BELLCOMM, INC. TR-68-710-1

NUMBER DENSITY OF MARTIAN CRATERS

January 29, 1968

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Work performed for Office of Manned Space Flight, National Aeronautics and Space Administration, under Contract NASW-417.

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ABSTRACT

The author has measured diameters of craters on the A.C.I.C. Cartographic Reduction of Frames 3-14 of the Mariner IV photographs of Mars. The incremental frequency distribution of diameters of craters larger than 20-30 km follows an inverse square law with density equal to that of craters on the lunar continents. This is in accord with the prediction that lunar continents and the Martian surface carry an "equilibrium" density of craters of meteoroidal impact origin. Details of fluctuations in crater counts support this conclusion.

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I. INTRODUCTION

Mariner IV discovered that Mars has a heavily cratered, surprisingly Moon-like surface. The major difference between craters on the Moon and on Mars is that the Martian craters seem to be very much shallower, possessing lower rims and flatter floors than "sharp" lunar craters of the same diameter. This flattening is usually attributed to processes which operate at the Martian surface, e.g., erosion by and deposition of wind blown sand. In spite of these surficial differences, it is possible that craters on the Moon and Mars have large-scale similarities.

A statistical method can help to answer this question. The expected number density of craters as a function of their diameter can be related to the crucial parameters of the processes by which craters are formed and then destroyed by obliteration, flooding, erosion and sedimentation. The consequences of the meteoroidal impact hypothesis have been previously worked out by Marcus (1) and are applied to the Mars data. The impact hypothesis, which adequately predicts the size distribution of large lunar craters in continental and mare regions, also adequately predicts the distribution of craters on Mars.

II. STATISTICS OF MARTIAN CRATERS

Early Martian crater counts by Leighton et al., (2)(3), based essentially on the first press kit photos, yielded a number density roughly intermediate between that of the lunar continents and lunar maria. Later counts by Binder (4), Bronshten (5), and Hartmann (6) essentially verified these densities, which were based on 70-110 craters for all of the usable Mariner IV photos (for this purpose, Frames 3 - 16). Although these densities of Martian craters were generally accepted, they were known to be low by a large factor (7) since only craters of relatively sharp and pristine appearance were counted, thus ignoring the many "ghost" craters which often were conspicuous in spite of their low relief.

The poor quality of the press kit photos and the knowledge that greatly improved photos would shortly be available from JPL discouraged earlier efforts on the part of the author to do a statistical analysis of the Martian craters. However, the Mariner IV cartographic reduction recently prepared by the Aeronautical Chart and Information Center (8) now gives us suitable material for statistical analyses. Although important problems of the subjective identification, interpretation, and measurement of craters remain, the A.C.I.C. reduction is still the best source of crater data.

Because their corners overlap, consecutive pairs of Mariner IV photographic frames are published on a single sheet mosaic, e.g. Frames 3-4, 5-6 etc., except for Frames 1 and 2. No frame taken after Frame 16 showed enough detail to merit cartographic reduction. The poor quality of photography on Frames 1, 2, 15 and 16 discouraged their use in this analysis.

The basic crater counts are given in Table 1 and displayed in Figures 1 - 7. (The reason for considering Frame 11 separately will be discussed later.) The counts are displayed in "incremental" rather than the more familiar "cumulative" form. As Hartmann (6)(9) has shown, incremental counts preserve the essential shape of the number density while giving a much better picture of the fluctuations than do the cumulative counts. Each bar on the graphs represents the number of craters between x and $x\sqrt{2}$ km diameter ($x = 2^{k/2}$ for k = 3, 4, 5 etc.) per square km. The parameters of the two forms are easily related in the case that we have an inverse power law type of diameter distribution:

Cumulative number of craters per unit area whose diameters exceeds x

$$= C\,x^{-s}$$

Incremental number of craters per unit area whose diameters are between x and $x\sqrt{2}$ km

$$= C\,x^{-s}\,(1 - 2^{-s/2})$$

where s and C are positive constants known as the population index and the density coefficient, respectively. On a log-log graph both cumulative and incremental counts of inverse power law type appear as a straight line with slope -s.

Lunar continents are known to be characterized by s = 2.0, C = 0.10 per sq km for craters larger than 1 km diameter (9). This includes both the Southern Highlands on the near side and the most densely cratered farside regions. The crater density on the lunar maria is much less, and also is characterized by a smaller population index, $s = 1.55$ to 1.80, and $C = 1 \times 10^{-3}$ to 2.5×10^{-3} per sq km, according to the mare being considered. The straight line on Figures 1 - 7 gives the lunar continental incremental density, $0.05\ x^{-2}$ per sq km, for craters of diameter x to $x\sqrt{2}$ km, which should be compared with the left ends of the bars on the graphs since the left end of the bar corresponds to the diameter x in the smoothed incremental density.

At large diameters the Martian crater counts are in excellent agreement with the lunar continental counts, taking into account the small number of observations (Table 1). We note first of all that the index $s = 2.0$ is justified <u>in every case</u>. The density coefficients may be slightly different from region to region. In Table 2 we record the ratio C (Mars)/C (lunar continents) of density of large craters on Mars and on the lunar continents, as well as the minimum diameter x_{min} at which an inverse square law is applicable to the Mars counts.

Note that the number densities in Figures 1 - 7 are not simply (number)/(area) from Table 1. There is an important practical reason for this. The author counted all craters for which any part of the perimeter extends into the Mariner frame. Consequently, the <u>true</u> area within which we are looking for craters of diameter x consists of the area of the photo plus an additional strip of width roughly x/2 which surrounds the region photographed. For lunar counts this is not usually important, but on the Mariner IV frames the craters are large compared to the size of the photo, thus the effective search area for large craters is significantly greater than the area photographed. This effect greatly modifies the estimated densities and should not be overlooked.

III. EQUILIBRIUM NUMBER DENSITY FOR THE IMPACT HYPOTHESIS

The author has developed a statistical theory of the formation and survival of craters which takes into account the randomness, in space and time, of the birth of primary and secondary craters, the destruction or obliteration of older craters by newer ones which form nearby, and their disappearances as a result of flooding or filling (1). It is assumed that the most important factor in the survival of large craters on the lunar continents and on Mars is the obliteration or coverage of old craters by new ones. It can be shown quite generally that if the size distribution of newborn craters does not change with time, the observed distribution will approach a statistical equilibrium state in which smaller craters are destroyed by larger ones at the same rate at which they are formed. It is therefore impossible to estimate, from the number of craters observed on a heavily cratered surface, the total number of craters which have been formed on that surface - a fatal defect in attempts to estimate the age of heavily cratered surfaces from crater statistics. (It is assumed that the equilibrium density has been reached for all large craters present in statistically significant numbers.)

The equilibrium expected number density $\xi(x)$ (expected number of craters of diameter x per unit area, per unit diameter interval) is simply related to the probability density $p(x)$ of crater diameter at the time of birth (1):

$$\xi(x) = p(x) \Big/ \left(\int_x^\infty \pi \, (y - x)^2 \, p(y) \, dy/4 \right)$$

It is assumed that a crater is a perfectly circular object which destroys everything within its perimeter, but leaves everything outside intact. We have also invoked the approximation that a crater is obliterated if and only if it is completely overlapped by a larger crater.

The meteoroidal impact hypothesis predicts that the functional form of p (x) is an inverse power law

$$p(x) = \gamma \, x^{-\gamma-1} \qquad \text{for } x > 1$$

$$p(x) = o \qquad \text{for } x < 1$$

where the size of the smallest "observable" crater is taken as the unit of distance (1 kilometer is convenient for our purposes). The constant γ is the product of two rather poorly known constants, the cumulative population index γ_1 of the masses of planetesimals which presumably bombarded the surface, and the exponent γ_2 in the scaling law which relates crater diameter x to the energy W of the explosion which caused the crater

$$x = (\text{constant}) \; W^{1/\gamma_2}$$

The most plausible ranges of parameter values are $0.6 \leq \gamma_1 \leq 0.8$, and $3 \leq \gamma_2 \leq 4$. The extreme range of possible values of γ is 1.8 to 3.2, but the most likely range is $2.1 \leq \gamma \leq 2.7$.

Assuming the inverse power law form for p(x) is correct, we derive an expected equilibrium density (for $\gamma>2$)

$$\xi(x) = 2\gamma(\gamma-1)\ (\gamma-2)/\pi x^3$$

an expected cumulative equilibrium density

$$\int_x^\infty \xi(y)\ dy = \gamma(\gamma-1)\ (\gamma-2)/\pi x^2$$

and an expected incremental equilibrium density

$$\int_x^{x\sqrt{2}} \xi(y)\ dy = \gamma(\gamma-1)\ (\gamma-2)/2\pi x^2$$

We first observe that for any $\gamma>2$, the cumulative or incremental equilibrium density is an inverse square law. Equating the observed lunar continental density coefficient to this value, we derive $\gamma = 2.13$, which is rather uncertain. Although the Martian crater statistics are too poor to permit us to determine definitely whether the index of the observed distribution is 2.00, the lunar continental observations are accurately characterized by this value. The lunar continental and Martian crater counts are therefore consistent with the hypothesis that these surfaces have an equilibrium density of primary impact craters. As usual, we cannot preclude the hypothesis that most of the large craters are of internal origin because there is no quantitative theory for the origin of endogeneous craters.

IV. FLUCTUATIONS IN CRATER COUNTS

The relatively small size of the Mariner IV sample introduces two severe problems in the accurate determination of crater number densities. In the first place, there would be an obvious problem of small number fluctuations even if the incremental counts obeyed Poisson statistics. Secondly, there is an additional source of variability because the numbers of craters of different sizes in a small region are strongly correlated, which follows directly from the assumption that smaller craters can be obliterated by larger ones. In regions with a relative surplus of recent large craters there is likely to be a dearth of smaller ones, and regions deficient in recent large craters will probably show a surplus of smaller ones. Theoretical studies (1) suggest that even in continental regions of the size of the Mariner IV frames (about 250 km) there may be a 20 to 30 per cent variation in the <u>average</u> density of even 10 to 100 km diameter craters, variation which is superimposed on the Poisson fluctuations. Thus the variability in C (Mars)/C (lunar continents) (Table 2) may still be consistent with similar histories of impact crater formation on the surfaces of the two planets.

Some slight anomalies in the crater counts may also be easily interpreted in terms of correlated fluctuations. Although the anomalies are not significant by themselves, they are uniformly consistent with the above theory. We discuss the photograph pairs individually.

Frames 3-4. No really large craters (greater than 110 km), thus smaller craters are somewhat more numerous than average. The terrain is somewhat rougher in Frame 3 than in Frame 4 (or so it appears) but the crater distributions are roughly similar.

Frames 5-6. There is an apparent deficiency of craters smaller than 45 km diameter. The crater loss may be intrinsic to this region and not due to poor photography, since some of the recent smaller craters are very clear and well resolved.

Frames 7-8. Good photography. All number densities are close to expected values. The rougher terrain in Frame 7 may lie in the Memnonia desert, and the smoother terrain in Frame 8 may lie in Mare Sirenum.

Frames 9-10. Also good photography. There are recent large craters in some abundance, but they all lie near the edges of Frame 10 and none are in the center of either frame. Thus, as in Frames 3-4, we would expect a surplus of smaller craters. These frames may fall in Mare Sirenum.

Frames 11-12. Frame 12 is definitely much "smoother" than Frame 11 and exhibits a much lower crater density; it is not possible to determine whether this is due to degraded photography or degraded topography. Frame 11 could fall within the light region Atlantis and Frame 12 within the dark region Mare Cimmerium. For this reason we have considered Frame 11 separately.

Frame 11. The observed distribution of large craters in this frame is close to the expected density, with no large fluctuations. The large, fairly sharp crater in the center of Frame 11 may have destroyed some old small craters when it was formed.

Frames 13-14. The shape of the distribution is very close to that expected. The lower overall density can be explained by poorer photography, which may cause the loss of shallow "ghost" craters.

We see that the large correlated fluctuations inherent in crater counts readily explain small variations in crater density and anomalous features in the distribution of large Martian craters. There are thus no statistical difficulties in concluding that large lunar continental craters and Martian craters have the same size distribution, and therefore have similar (presumably impact) histories.

1012-AHM-hjt A. H. Marcus

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TABLE I

NUMBER OF CRATERS WHOSE DIAMETER IS BETWEEN x AND $x\sqrt{2}$

Frame	x (km)													Area
	2.8	4	5.6	8	11.3	16	22.6	32	45	64	90	128	180	10^5 sq km
3-4	0	4	0	8	8	12	15	18	8	4	3	0	0	2.45
5-6	0	10	13	19	22	13	8	4	5	2	0	0	0	1.64
7-8	5	4	31	13	20	23	16	8	3	0	0	0	1	1.34
9-10	0	4	17	33	17	20	18	11	4	3	1	1	0	1.18
11-12	0	2	17	28	7	20	7	4	1	3	0	1	0	1.16
11	0	2	8	22	6	12	3	3	1	1	0	1	0	0.548
13-14	0	2	4	21	16	13	7	3	0	1	0	0	1(?)	1.00

TABLE II

FRAME	C(MARS)/C(LUNAR CONTINENTS)	x_{min} (km)
3-4	1.0	32
5-6	0.6	45
7-8	1.0	16
9-10	1.1	22
11-12	0.8	16
11	1.0	16
13-14	0.7	16

C (Mars)/C(lunar continents) is the ratio of the densities of large craters on Mars and on the lunar continents. x_{min} is the smallest diameter for which an inverse square law incremental frequency distribution is valid.

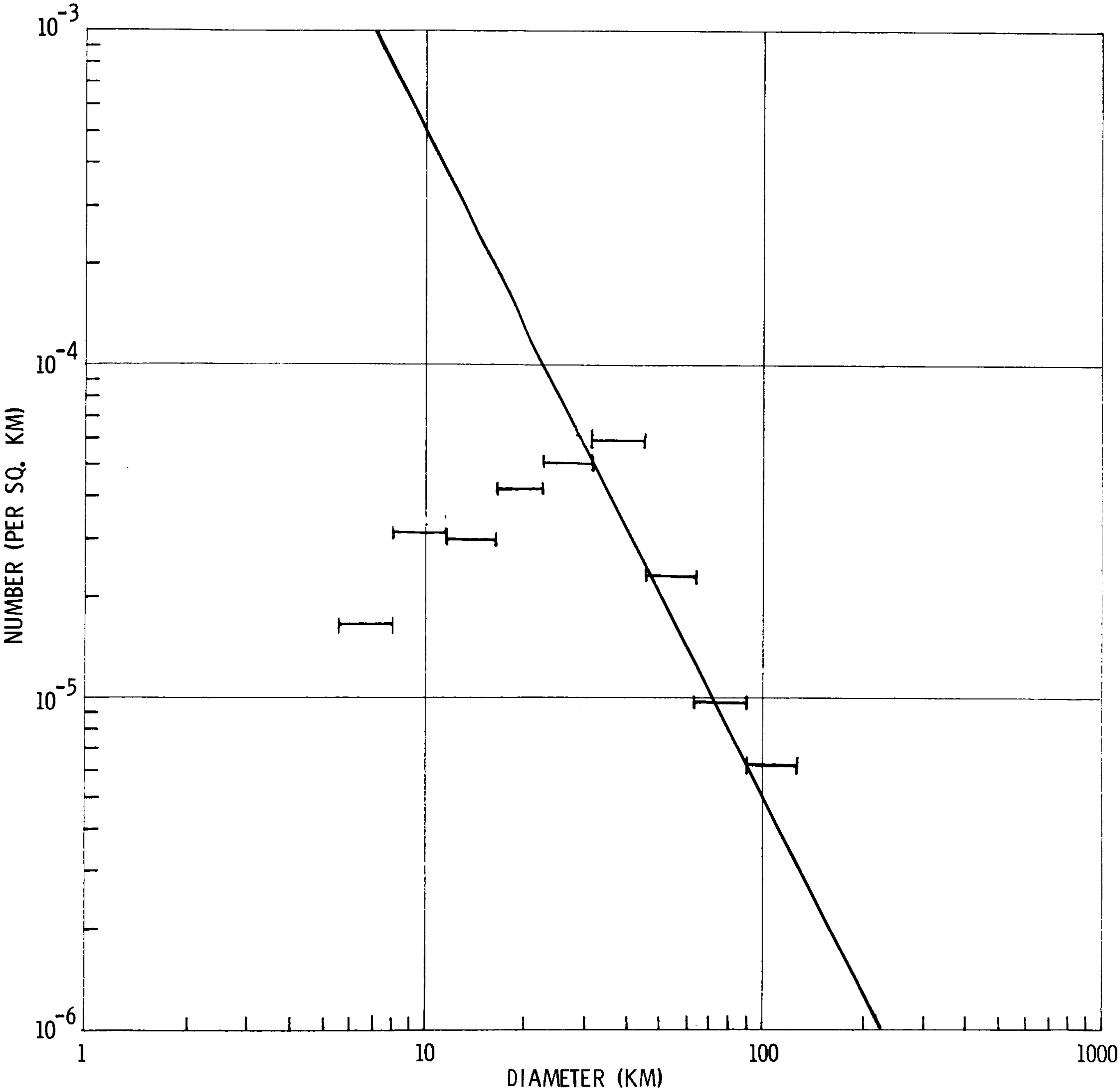

FIGURE 1 - FRAMES 3-4

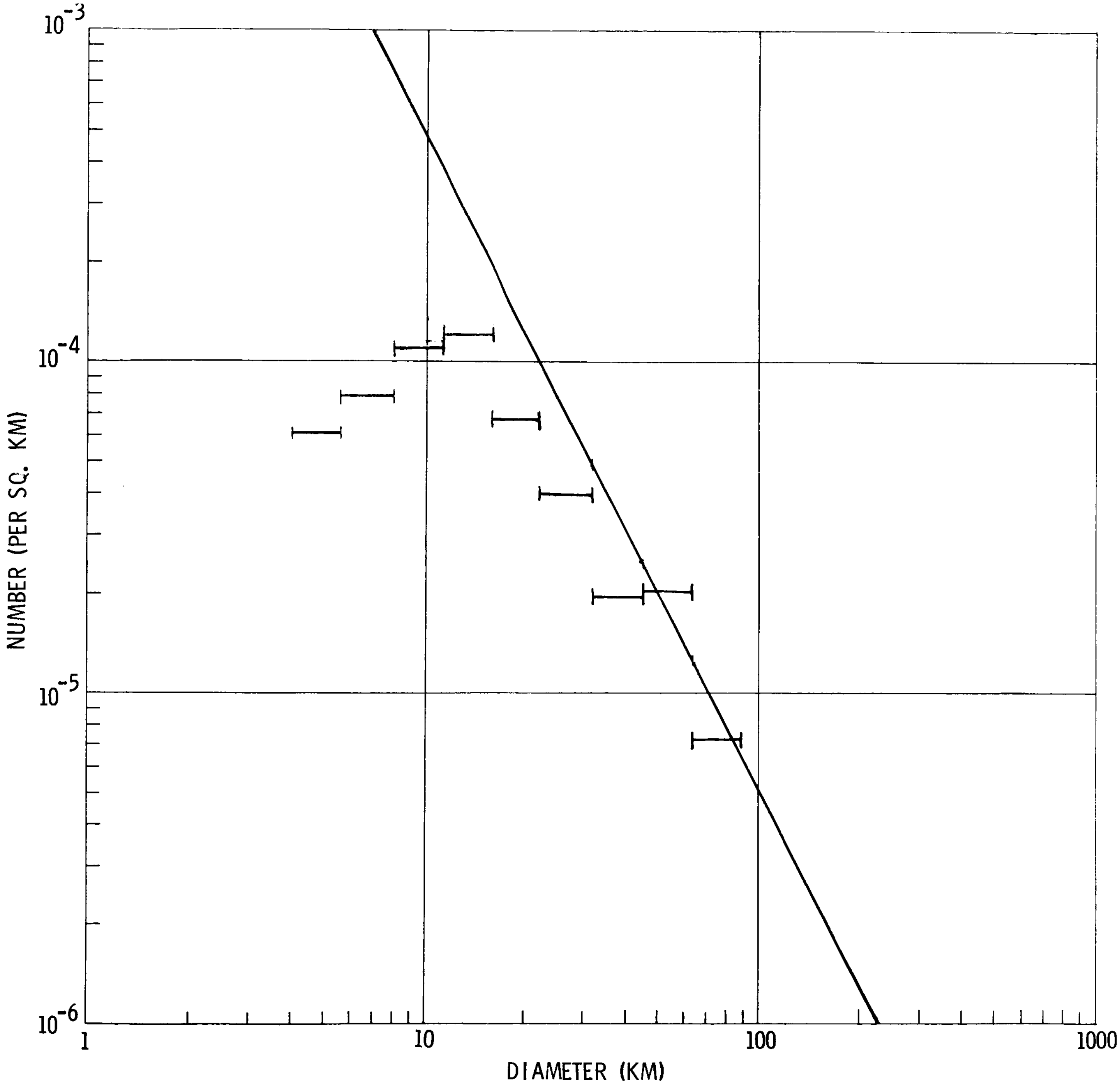

FIGURE 2 - FRAMES 5-6

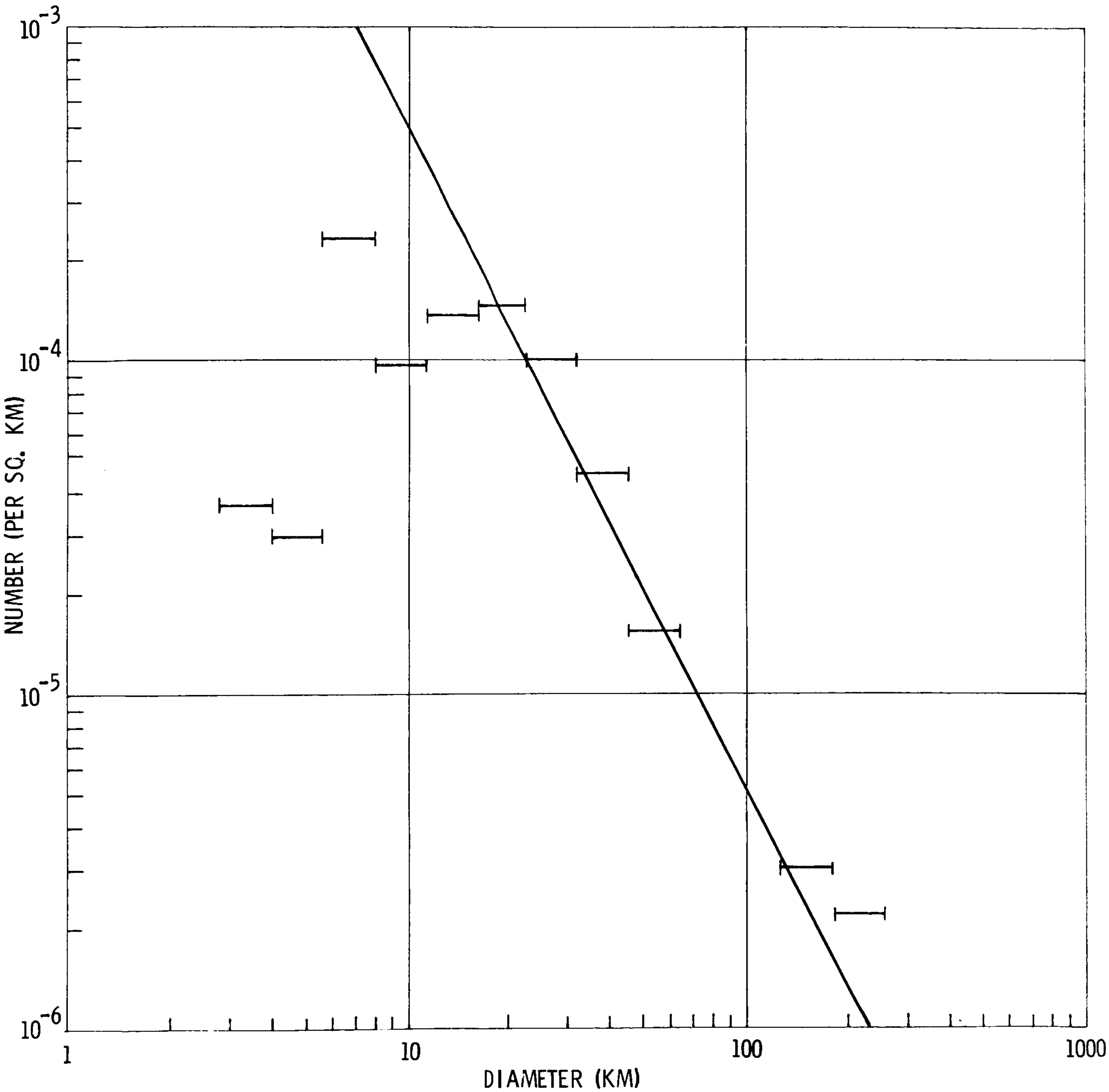

FIGURE 3 - FRAMES 7-8

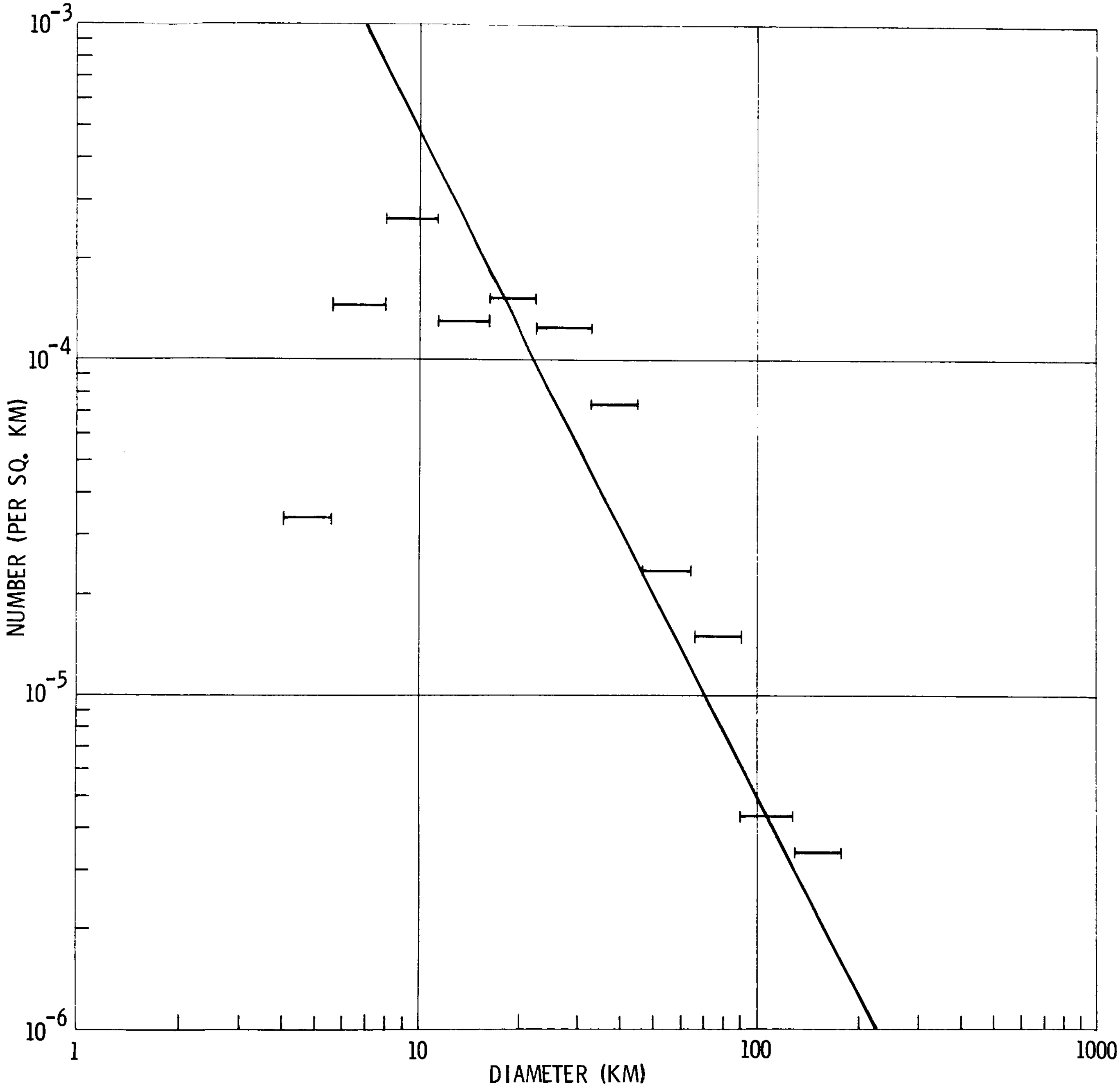

FIGURE 4 - FRAMES 9-10

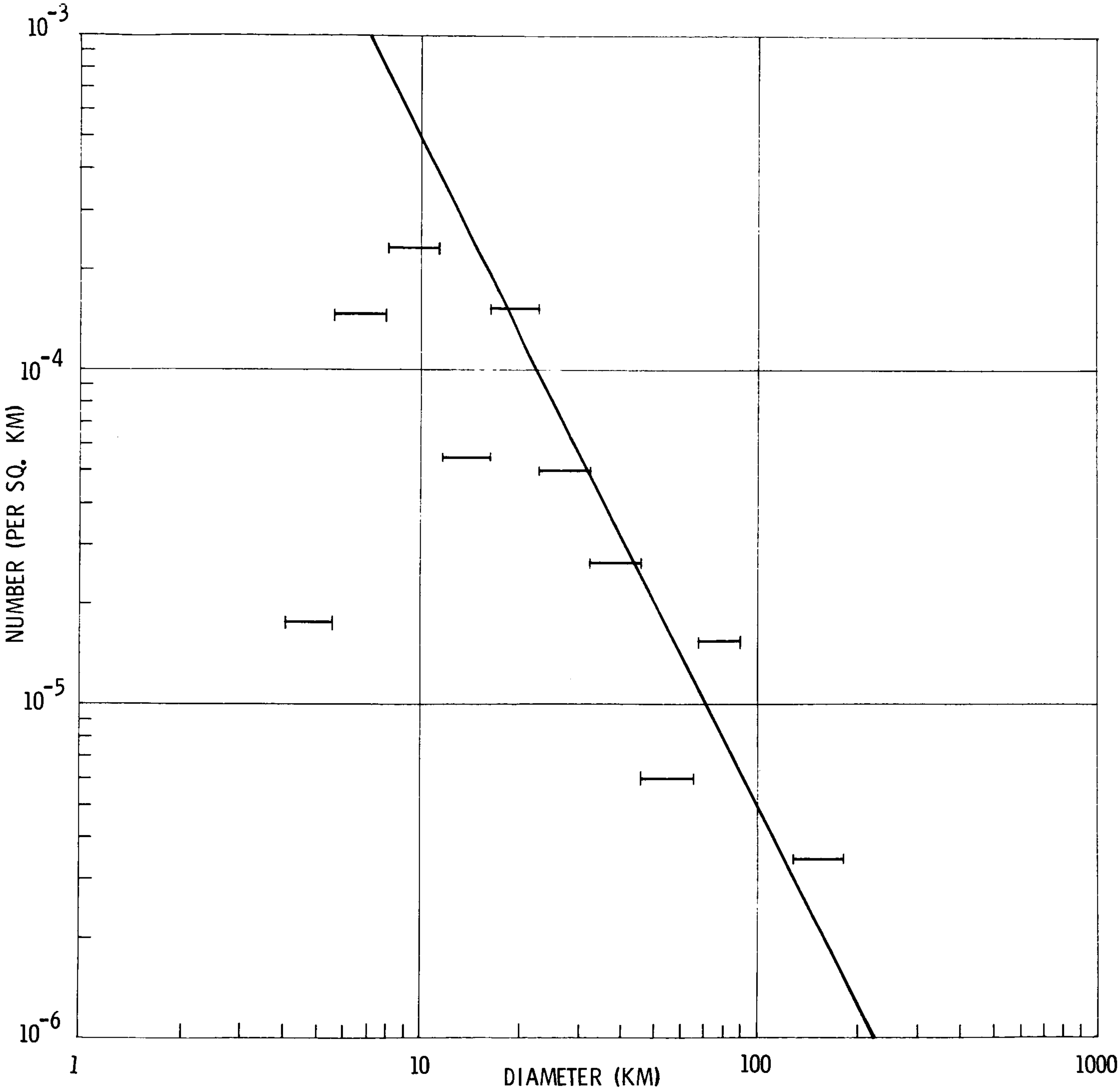

FIGURE 5 - FRAMES 11-12

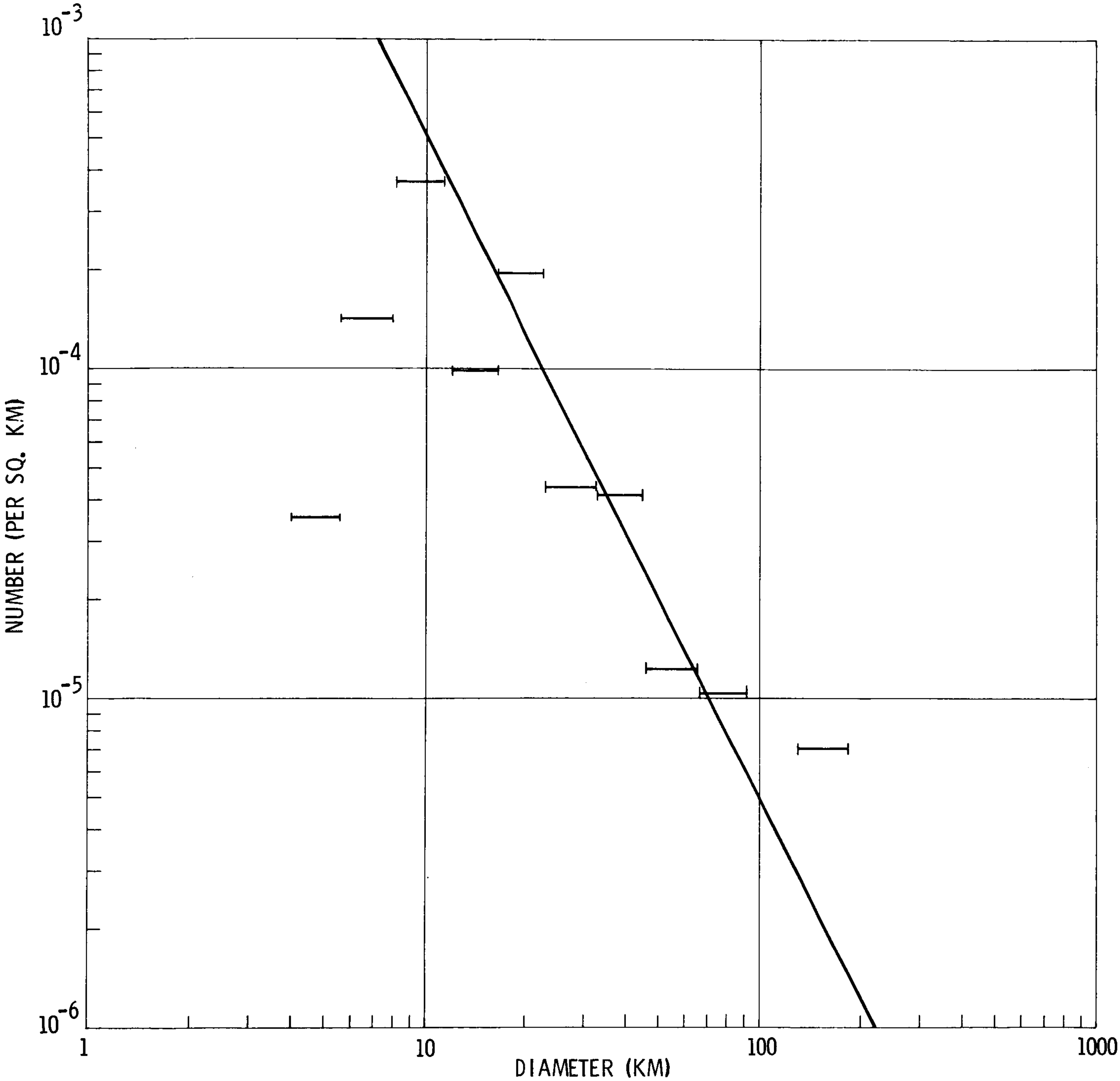

FIGURE 6 - FRAME 11 (TOP)

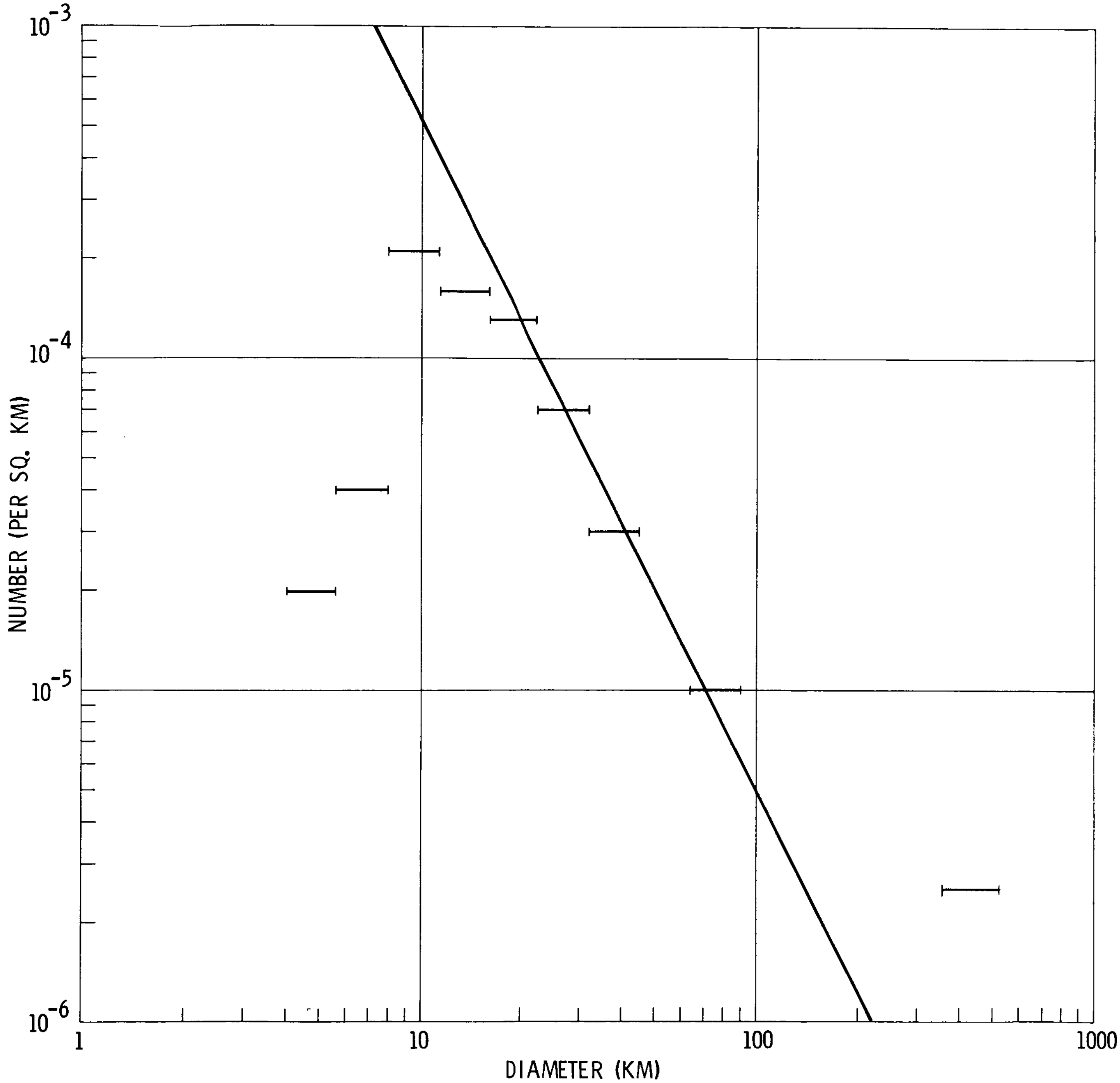

FIGURE 7 - FRAMES 13-14

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