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(Planetary and Space Science)

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ABSTRACT

It is pointed out that the excitation of geocoronal hydrogen at night by multiply scattered solar radiation occurs mainly at thousands of kilometers above the earth's surface. The nature of the hydrogen distribution below about 400 km is of secondary importance in the transport mechanism and is weakly excited. For this reason Balmer α excitation by Lyman β scattering can be predicted without serious error even if the large gradient in hydrogen density near 100 km is ignored. An approximate calculation involving plane parallel geometry shows that during solar minimum the expected nightglow Balmer α brightness has approximately the measured value of 2 Rayleighs.

Tinsley [1967] has recently questioned the applicability of my calculation Donahue,[1964] of the excitation of Balmer α by geocoronal hydrogen to the real atmosphere. His objection was based on my use of a model devised by Thomas [1962] for Lyman α calculations in which the density of hydrogen varied as r^{-5}. Such a model fits the expected hydrogen distribution well above 400 km when the exospheric temperature is 1250°. It fails, however, to reproduce the very rapid increase in hydrogen density expected at lower altitudes. In Thomas' model, for example, if the optical depth in Lyman α is unity above 120 km the density at 120 km is only 2.3×10^4 atoms/cm^3 compared to 1.5×10^6 atoms/cm^3 in a Kockarts and Nicolet [1963] model with the same optical depth. At 100 km the discrepancy is much worse - 2.3×10^4 atoms/cm^3 to be compared to about 1.9×10^7 atoms/cm^3.

In the Lyman α problem Thomas was concerned with calculating brightnesses above 120 km and took into account the effect of the mass of hydrogen below 120 km by imposing a boundary at 120 km which reflected Lyman α with the efficiency dictated by the albedo of 42 per cent which had been observed experimentally. Tinsely argues that in the case of Balmer α production at night by Lyman β transport there may be a significant number of Lyman β scatterings at low altitude (100 - 120 km) which contribute to the Balmer α observed on the earth's surface. Thus a calculation of the Balmer α brightness based on a model which terminates with a perfectly absorbing layer for Lyman β at 120 km might be seriously deficient.

This is in fact not likely to be the case. The reason is that, for geometrical reasons, the hydrogen atmosphere below 120 km hardly participates in the radiative transport responsible for the nightglow excitation. The property of geocoronal hydrogen which is essential for the efficacy of the mechanism of nightglow excitation by transport of resonance radiation is its great scale height. Above the anti-solar point, 180° from the sun, for example, the initial excitation of hydrogen by photons arriving there directly from the sunlit atmosphere has a maximum between 1000 km and 2000 km. At 200 km the excitation rate is reduced by a factor of 3 compared to the rate at 1200 km. At 120 km it has gone down by another factor of three. While multiple scattering does build up the excitation rate at low altitude the degree of excitation there is always small compared to that above 500 km. There will indeed be an impressive maximum in the volume excitation rate below 200 km but the integrated rate in the column between 100 and 200 km is very small compared to the integrated rate in the long, almost uniformly excited column of hydrogen between 200 km and 3000 km. It is the fact that the initial excitation occurs predominantly at very high altitude in the outermost third of the medium's total optical depth which is ultimately responsible for this phenomenon. Far more photons - after several scatterings - escape outward than penetrate into the bottom half of the medium below 150 km. This situation is even more aggravated in the case of Lyman β than in that of Lyman α because only 0.88 of the Lyman β photons survive each scattering. Furthermore, the effective base of the hydrogen for Lyman β scattering is considerably higher than the base for Lyman α. This is because of the greater opacity of O_2 for Lyman β. Only half of the diffuse Lyman β glow directed downward at 120 km penetrates below 110 km.

As an illustrative example consider a model in which the hydrogen distribution is normalized to 2.7×10^7 atoms/cm^3 at 100 km and in which the distribution is that appropriate to a 1500° exospheric temperature in the daytime and a 1000° temperature at night. For Lyman α the optical depth is 30 above 100 km in the day and 6 at night. The night time density profile, $\rho(z)$, is plotted in Fig. 2. Above the anti solar point (solar zenith angle of 180°) the volume rate of excitation has been calculated as a function of z per unit effective solar flux $\pi F_0 \sqrt{\pi}\, \Delta\nu_D$, where $\Delta\nu_D$ is the Doppler line half width and πF_0 is the solar flux in units of photons/cm^2 sec in unit frequency. This has been done by considering first the sunlit hydrogen, including the region beyond the terminator and solving approximately the radiative transfer equation for the volume rate of excitation there. The rate of excitation along the vertical column above the subsolar point by photons originating in the sunlit region and reaching that column without scattering is then calculated. This rate, normalized to unit cross section and unit effective solar flux $[\pi F_0 \sqrt{\pi}\ \Delta\nu_D]$ is plotted as ρS_0 in Fig. 2. It is also plotted per unit optical depth in units of $[\pi F_0 \sqrt{\pi}\ \Delta\nu_D]$ as S_0 in Fig. 1 as a function of optical depth and in Fig. 2 as a function of z. The small values of ρS_0 at low altitude are a result of two effects - the large optical depths along the paths to the highly excited sunlit regions near the terminator and the small solid angle subtended by these regions. At higher altitudes the transparency and the solid angles increase while the hydrogen density slowly decreases. The effect of decreasing density does not become dominant until an altitude of 1200 km is reached. Above that height ρS_0 drops rapidly although S_0 (in units of optical depth) increases all the way to $\tau = 0$ (Fig. 1).

To compute the steady state excitation rate generated by multiple scattering of these photons a plane parallel model of the atmosphere is next assumed in which the initial rate of excitation is S_0. The ultimate source function is calculated by solving the integral equation

$$S(\tau) = S_0(\tau) + \int S(\tau')\ H(\tau, \tau')\ d\tau' \qquad (1)$$

discussed, for example, by Donahue and Meier (1967). $H(\tau, \tau, \tau')\, d\tau'\, d\tau$ is the probability that a photon originating somewhere in $d\tau'$ at the level τ' is absorbed in a slab of width $d\tau$ at the level τ. $S(\tau)/(\pi F_o \sqrt{\pi}\ \Delta\nu_D)$ is plotted in Fig. 1 as a function of τ. In Fig. 2 both $S(\tau)/(\pi F_o \sqrt{\pi}\, \Delta\nu_D)$ (per unit optical depth) and $\rho(z)S(\tau)/(\pi F_o \sqrt{\pi}\, \Delta\nu_D)$ are plotted as functions of z. Multiplying this latter quantity by the line center cross section

$$\sigma_o = \frac{\pi e^2}{mc} \frac{f_{12}}{\sqrt{\pi}\ \Delta\nu_D} \tag{2}$$

and by the effective line center solar flux

$$\pi F_o \sqrt{\pi}\ \Delta\nu_D \tag{3}$$

gives the actual local volume excitation rate. This product is simply g_{12}, the number of solar Lα photons scattered per hydrogen atom before attenuation of the solar flux.

The source functions $S(\tau)$ and $S_o(\tau)$ are volume excitation rates divided by $\sigma_o\rho$. Hence they indicate the degree of excitation of the medium. The low degree of excitation below 200 km is evident in Fig. 2.

In Fig. 1 there is also plotted the reciprocal of the probability E that a photon will escape without scattering from a given altitude. This is essentially the mean number of scatterings a photon will suffer before escape. A good first approximation to the final source function S is the quantity S_o/E. The average number of scatterings the original family of photons (S_o) undergoes is 5.1. Most of these occur in the upper half of the medium.

The apparent integrated photon emission rate in a column whose axis is at an angle $\cos^{-1}\mu$ with the vertical is given by

$$4\pi I(\tau) = \int S(\tau')T(\tau,\tau')d\tau'/\mu \tag{4}$$

where the upper limit is the appropriate boundary at $\tau' = 0$ or $\tau' = \tau_o$ and $T(\tau, \tau')$ is the transmission function. Since

$$d\tau = \sigma_o \rho dz \tag{5}$$

this integral may also be written as

$$4\pi I(z) = \sigma_o \int_z \rho(z')S(\tau')T(\tau,\tau')dz'/\mu \tag{6}$$

It can be seen from Fig. 2 that when the transmission function is unity the column emission rate in the zenith at $\tau = 6$ receives minor contributions from the lower half of the medium. The value of the integral in units of $(\pi F_o)\ \sqrt{\pi}\ \Delta\nu_D$ is 0.205. Of this a fraction 0.95 comes from regions higher than 110 km and 0.88 from those above 120 km. In fact almost half of the photons originate in the first optical depth above 1500 km. An actual comparison with the results obtained by Thomas with his power law model and those resulting from the present sort of calculation confirm the expectation that only a small error results from the use of his approximation to the distribution.

To adapt these results to Lyman β we note that σ_o for Lyman α is a factor of 6 larger than that for Lyman β. Hence if there were 6 times as much hydrogen in the model ($\tau_o = 36$ for Lyman α) the present calculations of $S/(\pi F_o)\ \sqrt{\pi}\ \ \Delta\nu_D$ could be taken over for the excitation of the 3p level in a medium whose optical depth is 6 in Lyman β. Of course ρS would be 6 times as large but $\sigma_o \rho S$, the actual volume rate of excitation would remain the same, that is

$$\sigma_o(L\beta)6\rho S \equiv \sigma_o(L\alpha)\rho S \tag{7}$$

According to the NRL observations (Tousey et al, 1964) the quantity $\pi F_o \sqrt{\pi}\, \Delta\nu_D$ was 10^8 photons/cm^2 sec in unit frequency for Lβ in 1962. Since each excitation of the third level results in Balmer α emission 12 percent of the time the predicted Hα column emission rate observable on the surface of the earth is given by

$$4\,\pi I(H\alpha) = 0.2 \times 0.12 \times 10^8 = 2.4 \text{ Rayleighs} \tag{8}$$

Actually since the Hα emission causes a loss of Lβ photons at each scattering and about 5 scatterings occur before a Lβ photon gets entirely out of the medium $S(\tau)$ only builds up to about half its analagous Lα value and the Hα column emission rate predicted is only about 1.2 Rayleighs. This is to be compared with the 2 Rayleighs recently reported by Tinsley (1967) and Armstrong (1967).

Although results obtained by this method of calculation for other solar zenith angles are available and although they are appropriate to solar minimum when the night time hydrogen abundances may have been as large as those discussed here it is probably not worth while publishing them. The reason is that the plane parallel model is a poor approximation for hydrogen. Other calculations using proper geometry, a more realistic global hydrogen distribution, taking account of the 0.88 albedo on scattering and absorption by O_2 are now underway. For a quantitative comparison with experiment these are certainly preferable. Their results, however, do not alter the fundamental arguments presented here.

It should be noted here that in my previous paper on Hα excitation I used Thomas' (1962) Lα source functions. These are $\rho S/(\pi F)\sqrt{\pi}\Delta\nu_D$ rather than $S(\tau)$. When multiplied by σ_o for Lyman α they give the volume excitation rates (per unit $\pi F_o \sqrt{\pi}\ \Delta\nu_D$). Thus from his Fig. 14 the integral

$$\sigma_o(L\alpha) \int \rho S\ d\ z = 3.1 \times 10^{-2} \qquad (9)$$

gives the integrated emission rate in units of $\pi F_o \sqrt{\pi}\ \Delta\nu_D$ for any hydrogen line for which the medium has an optical depth of unity. In the case of Lyman β this leads to a predicted Balmer α column emission rate of 0.24R after allowance for the albedo of 0.88. My paper (Donahue, 1964) unfortunately gives a misleading explanation of the method of calculation as Tinsely has pointed out. The method is the one set forth here and the results as published are correct within the limitations of the model used. Hence, I still maintain that the large Balmer α emission rates reported for solar maximum do not appear to be consistent with the 1963 Lyman β solar line profile and the multiple scattering model. Their dependence on solar zenith angle is consistent with the model. The more recent measurements, on the other hand, do not seem to present any serious difficulties of interpretation. Thus a continuation of the observational program of Armstrong and Tinsley through the next few years and some determinations of the solar Lyman β line center flux during this period are certainly in order.

One final word of comment may be in order concerning Lβ nightglow fluxes in the E region. These are important as sources for ionization of O_2 at night. It is not proper to infer these by scaling the Balmer α

arriving at the ground. The reason is again the great altitude of the excitation. Lyman β is seriously attenuated in reaching the 120 km region. For example, the Lyman β apparent column emission rate in the zenith at 120 km for the case discussed here is not 6R but only about 3.3R. Similarly for a solar zenith angle of 135° the present type of calculation would predict a zenith emission rate of 5R for Hα and only 15R of Lβ at 120 km. This point has been properly handled in the approximate treatment of the night time E region by Ogawa and Tohmatsu (1967).

Acknowledgements

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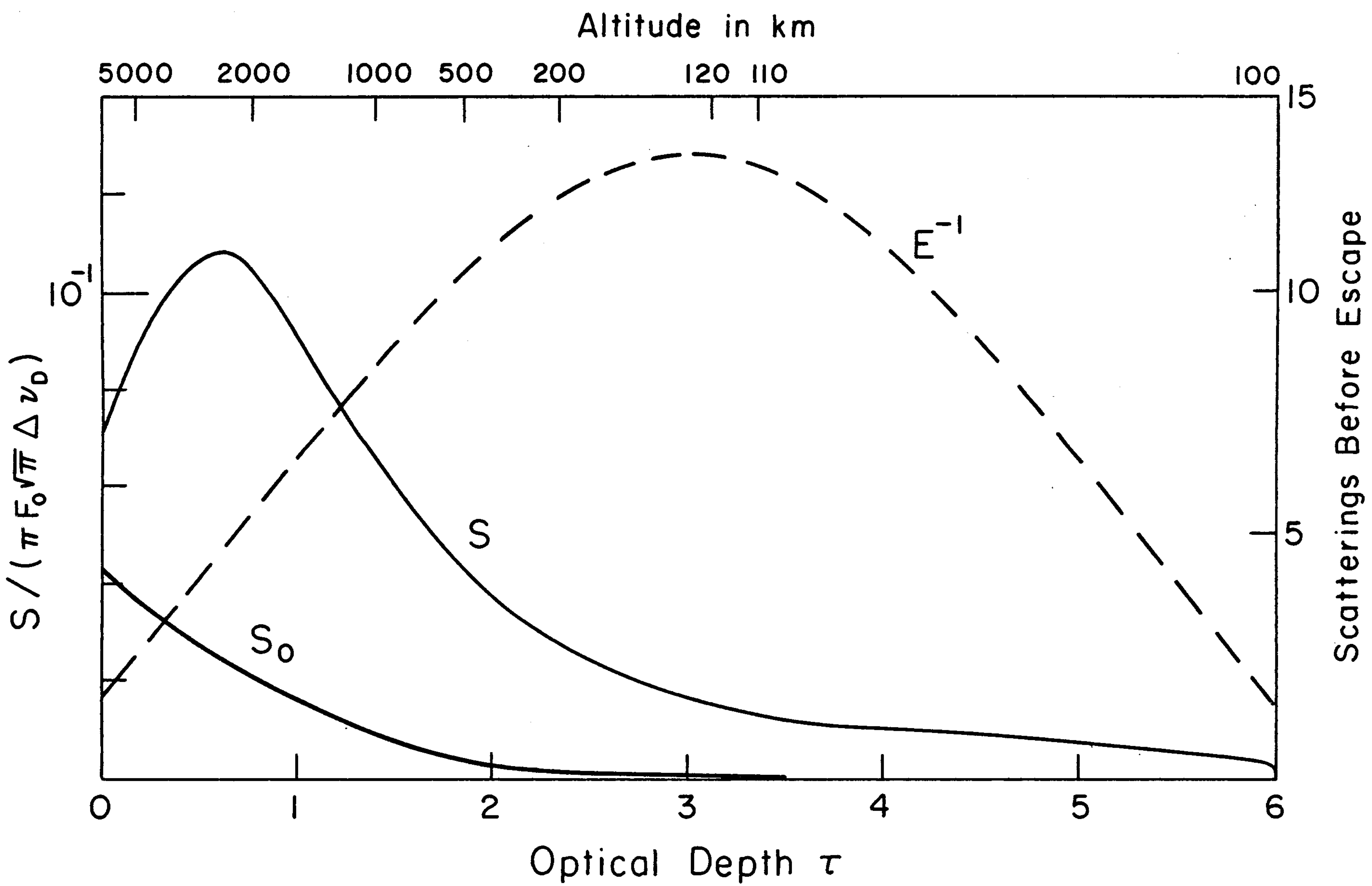
Figure Captions

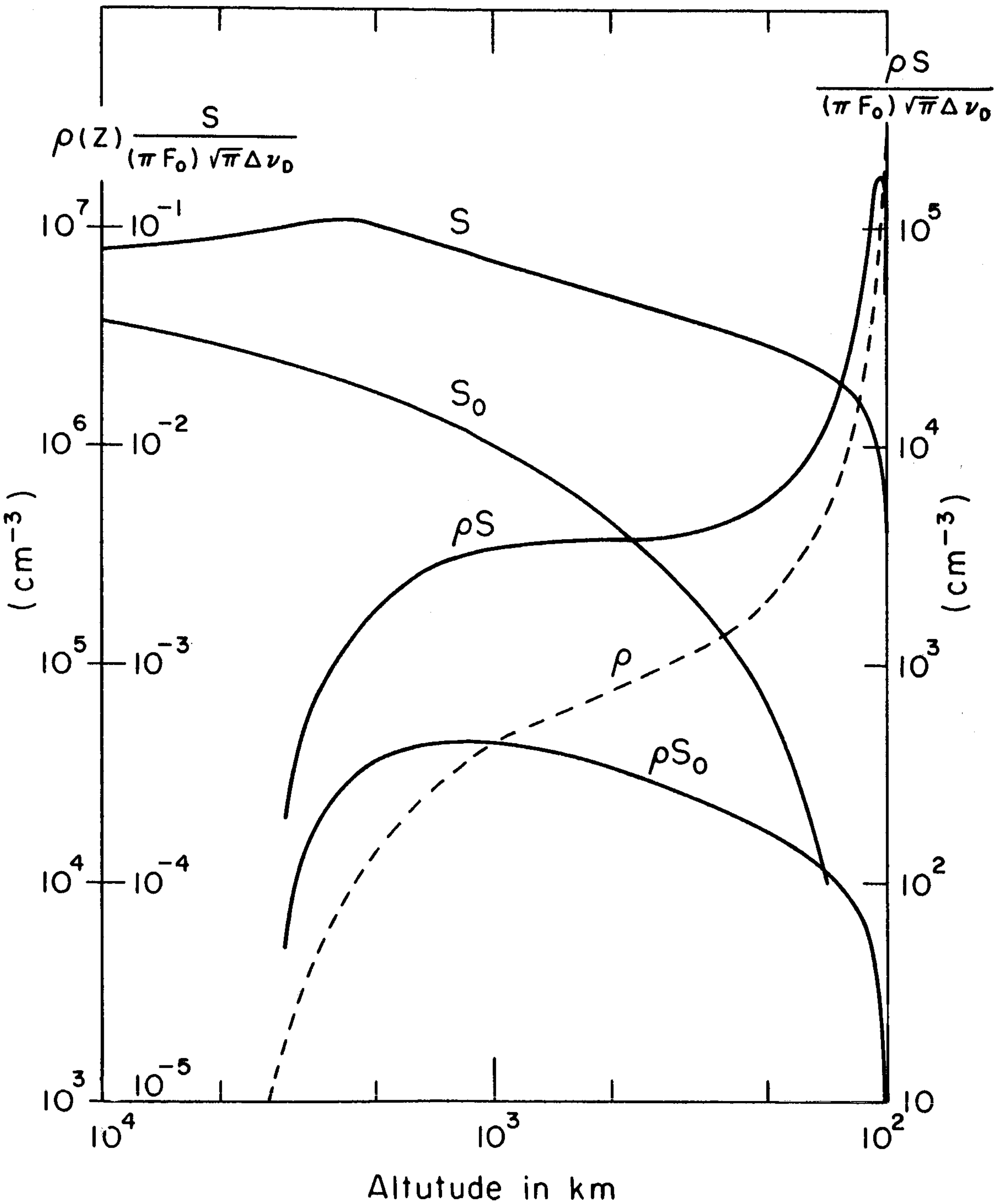
Fig. 1 Source functions (initial and final) and most probable number of scatterings before escape as functions of optical depth in a Kockarts and Nicolet (1963) hydrogen model of optical Depth 6, solar zenith angle 180°.

Fig. 2 Hydrogen Density, Lyman α source functions (Degree of excitation) and volume excitation rates normalized to unit cross section and effective solar flux as functions of altitude, 180° from the sun.



Altitude in km
5000
2000
1000
500
200
120
110
100
10^{-1}
$S/(\pi F_0 \sqrt{\pi}\,\Delta\nu_D)$
15
10
5
Scatterings Before Escape
E^{-1}
S
S_0
0
1
2
3
4
5
6
Optical Depth τ

Figure 1



ρ(Z) S / (πF₀) √π Δν_D
ρS / (πF₀) √π Δν_D
10^7
10^-1
10^6
10^-2
10^5
10^-3
10^4
10^-4
10^3
10^-5
10^5
10^4
10^3
10^2
10
(cm^-3)
(cm^-3)
S
S₀
ρS
ρ
ρS₀
10^4
10^3
10^2
Altutude in km

Figure 2