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EXPLORING IN AEROSPACE ROCKETRY

9. ROCKET TRAJECTORIES, DRAG, AND STABILITY

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Presented to Lewis Aerospace Explorers
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9. ROCKET TRAJECTORIES, DRAG, AND STABILITY

Roger W. Luidens*

TRAJECTORIES

The three phases of a typical model rocket flight are powered flight, coasting, and parachute descent. These phases, shown in figure 9-1, are analyzed in the following discussion. (Vertical flight has already been discussed in chapter 3. However, the current

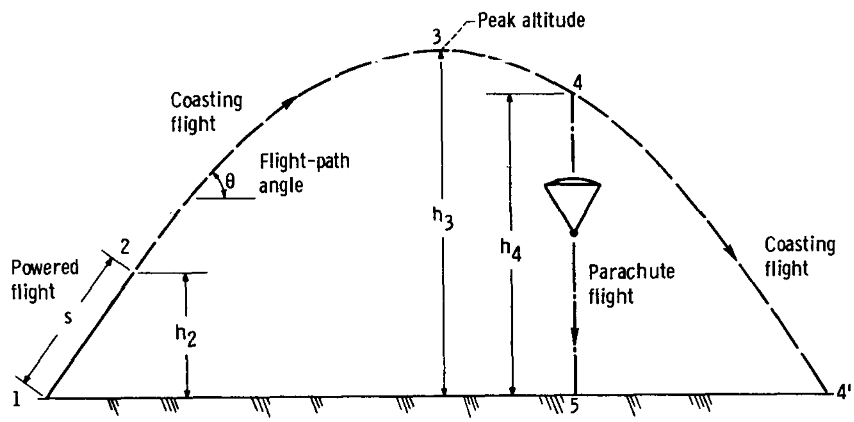

Figure 9-1. - Typical model rocket flight.

discussion concerns the more general, oblique trajectory.) The analysis depends primarily on the use of Newton's law, which states that the net force, or the unbalanced force, F applied to a body is equal to the mass m of the body multiplied by its acceleration A.

$$F = mA \tag{1}$$

where F is in pounds, m is in slugs ($m \equiv W/g$, where W is the weight of the object in lb, and g is the acceleration due to gravity in ft/sec^2), and A is in feet per second per second ($A \equiv \Delta V/\Delta t$, or time rate of change of velocity).

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According to Newton's law, if there are no unbalanced forces, then there is no acceleration; and, conversely, if the acceleration is zero, then the forces are balanced. A zero acceleration means that the mass m is moving at a constant velocity which may or may not be zero.

Powered Flight

In order to simplify the analysis, it is assumed that the entire powered flight is accomplished with a single stage. However, the equations can be applied to multistage rockets, as will be explained at the end of this section. Also for the sake of simplicity, the effects of drag are not considered.

For those who are unfamiliar with trigonometry, some simple definitions will be useful here. The functions (sine, cosine, and tangent) are defined as ratios of sides of a right-angled triangle, as shown in figure 9-2.

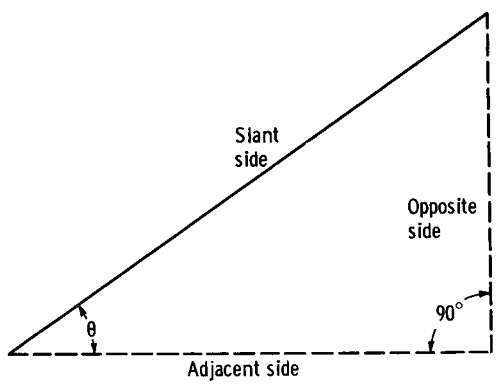


Figure 9-2. - Right-angled triangle as it applies to definitions of trigonometric functions.

Thus,

$$\sin \theta = \frac{\text{Side opposite } \theta}{\text{Slant side}}$$

$$\cos \theta = \frac{\text{Side adjacent to } \theta}{\text{Slant side}}$$

$$\tan \theta = \frac{\text{Side opposite } \theta}{\text{Side adjacent to } \theta}$$

These functions depend only on the angle θ, provided that the angle opposite the slant

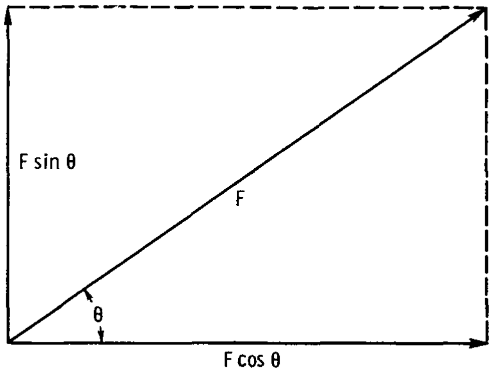


Figure 9-3. - Vector relations.

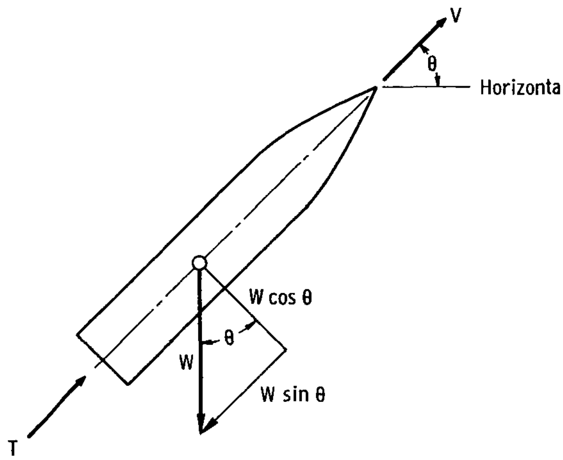


Figure 9-4. - Forces on a rocket during powered flight.

side (hypotenuse) is 90°.

Trigonometric functions are useful in vector considerations. (A vector is any quantity or characteristic that has both magnitude and direction, such as an applied force or the velocity of an object.) For example, a force F applied along the slant side at an angle θ is equivalent to, or can be replaced by, a force along the adjacent side F cos θ and a force parallel and equal to the opposite side F sin θ. This relation is illustrated in figure 9-3. Usually, but not necessarily, the adjacent side is considered horizontal, and the opposite side is considered vertical; in all cases, however, the two components are perpendicular to each other.

As shown in figure 9-4, the net force F applied to the rocket in the line of flight is the rocket thrust T minus (because it is the direction opposite to the thrust) the component of the rocket weight in the line of flight, W sin θ. The flight-path angle θ is measured from the horizontal. So, in the line of flight

$$F = T - (W \sin \theta) \tag{2}$$

The mass m of the rocket is

$$m = \frac{W}{g} \tag{3}$$

By substitution of equations (2) and (3) into equation (1), the acceleration A of the rocket along the line of flight is found to be

$$A = \left(\frac{T}{W} - \sin \theta\right) g \tag{4}$$

The weight, and therefore the mass, of the rocket varies during the powered flight because of the "burning" and exhausting of the propellant. Thus, the acceleration varies. The acceleration at the beginning of the powered flight (point 1 of fig. 9-1) is

$$A_1 = \left(\frac{T_1}{W_1} - \sin \theta_1\right) g \tag{5}$$

The acceleration at the end of the powered flight (point 2, fig. 9-1) is

$$A_2 = \left(\frac{T_2}{W_2} - \sin \theta_2\right) g \tag{6}$$

where

$$W_2 = W_1 \text{ minus weight of propellant burned}$$

The average acceleration A_{av} is

$$A_{av} = \frac{A_1 + A_2}{2} \tag{7}$$

From the definition of acceleration, the change in velocity ΔV is

$$\Delta V \equiv V_2 - V_1 = A_{av}(t_2 - t_1) \tag{8}$$

where $t_2 - t_1 \equiv t_b$ is the rocket burning time, and V_1 for a one-stage rocket is zero.

The average velocity V_{av} during powered flight is

$$V_{av} = \frac{V_1 + V_2}{2} = V_1 + \frac{\Delta V}{2} \tag{9}$$

The distance travelled Δs during powered flight is then

$$\Delta s = V_{av} t_b \tag{10}$$

and the altitude reached by the end of powered flight is

$$\Delta h = h_2 - h_1 = \Delta s \sin\left(\frac{\theta_1 + \theta_2}{2}\right) \tag{11}$$

The velocity increase due to rocket thrust (eq. (8)) can be written in other forms. If a rocket engine is test fired, the thrust can be plotted as a function of time, as shown in figure 9-5. This is known as the thrust-against-time history, or simply thrust-time

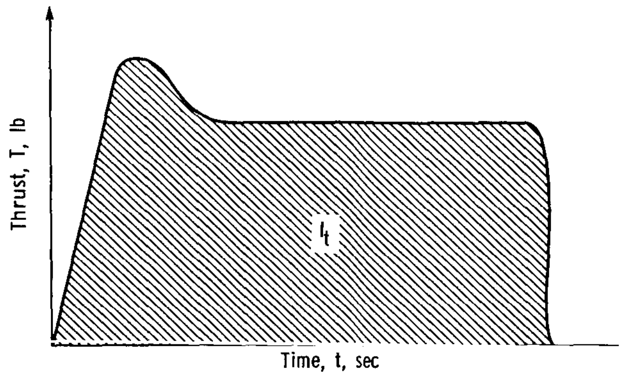


Figure 9-5. - Thrust-time history of rocket engine.

history (discussed in chapter 6). The area under the thrust-time curve represents the total impulse I_t of the rocket (discussed in chapters 2 and 15). The change of velocity can be stated in terms of I_t as follows:

$$\Delta V = V_2 - V_1 = \frac{I_t}{\dfrac{W_1 + W_2}{2}} - gt_b \sin\frac{\theta_1 + \theta_2}{2} \tag{12a}$$

If in figure 9-5 the thrust is constant during the burning time, then ΔV can be written in the commonly seen form

$$\Delta V = I_{sp}g \ln\frac{W_1}{W_2} - gt_b \sin\frac{\theta_1 + \theta_2}{2} \tag{12b}$$

where ln means "natural logarithm of" and I_{sp} is the specific impulse (discussed in chapters 2, 6, 11, and 15). The altitude may then be calculated as before, starting with equation (9).

The preceding equations and discussion have been based on the assumption that the powered flight was accomplished with only a single stage. However, these same equations also can be applied in the analysis of the powered flight of a multistage rocket.

Consider a rocket consisting of a first stage designated by the subscript a and a second stage designated by the subscript b. Apply equations (5) to (12) to the first stage. The resulting values at the end of the first stage of powered flight are V_{2_a}, from equation (8), and h_{2_a}, from equation (11). For efficient staging (i.e., to achieve maximum velocity after the two stages have burned), the second stage should ignite immediately after the burnout of the first stage. Thus, the conditions at the end of the first stage (conditions designated by subscript 2_a) become the conditions for the beginning of the second stage (conditions designated by the subscript 1_b); that is, $V_{1_b} = V_{2_a}$, and $h_{1_b} = h_{2_a}$. Now, equations (5) to (12) may be applied a second time. Coasting flight then begins at burnout of the second stage.

Coasting Flight

The coasting trajectory of a rocket (point 2 to point 4' in fig. 9-1) may be analyzed by considering separately the vertical and horizontal components of the velocity. For the sake of simplicity, the effects of drag are ignored in this analysis.

Vertical component of velocity. - This part of the flight is most easily understood by first considering the rocket at peak altitude (point 3 in fig. 9-1 and fig. 9-6). Here, the vertical velocity V_v is zero, the vertical distance, or altitude, is h_3, and the time is t_3. The only force acting on the rocket is that due to gravitation, and the acceleration of the rocket is thus the acceleration due to gravity.

$$A = g = 32.2 \text{ (ft/sec)/sec} \tag{13}$$

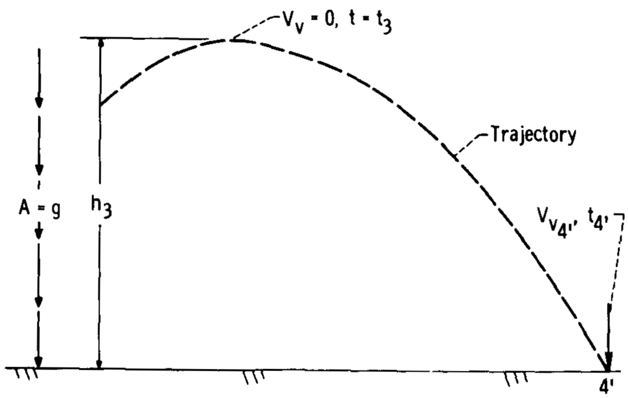


Figure 9-6. - Vertical components of acceleration, velocity, and altitude during coasting part of rocket trajectory.

The velocity of the rocket when it reaches the ground at the end of coasting (point 4' in fig. 9-6) is the acceleration g multiplied by the coasting time from peak altitude $t_{c'}$ (where $t_{c'} = t_{4'} - t_3$):

$$V_{v_{4'}} = gt_{c'} \tag{14}$$

The distance traveled h_3 is

$$h_3 = V_{v_{av}} t_{c'} = \frac{V_{v_{4'}} + 0}{2} t_{c'} \tag{15}$$

or, by use of equation (14) in equation (15)

$$h_3 = \frac{1}{2} gt_{c'}^2 \tag{16}$$

A more useful equation for the vertical velocity at the end of coasting may be obtained by another combination of equations (14) and (15). If the altitude h_3 is known, then

$$V_{v_{4'}} = \sqrt{2gh_3} \tag{17}$$

The ascending leg of the coasting trajectory is similar to the descending leg; that is, one leg is a mirror image of the other leg in a vertical plane through the peak altitude. The vertical velocity V_{v_2} at the end of powered flight (at burnout) can be determined by calculations which will be discussed in the section Relating the vertical and horizontal velocity components. Equation (17) can be rearranged to give the altitude increase above the burnout altitude. So, the peak altitude is

$$h_3 = h_2 + \frac{V_{v_2}^2}{2g} \tag{18}$$

Horizontal component of velocity. - The horizontal component of velocity is shown in figure 9-7. Because there is no force acting in the horizontal direction during coasting flight (gravity acts only in the vertical direction), the horizontal component of velocity V_h remains constant even though the vertical component of velocity changes. Therefore,

$$V_{h_2} = V_{h_3} = V_{h_{4'}} = V_h \tag{19}$$

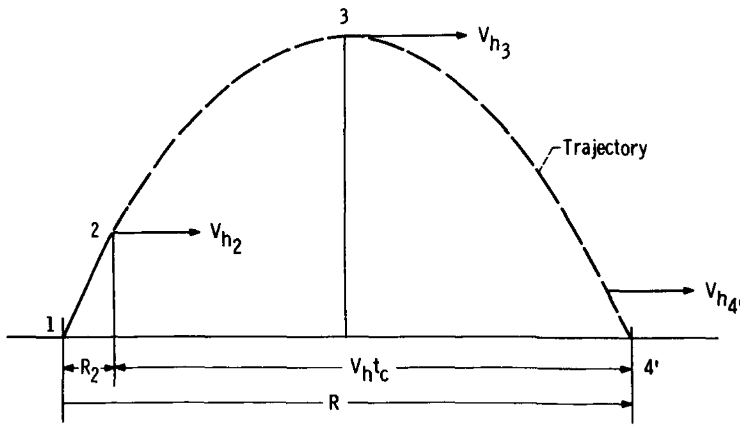


Figure 9-7. - Horizontal components of velocity and distance during coasting part of rocket trajectory.

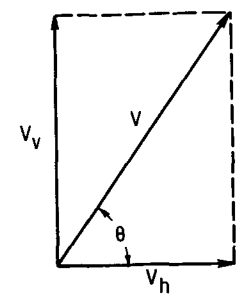


Figure 9-8. - Relation between vertical component, horizontal component, and total rocket velocities.

The horizontal distance, or range, during coasting is

$$R_c = V_h t_c \tag{20}$$

where t_c is the time in coasting flight from the end of burnout either to the point of parachute deployment ($t_c = t_4 - t_2$) or to the ground, if no parachute is used, ($t_c = t_{c'} = t_{4'} - t_2$). The total range from launch is $R = R_2 + R_c$.

Relating the vertical and horizontal velocity components. - In general, the vertical and horizontal components of velocity are related as shown in figure 9-8. In this figure, V is the velocity of the rocket along its flight path, and θ is the flight-path angle measured from the horizontal. The velocities shown in figure 9-8 are related as follows:

$$\tan\theta = \frac{V_v}{V_h} \tag{21}$$

and

$$V_v^2 + V_h^2 = V^2 \tag{22}$$

Sometimes it is convenient to write these relations in other forms. For example,

$$V_{v_2} = V_2 \sin\theta_2 \tag{23}$$

and

$$V_{h_2} = V_2 \cos\theta_2 \tag{24}$$

When these equations are applied at burnout (point 2 in fig. 9-1), as is indicated in equations (23) and (24), then equation (23) gives the value required in equation (18), and equation (24) gives the value required in equation (20).

The coasting flight path described by the preceding equations (zero drag assumed) is a parabola.

Parachute Flight

When a parachute is deployed (point 4 in fig. 9-1), the vehicle quickly reaches its terminal velocity, which is a condition of no acceleration (constant velocity). As is shown in figure 9-9, there are now only two forces acting on the vehicle, and they are in equi-

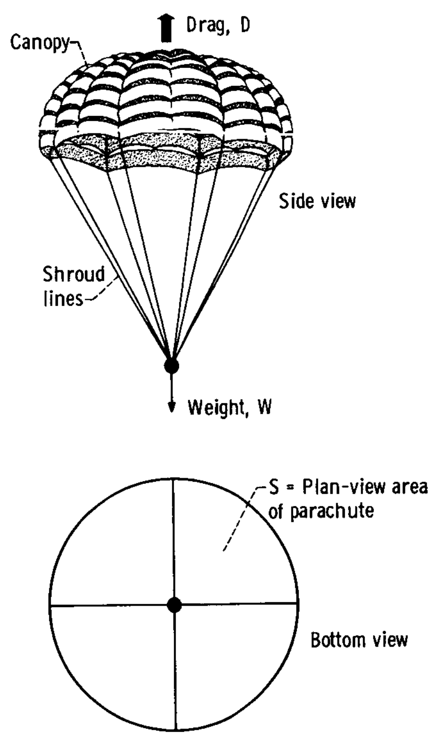


Figure 9-9. - Forces on a rocket during parachute descent.

librium (equal and opposite). These forces are the aerodynamic drag D_p of the parachute and the weight W of the vehicle. It should be recalled that Newton's law states that when the forces are in equilibrium, the acceleration is zero; this law does not state that the velocity is zero. Since the drag increases with velocity, there is just one velocity at which the drag equals the weight, and that is the terminal velocity. The aerodynamic drag of the parachute, in pounds, can be expressed as

$$D_p = C_{D_p} \frac{\rho V_p^2}{2} S_p \tag{25}$$

where C_{D_p} is the aerodynamic drag coefficient of the parachute approximate value of 1.3), ρ is the air density in slugs per cubic foot (mass in slugs is weight in pounds divided by acceleration due to gravity in feet per second per second), V_p is the terminal velocity in feet per second of the parachute along its flight path, and S_p is the reference area for the drag coefficient of the parachute (plan-view area of parachute in square feet). By equating the drag to the vehicle weight, equation (25) may be solved for the terminal velocity V_p, which is a vertical velocity:

$$V_p = \sqrt{\frac{2W}{S_p \rho C_{D_p}}} \tag{26}$$

The time for the parachute to reach the ground t_p is

$$t_p = t_5 - t_4 = \frac{h_4}{V_p} \tag{27}$$

General Equations of Motion

The previous equations have been generated for special parts of the flight path. Equations of motion which are completely general and applicable to all phases of the flight may be found as follows:

If the forces in the direction of flight in figure 9-10, including the drag that was previously omitted, are summed, the general vehicle acceleration can be determined to be

$$A = \left(\frac{T - C_D \frac{\rho V^2}{2} S}{W} - \sin \theta \right) g \tag{28}$$

where in equation (26) S is the reference area for the drag coefficient.

The change in velocity ΔV in an increment of time Δt is

$$\Delta V = A \, \Delta t \tag{29}$$

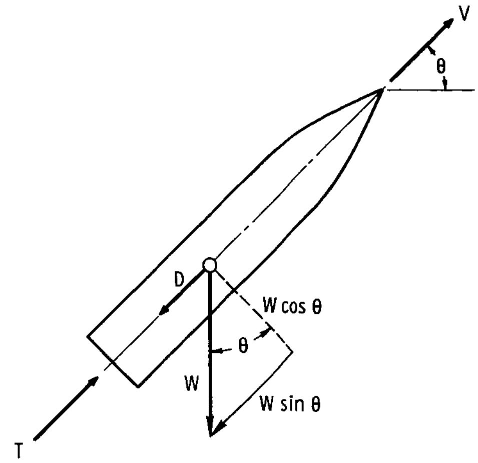


Figure 9-10. - Forces acting on a rocket.

and the velocity at the end of Δt is

$$V_{t+\Delta t} = V_t + \Delta V \tag{30}$$

The increment of distance traveled is

$$\Delta s = \left(V_t + \frac{1}{2}\Delta V\right)\Delta t \tag{31}$$

The rocket weight at the end of Δt is

$$W_{t+\Delta t} = W_t - \dot{W}\,\Delta t \tag{32}$$

where $\dot{W}$ is the weight flow rate of the propellant and

$$\dot{W} = \frac{T}{I_{sp}} \tag{33}$$

where I_{sp} is the rocket specific impulse. (Weight flow rate and specific impulse are discussed in chapter 2.)

From the summation of forces normal (perpendicular) to the flight path, the change in path angle $\Delta\theta$ is

$$\Delta\theta = \frac{g \cos\theta \, \Delta t}{V} \tag{34}$$

and the path angle at the end of Δt is

$$\theta_{t+\Delta t} = \theta_t + \Delta\theta \tag{35}$$

These equations may be integrated (i.e., used repeatedly) over successive steps in time to determine the rocket flight path. This is usually done by an electronic computer, but it may also be done by hand calculations. For near-vertical flight, $\sin\theta$ in equation (28) is 1.0, and equations (34) and (35) are not required.

Orbital Flight

The terminal velocity of the parachute was found by equating the vehicle weight to its drag. The two forces were in equilibrium. Orbital flight is an equilibrium between the centrifugal force and gravity, as shown in figure 9-11.

The centrifugal force F_c is given by the equation

$$F_c = \frac{mV^2}{r} \tag{36}$$

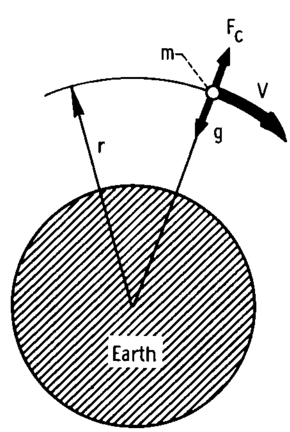


Figure 9-11. - Balance of forces in orbital flight.

where r is the radius of curvature of the orbital path and is only slightly larger than the radius of curvature of the Earth's surface. (In the equations of the preceding sections the curvature of the Earth's surface has been ignored because the effect of this curvature on the trajectory of a model rocket is negligible.) For orbital flight, the centrifugal force F_c must be equal to the pull of gravity mg, so that

$$\frac{mV^2}{r} = mg \tag{37}$$

The velocity of a circular orbit about the Earth V_c is then found by rearranging equation (37)

$$V_c = \sqrt{gr} \tag{38}$$

The gravity of the Earth is about 32.2 feet per second per second, and the radius is about 4000 miles. Therefore, the velocity for a low circular orbit is

$$V_c = \sqrt{32.2 \times 4000 \times 5280} = 26\ 000\ \text{ft/sec}$$

or 17 800 miles per hour. These numbers are only approximate. The circular velocity V_c is the velocity that must be provided by a rocket to achieve orbital flight.

To escape from the Earth's gravity, an additional 41 percent in velocity is required. With this velocity, the rocket will occupy essentially the same orbit about the Sun as does the Earth. To go to another planet, a still higher velocity is required. To go to the Moon, a slightly lower velocity is sufficient because the Moon is in orbit about the Earth.

Orbital flight may be viewed in another way to relate it to more familiar ideas of trajectories. Consider an imaginary cannon on top of an imaginary mountain that extends out of the Earth's atmosphere, as shown in figure 9-12. The cannon points horizontally. A small powder charge will send the cannon ball a short distance before it falls to the Earth's surface (trajectory 1). A larger charge of powder will send it a greater distance before it falls to the Earth's surface (trajectory 2). With a sufficiently large charge, the cannon ball will never fall to the Earth's surface, because the Earth's surface curves away at the same rate the cannon ball is "falling." The cannon ball is then in orbit (trajectory 3).

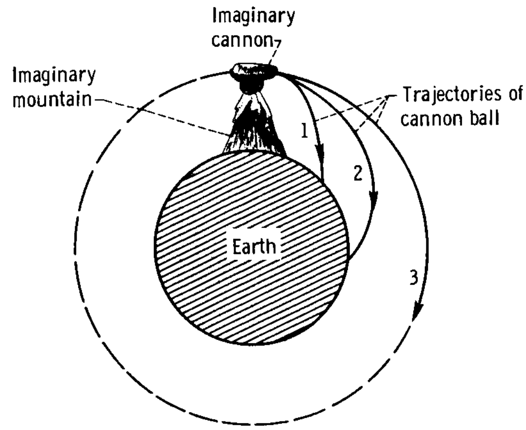


Figure 9-12. - Another view of orbital flight.

DRAG

The rocket drag affects the rocket flight path and maximum altitude. The rocket drag consists of friction drag and form drag (fig. 9-13). The friction drag is the molasseslike effect of the air on the vehicle as it passes through the air. This friction drag D_f may be estimated by the equation

$$D_f = C_f \frac{\rho V^2}{2} \times \text{Surface area of rocket} \tag{39}$$

where C_f is the friction-drag coefficient. For a typical model rocket, C_f is 0.0045 for turbulent skin friction (for rough body surface), and it is 0.0015 for laminar flow (for smooth body surface).

The form drag is the result of low pressures acting on the rear or base areas of the rocket because of poor streamlining which leads to flow separation and turbulent air. The form drag can be determined by measuring the base pressure in a wind tunnel.

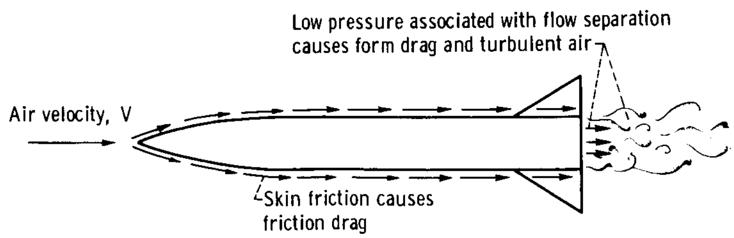


Figure 9-13. - Drag forces on a rocket.

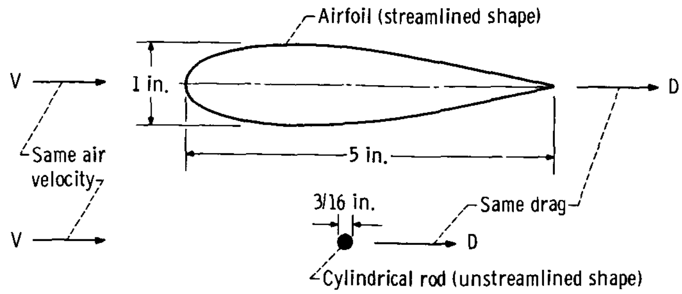


Figure 9-14. - Streamlining minimizes drag.

The importance of good streamlining is illustrated in figure 9-14. The two shapes shown in the figure have the same drag. Obviously, a cylinder with its axis normal to the flow direction is a high-drag shape.

STABILITY

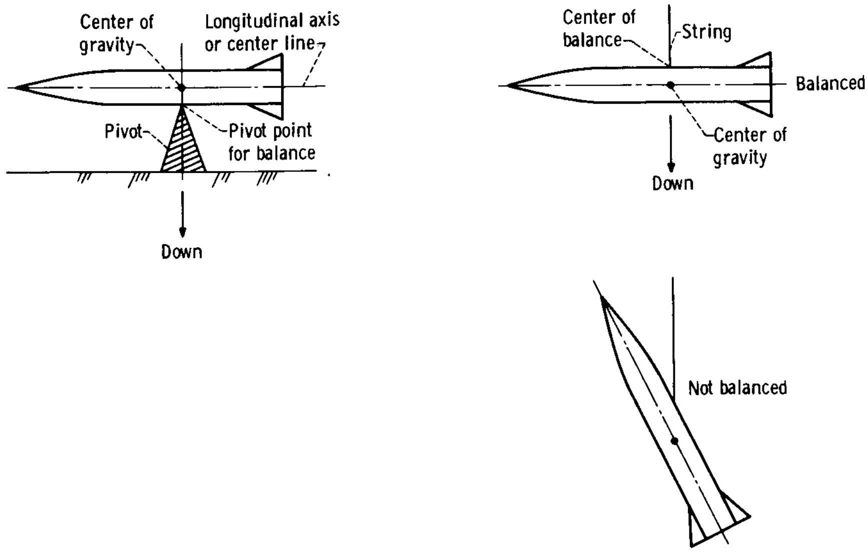
Basic Concepts

Center of gravity. - The center of gravity, C. G., is the point on a body where all its mass (or weight) can be considered to be concentrated. A spinning or rotating body which is not under the influence of aerodynamic forces or mechanical constraints (e.g., a body thrown into the air so that it is spinning freely) will rotate about its center of gravity. Under static conditions, a body balances about its center of gravity.

The C. G. of a model rocket can be determined by finding its center of balance. The center of balance can be found by either of the methods shown in figure 9-15. In figure 9-15(a), the C. G. lies at the intersection of the longitudinal axis of the rocket and a vertical line through the pivot point at which balance is achieved. In figure 9-15(b), the C. G. lies at the intersection of the longitudinal axis of the rocket and the extension of the line of the string. (The rocket should be built to be symmetrical about its longitudinal axis in weight, thrust, and aerodynamics.)

Center of pressure. - The aerodynamic forces act on all the external surfaces of the rocket to yield lift, side force, and drag. The point on the body where all these forces can be considered to be acting (concentrated) is the center of pressure, C. P.

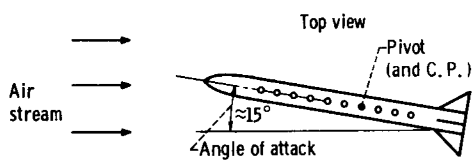
The most accurate way to determine the C. P. is in a wind tunnel or any airstream, as shown in figure 9-16. First, the rocket model is mounted on a pivot and is alined so that its longitudinal axis is parallel to the direction of the airstream and its nose is pointing upstream. In this position the model has zero angle of attack. (The angle of attack is formed by the longitudinal axis of the rocket and a line in the direction of the air-



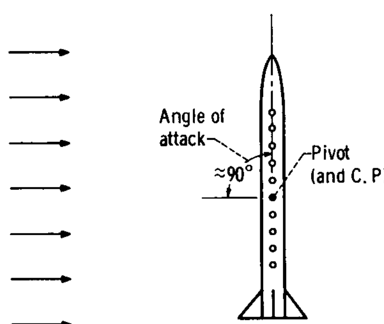
(a) By resting model on pivot.

(b) By suspending model on string.

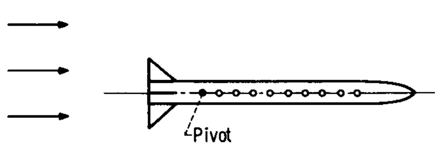
Figure 9-15. - Methods of determining center of balance (and center of gravity) of model rocket.



(a) Low-angle-of-attack center of pressure.



(b) High-angle-of-attack center of pressure.



(c) Pivot behind both high- and low-angle-of-attack centers of pressure.

Figure 9-16. - Centers of pressure of model rocket.

stream.) Next, the tail of the rocket is displaced to one side so that the rocket has a small angle of attack of approximately 15° (fig. 9-16(a)). If the airstream causes the model to return to zero angle of attack, then the pivot point is ahead of the C. P. Several more rearward pivot locations are tried until one is found at which the model rocket no longer has the tendency to return to zero angle of attack. This pivot point, then, is the low-angle-of-attack C. P.

A similar procedure may be followed to obtain the high-angle-of-attack C. P. In this case the rocket model is displaced to a high angle of attack of almost 90° (fig. 9-16(b)). The pivot point at which the model maintains the high angle of attack to which it is displaced is the high-angle-of-attack C. P. The high-angle-of-attack C. P. is generally ahead of the low-angle-of-attack C. P.

If the pivot point were moved to the rear of the low-angle-of-attack C. P., then a displacement of the model from zero angle of attack to any other angle of attack would result in the rocket pointing downstream (fig. 9-16(c)).

A method of estimating the location of the C. P. of a model rocket without a wind-tunnel test is to locate its center of lateral area. The high-angle-of-attack C. P. is very close to the center of lateral area. The procedure for finding the center of lateral area is to cut from a piece of cardboard the side outline (or shadow) of the model rocket. The center of lateral area of this cardboard outline is also its center of gravity. The center of gravity can be determined by the method already described in the section Center of gravity.

Positive Static Stability

Positive static stability is a property of a rocket such that when the rocket is disturbed from zero angle of attack, it tends to return to zero angle of attack. Since the rocket rotates about its C. G. and the aerodynamic restoring forces act at the C. P., the relative positions of these two points determine the stability of the rocket. If the C. P. is behind the C. G., the rocket has positive static stability. If the C. P. and C. G. are at the same point, the rocket has neutral stability. If the C. P. is ahead of the C. G., the rocket has negative stability (i. e., the rocket is unstable).

For a stable rocket, the relative locations of the C. G., the center of lateral area, the high-angle-of-attack C. P., and the low-angle-of-attack C. P. are shown in figure 9-17. Positive stability is essential for a predictable flight path. An unstable rocket can be a hazard to the persons launching or observing the rocket because its flight path cannot be predicted and it will not fly in the direction in which it is aimed. To ensure positive stability for all angles of attack, a model rocket should be designed so that the C. G. is located ahead of the high-angle-of-attack C. P. by a distance of 1 caliber (di-

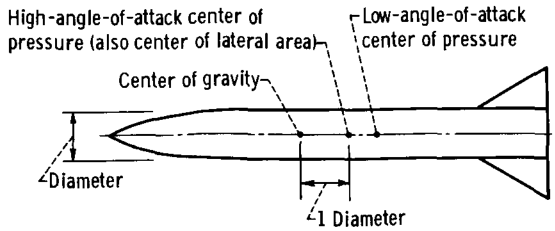


Figure 9-17. - Designing a rocket for positive static stability.

ameter of rocket), as shown in figure 9-17. The locations of the C.G. and the C.P. can be controlled by varying the size of the tail fins and/or the weight distribution of the model. Using the center of lateral area as the C.P. and locating the C.G. a distance of 1 diameter ahead of the C.P. generally results in a conservative and stable design.

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