To be submitted to the Proceedings of the 1965 Eclipse Symposium, Ames R.C., Dec. 16-17, 1965.

# A NOTE ON THE RATE OF IMAGE ROTATION APPARENT TO AIRBORNE OBSERVERS

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The diurnal rotation of the earth causes apparent rotation of an extended image about the line of sight that is quite familiar to observers of the polar skies(1). Actually, in inertial space it is the image that is stationary while the observer and his coordinate system rotate beneath it. Thus it is possible to eliminate the apparent rotation with the proper counter rotation of the detector plane, a technique rather familiar to astronomers. However, aircraft and certain earth oriented satellite observation platforms can materially increase the rate of image rotation from that due to just the diurnal rotation. For this reason it is of interest to present a particular solution of the celestial triangle for the rate of apparent image rotation.

The method of solution is suggested by Smart(2), and is fairly straightforward. In figure 1, the observer's zenith (Z) and the object of observation (S) are two apexes of the celestial triangle. Of particular interest is the parallactic angle (Q), for it is the angle on the celestial sphere at S between the planes containing the observer's local vertical and the north direction. As photographic plates or other earth oriented detectors generally maintain a fixed relationship to the plane

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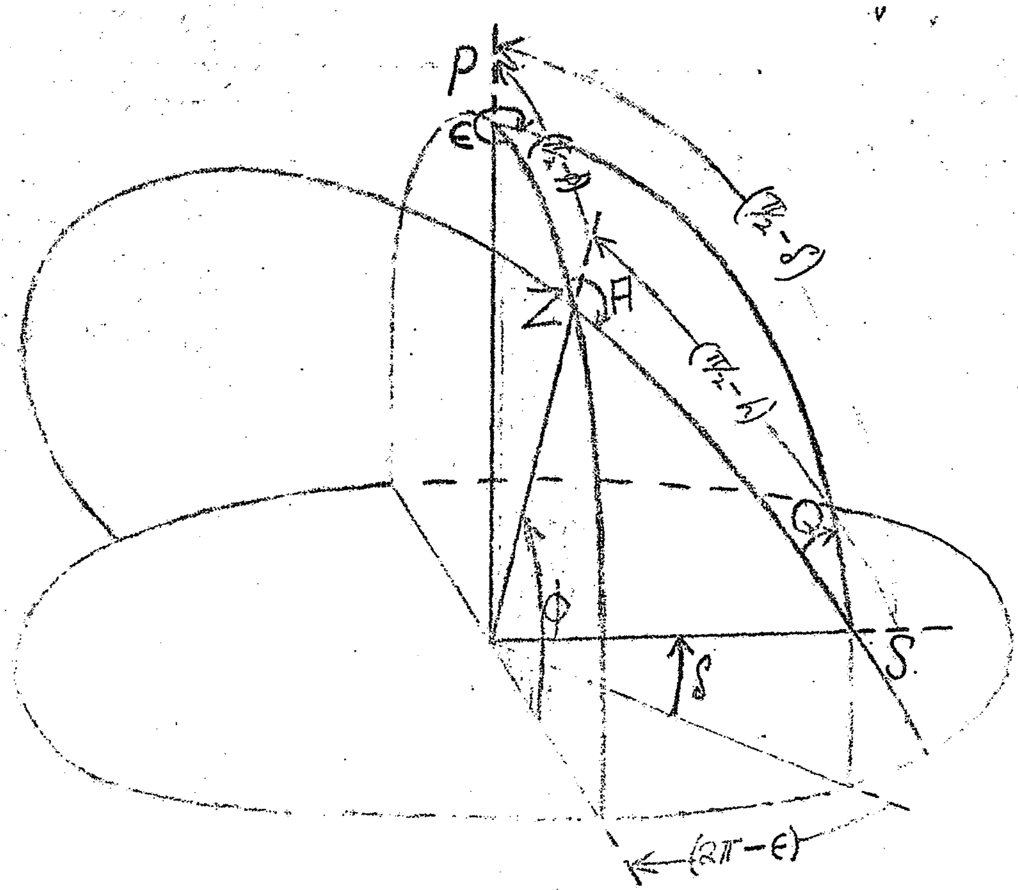
Figure 1. The Celestial Triangle east of the local meridian




Angles of the Triangle:

$Z = 360^{\circ} - \epsilon$

A = Azimuth of object

Q = Parallactic angle

($\epsilon$ = Local hour angle of object)

Opposite Sides:

$(\frac{\pi}{2}-h)$ = Co-elevation of object

$(\frac{\pi}{2}-\delta)$ = Co-declination of object

$(\frac{\pi}{2}-\phi)$ = Co-latitude of observer

[redacted]

containing local vertical, the rate of change of Q is the rate of apparent image rotation. It is desirable to solve for $dQ/dt$ only in terms of the known parameters of latitude ($\phi$), declination ($\delta$), local hour-angle ($\epsilon$), latitude rate $d\phi/dt$, and the total rate of change from all causes of local hour-angle, $d\epsilon/dt$. Application of the Sine Formula and then the Cosine Formula to the spherical triangle PZS produces equations 1 and 2 respectively.

$$\mathrm{Sin}\ Q = -\frac{\mathrm{Cos}\phi}{\mathrm{Cos}\ h}\ \mathrm{Sin}\epsilon \tag{1}$$

$$\mathrm{Cos}\ Q = \frac{\mathrm{Sin}\phi - \mathrm{Sin}\ h\ \mathrm{Sin}\delta}{\mathrm{Cos}\delta\ \ \mathrm{Cos}\ h} \tag{2}$$

Hence, equation 3:

$$\mathrm{Tan}\ Q = -\frac{\mathrm{Cos}\delta\ \mathrm{Cos}\phi\ \mathrm{Sin}\epsilon}{\mathrm{Sin}\phi - (\mathrm{Sin}\ h\ \mathrm{Sin}\delta)} \tag{3}$$

Again the Cosine Formula in triangle PZS gives the elevation (h) as

$$\text{Sin } h = \text{Sin}\phi \ \text{Sin}\delta + \text{Cos}\phi \ \text{Cos}\delta \ \text{Cos}\epsilon \tag{4}$$

Elimination of h from equation 3 gives the desired result in terms of the time dependent variables $\phi$ and $\epsilon$ only:

$$\text{Tan } Q = - \frac{\text{Cos}\phi \ \text{Sin}\epsilon}{\text{Sin}\phi \ \text{Cos}\delta - \text{Sin}\delta \ \text{Cos}\phi \ \text{Cos}\epsilon} \tag{5}$$

Taking the total derivative and applying standard trigonometric substitutions results in the final expression for the rate of apparent image rotation:

$$\frac{dQ}{dt} = - \frac{\text{Cos}^2 Q \ \text{Cos}\phi \ \left(\text{Cos}\epsilon \ \text{Sin}\phi \ \text{Cos}\delta - \text{Cos}\phi \ \text{Sin}\delta\right)}{(\text{Sin}\phi \ \text{Cos}\delta - \text{Cos}\phi \ \text{Sin}\delta \ \text{Cos}\epsilon)^2} \ \frac{d\epsilon}{dt} + \frac{\text{Cos}^2 Q \ \text{Sin}\epsilon \ \text{Cos}\delta}{(\text{Sin}\phi \ \text{Cos}\delta - \text{Cos}\phi \ \text{Sin}\delta \ \text{Cos}\epsilon)^2} \ \frac{d\phi}{dt} \tag{6}$$

Here equations 2 and 4 must be used to evaluate numerically Cos Q for the general case. It is immediately evident that there is a large singularity in the solution for $\phi = \delta$ when $\epsilon \rightarrow 0$; i.e. at the observer's zenith. This is correct for it corresponds to the case of one apex of a plane right triangle collapsing through the right angle apex. In this case however, $Q \gtrsim \pi/2$ for $\epsilon$ small but not zero, so that for regions near the singularity the solution is still well behaved. There is another normal singularity for $\phi = \delta \rightarrow \pi/2$, where the spherical triangle collapses.

It may be illuminating to evaluate equation 6 for the circumstances of the 1965 eclipse as observed from the NASA CV-990. The inputs for mid-totality are:

$\phi = -1.7^{\circ}$ $\qquad$ $h = +65^{\circ}$

$\delta = +22^{\circ}$ $\qquad$ $\epsilon = 9^{\circ}$ ($\sim$12:25 local time)

For ease in application to relatively short straight flight paths, $d\epsilon/dt$ and $d\phi/dt$ can be evaluated as

$$\frac{d\epsilon}{dt} = \left(1 + \mathrm{Sin}\, C_j \frac{S_j}{V_e}\right) 15 \frac{\mathrm{arcmin}}{\mathrm{min}} \tag{7}$$

and

$$\frac{d\phi}{dt} = S_j \,\mathrm{Cos}\, C_j \tag{8}$$

Here 15 arcmin/min is the diurnal rotation rate, $V_e$ is the linear velocity of the earth's surface at the equator, $S_j$ is the ground speed of the aircraft, and $C_j$ is the true course followed.

For $S_j$ = 510 kts. and $C_j = 78^o$, the solution becomes:

$$\frac{dQ}{dt} = + \underbrace{(15 \ \frac{\text{arcmin}}{\text{min}})\ (2.23)}_{\text{surface rate}}\ \underbrace{(1.55)}_{\substack{\text{aircraft}\\ \text{speed}\\ \text{factor}}} + \underbrace{0.73}_{\text{geometry}}\ \underbrace{(1.75 \ \frac{\text{arcmin}}{\text{min}})}_{\frac{d\Phi}{dt}} = 53.3 \ \frac{\text{arcmin}}{\text{min}}$$

Thus the image did appear to rotate 8.6 degrees during the 9.70 minutes of airborne totality. On a one minute photographic exposure at 10 $R_{\odot}$, this would cause a 2.3 arcminute azimuthal blur. This is a significant distortion as many coronal features have one dimension smaller than 1 arcminute in subtense.

Conveniently for airborne eclipse observers and others who generally observe near local noon, equation 6 approaches the rather concise limit given in equation 9 as $\epsilon$ approaches zero. Here it is assumed that $\phi \neq \delta$.

$$\lim_{(\epsilon \to 0_{\pm})} \frac{dQ}{dt} = - \frac{\cos\phi}{\sin(\phi - \delta)} \frac{d\epsilon}{dt} \qquad (9)$$

This also serves as a check of the solution, since for $\Phi$ constant and not equal to $\delta$, h is approximately constant and equal to $\pi/2 - (\phi - \delta)$ as $\epsilon \to 0_{\pm}$ .

This permits simple differentiation of equation 1, which also produces equation 9. Applying equation 9 to the above circumstances of the 1965 eclipse gives $dQ/dt = 58$ arcmin/min or 10 percent error in this case.

Because the image is steady in inertial space, a properly oriented inertially stabilized platform can directly compensate for the actual platform rotation. It is hoped that these remarks may be of use to those experimenters contemplating prolonged observations of extended images from aircraft platforms.

References:

1. J. J. Nassau, Practical Astronomy, p. 7, McGraw-Hill, New York 1948
2. W. M. Smart, Textbook on Spherical Astronomy, p. 48, Cambridge University Press, London 1962