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ATTENUATION AND DISPERSION OF LONGITUDINAL WAVES IN VISCOUS, PARTLY IONIZED GASES

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ATTENUATION AND DISPERSION OF LONGITUDINAL WAVES
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The propagation of longitudinal waves in three fluid, partly ionized gases is studied as a function of the collision and plasma-to-wave-frequency ratios. A collisional approach is taken with the full Boltzmann equations (together with Maxwell's equations) serving as the starting point, the intention being to describe wave dispersion and absorption in the collisional regime, and to include the viscous and heat conduction effects on the wave propagation from the beginning as an integral part of the theory. While wave propagation in both the collisionless and continuum regimes has been extensively studied, collisional kinetic investigations have been more limited [1]-[4] and for the three fluid gas, the full Boltzmann equation approach employed in this paper for studying wave propagation has the advantage that all three fluid collision frequencies are used rather than only the neutral one of [3], and are obtained directly from the collision integrals. Since viscosity is a molecular interaction phenomenon, this direct identification with the collision integrals is consistent and desirable, the self collision frequencies determining the equilibration of the individual fluids and the interparticle collision frequencies determining the effective coupling between the fluids. On the other hand, the complicated equations that result from this approach lead to possible disadvantages: In order to simplify the equations we assume Maxwell molecule interactions involving the neutral particles and assume a long- and short-range Coulomb force (cut off at the Debye length) for interactions among the charged particles. While these are certainly approximations to the true law of interaction, the error introduced is probably not very large as has been discussed previously [4]. The equations are further simplified by assuming that the fluids are initially (prior to being perturbed) at the same temperature. This common temperature assumption leads to large ion wave damping because of phase mixing as the ion wave and particle speeds become comparable [1]. However, since this occurs predominantly in the Landau regime, for the purposes of this paper which is intended to determine the amount of collisional damping and collisional dispersion, the restriction appears not to be too severe.

For our collisional study we expand each of the velocity distribution functions, satisfying the corresponding Boltzmann equation, in Sonine polynomials and solid harmonics (expansions about equilibrium), and obtain the coupled set of equations for longitudinal waves shown below. The notations used are the δ-functions δ_q which are zero unless the interaction is between charged species in which case it is equal to unity and $\delta_{s,q} = 1$ if s refers to an electron or ion; otherwise it is zero; a propagation

*Presented at VIII International Conference on Phenomena in Ionized Gases, Vienna, Aug. 27-Sept. 2, 1967 constant $k_s = KC_{os}/\omega$ normalized with respect to the adiabatic s-fluid sound speed $C_{os} = \sqrt{\frac{5}{3}\, k_B\, T_0/m_s}$; complex frequencies $\Omega_{e,i,n} = (\omega^2_{e,i,n} + i\omega\nu^{(1)}_{e,i,n})^{1/2}$ where $\omega_n \equiv 0$ and $\omega_{e,i}$ are the usual electron and ion plasma frequencies and where $\nu_e^{(1)}$, $\nu_i^{(1)}$, and $\nu_n^{(1)}$ are the total electron, ion and neutral effective cross-collision frequencies for momentum transfer, respectively; the effective self collision frequencies are included in the $\nu^{(2)}_{st}$ occurring in the higher order equations (3) and (4) below. Finally $\rho_{os} = m_s n_{os}$ is the equilibrium mass density of the s-fluid, s denoting the electron, ion, or neutral particles:

$$A_{01s}\left(1-\frac{3}{5}k_s^2-\frac{\Omega_s^2}{\omega^2}\right)+\sum_{t\neq s}\frac{m_s\rho_{os}}{m_t\rho_{ot}}A_{01t}\left(\frac{m_t}{m_s}\omega_t^2\delta_{s,q}+i\frac{\nu^{(1)}_{st}}{\omega}\right)$$
$$+\sqrt{\frac{6}{5}}\,k_s\left(A_{10s}-\frac{2}{5}A_{02s}\right)+\frac{3}{5}\,i\,\frac{\nu^{(1)}_{st}}{\omega}\,\frac{m_s}{m_s+m_t}\left(\frac{m_s}{m_t}A_{11t}-A_{11s}\right)\delta_q=0 \tag{1}$$

$$A_{10s}\left(1-2\sum_{t\neq s}\frac{m_s}{m_s+m_t}\,i\,\frac{\nu^{(1)}_{st}}{\omega}\right)+2\sum_{t\neq s}\frac{(m_s^3/m_t)^{1/2}}{m_s+m_t}\,\frac{\rho_{os}}{\rho_{ot}}\,i\,\frac{\nu^{(1)}_{st}}{\omega}A_{10t}$$
$$+\sqrt{\frac{2}{15}}\,k_s\left(A_{01s}-A_{11s}\right)=0 \tag{2}$$

$$A_{02s}\left[1-i\frac{\nu^{(2)}_{ss}}{\omega}-\sum_{t\neq s}\frac{m_t}{m_s+m_t}\left(i\frac{\nu^{(2)}_{st}}{\omega}+2\frac{m_s}{m_t}\,i\,\frac{\nu^{(1)}_{st}}{\omega}\right)\right]$$
$$+\sqrt{\frac{8}{15}}\,k_s\left(A_{11s}-\frac{5}{2}A_{01s}\right)-\sum_{t\neq s}A_{02t}\,\frac{(m_s^3/m_t)^{1/2}}{m_s+m_t}$$
$$\cdot\,\frac{\rho_{os}}{\rho_{ot}}\left(i\frac{\nu^{(2)}_{st}}{\omega}-2\,i\frac{\nu^{(1)}_{st}}{\omega}\right)=0 \tag{3}$$

$$A_{11s}\left\{1-\frac{2}{3}\,i\,\frac{\nu^{(2)}_{ss}}{\omega}-\sum_{t\neq s}\frac{m_s m_t}{(m_s+m_t)^2}\left[\frac{4}{3}\,i\,\frac{\nu^{(2)}_{st}}{\omega}+\frac{m_s}{m_t}\,i\,\frac{\nu^{(1)}_{st}}{\omega}\right.\right.$$
$$\left.\left.\cdot\left(3+\frac{m_t^2}{m_s^2}+\frac{3}{10}\frac{m_t^2}{m_s^2}\delta_q\right)\right]\right\}-\sum_{t\neq s}A_{11t}\left(\frac{m_s}{m_s+m_t}\right)^2 \tag{4}$$

$$\cdot \frac{\rho_{os}}{\rho_{ot}}\left[\frac{4}{3}\, i \frac{\nu_{st}^{(2)}}{\omega} - i\frac{\nu_{st}^{(1)}}{\omega}\left(4+\frac{3}{10}\,\delta_q\right)\right] - \sqrt{\frac{15}{2}}\, k_s \left(A_{10s} - \frac{4}{25}A_{02s}\right)$$

$$+\left(\frac{5}{2}\,\frac{\omega_s^2}{\omega^2} + \frac{3}{2}\,\frac{m_t}{m_s+m_t}\, i \,\frac{\nu_{st}^{(1)}}{\omega}\right)\left(\frac{m_s\rho_{os}}{m_t\rho_{ot}}\,A_{01t} - A_{01s}\right)\delta_q = 0 \qquad (4)$$

From this set of 12 equations, 3 each for the momentum (A_{01}), energy (A_{10}), stress (A_{02}) and heat flow (A_{11}), we immediately form the longitudinal wave dispersion relation as the determinantal equation DET A = 0 with the elements of this determinant being just the coefficients of the $A_{p\ell s}$ terms. The dispersion relation has been solved numerically for the propagation constant over a large range of collision, plasma, and wave frequency ratios for several different plasmas ranging from slightly ionized to fully ionized, and including several plasmas which are intended to approximate some experimental situations [5]. Because of limited space, we show here only the results of a couple of cases obtained: The neutral wave in a slightly ionized gas and the ion wave in a fully ionized gas (figures 1 - 4). These examples, though picked for their relative simplicity, do, however, illustrate the effects of collisional coupling on the wave propagation and the general result of a maximum in the absorption constant when the self collision frequency is approximately equal to the wave frequency. Further information is obtained by noting several limiting cases that were taken. In the limit of very large self-collision-to-wave-frequency ratios, the dispersion relation reduces to the inviscid dispersion relations of [4] and, in the fully ionized case, of [6], since the viscosity vanishes in this limit where $\nu_{ss}/\omega \to \infty$. If we also idealize to the case of negligible interparticle collision frequencies in this inviscid limit, we obtain just the usual nondissipative adiabatic dispersion relations discussed in [2].

In all cases solution of the dispersion relation yields the six roots $C_{os}/C = 0.606$ and 1.59 when the collision-to-wave-frequency ratios are sufficiently small (a result also obtained for neutral gases by

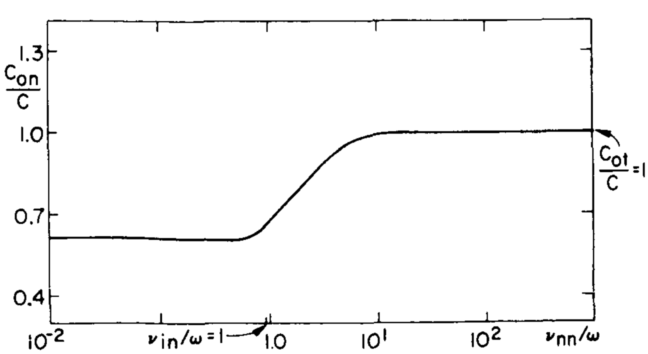

Figure 1. - Neutral particle wave speed

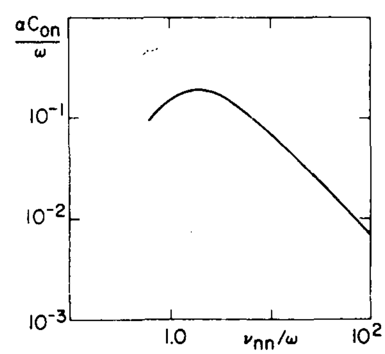

Figure 2. - Neutral particle wave absorption

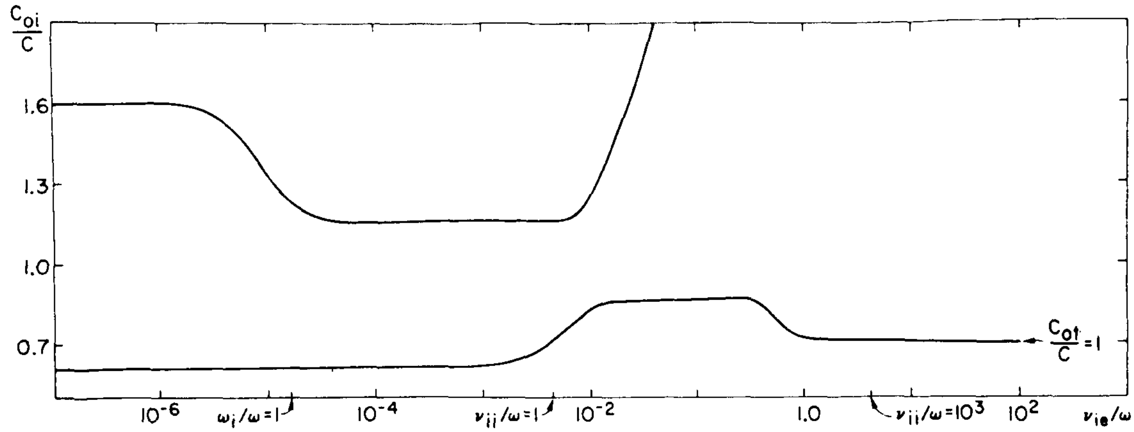

Figure 3. - Ion wave speeds

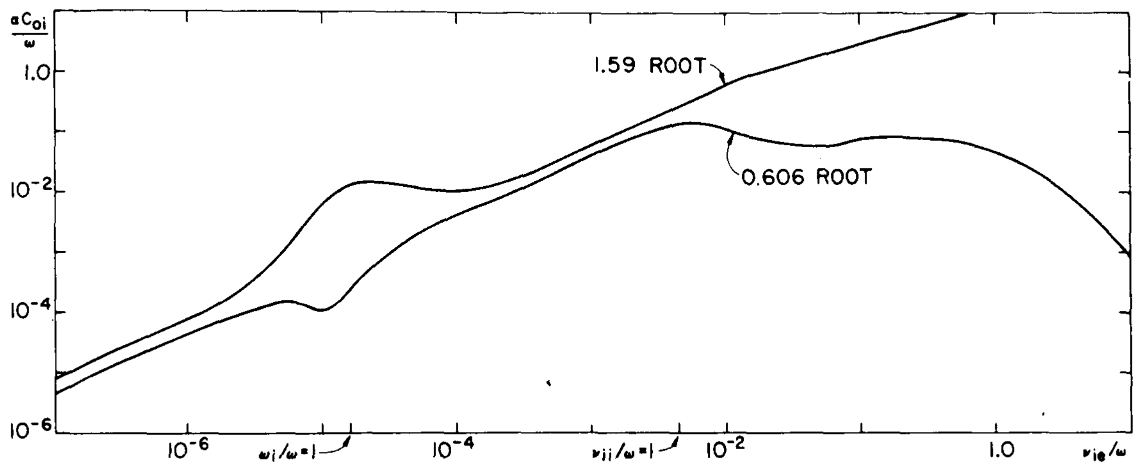

Figure 4. - Ion wave absorption functions

Grad [7] in this limit); these roots will serve as convenient labels for the waves picked out for discussion here. In figures 1 and 2 we show the $C_{on}/C = 0.606$ root for a slightly ionized gas (the neutral to ionized particle density was taken as 2×10^4) as a function of the neutral-neutral collision-to-wave-frequency ratio ν_{nn}/ω. We find an absorption peak around $\nu_{nn}/\omega \approx 1$ where the individual collisions are most effective in extracting energy from the wave; as ν_{nn}/ω (and also ν_{in}/ω) further increases to values greater than unity, the wave speed approaches the total adiabatic sound speed $C_{ot} = \sqrt{\frac{5}{3} P_{ot} / \rho_{ot}}$ appropriate for the slightly ionized collision-dominated gas.

In figures 3 and 4 we show both ion wave roots $C_{oi}/C = 0.606$ and 1.59 of the fully ionized example plotted as a function of the ion-electron collision to wave frequency ratio ν_{ie}/ω (or, as indicated in the figures, as a function of ν_{ii}/ω and ω_i/ω, as well). The roots exhibit marked differences in their dependence on these ratios. The slow root (1.59) has a strong phase speed dependence on ω_i and becomes strongly damped just as the collisional regime becomes established ($\nu_{ii}/\omega > 1$). The fast root, on the other hand, shows no phase speed dependence upon the ion plasma frequency ω_i, maintaining a constant speed until $\nu_{ii}/\omega \approx 1$ at which point the wave absorption shows its characteristic maximum and the phase speed decreases to a value near the adiabatic ion sound speed. As the ions and electrons become coupled through collisions for $\nu_{ie}/\omega > 1$, the absorption correctly approaches zero and the wave travels with the adiabatic sound speed C_{ot} as it should in the collision-coupled gas.

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