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ANALYTICAL INVESTIGATION OF SUPERSONIC TURBOMACHINERY BLADING

I - Computer Program for Blading Design

by Louis J. Goldman and Vincent J. Scullin

Lewis Research Center
Cleveland, Ohio

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CONTENTS

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SUMMARY

A FORTRAN IV computer program for the design of supersonic blading based on establishing vortex flow within the blade passage is presented. The method of characteristics, as applied to the two-dimensional isentropic flow of a perfect gas, was utilized for the blade design. The equations necessary for the design are developed. The information required for the program consists of an inlet flow angle, specification of the inlet, outlet, and lower- and upper-circular-surface Mach numbers, and the specific-heat ratio. The program output consists of the blade coordinates and, if desired, a printer plot of the blade profile and flow passage. In addition, supersonic starting and flow separation calculations are performed by the program and obtained as output. An example is included to indicate the use of the program and the results obtainable.

INTRODUCTION

Supersonic compressors and turbines are employed in special circumstances because of their simplicity and low weight. A recent application for a supersonic turbine involves the hydrogen-fueled open-cycle auxiliary space power system described in reference 1. If the highest practical efficiency is to be obtained from supersonic compressors or turbines, proper design methods must be available.

A method for designing supersonic blade sections based on two-dimensional isentropic flow is given in reference 2. The method consists of converting the uniform parallel flow at the blade inlet into a vortex flow field, turning the vortex flow, and reconverting to a uniform parallel flow at the blade exit. The application of this design procedure involves specification of the inlet and outlet Mach numbers, the lower- (or concave) surface Mach number, the upper- (or convex) surface Mach number, the inlet flow angle, and the specific-heat ratio of the working fluid. In general, a wide range of designs is possible by selection of these parameters. Guidance in the selection of a blade design is obtained by considering blade shape, solidity, and supersonic starting and flow separation problems. In reference 2 the effect of some of the design parameters for low Mach numbers and a specific-heat ratio of 1.4 is examined.

In view of the interest in hydrogen-fueled auxiliary space power systems, an analysis was conducted to gain a better understanding of the effects of the design parameters on the resulting blade geometry and to extend the results of reference 2 to levels of interest for such systems. In reference 3, the effect of surface Mach numbers, inlet flow angle, and specific-heat ratio on the geometric characteristics of supersonic impulse turbine-blade sections is investigated over an inlet Mach number range of 1.5 to 5.0. Blade design limitations resulting from supersonic starting and flow separation problems are also considered. In the present report, a description and a FORTRAN IV listing of a computer program for the design of blading applicable for any supersonic Mach number level and specific-heat ratio are presented. Supersonic starting and flow separation calculations are also performed by the program. An example is included to indicate the use of the program and the results obtainable. The report is organized so that those persons desiring to use the program need only read the sections METHOD OF ANALYSIS, DESCRIPTION OF INPUT, and DESCRIPTION OF OUTPUT. All necessary information pertaining to the program itself is contained in the sections DESCRIPTION OF INPUT, DESCRIPTION OF OUTPUT, and PROGRAM DESCRIPTION.

SYMBOLS

A	area, ft^2 (m^2)
a	speed of sound, ft/sec (m/sec)
C	reduction in maximum weight flow due to two-dimensional flow (eq. (34b))
C^*	dimensionless blade chord, chord/r^*
$f(R^*)$	function defined by eq. (10b)
G^*	dimensionless blade spacing, spacing/r^*
g_c	dimensional conversion constant, 32.17 ft-lb/(lb)(sec^2) (1 kg-m/(N)(sec^2))
h	blade height, ft (m)
j	index for upper surface of blade
K^*	dimensionless vortex constant defined by eq. (23)

K^*_{max}	value of K^* for which weight flow is maximum
k	index for lower surface of blade
M	Mach number, V/a
M^*	demensionless velocity or critical velocity ratio, V/V_{cr}
$(M^*_i)_{max}$	maximum inlet velocity ratio for supersonic starting
$(M^*_l)_{min}$	minimum lower-surface velocity ratio from separation criterion
$(M^*_u)_{max}$	maximum upper-surface velocity ratio from separation criterion
m	slope of Mach line
$\overline{m}$	slope of wall segment
p	static pressure, lb/ft^2 (N/m^2)
$(p_l)_{max}$	maximum lower-surface static pressure from separation criterion
Q	vortex flow parameter (eq. 34a))
R	radius in vortex field, ft (m)
R^*	dimensionless radius in vortex field, R/r^*
r^*	radius of sonic velocity streamline in vortex field, ft (m)
V	velocity, ft/sec (m/sec)
V_{cr}	critical velocity, ft/sec (m/sec)
w	weight flow, lb/sec (kg/sec)
w_{max}	maximum weight flow, lb/sec (kg/sec)
X^*	dimensionless X-coordinate of blade (fig. 1), X/r^*
x^*	dimensionless x-coordinate of transition arc (fig. 1), x/r^*
Y^*	dimensionless Y-coordinate of blade (fig. 1), Y/r^*
y^*	dimensionless y-coordinate of transition arc (fig. 1), y/r^*
α	circular arc turning angle, rad
β	flow angle outside blade passage, rad
γ	specific-heat ratio
θ	total flow turning angle, rad
μ	Mach angle, rad

ν	Prandtl-Meyer angle, angle through which flow must turn from Mach 1 to required Mach number, rad
$\Delta\nu$	incremental flow turning, rad
$(\nu_i)_{max}$	maximum inlet Prandtl-Meyer angle for supersonic starting, rad
$(\nu_l)_{min}$	minimum lower-surface Prandtl-Meyer angle from separation criterion, rad
$(\nu_u)_{max}$	maximum upper-surface Prandtl-Meyer angle from separation criterion, rad
ρ	density, lb/ft^3 (kg/m^3)
σ	blade solidity, C^*/G^*
φ	velocity direction angle, rad

Subscripts:

d	downstream of normal shock
i	blade inlet
j	index for upper surface of blade
k	index for lower surface of blade
l	lower surface of blade
max	maximum
min	minimum
o	blade outlet
u	upper surface of blade

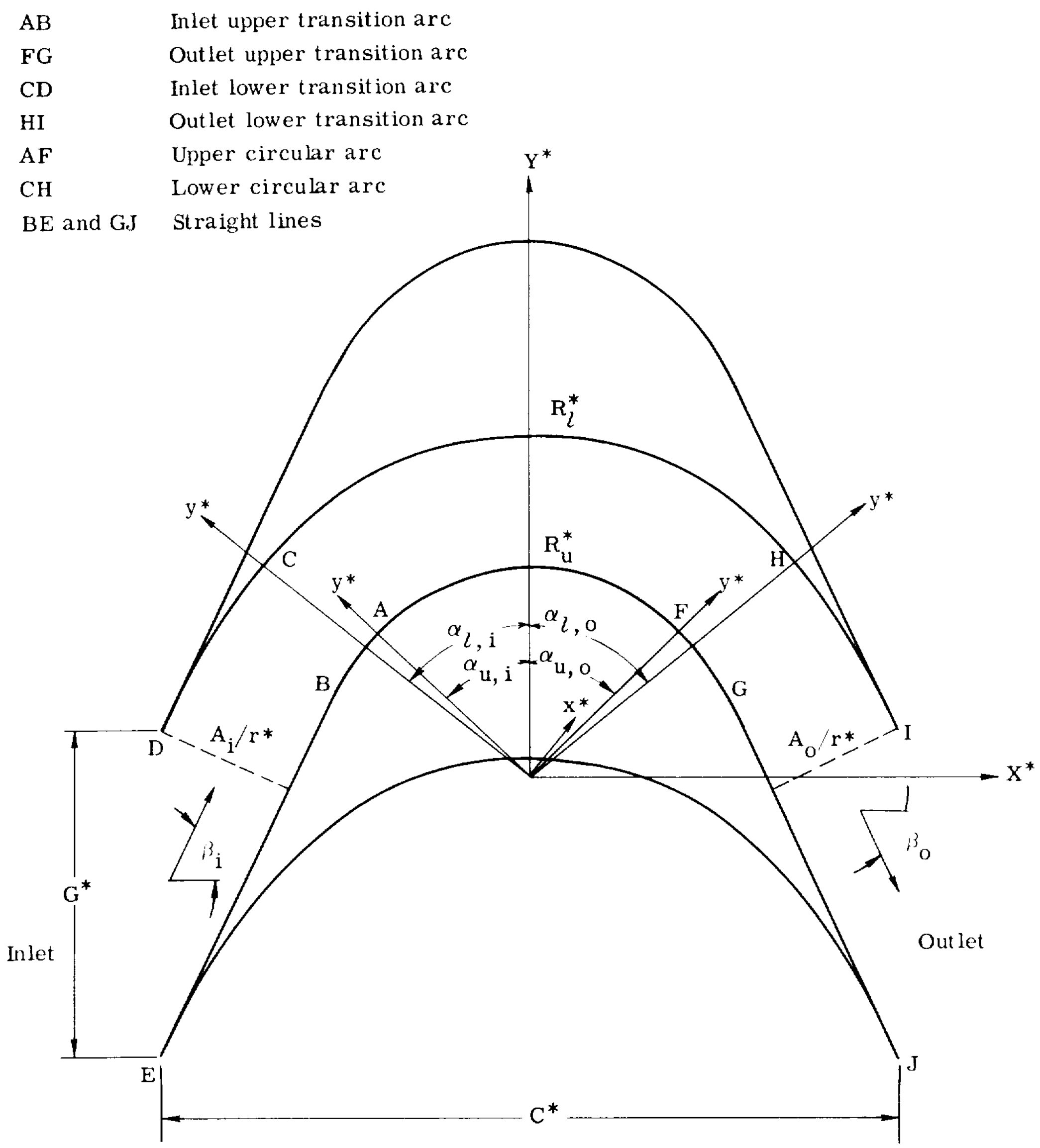
Superscripts:

	total-state conditions

METHOD OF ANALYSIS

Blade Description

The design of supersonic blade sections described herein is based on establishing vortex flow within the blade passage by a procedure analogous to that given in reference 2. The blade so designed consists essentially of three major parts: (1) inlet transition arcs, (2) circular arcs, and (3) outlet transition arcs. A typical blade, with pertinent nomenclature noted, is shown schematically in figure 1. The inlet transition arcs



AB	Inlet upper transition arc
FG	Outlet upper transition arc
CD	Inlet lower transition arc
HI	Outlet lower transition arc
AF	Upper circular arc
CH	Lower circular arc
BE and GJ	Straight lines

Figure 1. - Typical supersonic blade section. (All coordinates are made dimensionless by dividing by r*.)

(lower and upper) are required to convert the assumed uniform parallel flow at the blade inlet into vortex flow. The concentric circular arcs turn and maintain the vortex flow. Finally, the outlet transition arcs reconvert the vortex flow into uniform parallel flow at the blade exit. Straight-line segments parallel to the inlet and outlet flow directions complete the blade profile. Methods for creating a finite thickness at the leading and trailing edges are given in reference 2.

In general, the inlet lower transition arc reduces the Mach number from its value at the blade inlet M_i to a preselected value of the lower-surface Mach number M_l, whereas the inlet upper transition arc increases the Mach number to a preselected value

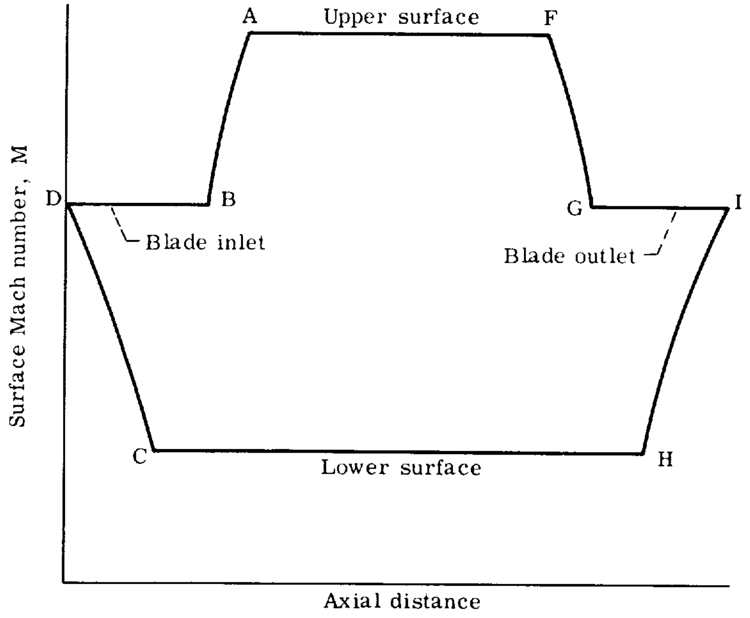


Figure 2. - Surface Mach number variation for typical blade section.

of the upper-surface Mach number M_u. The surface Mach numbers remain constant, at these preselected values, on the lower and upper circular arcs. At the outlet region the procedure is reversed. The surface Mach number variation is shown in figure 2 for a typical blade.

The amount of flow turning produced by either the lower or upper surface of the blade consists, in general, of two parts (fig. 1): (1) the turning produced by the transition arcs and (2) the turning produced by the circular arcs. When isentropic flow turning at supersonic speeds is considered, it is convenient to introduce the Prandtl-Meyer angle ν, which is defined as the angle through which the flow must turn from Mach 1 to the required Mach number. The flow turning produced by a transition arc is then equal to differences in Prandtl-Meyer angles and is $\nu_i - \nu_l$ and $\nu_o - \nu_l$ for the inlet and outlet lower transition arcs and $\nu_u - \nu_i$ and $\nu_u - \nu_o$ for the inlet and outlet upper transition arcs, respectively. The turning produced by the inlet or outlet transition arcs cannot exceed the inlet or outlet flow angle β_i or β_o, respectively. The relation between Prandtl-Meyer angle ν and Mach number M is given by the following equations (ref. 4):

$$\nu = \frac{\pi}{4}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right) + \frac{1}{2}\left\{\sqrt{\frac{\gamma + 1}{\gamma - 1}} \arcsin\left[(\gamma - 1)M^{*2} - \gamma\right] + \arcsin\left(\frac{\gamma + 1}{M^{*2}} - \gamma\right)\right\} \tag{1}$$

where the dimensionless velocity or critical velocity ratio M^* is

$$M^* = \left(\frac{\frac{\gamma + 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2}\right)^{1/2} \tag{2}$$

This relation is shown in figure 3 for specific-heat ratios of 1.3, 1.4, and 1.66. As the Mach number approaches infinity, ν approaches an upper limit of $\frac{\pi}{2}\left[\sqrt{(\gamma + 1)/(\gamma - 1)} - 1\right]$.

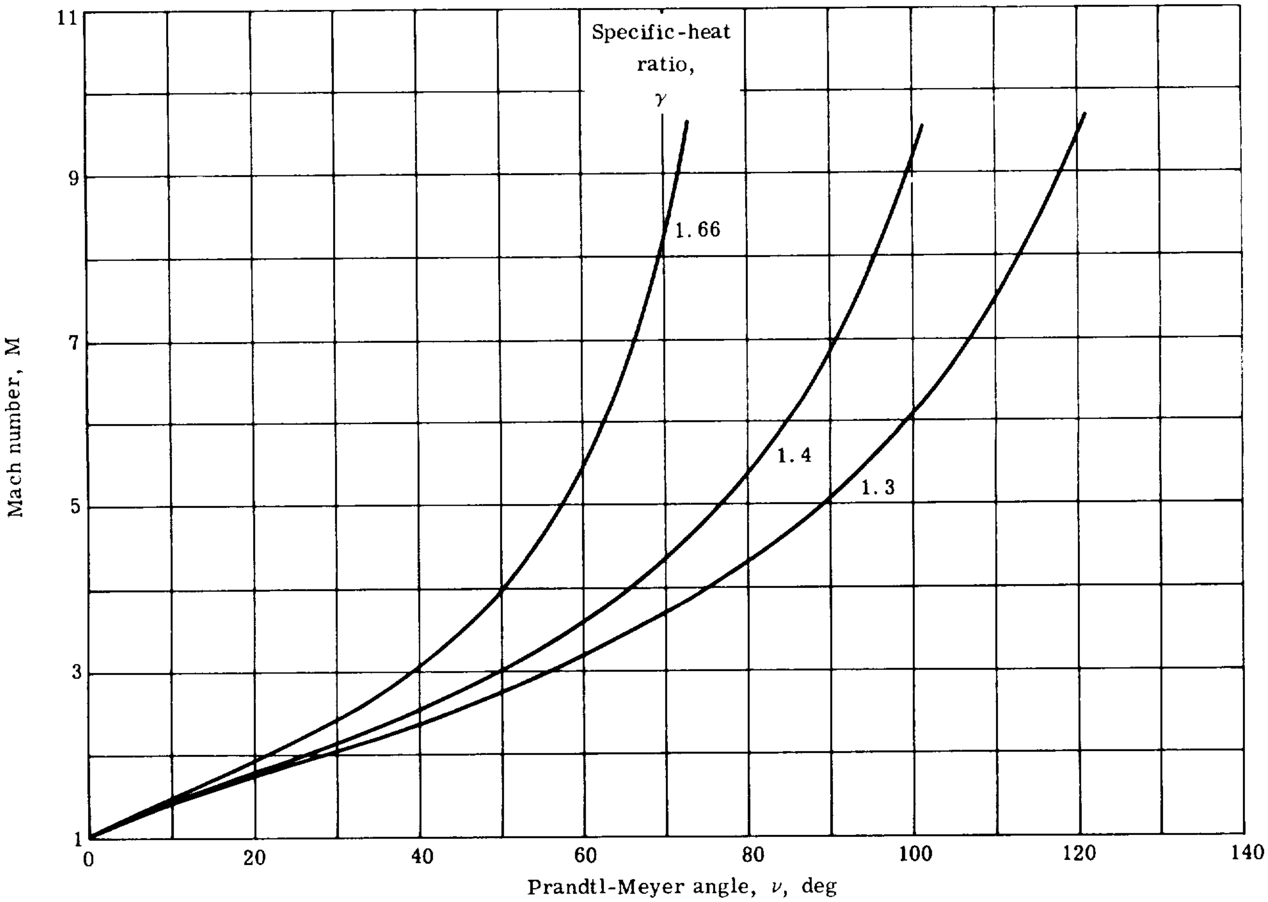


Figure 3. - Variation of Prandtl-Meyer angle with Mach number for different specific-heat ratios.

Blade Design

The method of characteristics as applied to the two-dimensional isentropic flow of a perfect gas is utilized in the design of the supersonic blade sections. A description of the method of characteristics is given in references 4 and 5, and its application to the design of supersonic blade sections is given in references 2 and 6. For purposes of calculation, the flow field is considered to be divided into small regions, in each of which the flow properties are assumed to be constant. If adjacent regions are to differ slightly in properties, then the boundary between the regions must be characteristic lines and can also be shown (ref. 5) to be Mach lines. Therefore, each region is, in general, bounded either by a Mach line or a physical boundary. In figure 4 the flow field for a typical blade passage is divided by characteristic lines into a finite number of regions. The vortex-flow region is bounded by the circular arcs and the outermost vortex characteristics AE and CE. The transition arcs are composed of straight-line segments, and within each region bounded by these segments the flow is constrained to follow the wall direction. The mathematical equations necessary to define the blade are developed in the following sections.

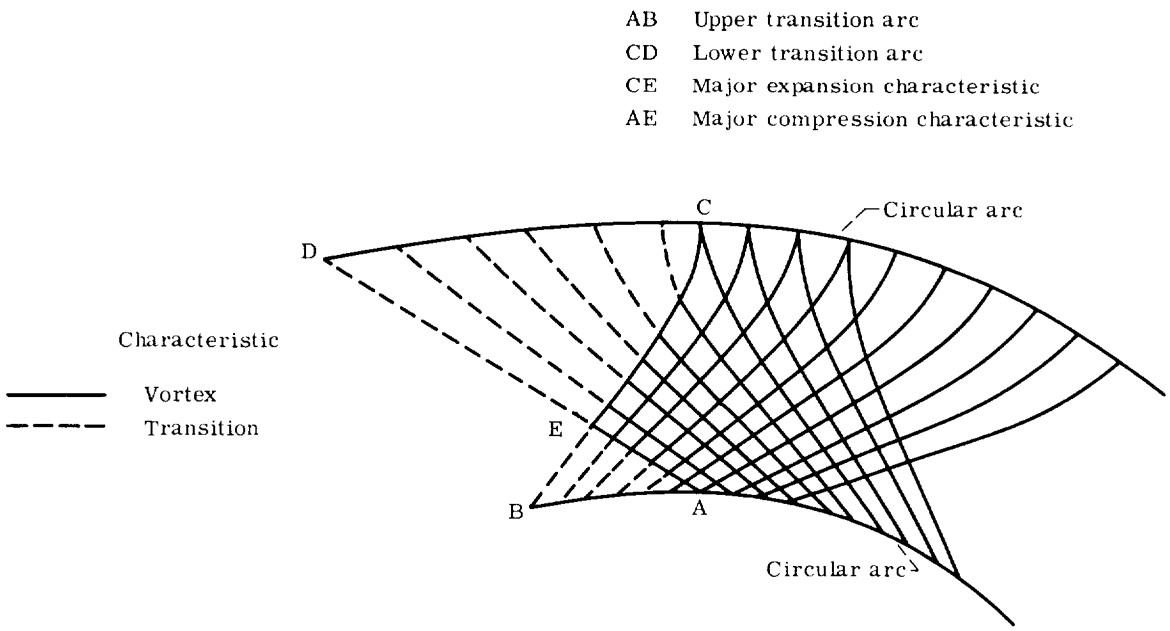


Figure 4. - Characteristic network within blade passage.

<u>Circular arcs</u>. - Within the concentric circular arcs, vortex flow exists; therefore,

$$VR = \text{Constant} \tag{3}$$

where V is the velocity and R is the radius in the vortex field. In this report, dimensionless parameters are used whenever possible; if this procedure is followed, equation (3) can be rewritten as

$$\left(\frac{V}{V_{cr}}\right)\left(\frac{R}{r^*}\right) = \frac{\text{Constant}}{V_{cr}r^*} \tag{4}$$

where V_{cr} is the critical velocity and r^* is the radius of the sonic velocity streamline in the vortex field. At $R = r^*$, $V = V_{cr}$; therefore, the constant is $V_{cr}r^*$. Equation (4) then becomes

$$M^*R^* = 1 \tag{5}$$

where $M^* = V/V_{cr}$ is the dimensionless velocity and $R^* = R/r^*$ is the dimensionless radius in the vortex field. The Prandtl-Meyer angle ν is related to M^* through equation (1). Therefore, once ν_l and ν_u are specified, M_l^* and M_u^* are fixed, and the circular arc radii R_l^* and R_u^* are determined from equation (5). The amount of lower circular arc turning for the inlet and outlet portions of the blade $\alpha_{l,i}$ and $\alpha_{l,o}$, respectively, are

$$\alpha_{l,i} = \beta_i - (\nu_i - \nu_l) \tag{6a}$$

and

$$\alpha_{l,o} = \beta_o + (\nu_o - \nu_l) \tag{6b}$$

Angles measured in the counterclockwise direction are considered positive. With this convention, the inlet flow angle β_i is positive, and the outlet flow angle β_o is negative. Similarly, for the upper circular arc

$$\alpha_{u,i} = \beta_i - (\nu_u - \nu_i) \tag{7a}$$

and

$$\alpha_{u,o} = \beta_o + (\nu_u - \nu_o) \tag{7b}$$

The circular arcs are completely described by specification of ν_i, ν_o, ν_l, ν_u, and β_i.

The outlet flow angle β_o does not have to be specified because it can be related to M_i, M_o, and β_i from the following consideration.

Because the inlet and outlet blade spacing is the same, the inlet and outlet blade passage areas A_i and A_o, respectively, are related by geometry (see fig. 1) according to the equation

$$\frac{A_i}{A_o} = \frac{\cos \beta_i}{\cos \beta_o} \tag{8}$$

The area ratio A_i/A_o can be obtained from the continuity equation (ref. 5) with the result that equation (8) becomes

$$\beta_o = -\text{arc cos}\left\{\left[\frac{M_i}{M_o}\left(\frac{1 + \frac{\gamma - 1}{2} M_o^2}{1 + \frac{\gamma - 1}{2} M_i^2}\right)^{(\gamma+1)/2(\gamma-1)}\right]\cos \beta_i\right\} \tag{9}$$

Lower transition arcs. - The lower surface is composed of an inlet and outlet transition arc and a circular arc (fig. 1). For symmetric blades (i.e., $\nu_i = \nu_o$), the two transition arcs are identical, and, therefore, only one needs to be calculated. For asymmetric blades, the two arcs are not identical, one being smaller (less turning) than the other. However, the smaller transition arc corresponds to a portion of the larger arc, and, consequently, only the larger arc need be calculated. If ν_i is greater than ν_o, the inlet transition arc is the larger of the two arcs. For simplicity, the inlet transition arc is assumed to be the larger arc in the following discussion.

In figure 5, the lower transition arc is shown, and the nomenclature used in the computer program is indicated. The calculations are performed with respect to the nondimensional axes x^* and y^* (where the x- and y-coordinates are made dimensionless by dividing by r^*). The transition arc coordinates are generated in a sequential manner (starting at $x^* = 0$, $y^* = R_l^*$) by obtaining the intersection of the straight-line wall segments and straight Mach lines for a specified small change in flow turning. The Mach lines are determined from the outermost or major vortex-expansion characteristic, and the wall segments are determined from the flow direction. After the transition arc coordinates are calculated, they are rotated through an angle of $\alpha_{l,i}$ to obtain the coordinates of interest in the blade design (see fig. 1).

For the vortex region, it can be shown (ref. 2) that the velocity direction φ and the dimensionless radius R^* are related along the characteristic line by the equation

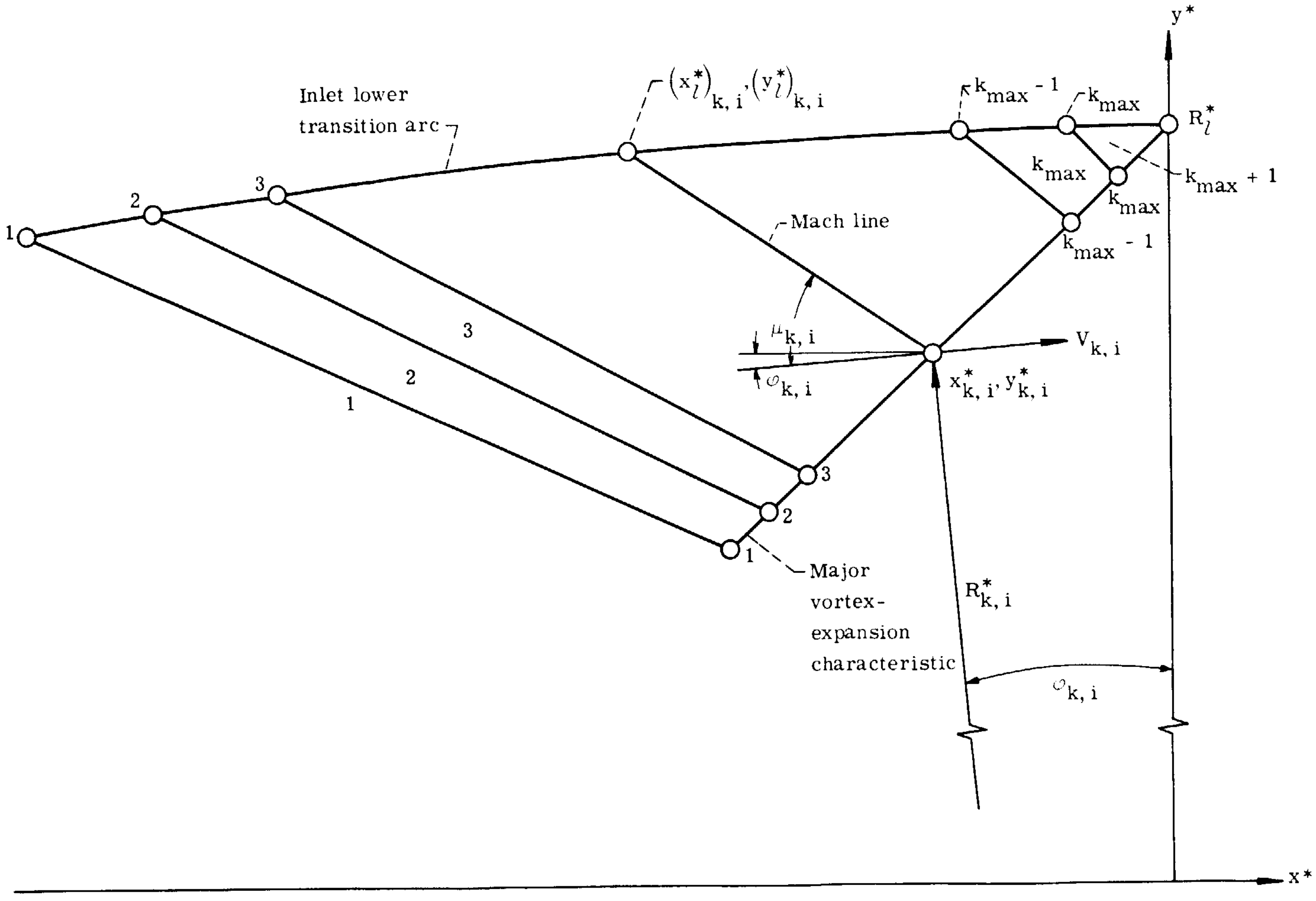


Figure 5. - Nomenclature used for calculation of inlet lower transition arc.

$$\varphi = \pm \frac{1}{2} f(R^*) + \text{Constant} \tag{10a}$$

where

$$f(R^*) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \arcsin\left(\frac{\gamma - 1}{R^{*2}} - \gamma\right) + \arcsin\left[(\gamma + 1)R^{*2} - \gamma\right] \tag{10b}$$

Two families of characteristics exist: the positive sign in equation (10a) gives the expansion lines, and the negative sign gives the compression lines. The major vortex-expansion-characteristic equation is

$$\varphi = \frac{1}{2}\left[f(R^*) - f(R_l^*)\right] \tag{11}$$

since the boundary condition at $x^* = 0$ is that $\varphi = 0$ and $R^* = R_l^*$. If the flow field is divided into small regions, the flow direction at any point along the characteristic line (as given by eq. (11)) may be considered to be equal to the flow direction within the adjacent flow region, as indicated in figure 5. For k transition arc segments, each of which produces $\Delta\nu$ degrees of turning, the flow direction within any flow segment $\varphi_{k,i}$ is given by

$$\varphi_{k,i} = \nu_i - \nu_l - (k - 1)\Delta\nu = \varphi_{k+1,i} + \Delta\nu \tag{12}$$

where k is an integer that varies from 1 to $\left[(\nu_i - \nu_l)/\Delta\nu\right] + 1$. At $k = \left[(\nu_i - \nu_l)/\Delta\nu\right] + 1$, the flow direction is 0, and at $k = 1$, it is $\nu_i - \nu_l$. Equating equations (11) and (12) and eliminating $f(R_l^*)$ through equations (1), (5), and (10b) result in

$$f(R_{k,i}^*) = 2\nu_i - \frac{\pi}{2}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right) - 2(k - 1)\Delta\nu \tag{13}$$

which relates the dimensionless radius R^* to the incremental flow turning $\Delta\nu$ along the major vortex-expansion characteristic. At $k = \left[(\nu_i - \nu_l)/\Delta\nu\right] + 1$, $R_{k,i}^* = R_l^*$; as k is decreased, $R_{k,i}^*$ decreases and is obtained by the simultaneous solution of equations (10b) and (13) by an iterative procedure. Since $R_{k,i}^*$ can be determined for any value of k, the x^*, y^* coordinates along the major expansion characteristic can be obtained from

$$x^*_{k,i} = -R^*_{k,i} \sin \varphi_{k,i} \tag{14a}$$

and

$$y^*_{k,i} = R^*_{k,i} \cos \varphi_{k,i} \tag{14b}$$

These coordinates are also points on the straight Mach lines. The equation specifying the Mach line is therefore determined once the slope is obtained, which is easily accomplished because the Mach line is inclined at the Mach angle μ to the velocity direction (see fig. 5). The Mach line, therefore, makes an angle of $\varphi + \mu$ with respect to the x^* axis. It is possible to define the Mach line at the mean Mach angle to the mean flow direction (ref. 5) so that the slope of the Mach line $m_{k,i}$ is given by

$$m_{k,i} = \tan\left(\frac{\varphi_{k,i} + \varphi_{k+1,i}}{2} + \frac{\mu_{k,i} + \mu_{k+1,i}}{2}\right) \tag{15a}$$

where

$$\mu_{k,i} = -\text{arc sin}\left(\frac{1}{M_{k,i}}\right) = -\text{arc sin}\left[\sqrt{\left(\frac{\gamma + 1}{2}\right)R^{*2}_{k,i} - \left(\frac{\gamma - 1}{2}\right)}\right] \tag{15b}$$

The equation of the Mach line is therefore

$$y^* = m_{k,i}(x^* - x^*_{k,i}) + y^*_{k,i} \tag{16}$$

where k varies from 1 to $k_{max} = (\nu_i - \nu_l)/\Delta\nu$. The equation of the transition arc segment is more easily determined since each segment is a straight line parallel to the velocity direction φ. The slope of the wall segment $\overline{m}_{k,i}$ is then

$$\overline{m}_{k,i} = \tan \varphi_{k+1,i} \tag{17}$$

and the equation for the wall segment is

$$y^* = \overline{m}_{k,i}\left[x^* - (x^*_l)_{k+1,i}\right] + (y^*_l)_{k+1,i} \tag{18}$$

where k varies from 1 to k_{max} and x_l^* and y_l^* are the lower transition arc coordinates. The values of x_l^* and y_l^* are known at $k = (\nu_i - \nu_l)/\Delta\nu + 1 = k_{max} + 1$, where $x_l^* = 0$ and $y_l^* = R_l^*$. The remaining transition coordinates are generated by finding the intersection of the Mach lines with the wall segments starting at $k = k_{max}$ and sequentially decreasing k until $k = 1$. The intersection of the two straight lines is given by

$$(x_l^*)_{k,i} = \frac{\left[(y_l^*)_{k+1,i} - \overline{m}_{k,i}(x_l^*)_{k+1,i}\right] - (y^*_{k,i} - m_{k,i}x^*_{k,i})}{m_{k,i} - \overline{m}_{k,i}} \tag{19a}$$

and

$$(y_l^*)_{k,i} = \frac{m_{k,i}\left[(y_l^*)_{k+1,i} - \overline{m}_{k,i}(x_l^*)_{k+1,i}\right] - \overline{m}_{k,i}(y^*_{k,i} - m_{k,i}x^*_{k,i})}{m_{k,i} - \overline{m}_{k,i}} \tag{19b}$$

The transition arc coordinates obtained from equation (19) are rotated through an angle $\alpha_{l,i}$ resulting in the X^*, Y^* coordinates of interest in the blade design. The rotated coordinates X_l^* and Y_l^* are obtained from

$$(X_l^*)_{k,i} = (x_l^*)_{k,i} \cos \alpha_{l,i} - (y_l^*)_{k,i} \sin \alpha_{l,i} \tag{20a}$$

and

$$(Y_l^*)_{k,i} = (x_l^*)_{k,i} \sin \alpha_{l,i} + (y_l^*)_{k,i} \cos \alpha_{l,i} \tag{20b}$$

Upper transition arcs. - The upper surface (like the lower surface) is composed, in part, of an inlet and outlet transition arc, only one of which must be calculated. For simplicity, it is again assumed that the inlet transition arc is the larger of the two arcs. (For the upper arc, this requires that ν_o be greater than ν_i.) The upper transition arc is shown schematically in figure 6, and the pertinent nomenclature is noted. The procedure employed to calculate the upper transition arc is analogous to that used for the lower transition arc; the resulting equations which are of similar form are not repeated herein. The subscript j is used to represent the upper transition arc coordinates where j varies from 1 to $j_{max} = (\nu_u - \nu_i)/\Delta\nu$.

Geometric parameters. - After the blade calculations have been performed, a number of blade parameters of interest, including blade solidity, spacing, chord, and total

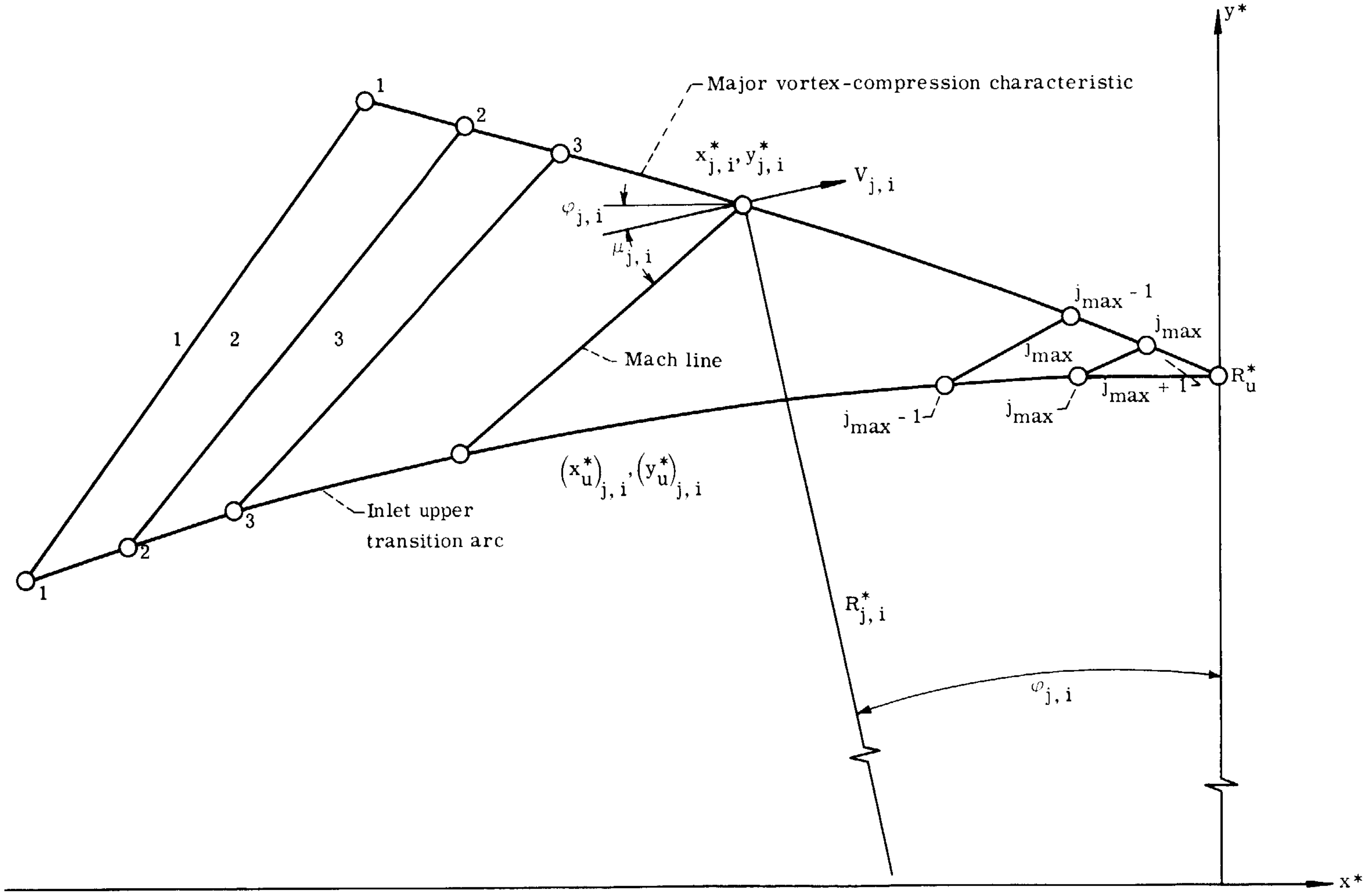


Figure 6. - Nomenclature used for calculation of inlet upper transition arc.

flow turning angle, are calculated. The blade spacing G^* and chord C^* are obtained from the blade coordinates, and the solidity is obtained from the ratio C^*/G^*. The total flow turning angle θ is obtained from the inlet and outlet flow angles. The blade coordinates are also translated by G^* so that coordinates for two complete blades are obtained.

Design Limitations

The design limitations (i.e., the constraints on the choice of ν_l and ν_u for specified ν_i, ν_o, β_i, and γ) imposed by consideration of supersonic starting and flow separation problems have been discussed in reference 3. These limitations are calculated by the procedure described in the following paragraphs and are given as output from the computer program presented herein.

Supersonic starting. - The problem of establishing supersonic flow on startup is discussed in reference 7 for supersonic compressors. For supersonic turbines, the resulting design limitations due to starting are presented in reference 2, where it is assumed that a normal shock wave spans the blade inlet at the instant of startup. Under this condition it is necessary to ensure that the weight flow can pass through the turbine. The maximum value of the inlet Prandtl-Meyer angle $(\nu_i)_{max}$ is determined by first finding the maximum weight flow through the blade passage, while taking into account the normal shock losses. This maximum weight flow is then equated to the flow rate after the shock has passed through the passage.

The weight flow through the passage is obtained by integrating the continuity equation in the vortex region

$$w = h \int_{R_u}^{R_l} \rho V \, dR \tag{21}$$

where ρ is the density, V is the velocity, and h is the blade height. The density can be written as (ref. 2)

$$\rho = \rho'_{i,d}\left[1 - \left(\frac{\gamma - 1}{2}\right)\left(\frac{V}{a'_{i,d}}\right)^2\right]^{1/(\gamma-1)} \tag{22}$$

where $\rho'_{i,d}$ and $a'_{i,d}$ are the density and the speed of sound, respectively, just downstream of the shock, and are evaluated at total conditions. For perfect gases, the total temperature is constant through a normal shock so that $a'_{i,d} = a'_i$. Utilizing equation (3) and the definition

$$K^* = \sqrt{\frac{\gamma - 1}{2}}\left(\frac{VR}{a'_i R_l}\right) \tag{23}$$

results in equation (21) in the following form:

$$w = ha'_i \rho'_{i,d} R_l \sqrt{\frac{2}{\gamma - 1}} \int_{R_u}^{R_l} \left(1 - \frac{K^{*2} R_l^2}{R^2}\right)^{1/(\gamma-1)} \frac{K^*}{R}\, dR \tag{24}$$

Differentiating equation (24) with respect to K^* and setting the result equal to 0 give the value of K^* (denoted as K^*_{max}) for which the weight flow is a maximum. This procedure gives

$$\frac{dw}{dK^*} = ha'_i \rho'_{i,d} R_l \sqrt{\frac{2}{\gamma - 1}} \int_{R_u}^{R_l} \left[\left(1 - \frac{K^{*2}_{max} R_l^2}{R^2}\right)^{1/(\gamma-1)} - \frac{2}{\gamma - 1}\left(\frac{K^*_{max} R_l}{R}\right)^2 \left(1 - \frac{K^{*2}_{max} R_l^2}{R^2}\right)^{(2-\gamma)/(\gamma-1)}\right] \frac{dR}{R} \equiv 0 \tag{25}$$

Changing the variable from R to M^* gives, for equation (25),

$$\int_{M_l^*}^{M_u^*} \left[1 - \left(\frac{K_{max}^*}{M_l^*}\right)^2 M^{*2}\right]^{1/(\gamma-1)} \frac{dM^*}{M^*}$$

$$= \frac{2}{\gamma - 1} \int_{M_l^*}^{M_u^*} \left(\frac{K_{max}^*}{M_l^*}\right)^2 \left[1 - \left(\frac{K_{max}^*}{M_l^*}\right)^2 M^{*2}\right]^{(2-\gamma)/(\gamma-1)} M^*\, dM^* \qquad (26)$$

Integrating the right side of equation (26) results in

$$\int_{M_l^*}^{M_u^*} \left[1 - \left(\frac{K_{max}^*}{M_l^*}\right)^2 M^{*2}\right]^{1/(\gamma-1)} \frac{dM^*}{M^*} = \left(1 - K_{max}^{*2}\right)^{1/(\gamma-1)} - \left[1 - K_{max}^{*2}\left(\frac{M_u^*}{M_l^*}\right)^2\right]^{1/(\gamma-1)}$$

(27)

Similarly, equation (24) in terms of M^* becomes

$$w_{max} = r^* h a_i' \rho_{i,d}' \sqrt{\frac{2}{\gamma - 1}} \int_{M_l^*}^{M_u^*} \frac{K_{max}^*}{M_l^*} \left[1 - \left(\frac{K_{max}^*}{M_l^*}\right)^2 M^{*2}\right]^{1/(\gamma-1)} \frac{dM^*}{M^*} \qquad (28)$$

The value of K_{max}^* is determined from equation (27) by an iterative procedure (for given values of M_l^* and M_u^*), and then the maximum weight flow is obtained from equation (28). After the shock has passed through the turbine, the weight flow can again be obtained from equation (21), which is rewritten by substituting $VR = V_{cr} r^*$ and changing the variable from R to M^* to give

$$w = r^* h \int_{M_l^*}^{M_u^*} \rho V_{cr} \frac{dM^*}{M^*} \qquad (29)$$

Substituting the following relations (ref. 5) into equation (29)

$$\rho = \rho_i' \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma - 1)} \left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M^{*2} \right)^{1/(\gamma - 1)} \tag{30}$$

and

$$V_{cr} = a_i' \sqrt{\frac{2}{\gamma + 1}} \tag{31}$$

gives

$$w = r^* h a_i' \rho_i' \sqrt{\frac{2}{\gamma + 1}} \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma - 1)} \int_{M_l^*}^{M_u^*} \left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M^{*2} \right)^{1/(\gamma - 1)} \frac{dM^*}{M^*} \tag{32}$$

where the term multiplying $r^* h a_i' \rho_i'$ is defined as the weight-flow parameter. Equating the two weight flows, equations (28) and (32), results in

$$\frac{Q}{1 - C} = \frac{\rho_{i,d}'}{\rho_i'} = \frac{p_{i,d}'}{p_i'} \tag{33}$$

where

$$Q = \frac{M_l^* M_u^*}{M_u^* - M_l^*} \int_{M_l^*}^{M_u^*} \left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M^{*2} \right)^{1/(\gamma - 1)} \frac{dM^*}{M^*} \tag{34a}$$

and

$$C = 1 - \sqrt{\frac{\gamma + 1}{\gamma - 1}} \left(\frac{\gamma + 1}{2} \right)^{1/(\gamma - 1)} \left(\frac{M_u^*}{M_u^* - M_l^*} \right) \int_{M_l^*}^{M_u^*} K_{max}^* \left[1 - \left(\frac{K_{max}^*}{M_l^*} \right)^2 M^{*2} \right]^{1/(\gamma - 1)} \frac{dM^*}{M^*} \tag{34b}$$

The parameter C has been shown in reference 2 to be the reduction in maximum flow rate caused by two-dimensional flow. The quantity $p'_{i,d}/p'_i$ is the total pressure recovery for a normal shock and is given by (ref. 5)

$$\frac{p'_{i,d}}{p'_i} = \left(M_i^*\right)_{max}^{2\gamma/(\gamma-1)} \left[\frac{1 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\left(M_i^*\right)_{max}^2}{\left(M_i^*\right)_{max}^2 - \frac{\gamma - 1}{\gamma + 1}}\right]^{1/(\gamma-1)} \tag{35}$$

The maximum inlet velocity ratio $(M_i^*)_{max}$ is obtained (for given values of M_l^* and M_u^*) by simultaneous solution of equations (33) and (35), by using the definition in equations (34a) and (34b), and by obtaining K^*_{max} from equation (27). The maximum inlet Prandtl-Meyer angle for supersonic starting $(\nu_i)_{max}$ is then obtained from equation (1).

Flow separation. - Analysis of supersonic blade sections (ref. 3) has shown the desirability of maintaining high surface Mach numbers to alleviate the problem of supersonic starting. Under these conditions, however, adverse pressure gradients created on the blade surfaces would be expected to cause flow separation and, consequently, poor performance. Experimental investigation of simple shapes with incompressible flow at fairly high pressure gradients (ref. 8) has indicated that if the coefficient of pressure recovery (defined as the ratio of the pressure rise to the dynamic pressure at the initial point) is less than about 1/2, flow separation may be avoided. This criterion was used in the analysis presented in reference 3 to give some indication of the design restrictions due to flow separation. For supersonic velocities the separation value of the coefficient of pressure recovery may be less than 1/2. The calculational procedure is as follows.

Flow separation can occur on both the lower and upper surfaces of the blade, but since the calculational procedure is similar for both cases only the derivation for the lower surface is presented. The flow separation criterion can be written as

$$\frac{(p_l)_{max} - p_i}{\frac{\rho_i V_i^2}{2g_c}} = \frac{1}{2} \tag{36}$$

where $(p_l)_{max}$ is the maximum lower-surface pressure possible (for given inlet conditions) without causing separation. Equation (36) can be rewritten in the form

$$\frac{(p_l)_{max}}{p_i'} = \frac{p_i}{p_i'}\left[1 + \frac{1}{2}\left(\frac{\frac{1}{2g_c}\rho_i V_i^2}{p_i}\right)\right] \tag{37}$$

where the equation has been divided by the inlet total pressure p_i'. Substituting the following relations (ref. 9) into equation (37)

$$\frac{p}{p'} = \left(1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}\right)^{\gamma/(\gamma-1)} \tag{38a}$$

and

$$\frac{\frac{1}{2g_c}\rho V^2}{p} = \frac{\frac{\gamma}{\gamma + 1} M^{*2}}{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}} \tag{38b}$$

and simplifying result in

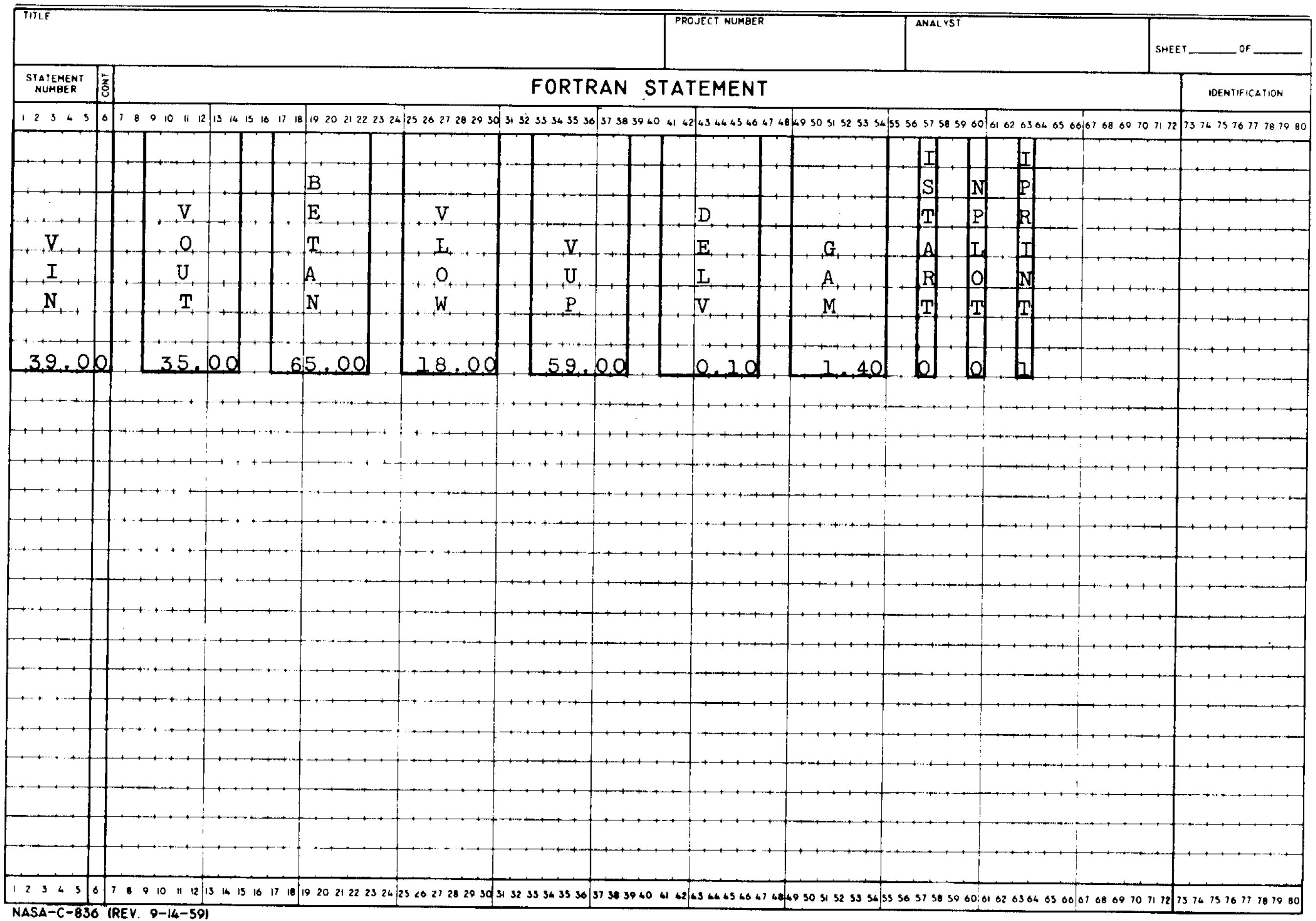
$$(M_l^*)_{min} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\left\{1 - \left(1 - \frac{\gamma - 1}{\gamma + 1} M_i^{*2}\right)\left[1 + \frac{1}{2}\left(\frac{\frac{\gamma}{\gamma + 1} M_i^{*2}}{1 - \frac{\gamma - 1}{\gamma + 1} M_i^{*2}}\right)\right]^{(\gamma-1)/\gamma}\right\}^{1/2} \tag{39}$$

Similarly, applying the same criterion to the upper surfaces gives

$$M_o^* = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\left\{1 - \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(M_u^*\right)_{max}^2\right]\left[1 + \frac{1}{2}\left(\frac{\frac{\gamma}{\gamma + 1}\left(M_u^*\right)_{max}^2}{1 - \frac{\gamma - 1}{\gamma + 1}\left(M_u^*\right)_{max}^2}\right)\right]^{(\gamma-1)/\gamma}\right\}^{1/2} \tag{40}$$

The corresponding Prandtl-Meyer angles $(\nu_l)_{min}$ and $(\nu_u)_{max}$ are obtained from equation (1).

TABLE I. - INPUT FORMAT FOR FORTRAN COMPUTER PROGRAM



TITLE		PROJECT NUMBER	ANALYST	SHEET _____ OF _____	
STATEMENT NUMBER	CONT	FORTRAN STATEMENT			IDENTIFICATION

VIN	VOUT	BETAN	VLOW	VUP	DELV	GAM	ISTART	NPLOT	IPRINT
39.00	35.00	65.00	18.00	59.00	0.10	1.40	0	0	1

NASA-C-836 (REV. 9-14-59)

CD-6729

DESCRIPTION OF INPUT

The input for the computer program consists of an inlet flow angle, several Prandtl-Meyer angles, the specific-heat ratio γ, and an angular increment. In addition, three optional switches must be set which regulate the content and form of the output. All the input parameters, except γ and the switches, must be specified in degrees. The input variables are as follows:

BETAN	inlet flow angle, β_i
DELV	flow turning increment (recommended value, 0.1), $\Delta\nu$
GAM	specific-heat ratio, γ
IPRINT	a value of 0 will result in printing of rotated blade coordinates X^* and Y^*; a value of 1 will cause both rotated and unrotated coordinates X^*, Y^* and x^*, y^* (see fig. 1) to be printed
ISTART	a value of 0 will cause both starting and blade design calculations to be printed out; a value of 1 will cause only starting calculations to be performed and printed out
NPLOT	a value of 0 will cause blade profile and flow passage to be plotted; a value of 1 will suppress the plot
VIN	inlet Prandtl-Meyer angle, ν_i
VLOW	lower-surface Prandtl-Meyer angle, ν_l
VOUT	outlet Prandtl-Meyer angle, ν_o
VUP	upper-surface Prandtl-Meyer angle, ν_u

The flow turning increment $\Delta\nu$ must be specified so that $(\nu_i - \nu_l)/\Delta\nu$, $(\nu_o - \nu_l)/\Delta\nu$, $(\nu_u - \nu_i)/\Delta\nu$, and $(\nu_u - \nu_o)/\Delta\nu$ are all integers. Table I shows a sample input card.

DESCRIPTION OF OUTPUT

An example of the output obtained from the program is shown in table II. The output corresponds to the input data shown in table I and consists of tables of coordinates for the description of two blade sections, supersonic starting and flow separation parameters, and a plot of the blade profile and flow passage. This example required approximately 0.2 minute of computer running time. Each section of the output has been numbered to correspond to the following description:

(1) The first output of the program is a listing of the supersonic starting parameters.

If ISTART=0, the program will continue with the blade design calculations. If ISTART=1, no further output is obtained.

(2) The next output is a listing of all the input data plus the value of the calculated outlet flow angle.

(3) If IPRINT=1, the following tables are printed:

(a) Unrotated coordinates of the lower- and upper-surface transition arcs

(b) Coordinates of the lower- and upper-surface circular arcs

(c) Coordinates of the upper-surface straight-line segments

This output is, in general, not of interest except for debugging purposes and may be omitted by setting IPRINT=0.

(4) The next output is tables of the rotated coordinates of the lower and upper transition arcs. In addition, these coordinates are translated by the value of the blade spacing so that coordinates for two blade sections are obtained. Every tenth calculation point is printed if $\Delta\nu < 0.2^{\circ}$; otherwise every calculation point is printed.

(5) The next output is a listing of miscellaneous parameters including

(a) Inlet, outlet, and surface dimensionless velocities or critical velocity ratios and Mach numbers

(b) Dimensionless blade spacing, chord and solidity

(c) Separation limitation for the lower- and upper-surface Prandtl-Meyer angles

(6) If NPLOT=0, the final output is a printer plot of the blade profile and the flow passage. If NPLOT=1, this output is omitted.

TABLE II. - SAMPLE OUTPUT

```
                                        DESIGN OF SUPERSONIC BLADES

                                  CALCULATIONS FOR SUPERSONIC STARTING

                                        WEIGHT-FLOW PARAMETER =    0.0516

1                  K*(MAX) =    0.3447       C =    0.0109       Q =    0.4701      M*(I(MAX)) =    1.8449

   THE MAXIMUM DESIGN VALUE FOR V(IN) IS   40.5161 DEG  WHEN V(LOW) IS   18.0000 DEG,  V(UP) IS   59.0000 DEG,  GAMMA IS  1.4000

                                              DESIGN PARAMETERS

2  BETA(IN) = 65.0000 DEG    V(IN) = 39.0000 DEG       V(UP) =  59.0000 DEG       V(OUT) = 35.0000 DEG    BETA(OUT) = -68.7395 DEG

                 DELTA V =  0.1000 DEG              V(LOW) = 18.0000 DEG               GAMMA =  1.4000


                                              LOWER SURFACE

                                        UNROTATED TRANSITION ARCS

     INLET K     X*(LOW)      Y*(LOW)                                                         Y*(LOW)     X*(LOW)    OUTLET K
       211       0.           0.6729                                                          0.6729     -0.            171
       201      -0.0237       0.6727                                                          0.6727      0.0237        161
       191      -0.0480       0.6721                                                          0.6721      0.0480        151
       181      -0.0728       0.6711                                                          0.6711      0.0728        141
       171      -0.0981       0.6695                                                          0.6695      0.0981        131
       161      -0.1241       0.6675                                                          0.6675      0.1241        121
       151      -0.1507       0.6650                                                          0.6650      0.1507        111
       141      -0.1780       0.6619                                                          0.6619      0.1780        101
       131      -0.2061       0.6582                                                          0.6582      0.2061         91
       121      -0.2349       0.6539                                                          0.6539      0.2349         81
3      111      -0.2646       0.6490                                                          0.6490      0.2646         71
       101      -0.2952       0.6433                                                          0.6433      0.2952         61
        91      -0.3268       0.6369                                                          0.6369      0.3268         51
        81      -0.3594       0.6297                                                          0.6297      0.3594         41
        71      -0.3931       0.6217                                                          0.6217      0.3931         31
        61      -0.4280       0.6127                                                          0.6127      0.4280         21
        51      -0.4643       0.6027                                                          0.6027      0.4643         11
        41      -0.5019       0.5915                                                          0.5915      0.5019          1
        31      -0.5410       0.5792
        21      -0.5817       0.5657
        11      -0.6242       0.5507
         1      -0.6686       0.5341

                                     ROTATED AND TRANSLATED TRANSITION ARCS

     INLET K      X*(LOW)       Y*(LOW)     Y*(LOW)-G*                 Y*(LOW)-G*     Y*(LOW)      X*(LOW)     OUTLET K
       211       -0.4675        0.4841      -0.0703                    -0.1377        0.4167       0.5284        171
       201       -0.4844        0.4674      -0.0869                    -0.1564        0.3980       0.5429        161
       191       -0.5014        0.4502      -0.1042                    -0.1758        0.3785       0.5575        151
       181       -0.5185        0.4322      -0.1222                    -0.1960        0.3584       0.5720        141
       171       -0.5357        0.4135      -0.1409                    -0.2168        0.3375       0.5865        131
       161       -0.5530        0.3940      -0.1604                    -0.2385        0.3159       0.6010        121
       151       -0.5704        0.3736      -0.1808                    -0.2610        0.2934       0.6155        111
       141       -0.5879        0.3524      -0.2019                    -0.2843        0.2701       0.6300        101
       131       -0.6055        0.3303      -0.2241                    -0.3086        0.2458       0.6445         91
       121       -0.6233        0.3072      -0.2472                    -0.3339        0.2205       0.6590         81
4      111       -0.6412        0.2830      -0.2714                    -0.3603        0.1941       0.6735         71
       101       -0.6593        0.2577      -0.2967                    -0.3878        0.1666       0.6880         61
        91       -0.6775        0.2312      -0.3232                    -0.4166        0.1378       0.7025         51
        81       -0.6960        0.2033      -0.3510                    -0.4466        0.1078       0.7170         41
        71       -0.7146        0.1741      -0.3803                    -0.4781        0.0763       0.7316         31
        61       -0.7335        0.1434      -0.4110                    -0.5111        0.0433       0.7461         21
        51       -0.7526        0.1110      -0.4434                    -0.5457        0.0086       0.7607         11
        41       -0.7720        0.0769      -0.4775                    -0.5822       -0.0278       0.7753          1
        31       -0.7915        0.0409      -0.5135
        21       -0.8114        0.0028      -0.5516
        11       -0.8315       -0.0375      -0.5919
         1       -0.8520       -0.0802      -0.6346
```

TABLE II. - Continued. SAMPLE OUTPUT

```
                              CIRCULAR ARCS

   X*C(LOW)    Y*C(LOW)                             X*C(LOW)    Y*C(LOW)-G*
  -0.4675      0.4841                              -0.4675     -0.0703
  -0.4589      0.4921                              -0.4589     -0.0622
  -0.4503      0.5001                              -0.4503     -0.0543
  -0.4415      0.5079                              -0.4415     -0.0465
  -0.4326      0.5155                              -0.4326     -0.0389
  -0.4235      0.5230                              -0.4235     -0.0314
  -0.4143      0.5303                              -0.4143     -0.0241
  -0.4050      0.5374                              -0.4050     -0.0170
  -0.3955      0.5444                              -0.3955     -0.0100
  -0.3860      0.5512                              -0.3860     -0.0032
  -0.3763      0.5579                              -0.3763      0.0035
  -0.3665      0.5644                              -0.3665      0.0100
  -0.3566      0.5707                              -0.3566      0.0163
  -0.3466      0.5768                              -0.3466      0.0224
  -0.3365      0.5828                              -0.3365      0.0284
  -0.3262      0.5886                              -0.3262      0.0342
  -0.3159      0.5942                              -0.3159      0.0398
  -0.3055      0.5996                              -0.3055      0.0452
  -0.2950      0.6048                              -0.2950      0.0504
  -0.2844      0.6099                              -0.2844      0.0555
  -0.2737      0.6146                              -0.2737      0.0604
  -0.2629      0.6194                              -0.2629      0.0651
  -0.2521      0.6239                              -0.2521      0.0695
  -0.2412      0.6282                              -0.2412      0.0738
  -0.2302      0.6323                              -0.2302      0.0780
  -0.2191      0.6363                              -0.2191      0.0819
  -0.2079      0.6400                              -0.2079      0.0856
  -0.1967      0.6435                              -0.1967      0.0891
  -0.1855      0.6469                              -0.1855      0.0925
  -0.1742      0.6500                              -0.1742      0.0956
  -0.1628      0.6529                              -0.1628      0.0986
  -0.1514      0.6557                              -0.1514      0.1013
  -0.1399      0.6582                              -0.1399      0.1038
  -0.1284      0.6606                              -0.1284      0.1062
  -0.1169      0.6627                              -0.1169      0.1083
  -0.1053      0.6646                              -0.1053      0.1103
  -0.0937      0.6664                              -0.0937      0.1120
  -0.0820      0.6679                              -0.0820      0.1135
  -0.0703      0.6692                              -0.0703      0.1149
  -0.0586      0.6704                              -0.0586      0.1160
  -0.0469      0.6713                              -0.0469      0.1169
  -0.0352      0.6720                              -0.0352      0.1176
  -0.0235      0.6725                              -0.0235      0.1181
  -0.0117      0.6728                              -0.0117      0.1184
   0.0000      0.6729                               0.0000      0.1185
   0.0117      0.6728                               0.0117      0.1184
   0.0235      0.6725                               0.0235      0.1181
   0.0352      0.6720                               0.0352      0.1176
   0.0469      0.6713                               0.0469      0.1169
   0.0586      0.6704                               0.0586      0.1160
   0.0703      0.6692                               0.0703      0.1149
   0.0820      0.6679                               0.0820      0.1135
   0.0937      0.6664                               0.0937      0.1120
   0.1053      0.6646                               0.1053      0.1103
   0.1169      0.6627                               0.1169      0.1083
   0.1284      0.6606                               0.1284      0.1062
   0.1399      0.6582                               0.1399      0.1038
   0.1514      0.6557                               0.1514      0.1013
   0.1628      0.6529                               0.1628      0.0986
   0.1742      0.6500                               0.1742      0.0956
   0.1855      0.6469                               0.1855      0.0925
   0.1967      0.6435                               0.1967      0.0891
   0.2079      0.6400                               0.2079      0.0856
   0.2191      0.6363                               0.2191      0.0819
   0.2302      0.6323                               0.2302      0.0780
   0.2412      0.6282                               0.2412      0.0738
   0.2521      0.6239                               0.2521      0.0695
   0.2629      0.6194                               0.2629      0.0651
   0.2737      0.6146                               0.2737      0.0604
   0.2844      0.6099                               0.2844      0.0555
   0.2950      0.6048                               0.2950      0.0504
   0.3055      0.5996                               0.3055      0.0452
   0.3159      0.5942                               0.3159      0.0398
   0.3262      0.5886                               0.3262      0.0342
   0.3365      0.5828                               0.3365      0.0284
   0.3466      0.5768                               0.3466      0.0224
   0.3566      0.5707                               0.3566      0.0163
   0.3665      0.5644                               0.3665      0.0100
   0.3763      0.5579                               0.3763      0.0035
   0.3860      0.5512                               0.3860     -0.0032
   0.3955      0.5444                               0.3955     -0.0100
   0.4050      0.5374                               0.4050     -0.0170
   0.4143      0.5303                               0.4143     -0.0241
   0.4235      0.5230                               0.4235     -0.0314
   0.4326      0.5155                               0.4326     -0.0389
   0.4415      0.5079                               0.4415     -0.0465
   0.4503      0.5001                               0.4503     -0.0543
   0.4589      0.4921                               0.4589     -0.0622
   0.4675      0.4841                               0.4675     -0.0703
   0.4758      0.4758                               0.4758     -0.0786
   0.4841      0.4675                               0.4841     -0.0865
   0.4921      0.4589                               0.4921     -0.0954
   0.5001      0.4503                               0.5001     -0.1041
   0.5079      0.4415                               0.5079     -0.1129
   0.5155      0.4326                               0.5155     -0.1218
   0.5230      0.4235                               0.5230     -0.1309
   0.5284      0.4167                               0.5284     -0.1377
```

3

TABLE II. - Continued. SAMPLE OUTPUT

```
                              UPPER SURFACE
                        UNROTATED TRANSITION ARCS

   INLET J      X*(UP)      Y*(UP.
     201        0.          0.483
     191       -0.0166      0.4831
     181       -0.0326      0.4827
     171       -0.0481      0.4821
     161       -0.0631      0.4812
     151       -0.0778      0.4800
     141       -0.0921      0.4787
     131       -0.1060      0.4771
     121       -0.1197      0.4753
     111       -0.1331      0.4733
3    101       -0.1462      0.4711
      91       -0.1591      0.4687
      81       -0.1718      0.4662
      71       -0.1843      0.4634
      61       -0.1967      0.4605
      51       -0.2089      0.4573
      41       -0.2209      0.4540
      31       -0.2328      0.4505
      21       -0.2446      0.4468
      11       -0.2563      0.4429
       1       -0.2678      0.4388
```

```
   Y*(UP)      X*(UP)     OUTLET J
   0.4833     -0.           241
   0.4831      0.0166       231
   0.4827      0.0326       221
   0.4821      0.0481       211
   0.4812      0.0631       201
   0.4800      0.0778       191
   0.4787      0.0921       181
   0.4771      0.106^       171
   0.4753      0.1197       161
   0.4733      0.1331       151
   0.4711      0.1462       141
   0.4687      0.1591       131
   0.4662      0.1718       121
   0.4634      0.1843       111
   0.4605      0.1967       101
   0.4573      0.2089        91
   0.4540      0.2209        81
   0.4505      0.2328        71
   0.4468      0.2446        61
   0.4429      0.2563        51
   0.4388      0.2678        41
   0.4345      0.2793        31
   0.4301      0.2906        21
   0.4254      0.3019        11
   0.4206      0.3130         1
```

```
                  ROTATED AND TRANSLATED TRANSITION ARCS

   INLET J       X*(UP)       Y*(UP)      Y*(UP)+G*
     201        -0.3417       0.3417       0.8961
     191        -0.3533       0.3299       0.8843
     181        -0.3644       0.3183       0.8727
     171        -0.3749       0.3069       0.8613
     161        -0.3845       0.2956       0.8500
     151        -0.3944       0.2844       0.8388
     141        -0.4036       0.2734       0.8278
     131        -0.4123       0.2624       0.8168
     121        -0.4207       0.2515       0.8059
     111        -0.4288       0.2406       0.7950
4    101        -0.4365       0.2298       0.7841
      91        -0.4440       0.2189       0.7733
      81        -0.4511       0.2081       0.7625
      71        -0.4580       0.1973       0.7517
      61        -0.4647       0.1865       0.7409
      51        -0.4711       0.1757       0.7300
      41        -0.4772       0.1648       0.7192
      31        -0.4832       0.1539       0.7083
      21        -0.4889       0.1429       0.6973
      11        -0.4944       0.1319       0.6863
       1        -0.4997       0.1209       0.6753
```

```
   Y*(UP)+G*     Y*(UP)       X*(UP)     OUTLET J
    0.8977       0.3433       0.3402       241
    0.8859       0.3315       0.3518       231
    0.8744       0.3200       0.3629       221
    0.8630       0.3086       0.3735       211
    0.8517       0.2974       0.3835       201
    0.8406       0.2862       0.3931       191
    0.8296       0.2752       0.4023       181
    0.8187       0.2643       0.4111       171
    0.8078       0.2534       0.4196       161
    0.7969       0.2426       0.4277       151
    0.7861       0.2317       0.4355       141
    0.7753       0.2210       0.4430       131
    0.7646       0.2102       0.4502       121
    0.7538       0.1994       0.4571       111
    0.7430       0.1886       0.4638       101
    0.7322       0.1778       0.4703        91
    0.7213       0.1670       0.4765        81
    0.7105       0.1561       0.4825        71
    0.6995       0.1452       0.4882        61
    0.6886       0.1342       0.4938        51
    0.6775       0.1232       0.4991        41
    0.6665       0.1121       0.5042        31
    0.6553       0.1009       0.5091        21
    0.6441       0.0897       0.5139        11
    0.6328       0.0784       0.5184         1
```

TABLE II. - Continued. SAMPLE OUTPUT

```
                              CIRCULAR ARCS

   X*C(UP)     Y*C(UP)                          X*C(LP)     Y*C(LP)+G*
   -0.3417      0.3417                          -0.3417      0.8961
   -0.3357      0.3476                          -0.3357      0.9020
   -0.3296      0.3534                          -0.3296      0.9078
   -0.3234      0.3591                          -0.3234      0.9135
   -0.3171      0.3647                          -0.3171      0.9191
   -0.3106      0.3702                          -0.3106      0.9246
   -0.3041      0.3756                          -0.3041      0.9300
   -0.2975      0.3808                          -0.2975      0.9352
   -0.2908      0.3860                          -0.2908      0.9403
   -0.2841      0.3910                          -0.2841      0.9454
   -0.2772      0.3959                          -0.2772      0.9503
   -0.2702      0.4007                          -0.2702      0.9550
   -0.2632      0.4053                          -0.2632      0.9597
   -0.2561      0.4098                          -0.2561      0.9642
   -0.2489      0.4142                          -0.2489      0.9686
   -0.2416      0.4185                          -0.2416      0.9729
   -0.2343      0.4227                          -0.2343      0.9771
   -0.2269      0.4267                          -0.2269      0.9811
   -0.2194      0.4306                          -0.2194      0.9850
   -0.2119      0.4344                          -0.2119      0.9887
   -0.2042      0.4380                          -0.2042      0.9924
   -0.1966      0.4415                          -0.1966      0.9959
   -0.1888      0.4449                          -0.1888      0.9992
   -0.1810      0.4481                          -0.1810      1.0025
   -0.1732      0.4512                          -0.1732      1.0056
   -0.1653      0.4541                          -0.1653      1.0085
   -0.1573      0.4569                          -0.1573      1.0113
   -0.1493      0.4596                          -0.1493      1.0140
   -0.1413      0.4622                          -0.1413      1.0165
   -0.1332      0.4646                          -0.1332      1.0189
   -0.1251      0.4668                          -0.1251      1.0212
   -0.1169      0.4689                          -0.1169      1.0233
   -0.1087      0.4709                          -0.1087      1.0253
   -0.1005      0.4727                          -0.1005      1.0271
   -0.0922      0.4744                          -0.0922      1.0288
   -0.0839      0.4759                          -0.0839      1.0303
   -0.0756      0.4773                          -0.0756      1.0317
   -0.0673      0.4786                          -0.0673      1.0330
   -0.0589      0.4797                          -0.0589      1.0341
   -0.0505      0.4806                          -0.0505      1.0350
   -0.0421      0.4814                          -0.0421      1.0358
   -0.0337      0.4821                          -0.0337      1.0365
   -0.0253      0.4826                          -0.0253      1.0370
   -0.0169      0.4830                          -0.0169      1.0374
   -0.0084      0.4832                          -0.0084      1.0376
3   0.0000      0.4833                           0.0000      1.0377
    0.0084      0.4832                           0.0084      1.0376
    0.0169      0.4830                           0.0169      1.0374
    0.0253      0.4826                           0.0253      1.0370
    0.0337      0.4821                           0.0337      1.0365
    0.0421      0.4814                           0.0421      1.0358
    0.0505      0.4806                           0.0505      1.0350
    0.0589      0.4797                           0.0589      1.0341
    0.0673      0.4786                           0.0673      1.0330
    0.0756      0.4773                           0.0756      1.0317
    0.0839      0.4759                           0.0839      1.0303
    0.0922      0.4744                           0.0922      1.0288
    0.1005      0.4727                           0.1005      1.0271
    0.1087      0.4709                           0.1087      1.0253
    0.1169      0.4689                           0.1169      1.0233
    0.1251      0.4668                           0.1251      1.0212
    0.1332      0.4646                           0.1332      1.0189
    0.1413      0.4622                           0.1413      1.0165
    0.1493      0.4596                           0.1493      1.0140
    0.1573      0.4569                           0.1573      1.0113
    0.1653      0.4541                           0.1653      1.0085
    0.1732      0.4512                           0.1732      1.0056
    0.1810      0.4481                           0.1810      1.0025
    0.1888      0.4449                           0.1888      0.9992
    0.1966      0.4415                           0.1966      0.9959
    0.2042      0.4380                           0.2042      0.9924
    0.2119      0.4344                           0.2119      0.9887
    0.2194      0.4306                           0.2194      0.9850
    0.2269      0.4267                           0.2269      0.9811
    0.2343      0.4227                           0.2343      0.9771
    0.2416      0.4185                           0.2416      0.9729
    0.2489      0.4142                           0.2489      0.9686
    0.2561      0.4098                           0.2561      0.9642
    0.2632      0.4053                           0.2632      0.9597
    0.2702      0.4007                           0.2702      0.9550
    0.2772      0.3959                           0.2772      0.9503
    0.2841      0.3910                           0.2841      0.9454
    0.2908      0.3860                           0.2908      0.9403
    0.2975      0.3808                           0.2975      0.9352
    0.3041      0.3756                           0.3041      0.9300
    0.3106      0.3702                           0.3106      0.9246
    0.3171      0.3647                           0.3171      0.9191
    0.3234      0.3591                           0.3234      0.9135
    0.3296      0.3534                           0.3296      0.9078
    0.3357      0.3476                           0.3357      0.9020
    0.3402      0.3433                           0.3402      0.8977
```

TABLE II. - Concluded. SAMPLE OUTPUT

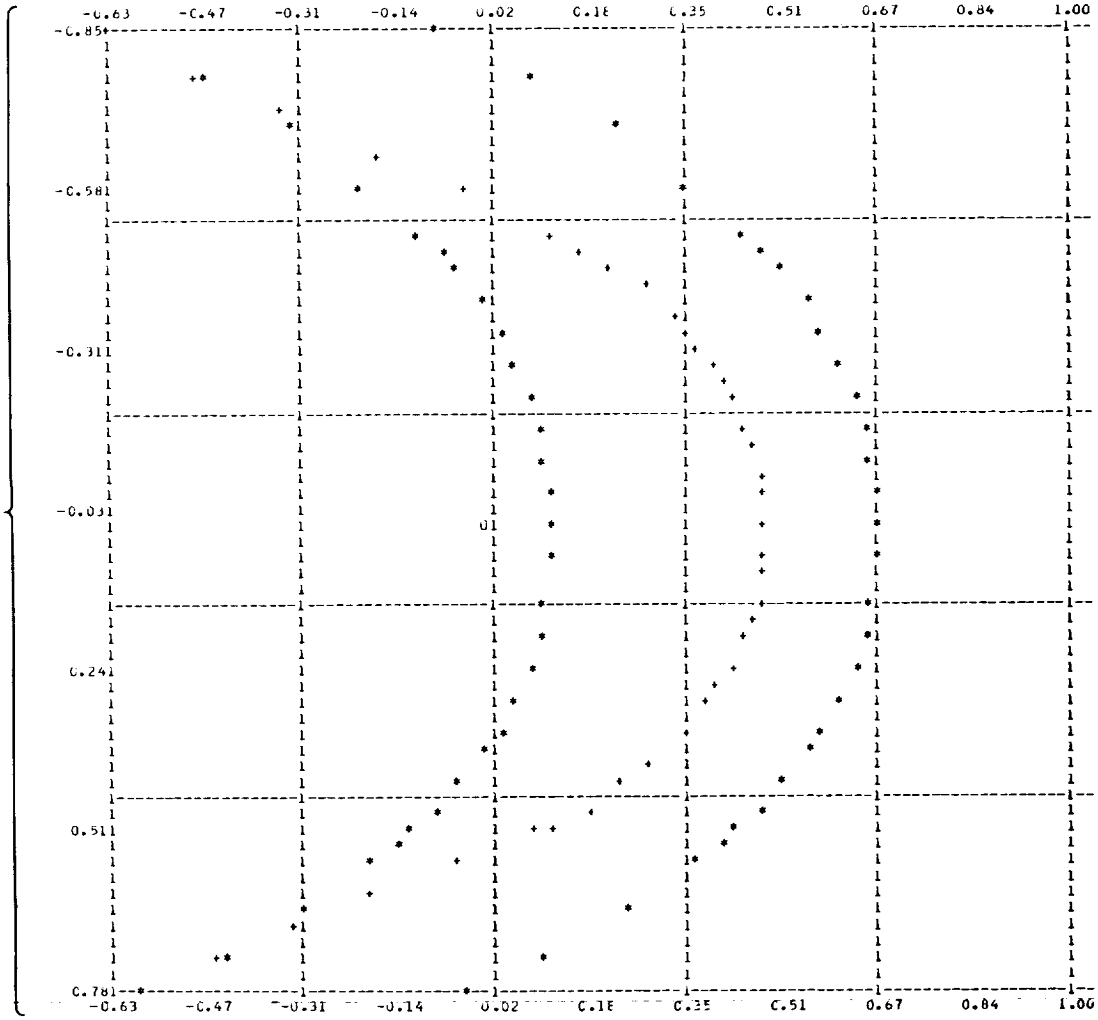
3

```
                                  STRAIGHT LINES

X*S(IN)     Y*S(IN)    Y*S(IN)+G*                           Y*S(OUT)+G*   Y*S(OUT)      X*S(OUT)
-0.4997     0.1209      0.6753                                0.6328       0.0784       0.5184
-0.5231     0.0705      0.6249                                0.5888       0.0344       0.5355
-0.5466     0.0202      0.5745                                0.5448      -0.0096       0.5526
-0.5701    -0.0302      0.5242                                0.5008      -0.0536       0.5698
-0.5936    -0.0806      0.4738                                0.4568      -0.0976       0.5869
-0.6171    -0.1309      0.4234                                0.4127      -0.1416       0.6040
-0.6406    -0.1813      0.3731                                0.3687      -0.1857       0.6211
-0.6641    -0.2317      0.3227                                0.3247      -0.2297       0.6383
-0.6875    -0.2820      0.2723                                0.2807      -0.2737       0.6554
-0.7110    -0.3324      0.2220                                0.2367      -0.3177       0.6725
-0.7345    -0.3828      0.1716                                0.1927      -0.3617       0.6896
-0.7580    -0.4331      0.1212                                0.1486      -0.4057       0.7068
-0.7815    -0.4835      0.0709                                0.1046      -0.4498       0.7239
-0.8050    -0.5339      0.0205                                0.0606      -0.4938       0.7410
-0.8285    -0.5842     -0.0299                                0.0166      -0.5378       0.7581
-0.8520    -0.6346     -0.0802                               -0.0274      -0.5818       0.7753
```

5

```
                                  MISCELLANEOUS PARAMETERS

            THE MINIMUM LOWER SURFACE PRANDTL-MEYER ANGLE PREDICTED BY SEPARATION CONDITIONS IS   19.3009 DEG

            THE MAXIMUM UPPER SURFACE PRANDTL-MEYER ANGLE PREDICTED BY SEPARATION CONDITIONS IS   56.8768 DEG

                  M*(IN) =   1.8240   M(IN) =   2.4947          M(OUT) =   2.3287    M*(OUT) =   1.7669

R*(LOW) =  0.6729    M*(LOW) =   1.4860   M(LOW) =   1.7065          M(UP) =   3.5297     M*(UP) =   2.0692  R*(UP) =   0.4833

        THETA = 133.7395 DEG           G* =   0.5544            C* =   1.6281            SIGMA =   2.9367
```

6



PROGRAM DESCRIPTION

Main Program

The main program generates a table of the inlet and outlet transition arcs of the upper, lower, and translated curves. It also computes several parameters which are pertinent to the blade description, such as Mach numbers and radii, and, as an option, plots a blade profile and flow channel. The plotting is done by subroutine PLOTMY, (ref. 10). The program variables are

ALPH	fixed angle
ALPHLN	rotation angle for lower-curve inlet transition arc, $\alpha_{l,i}$
ALPHLO	rotation angle for lower-curve outlet transition arc, $\alpha_{l,o}$
ALPHUI	rotation angle for upper-curve inlet transition arc, $\alpha_{u,i}$
ALPHUO	rotation angle for upper-curve outlet transition arc, $\alpha_{u,o}$
ALPHUP	temporary storage
ALPLOW	temporary storage
ANGLE	logical switch
BETAN	see INPUT
BETAT	outlet flow angle, β_o
CONVER	conversion factor for degrees to radians
COSALN	cosine of ALPHLN
COSALO	cosine of ALPHLO
COSAUI	cosine of ALPHUI
COSAUO	cosine of ALPHUO
CSTAR	blade chord, C^*
DALPH	angle increment
DELF	see subroutine ROOT
DELV	see INPUT
DELXI	1/15 length of straight-line portion of upper-curve inlet arc
DELXO	1/15 length of straight-line portion of upper-curve outlet arc
EMJ	slope of Mach lines, m_j

EMK	slope of Mach lines (eq. (15)), m_k
EMWJ	slope of wall segments, $\overline{m}_j$
EMWK	slope of wall segments (eq. (17)), $\overline{m}_k$
F(V, FN)	internally defined function (eq. (13)), $f(R^*)$
FLO	see subroutine START
FN	floating point index
FOFX	see subroutine ROOT
FUP	see subroutine START
GAM	ratio of specific heats, γ
GAMEXP	1/(GAM - 1)
GAMM1	(GAM - 1)/2
GAMP1	(GAM + 1)/2
GRTY	dummy name
GSTAR	blade spacing, G^*
I	counter
IPRINT	see INPUT
ISTART	see INPUT
J	variable index for upper curve
JJ	variable index
JDEX	variable index
JMAXN	number of points on upper-curve inlet transition arc
JMAXO	number of points on upper-curve outlet transition arc
JMN	maximum of JMAXO and JMAXN
JN	variable index
JNDEX	number of upper-curve transition arc points to be printed
JNN	variable index
JO	variable index
JOO	variable index
K	variable index for lower curve

KK	variable index
KKK	array required by PLOTMY (see ref. 10)
KDEX	variable index
KMAXN	number of points on lower-curve inlet transition arc
KMAXO	number of points on lower-curve outlet transition arc
KMN	maximum of KMAXN and KMAXO
KN	variable index
KNDEX	number of lower-curve transition arc points to be printed
KNN	variable index
KO	variable index
KOO	variable index
KOUNT	counter
L	counter
LLL	NP1 + NP2
LSTORE	number of points saved per 5° of turning
LSTR	temporary storage
M	counter
MAXN	integer constant
MAXO	integer constant
N	variable index
NPER	variable governing selective storage
NPLOT	see INPUT
NP1	number of points on lower curve which have been saved for plotter
NP2	number of points on translated lower curve which have been saved for plotter
NP3	number of points on upper curve which have been saved for plotter
NSUM	total number of points which have been stored for plotter
NUM	counter
P	array required by PLOTMY (see ref. 10)
PERM	$[(GAM + 1)/(GAM - 1)]^{1/2}$

PHIJ	flow direction, φ_j
PHIJP1	previous value of PHIJ, φ_{j+1}
PHIK	flow direction (eq. (12)), φ_k
PHIKP1	previous value of PHIK, φ_{k+1}
R	array for storing radii of major vortex-compression-characteristic points, R_j^*
RA	array for storing radii of major vortex-expansion-characteristic points, R_k^*
RECONV	conversion factor for radians to degrees
RIN	1/SSMIN
RLOW	radius of circular arc of lower curve as calculated in JOKOS, R_l^*
ROUT	1/SSMOUT
RUP	radius of upper-curve circular arc, R_u^*
SAME	see subroutine START
SIGMA	blade solidity, σ
SINALN	sine of ALPHLN
SINALO	sine of ALPHLO
SINAUI	sine of ALPHUI
SINAUO	sine of ALPHUO
SM	temporary storage for Mach numbers, M
SMIN	inlet Mach number, M_i
SMLOW	lower-surface Mach number, M_l
SMOUT	outlet Mach number, M_o
SMS	temporary storage for velocity ratio, M^*
SMUP	upper-surface Mach number, M_u
SSMIN	inlet velocity ratio, M_i^*
SSMLOW	lower-surface velocity ratio, M_l^*
SSMOUT	outlet velocity ratio, M_o^*
SSMUP	upper-surface velocity ratio, M_u^*
TANBI	tangent of BETAN
TANBO	tangent of BETAT

TEMP	temporary storage
TEMPP	temporary storage
TEMPPP	temporary storage
THETA	total flow turning angle, θ
TR	temporary storage for radii
TX	temporary storage for unrotated x*-coordinates
TXLO	temporary storage for values of XLOW(I)
TXUP	temporary storage for values of XUP(I)
TY	temporary storage for unrotated y*-coordinates
TYLO	temporary storage for values of YLOW(I)
TYUP	temporary storage for values of YUP(I)
UMJ	Mach angle, μ_j
UMJP1	previous value of UMJ, μ_{j+1}
UMK	Mach angle (eq. (15)), μ_k
UMKP1	previous value of UMK, μ_{k+1}
V	temporary storage for Prandtl-Meyer angles, ν
VIMAX	see subroutine START
VIN	see INPUT
VLOW	see INPUT
VLSPMN	minimum lower-surface Prandtl-Meyer angle from separation criterion, $(\nu_l)_{min}$
VNL	VIN-VLOW
VOL	VOUT-VLOW
VOUT	see INPUT
VUI	VUP-VIN
VUMAX	$\frac{\pi}{2}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right)$
VUP	see INPUT

VUSPMX	maximum upper-surface Prandtl-Meyer angle from separation criterion, $(\nu_u)_{max}$
VUT	VUP-VOUT
X0	see subroutine ROOT
X1	see subroutine ROOT
X2	see subroutine ROOT
XCG	X*-coordinate of a translated-curve circular arc point
XCLOW	X*-coordinate of a lower-curve circular arc point
XCUP	X*-coordinate of an upper-curve circular arc point
XDOWN	array for storing X*-coordinate of points to be plotted
XINTL	see subroutine ROOT
XLOW	array for storing x*-coordinate of unrotated lower transition arc points, $(x^*_l)_k$
XLOWN	array for storing X*-coordinate of lower-curve rotated inlet transition arc points, $(X^*_l)_{k,i}$
XLOWO	array for storing X*-coordinate of lower-curve rotated outlet transition arc points, $(X^*_l)_{k,o}$
XMLOW	temporary storage for values of -XLOW(I)
XMUP	temporary storage for values of -XUP(I)
XSIN	X*-coordinate of a point on inlet straight-line portion of upper curve
XSOUT	X*-coordinate of a point on outlet straight-line portion of upper curve
XUP	array for storing x*-coordinate of unrotated upper transition arc points, $(x^*_u)_j$
XUPN	array for storing X*-coordinate of upper-curve rotated inlet transition arc points, $(X^*_u)_{j,i}$
XUPO	array for storing X*-coordinate of upper-curve rotated outlet transition arc points, $(X^*_u)_{j,o}$
YACRS1	array for storing Y*-coordinates of points for plotter
YACRS2	array for temporary storage of Y*-coordinate of translated lower-curve points for plotter

YCG	Y*-coordinate of a translated-curve circular arc point
YCLOW	Y*-coordinate of a lower-curve circular arc point
YCUP	Y*-coordinate of an upper-curve circular arc point
YLASTI	Y*-coordinate of first point on inlet side of upper curve
YLASTO	Y*-coordinate of first point on outlet side of upper curve
YLOW	array for storing y*-coordinate of unrotated lower transition arc points, $(y_{\ell}^{*})_{k}$
YLOWN	array for storing Y*-coordinate of lower-curve rotated inlet transition arc points, $(Y_{\ell}^{*})_{k,i}$
YLOWO	array for storing Y*-coordinate of lower-curve rotated outlet transition arc points, $(Y_{\ell}^{*})_{k,o}$
YNG	temporary storage for Y*-coordinate of a point on translated-curve inlet transition arc
YSIN	Y*-coordinate of a point on inlet straight-line portion of upper curve
YSNG	Y*-coordinate of a point on inlet straight-line portion of translated upper curve
YSOUT	Y*-coordinate of a point on outlet straight-line portion of upper curve
YSTG	Y*-coordinate of a point on outlet straight-line portion of translated upper curve
YTG	temporary storage for Y*-coordinate of a point on translated-curve outlet transition arc
YUP	array for storing y*-coordinate of unrotated upper transition arc points, $(y_{u}^{*})_{j}$
YUPN	array for storing Y*-coordinate of upper-curve rotated inlet transition arc points, $(Y_{u}^{*})_{j,i}$
YUPO	array for storing Y*-coordinate of upper-curve rotated outlet transition arc points, $(Y_{u}^{*})_{j,o}$

Subroutine ROOT

Subroutine ROOT is a general routine for finding the roots of equations and is derived from the "half-interval search" method described in reference 11. This method depends on successively halving an interval which is known to contain the desired root. Subroutine ROOT is used to calculate R^* from equation (10b).

A call to ROOT has the form CALL ROOT (X0, X2, XINTL, FOFX, FUNC, X1), where the elements of the call vector are

X0	lower bound of initial root interval
X2	upper bound of initial root interval
XINTL	initial estimate of value of root
FOFX	given value of dependent variable
FUNC	externally defined function
X1	value of root

Other program variables are

A	FOFX-F2
DELF	convergence criterion
F0	function FUNC evaluated at XX0
F2	function FUNC evaluated at XX2
FX	function FUNC evaluated at X
KOUNT	count of number of iterations performed
X	temporary storage for present estimate of root
XX0	present value of lower bound of root interval
XX2	present value of upper bound of root interval

Subroutine START

Subroutine START is used to compute the maximum value of the inlet Prandtl-Meyer angle $(\nu_i)_{max}$ for supersonic starting for various lower- and upper-surface Prandtl-Meyer angles. Several other parameters of interest are computed by START and are printed as output. Among these parameters are the vortex constant for maximum weight flow K^*_{max}, the reduction in weight flow due to two-dimensional flow C, and the maximum value of the inlet velocity ratio for supersonic starting $(M_i^*)_{max}$.

A call to START has the form CALL START (VLOW, FLO, VUP, FUP, VIMAX), where the elements of the call vector are

VLOW	lower-surface Prandtl-Meyer angle, ν_l

FLO	eq. (10b) evaluated at R_l^*
VUP	upper-surface Prandtl-Meyer angle, ν_u
FUP	eq. (10b) evaluated at R_u^*
VIMAX	maximum inlet Prandtl-Meyer angle for supersonic starting, $(\nu_i)_{max}$

Other program variables are

BINTGR	value of integral (eq. (27b))
C	reduction in maximum weight flow due to two-dimensional flow (eq. (34b))
CINTGR	value of integral (eq. (32))
FINTL	eq. (27) evaluated at $K_{max}^* = \text{XINTL}$
F0	eq. (27) evaluated at $K_{max}^* = \text{X0}$
F2	eq. (27) evaluated at $K_{max}^* = \text{X2}$
FOFX	eq. (27) evaluated at $K_{max}^* = \text{XAMK}$
Q	eq. (34a)
RATIO	ratio of Q to 1 - C
RLOW	radius of circular portion of lower curve, R_l^*
RUP	radius of circular portion of upper curve, R_u^*
SAME	square of ratio of XAMK to SSMIOW
SLOPE	slope of line
SSMIAX	maximum value of entering velocity ratio for starting, $(M_i^*)_{max}$
SSMLOW	lower-surface velocity ratio, M_l^*
SSMUP	upper-surface velocity ratio, M_u^*
WSTAR	weight-flow parameter (eq. (32))
XAMK	vortex constant for maximum weight flow, K_{max}^*
XINTL	initial estimate of a parameter
X0	lower bound of a parameter
X2	upper bound of a parameter
YINCPT	y-intercept of a line

Subroutine MSSTAR

Subroutine MSSTAR is used to determine the minimum lower-surface Prandtl-Meyer angle and the maximum upper-surface Prandtl-Meyer angle from separation considerations. This subroutine uses ROOT and ADSTR in the calculation of these angles.

A call to MSSTAR has the form CALL MSSTAR (M, N, VSSTAR), where the elements of the call vector are

M	inlet or outlet dimensionless velocity, M_i^* or M_o^*, respectively
N	variable switch
VSSTAR	Prandtl-Meyer angle from separation criterion, $(\nu_l)_{min}$ or $(\nu_u)_{max}$

The other variables used by MSSTAR are

A	$\frac{\pi}{4}\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right)$
B	$\frac{1}{2}\sqrt{\frac{\gamma + 1}{\gamma - 1}}$
C	$\gamma - 1$
D	$\gamma + 1$
MS	velocity ratio from separation criterion, $(M_l^*)_{min}$ or $(M_u^*)_{max}$
X0	see subroutine ROOT
X2	see subroutine ROOT
XINTL	see subroutine ROOT
FOFX	see subroutine ROOT
SQRDMS	$(MS)^2$

Subroutine SIMPS1

This function subprogram is used to perform numerical integration of explicit functions of one variable. The integration is performed by a modification of Simpson's rule, in which a sufficient number of intervals is used to assure six or more significant figures in the result.

A call to SIMPS1 has the form ANSWER = SIMPS1 (XMIN, XMAX, FUNC1, KER), where the elements of the call vector are

XMIN	lower limit of integration
XMAX	upper limit of integration
FUNC1	externally defined function of a single variable
KER	storage for flagging result if necessary

Other program variables are

A	array for storing functional values at certain partition points
ANS	sum of subapproximations
B	array for storing functional values at certain partition points
C	array for storing functional values at certain partition points
E	array for storing difference terms
FRAC	variable tolerance used for subapproximations
H	distance between successive points of partition
K	variable index
N	counter
NE	equivalent to E
NTEST	equivalent to TEST
P	array for storing successive subapproximations
Q	sum of difference terms
SIMPS1	value of desired integral
TEST	tester for subapproximations
T	tolerance for difference terms
V	array for storing partition points of interval

Function Subprograms

The following function subprograms are used intermittently throughout the main program and subroutines:

FUNCTION ALFUNC (A, B, Y), defined by ALFUNC where

$$\text{ALFUNC} = \frac{1}{Y}\left(A - BY^2\right)^{1/(\gamma-1)}$$

FUNCTION CFACT (Y), defined by CFACT where

$$\mathrm{CFACT} = \frac{1}{Y}\left[1 - \left(\frac{K^*_{max}}{M^*_l}\right)^2 Y^2\right]^{1/(\gamma-1)}$$

FUNCTION QFACT (Y), defined by QFACT where

$$\mathrm{QFACT} = \frac{1}{Y}\left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} Y^2\right)^{1/(\gamma-1)}$$

FUNCTION FRAT (Y), defined by FRAT where

$$\mathrm{FRAT} = \frac{2\gamma}{Y^{\gamma-1}}\left(\frac{\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} Y^2}{\frac{\gamma + 1}{2} Y^2 - \frac{\gamma - 1}{2}}\right)^{1/(\gamma-1)}$$

FUNCTION FOFRS (X), defined by FOFRS where

$$\mathrm{FOFRS} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\ \mathrm{arc\ sin}\left(\frac{\gamma - 1}{X^2} - \gamma\right) + \mathrm{arc\ sin}\left[(\gamma + 1)X^2 - \gamma\right]$$

FUNCTION FKMAX (Y, L), defined by FKMAX where

$$\mathrm{FKMAX} = \int_{M^*_l}^{M^*_u} \left[1 - \left(\frac{Y}{M^*_l}\right)^2 Z^2\right]^{1/(\gamma-1)} \frac{dZ}{Z} + \left[1 - Y^2\left(\frac{M^*_u}{M^*_l}\right)^2\right]^{1/(\gamma-1)} - (1 - Y^2)^{1/(\gamma-1)}$$

and L is an optional switch.

FUNCTION ADSTR (X), defined by ADSTR where

$$\mathrm{ADSTR} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\left\{1 - \left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right)X^2\right]\left[1 + \frac{1}{2}\left(\frac{\frac{\gamma}{\gamma + 1} X^2}{1 - \frac{\gamma - 1}{\gamma + 1} X^2}\right)\right]^{(\gamma-1)/\gamma}\right\}^{1/2}$$

PROGRAM LISTING

```
      COMMON/EXPALF/GAMEXP
      COMMON/ROOTS/DELF
      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY

      DIMENSION R(800),RA(800),XLOW(800),YLOW(800),XLOWN(800),YLOWN(800)
     1,XUP(800),YUP(800),XLOWO(800),YLOWO(800),XUPN(800),YUPN(800),
     2XUPO(800),YUPO(800)
      DIMENSION XDOWN(400),YACRS1(400),YACRS2(200),KKK(14),P(20)
      LOGICAL ANGLE

      EXTERNAL FOFRS

      F(V,FN) = (2.*V) - ((3.14159265/2.)*(PERM-1.)) - (2.*(FN-1.)*DELV)

CC    INPUT  AND  TITLE
C          ISTART=0 FOR STARTING AND DESIGN   ISTART=1 FOR STARTING ONLY
C          NPLOT=0 IF PLOT IS DESIRED   NPLOT=1 IF PLOT IS NOT DESIRED
C          IPRINT=0 PRINT ROTATED COORDINATES ONLY   IPRINT=1 PRINT UNROTATED
C                                                    AND ROTATED COORDINATES

    1 READ (5,11) VIN,VOUT,BETAN,VLOW,VUP,DELV,GAM,ISTART,NPLOT,IPRINT
   11 FORMAT ( 7(F6.2,2X),3(I1,2X) )
      WRITE (6,99)
   99 FORMAT (1H1,38X,53HD E S I G N   O F   S U P E R S O N I C   B L A
     1 D E S)

CC    CONVERSION FACTORS  AND  CONSTANTS
      CONVER = .174532925E-01
      RECONV = 57.2957796
C          ONE POINT WILL BE PRINTED FOR EVERY NPER POINTS CALCULATED
      IF (DELV .GE. 0.2) GO TO 12
      NPER = 10
      GO TO 13
   12 NPER = 1
   13 GAMP1 = (GAM + 1.)/2.
      GAMM1 = (GAM-1.)/2.
      GAMEXP = 1./(GAM-1.)
      PERM = SQRT(GAMP1/GAMM1)
      DELF = 0.000001
      XO = 1./PERM
      X2 = 0.999999999
      XINTL = (XO + X2)/2.
      LSTORE = (5.0/DELV)/FLOAT(NPER)
      DALPH = 1.0*CONVER

      ANGLE = .TRUE.
      IF (VLOW .LE. AMIN1(VUP,VIN,VOUT)) GO TO 120
      WRITE (6,119)
```

```
 119 FORMAT (//31X,70HV(LOW) MUST BE LESS THAN OR EQUAL TO THE MINIMUM
    1OF V(UP),V(IN),V(OUT))
     ANGLE = .FALSE.

 120 IF (VUP .GE. AMAX1(VIN,VOUT)) GO TO 118
     WRITE (6,117)
 117 FORMAT (//33X,66HV(UP) MUST BE GREATER THAN OR EQUAL TO THE MAXIMU
    1M OF V(IN),V(OUT))
     ANGLE = .FALSE.

 118 VUMAX = (3.14159265/2.)*(PERM-1.)*RECONV
     IF (VUP .LE. VUMAX) GO TO 116
     WRITE (6,115) VUMAX
 115 FORMAT (//41X,37HV(UP) MUST BE LESS THAN V(UP)(MAX) = ,F9.4,4H DEG
    1)
     ANGLE = .FALSE.

 116 IF (.NOT. ANGLE) GO TO 1

CC    PARAMETERS  FOR  STARTING

      VLOW = VLOW*CONVER
      FLO = F(VLOW,1.0)
      VUP = VUP*CONVER
      FUP = F(VUP,1.0)
      CALL START (VLOW,FLO,VUP,FUP,VIMAX)

      IF (ISTART .NE. 0) GO TO 1

      WRITE (6,97)
   97 FORMAT (//58X,17HDESIGN PARAMETERS)

CC    MISCELLANEOUS  CALCULATIONS
      DELV = DELV*CONVER
      FN = 1.
      V = VIN*CONVER
      DO 4 I=1,2
      FOFX = F(V,FN)
      CALL ROOT (X0,X2,XINTL,FOFX,FOFRS,X1)
      IF (I .EQ. 2) GO TO 4
      RIN = X1
      V = VOUT*CONVER
    4 CONTINUE
      ROUT = X1

      SSMIN = 1./RIN
      CALL MSSTAR (SSMIN,0,VLSPMN)
      SSMOUT = 1./ROUT
      CALL MSSTAR (SSMOUT,1,VUSPMX)

      SMS = SSMIN
      I = 1
   16 SM = SQRT(((1./GAMP1)*SMS*SMS)/(1.-(GAMM1/GAMP1)*SMS*SMS))
      GO TO (17,18,19,20),I
   17 SMIN = SM
      SMS = SSMOUT
```

```
      I = 2
      GO TO 16
   18 SMOUT = SM
      TEMP = (((GAMM1*SMOUT*SMOUT)+1.)/((GAMM1*SMIN*SMIN)+1.))**(GAMP1
     1/(2.*GAMM1))
      BETAN = BETAN*CONVER
      BETAT = -ARCOS(COS(BETAN)*(SMIN/SMOUT)*TEMP)
      BETAT = BETAT*RECONV
      BETAN = BETAN*RECONV
      DELV = DELV*RECONV

CC    PRINT  ALL  DESIGN  PARAMETERS
      WRITE (6,95) BETAN,VIN,VUP,VOUT,BETAT
   95 FORMAT (/2X,11HBETA(IN) = ,F7.4,4H DEG,4X,8HV(IN) = ,F7.4,4H DEG,
     16X,8HV(UP) = ,F8.4,4H DEG,7X,9HV(OUT) = ,F7.4,4H DEG,4X,12HBETA(OU
     2T) = ,F8.4,4H DEG)
      WRITE (6,94) DELV, VLOW, GAM
   94 FORMAT (/20X,10HDELTA V = ,F7.4,4H DEG,11X,9HV(LOW) = ,F7.4,4H DEG
     1,11X,8HGAMMA = ,F7.4)

CC    CONVERT  FROM  DEGREES  TO  RADIANS
      VIN = VIN*CONVER
      VOUT = VOUT*CONVER
      VUP = VUP*CONVER
      VLOW = VLOW*CONVER
      BETAN = BETAN*CONVER
      BETAT = BETAT*CONVER
      DELV = DELV*CONVER

CC    CHOOSE LONGEST TRANSITION ARC OF LOWER SURFACE
      VNL = VIN - VLOW
      KMAXN = (VNL/DELV) + 0.5
      VOL = VOUT - VLOW
      KMAXO = (VOL/DELV) + 0.5
      KMN = MAX0(KMAXN,KMAXO)
      V = AMAX1(VIN,VOUT)

CC    CALCULATE  R*(LOW)=RLOW,  M*(LOW)=SSMLOW,  M(LOW)=SMLOW
      IF (VLOW .EQ. 0.0) GO TO 2
      FN = KMN + 1
      FOFX = F(V,FN)
      CALL ROOT (X0,X2,XINTL,FOFX,FOFRS,RLOW)
      GO TO 3
    2 RLOW = 1.0
    3 SSMLOW = 1./RLOW
      SMS = SSMLOW
      I = 3
      GO TO 16
   19 SMLOW = SM

CC    SET INITIAL POINTS FOR LOWER ARC CALCULATIONS
      KNDEX = KMN/NPER
      KDEX = KNDEX
      RA(KDEX+1) = RLOW
      XLOW(KDEX+1) = 0.0
      YLOW(KDEX+1) = RLOW
```

```
      PHIKP1 = -(V-VLOW) + FLOAT(KMN)*DELV
      UMKP1 = ARSIN(SQRT(GAMP1*RLOW*RLOW - GAMM1))
      TXLO = XLOW(KDEX+1)
      TYLO = YLOW(KDEX+1)
      ALPHLN = VNL - BETAN
      ALPHLO = -(VOL+BETAT)

      IF (ALPHLN .LE. 0.0  .AND.  ALPHLO .GE. 0.0) GO TO 110
      ANGLE = .FALSE.
      WRITE (6,111)
  111 FORMAT (//27X,79HV(LOW) MUST BE GREATER THAN OR EQUAL TO V(IN) - B
     1ETA(IN) AND V(OUT) + BETA(OUT))

CC    CHOOSE LONGEST TRANSITION ARC OF UPPER SURFACE
  110 VUT = VUP - VOUT
      JMAXO = (VUT/DELV)+0.5
      VUI = VUP - VIN
      JMAXN = (VUI/DELV)+0.5
      JMN = MAX0(JMAXO,JMAXN)
      V = AMIN1(VOUT,VIN)

CC    CALCULATE  R*(UP)=RUP,  M*(UP)=SSMUP,  M(UP)=MUP
      FN = -(JMN+1) + 2
      FOFX = F(V,FN)
      CALL ROOT (X0,X2,XINTL,FOFX,FOFRS,RUP)
      SSMUP = 1./RUP
      SMS = SSMUP
      I = 4
      GO TO 16
   20 SMUP = SM

CC    SET INITIAL POINTS FOR UPPER ARC CALCULATIONS
      JNDEX = JMN/NPER
      JDEX = JNDEX
      R(JDEX+1) = RUP
      XUP(JDEX+1) = 0.0
      YUP(JDEX+1) = RUP
      PHIJP1 = -(VUP-V) + FLOAT(JMN)*DELV
      UMJP1 = ARSIN(SQRT(GAMP1*RUP*RUP - GAMM1))
      TXUP = XUP(JDEX+1)
      TYUP = YUP(JDEX+1)
      ALPHUI = VUI - BETAN
      ALPHUO = -(VUT+BETAT)

      IF (ALPHUI .LE. 0.0  .AND.  ALPHUO .GE. 0.0) GO TO 112
      ANGLE = .FALSE.
      WRITE (6,113)
  113 FORMAT (//28X,75HV(UP) MUST BE LESS THAN OR EQUAL TO V(IN) + BETA(
     1IN) AND V(OUT) - BETA(OUT))
  112 IF (.NOT. ANGLE) GO TO 1

      IF (VIN .EQ. VLOW  .AND.  VLOW .EQ. VOUT) GO TO 100

C****CALCULATE COORDINATES FOR LOWER TRANSITION ARC - UNROTATED
      KDEX = KNDEX + 1
      NUM = 0
```

```
      V = AMAX1(VIN,VOUT)
      DO 30 KK=1,KMN
      K = (KMN+1) - KK
      NUM = NUM + 1
      PHIK = PHIKP1 - DELV
      FN = K
      FOFX = F(V,FN)
      CALL ROOT (XO,X2,XINTL,FOFX,FOFRS,TR)
      TX = TR*SIN(PHIK)
      TY = TR*COS(PHIK)
      EMWK = TAN(-PHIKP1)
      UMK = ARSIN(SQRT(GAMP1*TR*TR - GAMM1))
      EMK = -TAN((PHIK+UMK+PHIKP1+UMKP1)/2.)
      TEMP = TYLO - EMWK*TXLO
      TEMPP = TY - EMK*TX
      TEMPPP = EMK - EMWK
      TXLO = (TEMP - TEMPP)/TEMPPP
      TYLO = ((EMK*TEMP) - (EMWK*TEMPP))/TEMPPP
      PHIKP1 = PHIK
      UMKP1 = UMK

CC        SAVE EVERY =NPER-TH= POINT
      N = NUM - (NUM/NPER)*NPER
      IF (N .GT. 0) GO TO 30
      KDEX = KDEX - 1
      RA(KDEX) = TR
      XLOW(KDEX) = TXLO
      YLOW(KDEX) = TYLO

   30 CONTINUE

C****CALCULATE COORDINATES FOR LOWER TRANSITION ARC - ROTATED
  100 KDEX = KNDEX + 1
      KMN = KMN + 1
      SINALN = SIN(ALPHLN)
      COSALN = COS(ALPHLN)
      SINALO = SIN(ALPHLO)
      COSALO = COS(ALPHLO)
      KN = (KMAXN/NPER) + 2
      KO = (KMAXO/NPER) + 2

      DO 40 KK=1,KDEX
      K = (KDEX+1) - KK
      KN = KN - 1
      KO = KO - 1
      IF (KN .LE. 0) GO TO 401
      XLOWN(KN) = YLOW(K)*SINALN + XLOW(K)*COSALN
      YLOWN(KN) = YLOW(K)*COSALN - XLOW(K)*SINALN
  401 IF (KO .LE. 0) GO TO 40
      XLOWO(KO) = YLOW(K)*SINALO - XLOW(K)*COSALO
      YLOWO(KO) = YLOW(K)*COSALO + XLOW(K)*SINALO
   40 CONTINUE

      IF (VIN .EQ. VUP  .AND.  VUP .EQ. VOUT) GO TO 200

C****CALCULATE COORDINATES FOR UPPER TRANSITION ARC - UNROTATED
```

```
      JDEX = JNDEX + 1
      NUM = 0
      V = AMIN1(VOUT,VIN)
      DO 41 JJ=1,JMN
      J = (JMN+1) - JJ
      NUM = NUM + 1
      PHIJ = PHIJP1 - DELV
      FN = -J + 2
      FOFX = F(V,FN)
      CALL ROOT (X0,X2,XINTL,FOFX,FOFRS,TR)
      TX = TR*SIN(PHIJ)
      TY = TR*COS(PHIJ)
      EMWJ = TAN(-PHIJP1)
      UMJ = ARSIN(SQRT(GAMP1*TR*TR - GAMM1))
      EMJ = TAN((-PHIJ+UMJ-PHIJP1+UMJP1)/2.)
      TEMP = TYUP - EMWJ*TXUP
      TEMPP = TY - EMJ*TX
      TEMPPP = EMJ - EMWJ
      TXUP = (TEMP - TEMPP)/TEMPPP
      TYUP = ((EMJ*TEMP) - (EMWJ*TEMPP))/TEMPPP
      PHIJP1 = PHIJ
      UMJP1 = UMJ
CC        SAVE EVERY =NPER-TH= POINT
      N = NUM - (NUM/NPER)*NPER
      IF (N .GT. 0) GO TO 41
      JDEX = JDEX - 1
      R(JDEX) = TR
      XUP(JDEX) = TXUP
      YUP(JDEX) = TYUP
   41 CONTINUE

C****CALCULATE COORDINATES FOR UPPER TRANSITION ARC - ROTATED
  200 JDEX = JNDEX + 1
      JMN = JMN + 1
      SINAUI = SIN(ALPHUI)
      COSAUI = COS(ALPHUI)
      SINAUO = SIN(ALPHUO)
      COSAUO = COS(ALPHUO)
      JN = (JMAXN/NPER) + 2
      JO = (JMAXO/NPER) + 2

      DO 47 JJ=1,JDEX
      J = (JDEX+1) - JJ
      JO = JO - 1
      JN = JN - 1
      IF (JO .LE. 0) GO TO 471
      XUPO(JO) = YUP(J)*SINAUO - XUP(J)*COSAUO
      YUPO(JO) = YUP(J)*COSAUO + XUP(J)*SINAUO
  471 IF (JN .LE. 0) GO TO 47
      XUPN(JN) = YUP(J)*SINAUI + XUP(J)*COSAUI
      YUPN(JN) = YUP(J)*COSAUI - XUP(J)*SINAUI
   47 CONTINUE

CC   CALCULATE G* - THE DIMENSIONLESS BLADE SPACING
      TANBI = TAN(BETAN)
      YLASTI = YUPN(1) + TANBI*(XLOWN(1) - XUPN(1))
```

```
      GSTAR = YLOWN(1) - YLASTI

CC    TITLES
      WRITE (6,93)
   93 FORMAT (///54X,25HL O W E R   S U R F A C E)
      IF (IPRINT .EQ. 0) GO TO 844
      WRITE (6,88)
   88 FORMAT (/54X,25HUNROTATED TRANSITION ARCS)
      WRITE (6,87)
   87 FORMAT (//2X,8H INLET K,4X,8HX*(LOW) ,3X,8HY*(LOW) ,68X,8HY*(LOW)
     1,3X,8HX*(LOW) ,2X,8HOUTLET K)

C****PRINT COORDINATES FOR LOWER TRANSITION ARC - UNROTATED
      KDEX = KNDEX + 2
      DO 51 KK=1,KMN,NPER
      K = (KMN+1) - KK
      KN = (KMAXN+2) - KK
      KO = (KMAXO+2) - KK
      KDEX = KDEX - 1
      XMLOW = -XLOW(KDEX)
      IF (KN .GT. 0  .AND.  KO .GT. 0) GO TO 510
      IF (KN .LE. 0) GO TO 511
      WRITE (6,861) KN,XLOW(KDEX),YLOW(KDEX)
  861 FORMAT (4X,I4,5X,F8.4,3X,F8.4)
      GO TO 51
  511 WRITE (6,863) YLOW(KDEX),XMLOW,KO
  863 FORMAT (100X,F8.4,3X,F8.4,5X,I4)
      GO TO 51
  510 WRITE (6,860) KN,XLOW(KDEX),YLOW(KDEX),YLOW(KDEX),XMLOW,KO
  860 FORMAT (4X,I4,5X,F8.4,3X,F8.4,68X,F8.4,3X,F8.4,5X,I4)
   51 CONTINUE

CC    TITLES
  844 WRITE (6,92)
   92 FORMAT (1HL,46X,38HROTATED AND TRANSLATED TRANSITION ARCS)
      WRITE (6,84)
   84 FORMAT (//2X,8H INLET K,6X,7HX*(LOW),7X,7HY*(LOW),6X,10HY*(LOW)-G*
     1,28X,10HY*(LOW)-G*,5X,7HY*(LOW),7X,7HX*(LOW),5X,8HOUTLET K)

C****PRINT COORDINATES FOR LOWER TRANSITION ARC - ROTATED
      M = 1
      XDOWN(M) = XLOWN(1)
      YACRS1(M) = YLOWN(1)
      YACRS2(M) = YLOWN(1) - GSTAR
      M = M + 1
      XDOWN(M) = XLOWO(1)
      YACRS1(M) = YLOWO(1)
      YACRS2(M) = YLOWO(1) - GSTAR

CC    STORE POINTS FOR PLOTTER - ONE POINT FOR EVERY FIVE DEGREES OF TURNING
      MAXN = (KMAXN/NPER) + 1
      MAXO = (KMAXO/NPER) + 1
      KNN = MAXN + 1
      KOO = MAXO + 1
      I = 0
```

```
      DO 55 KK=1,KMN,NPER
      KN = (KMAXN+2) - KK
      KO = (KMAXO+2) - KK
      KNN = KNN - 1
      KOO = KOO - 1
      I = I + 1
      LSTR = LSTORE*I
      IF (KN .GT. 0  .AND.  KO .GT. 0) GO TO 550
      IF (KN .LE. 0) GO TO 551
      IF (LSTR .GT. MAXN) GO TO 559
      M = M + 1
      XDOWN(M) = XLOWN(LSTR)
      YACRS1(M) = YLOWN(LSTR)
      YACRS2(M) = YLOWN(LSTR) - GSTAR
  559 YNG = YLOWN(KNN) - GSTAR
      WRITE (6,831) KN,XLOWN(KNN),YLOWN(KNN),YNG
  831 FORMAT (4X,I4,7X,F8.4,6X,F8.4,6X,F8.4)
      GO TO 55
  551 IF (LSTR .GT. MAXO) GO TO 557
      M = M + 1
      XDOWN(M) = XLOWO(LSTR)
      YACRS1(M) = YLOWO(LSTR)
      YACRS2(M) = YLOWO(LSTR) - GSTAR
  557 YTG = YLOWO(KOO) - GSTAR
      WRITE (6,833) YTG,YLOWO(KOO),XLOWO(KOO),KO
  833 FORMAT (81X,F8.4,6X,F8.4,6X,F8.4,7X,I4)
      GO TO 55
  550 YNG = YLOWN(KNN) - GSTAR
      YTG = YLOWO(KOO) - GSTAR
      IF (LSTR .GT. MAXN) GO TO 558
      M = M + 1
      XDOWN(M) = XLOWN(LSTR)
      YACRS1(M) = YLOWN(LSTR)
      YACRS2(M) = YLOWN(LSTR) - GSTAR
  558 IF (LSTR .GT. MAXO) GO TO 556
      M = M + 1
      XDOWN(M) = XLOWO(LSTR)
      YACRS1(M) = YLOWO(LSTR)
      YACRS2(M) = YLOWO(LSTR) - GSTAR
  556 WRITE (6,830) KN,XLOWN(KNN),YLOWN(KNN),YNG,YTG,YLOWO(KOO),XLOWO(KO
     1O),KO
  830 FORMAT (4X,I4,7X,F8.4,6X,F8.4,6X,F8.4,30X,F8.4,6X,F8.4,6X,F8.4,7X,
     1I4)
   55 CONTINUE

      M = M + 1
      XDOWN(M) = XLOWN(MAXN)
      YACRS1(M) = YLOWN(MAXN)
      YACRS2(M) = YLOWN(MAXN) - GSTAR
      M = M + 1
      XDOWN(M) = XLOWO(MAXO)
      YACRS1(M) = YLOWO(MAXO)
      YACRS2(M) = YLOWO(MAXO) - GSTAR

C****CIRCULAR ARC (LOWER)
      IF (IPRINT .EQ. 0) GO TO 810
```

```
      WRITE (6,81)
   81 FORMAT (//60X,13HCIRCULAR ARCS//40X,8HX*C(LOW),3X,8HY*C(LOW),16X,
     1 8HX*C(LOW),3X,11HY*C(LOW)-G*)

  810 M = M + 1
      XDOWN(M) = 0.0
      YACRS1(M) = RLOW
      YACRS2(M) = RLOW - GSTAR
      THETA = (BETAN - BETAT)*RECONV
      ALPH = ALPHLO + DALPH
      ALPLOW = ALPHLN
      KOUNT = 0
   60 XCLOW = RLOW*SIN(ALPLOW)
      YCLOW = RLOW*COS(ALPLOW)
      XCG = XCLOW
      YCG = YCLOW - GSTAR
      KOUNT = KOUNT + 1

      IF (KOUNT .NE. LSTORE) GO TO 601
      KOUNT = 0
      M = M + 1
      XDOWN(M) = XCLOW
      YACRS1(M) = YCLOW
      YACRS2(M) = YCG

  601 IF (IPRINT .EQ. 0) GO TO 800
      WRITE (6,80) XCLOW,YCLOW,XCLOW,YCG
   80 FORMAT (39X,F8.4,3X,F8.4,16X,F8.4,3X,F8.4)
  800 ALPLOW = ALPLOW + DALPH
      IF (ABS(ALPH-ALPLOW) .LE. 0.001) GO TO 56
      IF (ALPHLO .LT. ALPLOW  .AND.  ALPLOW .LT. ALPH) ALPLOW = ALPHLO
      GO TO 60

CC    STORE THE TRANSLATED LOWER ARC FOR PLOTTER
   56 NP1 = M
      DO 2000 I=1,NP1
      M = M + 1
      XDOWN(M) = XDOWN(I)
 2000 YACRS1(M) = YACRS2(I)
      NP2 = NP1

CC    TITLES
      WRITE (6,79)
   79 FORMAT (1H1,53X,25HU P P E R   S U R F A C E)
      IF (IPRINT .EQ. 0) GO TO 700
      WRITE (6,74)
   74 FORMAT (/54X,25HUNROTATED TRANSITION ARCS)
      WRITE (6,73)
   73 FORMAT (//2X,8H INLET J,3X,8H  X*(UP),3X,8H  Y*(UP),68X,8H  Y*(UP)
     1,3X,8H  X*(UP),3X,8HOUTLET J)

C****PRINT COORDINATES FOR UPPER TRANSITION ARC - UNROTATED
      JDEX = JNDEX + 2
      DO 65 JJ=1,JMN,NPER
      J = (JMN+1) - JJ
      JO = (JMAXO+2) - JJ
```

```
      JN = (JMAXN+2) - JJ
      JDEX = JDEX - 1
      XMUP = -XUP(JDEX)
      IF (JN .GT. 0  .AND.  JO .GT. 0) GO TO 650
      IF (JN .LE. 0) GO TO 651
      WRITE (6,720) JN,XUP(JDEX),YUP(JDEX)
  720 FORMAT (4X,I4,5X,F8.4,3X,F8.4)
      GO TO 65
  651 WRITE (6,723) YUP(JDEX),XMUP,JO
  723 FORMAT (100X,F8.4,3X,F8.4,5X,I4)
      GO TO 65
  650 WRITE (6,721) JN,XUP(JDEX),YUP(JDEX),YUP(JDEX),XMUP,JO
  721 FORMAT (4X,I4,5X,F8.4,3X,F8.4,68X,F8.4,3X,F8.4,5X,I4)
   65 CONTINUE

CC    TITLES
  700 WRITE (6,78)
   78 FORMAT (1HL,46X,38HROTATED AND TRANSLATED TRANSITION ARCS)
      WRITE (6,70)
   70 FORMAT (//2X,8H INLET J,6X,7H X*(UP),7X,7H Y*(UP),6X, 9HY*(UP)+G*,
     129X,9HY*(UP)+G*,6X,7H Y*(UP),7X,7H X*(UP),5X,8HOUTLET J)

      L = NP1 + NP2

C****PRINT COORDINATES FOR UPPER TRANSITION ARC - ROTATED
CC    STORE POINTS FOR PLOTTER - ONE POINT FOR EVERY FIVE DEGREES OF TURNING
      L = L + 1
      XDOWN(L) = XUPN(1)
      YACRS1(L) = YUPN(1)
      L = L + 1
      XDOWN(L) = XUPO(1)
      YACRS1(L) = YUPO(1)
      MAXO = (JMAXO/NPER) + 1
      MAXN = (JMAXN/NPER) + 1
      JOO = MAXO + 1
      JNN = MAXN + 1
      I = 0

      DO 303 JJ=1,JMN,NPER
      JO = (JMAXO+2) - JJ
      JN = (JMAXN+2) - JJ
      JOO = JOO - 1
      JNN = JNN - 1
      I = I + 1
      LSTR = LSTORE*I
      IF (JN .GT. 0  .AND.  JO .GT. 0) GO TO 3030
      IF (JN .LE. 0) GO TO 3031
      IF (LSTR .GT. MAXN) GO TO 688
      L = L + 1
      XDOWN(L) = XUPN(LSTR)
      YACRS1(L) = YUPN(LSTR)
  688 YNG = YUPN(JNN) + GSTAR
      WRITE (6,68) JN,XUPN(JNN),YUPN(JNN),YNG
   68 FORMAT (4X,I4,7X,F8.4,6X,F8.4,6X,F8.4)
      GO TO 303
 3031 IF (LSTR .GT. MAXO) GO TO 689
```

```
      L = L + 1
      XDOWN(L) = XUPO(LSTR)
      YACRS1(L) = YUPO(LSTR)
  689 YTG = YUPO(JOO) + GSTAR
      WRITE (6,683) YTG,YUPO(JOO),XUPO(JOO),JO
  683 FORMAT (81X,F8.4,6X,F8.4,6X,F8.4,7X,I4)
      GO TO 303
 3030 YNG = YUPN(JNN) + GSTAR
      YTG = YUPO(JOO) + GSTAR
      IF (LSTR .GT. MAXN) GO TO 670
      L = L + 1
      XDOWN(L) = XUPN(LSTR)
      YACRS1(L) = YUPN(LSTR)
  670 IF (LSTR .GT. MAXO) GO TO 671
      L = L + 1
      XDOWN(L) = XUPO(LSTR)
      YACRS1(L) = YUPO(LSTR)
  671 WRITE (6,680) JN,XUPN(JNN),YUPN(JNN),YNG,YTG,YUPO(JOO),XUPO(JOO),
     1JO
  680 FORMAT (4X,I4,7X,F8.4,6X,F8.4,6X,F8.4,30X,F8.4,6X,F8.4,6X,F8.4,7X,
     1I4)
  303 CONTINUE

      L = L + 1
      XDOWN(L) = XUPN(MAXN)
      YACRS1(L) = YUPN(MAXN)
      L = L + 1
      XDOWN(L) = XUPO(MAXO)
      YACRS1(L) = YUPO(MAXO)

C****CIRCULAR ARC (UPPER)
      IF (IPRINT .EQ. 0) GO TO 6700
      WRITE (6,67)
   67 FORMAT (//60X,13HCIRCULAR ARCS//40X,8HX*C(UP) ,3X,8HY*C(UP) ,16X,
     1 8HX*C(UP) ,3X,10HY*C(UP)+G*)

 6700 L = L + 1
      XDOWN(L) = 0.0
      YACRS1(L) = RUP

      ALPH = ALPHUO + DALPH
      ALPHUP = ALPHUI
      KOUNT = 0
  305 XCUP = RUP*SIN(ALPHUP)
      YCUP = RUP*COS(ALPHUP)
      XCG = XCUP
      YCG = YCUP + GSTAR
      KOUNT = KOUNT + 1

      IF (KOUNT .NE. LSTORE) GO TO 672
      KOUNT = 0
      L = L + 1
      XDOWN(L) = XCUP
      YACRS1(L) = YCUP

  672 IF (IPRINT .EQ. 0) GO TO 660
```

```
      WRITE (6,66) XCUP,YCUP,XCUP,YCG
   66 FORMAT (39X,F8.4,3X,F8.4,16X,F8.4,3X,F8.4)
  660 ALPHUP = ALPHUP + DALPH
      IF (ABS(ALPH-ALPHUP) .LE. 0.001) GO TO 306
      IF (ALPHUO .LT. ALPHUP  .AND.  ALPHUP .LT. ALPH) ALPHUP = ALPHUO
      GO TO 305

C****CALCULATE COORDINATES FOR STRAIGHT LINE PORTION OF UPPER ARC
CC    FIFTEEN POINTS ARE CALCULATED  FOR PLOTTING PURPOSES
  306 IF (IPRINT .EQ. 0) GO TO 3070
      WRITE (6,307)
  307 FORMAT (//59X,14HSTRAIGHT LINES//5X,8H X*S(IN),5X,8H Y*S(IN) ,3X,
     110HY*S(IN)+G*,54X,11HY*S(OUT)+G*,2X,8HY*S(OUT),5X,8HX*S(OUT))
 3070 KOUNT = -1
      DELXI = ( XUPN(1) - XLOWN(1) )/15.
      DELXO = ( XLOWO(1) - XUPO(1) )/15.
      XSIN = XUPN(1)
      YSIN = YUPN(1)
      XSOUT = XUPO(1)
      YSOUT = YUPO(1)
      TANBO = TAN(BETAT)
      GO TO 309
  310 XSIN = XSIN - DELXI
      XSOUT = XSOUT + DELXO
      YSIN = YUPN(1) + TANBI*(XSIN - XUPN(1))
      YSOUT = YUPO(1) + TANBO*(XSOUT - XUPO(1) )
  309 YSNG = YSIN + GSTAR
      YSTG = YSOUT + GSTAR
      IF (XSIN .LE. XLOWN(1) ) GO TO 312
      KOUNT = KOUNT + 1
      N = KOUNT - (KOUNT/3)*3
      IF (N .GT. 0) GO TO 673
      L = L + 1
      XDOWN(L) = XSIN
      YACRS1(L) = YSIN
  673 IF (IPRINT .EQ. 0) GO TO 3133
      WRITE (6,313) XSIN,YSIN,YSNG
  313 FORMAT (5X,F8.4,4X,F8.4,4X,F8.4)
 3133 IF (XSOUT .GE. XLOWO(1) ) GO TO 310
      IF (N .GT. 0) GO TO 674
      L = L + 1
      XDOWN(L) = XSOUT
      YACRS1(L) = YSOUT
  674 IF (IPRINT .EQ. 0) GO TO 310
      WRITE (6,315) YSTG,YSOUT,XSOUT
  315 FORMAT (1H+,93X,F8.4,4X,F8.4,4X,F8.4)
      GO TO 310
  312 IF (XSOUT .GE. XLOWO(1) ) GO TO 311
      IF (IPRINT .EQ. 0) GO TO 310
      WRITE (6,321) YSTG,YSOUT,XSOUT
  321 FORMAT (94X,F8.4,4X,F8.4,4X,F8.4)
      GO TO 310

  311 NP3 = L - (NP1 + NP2)

      NSUM = NP1 + NP2 + NP3 + 1
```

```
      XDOWN(NSUM) = 0.0
      YACRS1(NSUM) = 0.0

C****MISCELLANEOUS  CALCULATIONS
      WRITE (6,622)
  622 FORMAT (//54X,24HMISCELLANEOUS PARAMETERS//)
      YLASTO = YUPO(1) + TANBO*(XLOWO(1) - XUPO(1))
      CSTAR  = SQRT( ((XLOWO(1) - XLOWN(1))**2) + ((YLOWO(1) - YLOWN(1))
     1**2) )
      SIGMA  = CSTAR /GSTAR
      WRITE (6,999) VLSPMN,VUSPMX
  999 FORMAT (17X,84HTHE MINIMUM LOWER SURFACE PRANDTL-MEYER ANGLE PREDI
     1CTED BY SEPARATION CONDITIONS IS ,F9.4,4H DEG//17X,84HTHE MAXIMUM
     2UPPER SURFACE PRANDTL-MEYER ANGLE PREDICTED BY SEPARATION CONDITIO
     3NS IS ,F9.4,4H DEG)
      WRITE (6,1000) SSMIN,SMIN,SMOUT,SSMOUT
 1000 FORMAT (/25X, 9HM*(IN) = ,F8.4,2X,9H M(IN) = ,F8.4,10X,9HM(OUT) =
     1,F8.4,5X,10HM*(OUT) = ,F8.4)
      WRITE (6,1001) RLOW,SSMLOW,SMLOW,SMUP,SSMUP,RUP
 1001 FORMAT (/2X,9HR*(LOW) =,F8.4,5X,10HM*(LOW) = ,F8.4,2X,9HM(LOW) = ,
     1F8.4,10X,9H M(UP) = ,F8.4,5X,10H M*(UP) = ,F8.4,2X,9HR*(UP) = ,
     2F8.4)
      WRITE (6,1002) THETA,GSTAR,CSTAR,SIGMA
 1002 FORMAT (/11X,8HTHETA = ,F8.4,4H DEG,12X,5HG* = ,F8.4,13X,5HC* = ,
     1F8.4,11X,8HSIGMA = ,F8.4)

      IF (NPLOT .NE. 0) GO TO 1

CC    IF PLOTMY IS NOT AVAILABLE, REMUVE THE FOLLOWING CARDS
C****MULTIPLE PLOT  -  START
      LLL = NP1 + NP2
      CALL SORTXY (XDOWN(1),YACRS1(1),NP1)
      CALL SORTXY (XDOWN(NP1+1),YACRS1(NP1+1),NP2)
      CALL SORTXY (XDOWN(LLL+1),YACRS1(LLL+1),NP3)
      P(1) = 5.0
      P(3) = 12.0
      P(4) = 20.0
      P(11) =((1. - AMIN1(YACRS1(1),YACRS1(NP1+1),YACRS1(LLL+1)))/100.)*
     1(10.**4)
      P(6) = 2.0
      P(7) = AMIN1( XDOWN(1),XDOWN(NP1+1),XDOWN(LLL+1) ) *(10.**4)
      P(8) = P(11)*(5./3.)
      P(9) = 2.0
      P(10) = AMIN1(YACRS1(1),YACRS1(NP1+1),YACRS1(LLL+1))*(10.**4)
      KKK(1) = 55
      KKK(2) = 4
      KKK(3) = NP1
      KKK(5) = NP2
      KKK(7) = NP3
      KKK(9) = 1
      DATA KKK(4),KKK(6),KKK(8)/1H*,1H*,1H+/,KKK(10)/1HO/
      CALL PLOTMY (XDOWN,YACRS1,KKK,P)
C****MULTIPLE PLOT  -  STOP
      GO TO 1

      END
```

```
$IBFTC ROO     LIST

      SUBROUTINE ROOT (X0,X2,XINTL,FOFX,FUNC,X1)
      COMMON/ROOTS/DELF
      DOUBLE PRECISION X,XX0,XX2

C     WE ARE SEEKING AN X SUCH THAT FUNC(X) = FOFX WHERE FOFX IS A KNOWN
C     FUNCTIONAL VALUE
C       1  LOCATE FOFX IN (F0,FX) OR (FX,F2) WHERE FX IS THE PREVIOUS
C          APPROXIMATION TO FOFX
C       2  LET X = 1/2(XX0+X) OR X = 1/2(X+XX2)
C       3  IS FUNC(X) = FOFX =  IF NOT, REPEAT PROCEDURE

      XX0 = X0
      XX2 = X2
      F0 = FUNC(XX0)
      F2 = FUNC(XX2)
      IF ( FOFX .LT. F0 .AND. FOFX .LT. F2  .OR.  FOFX .GT. F0 .AND.
     1FOFX .GT. F2 ) GO TO 1005

      IF ( ABS(FOFX-F0) .LE. DELF) GO TO 1007
      IF ( ABS(FOFX-F2) .LE. DELF) GO TO 1008

      X = XINTL
      KOUNT = 0
 1000 X1 = X
      KOUNT = KOUNT + 1
      A = FOFX - F2
      FX = FUNC(X)
      IF (KOUNT .GE. 60) WRITE (6,1004) KOUNT,X,FX,FOFX
 1004 FORMAT (1HL,9H  KOUNT   ,G16.9,9H    X    ,G16.9,9H   FX    ,G16.9,
     19H   FOFX   ,G16.9)
      IF (ABS(FX-FOFX) .LE. DELF) RETURN
      IF (KOUNT .EQ. 75) GO TO 1002
      IF (A*(FX-FOFX) .LT. 0.) GO TO 1001
      XX0 = X
      X = (X+XX2)/2.
      GO TO 1000
 1001 XX2 = X
      X = (XX0+X)/2.
      F2 = FX
      GO TO 1000

 1002 WRITE (6,1003)
 1003 FORMAT (//30X,62H75 ITERATIONS HAVE BEEN PERFORMED WITHOUT CONVERG
     1ING TO A ROOT)
      RETURN

 1005 WRITE (6,1006) FOFX
 1006 FORMAT (//10X,7HF(X) = ,G16.9,31H IS OUTSIDE OF SPECIFIED LIMITS)
      RETURN

 1007 X1 = X0
      RETURN

 1008 X1 = X2
      RETURN

      END
```

```
$IBFTC STARTT  LIST

      SUBROUTINE START (VLOW,FLO,VUP,FUP,VIMAX)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,BINTGR
      EXTERNAL CFACT,QFACT,FRAT,FOFRS,FKMAX

      X0 = 1./PERM
      X2 = 0.999999999
      XINTL = (X0 + X2)/2.

      IF (VLOW .EQ. 0.0) GO TO 70
      CALL ROOT (X0,X2,XINTL,FLO,FOFRS,RLOW)
      GO TO 71
   70 RLOW = 1.0
   71 SSMLOW = 1./RLOW
      CALL ROOT (X0,X2,XINTL,FUP,FOFRS,RUP)
      SSMUP = 1./RUP

      IF (SSMLOW .EQ. SSMUP) GO TO 40

C     FKMAX(X) IS LINEAR IN A NEIGHBORHOOD OF X WHEN X IS SUCH THAT FKMAX(X)=0
C     USE GOOD INITIAL ESTIMATE PLUS LINEARITY TO FIND X SUCH THAT FKMAX(X)=0

      XINTL = (1./PERM)*SQRT( SSMLOW/SSMUP )
      X0 = XINTL - 0.005
      F0 = FKMAX(X0,0)
      X2 = XINTL + 0.001
      F2 = FKMAX(X2,0)
      SLOPE = (F2 - F0)/(X2 - X0)
      FINTL = FKMAX(XINTL,0)
      YINCPT = FINTL - SLOPE*XINTL
      XAMK = -YINCPT/SLOPE
      FOFX = FKMAX(XAMK,1)
      IF (ABS(FOFX) .GT. 0.00009) WRITE (6,60) FOFX,XAMK
   60 FORMAT (//29X,35HSEARCH FOR ROOT FAILED      F(X) = ,G16.9,7H    X
     1= ,G16.9)

      SAME = (XAMK/SSMLOW)*(XAMK/SSMLOW)
      C = 1. - PERM*(GAMP1**(1./(GAM-1.)))*(SSMUP/(SSMUP-SSMLOW))*XAMK*
     1BINTGR
      CINTGR = SIMPS1(SSMLOW,SSMUP,QFACT,K)
      Q = (SSMLOW*SSMUP/(SSMUP-SSMLOW))*CINTGR
      RATIO = Q/(1. - C)
      GO TO 50
   40 XAMK = 1./PERM
      RATIO = SSMUP*SSMUP*QFACT(SSMUP)
      C = 0.0
      Q = 0.0
   50 X0 = 1.0
      X2 = PERM
      XINTL = (X0 + X2)/2.
      CALL ROOT (X0,X2,XINTL,RATIO,FRAT,SSMIAX)
      VIMAX = (3.14159265/4.)*(PERM-1.) + (PERM/2.)*ARSIN(2.*GAMM1*
     1 SSMIAX*SSMIAX - GAM) + 0.5*ARSIN(2.*GAMP1/(SSMIAX*SSMIAX) - GAM)
```

```
      VIMAX = VIMAX*RECONV
      VLOW = VLOW*RECONV
      VUP = VUP*RECONV
      WSTAR = ((1./GAMP1)**(GAMP1/(2.*GAMM1)))*CINTGR

      WRITE (6,10)
   10 FORMAT (//48X,36HCALCULATIONS FOR SUPERSONIC STARTING)
      WRITE (6,90) WSTAR
   90 FORMAT (/50X,24HWEIGHT-FLOW PARAMETER = ,F9.4)
      WRITE (6,20) XAMK,C,Q,SSMIAX
   20 FORMAT(/20X,10HK*(MAX) = ,F9.4,5X,5H C = ,F9.4,5X,5H Q = ,F9.4,5X,
     113HM*(I(MAX)) = ,F9.4)
      WRITE (6,30) VIMAX,VLOW,VUP,GAM
   30 FORMAT (/4X,38HTHE MAXIMUM DESIGN VALUE FOR V(IN) IS ,F9.4,21H DEG
     1  WHEN V(LOW) IS ,F9.4,16H DEG,  V(UP) IS ,F9.4,16H DEG,  GAMMA IS
     2 ,F7.4)
      RETURN
      END
```

```
$IBFTC MSS      LIST
      SUBROUTINE MSSTAR (M,N,VSSTAR)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,D
      REAL M,MS
      EXTERNAL ADSTR

      A = 0.785398162*(PERM -1.)
      B = 0.5*PERM
      C = GAM -1.
      D = GAM + 1.
      IF(N .NE. 0) GO TO 1
      MS = ADSTR(M)
      IF (MS .LT. 1.) GO TO 3
      GO TO 2
    1 X0 = 1.
      X2 = PERM
      XINTL = (X0 + X2)/2.
      FOFX = M
      CALL ROOT (X0,X2,XINTL,FOFX,ADSTR,MS)
    2 SQRDMS = MS*MS
      VSSTAR = ( A + B*ARSIN(C*SQRDMS-GAM) + 0.5*ARSIN(D/SQRDMS-GAM) )*
     1 RECONV
      RETURN

    3 VSSTAR = 0.
      RETURN

      END
```

```
$IBFTC SIMPS    LIST
      FUNCTION SIMPS1(XMIN,XMAX,FUNC1,KER)
      DIMENSION V(200),H(200),A(200),B(200),C(200),P(200),E(200),NE(200)
      EQUIVALENCE (E,NE),(TEST,NTEST)

      T=3.0E-5
      V(1)=XMIN
      H(1)=0.5*(XMAX-XMIN)
      A(1)=FUNC1(XMIN)
      B(1)=FUNC1(XMIN+H(1))
      C(1)=FUNC1(XMAX)
      P(1)=H(1)*(A(1)+4.0*B(1)+C(1))
      E(1)=P(1)
      ANS=P(1)
      N=1
      FRAC=2.0*T

    1 FRAC=0.5*FRAC
    2 TEST=ABS(FRAC*ANS)
      K=N
    3 DO 7 I=1,K
    4 IF (NTEST-IABS(NE(I))) 5,5,7
    5 N = N+1
      V(N)=V(I)+H(I)
      H(N)=0.5*H(I)
      A(N)=B(I)
      B(N)=FUNC1(V(N)+H(N))
      C(N)=C(I)
      P(N)=H(N)*(A(N)+4.0*B(N)+C(N))
      Q=P(I)
      H(I)=H(N)
      B(I)=FUNC1(V(I)+H(I))
      C(I)=A(N)
      P(I)=H(I)*(A(I)+4.0*B(I)+C(I))
      Q=P(I)+P(N)-Q
      ANS=ANS+Q
      E(I)=Q
      E(N)=Q
    6 IF (N-200) 7,13,13
    7 CONTINUE

    8 IF (N-K) 9,9,2
    9 Q = 0.0

   10 DO 11 I=1,N
   11 Q=Q+E(I)
   12 IF (ABS(Q)-T*ABS(ANS)) 14,14,1
13    KER=KER+1
   14 ANS=0.0
   15 DO 16 I=1,N
   16 ANS=ANS+P(I)

      SIMPS1=(ANS+Q/30.0)/3.0
   17 RETURN

      END
```

```
$IBFTC ALLFUN
      FUNCTION ALFUNC (A,B,Y)

      COMMON/EXPALF/GAMEXP
      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY

      ALFUNC = (1./Y)*((A - B*Y*Y)**GAMEXP)

      RETURN
      END
```

```
$IBFTC BAKE
      FUNCTION CFACT (Y)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY
      EXTERNAL ALFUNC

      CFACT = ALFUNC(1.,SAME,Y)

      RETURN
      END
```

```
$IBFTC CHARL
      FUNCTION QFACT(Y)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY
      EXTERNAL ALFUNC

      QFACT = ALFUNC(GAMP1,GAMM1,Y)

      RETURN
      END
```

```
$IBFTC DOGG
      FUNCTION FRAT(Y)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY
      EXTERNAL QFACT,ALFUNC

      FRAT = (Y**(GAM/GAMM1))*QFACT(Y)/ALFUNC(-GAMM1,-GAMP1,Y)

      RETURN
      END
```

```
$IBFTC FELI
       FUNCTION FOFRS (X)

       COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,GRTY
       DOUBLE PRECISION X

       ARG1 = 2.*GAMM1/(X*X) - GAM
       ARG2 = 2.*GAMP1*X*X - GAM
       IF (ABS(ARG1) .GT. 1.0  .OR.  ABS(ARG2) .GT. 1.0) WRITE (6,1) ARG1
      1,ARG2
     1 FORMAT (//14X,61HARGUMENT OF ARCSIN IS OUTSIDE DOMAIN OF DEFINITIO
      1N     ARG1 = ,G16.9,11H     ARG2 = ,G16.9)

       FOFRS = PERM*ARSIN(ARG1) + ARSIN(ARG2)

       RETURN
       END
```

```
$IBFTC GERT
       FUNCTION FKMAX(Y,L)

       COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,BINTGR
       EXTERNAL ALFUNC,CFACT

       SAME = (Y/SSMLOW)*(Y/SSMLOW)
       K = 0
       FKMAX = SIMPS1(SSMLOW,SSMUP,CFACT,K)
       IF (K .EQ. 1) WRITE (6,1)
     1 FORMAT (/10X,26HFAILURE TO INTEGRATE CFACT)
       IF (L .EQ. 1) BINTGR=FKMAX

       FKMAX = FKMAX + SSMUP*CFACT(SSMUP) - ALFUNC(1.,Y*Y,1.)

       RETURN
       END
```

```
$IBFTC STARM
      FUNCTION ADSTR(MSTAR)

      COMMON/FACTOR/PERM,SAME,GAM,GAMM1,GAMP1,SSMLOW,SSMUP,RECONV,D
      REAL MSTAR,M

      C = GAM - 1.
      E = C/D
      G = C/GAM
      H = GAM/D
      M = MSTAR*MSTAR

     ADSTR = PERM*SQRT( (1.-(1.-E*M)*((1.+0.5*((H*M)/(1.-E*M)))**G))  )

     RETURN
     END
```

Lewis Research Center,
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128-31-02-25-22.

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