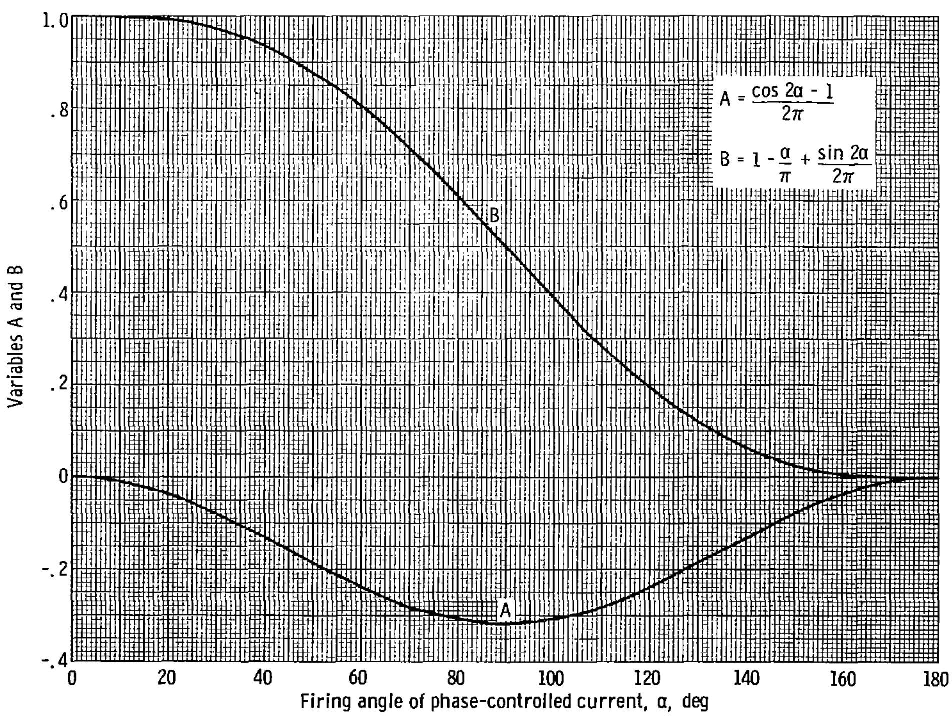


Figure 5. - Variation of A and B with firing angle.

The calculated values of I_{Tpu} and I_{T1pu} were used to determine the total harmonic distortion which, in per unit terms, becomes

$$THD = \frac{\left(I_{Tpu}^2 - I_{T1pu}^2\right)^{1/2}}{I_{T1pu}} \tag{39}$$

DISCUSSION OF TOTAL HARMONIC DISTORTION

The values of THD as a function of P_{Ppu} have been plotted in figures 6 to 8 for illustrative values of the parameters N and $\cos \theta_U$.

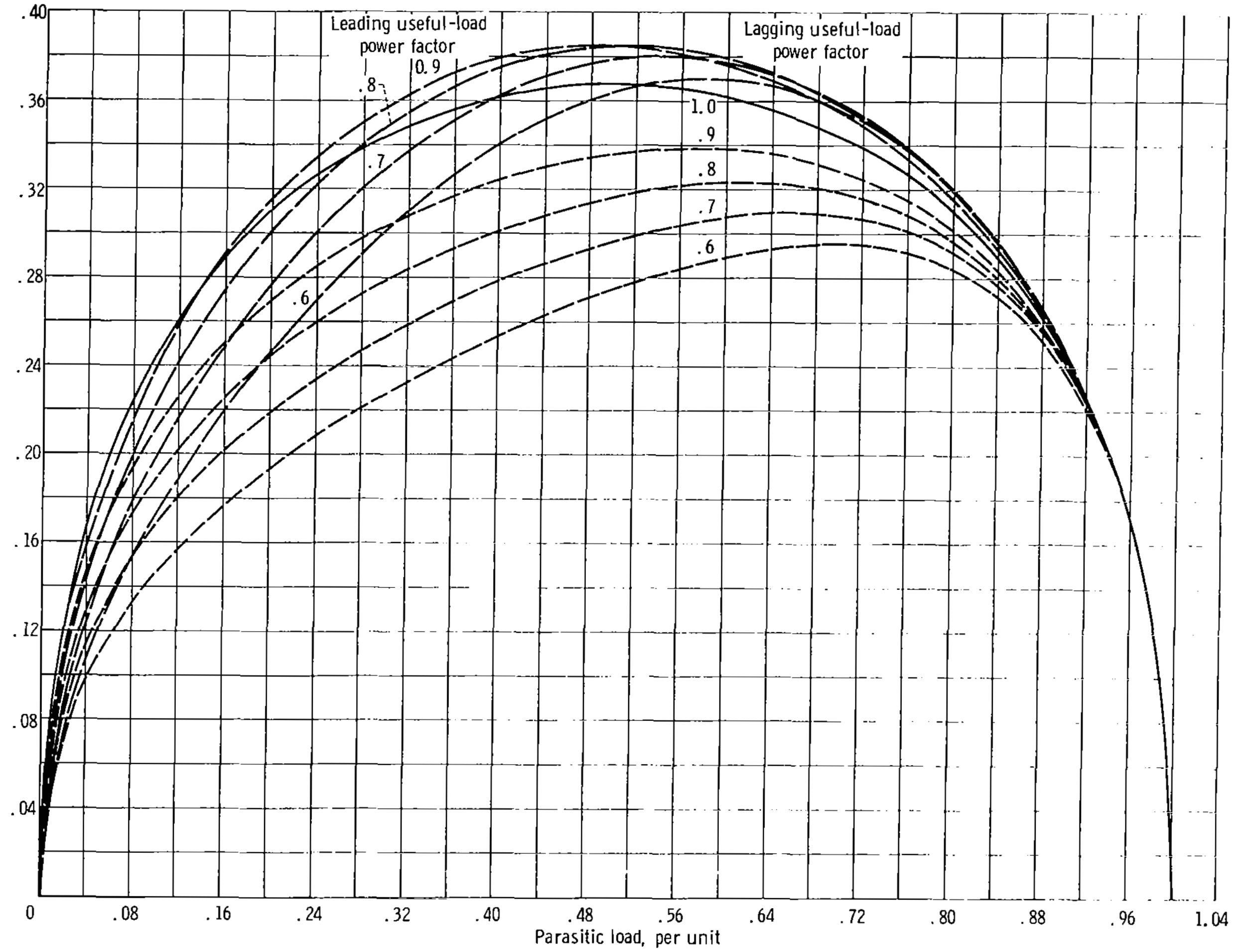
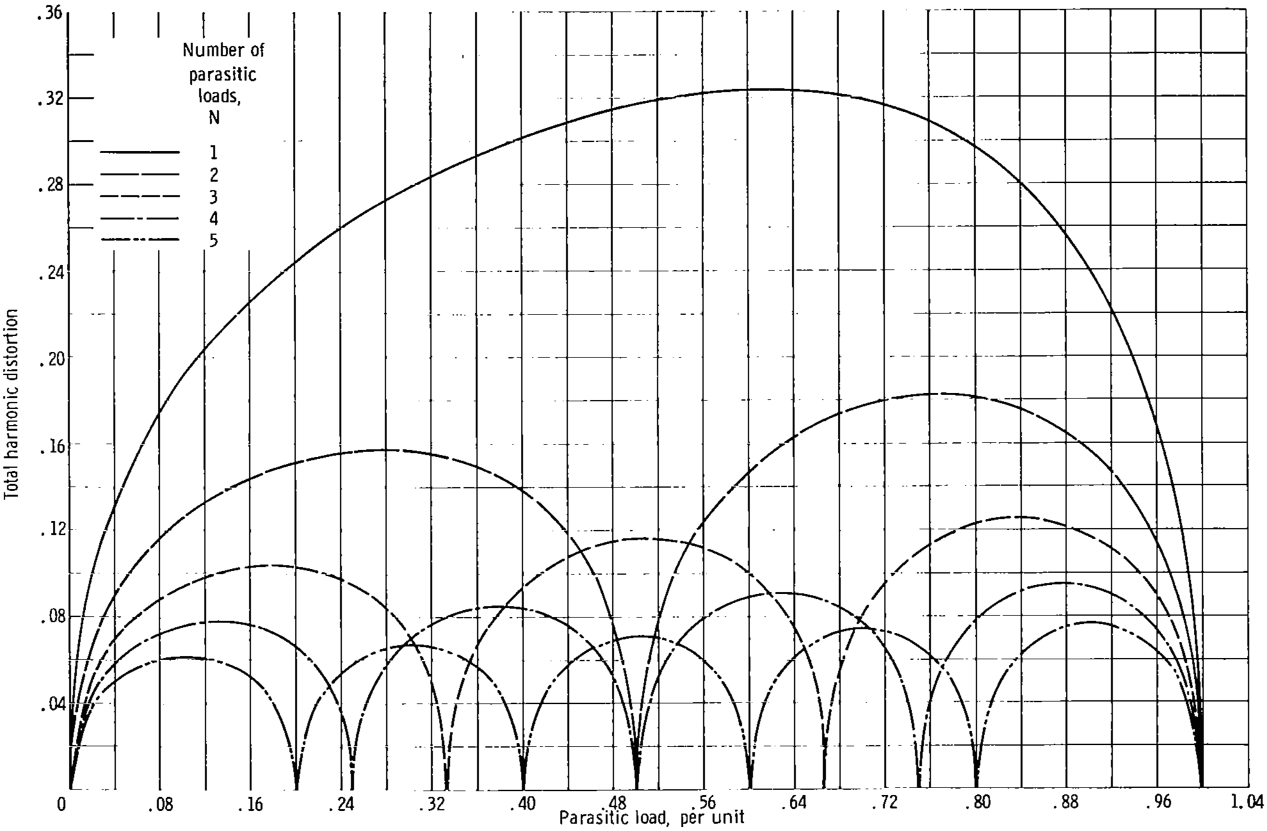
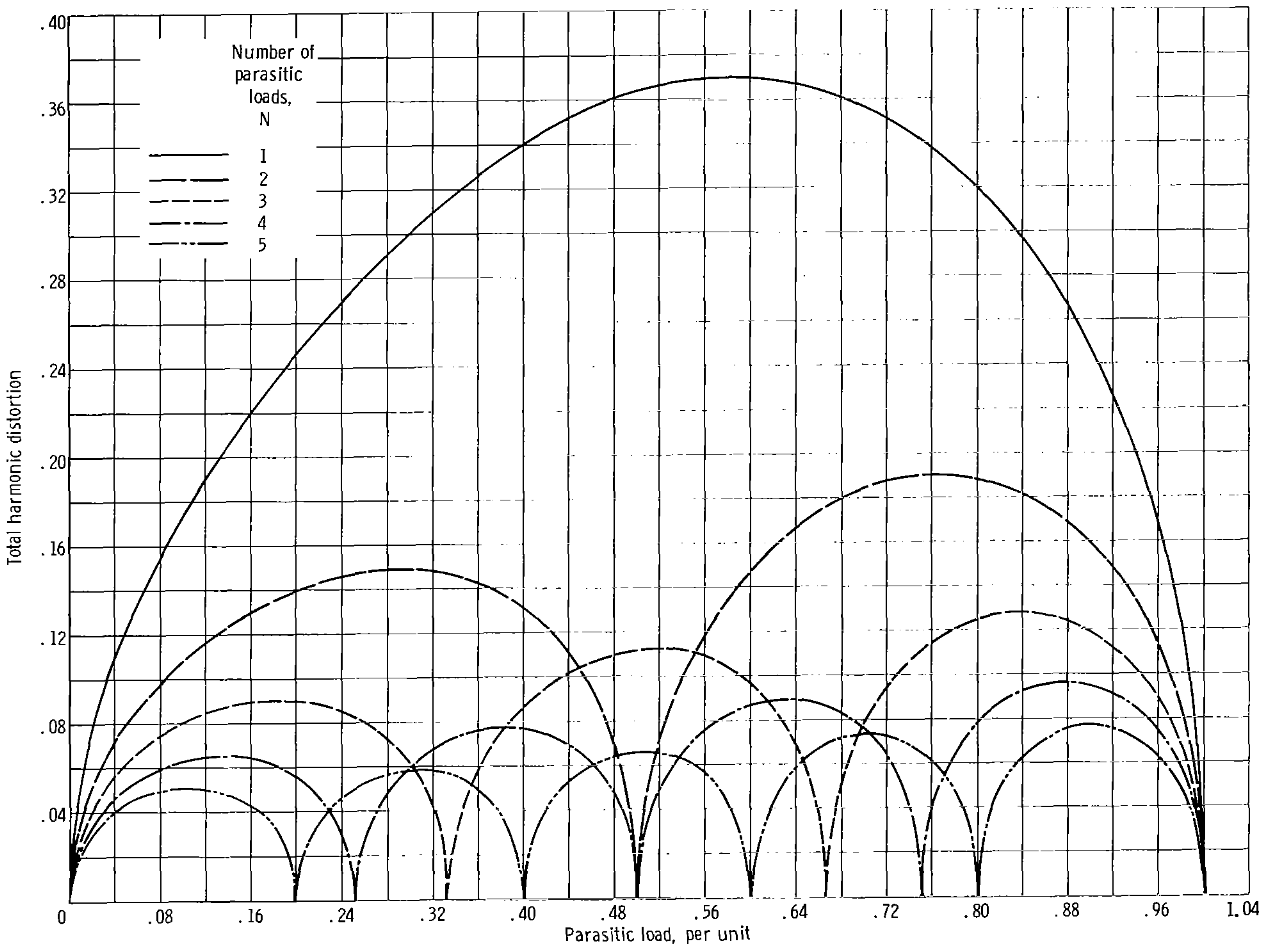


Figure 6. - Alternator current harmonic distortion for single parasitic load.



(a) Useful load power factor, 0.8 lagging.

Figure 7. - Alternator current harmonic distortion for N parallel parasitic loads.



(b) Useful load power factor, 0.6 leading.

Figure 7. - Concluded.

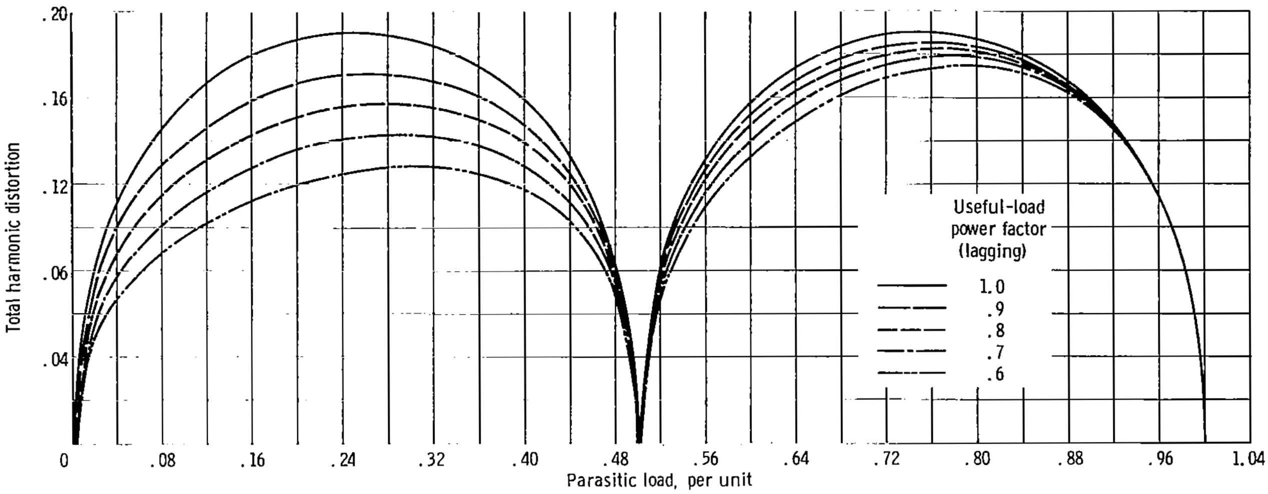


Figure 8. - Alternator current harmonic distortion for two parallel parasitic loads.

Two features of the variation of THD with parasitic loading observable in the plots presented in figures 6 to 8 are as follows:

(1) The peak THD is a function of the useful-load power factor.

(2) The peak THD is reduced by subdividing the total parasitic load into multiple phase-controlled parallel parasitic loads.

In figure 6 the total harmonic content of the alternator current is plotted against the per unit magnitude of the parasitic load with $N = 1$ and with the power factor of the useful load as parameter. As this power factor varies, the peak distortion is maximized. For the configuration of a single parasitic load ($N = 1$) the peak (worst case) value of THD occurs with a useful load of 0.84 leading power factor (not shown in figure). Under these conditions the peak THD is 39 percent. The peak is reduced only to 30 percent for a 0.6 lagging power factor useful load. These figures show that the current distortion is appreciable over the range of practical power factor loads.

Figure 7 illustrates the effect on the harmonic content of using more than one parasitic loading unit. Figure 7(a) shows the effect for a representative lagging power factor useful load, and figure 7(b) for a leading power factor useful load. In each case as the number of parallel loads increases, the maximum distortion is reduced. The greatest reduction occurs with the addition of a second parasitic-load resistor.

Figure 8 shows the variation with power factor of total harmonic distortion of alternator current when there are two parallel parasitic loads. Distortion varies with useful-load power factor in the same way as in the case of a single parasitic load shown in fig-

TABLE I. - MAXIMUM TOTAL HARMONIC DISTORTION OF ALTERNATOR CURRENT FOR SELECTED VALUES OF USEFUL-LOAD POWER FACTOR AND NUMBER OF PHASE-CONTROLLED PARASITIC LOADS

Useful-load power factor	Number of parasitic loads, N				
	1	2	3	4	5
Lagging					
0.6	0.2954	0.1747	0.1224	0.0937	0.0757
.7	.3097	.1790	.1240	.0944	.0761
.8	.3236	.1825	.1253	.0950	.0764
.9	.3388	.1858	.1263	.0954	.0766
1.0	0.3674	0.1903	0.1277	0.0960	0.0770
Leading					
0.9	0.3845	0.1926	0.1284	0.0963	0.0771
[a].8435	.3856	.1927	.1284	.0963	.0771
.8	.3850	.1926	.1284	.0963	.0771
.7	.3802	.1920	.1282	.0962	.0771
.6	.3703	.1902	.1277	.0960	.0769

[a]Worst case.

ure 6. In order to limit figure 8 to a range of useful-load power factors of primary interest, the variation of THD with only lagging power factor loads is depicted. The peak distortion occurs with a 0.84 leading power factor load. With this load the THD rises to 19 percent.

Table I summarizes the evaluation of the peak total harmonic distortion of alternator current for selected ranges of useful-load power factor and of multiple parallel parasitic loads.

Derivation of Expression for Apparent Power

The parasitic load appears to the power source (the alternator) to be an inductive load. The inductive character is shown by the lagging phase angle between the reference voltage e_G and the fundamental component of the parasitic-load current i_{P1}. This current is the sum of the n fully on parasitic-load currents and the fundamental component of the one partially on parasitic load.

$$i_{P1} = i'_{P1} + n i_{PR} \tag{40}$$

Substitution of equations (8) and (10) into equation (40) gives

$$i_{P1} = \sqrt{2}\, n I_{PR} \sin \omega t + \sqrt{2}\, I_{PR}\left(A^2 + B^2\right)^{1/2} \sin\left(\omega t + \tan^{-1} \frac{A}{B}\right)$$

$$= \sqrt{2}\, I_{PR}\left[A^2 + (B + n)^2\right] \sin\left(\omega t + \tan^{-1} \frac{A}{B + n}\right) \tag{41}$$

The phase angle $\tan^{-1} A/(B + n)$ is negative for all values of α, so that i_{P1} always lags e_G.

Because the parasitic load is, in effect, a reactive load, it imposes an additional apparent-power (volt-ampere) capacity requirement on the source.

Also the harmonic components of the current increase the necessary apparent-power capacity. This increase is manifest when the volt-ampere requirement is recognized as being equal to the product of the effective value of the source voltage and the effective value of the total current (ref. 2).

$$VA_G = E_G I_T \tag{42}$$

Equation (4) specifies that $I_T > I_{T1}$ when harmonics are present in I_T. Therefore, for a given value of I_{T1}, the apparent power must be greater when harmonics are present. In per unit terms

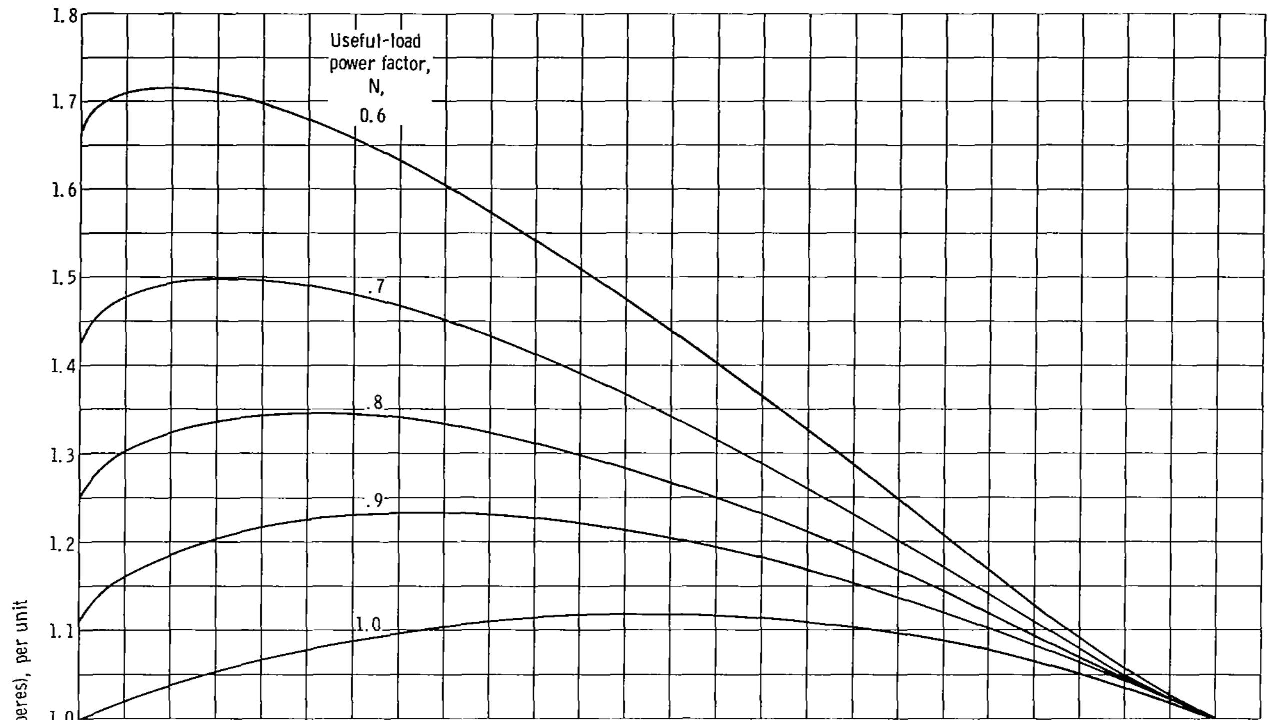
$$VA_{Gpu} = I_{Tpu} \tag{43}$$

and the electrical power source apparent-power requirement is equivalent in magnitude to the effective value of the total current.

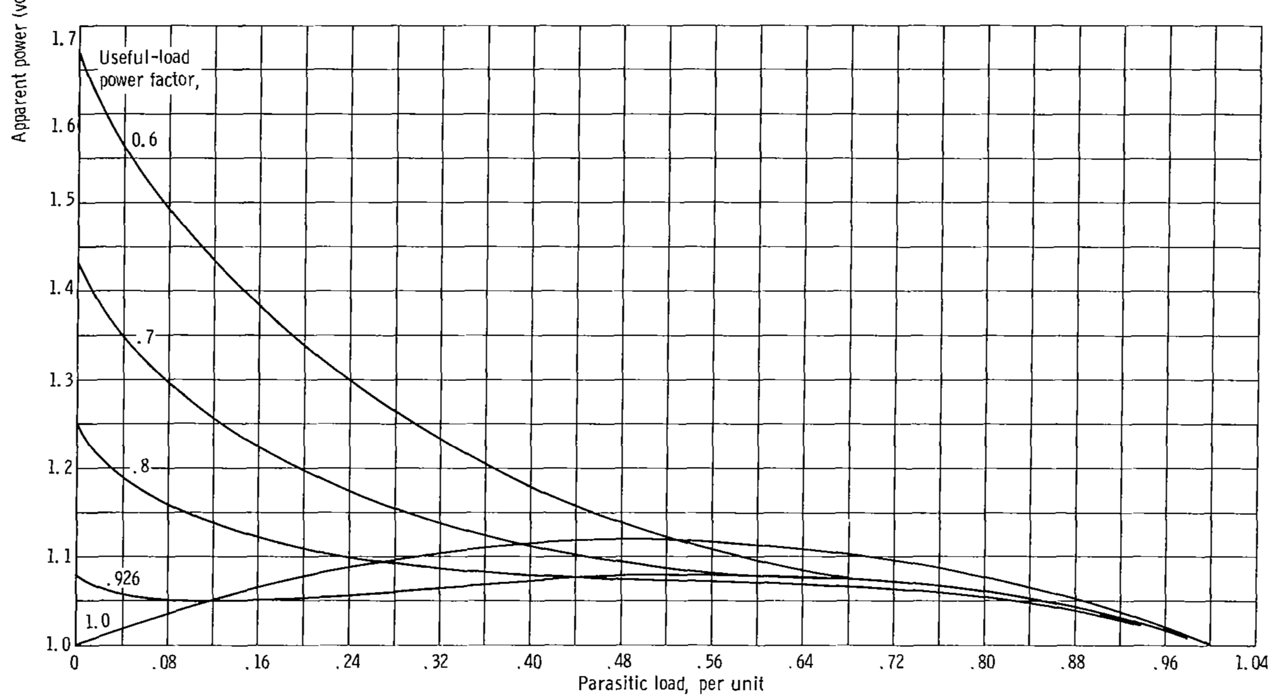
Discussion of Apparent-Power Requirements

Figures 9 to 12 are plots of apparent power (volt-amperes) against parasitic load with useful-load power factor and number of parallel parasitic loads as parameters. Examination of the plots of apparent power will indicate the significant features that

(1) the peak apparent power increases as the useful-load power factor decreases from one, lagging or leading



(a) Lagging power factor useful load.



(b) Leading power factor useful load.

Figure 9. - Alternator apparent power required with single parasitic load.

(2) there is a reduction in the apparent-power requirement when the total parasitic load is subdivided into multiple parasitic loads.

Point (1) is illustrated in figure 9(a) for a system with a lagging power factor useful load and a single parasitic load. As the useful-load power factor decreases, the maximum apparent-power requirement for the alternator increases.

Figure 9(b) shows the corresponding behavior of the apparent power for leading power factor useful loads: As the useful-load power factor decreases, the maximum apparent-power requirement for the alternator increases. An exception to point (1), however, occurs for values of useful-load power factor between 0.926 and 1.0. For power factors less than 0.926, the peak apparent-power point is the zero parasitic-load point. Between power factors of 0.926 and 1.0, the peak apparent power increases rather than decreases with increasing power factor. This performance indicates that, at a useful-load power factor of 0.926, a loading condition is reached at which the inductive character of the parasitic load equals the capacitive character of the useful load.

In every case with a lagging power factor, as shown in figure 9(a), the peak apparent power is greater than would be required for the useful load without the parasitic load. In fact, the variation of the apparent-power requirement of the parasitically loaded alternator relative to the apparent-power requirement of an alternator which will supply the power to the same useful load without a parasitic load is more relevant than the apparent-power requirement alone. This relative rating is the ratio

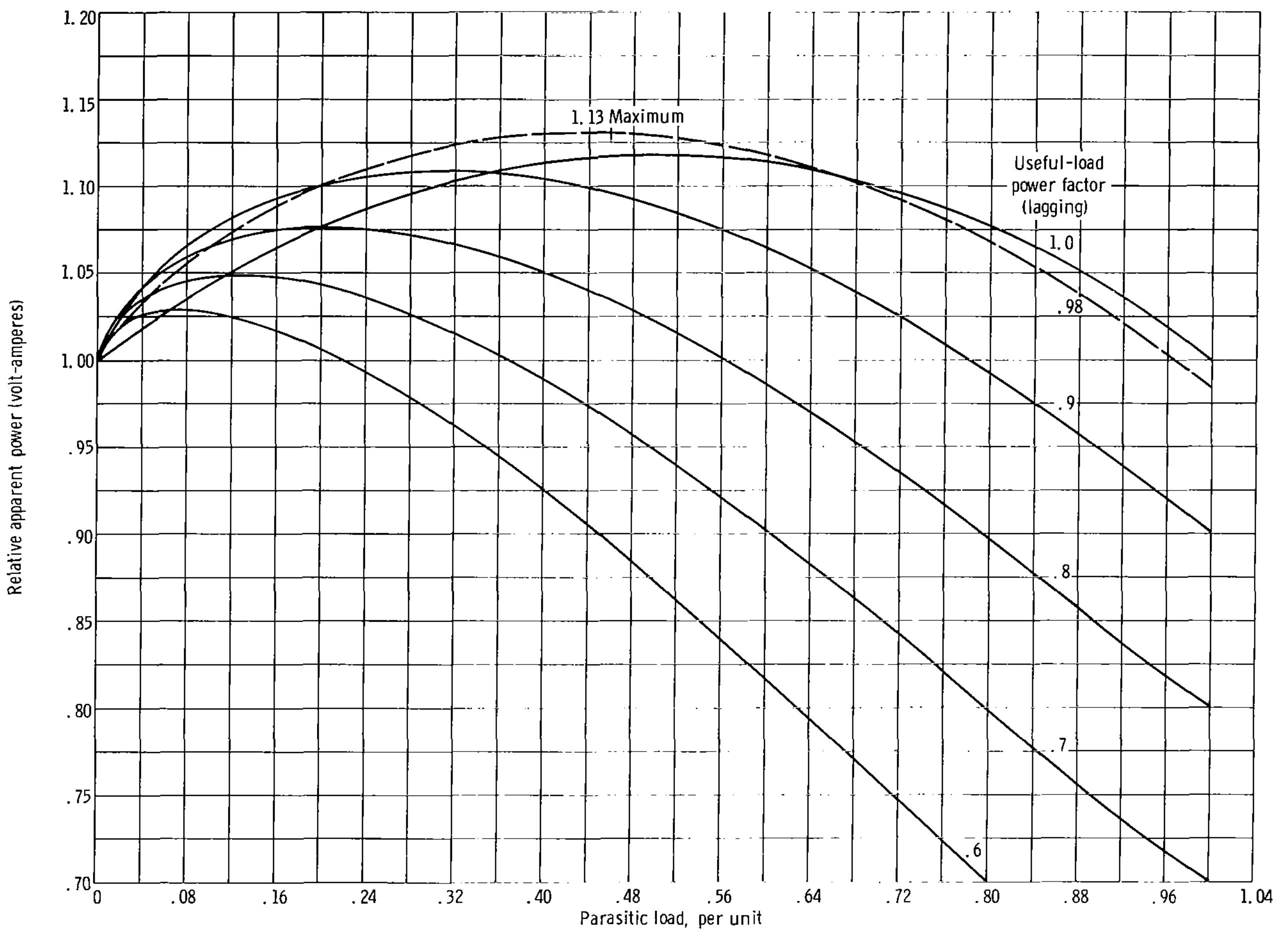
$$\text{Relative rating} = \frac{\text{Apparent-power requirement with parasitic load}}{\text{Apparent-power requirement without parasitic load}} \tag{44}$$

Relative ratings greater than one represent conditions for which the use of the parasitic load increases the apparent-power rating required of the alternator.

Figure 10(a) shows these relative values for the same loads as figure 9(a). A maximum ratio of 1.13 is reached when the power factor of the useful load is 0.98 lagging. This means that a 13 percent increase in apparent-power capacity is necessary when this useful load is used with a single parasitic load.

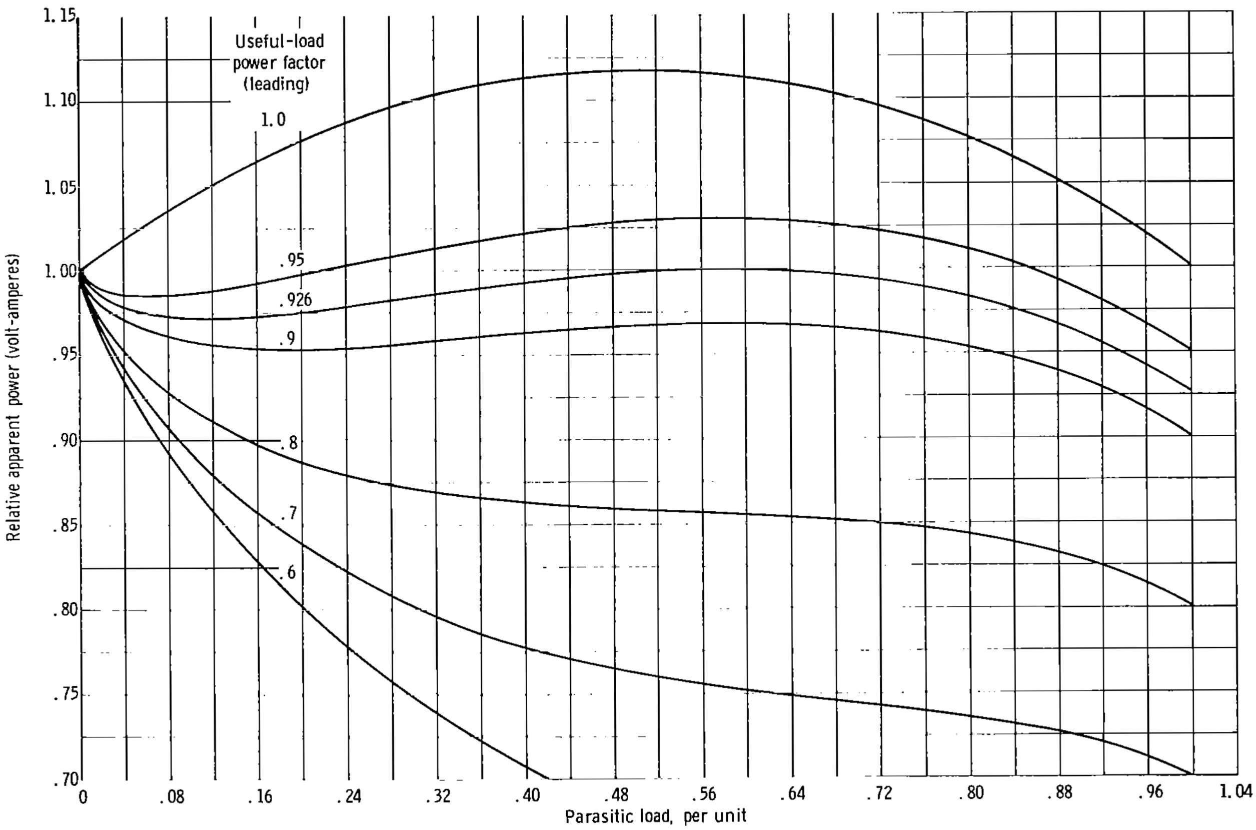
In the case of leading power factor useful loads, the maximum apparent-power requirement is equal to the requirement for the useful load without a parasitic load for all values of power factor less than 0.926. Figure 10(b) shows this behavior. For all useful loads with a leading power factor greater than 0.926 the maximum relative ratio exceeds one, indicating that in these cases a parasitic load increases the volt-ampere requirements of the alternator.

A similar variation of relative apparent power occurs with multiple parallel parasitic loads. Figure 11 illustrates this variation for $N = 2$. With two loads the peak ratio has decreased to 1.04 and occurs when the useful-load power factor is 0.96 lagging.



(a) Lagging power factor useful load.

Figure 10. - Alternator apparent power required with single parasitic load relative to alternator rating without parasitic load.



(b) Leading power factor useful load.

Figure 10. - Concluded.

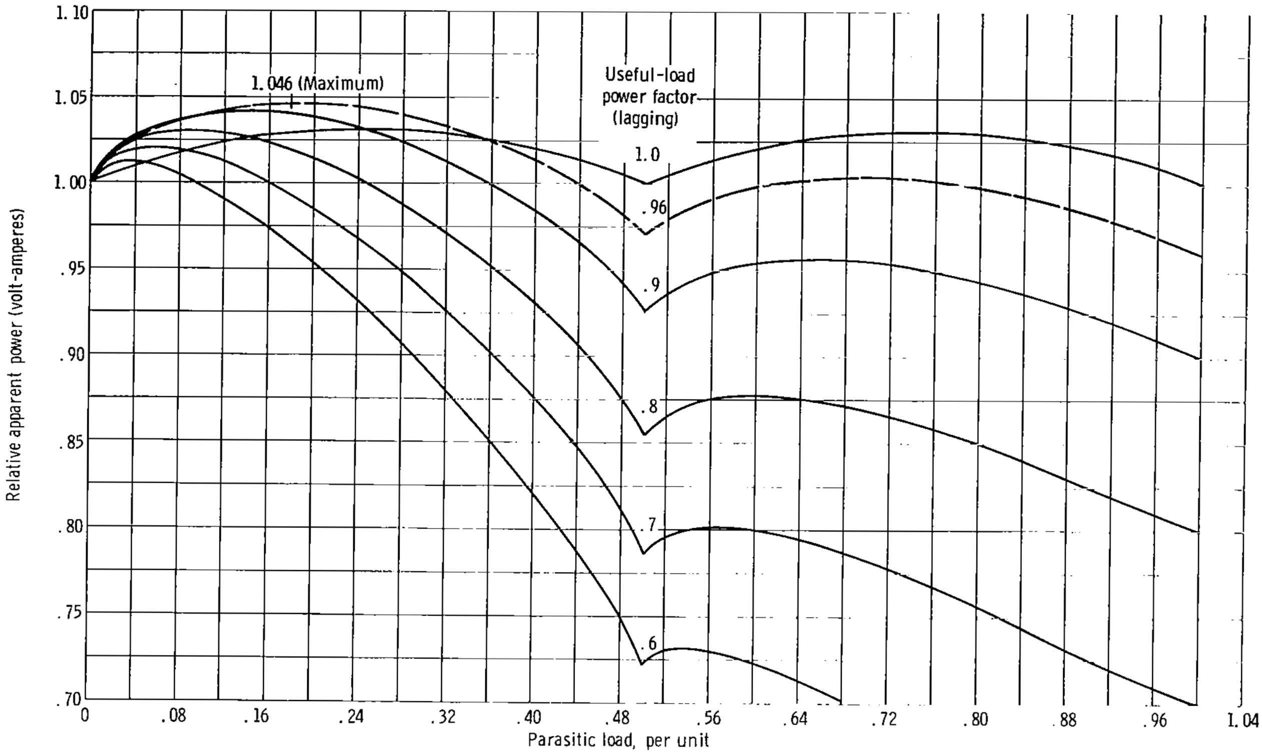


Figure 11. - Alternator apparent power required with two parallel parasitic loads relative to alternator rating without parasitic load.

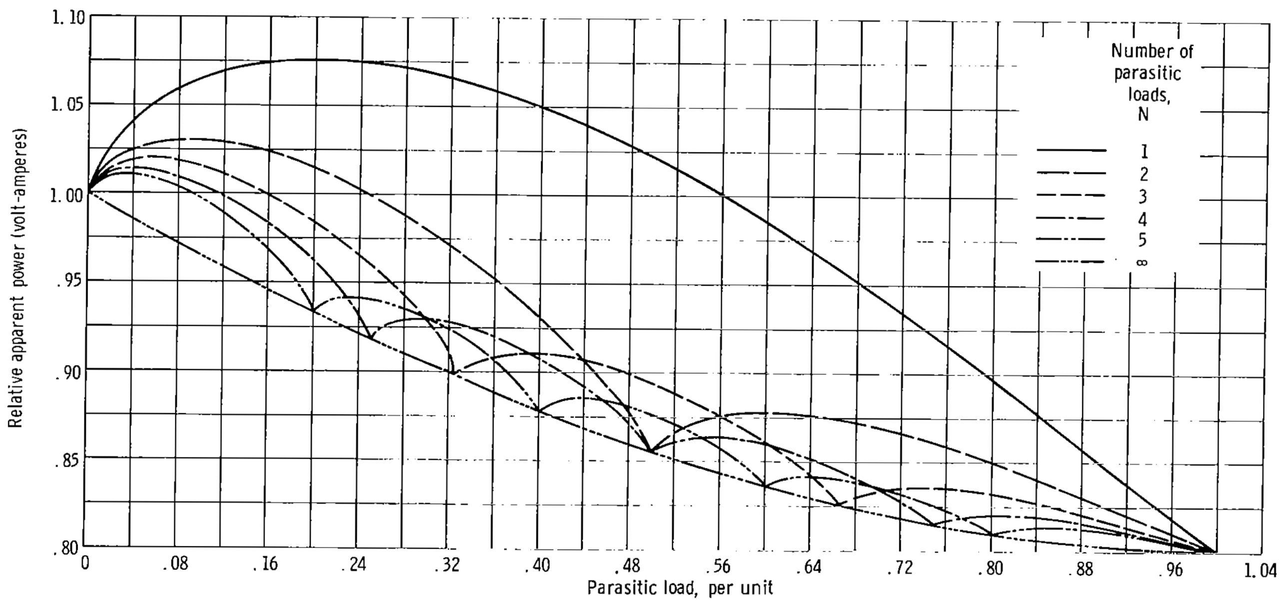


Figure 12. - Alternator apparent power required with N parallel parasitic loads relative to alternator rating without parasitic load. 0.8 lagging power factor useful load.

TABLE II. - MINIMUM APPARENT POWER REQUIRED OF ALTERNATOR WITH PHASE-CONTROLLED PARASITIC LOADS[a]

Useful-load power factor	Number of parasitic loads, N					
		1	2	3	4	5
Lagging						
0.6	1.6667	1.7150	1.6871	1.6800	1.6759	1.6730
.7	1.4286	1.4985	1.4575	1.4465	1.4416	1.4389
.8	1.2500	1.3452	1.2880	1.2733	1.2668	1.2627
.9	1.1111	1.2324	1.1576	1.1387	1.1306	1.1259
1.0	1.0000	1.1181	1.0307	1.0138	1.0078	1.0050
Leading						
0.9	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111
.8	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500
.7	1.4286	1.4286	1.4286	1.4286	1.4286	1.4286
.6	1.6667	1.6667	1.6667	1.6667	1.6667	1.6667

[a]Apparent power normalized to alternator rated power.

TABLE III. - MINIMUM APPARENT POWER REQUIRED OF ALTERNATOR WITH PHASE-CONTROLLED PARASITIC LOADS RELATIVE TO RATING OF ALTERNATOR WITHOUT PARASITIC LOAD

$$\left[\text{Relative rating} = \frac{\text{Apparent-power requirement with parasitic load}}{\text{Apparent-power requirement without parasitic load}}.\right]$$

Useful-load power factor lagging	Number of parasitic loads, N				
	1	2	3	4	5
1.0	1.1181	1.0307	1.0138	1.0078	1.0050
.9837	[a]1.1308	1.0108	1.0102	1.0009	.9968
.9600	1.1007	[a]1.0460	1.0229	1.0083	.9962
.9550	1.0813	1.0426	[a]1.0265	1.0168	1.0099
.9450	1.0646	1.0369	1.0253	[a]1.0184	1.0136
.9350	1.0547	1.0327	1.0234	1.0179	[a]1.0141
.9	1.1091	1.0418	1.0248	1.0175	1.0133
.8	1.0762	1.0304	1.0186	1.0134	1.0101
.7	1.0489	1.0203	1.0125	1.0091	1.0072
.6	1.0290	1.0122	1.0080	1.0055	1.0038

[a]Worst case for associated value of N.

Figure 12 is a plot of relative apparent power against parasitic load with N as a parameter. This figure shows how increasing the number of parallel parasitic loads reduces the magnitude of the relative rating ratio. As N increases, the maximum value of the ratio decreases, approaching a limiting value as N approaches infinity. The useful-load power factor selected for this figure as representative is 0.8 lagging; a similar variation is true for other power factors.

The peak apparent-power ratings for selected combinations of useful-load power factor and number of parallel parasitic loads have been tabulated in tables II and III. Table II shows the per unit apparent-power requirements; table III shows the relative ratings.

Alternator-Power Factor

The alternator-power factor is defined as the ratio of the alternator power output to the alternator apparent-power output (ref. 2).

$$pf_G = \frac{P_G}{VA_G} = \frac{1}{VA_{Gpu}} \tag{45}$$

In this analysis, therefore, alternator-power factor numerically equals the reciprocal of apparent power in per unit. Any discussion of the behavior of the apparent-power requirements of the alternator pertains to the behavior of the power factor of the alternator if the inversion is applied to the per unit volt-ampere requirement.

CONCLUSIONS

The use of a phase-controlled parasitic current to maintain a constant power load on an alternator effects an increase in the apparent-power capacity required of the alternator. A phase-controlled current is nonsinusoidal. The current not only contains harmonics, but also, as generally used, is reactive. These features are responsible for the greater alternator capacity required.

Analysis of the circuit configuration assumed for this study shows that with a 0.8 lagging power factor useful load, the apparent-power (volt-ampere) rating of the alternator must be increased 7.6 percent above the rating required without a parasitic load. With the same load the total harmonic distortion of the alternator current is 32 percent.

The size of the increase in alternator rating and the amount of current distortion generated can be reduced by using more than one parasitic load. With multiple smaller parasitic loads sequentially actuated, the effect of the nonsinusoidal load current is lessened. For the same 0.8 lagging power factor useful load, a second parasitic load used in parallel with the first reduces the required increase of the alternator capacity to 3.0 percent and the total harmonic distortion to 18 percent.

Additional division of the original parasitic load further reduces the undesirable effects. But the amount of improvement at each division decreases rapidly with increasing number of loads.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 28, 1967,
120-27-03-42-22.

APPENDIX A

SYMBOLS

A — $\frac{\cos 2\alpha - 1}{2\pi}$

a_m — amplitude of m^{th} harmonic cosine term of Fourier expansion

B — $1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}$

b_m — amplitude of m^{th} harmonic sine term of Fourier expansion

E_G — rms value of source voltage

e_G — instantaneous value of source voltage

I_H — rms value of total harmonic current

I_{Hpu} — per unit value of I_H

I_{PR} — rms value of current in a single fully conducting parasitic load

I'_{P1} — rms value of fundamental current component in a partially conducting parasitic load

I_T — rms value of total current

I_{Tm} — rms value of m^{th} harmonic of total current

I_{Tpu} — per unit value of I_T

I_{T0} — average value of I_T

I_{T1} — rms value of fundamental component of total current

I_{T1pu} — per unit value of I_{T1}

I_U — rms value of useful-load current

i_P — instantaneous value of total parasitic-load current

i'_P — instantaneous value of current in partially conducting parasitic load

i_{PR} — instantaneous value of current in single fully conducting parasitic load

i_{P1} — instantaneous value of fundamental component of total parasitic-load current

i'_{P1} — instantaneous value of fundamental component of current in partially conducting parasitic load

i_T — instantaneous value of total current

i_{T1} — instantaneous value of fundamental component of total current

i_U — instantaneous value of useful-load current

m — order of harmonic component of current

N — total number of parasitic loads in parasitic-loading system

$[NP_{Ppu}]$ — greatest integer in numerical value of product NP_{Ppu}

n number of parallel parasitic loads turned on fully

P_G average power provided by electrical power source (alternator)

P_P average power dissipated in total parasitic load

P'_P average power dissipated in partially conducting parasitic load

P_{Ppu} per unit value of P_P

P_U average power dissipated in useful load

P_{Upu} per unit value of P_U

pf_G power factor of electrical power source

R_P resistance of each parasitic load resistor

THD total harmonic distortion

t time

VA_G apparent power developed by electrical power source (alternator)

VA_{Gpu} per unit value of VA_G

α firing angle of phase-controlled current

θ_U useful-load power factor angle

ω angular frequency of source voltage

Subscripts:

pu per unit

APPENDIX B

FOURIER ANALYSIS OF PHASE-CONTROLLED CURRENT

The current variation depicted in figure 4(c) is defined by equation (9).

$$\left.\begin{aligned} i'_P &= 0 \Bigg|_{0,\ \frac{\pi}{\omega}}^{\frac{\alpha}{\omega},\ \frac{\alpha+\pi}{\omega}} \\ &= \sqrt{2}\, I_{PR} \sin \omega t \Bigg|_{\frac{\alpha}{\omega},\ \frac{\alpha+\pi}{\omega}}^{\frac{\pi}{\omega},\ \frac{2\pi}{\omega}} \end{aligned}\right\} \tag{B1}$$

This function may be represented by the following Fourier series:

$$i'_P(\omega t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t) \tag{B2}$$

where, since

$$i'_P(\omega t \pm \pi) = -i'_P(\omega t) \tag{B3}$$

$$a_m = \frac{2}{\pi} \int_0^{\pi} i'_P(\omega t) \cos m\omega t \, d(\omega t) \qquad (m = 0, 1, 2, \ldots) \tag{B4}$$

and

$$b_m = \frac{2}{\pi} \int_0^{\pi} i'_P(\omega t) \sin m\omega t \, d(\omega t) \qquad (m = 1, 2, 3, \ldots) \tag{B5}$$

The value of a_m and of b_m for all even integer values of m (including $m = 0$) is zero. Evaluation of the integrals of equations (B4) and (B5) gives the following values of a_m

and b_m for $m = 1$:

$$a_1 = \frac{\sqrt{2}\, I_{PR}}{2\pi} (\cos 2\alpha - 1) \tag{B6}$$

$$b_1 = \sqrt{2}\, I_{PR} \left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right) \tag{B7}$$

Thus the fundamental component of the current described by equation (B1) is

$$\begin{aligned} i'_{P1} &= \sqrt{2}\, I_{PR}\left(\frac{\cos 2\alpha - 1}{2\pi}\right) \cos \omega t + \sqrt{2}\, I_{PR}\left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right) \sin \omega t \\ &= \sqrt{2}\, I'_{P1} \sin\left(\omega t + \tan^{-1} \frac{\cos 2\alpha - 1}{2\pi - 2\alpha + \sin 2\alpha}\right) \end{aligned} \tag{B8}$$

where

$$I'_{P1} = I_{PR} \left[\left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right)^2 + \left(\frac{\cos 2\alpha - 1}{2\pi}\right)^2\right]^{1/2} \tag{B9}$$

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