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DIMENSIONAL ANALYSIS HANDBOOK
ARRANGEMENT RELATIONSHIPS FOR DIMENSIONAL ANALYSIS (ARDA)

Revised 1967 Edition

By

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NASA-ASEE Summer Faculty Fellowship Program
June 13-August 19, 1966

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ABSTRACT

Physical data obtained on various-sized equipment must be correlated in an orderly manner to establish general physical laws or equations governing the phenomena. Dimensional analysis presents a mathematical procedure for this correlation. Usual procedures have been modified in viewpoint and a so-called ARDA dimensional analysis method of mathematical attack is presented in encyclopedic format. The methods of application to such domains of physical knowledge as fluid drag, bubble mechanics and nucleation, convection heat transfer and tank pressurization.

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SYMBOLS AND NOTATION

Symbols. Symbols adopted as preferred are those that experience indicates are best to facilitate presentation of the subject and represent a composite adaption of the various terminology of the literature. Also given secondarily are symbols in extensive current use at NASA-MSFC and in current usage.

Detailed lists of symbols are under Symbols.

System of Units. In general the engineering system of units is used throughout because its familiarity aids in visualizing physical phenomena. Moreover, a generalized engineering system is used in which there is no restriction on the number of physical properties considered to be basic. Thus, relationships are expressed in lbf, lbm, ft and sec which might be termed an engineering FMLT system.

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SUMMARY

ARDA concepts in dimensional analysis, by introducing new viewpoints and procedures, make possible simpler and yet at the same time more comprehensive formulation of physical laws governing complex processes. The ARDA designation signifies Arrangement of Relationships in Dimensional Analysis (1). Introduction of this analytical technique minimizes the empirical data required, and the inherent similarity involved also permits small-scale model testing to replace many full-scale operating evaluations.

The method is applied to establish such general relationships as the domain of drag on bodies immersed in fluids.

The development differs somewhat from previous treatments. In the literature the classic work of Bridgman (1) in 1931 has been amplified by Langhaar in 1951 (2), Sedov in 1959 (3), and Ipsen (4) in 1960.

In general the previous attacks have been based on adoption of an MLT system of units with force F defined in terms of mass m, or an FLT system of units with m defined in terms of F. This adoption of a system of units having a limited number of fundamental dimensions has limited the scope of dimensional analysis to a series of somewhat individual solutions to special problems, with inherent confusion and limitation in comparing the findings of one investigator on one physical problem with those of a different investigator on a different physical problem.

With respect to dimensions, mass, force and weight are recognized as fundamental properties. The English engineering system is used in which these properties are always distinguished from each other and clearly indicated as lbm mass, lbf force and lbw weight, respectively.

An end result of dimensional analysis procedures is the obtaining of dimensionless numbers or parameters in which the units of the numerator are the same as the units of the denominator. Expressions involving these dimensionless numbers are perfectly general and may, therefore, be utilized in any consistent system of units.

An encyclopedic format is adopted so that each topic is discussed under its alphabetical title.

ACCELERATION. Newton's acceleration law written as a unit-consistent equation in engineering units is:

$$F = \left(\frac{m}{g_c}\right) a$$

$$F \text{ lbf} = \left(\frac{m \text{ lbm}}{32.2 \dfrac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(a \frac{\text{ft}}{\text{sec}^2}\right)$$

The modern practice of distinguishing mass from force by the use of the terms lbm or lbf is most important and has been adopted. This agrees with the modern metric system in which the force unit newton or N is clearly distinguished from the mass unit kg preferably written as kgmass or kgm. The force unit kg is no longer recommended but where used should be clearly designated as a force unit by use of the term kgf.

The previous equation does not contain weight w or gravity g. For a discussion of weight see under Weight.

ADVANCE RATIO. See Strouhal Number.

ASSOCIATIVE METHOD. The usual method in dimensional analysis is to assume that one property is a function of other properties and evaluate exponents to find a general relation of dimensionless numbers. This may be designated as the direct method.

The direct method. Many examples are given in this text in the domains of bubble mechanics, combustion, convection heat transfer, drag, elasticity, electromagnetic phenomena, magneto-hydrodynamics, nucleation, pressurization, propellor, etc.

As an example, for the drag domain if

$$\left(\frac{F}{A}\right) = C(T)^a (g)^b (\mu_f)^c (Lft)^d (v)^e (D)^f (g_c)^g (\rho)^h$$

Then,

$$Eu = C(We)^a (Fr)^b (Re)^c \left(\frac{L}{D}\right)^d$$

It will be observed from this example and from many other examples that the inclusion of certain physical properties in the first equation will lead to certain dimensionless numbers in the final relationship in which these physical properties are involved.

The property $\left(\frac{F}{A}\right)$ leads to the Euler number
The property T leads to the Weber number
The property g leads to the Froude number
The property μ_f leads to the Reynolds number
The property L leads to the $\left(\frac{L}{D}\right)$ number

The remaining properties are included in the preceding numbers so that all properties appear in the final result.

The other domains indicate many other relations as for example N_s in the Strouhal number, $\dot{U}$ in the Damkohler number, etc.

The associative method. An alternate method now suggests itself. In this method it appears that with experience one could examine the original properties and because of the known association of certain dimensionless numbers with physical properties, write the final equation directly in terms of usual basic dimensionless numbers. In the case of physical properties for which corresponding dimensionless numbers are not known it is necessary to go through the basic ARDA dimensional analysis procedure.

General law of dimensional analysis. A general law may be stated that for all phenomena the general property function equation

$$A = f(B, C, D, E \ldots.)$$

results in the general dimensionless number equation:

$$N = C(N_1)^a (N_2)^b (N_3)^c \ldots.$$

In any particular application or domain, only a few properties involved in that particular application or domain are included, thus, resulting in a dimensionless number equation with relatively few terms. These particular equations might be considered to be special cases of the general all inclusive equations where the exponents are zero so that the value of the property raised to the zero power is unity.

A study of the various domain equations with respect to the dimensionless number related to a physical quantity resulted in a tabulation given under the heading Dimensionless Numbers Associated with Properties.

It should be remembered that L and D are frequently interchangeable. L is usually a representative length which when a diameter is present is a D. In the form of L^2 an area A may be indicated. In the form of L^3 or LA a volume may be indicated. Frequently L/t is a velocity v.

BOND NUMBER.

Bo units in terms of weight.

$$Bo = \left(\begin{matrix}\text{Bond Number}\\ \text{dimensionless}\end{matrix}\right)$$

$$= \left(\frac{w}{TL}\right) = \frac{w\ \text{lbf}}{\left(T\ \frac{\text{lbf}}{\text{ft}}\right)\left(L\ \text{ft}\right)}$$

Bond number as a force ratio.

$$Bo = \frac{\text{Force of Gravity}}{\text{Surface Tension Force}}$$

$$= \frac{w}{TL}$$

Bo units in terms of density.

$$Bo = \frac{w}{TL} = \frac{\dot{w}}{TL}\ \frac{(Fr)}{(Fr)}$$

$$= \frac{\left[\frac{\left(\frac{m}{g_c}\right)g}{TL}\right]\frac{v^2}{gL}}{(Fr)} = \frac{\frac{\left(\frac{m}{V}\right)L\ v^2}{Tg_c}}{(Fr)} = \frac{(We)}{(Fr)}$$

$$= \frac{(We)}{(Fr)} = \frac{\left(\frac{\rho v^2 L}{T g_c}\right)}{\left(\frac{v^2}{gL}\right)}$$

$$= \frac{\rho L^2 g}{T g_c} \qquad \text{[sometimes used]}$$

Bo measures the effect of gravity and surface tension where velocities are insignificant.

We measures the effect of surface tension of moving fluids where velocity effects are of importance.

Fr measures the effect of gravity on moving fluids.

In same equation Bo or [(We)/(Fr)] can be used, but the use of all these would result in redundancy.

<u>BUBBLE MECHANICS DOMAIN.</u> ARDA analysis gives:

$$C = \text{fcn}\left(Fr,\ We,\ Re,\ Eu,\ Nu,\ Pr,\ \frac{L}{D},\ \frac{v_L}{v_b}\right)$$

<u>Derivation of general bubble mechanics domain.</u> The work of Steele (12, p. 15) is of interest.

$$\left(T\,\frac{\text{lbf}}{\text{ft}}\right) = C\left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)^a \left(D_b\,\text{ft}\right)^b \left(v_b\,\frac{\text{ft}}{\text{sec}}\right)^c \left(g\,\frac{\text{ft}}{\text{sec}^2}\right)^d \left(g_c\,\frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^e$$

$$\left(\mu_f\,\frac{\text{lbf sec}}{\text{ft}^2}\right)^f \left(P\,\frac{\text{lbf}}{\text{ft}^2}\right)^g \left(L\,\text{ft}\right)^h \left(q\,\frac{\text{Btu}}{\text{hr}}\right)^i \left(\Delta T\ \text{F abs}\right)^j$$

$$\left(v_L\,\frac{\text{ft}}{\text{sec}}\right)^k \left(D_c\,\text{ft}\right)^m \left(c_p\,\frac{\text{Btu}}{\text{lbm F abs}}\right)^n \left(k\,\frac{\text{Btu}}{\text{hr ft F abs}}\right)^r$$

$$\left(3600\,\frac{\text{sec}}{\text{hr}}\right)^s$$

where the usual notation is supplemented by

D_b and D_c = bubble and container diameters respectively

v_b and v_L = bubble velocity referred to fluid and to container wall respectively.

<u>ft</u> $-1 = -3a + b + c + d + e - 2f - 2g + h + k + m - r$
$= (+3f + 3g - 3 - 3n) + b(-2d - 2g - f - k + 2 + n) + d$
$+ (f + g - 1) - 2f - 2g + h + k + m + (-j + n)$
$0 = f - 1 - n + b - d + h + m - j$ $\qquad b = -f + 1 + n + d - h - m + j$

<u>sec</u> $0 = -c - 2d - 2e + f - k + s$
$= -c - 2d - 2f - 2g + 2 + f - k + n$ $\qquad c = -2d - 2g - f - k + 2 + n$

<u>lbm</u> $0 = a + e - n$
$= a + f + g - 1 - n$ $\qquad a = -f - g + 1 + n$

<u>lbf</u> $1 = -e + f + g$ $\qquad e = f + g - 1$

<u>Btu</u> $0 = n + r + i$ $\qquad s = n$

<u>hr</u> $0 = -r - s - i$
$0 = -j + n - n - i$ $\qquad i = -j$

<u>F abs</u> $0 = -n - r + j$ $\qquad i = r - n$

$$T = c(\rho)^{-f-g+1+n}(D_b)^{-f+1+n+d-h-m+j}(v_b)^{-2d-2g-f-k+2+n}(g)^d$$
$$(g_c)^{f+g-1}(\mu_f)^f(P)^g(L)^h(q)^{-j}(\Delta T)^j(v_L)^k(D_c)^m(c_p)^n(k)^{j-n}(3600)^n$$

$$\left(\frac{g_c T}{\rho v_b^2 D_b}\right) = C\left(\frac{g D_b}{v_b^2}\right)^d\left(\frac{\mu_f g_c}{\rho v_b D_b}\right)^f\left(\frac{P g_c}{\rho v_b^2}\right)^g\left(\frac{L}{D_b}\right)^h\left(\frac{k\,\Delta T}{q D_b}\right)^j\left(\frac{v_L}{v_b}\right)^k\left(\frac{D_c}{D_b}\right)^m$$
$$\left(\frac{3600\,\rho v_b D_b c_p}{k}\right)^n$$

$$\frac{1}{(We)} = C\left[\frac{\left(\frac{L}{D_b}\right)^h\left(\frac{v_L}{v_b}\right)^k\left(\frac{D_c}{D_b}\right)^m}{(Fr)^d(Re)^f(Eu)^g(Nu)_b^{\,j}(Pe)^n}\right]$$

where $(Pe) = (Re)\,(Pr)$

$$= C\left[\frac{\left(\frac{L}{D_b}\right)^h\left(\frac{v_L}{v_b}\right)^k\left(\frac{D_c}{D_b}\right)^m}{(Fr)^d(Re)^p(Eu)^g(Nu)_b(Pr)^q}\right]$$

$$= C\left[\frac{\left(\frac{L}{D_c}\right)^h\left(\frac{v_L}{v_b}\right)^k}{(Fr)^d\,(Re)^p\,(Eu)^g\,(Nu)_b{}^b\,(Pr)^q}\right]$$

where $m = -h$

$$C = (Fr)^a\,(We)^b\,(Re)^c\,(Eu)^d\,(Nu)_b{}^e\,(Pr)^f\left(\frac{L}{D_c}\right)^g\left(\frac{v_L}{v_b}\right)^h$$

where exponents are new.

This is the general drag equation discussed under Drag if the heat transfer terms and relative velocity terms are negligible, i.e.,

$$(Nu)_b{}^e = (Nu)_b{}^0 = 1$$

$$(Pr)^f = (Pr)^0 = 1$$

$$\left(\frac{v_L}{v_b}\right)^h = \left(\frac{v_L}{v_b}\right)^0 = 1$$

The terms $\left(\frac{D_c}{L}\right)$ and $\left(\frac{v_L}{v_b}\right)$ seldom appear in correlations.

Rohsenow correlation

$$C = (Re)^a\,(Pr)^b\,(Nu)^c$$

Foster-Zuber

$$Nu_b = 0.0015(Re)^{0.62}\,(Pr)$$

Usiskin and Seigil

$$0.02080 = \left(\frac{We}{Fr}\right) = Bo$$

Zuber

$$\frac{1}{6} = \frac{1}{(Nu)_b} \frac{1}{We} (Fr)$$

Cole

$$\frac{1}{(0.040)^2} = \frac{1}{(We)^{0.56}} (Fr)^{0.44}$$

ARDA derivation of simple bubble mechanics relation. The preceding general relation contains many factors. If less factors are included a simpler relation is obtained which may be considered to be the general domain equation with certain less important parameters omitted. As a result of ARDA dimensional analysis which follows:

$$C = [(Fr)^a (We)^b (Re)^c (Eu)^d]$$

The motion of a bubble or sphere in a fluid depends on properties of the fluid and conversion factors as follows:

F = force, lbf

L or D = diameter, ft.

Use D for diameter
L in velocity $\frac{L}{t}$
L^2 in pressure, $P = \frac{F}{L^2}$
L^3 in volume

v = velocity, $\frac{ft}{sec}$

$\rho = \Delta\left(\frac{m}{V}\right)$ = difference in density between inside and outside fluids, $\frac{lbm}{ft^3}$

$\mu = \mu_f$ = viscosity outside fluid, $\frac{lbf\ sec}{ft^2}$

g = gravity or acceleration field, $\frac{ft}{sec^2}$

g_c = conversion factor, $32.2 \dfrac{\text{lbm ft}}{\text{lbf sec}^2}$. This is an unvarying

conversion factor, used in the same manner as
$J = 778 \dfrac{\text{ft lbf}}{\text{Btu}}$

T = surface tension, $\dfrac{\text{lbf}}{\text{ft}}$

The relation obtained may also be valid as a general equation for interface or interface phenomena although some extension of theory may be required.

The general relationship may be written:

$$T = \text{fcn}(F,\ L,\ v,\ \rho,\ \mu,\ g,\ g_c), \text{ or}$$

$$T\left(\frac{\text{lbf}}{\text{ft}}\right) = C(F\ \text{lbf})^a\,(L\ \text{ft})^b\left(v\frac{\text{ft}}{\text{sec}}\right)^c\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)^d\left(\mu\ \frac{\text{lbf sec}}{\text{ft}^2}\right)^e\left(g\ \frac{\text{ft}}{\text{sec}^2}\right)^f$$

$$\left(g_c\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^g$$

The individual unit-properties and exponents on each side of the equation must be equal. For example, for lbf:

$$\text{lbf}^1 = \text{lbf}^{a+e-g}$$

Writing the equality for the exponents above, for each one of the unit-properties such as lbf, lbm, sec and ft:

<u>lbf</u> $\quad 1 = a + e - g \qquad\qquad g = a + e - 1$

<u>lbm</u> $\quad 0 = d + g \qquad\qquad d = -g = 1 - a - e$

<u>sec</u> $\quad 0 = -c + e - 2f - 2g \qquad\qquad c = e - 2f - 2g$
$\qquad = e - 2f + 2 - 2a - 2e$
$\qquad = -e - 2f + 2 - 2a$

<u>ft</u> $\quad -1 = b + c - 3d - 2e + f + g$
$-1 = b - e - 2f + 2 - 2a - 3 + 3a + 3e - 2e + f + a + e - 1$
$0 = b + e - f - 1 + 2a \qquad\qquad b = 1 + f - e - 2a$

Then,

$$T = C(F)^{a} (L)^{1 + f - e - 2a} (v)^{-e - 2f + 2 - 2a} (\rho)^{1 - a - e} (\mu)^{e}$$
$$(g)^{f} (g_c)^{a + e - 1}$$

$$\left[\frac{Tg_c}{\rho v^2 L}\right] = C\left[\frac{F\, g_c}{\rho L^2 v^2}\right]^a \left[\frac{\mu g_c}{\rho v L}\right]^e \left[\frac{gL}{v^2}\right]^f$$

$$(We)^{-1} = C(Eu)^a (Re)^{-e} (Fr)^{-f}$$

$$We = C\,\frac{(Re)^a (Fr)^b}{(Eu)^c}$$ General Bubble Mechanics Domain

where exponents have been re-designated.

This relation can also be written in the form of the general bubble mechanics domain equation with certain terms omitted.

$$C = (Fr)^a (We)^b (Re)^c (Eu)^d$$

<u>Simple bubble mechanics equations by association</u>. This is designated as the ARDA Associative Method. It is alternate to the ARDA basic method using applicable dimensionless numbers from the table on the following page.

If $T = \text{fcn}\,[F,\ L,\ v,\ \rho,\ \mu,\ g,\ g_c]$

$$T = \text{fcn}\left[(\mu) \quad (g) \quad \left(\frac{F}{L^2}\right) \quad \left(\begin{matrix}\text{Also}\\ v,\ \rho,\ g_c\end{matrix}\right)\right]$$

$$\left(\begin{matrix}\text{Surface}\\ \text{Tension}\\ \text{Related}\\ \text{Number}\end{matrix}\right) = \text{fcn}\left[\left(\begin{matrix}\text{Viscosity}\\ \text{Related}\\ \text{Number}\end{matrix}\right)\left(\begin{matrix}\text{Gravity}\\ \text{Related}\\ \text{Number}\end{matrix}\right)\left(\begin{matrix}\text{Pressure}\\ \text{Related}\\ \text{Number}\end{matrix}\right)\left(\begin{matrix}\text{Velocity,}\\ \text{Diameter}\\ \text{or Length}\\ g_c \text{ Related}\\ \text{Numbers}\end{matrix}\right)\right]$$

$$(We) = \text{fcn}[(Re)(Fr)(Eu)(Eu,\ Re,\ \text{or } Fr)]$$

$$= C[(Re)^a (Fr)^b (Eu)^c]$$

TABLE BASIC BUBBLE MECHANICS DIMENSIONLESS NUMBERS

$$We = \begin{pmatrix}\text{Weber}\\ \text{Number}\\ \text{Dimensionless}\end{pmatrix} = \begin{pmatrix}\text{Surface}\\ \text{Tension}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \left[\frac{\rho v^2 D}{T g_c}\right]$$

$$Eu = \begin{pmatrix}\text{Euler}\\ \text{Number}\\ \text{Dimensionless}\end{pmatrix} = \begin{pmatrix}\text{Pressure}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \left[\frac{P g_c}{\rho v^2}\right] \text{ where } P = \left(\frac{2T}{R}\right) \text{ for bubble}$$

$$C_D = (Eu) = \begin{pmatrix}\text{Drag}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \left[\frac{P g_c}{\rho v^2}\right] \text{ where } P = \left(\frac{F}{L^2}\right) \text{ for drag}$$

$$Re = \begin{pmatrix}\text{Reynolds}\\ \text{Number}\\ \text{Dimensionless}\end{pmatrix} = \begin{pmatrix}\text{Viscosity}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \left[\frac{\rho v D}{\mu g_c}\right]$$

$$Fr = \begin{pmatrix}\text{Froude}\\ \text{Number}\\ \text{Dimensionless}\end{pmatrix} = \begin{pmatrix}\text{Gravity}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \left[\frac{v^2}{g D}\right]$$

$$\begin{pmatrix}\text{Velocity}\\ \text{Related}\\ \text{Number}\end{pmatrix} = We \text{ or } (Eu = C_D)$$

$$\begin{pmatrix}\text{Diameter}\\ \text{or Length}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \text{Any of } We,\ Eu,\ C_D,\ Re,\ Fr$$

$$\begin{pmatrix}g_c\\ \text{Related}\\ \text{Number}\end{pmatrix} = \text{Any of } We,\ Eu,\ C_D,\ Re$$

or

$$(We)^d = C[(Re)^a (Fr)^b (Eu)^c]$$

To utilize this procedure one must know that the preceding dimensionless numbers are basic as determined by the basic ARDA procedure. However, having determined this for bubble analysis, perhaps the associations involved may be of use in the analysis of new problems.

Flow region 1. If $(Eu)^c = (Eu)^0 = 1 = (C_D)^0$ and $(We)^d = (We)^0 = 1$, the general fluid mechanics domain equation becomes:

$$(We)^0 = C \frac{(Re)^a (Fr)^b}{(C_D)^0}$$

Experimental evidence as embodied in Stokes Law (Ref. 10.a.16) indicates a = b = 1 and C = 9.

$$1 = 9[(Re)(Fr)] \qquad \text{for Re} < 2$$

$$= 9\left[\left(\frac{\mu g_c}{\rho L v}\right)\left(\frac{v^2}{gL}\right)\right]$$

$$= 9\left(\frac{\mu v g_c}{\rho L^2 g}\right) = 9(St)$$

where St (Stokes Number Dimensionless) is the name that will be assigned to this value.

Stokes law is generally accepted and is evidently defined by the following relations:

$$(We)^0 (C_D)^0 = 9(Re)(Fr)$$

or

$$(Re)(Fr) = \frac{1}{9} \qquad 9(St) = 1$$

Region 1
Stokes' Law
Re < 2

The first equation may be preferable in that it is stated that Stokes law $St = (Re)(Fr) = \frac{1}{9}$ is valid in Region 1, a region in fluid mechanics in which $(We)^d = (We)^0 = 1$ and $(C_D)^c = (C_D)^0 = 1$ or that Stokes law is independent of We and Eu or C_D. All the implications of this last statement are not fully understood at the present writing, but experimentally Stokes law appears to limit Region 1 to a region in which $Re < 2$.

The following experimental relation also quoted from Ref. 10 & 16 may also be of interest.

$$C_D = \left(\frac{24}{Re}\right)$$

Flow region 2. If $(We)^d = (We)^0 = 1$ and $(Fr)^b = (Fr)^0 = 1$, the general fluid mechanics domain equation becomes:

$$(We)^0 = C \frac{(Re)^a (Fr)^0}{(C_D)^c}$$

$$C_D = (C)^{1/c} (Re)^{a/c}$$

Experimental evidence by Lapple and Shephard (Ref. 10 & 16) indicates

$$C_D = \left(\frac{18.7}{2}\right)(Re)^{-0.68} = 9.35\,(Re)^{-0.68} \quad \text{for } 2 < Re < 720$$

for Re = 2 to Re = 720 for instance.

The 2 factor was introduced because of a 2 in the Lapple-Shephard drag formula.

Region 2 equation is then

$$\left.\begin{aligned} C_D &= 9.35 \frac{(Fr)^0}{(Re)^{0.68} (We)^0} \\ &= \frac{9.35}{(Re)^{0.68}} \end{aligned}\right\} \begin{array}{c} \text{Region 2} \\ 2 < Re < 720 \end{array}$$

Flow region 3. If $(Eu)^c = (Eu)^0 = 1$ and $(Fr)^b = (Fr)^0 = 1$, the general fluid mechanics domain equation becomes

$$(We) = C \frac{(Re)^a (Fr)^0}{(Eu)^0} = C$$

Experimental evidence by Kaissling and Rosenberg (Ref. 10 & 18) indicates

$$\mathrm{Re} = 1.91\left[\frac{D\rho T g_c}{\mu_f g_c}\right]^{\frac{1}{2}} = 1.91[(\mathrm{We})(\mathrm{Re})^2]^{\frac{1}{2}}$$

$$\left.\begin{aligned} \mathrm{We} &= \frac{1}{(1.91)^2} \\ (\mathrm{We}) &= \frac{(\mathrm{Re})^0(\mathrm{Fr})^0}{(1.91)^2(\mathrm{Eu})^0} \end{aligned}\right\} \begin{array}{l}\text{Region 3}\\ 720 < \mathrm{Re}\end{array}$$

Evidently $(\mathrm{Re})^a$ also $= (\mathrm{Re})^0 = 1$.

Flow region 4. If $(\mathrm{Re})^a = (\mathrm{Re})^0 = 1$ and $(\mathrm{Eu})^c = (\mathrm{Eu})^0 = 1$, the general fluid mechanics domain equation becomes:

$$(\mathrm{We}) = C\,\frac{(\mathrm{Re})^0\,(\mathrm{Fr})^b}{(\mathrm{Eu})^0} = C(\mathrm{Fr})^b \qquad \text{Region 4A}$$

If b = 1

$$C = \frac{\mathrm{We}}{\mathrm{Fr}} = \mathrm{Bo} = \text{Bond Number.}$$

This equation was obtained by another derivation under the heading Bond Number.

According to Fritz (Ref. 10, p 20) for Re > for liquid N_2 at 14.7 psia 139 F abs

$$1 = 1.20\left[\frac{Tg\,g_c}{\rho v^4}\right]^{0.25}$$

$$1 = (1.20)^4\left[\frac{Tg\,g_c}{\rho v^4}\right] = (1.20)^4\left[\frac{Tg_c}{\rho v^2 D}\right]\left[\frac{gD}{v^2}\right]$$

$$= (1.20)^4\,\frac{1}{(\mathrm{We})(\mathrm{Fr})}$$

$$\left.\begin{aligned} (\mathrm{We}) &= \frac{(1.20)^4}{(\mathrm{Fr})} \\ (\mathrm{We}) &= (1.20)^4\,\frac{(\mathrm{Re})^0(\mathrm{Fr})^{-1}}{(\mathrm{Eu})^0} \end{aligned}\right\} \begin{array}{l}\text{Region 4}\\ \mathrm{Re} > 2500\end{array}$$

Summary of regions in flow domain. The preceding is summarized in a table as follows:

TABLE GENERAL BUBBLE MECHANICS DOMAIN

General Equation is: $$(We)^d = C\,\frac{(Re)^a\,(Fr)^b}{(Eu)^c} = C\,\frac{(Re)^a\,(Fr)^b}{(C_D)^c}$$

Dimensionless Number		Weber (We) $\left[\frac{\rho v^2 D}{T g_c}\right]$	Reynolds (Re) $\left[\frac{\rho v D}{\mu_f g_c}\right]$	Froude (Fr) $\left[\frac{v^2}{gD}\right]$	Euler (Eu) = (C_D) $\left[\frac{P g_c}{\rho v^2}\right]$	
Principal Controlling Factor in Dimensionless Number		T Surface Tension	μ_f Viscosity	g Gravity	Eu $P = \frac{2T}{R}$ Bubble Pressure	$C_D = Eu$ $P = \frac{F}{L^2}$ Drag
Region						
Solid Sphere Re < 2	1. μ and g control v Stokes Law	1 $(We)^0 = 1$	= $C(Re)^1$ = C(St) (Stokes Law)	$(Fr)^1$		1 $(C_D)^0 = 1$
Bubble in Dense Fluid	2. μ controls v g and σ negligible (Re 2 to 720)	1 $(We)^0 = 1$	Re (Lapple and Shephard Equation)	$(Fr)^0 = 1$	= 1	$C(C_D)^c$
	3. T controls v μ, g and P negligible (Re 720 to 2500)	(We) =	C(1) (Kaissling and Rosenberg Tests) $(Re)^0 = 1$	$(Fr)^0 = 1$	$(Eu)^0 = 1$	
Bubble in Gas or Vapor Interface Phenomena	4. T and g important μ and P negligible (Re > 2500)	(We) =	$(Re)^0 = 1$	$C(Fr)^b$ (Bond Number - Fritz Relation)	$(Eu)^0 = 1$	

The empirical equations for the four regions are:

1. $(Re)(Fr) = \frac{1}{9}$

2. $(Re)(C_D)^{1/0.68} = (9.35)^{1/0.68}$

3. $(We) = \frac{1}{(1.91)^2}$

4. $(We)(Fr) = (1.20)^4$

The preceding may be combined into the following general equation:

$$\left(\frac{(C_D)^{1/0.68}}{(9.35)^{1/0.68}(1.91)^2(1.20)^4(9)}\right)^c \left((1.91)^2(1.20)^4(9)(Re)\right)^a$$

$$\left(\frac{Fr}{(1.91)^2(1.20)^4}\right)^b \left((1.91)^2 We\right)^d = 1$$

where for

region 1	a = b = 1	c = d = 0
region 2	a = c = 1	b = d = 0
region 3	d = 1	a = b = c = 0
region 4	b = d = 1	a = c = 0

This appears to indicate that a general domain equaticn should have the form

$$(C\,C_D)^c (C\,Re)^a (C\,Fr)^b (C\,We)^d = C$$

where C represents different constants rather than the more usual form

$$We = C\,(Re)^a (Fr)^b (Eu)^c$$

Similarity and model testing require that the preceding dimensionless relationships be obtained. The table is useful in indicating test possibilities in each of the four regions and procedures in correlation.

BUBBLE PRESSURE. A stationary bubble immersed in its own fluid has an internal pressure P_T due to surface tension.

$$P_T = \frac{4T}{D}$$

If the bubble is rising or otherwise moving with a velocity v it also contains a velocity pressure P_v.

$$P_v = \frac{1}{2} \frac{\rho v^2}{g_c}$$

The bubble can be made to rise by immersing it in a fluid in which it will be buoyed-up by a force equal to the weight of the displaced fluid. The buoyant pressure P_B or weight per unit area on the bottom is the weight of displaced fluid divided by the projected area.

$$\left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right)$$

$$w = m\left(\frac{g}{g_c}\right) = \left(\frac{m}{V}\right)\frac{gV}{g_c} = \rho\,\frac{gV}{g_c}$$

$$P_w = \frac{w}{A} = \rho\left(\frac{g}{g_c}\right)\frac{V}{A} = \rho\left(\frac{g}{g_c}\right)\frac{\frac{4}{3}\pi R^3}{\pi R^2}$$

$$= \rho\left(\frac{g}{g_c}\right)\frac{4R}{3} = \rho\left(\frac{g}{g_c}\right)\frac{2D}{3}$$

Internal Bubble Pressure = Bouyant Outside Pressure

$$P_T + P_v = P_B$$

$$\frac{4T}{D} + \frac{1}{2}\frac{\rho v^2}{g_c} = \rho\left(\frac{g}{g_c}\right)\frac{2D}{3}$$

This can be written

$$T = f(D,\ \rho,\ v,\ g_c,\ g)$$

By ARDA dimensional analysis

$$\left(T\,\frac{\text{lbf}}{\text{ft}}\right) = C(D\,\text{ft})^a \left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)^b \left(v\,\frac{\text{ft}}{\text{sec}}\right)^c \left(g_c\,\frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^d \left(g\,\frac{\text{ft}}{\text{sec}^2}\right)^e$$

For:

lbf	$1 = -d$	$d = -1$
lbm	$0 = b + d$	$b = -d = 1$
sec	$0 = -c - 2d - 2e$	$c = -2d - 2e = 2 - 2e$
ft	$-1 = a - 3b + c + d + e$	
	$= a - 3 + 2 - 2e - 1 + e$	
	$= a - 2 - e$	$a = 1 + e$

$$T = C(D)^{1+e}(\rho)^1(v)^{2-2e}(g_c)^{-1}(g)^e$$

$$\left[\frac{Tg_c}{D\rho v^2}\right] = C\left[\frac{Dg}{v^2}\right]^e$$

If $e = 1$

$$\left(\frac{v^2}{Dg}\right) = C\left(\frac{D\rho v^2}{Tg_c}\right)$$

$$C = \frac{\left(\dfrac{D\rho v^2}{Tg_c}\right)}{\left(\dfrac{v^2}{Dg}\right)} = \frac{(\text{We})}{(\text{Fr})} = \frac{(\text{Weber Number})}{(\text{Froude Number})} = \text{Bo} = (\text{Bond Number})$$

If $D = L$

$$C = \left(\frac{L\left(\dfrac{m}{L^3}\right)v^2}{Tg_c}\right)\left(\frac{Lg}{v^2}\right) = \left(\frac{mg}{TLg_c}\right) = \text{Bo} = (\text{Bond Number})$$

where Bo = Bond Number, used frequently in correlations by many authors.

Reference to the general bubble mechanics equation shows that this is the same result obtained in region 4 in the general relation where $(\text{Re})^0 = 1$ and $(\text{Eu})^0 = 1$.

The preceding would seem to indicate that the pressures in a bubble vary due to surface tension, velocity and weight with velocity v and diameter D and that one measure of these is (We/Fr) = Bo. It may be that the Bond Number Bo may be a measure of bubble stability.

BUCKINGHAM PI THEOREM. See Pi Theorem.

BUOYANCY NUMBER. This number, which is a natural convection term occurring in the general convection equation, does not seem to have been given a name

$$Bu = \begin{pmatrix}\text{Buoyancy Number} \\ \\ \text{dimensionless}\end{pmatrix}$$

$$= \frac{L^2 w B \Delta T}{\mu_f V v} = \frac{\left(L^2\ ft^2\right)\left(w\ lbf\right)\left(B\ \frac{1}{F\ abs}\right)\left(\Delta T\ F\ abs\right)}{\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(V\ ft^3\right)\left(v\ \frac{ft}{sec}\right)}$$

$$Bu = \frac{L^2 mgB\Delta T}{\mu_f g_c Vv} \qquad \left[\text{where } w = \frac{mg}{g_c}\right]$$

$$= \frac{L^2 \rho g B \Delta T}{(\mu_f g_c) v} = \frac{\left(L\ ft^2\right)\left(\rho\ \frac{lbm}{ft^3}\right)\left(g\ \frac{ft}{sec^2}\right)\left(B\ \frac{1}{F\ abs}\right)\left(\Delta T\ F\ abs\right)}{\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(32.2\ \frac{lbm\ ft}{lbf\ sec^2}\right)\left(v\ \frac{ft}{sec}\right)}$$

$$= \frac{\left(\frac{\rho v L}{\mu_f g_c}\right) B\Delta T}{\left(\frac{v^2}{Lg}\right)} = \frac{(Re) B\Delta T}{(Fr)}$$

The last equation indicates that $B\Delta T$ is the basic dimensionless number rather than Bu.

It will also be shown that

$$Bu = \left(\frac{Re}{Fr}\right) B\Delta T = \left(\frac{Re}{Fr}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{g}{g_c}\right)$$

Buoyancy force per unit volume. If a bubble of volume V and of weight density $(w/V)_i$ be immersed in a surrounding fluid of weight density $(w/v)_o$, Archimedes' principle is that a body immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid. The net force is the buoyancy force less the weight force.

$$\left(\frac{F}{V}\right)_{\Delta d} = \left(\frac{w}{V}\right)_o - \left(\frac{w}{V}\right)_i$$

If the inside fluid now increases ΔT above the initial temperature and $B(ft^3/ft^3\,F)$ is the coefficient-of-volume expansion, the increase in volume per unit volume is $(B\Delta T)$. The additional buoyant force upward is then

$$\left(\frac{F}{V}\right)_{\Delta t} = (B\Delta T)\left(\frac{w}{V}\right)_o = \frac{wB\Delta T}{V_o} \quad .$$

There is no additional weight force downward because there was no increase of inside weight resulting from the volume expansion. The total net buoyancy force is then

$$\left(\frac{F}{V}\frac{lbf}{ft^3}\right) = \left(\frac{F}{V}\right)_{\Delta d} + \left(\frac{F}{V}\right)_{\Delta t} = \left(\frac{w}{V_o}\right) - \left(\frac{w}{V}\right)_i + \left(\frac{wB\Delta T}{V_o}\right)$$

$$\left(\frac{F}{V}\right) = \left[\left(\frac{w}{V_o}\right)(1 + B\Delta T) - \left(\frac{w}{v}\right)_i\right]$$ for inside heated ΔT above different outside fluid

$$\left(\frac{F}{V}\right) = \left(\frac{w}{V}\right)_o - \left(\frac{w}{V_1}\right)$$ for no ΔT and different fluids

$$\left(\frac{F}{V}\right) = (B\Delta T)\left(\frac{w}{V}\right) = \frac{wB\Delta T}{V}$$ for ΔT inside above same outside fluid

In the preceding, $\left(\frac{w}{V}\right) = \left(\frac{m}{V}\right)\frac{g}{g_c}$, thus the buoyancy force due to weight densities is frequently called a gravity force and written in terms of mass densities $\left(\frac{m}{V}\right) = \rho$ or

$$\left(\frac{F}{V}\right) = B\Delta T\left(\frac{m}{V}\right)\frac{g}{g_c} = (B\Delta T)\rho\left(\frac{g}{g_c}\right)$$

<u>Buoyancy force per unit mass.</u>

$$\left(\frac{F}{m}\right) = \frac{\left(\frac{F}{V}\right)}{\left(\frac{m}{V}\right)} = \frac{B\Delta T\rho\left(\frac{g}{g_c}\right)}{\rho} = \left[B\Delta T\left(\frac{g}{g_c}\right)\right]\frac{lbf}{lbm}$$

$$\left(\frac{F}{m}\right) = \left(B\frac{ft^3}{ft^3\ F\ abs}\right)\left(\Delta T\ Fabs\right)\left(\frac{g\ \dfrac{ft}{sec^2}}{g_c\ \dfrac{lbm\ ft}{lbf\ sec^2}}\right)$$

$$\left(\frac{F}{m}\right) = \frac{B\rho\Delta T}{\rho}\left(\frac{g}{g_c}\right)$$

$$= \left(\frac{\Delta\rho}{\rho}\right)\left(\frac{g}{g_c}\right)$$

Buoyancy number by dimensional analysis. It is desired to develop a Buoyancy Number to express the effect of natural convection resulting from a body of fluid of volume V using in temperature ΔT above the surrounding temperature under the action of a buoyancy force $\left(\frac{F}{V}\right) = \left(\frac{wB\Delta T}{v}\right)$. By basic ARDA procedure

$$Bu = \begin{pmatrix}\text{Buoyancy Number}\\ \text{dimensionless}\end{pmatrix}$$

$$= f\left(\frac{F}{V},\ L,\ v,\ \mu_f\right)$$

$$= C\left(\frac{wB\Delta T}{V}\ \frac{lbf}{ft^3}\right)^a \left(L\ ft\right)^b \left(v\ \frac{ft}{sec}\right)^c \left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)^d$$

lbf	$0 = a + d$	$d = -a$
sec	$0 = -c + d$	$c = d = -a$
ft	$0 = -3a + b + c - 2d$	$b = 3a - c + 2d$
		$= 3a + a - 2a$
		$= 2a$

$$Bu = C\left(\frac{wB\Delta T}{V}\right)^a (L)^{2a} (v)^{-a} (\mu_f)^{-a}$$

$$= C\left(\frac{L^2 wB\Delta T}{\mu_f vV}\right)^a$$

where C and a can both equal 1.

Examining the buoyancy number

$$Bu = \left[\frac{L^2 wB\Delta T}{\mu_f vV}\right] = \frac{\left[\dfrac{wB\Delta T}{\left(\frac{V}{L}\right)\frac{v}{L}}\right]}{\mu_f} = \frac{\left[\dfrac{wB\Delta T}{A\left(\frac{v}{L}\right)}\right]}{\left[\dfrac{F_{drag}}{A\left(\frac{v}{L}\right)}\right]} = \frac{(F)\ \text{buoyancy}\ \Delta T}{(F)\ \text{viscous drag}}$$

As was the case with Re these forces are per unit area per unit velocity per unit distance.

CAPILLARY TUBE. For flow through a capillary tube (Ref. 24, p. 16).

If $\quad \dot{V}\ \frac{ft^3}{sec} = C(D\ ft)^a (L\ ft)^b \left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)^c \left(\Delta P\ \frac{lbf}{ft^2}\right)^d$

sec	$-1 = c$	$c = 1$
lbf	$0 = c + d$	
	$= -1 + d$	$d = 1$
ft	$3 = a + b - 2c + 2d$	
	$= a + b + 2 - 2$	$a = 3 - b$

$$\dot{V} = C(D)^{3-b}(L)^b(\psi)^{-1}(\Delta P)^1$$

$$\frac{\dot{V}\mu_f}{D^3\Delta P} = C\left(\frac{L}{D}\right)^b$$

CAUCHY NUMBER. This parameter occurs in elasticity problems involving modulus of elasticity, E. When it occurs with the shear modulus S it is known as the Fanning number.

For solids Youngs modulus of elasticity is used for E. For compressible fluids Murphy (Ref. 28, p. 145) suggests the bubble modulus of elasticity (see Modulus of Elasticity).

Ca Units

$$Ca = \left(\begin{matrix}\text{Cauchy Number}\\ \text{dimensionless}\end{matrix}\right)$$

$$= \frac{\rho v^2}{E g_c} = \frac{\left(\rho\ \frac{lbm}{ft^3}\right)\left(v^2\ \frac{ft^2}{sec^2}\right)}{\left(E\ \frac{lbf}{ft^2}\right)\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)}$$

Murphy (Ref. 28, p. 1670) shows

$$Ca = (Ma)^2 = \frac{\rho v^2}{E g_c}$$

Cauchy Number as a force ratio. The Cauchy Number may be expressed as a ratio of the force of deceleration or inertia force to a compression force.

$$Ca = \frac{(\text{Force of Deceleration})}{(\text{Force of Compression})} = \frac{\left(\frac{m}{g_c}\right)a}{EA}$$

$$= \frac{m\left(\frac{v}{t}\right)}{EAg_c} = \frac{\left(\frac{m}{AL}\right)A\left(\frac{L}{t}\right)v}{EAg_c} = \left(\frac{\rho v^2}{E g_c}\right)$$

CAVITATION. Increases in fluid velocity are accompanied by reduction in static pressure. When the velocity in a liquid has increased to a value such that the local static pressure is reduced to the vapor pressure of the liquid, the liquid boils to form bubbles on cavities.

Pump cavitation equation. In a pump inlet the allowable pressure reduction P of a fluid because of velocity may be established.

If: $P = (P_1 - P_v) = (\text{total inlet pressure}) - (\text{vapor pressure})$

$$= (P_{atm} - P_{suction}) - (P_{vapor})$$

$\Delta P = (P_2 - P_1) = (\text{pressure rise across pump})$

$N = \text{pump rpm}$

$v = \pi ND = \text{pump impeller tip velocity, ft per min}$

$$\left(P \frac{lb}{ft^2}\right) = C\left(\Delta P \frac{lb}{ft^2}\right)^a \left(N \frac{1}{sec}\right)^b \left(\rho \frac{lbm}{ft^3}\right)^c \left(D\ ft\right)^d \left(g_c \frac{lbm\ ft}{lbf\ sec^2}\right)$$

lbm	$0 = c + e$	$e = -c$
sec	$0 = -b - 2e$	
	$= -b + 2c$	$b = 2c$
lbf	$1 = a - e$	
	$= a + c$	$a = 1 - c$
ft	$-2 = -2a - 3c + d + e$	
	$= -2 + 2c - 3c + d - c$	
	$0 = -2c + d$	$d = 2c$

$$\Delta P = C(P)^{1-c}(N)^{2c}(\rho)^{c}(D)^{2c}(g_c)^{-c}$$

$$\left(\frac{Pg_c}{\rho N^2 D^2}\right)^c = C\left(\frac{P}{\Delta P}\right)$$

$$\left(\frac{Pg_c}{\rho N^2 D^2}\right) = C\left(\frac{P}{\Delta P}\right)^a = C(Th)^a \qquad \text{(where C and a are new numbers)}$$

The preceding relation is given by Langhaar (Ref. 2, p. 114) where Th is the Thomas number. An alternate form given by Pankhurst (Ref. 24, p. 92) is

$$\frac{Pg_c}{\rho\pi^2 N^2 D^2} = \frac{Pg_c}{\rho v^2} = C(Th)^a$$

$$Eu = C(Th)^a$$

Pipe line cavitation. Murphy (Ref. 28, p. 144) suggests use of C_n with P defined as the vapor pressure for pipe line cavitation analysis.

COMBUSTION DOMAIN. ARDA analysis gives

$$C = \text{fcn}(Pr, Sc, Fr, Eu, Hu, Da, Ec, Re, SF)$$

Derivation of combustion domain. It is desired to develop by ARDA dimensional analysis a general combustion equation (Ref. 24, p. 110) to include the physical processes governed by the following physical properties and conversion factors:

1. k = thermal conductivity, Btu per hr ft^2 per F per ft
2. D_m = mass diffusivity of one substance into another, ft^2/hr
3. g = gravity, ft per sec^2
4. P = pressure, lbf per ft^2
5. q = heat value, Btu per lbm
6. $\dot{U}$ = reaction rate, lbm per sec per lbm
7. J = 778 ft lbf per Btu for heat conversion into mechanical energy
8. 3600 = 3600 sec per hr for mixed units of sec and hr
9. c_p = specific heat at constant pressure, Btu per lbm F
10. D = diameter, ft
11. v = velocity, ft per sec
12. μ_m = viscosity expressed in mass units, lbm per ft hr
13. ΔT = temperature rise or value above given base, F
14. $\rho = (w/V)$ = mass density, lbm per ft^3
15. g_c = conversion factor = 32.2 lbm ft per lbf sec^2
16. L = length dimension, ft

The relation between these factors is:

$$1 = C\left(k \frac{\text{Btu}}{\text{hr ft F}}\right)^a \left(D_m \frac{\text{ft}^2}{\text{hr}}\right)^b \left(g \frac{\text{ft}}{\text{sec}^2}\right)^c \left(P \frac{\text{lbf}}{\text{ft}^2}\right)^d \left(q \frac{\text{Btu}}{\text{lbm}}\right)^e \left(\dot{U} \frac{1}{\text{sec}}\right)^f$$

$$\left(J \frac{\text{ft lbf}}{\text{Btu}}\right)^g \left(3600 \frac{\text{sec}}{\text{hr}}\right)^h \left(c_p \frac{\text{Btu}}{\text{lbm F}}\right)^i \left(D \text{ ft}\right)^j \left(v \frac{\text{ft}}{\text{sec}}\right)^k \left(\mu_m \frac{\text{lbm}}{\text{ft hr}}\right)^m$$

$$\left(\Delta T F\right)^n \left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)^p \left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^q \left(L \text{ ft}\right)^r$$

Equating exponents:

hr	$0 = -a - b - h - m$	$m = -a - b - h$
lbf	$0 = d + g - q$	$q = +d + g$
Btu	$0 = a + e - g + i$	$i = -a - e + g$
sec	$0 = -2c - f + h - k - 2q$	
	$= -2c - f + h - k - 2d - 2g$	$k = -2c - 2d - f - 2g + h$
lbm	$0 = -e - i + m + p + q$	
	$= -e + a + e - g - a - b - h$	
	$+ p + d + g$	$p = b - d + h$
F	$0 = -a - i + n$	
	$= -a + a + e - g + n$	$n = g - e$
ft	$0 = -a + 2b + c - 2d + g + j$	
	$+ k - m - 3p + q + r$	
	$0 = -a + 2b + c - 2d + g + j$	
	$- 2c - 2d - 2g - f + h + a$	
	$+ b + h - 3b + 3d - 3h$	
	$+ d + g + r$	
	$0 = c - f - h + j + r$	$r = c + f + h - j$

Substituting in original equation

$$1 = C(k)^a (D_m)^b (g)^c (P)^d (q)^e (\dot{U})^f (J)^g (3600)^h (C_p)^{-a - e + g} (D)^j$$

$$(v)^{-2c - 2d - f - 2g + h} (\mu_m)^{-a - b - h} (\Delta T)^{g - e} (\rho)^{b - d + h}$$

$$(g_c)^{d + g} (L)^{c + f + h - j}$$

Collecting and identifying terms:

$$
1 = \left[\underbrace{\frac{\mu_m c_p}{k}}_{\substack{= \text{Pr} = \text{Prandtl} \\ \text{Physical Properties}}} \right]^{-a} \left[\underbrace{\frac{\mu_m}{\rho D_m}}_{\substack{= \text{Sc} = \text{Schmidt} \\ \text{Diffusion}}} \right]^{-b} \left[\underbrace{\frac{Lg}{v^2}}_{\substack{= \text{Fr} = \text{Froude} \\ \text{Gravity}}} \right]^{c}
$$

$$
\left[\underbrace{\frac{pg_c}{\rho v^2}}_{\substack{= \text{Eu} = \text{Euler} \\ \text{Pressure}}} \right]^{d} \left[\underbrace{\frac{q}{c_p \Delta T}}_{\substack{= \text{Hv} = \text{Heat Value} \\ \text{Heat Release}}} \right]^{e} \left[\underbrace{\frac{L\dot{U}}{v}}_{\substack{= \text{Da} = \text{Damkohler} \\ \text{Chemical Reaction Ratio}}} \right]^{f}
$$

$$
\left[\underbrace{\frac{v^2}{c_p \Delta T g_c J}}_{\substack{= \text{Ec} = \text{Eckert} \\ \text{Heat to KE}}} \right]^{-g} \left[\underbrace{\frac{\rho v L(3600)}{\mu_m}}_{\substack{= \text{Re} = \text{Reynolds} \\ \text{Viscosity}}} \right]^{h} \left[\underbrace{\frac{L}{D}}_{\substack{= \text{SF} \\ = \text{Shape Factor}}} \right]^{-j}
$$

$$
C = (\text{Pr})^{-a}(\text{Sc})^{-b}(\text{Fr})^{c}(\text{Eu})^{d}(\text{Hv})^{e}(\text{Da})^{f}(\text{Ec})^{-g}(\text{Re})^{h}(\text{SF})^{-j}
$$

Combustion regions. If certain phenomena are not present then the terms representing the phenomena are absent. For example, if there is diffusion present, the Schmidt number is absent or

$$
(\text{Sc})^{-b} = (\text{Sc})^{o} = 1
$$

Combined terms. For specific cases the terms are combined to give other less basic numbers. For example the Damkohler II parameter equals [(Da)(Re)(Sc)]. The Lewis number equals (Pr)/(Sc), etc.

Combustion domain by associative method. Knowing the physical properties that are to be included the appropriate dimensionless numbers including these properties are included to permit the immediate writing of the equations as shown by the tabulation.

Property		Corresponding Dimensionless Number
1	k	Prandtl
2	D_m	Schmidt
3	g	Froude
4	P	Euler
5	q	Heat Value
6	$\dot{U}$	Damkohler
7	J	Eckert
8	3600	Included in other numbers
9	c_p	Included in other numbers
10	D	Included in other numbers
11	v	Included in other numbers
12	μ_m	Reynolds
13	ΔT	(Included in Eckert)
14	ρ	(Included in other numbers)
15	g_c	(Included in other numbers)
16	L	Shape Factor

CONSTANT. A pure constant is a dimensionless number, such is the 778 in the dimensional constant or conversion factor $J = 778 \frac{\text{ft lb}}{\text{Btu}}$.

CONVECTION HEAT TRANSFER. ARDA analysis gives

$$Nu = \text{fcn}\,(Re,\ Pr,\ Bu,\ \frac{L}{D})$$

Derivation of convection heat transfer domain. The general convection domain equation is established by basic ARDA dimensional analysis. The convection conductance h_c depends on properties of the fluid and conversion factors as follows:

1. $\rho = (m/V)$ = mass density, lbm/ft^3
2. c_p = specific heat per unit mass, Btu/lbm F
3. $wB\,\Delta T/V$ = buoyant force per unit volume, lbf/ft^3. Determination of this buoyant force is based on the principle that a volume immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid. The volume increase per unit volume of a part of the fluid, resulting from a temperature rise ΔT °F is $B(\Delta T)$ where B is the coefficient of thermal volume expansion, ft^3/ft^3 F = 1/F. If the weight density (w/V) is multiplied by $B(\Delta T)$, the weight of the fluid displaced by this increase in volume is obtained, which is the buoyant force per unit volume.
4. L = size factor, length in ft
5. D = tube diameter, ft
6. v = fluid velocity produced by forced flow rather than buoyance, ft/sec
7. μ_f = viscosity in force units, lbf sec/ft^2
8. k = thermal conductivity, Btu/hr ft F
9. g_c = mass acceleration constant = 32.2 lbm ft/lbf sec^2. Introduced because flow may be expected in which lbm masses may be expected to be accelerated by lbf forces which will then require the g_c relation between lbm and lbf in the $F = ma/g_c$ law. That is to say, because the $F = ma/g_c$ law is involved in the flow phenomena, it is introduced by use of the g_c conversion factor.
10. (3600 sec/hr) = time conversion factor, introduced because some properties are based on seconds, others on hours.

<u>General convection equation.</u> The relationship between h_c and these factors may be written using usual units, if C is a dimensionless numerical constant and a, b, c, etc. are exponents, as

$$\left(h_c \frac{\text{Btu}}{\text{hr ft}^2\text{ F}}\right) = C\left(\frac{m}{V}\frac{\text{lbm}}{\text{ft}^3}\right)^a \left(c_p \frac{\text{Btu}}{\text{lbm F}}\right)^b \left(\frac{wB\Delta T}{V}\frac{\text{lbf}}{\text{ft}^3}\right)^c \left(L \text{ ft}\right)^d \left(D \text{ ft}\right)^e$$

$$\left(v \frac{\text{ft}}{\text{sec}}\right)^f \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)^g \left(k \frac{\text{Btu}}{\text{hr ft F}}\right)^h \left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^i$$

$$\left(3600 \frac{\text{sec}}{\text{hr}}\right)^j$$

$$Nu = C(Re)^a (Pr)^b (Bu)^c \left(\frac{L}{D}\right)^d$$ General Convection Equation Domain

Region 1 with high velocity. For the convection domain three general heat transfer regions may be recognized. Region 1 may be designated as one with high velocity. This high velocity occurs with forced flow and would be turbulent flow with buoyancy effects negligible. For buoyancy effects to be negligible $\left(\frac{wB\Delta T}{V}\right)^c$ in the basic equation must equal $\left(\frac{wB\Delta T}{V}\right)^c = 1$ or $c = 0$. Then in the general convection domain equation $(Bu)^c = (Bu)^o = 1$ to yield the usual turbulent flow Nusselt relations.

$$Nu = C\left[(Re)^a (Pr)^b \left(\frac{L}{D}\right)^d\right]$$

For long tubes the effect of $\left(\frac{L}{D}\right)^d$ is negligible or $\left(\frac{L}{D}\right)^d = \left(\frac{L}{D}\right)^o = 1$ or $d = 0$.

$$Nu = C[(Re)^a (Pr)^b]$$

This is the usual convection equation for turbulent flow (Ref. 20) (Ref. 21, p. 618).

Region 2 with low velocity. This occurs with forced flow. If the velocity is low enough for laminar flow, then viscosity effects as well as buoyancy effects must be negligible. In the basic heat transfer domain equation: for viscosity effects to be negligible $\mu_f^g = \mu_f^o = 1$ or $g = 0$; for buoyancy effects to be negligible $\left(\frac{wB\Delta T}{V}\right)^c = \left(\frac{wB\Delta T}{V}\right)^o = 1$ or $c = 0$. In the derivation of the general convection domain equation $g = b - a - c$. Thus, $0 = b - a - 0$ and $b = a$. The general convection domain equation with $b = a$ and $(Bu)^c = (Bu)^o = 1$ becomes

$$(Nu) = C[(Re)(Pr)]^a \left(\frac{L}{D}\right)^d$$

$$= C'\left[\frac{\pi}{4}(Re)(Pr)\,\frac{D}{L}\right]^a \left(\frac{L}{D}\right)^{d+a}$$

The individual unit properties and exponents on each side of the equation must be equal. For example, for lbm

$$lbm^{o} = lbm^{a-b+i}$$

Writing the equality for the exponents for each one of the unit properties such as lbm, lbf, etc. in turn

lbm	$0 = a - b + i$	$i = b - a$
lbf	$0 = c + g - i = c + g - b + a$	$g = b - a - c$
F	$-1 = -b - h$	
Btu	$1 = b + h$	$h = 1 - b$
hr	$-1 = -h - j$	
	$-1 = -1 + b - j$	$j = b$
sec	$0 = -f + g - 2i + j$	
	$= -f + b - a - c - 2b + 2a + b$	$f = a - c$
ft	$-2 = -3a - 3c + d + e + f - 2g - h + i$	
	$= -3a - 3c + d + e + a - c - 2b + 2a + 2c - 1 + b + b - a$	
	$= -a - 2c + d + e - 1$	$e = a - 1 + 2c - d$

Substituting these values of i, g, h, j, f and e in the original equation

$$h_c = C\left(\frac{m}{V}\right)^a \left(c_p\right)^b \left(\frac{wB\Delta t}{V}\right)^c \left(L\right)^d \left(D\right)^{a-1+2c-d} \left(v\right)^{a-c} \left(\mu_f\right)^{b-a-c}$$

$$\left(k\right)^{1-b} \left(g_c\right)^{b-a} \left(3600\right)^b$$

Rearranging:

$$\left(\frac{h_c D}{k}\right) = C\left(\frac{Dv\,\frac{m}{V}}{\mu_f\, g_c}\right)^a \left[\frac{c_p(3600)\mu_f\, g_c}{k}\right]^b \left(\frac{D^2 wB\Delta T}{\mu_f\, vV}\right)^c \left(\frac{L}{D}\right)^d$$

Checking units in each of the groups in the preceding equation shows the important fact that each group is dimensionless in that the units of the numerator cancel the units of the denominator. Such groups occur frequently in heat transfer and fluid flow and are called dimensionless numbers. Most of these dimensionless numbers have been given names except the fourth which will be defined as the buoyancy number Bu. Thus the previous equation may be written:

If the effect of the $\left(\frac{L}{D}\right)^{d+a}$ term is negligible, as for example with long tubes, $\left(\frac{L}{D}\right)^{d+a} = \left(\frac{L}{D}\right)^{o} = 1$. The preceding equation becomes the usual laminar flow relation.

$$
\begin{aligned}
Nu &= C'\left[\frac{\pi}{4}(Re)(Pr)\frac{D}{L}\right]^a \\
&= C'\left[\frac{\pi}{4}\left(\frac{Dv\,\frac{m}{V}}{\mu_f\,g_c}\right)\left(\frac{3600\;c_p\,\mu_f\,g_c}{k}\right)\frac{D}{L}\right]^a \\
&= C'\left[\left(\frac{\pi}{4}\,D^2\;3600\,v\,\frac{m}{V}\right)\left(\frac{c_p}{kL}\right)\right]^a \\
&= C'\left[\dot{m}\,\frac{c_p}{kL}\right]^a \quad \text{where} \left(\frac{\pi}{4}\,D^2\;3600\,v\right) = \frac{V}{hr} \\
&= C'\,[Gz]^a
\end{aligned}
$$

This is a well known convection equation for laminar flow (Ref. 20, Ref. 21, p. 621) where $\dot{m}$ = mass flow rate, lbm/hr and Gz = Graetz number, dimensionless (Ref. 22, p. 228).

Region 3 natural convection. If natural convection is assumed as not applicable to tubes of diameter D having a forced velocity v, then the terms involving D and v in the initial derivation of the general convection equation must have negligible effect or $D^e = 1 = D^o$ so that e = 0 and $v^f = 1 = f^o$ so that f = 0. Experiment seems to show that b = a.

Substituting these values of e, f and b in equations for the exponents in the derivation of the general convection equation

$f = a - c$	$e = a - 1 + 2c - d$
$0 = a - c$	$0 = a - 1 + 2a - d$
$c = a$	$d = 3a - 1$

Substituting these values of b, c and d in the general convection equation domain:

$$\left(\frac{h_c D}{k}\right) = C\left(\frac{Dv\,\frac{m}{V}}{\mu_f\, g_c}\right)^a \left(\frac{D^2 wB\Delta T}{\mu_f\, vV}\right)^a \left(Pr\right)^a \left(\frac{L}{D}\right)^{3a-1}$$

$$\left(\frac{h_c L}{k}\right) = C\left[\left(\frac{Lv\,\frac{m}{V}}{\mu_f\, g_c}\right)\left(\frac{L^2 wB\Delta T}{\mu_f\, vV}\right)\left(Pr\right)\right]^a$$

$$(Nu) = C\left[\left(\frac{\frac{m}{V}}{\mu_f\, g_c}\right)\left(\frac{wB\Delta T}{\mu_f}\right)\left(Pr\right)\right]^a \qquad \text{where } L^3 = V$$

$$= C\left[\left(\frac{\text{density}}{\text{viscosity}}\right)\left(\frac{\text{buoyancy}}{\text{viscosity}}\right)\left(Pr\right)\right]^a$$

$$= C\left[\frac{L^3\left(\frac{m}{V}\right)\left(wg_c\right)B\Delta T}{\mu_f^{\,2}\, g_c^{\,2}\, V}\left(Pr\right)\right]^a$$

$$= C\left[\frac{L^3\left(\frac{m}{V}\right) mg\, B\Delta T}{\mu_f^{\,2}\, g_c^{\,2}\, V}\left(Pr\right)\right]^a \qquad \text{where } wg_c = mg$$

$$= C\left[\frac{L^3\left(\frac{m}{V}\right)^2 gB\Delta T}{\mu_f^{\,2}\, g_c^{\,2}}\left(Pr\right)\right]^a$$

$$= C[(Gr)(Pr)]^a$$

(Ref. 20, p. 171), (Ref. 21, p. 624), (Ref. 22, p. 373)

$$= C\left[\frac{L^3\left(\frac{m}{V}\right)^2 gB\Delta T}{\mu_f^{\,2}\, g_c^{\,2}} \cdot \frac{c_p\, 3600\, \mu_f\, g_c}{k}\right]^a$$

$$= C\left[\frac{L^3\left(\frac{m}{V}\right)^2 (3600)\, gB\Delta T c_p}{k\, \mu_f\, g_c}\right]^a$$

$$= C(Ra)^a$$

where:

dimensionless numbers are in terms of L rather than D
Gr = Grashof number, dimensionless (22, p. 228)
Ra = Rayleigh number, dimensionless
= (Gr)(Pr) = (Re)(Bu)(Pr) (22, p. 373)

The equation $Nu = C[(Gr)(Pr)]^a$ is the usual natural convection equation. The form $Nu = C(Ra)^a$ is less well known (22).

Summary. Results in the three regions may be summarized in the table.

Convection heat transfer by associative method. This is alternate to the ARDA basic method, using applicable dimensionless numbers from Table 2.

If $$h = \text{fcn}\left[D,\ v,\ \rho,\ \mu,\ k,\ c_p,\ \frac{wB\,\Delta T}{V},\ L,\ g_c,\ 3600\,\frac{\text{sec}}{\text{hr}}\right]$$

$$= \text{fcn}\left[(\mu),\ (k,\ c_p)\left(\frac{wB\,\Delta T}{V}\right)\left(\frac{L}{D}\right)\left(\text{Also } v,\ \rho,\ g_c,\ 3600\right)\right]$$

$$\begin{pmatrix}\text{Convection}\\ \text{Conductance}\\ \text{Related}\\ \text{Number}\end{pmatrix} = \text{fcn}\left[\begin{pmatrix}\text{Viscosity}\\ \text{Related}\\ \text{Number}\end{pmatrix}\begin{pmatrix}\text{Heat}\\ \text{Transfer}\\ \text{Properties}\\ \text{Number}\end{pmatrix}\begin{pmatrix}\text{Heat}\\ \text{Buoyancy}\\ \text{Number}\end{pmatrix}\begin{pmatrix}\text{Shape}\\ \text{Factor}\\ \text{Number}\end{pmatrix}\begin{pmatrix}\text{Numbers}\\ \text{Including}\\ v,\ \rho,\ g_c,\\ 3600\end{pmatrix}\right]$$

$$Nu = \text{fcn}\left[(Re)(Pr)(Bu)\left(\frac{L}{D}\right)(Re)\right]$$

$$Nu = c\left[(Re)^a\,(Pr)^b (Bu)^g \left(\frac{L}{D}\right)^h\right]$$

The limitations of either method in general use are not at present determined. It appears that use of both methods will be of value in the solution of particular problems.

CONVERSION. Conversion from given units to other units is accomplished by the use of conversion relationships.

Conversion equation. A conversion equation is fundamental. It is arbitrarily adopted and requires no proof. 1 ft = 12 in.

The constant 12 may be considered to be a proportionality factor. It is a number and is completely dimensionless.

Region and Type of Convection	Properties: Dominant	Properties: Negligible	General Convection Domain Equation
			$Nu = \left(\text{Nusselt Number}\right) = \left(\frac{h_c D}{k}\right)$
			$= C(Re)^a \quad (Pr)^b \quad (Bu)^c \quad \left(\frac{L}{D}\right)^d$ $= C\left(\text{Reynolds Number}\right)^a \left(\text{Prandtl Number}\right)^b \left(\text{Buoyancy Number}\right)^c \left(\text{Shape Factor}\right)^d$ $= C\left(\frac{Dv\rho}{\mu_f g_c}\right)^a \left(\frac{c_p(3600)\mu_f g_c}{k}\right)^b \left(\frac{D^2 wB\Delta T}{\mu_f v V}\right)^c \left(\frac{L}{D}\right)^d$
1. Forced Turbulent Re Above 2100	$\left(\frac{D}{\mu_f}\right)$ $(v\rho)$	(Bu) $\left(\frac{L}{D}\right)$	$= C(Re)^a \quad (Pr)^b \quad (Bu)^o \quad \left(\frac{L}{D}\right)^o$ $= C(Re)^a \quad (Pr)^b = C(Pe)^a$ if $b = a$ $= C\left[\left(\frac{D}{\mu_f g_c}\right)(v\rho)\right]^a$ if $(Pr)^b = C$ $= C\left[\left(\frac{\text{diameter}}{\text{viscosity}}\right)\left(\frac{\text{mass}}{\text{sec ft}^2}\right)\right]^b$
Pe = Peclet Number			
2. Forced Laminar Re Below 2100	$\dot{m}$ $\left(\frac{c_p}{kL}\right)$	(Bu) μ_f	$= C\left[\frac{\pi}{4}(Re)(Pr)\left(\frac{D}{L}\right)\right]^a \quad (Bu)^o \quad \left(\frac{L}{D}\right)^{d+a}$ $= C(Gz)^a$ if $\left(\frac{L}{D}\right)^{d+a} = \left(\frac{L}{D}\right)^o = 1$ $= C\left[(\dot{m})\left(\frac{c_p}{kL}\right)\right]^a$ $= C\left[\frac{\text{mass}}{\text{hr}}\left(\frac{\text{specific heat}}{\text{conductivity ft}}\right)\right]$
Gz = Graetz Number			
3. Natural Convection	$\frac{\rho}{\mu_f}$ $\left(\frac{gB\Delta T}{\mu_f}\right)$	D v	$= C[(Re) \quad (Pr) \quad (Bu)]^a \quad \left(\frac{L}{D}\right)^{3a-1}$ $= C[(Gr)(Pr)]^a \quad = C[Ra]^a$ $= C[(Gr)]^a$ if $(Pr) = C$ $= C\left[\left(\frac{\rho}{\mu_f g_c}\right)\left(\frac{wB\Delta T}{\mu_f}\right)\right]^a$ $= C\left[\left(\frac{\text{density}}{\text{viscosity}}\right)\left(\frac{\text{buoyancy}}{\text{viscosity}}\right)\right]^a$ Alternate Form $Nu = fcn\left(St, Bu, \frac{L}{D}\right)$ $St = fcn\left(Bu \frac{L}{D}\right)$
Gr = Grashof Number Ra = Raleigh Number St = Staunton Number			

TABLE 2. SOME BASIC DIMENSIONLESS NUMBERS

Case No.	Process	Law or Proof	Association of Dimensions in a Dimensionless Number: Dimensionless Number	Symbol	Name
1	Mass Under Acceleration	$F = \frac{ma}{g_c}$	$\left(\frac{ma}{g_c F}\right)$	Ne	Newton Law in Dimensionless Form
2	Mass Flow Influenced by Viscosity	Dimensional Analysis	$\left(\frac{DG}{\mu_f}\right) = \left[\frac{Dv\rho}{\mu_f g_c}\right]$	Re	Reynolds
3	Surface Heat Flow	Dimensional Analysis	$\left(\frac{hD}{k}\right)$	Nu	Nusselt
4	Buoyancy Force on Hot Mass in Gravity Field	Natural Convection	$\left[\frac{D^2 w B \Delta T}{\mu_f v V}\right]$	Bu	Natural Convection Number
5	Length to Diameter Ratio Shape Factor	Dimensional Analysis	$\left(\frac{L}{D}\right)$	$\frac{L}{D}$	Shape Factor
6	Equivalent Diameter	For Circle $\frac{4A}{P} = D$	$\left(\frac{4A}{DP}\right)$		Equivalent Diameter
7	Transient Heat Conduction	Dimensional Analysis	$\left[\frac{kt}{c_p \rho L^2}\right]$	Fo	Fourier
8	Heat Transfer Physical Properties	Dimensional Analysis	$\left(\frac{c_p (3600) \mu_f g_c}{k}\right)$	Pr	Prandtl

A conversion equation contains no unknowns. It is universally valid. The terms have both numerical and units values. The numerical values on both sides are different. The units values on both sides are different.

The previous conversion equation may also be written as a conversion factor in the form

$$1 = \frac{12 \text{ in.}}{\text{ft}} = 12 \frac{\text{in.}}{\text{ft}}$$

Conversion factors. Conversion from given units to other units is best done by the use of conversion equations expressed as conversion factors. Some selected conversion equations adopted by the U. S. Bureau of Standards with corresponding conversion factors to slide rule accuracy are:

$1 \text{ in.} = 0.0254 \text{ m}$	(Ref. 33, p. 9)
$1 = \left(0.0254 \frac{\text{m}}{\text{in.}}\right)$	[conversion factor]
$1 \text{ lbf} = 4.4482216152605 \text{ Newton}$	(Ref. 33, p. 9)
$1 = \left(4.45 \frac{\text{N}}{\text{lbf}}\right)$	[conversion factor]
$1 \text{ kgf} = 9.80665 \text{ Newton}$	(Ref. 33, p. 9)
$1 = \left(9.81 \frac{\text{N}}{\text{kgf}}\right)$	[conversion factor]
$1 \text{ lbm} = 0.45359237 \text{ kgm}$	(Ref. 33, p. 9)
$1 = \left(0.454 \frac{\text{kgm}}{\text{lbm}}\right)$	[conversion factor]
$1 \text{ ft} = 0.3048 \text{ m}$	(Ref. 33, p. 8)
$1 = 0.3048 \frac{\text{m}}{\text{ft}}$	[conversion factor]

A conversion factor is so called because it is a multiplying term used to convert from given units to other units.

Conversion example. As an example of conversion, it is required to convert 1.00 psi to N per sq m. Conversion factors are used as required so that the units cancel to give the desired answer.

$$P = 1.00 \text{ psi}$$

$$P = \frac{\left(1.00 \frac{\text{lbf}}{\text{in.}^2}\right)\left(4.45 \frac{\text{N}}{\text{lbf}}\right)}{\left(0.0254^2 \frac{\text{m}^2}{\text{in.}^2}\right)}$$

$$= 6895 \frac{\text{N}}{\text{m}^2}$$

The conversion equation for pressure is thus

$$1 \frac{\text{lb}}{\text{in.}^2} = 6895 \frac{\text{N}}{\text{m}^2}$$

The corresponding conversion factor is thus

$$1 = \left(6895 \frac{\frac{\text{N}}{\text{m}^2}}{\frac{\text{lb}}{\text{in.}^2}}\right) = \left(6895 \frac{\frac{\text{N}}{\text{m}^2}}{\text{psi}}\right)$$

More accurately $\quad 1 = \left(6894.7572 \frac{\left(\frac{\text{N}}{\text{m}^2}\right)}{\text{psi}}\right)$ (Ref. 33, p. 17)

Conversion factors g_c. Most conversion equations and conversion factors as previously given are between the units of the same kind of property or dimension. Thus

$$12 \text{ in.} = 1 \text{ ft}$$

Another type of conversion factor occurs in conversion between different kinds of properties. These conversion factors may have definite symbols such as g_c which is essentially a conversion factor with numerical and units part for converting lbf to lbm.

$$g_c = \left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right) = 1$$

This conversion factor was obtained dimensionally from the $F = \left(\frac{m}{g_c}\right)a$ equation. Its numerical value was established so as to have 1 lbm weigh 1 lbf under standard gravity acceleration $g = 32.2$ ft/sec^2.

The g_c factor operates like any other conversion factor. For example to express μ_f viscosity in μ_m mass units.

$$\mu_f = 100 \frac{\text{lbf sec}}{\text{ft}^2}$$

$$\mu_m = \left(1.00 \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)\left(3600 \frac{\text{sec}}{\text{hr}}\right)$$

$$= 116{,}000 \frac{\text{lbm}}{\text{ft hr}}$$

The g_c conversion factor must not be confused with the mathematical expression of a physical law such as

$$F = \left(\frac{m}{g_c}\right)a$$

where the terms F, m, a represent terms having numerical value and units, the numerical part varying with the problem.

In many systems g_c has been assigned a numerical value of one. Important g_c conversion factors are

$$1 = g_c = \left(32.1740 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)$$ (Ref. 32, p. xvi)

$$1 = g_c = \left(1 \frac{\text{slug mass ft}}{\text{lbf sec}^2}\right)$$

$$1 = g_c = \left(1 \frac{\text{lbm ft}}{\text{poundal force sec}^2}\right)$$

$$1 = g_c = \left(980.665 \frac{\text{gram mass cm}}{\text{gram force sec}^2}\right)$$ (Ref. 32, p. xvi)

$$1 = g_c = \left(980.665 \frac{\text{kg mass cm}}{\text{kg force sec}^2}\right)$$

$$1 = g_c = \left(1 \frac{\text{kg mass m}}{\text{Newton force sec}^2}\right) \qquad \text{(Ref. 33, p. 3)}$$

$$1 = g_c = \left(1 \frac{\text{gram mass cm}}{\text{dyne force sec}^2}\right)$$

Conversion factors J. Another group of conversion factors applies, usually relating energy as work to energy as heat.

$$1 = J = 778 \frac{\text{ft lbf}}{\text{Btu}}$$

$$1 = J = \left(1 \frac{\text{Newton meter}}{\text{joule}}\right) \qquad \text{(Ref. 33, p. 3)}$$

$$1 = J = 1.3558179 \frac{\text{joule}}{\text{ft lbf}} \qquad \text{(Ref. 33, p. 14)}$$

Energy conversion.

$$1 = \left(1.05504 \frac{\text{joule}}{\text{Btu}}\right) \qquad \text{(Ref. 33, p. 7)}$$

$$1 = \left(4.1868 \frac{\text{joule}}{\text{cal}}\right) \qquad \text{(Ref. 33, p. 7)}$$

$$1 = \left(1 \frac{\text{joule}}{\text{watt sec}}\right)$$

$$1 = \left(1 \frac{\text{joule}}{\text{amp volt sec}}\right)$$

Conversion factors, general. Important unity conversion factors are given for reference. These permit easy conversion from given units to other units as may be desired. Lbm denotes lb mass, lbf denotes lb force.

Length Conversion:

$$1 = \left(5280 \frac{ft}{mile}\right) \quad 1 = \left(12 \frac{in.}{ft}\right) \quad 1 = \left(1000 \frac{mils}{in.}\right) \quad 1 = \left(\frac{39.37\ in.}{m}\right)$$

$$1 = \left(3.281 \frac{ft}{meter}\right) \quad 1 = \left(2.54 \frac{cm}{in.}\right) \quad 1 = \left(10 \frac{mm}{cm}\right) = \left(10^{10} \frac{\mathring{A}}{m}\right)$$

$$1 = \left(10^3 \frac{meter}{km}\right) \quad 1 = \left(10 \frac{mm}{cm}\right) \quad 1 = \left(10^{10} \frac{angstrom}{meter}\right)$$

$$1 = \left(10^2 \frac{cm}{meter}\right) \quad 1 = \left(10^3 \frac{mm}{meter}\right)$$

$$1 = \left(10^6 \frac{micron}{meter}\right) = \frac{\left(10^6 \frac{micron}{meter}\right)}{\left(3.28 \frac{ft}{meter}\right)\left(12 \frac{in.}{ft}\right)} = \left(25,400 \frac{micron}{in.}\right)$$

Weight Conversion:

$$1 = \left(16 \frac{oz}{lb}\right) \quad 1 = \left(\frac{gram}{980\ dynes}\right) \quad 1 = \left(1000 \frac{gm}{kg}\right)$$

$$1 = \left(2000 \frac{lb}{ton}\right) \quad 1 = \left(2.205 \frac{lb}{kg}\right) \quad 1 = \left(453 \frac{gm}{lb}\right)$$

Time Conversion:

$$1 = \left(60 \frac{sec}{min}\right) \quad 1 = \left(3600 \frac{sec}{hr}\right)$$

$$1 = \left(60 \frac{min}{hr}\right) \quad 1 = \left(24 \frac{hr}{day}\right)$$

Temperature

$T\,^\circ R = T\,^\circ F\ abs = t\,^\circ C + 460 \qquad T\,^\circ K = T\,^\circ C + 273$

Area Conversion:

$$1 = \left(144 \frac{in.^2}{ft^2}\right)$$

Volume Conversion:

$$1 = \left(231 \frac{in.^3}{gal}\right) \quad 1 = \left(4 \frac{quart}{gal}\right) \quad 1 = \left(32 \frac{oz}{quart}\right)$$

$$1 = \left(1.057 \frac{liter}{quart}\right)$$

$$1 = \left(1000 \frac{cm}{liter}\right)$$

Velocity Conversion:

$$1 = \left(1.151 \frac{\left(\frac{\text{mile}}{\text{hr}}\right)}{\text{knot}}\right)$$

$$1 = \left(\frac{186,330 \frac{\text{mile}}{\text{sec}}}{\text{vel of light}}\right) \qquad 1 = \left(\frac{2.99793 \times 10^{10} \frac{\text{cm}}{\text{sec}}}{\text{vel of light}}\right)$$

Energy Conversion:

$$1 = J = \left(778 \frac{\text{ft lbf}}{\text{Btu}}\right) = \left(107.6 \frac{\text{kgf m}}{\text{Btu}}\right)$$

$$1 = \left(33,000 \frac{\text{ft lbf}}{\text{min hp}}\right) \qquad 1 = \left(746 \frac{\text{watts}}{\text{hp}}\right)$$

$$1 = \left(2545 \frac{\text{Btu}}{\text{hp hr}}\right)$$

$$1 = \left(3413 \frac{\text{Btu}}{\text{kw hr}}\right) = \left(3,600,000 \frac{\text{joule}}{\text{kw hr}}\right) = \left(3.60 \times 10^{13} \frac{\text{dyne cm}}{\text{kw hr}}\right)$$

$$1 = \left(10^{7} \frac{\text{erg}}{\text{joule}}\right) = \left(10^{7} \frac{\text{dyne cm}}{\text{joule}}\right) = \left(1 \frac{\text{sec watt}}{\text{joule}}\right) = 1 = \left(4.184 \frac{\text{joule}}{\text{cal}}\right)$$

$$1 = \left(1 \frac{\text{dyne cm}}{\text{erg}}\right)$$

$$1 = \left(252 \frac{\text{cal}}{\text{Btu}}\right) = \left(1060 \frac{\text{joule}}{\text{Btu}}\right)$$

Mass Conversion:

$$1 = \left(32.2 \frac{\text{lbm}}{\text{slug mass}}\right)$$

Force Conversion:

$$1 = \left(32.2 \frac{\text{poundal force}}{\text{lbf}}\right)$$

$$1 = \left(981 \frac{\text{dyne force}}{\text{gram force}}\right) = \left(\frac{2.25 \text{ lbf}}{1,000,000 \text{ dyne force}}\right) = \left(4.45 \frac{\text{Newton}}{\text{lbf}}\right)$$

Pressure Conversion:

(atm refers to standard atmospheric pressure at sea level)

$$1 = \left(14.7 \frac{\text{psi}}{\text{atm}}\right) \qquad 1 = \left(34.0 \frac{\text{ft water}}{\text{atm}}\right)$$

$$1 = \left(29.9 \frac{\text{in. Hg}}{\text{atm}}\right) \quad 1 = \left(760 \frac{\text{mm Hg}}{\text{atm}}\right) \quad 1 = \left(\frac{1,013,250 \frac{\text{dyne}}{\text{cm}^2}}{\text{atm}}\right)$$

Circle: Area $= \pi R^2 = \frac{\pi D^2}{4}$

Perimeter $= 2\pi R = \pi D$

Sphere: Surface $= 4\pi R^2 = \pi D^2$

Volume $= \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$

Cylinder: Surface $= \pi DL + 2\pi R^2$

Volume $= \pi R^2 L$

Metric conversion. See metric system.

DAMKOHLER NUMBERS. A quantity involving chemical reaction rate.

Basic Damkohler Number. It is assumed that $\dot{U}$ is a driving potential per unit velocity per unit distance for chemical reaction in the same manner that viscosity, μ_f, is a driving force (equal to drag force) per unit velocity per unit distance for flow.

Thus, for driving potential

Fluid Flow	Chemical Reaction
$\dfrac{\mu_f\left(\dfrac{\text{lbf force}}{\text{ft}^2\text{ area}}\right)}{\left(\dfrac{\frac{\text{ft}}{\text{sec}}\text{ velocity}}{\text{ft distance}}\right)}$	$\dfrac{\dfrac{\dot{U}\left(\frac{\text{lbm}}{\text{sec}}\right)}{\text{lbm}}}{\left(\dfrac{v\,\frac{\text{ft}}{\text{sec}}\text{ velocity}}{L\text{ ft distance}}\right)}$

The chemical driving force is therefore $\dfrac{\dot{U}}{\left(\frac{v}{L}\right)} = \dfrac{L\dot{U}}{v}$ which inspection shows to be dimensionless. This is the Damkohler parameter 1 which we will call the Damkohler parameter.

$$\text{Dal} = \text{Da} = \left(\begin{array}{c}\text{Damkohler Number}\\ \\ \text{dimensionless}\end{array}\right)$$

$$= \frac{L\dot{U}}{v} = \frac{(L\text{ ft})\left(\dot{U}\,\frac{\text{lbm}}{\text{lbm sec}}\right)}{\left(v\,\frac{\text{ft}}{\text{sec}}\right)}$$

Damkohler Number. There are five dimensionless groups (6). The first Da1 expresses a basic chemical driving potential and will be termed the Damkohler number. The other groups are redundant in that they are expressible in terms of more basic groups.

$$\mathrm{Da1} = \mathrm{Da} = \left(\frac{L\dot{U}}{v}\right) = \left(\frac{L\ \mathrm{ft}\ \dot{U}\ \frac{\mathrm{lbm}}{\mathrm{lbm\ sec}}}{v\ \frac{\mathrm{ft}}{\mathrm{sec}}}\right)$$

$$\mathrm{Da2} = (\mathrm{Da})(\mathrm{Re})(\mathrm{Sc})$$

$$\left(\frac{L\dot{U}}{v}\right)\left(\frac{Lv\rho}{\mu_m}\right)\left(\frac{\mu_m}{\rho Dm}\right) = \left(\frac{L^2\dot{U}}{D}\right) = \left(\frac{L\ \mathrm{ft}^2\ \dot{U}\ \frac{\mathrm{lbm}}{\mathrm{lbm\ sec}}}{D\ \frac{\mathrm{ft}^2}{\mathrm{sec}}}\right)$$

$$\mathrm{Da3} = (\mathrm{Da})(\mathrm{Hv})$$

$$= \left(\frac{L\dot{U}}{v}\right)\left(\frac{q}{c_p T}\right) = \frac{qL\dot{U}}{v\,c_p T}$$

$$= \frac{\left(q\ \frac{\mathrm{Btu}}{\mathrm{lbm}}\right)\left(L\ \mathrm{ft}\right)\left(D\ \frac{1}{\mathrm{sec}}\right)}{\left(v\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)\left(c_p\ \frac{\mathrm{Btu}}{\mathrm{lbm\ F}}\right)\ (\mathrm{TF})}$$

$$\mathrm{Da4} = (\mathrm{Re})(\mathrm{Pr})(\mathrm{Da3})(3600) = (\mathrm{Re})(\mathrm{Pr})(\mathrm{Da})(\mathrm{Hv})(3600)$$

$$= \left(\frac{\rho L v}{\mu_m}\right)\left(\frac{\mu_m c_p}{k}\right)\left(\frac{L\dot{U}}{v}\right)\left(\frac{q}{c_p T}\right)\left(3600\right)$$

$$= \left(\frac{\rho L^2\dot{U}q}{kT}\right)\left(3600\right)$$

$$= \frac{\left(\rho\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)\left(L^2\ \mathrm{ft}^2\right.\ \left(\dot{U}\ \frac{\mathrm{lbm}}{\mathrm{lbm\ sec}}\right.\ \left(q\ \frac{\mathrm{Btu}}{\mathrm{lbm}}\right.\ \left(3600\ \frac{\mathrm{sec}}{\mathrm{hr}}\right)}{\left(k\ \frac{\mathrm{Btu\ ft}}{\mathrm{hr\ ft}^2\ \mathrm{F\ abs}}\right)\left(T\ \mathrm{F\ abs}\right)}$$

$$\mathrm{Da5} = \mathrm{Re}$$

DIFFUSIVITY. The three principal diffusivities are listed below. All have the reduced units (ft^2/hr), but in this form the physical nature of the property is not indicated. Equivalent or unreduced units are required to indicate the physical nature of the quantity. Thermal diffusivity and viscosity are separately discussed elsewhere. Mass diffusivity is discussed in a following section.

DIFFUSIVITY

Thermal Diffusivity	Momentum Diffusivity (Kinematic Viscosity)	Mass Diffusivity (Molecular Diffusivity)
$a\ \frac{ft^2}{hr}$	$\left(\frac{\mu_m}{\rho}\right)\frac{ft^2}{hr}$	$\left(D_m\right)\frac{ft^2}{hr}$
$= \left(\frac{k}{\rho\, c_p}\right) = \frac{\left(k\ \frac{Btu\ ft}{hr\ ft^2\ F}\right)}{\left(\rho\ \frac{lbm}{ft^3}\right)\left(c_p\ \frac{Btu}{lbm\ F}\right)}$	$= \left(\frac{\mu_m\ \frac{lbm}{ft\ hr}}{\rho\ \frac{lbm}{ft^3}}\right)$	$= D_m \frac{\left(\frac{lbm}{hr\ ft^2}\ \text{diffusion}\right)}{\left[\frac{\left(\frac{lbm}{ft^3}\ \text{density}\right)}{(\text{ft thickness})}\right]}$
	$= \left(\frac{\mu_f\ g_c\ 3600}{\rho}\right)$	$= D_m \frac{\left(\frac{lbm}{hr\ ft^2}\right)}{\left(\frac{lbm}{ft^3\ ft}\right)}$
	$\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)$	$= D_m \frac{lbm\ ft^3\ ft}{hr\ ft^2\ lbm}$
	$\left(32.2\ \frac{lbm\ ft}{lbf\ sec^2}\right)$	
	$\frac{\left(3600\ \frac{sec}{hr}\right)}{\left(\rho\ \frac{lbm}{ft^3}\right)}$	

Mass diffusivity. Mass diffusivity D is a property similar to thermal conductivity k and viscosity μ_f as shown in the table of basic definitions or as used in a flux equation in the second tabulation.

SIMILAR PROPERTIES k, μ_f, D

k Thermal Conductivity	μ_f (Absolute) Viscosity	D_m (Mass) Diffusivity
$k \dfrac{\left(\dfrac{Btu}{hr} \text{ heat rate}\right)}{\left[\dfrac{(ft^2 \text{ area})}{\left(\dfrac{ft \text{ thickness}}{F \text{ temp. diff.}}\right)}\right]}$ $= k \dfrac{\left(\dfrac{Btu}{hr}\right)}{\left(\dfrac{ft^2\ F}{ft}\right)}$ $= k \dfrac{Btu\,ft}{hr\,ft^2\ F}$ $= k \dfrac{Btu}{hr\ ft\ F}$	$\mu_f \dfrac{\left(\dfrac{lbf \text{ force}}{ft^2 \text{ area}}\right)}{\left(\dfrac{\dfrac{ft}{sec} \text{ velocity}}{ft \text{ thickness}}\right)}$ $= \mu_f \dfrac{\left(\dfrac{lbf}{ft^2}\right)}{\left(\dfrac{ft}{ft\ sec}\right)}$ $= \mu_f \dfrac{lbf\ ft\ sec}{ft^2\ ft}$ $= \mu_f \dfrac{lbf\ sec}{ft^2}$	$D_m \dfrac{\left(\dfrac{\dfrac{lbm}{hr} \text{ diffusion}}{ft^2 \text{ area}}\right)}{\left(\dfrac{\dfrac{lbm}{ft^3} \text{ density}}{ft \text{ thickness}}\right)}$ $= D_m \dfrac{\left(\dfrac{lbm}{hr\ ft^2}\right)}{\left(\dfrac{lbm}{ft^3\ ft}\right)}$ $= D_m \dfrac{lbm\ ft^3\ ft}{hr\ ft^2\ lbm}$ $= D_m \dfrac{ft^2}{hr}$
	$\mu_m = \mu_f\, g_c\, (3600) \dfrac{lbm\,ft}{ft^2\ hr}$	$\rho\, D_m = \dfrac{lbm\ ft^2}{ft^3\ hr}$

DEFINING FLUX EQUATION (REF. 31, p. 6-6)

Property k	Property μ_f	Property D
Fourier Law	Newton Law	Fick Law
	$\frac{ma}{A} = \frac{F g_c}{A}$ $\frac{mv}{At} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{v}{L}\right)} g_c \left[\frac{v}{L}\right]$	
Heat Flux $\frac{\dot{q}\,\frac{Btu}{hr}}{A\ ft^2}$ or $\left(\frac{\dot{q}}{A}\right)\frac{Btu}{ft^2\ hr}$ $= -\left(k\,\frac{Btu\ ft}{hr\ ft^2\ F}\right)\left(\frac{dT\ F}{dL\ ft}\right)$ $= -k\left(\frac{dT}{dL}\right)\frac{Btu}{hr\ ft^2}$ $= -\rho c\left(\frac{k}{\rho c}\right)\left(\frac{dT}{dL}\right)$ $= -\rho ca\left(\frac{dT}{dL}\right)$	Momentum Flux $\frac{\left(\dot{m}\,\frac{lbm}{hr}\right)\left(v\,\frac{ft}{hr}\right)}{A\ ft^2}$ or $\left(\frac{\dot{m}v}{A}\right)\frac{lbm\ ft}{hr\ ft^2\ hr}$ or $\left(\frac{\dot{m}v}{A}\right)\frac{lbm}{ft\ hr^2}$ $= -\left(\mu_m\,\frac{lbm}{ft\ hr}\right)\left(\frac{dv\ \frac{ft}{hr}}{dL\ ft}\right)$ $= -\mu_m\left(\frac{dv}{dL}\right)\frac{lbm}{ft\ hr^2}$ $= -\mu_f\, g_c\, 3600\left(\frac{dv}{dL}\right)$ $\frac{lbm}{ft\, hr^2}$	Mass Flux $\frac{\dot{m}\,\frac{lbm}{hr}}{A\ ft^2}$ or $\left(\frac{\dot{m}}{A}\right)\frac{lbm}{ft^2\ hr}$ $= -\left(\rho\,\frac{lbm}{ft^3}\right)\left(D\,\frac{ft^2}{hr}\right)\left(\frac{dr\ \frac{lbm}{lbm}}{dL\ ft}\right)$ $= -\rho D\left(\frac{dr}{dL}\right)\frac{lbm}{ft^2\ hr}$ $\left[r = \text{reaction}\ \frac{lbm}{lbm}\right]$

DIMENSIONAL ANALYSIS. Dimensional analysis is an important tool in the formulation of complex physical laws. Dimensional analysis consists of:

1. Deciding the dimensions present in a given process
2. Establishing the relationships between these dimensions when expressing a physical law in mathematical terms.

General. To consider certain simple processes, the laws expressing them when written as mathematical unit-consistent or unit-homogeneous equations indicate association of dimension in certain ways.

It has been customary in dimensional analysis to use letter symbols to represent generalized total dimensions such as L for length, F for force, etc. While this eliminates the immediate necessity for adoption of a unit-dimension system, as far as the dimensional analysis is concerned, any actual calculations must be made in a system. Actually it is simpler to adopt such a system from the beginning and thus eliminate use of the intermediate generalized dimension system. For example, consider that L represents L ft where the L used with ft represents a number and L used alone represents both a number and a unit dimension of feet. In this book the so-called engineering system of units is used.

Thus ARDA dimensional analysis is an extension of usual procedures to use engineering dimensional units rather than general properties to simplify and to promote better understanding and correlation.

This arrangement of relationships in dimensional analysis puts more emphasis on the nature of properties and less on the mathematical equivalent of a property. That is lbf is a force and is so used whereas lbf defined as MLt^{-2} is rather mathematical than physical, particularly if no acceleration force is involved in the process.

The ARDA principle of Arrangement of Relations in Dimensional Analysis is based on the premise that every equation must be dimensionally consistent. The units and exponents on the left-hand side must equal the units and exponents on the right-hand side of the equations. This principle is applicable to the equation as a whole and is also applicable to each one of the unit-properties.

Considerable attention must be paid to the nature of basic dimensions, definitions and quantities. These are discussed in detail under their individual alphabetical headings.

The general procedures of dimensional analysis are best illustrated by specific examples such as the drag domain, etc. given elsewhere.

Dimensions to powers. It may be shown (28, p. 21) that any measurable phenomenon may be evaluated in terms of the causative factors or properties in the form of an equation involving the properties to exponents or powers.

Thus, if $\quad A = f(BCD \ldots .)$

then $\quad A = C(B^b C^c D^d \ldots .)$

where first C is a constant and A, B, C, etc. are properties such as ρ, k, μ, etc. and a, b, c, etc. are exponents.

For example, for a freely falling body

if $\quad L = f(w,\ t,\ g)$

then $\quad L\ ft = C(w\ lbf)^a (t\ sec)^b \left(g\ \frac{ft}{sec^2}\right)^c$.

lbf	$0 = a$	$a = 0$
ft	$1 = c$	$c = 1$
t	$0 = b - 2c$ $= b - 2$	$b = 2$

Thus $\quad L = C(w)^0\,(t)^2\,(g)^1$

$$L = C\,gt^2$$

Dimensionless number equation. (1) Any dimensionally consistent equation (having units on left-hand side equal to units on right-hand side, perhaps after cancellation) can form a dimensionless number.

$$A^m = CB^n$$

forms

$$1 = C\left(\frac{B^m}{A^n}\right) \text{ which is dimensionless.}$$

(2) Also if

$$A = CB^n C^p D^q E^r \ldots .$$

this may be rearranged into the form

$$N = C(N_1{}^a\ N_2{}^b\ N_3{}^c \ldots .)$$

where N represents some dimensionless number combination to some power of $A^m\,B^n\,C^p$ such as (B^n/A^m) etc. This is true because the final equation, composed of individual dimensionless numbers, must be dimensionless or unit-consistent as a whole.

Physical equations involving powers. Certain other principles, useful in dimensional analysis, are applicable to the type of equation.

$$A = f(B, C, D. \dots)$$

$$= C(B^n C^p D^q \dots)$$

$$N = C(N_1{}^a N_2{}^b N_3{}^c \dots)$$

$$= f(N_1 N_2 N_3 \dots)$$

(1) If any term B, C, etc. has the same dimensions, their ratio may be written directly as a dimensionless number $N_1 = (B/C)$.

(2) Any dimensionless number term $N_1{}^a$, etc. where a is unknown may be replaced by any plus or minus power of the N_1 term.

Example $N_1{}^a$ may be replaced by $N_1{}^{-d}$.

$N = C(N_1{}^a N_2{}^b N_3{}^c$ can be written in any of the forms

$$N^d = C(N_1{}^a N_2{}^b N_3{}^c \dots)$$

$$1 = C(N^a N_1{}^b N_2{}^c \dots)$$

$$1 = C(N_1{}^c N_2{}^b N_3{}^a \dots)$$

This is because the exponents a, b, c are unknown and any symbol may be used to represent them.

(3) Any dimensionless number may be multiplied by any numerical constant because the C term preceding the expression represents any unknown numerical constant.

Example

$$N = C(N_1{}^a N_2{}^b N_3{}^c \dots)$$

$$= C(CN_1{}^a)(CN_2{}^b)(CN_3{}^c) \dots$$

where C represents any unknown numerical value (each value is different).

(4) Two or more dimensionless numbers may be combined to form a new dimensionless number.

Example

$$(N_1 N_2) = N_5.$$

DIMENSIONAL ANALYSIS ARDA. The fundamental theorems governing the Arrangement of Relationships in Dimensional Analysis can be formulated as follows. As such they are appreciably more informative than the Pi theorem commonly used in dimensional analysis.

ARDA theorem I. Every equation should be unit-consistent in that the units on the left hand side should equal the units on the right hand side.

Example: $$\left(\dot{w}\,\frac{\text{lbm}}{\text{sec}}\right) = \left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(v\,\frac{\text{ft}}{\text{sec}}\right)\left(A\ \text{ft}^2\right)$$

Example: $$F\ \text{lbf} = \left(\frac{m\ \text{lbm}}{g_c\,\dfrac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(a\,\frac{\text{ft}}{\text{sec}^2}\right)$$

The second equation indicates that where constants such as g_c are involved they have both a numerical and a units part.

ARDA theorem II. In the presentation of a physical law

if $X = \text{fcn}\,(ABCD \ldots.\ MNPQ \ldots.)$

then $X = C A^a B^b C^c D^d \ldots.\ M^m N^n P^p Q^q$.

Mathematical proofs of this appear somewhat formidable (28, p. 21) and will not be given here.

ARDA theorem III. This is an extension of the Pi theorem in a more useful form.

If $X = C A^a B^b C^c D^d \ldots.\ M^m N^n P^p Q^q \ldots.$ represents a valid physical relationship, then dimensional number products can be found.

$$\frac{XN}{MQ} = C\left(\frac{AM}{PQ}\right)^a\left(\frac{B}{MN}\right)^b\left(\frac{CP}{MNQ}\right)^c\left(\frac{DM}{NP}\right)^d$$

in which the properties A, B, C, D each appear in only one of the dimensionless parameters such as (AM/PQ) combined in various manners with other terms such as MNPQ appearing in dimensionless parameter more than once. The exponents mnpq of properties that appear more than once in dimensionless parameters will not appear in the final equation.

It should be noted that if the original expression does not contain the proper terms, sufficient terms, or contains too many terms, the relationship is not valid and it will not be possible to form the second equation. A proper estimate of the properties involved in the original expression is vital.

Example for the drag domain

if $\left(\frac{F}{A}\right) = C(T)^a(g)^b(\mu_f)^c(L)^d(v)^e(D)^f(g_c)^g(\rho)^h$

then application of theorem III with regard to units will yield by ARDA procedure

$$\left(\frac{F g_c}{\rho A v^2}\right) = C\left(\frac{\rho v^2 D}{T g_c}\right)^{-a}\left(\frac{v^2}{Dg}\right)^{-b}\left(\frac{\rho v D}{\mu_f g_c}\right)^{-c}\left(\frac{L}{D}\right)^d$$

$$(Eu) = C(We)^{-a}(Fr)^{-b}(Re)^{-c}\left(\frac{L}{D}\right)^d$$

ARDA theorem IV. Each basic dimensionless number defines the effect of a physical property.

Example for the drag domain

if $\left(\frac{F}{A}\right) = C(T)^a(g)^b(\mu_f)^c(L)^d(v)^e(D)^f(g_c)^g(\rho)^h$

then $\left(\frac{F g_c}{\rho A v^2}\right) = C\left(\frac{\rho v^2 D}{T g_c}\right)^{-a}\left(\frac{v^2}{Dg}\right)^{-b}\left(\frac{\rho v D}{\mu_f g_c}\right)^{-c}\left(\frac{L}{D}\right)^d$

$$(Eu) = C(We)^{-a}(Fr)^{-b}(Re)^{-c}\left(\frac{L}{D}\right)^d$$

Comparing the final equation with the original equation evidently

Eu involves the effect of force F
We involves the effect of gravity g
Re involves the effect of viscosity μ_f
$\left(\frac{L}{D}\right)$ involves the effect of length L

Thus, each basic dimensionless number appears to be related to a specific physical property or dimension.

This is not complete, in that the remaining physical properties such as v D g_c ρ are involved in two or more dimensionless numbers; but if they are so involved, it may be assumed that their effect is included in the final formulation.

Other ARDA principles. Consider that all dimensions may be present if they are involved in the physical process.

In particular, for flow of any kind, if lbf and lbm are present because of the operation of the $F = (M/g_c)a$ law it is necessary to introduce g_c as one of the dimensional analysis terms. If there is no mass accelerated in the process the g_c term is not needed.

If work energy is convertible into heat energy the J = 778 (ft lbf/Btu) is introduced. If this energy conversion is not present, this J term is not needed.

If two units of the same property are involved, a conversion factor such as 3600 (sec/hr) may be needed. This conversion factor is needed when some terms such as k (Btu ft/hr ft^2 F) are in hour units and other terms such as v (ft/sec) are in second units.

Redundant dimensionless numbers. Dimensionless numbers obtained in the preceding manner are termed basic. An equation containing a basic dimensionless number more than once such as the following is redundant in that it can be reduced to a simpler form in which each basic dimensionless number enters only once.

Example: $Nu = C(Re)^a(Gr)^b(Pr)^{-c}$

appears to be redundant because Ra = (Re)(Nc)(Pr) to give

$$Nu = C(Re)^a(Re)^b(Nc)^b(Pr)^b(Pr)^{-c}$$

which could be better expressed in the form

$$Nu = C(Re)^{a+b}(Nc)^b(Pr)^{b-c}$$

Redesignation of exponents. Dimensional analysis does not usually reveal the numerical value of C, a, b, c etc. As such, these symbols represent values to be empirically determined. They can be positive or negative and they can be redesignated.

Example: $X = C(A)^{a+b}(B)^b(C)^{b-c}$

can be rewritten to the general form

$$X = C(A)^a(B)^b(C)^c \ldots.$$

or

$$1 = C(A)^a(B)^b(C)^c(X)^d \ldots.$$

which can be represented

$$C = \text{fcn}(A,\ B,\ C,\ X\ldots)$$

DIMENSIONAL RELATIONSHIPS. Properties or dimensions may be related to each other by equations expressing physical laws. One of the most important of these is the Newton Law.

$$F = Cma$$

$$F = \frac{ma}{g_c}$$

where c is a dimensional constant $= \frac{1}{g_c}$ as defined elsewhere.

In a process in which a force F gives an acceleration a to a mass m it is clearly possible to use this mathematical relation to express force in terms of mass and acceleration, and certain amounts of one correspond to certain amounts of others, but whenever force itself is specifically considered it is a force (push) and not a mass (amount of matter). Viewed in another way, in a process in which mass and acceleration are not present, force is always a force and not a mass, or acceleration and mass and force should not enter the problem in any way.

A similar argument holds for the mechanical equivalent of heat J.

$$H = CFL$$

where C is a dimensional constant $= \frac{1}{J} = \frac{1}{778\ \frac{\text{ft lb}}{\text{Btu}}}$.

In a process in which force F acts through a length or distance L a heat H may be produced. (Under very definite limitations H may be converted into F times L). Thus it is possible to express heat in terms of force and length, but when heat itself is specifically considered it is energy and not a force and distance. Viewed in the second aspect, in a process in which force is not present, heat is heat and force should not enter into the problem.

DIMENSIONAL CONSTANT. A dimensional constant has both numerical value and dimensional units. J = 778 ft lb/Btu is a dimensional constant.

A dimensionless constant is a numerical value such as 778 in the preceding dimensional constant.

Most equations have dimensional constants although the units are not always clearly defined. In the equation F = Kma, K is a dimensional constant having units such as to make the equation dimensionally consistent (units on left-hand side of equation equal to units on the right-hand side of the equation).

DIMENSIONAL SYSTEMS. Also see Systems of Units. The adoption of various fundamental dimensional systems has severely limited scope of dimensional analysis to a series of somewhat individual solutions to special problems with inherent confusion and limitation in comparing the findings of one investigator on one physical problem with those of a different investigator on a different physical problem.

A more general approach is desirable. Let the principle be adopted that there is no arbitrarily fixed number of basic dimensions. Basic dimensions are properties that are entirely different from each other in their physical natures. Thus force, mass, and heat are different dimensions and should not be defined in terms of each other in a basic method of dimensional analysis.

This is in accord with newer thermodynamics texts (6, 7) using what might be termed an FMLT system of engineering units in which force lbf is a dimensional property distinct from mass lbm.

A corollary is that a dimension is a basic property different from any other dimension and, therefore, not physically a combination of other dimensions, although in any particular process in which the dimensions are all present they may be related by a mathematical forumula. Thus, if force, mass, time, and length are all present in a given process, it is proper to conclude that the physical law expressing their interaction is also present. Thus, if a force producing acceleration of a mass is present (Newton's Law) it is proper to include the conversion factor $g_c = 32.2$ lbm ft/lbf sec^2.

DIMENSIONLESS NUMBERS. These numbers, resulting from dimensional analysis, are dimensionless in the sense that the units of the numerator are the same as or cancel with the units of the denominator.

The important dimensionless numbers will be tabulated here. Many of them are discussed in detail under their alphabetical headings. Examination of the derivation of the various domain equations indicates that many of the dimensionless numbers are associated with the presence of a given physical property. These properties are also included in the following list.

Dimensionless Number	Associated Property
Angle ratio $= \frac{\phi}{\theta}$	Angles, related
Bo = Bond $= \left(\frac{We}{Fr}\right) = \left(\frac{w}{TL}\right)$	Bubbles
Bu = Buoyancy $\begin{cases} = \left(\frac{L^2 wB\Delta T}{\mu_f \bar{V} v}\right) \\ = \frac{(Re)}{(Fr)}\left(\frac{\Delta\rho}{\rho}\right) \end{cases}$	Buoyant force $= \left(\frac{wB\Delta T}{V}\right)$
Ca = Cauchy $= \left(\frac{\rho v^2}{E g_c}\right)$	Elasticity modulus = E
C_D = Drag Coefficient = 2Eu	
Da = Damkohler $= \left(\frac{L\dot{U}}{v}\right)$ = Da1	Chemical reaction rate = $\dot{U}$
Da2 = Damkohler 2 = (Da)(Re)(Sc)	Combustion

$Ec = \text{Eckert} = \left(\dfrac{v^2}{c_p \Delta T\, g_c\, J}\right)$ — KE to heat = J

$Em1 = \text{Electromagnetic 1} = (\mu_p E v^2)$ — Magnetic permittivity = ϵ

$Em2 = \text{Electromagnetic 2} = \left(\dfrac{Jm\,E}{\mu_p H v}\right)$ — Electromagnetic field = $\dfrac{E}{H}$

$Em3 = \text{Electromagnetic 3} = \left(\dfrac{\mu_p H^2 g_c}{\rho v^2}\right)$

Fluid flow, gravity see gravity

Fluid flow, viscous see viscosity

$Eu = \text{Euler} = \left(\dfrac{F g_c}{\rho L^2 v^2}\right) = \left(\dfrac{P g_c}{\rho v^2}\right) = \left(\dfrac{C_D}{2}\right)$ — Force = F; Pressure = P

$Fd1 = \text{Fluid Dynamics 1} = \dfrac{(We)^3}{(Fr)(Re)^4} = \dfrac{(Bo)(We)^2}{(Re)^4} = \dfrac{g\,\mu_f^{\,4}}{\rho T^3 g_c^{\,3}}$

$Fo = \text{Fourier} = \left(\dfrac{kt}{c_p \rho L^2}\right)$ — Heat flow time = t

$Fr = \text{Froude} = \left(\dfrac{v^2}{gL}\right)$ — Gravity = g

Frequency, see speed

$Gr = \text{Grashof} = \dfrac{(Re)^2 (B\Delta T)}{(Fr)} = \left(\dfrac{\rho^2 L^3 g B \Delta T}{\mu_f^{\,2} g_c^{\,2}}\right)$

$Gz = \text{Graetz} = \left[\dfrac{\pi}{4} (Re)(Pr) \dfrac{D}{L}\right] = \left(\dfrac{\dot{m} c_p}{kL}\right)$

$Ha = \text{Hartmann} = \left(\dfrac{\mu_p^{\,2} H^2 g_c}{\rho v^2}\right)$

$Hv = \text{Heat Value} = \left(\dfrac{q}{c_p \Delta T}\right)$ — Heat value = q

$Ja = \text{Jacob} = \left(\dfrac{h_f\, g}{c_p \Delta T}\right)$ — Evaporation enthalpy = h_f

$k = \text{Specific Heat Ratio} = \left(\dfrac{c_p}{c_v}\right)$

$\dfrac{L}{D}, \dfrac{w}{D}, \dfrac{e}{L}, \dfrac{w}{L}$, etc. — Length, repeated

$Le = \text{Lewis} = \dfrac{(Pr)}{(Sc)} = \left(\dfrac{\rho c_p D_m}{k}\right)$

$Ma = \text{Mach} = \dfrac{v_1}{v}, \dfrac{v_2}{v}$, etc. — Velocity, repeated = v_1, v_2, etc.

Nu = Nusselt = $\left(\frac{hD}{k}\right)$ — Heat surface conductance = h

Pe = Peclet = (Re)(Pr) = $\left(\frac{3600\, c_p \rho v D}{k}\right)$

Pm = Magnetic Prandtl = $\frac{(Rm)}{(Re)} = \left(\frac{\mu_p \sigma \mu_f g_c}{J_m \rho}\right)$

Pr = Prandtl = $\left(\frac{3600\, c_p \mu_f g_c}{k}\right)$ — Heat flow = $(c_p \mu_p k)$

Ra = Rayleigh = (Gr)(Pr) = $\left(\frac{3600\, D^3 \rho^2 g B \Delta T\, c_p}{k \mu_f g_c}\right)$

Re = Reynolds = $\left(\frac{\rho v D}{\mu_f g_c}\right) = \left(\frac{\rho v L}{\mu_f g_c}\right)$ — Viscosity μ_f

RF = Roughness Factor = $\frac{e}{D}$ or $\frac{e}{L}$ — Roughness height = e

Rm = Magnetic Reynolds = $\left(\frac{\mu_p \sigma L V}{J_m}\right)$ — Electrical conductivity = σ

Sc = Schmidt = $\left(\frac{\mu_m}{\rho D_m}\right)$ — Mass diffusivity = D_m

Slenderness Ratio = $\left(\frac{L}{D}\right)$ — = Shape factor

SF = Shape Factor = $\left(\frac{L}{D}\right)$

Sh = Strouhal = $\left(\frac{N_s L}{v}\right)$ — Rotary or cycle speed = N_s

St = Stanton = $\frac{(Nu)}{(Pr)(Re)} = \left(\frac{h}{3600\, c_p \rho v}\right)$

Th = Thoma = $\frac{P}{\Delta P}$

Vi = Vibration Number = $\frac{W}{D}$ — Amplitude of vibration = W

We = Weber = $\left(\frac{\rho v^2 D}{T g_c}\right)$ — Surface tension = T

DIMENSIONLESS NUMBERS AS RATIOS OF FORCES. Many dimensionless numbers may be derived from force ratios, which gives some conception of the significance of the dimensionless number. The force of acceleration is also known as the inertia force.

Re = Reynolds Number, a viscous force parameter

$$= \frac{\text{(Force of Deceleration)}}{\text{(Viscous Drag Force)}} = \frac{\left(\frac{m}{g_c}\right)a}{\mu_f A\left(\frac{v}{L}\right)}$$

$$= \frac{m\left(\frac{v}{t}\right)}{\mu_f A\left(\frac{v}{L}\right) g_c} = \frac{\left(\frac{m}{AL}\right) A\left(\frac{v}{L}\right)\left(\frac{L}{t}\right) L}{\mu_f A\left(\frac{v}{L}\right) g_c} = \frac{\rho v L}{\mu_f g_c}$$

Eu = Euler Number, a pressure force parameter

$$= \frac{\text{(Force of Pressure)}}{\text{(Force of Acceleration)}} = \frac{PA}{\left(\frac{m}{g_c}\right)a}$$

$$= \frac{PA\,g_c}{m\left(\frac{L}{t^2}\right)} = \frac{PA\,g_c}{\frac{m}{(AL)} A\left(\frac{L^2}{t^2}\right)} = \frac{P g_c}{\rho v^2}$$

Fr = Froude Number, a gravity force parameter

$$= \frac{\text{(Force of Acceleration)}}{\text{(Force of Gravity)}} = \frac{\left(\frac{m}{g_c}\right)a}{\left(\frac{m}{g_c}\right)g}$$

$$= \frac{\left(\frac{v}{t}\right)}{g} = \frac{v\left(\frac{L}{t}\right)}{gL} = \frac{v^2}{gL}$$

We = Weber Number, a surface tension force parameter

$$We = \frac{\text{(Force of Acceleration)}}{\text{(Force of Surface Tension)}} = \frac{\left(\frac{m}{g_c}\right)a}{TL}$$

$$= \frac{m\left(\frac{v}{t}\right)}{TL\,g_c} = \frac{\frac{m}{L^3} L\left(\frac{L}{t}\right) vL}{TL\,g_c} = \frac{\rho v^2 L}{T\,g_c}$$

Ca = Cauchy Number, an elasticity parameter

$$= \frac{\text{(Force of Deceleration)}}{\text{(Force of Compression)}} = \frac{\left(\frac{m}{g_c}\right) a}{EA}$$

$$= \frac{m\left(\frac{v}{t}\right)}{EA\, g_c} = \frac{\left(\frac{m}{AL}\right) A \left(\frac{L}{t}\right) v}{EA\, g_c} = \frac{\rho v^2}{E\, g_c}$$

Dimensionless numbers as energy ratios. Any of the preceding dimensionless numbers viewed as force ratios can be converted to energy ratios by multiplying numerator and denominator each by L, since energy = W = FL.

The force to produce acceleration times distance becomes a kinetic energy.

Dimensionless numbers as stress ratios. Any of the preceding dimensionless numbers viewed as force ratios can be converted to stress ratios by dividing numerator and denominator each by A, because stress = S = F/A.

DOMAINS. In dimensional analysis the various physical phenomena may be divided into various domains which are discussed in detail under the respective headings. Important domains so discussed are:

- Bubble Mechanics
- Combustion
- Convection Heat Transfer
- Drag
- Elasticity
- Electromagnetic
- Flow
- Magnetohydrodynamic
- Nucleation
- Pressurization
- Propellor
- Pump
- Vibration

Dimensional analysis examples. ARDA dimensional analysis has been applied to physical phenomena domains as previously listed and also to the following topics discussed in more limited scope.

Bubble Pressure
Buoyancy Number
Falling Body
Fluid Drag of Viscous Fluid
Orifice Flow Produced by Gravity and Pressure
Shear Stress in Pipe

DRAG COEFFICIENT. Although not usually so defined, the drag coefficient C_D is 2 (Eu) when Eu is the Euler Number. It seems advisable to so consider it.

$$C_D = \begin{pmatrix}\text{Drag Coefficient}\\ \text{Dimensionless}\end{pmatrix}$$

$$= 2\ \text{Eu} = \left(2\,\frac{Pg_c}{\rho v^2}\right)$$

$$= \frac{\left(\frac{F}{A}\right)}{\left[\frac{1}{2}\,\frac{\left(\frac{m}{g_c}\right)v^2}{V}\right]} = \frac{\left[\frac{\text{Drag Force}}{A}\ \frac{\text{lbf}}{\text{ft}^2}\right]}{\left[\frac{\text{Kinetic Energy}}{V}\ \frac{\text{ft lbf}}{\text{ft}^3}\right]}$$

$$= \frac{\left[\frac{\text{Drag Force}}{A}\right]}{\left[\frac{\text{Inertial Force}}{A}\right]}$$

$$= \frac{(\text{Drag Force})}{(\text{Inertia Force})}$$

DRAG DOMAIN. ARDA analysis gives

$$\text{Eu} = f(\text{We},\ \text{Fr},\ \text{Re},\ \frac{L}{D})$$

Summary. Dimensional analysis is used to obtain the general fluid-drag relation

$$\begin{pmatrix}\text{Euler}\\ \text{Number}\end{pmatrix} = C\begin{pmatrix}\text{Weber}\\ \text{Number}\end{pmatrix}^{-a}\begin{pmatrix}\text{Froude}\\ \text{Number}\end{pmatrix}^{-b}\begin{pmatrix}\text{Reynolds}\\ \text{Number}\end{pmatrix}^{-c}\left(\frac{L}{D}\right)^{d}$$

or

$$Eu = C(We)^{-a}(Fr)^{-b}(Re)^{-c}\left(\frac{L}{D}\right)^{d}$$

This equation is applicable for regions or regimes extending from those of very low Reynolds numbers in which Stokes' law is valid, to regions having large Reynolds numbers. The drag of ships is also included as a region. These regions emerge or recede as certain properties become dominant or decrease in importance. This general equation permits study of the inter-relationship of regions and permits an overall correlation which should enable better understanding of fundamental principles governing flow drag phenomena.

General drag equation. The general equation for the drag on bodies immersed in fluids is dependent on properties of the fluid and conversion factors as follows:

1. F = drag force, lbf, always associated with an area
2. A = area, ft^2
3. T = surface tension, lbf/ft
4. g = gravity or acceleration field, ft/sec^2
5. μ_f = viscosity, lbf sec/ft^2
6. L = length, ft
7. v = velocity, ft/sec
8. D = diameter, ft
9. g_c = acceleration constant, 32.2 lbm $ft/lbf\ sec^2$. Introduced because in turbulent flow lbm masses are accelerated by lbf forces, i.e., the $F = (m/g_c)a$ law is involved in flow phenomena.

10. $\rho = \Delta(m/V)$ = difference in mass density between inside and outside fluids or between a solid and a fluid or of a fluid alone if only one fluid is present, lbm/ft^3.

The relationship between F and these factors may be written in usual units with C as a dimensionless numerical constant and a, b, c, etc. as exponents, as

$$\left(\frac{F}{A}\frac{lbf}{ft^2}\right) = C\left(T\,\frac{lbf}{ft}\right)^a\left(g\,\frac{ft}{sec^2}\right)^b\left(\mu_f\,\frac{lbf\ sec}{ft^2}\right)^c (L\,ft)^d\left(v\,\frac{ft}{sec}\right)^e (D\,ft)^f$$

$$\left(g_c\,\frac{lbm\,ft}{lbf\ sec^2}\right)^g\left(\rho\,\frac{lbm}{ft^3}\right)^h$$

The individual unit-properties and exponents on each side of the equation must be equal. For example, for lbf

$$(lbf)^1 = (lbf)^{a+c-g}$$

Writing successively the equality for the exponents for each one of the unit-properties such as lbf, lbm, etc.

<u>lbf</u>	$1 = a + c - g$	$g = a + c - 1$
<u>lbm</u>	$0 = g + h$	$h = -g = -a - c + 1$
<u>sec</u>	$0 = -2b + c - e - 2g$	$e = -2b + c - 2g$
		$= -2b + c - 2a - 2c + 2$
		$= -2a - 2b - c + 2$

<u>ft</u> $\quad -2 = -a + b - 2c + d + e + f + g - 3h$

$$= -a + b - 2c + d - 2a - 2b - c + 2 + f + a + c - 1 + 3a + 3c - 3$$

$$= a - b + c + d - 2 + f \qquad f = -a + b - c - d$$

Substituting these values of g, d, e and f in the original equation

$$\left(\frac{F}{A}\right) = C(T)^{a}(g)^{b}(\mu_f)^{c}(L)^{d}(v)^{-2a-2b-c+2}(D)^{-a+b-c-d}$$

$$(g_c)^{a+c-1}(\rho)^{-a-c+1}$$

Rearranging

$$\left(\frac{F g_c}{\rho A v^2}\right) = C\left(\frac{T g_c}{\rho v^2 D}\right)^{a}\left(\frac{Dg}{v^2}\right)^{b}\left(\frac{\mu_f g_c}{\rho v D}\right)^{c}\left(\frac{L}{D}\right)^{d}$$

Checking units in each of the groups in the preceding equation shows that each group is dimensionless in that the units of the numerator cancel the units of the denominator. Such groups occur frequently in fluid flow and heat transfer and are called dimensionless numbers. These dimensionless numbers have been given names. Thus the previous equation may be written:

$$(Eu) = \left(\begin{matrix}\text{Euler}\\ \text{Number}\end{matrix}\right) = \frac{C\left(\frac{L}{D}\right)^{d}}{\left(\begin{matrix}\text{Weber}\\ \text{Number}\end{matrix}\right)^{a}\left(\begin{matrix}\text{Froude}\\ \text{Number}\end{matrix}\right)^{b}\left(\begin{matrix}\text{Reynolds}\\ \text{Number}\end{matrix}\right)^{c}}$$

$$= C(We)^{-a}(Fr)^{-b}(Re)^{-c}\left(\frac{L}{D}\right)^{d} \qquad \left[\begin{matrix}\text{General}\\ \text{Drag}\\ \text{Equation}\end{matrix}\right]$$

Drag equation by ARDA associative method. With experience examination of the defining equation enables direct writing of the dimensionless number equation.

$$\left(\frac{F}{A}\right) = f(T,\ g,\ \mu_f,\ L,\ v,\ D,\ \rho)$$

By inspection

$\left(\frac{F}{A}\right)$ involves the force dimensionless number Eu

T involves the surface tension number We

g involves the gravity dimensionless number Fr

μ involves the viscosity dimensionless number Re

L and D result in the dimensionless number L/D

The required relationship is then

$$\mathrm{Eu} = f\left(\mathrm{We}, \mathrm{Fr}, \mathrm{Re}, \frac{L}{D}\right) \quad \text{or Eu}$$

or

$$\mathrm{Eu} = C(\mathrm{We})^{a}(\mathrm{Fr})^{b}(\mathrm{Re})^{c}\left(\frac{L}{D}\right)^{d}$$

Flow region 1. Flow at very low Reynolds numbers is considered as a first region, applicable to slowly falling rigid bodies immersed in fluids. For laminar flow the drag (F/A) is not proportional to velocity squared, v^2, in the Euler number, so the effect of Eu must be negligible or (Eu) = 1. Also the shape of the solid body is not determined by surface tension, T, so that $(T)^a = (T)^o = 1$ or a = 0. The general equation then becomes:

$$\mathrm{Eu} = 1 = C(\mathrm{We})^{-o}(\mathrm{Fr})^{-b}(\mathrm{Re})^{-c}\left(\frac{L}{D}\right)^{d}$$

The values of C, b, c and d must be determined experimentally. For Re < 2, experiment indicates that C = 18, b = -1 and c = 1 for a sphere for which L = D so that $\left(\frac{L}{D}\right)^d = \left(\frac{D}{D}\right)^d = 1$.

$$1 = 18(\mathrm{Fr})^{1}(\mathrm{Re})^{-1} \qquad \left[\begin{array}{c}\text{or } 1 = 18\ \mathrm{Fr/Re} \text{ if Re} \\ \text{is based on D}\end{array}\right]$$

$$= 18\left(\frac{v^2}{Dg}\right)\left(\frac{\mu_f\, g_c}{\rho v D}\right)$$

$$= 18\left(\frac{\mu_f\, v}{\rho D^2}\right)\frac{g_c}{g} \qquad \left[\text{where } \frac{D^2}{4} = R^2\right]$$

$$\rho\frac{g}{g_c} = \Delta\left(\frac{m}{V}\right)\frac{g}{g_c} = 18\left(\frac{\mu_f\, v}{4R^2}\right)$$

$$\Delta\left(\frac{w}{V}\right) = 18\left(\frac{\mu_f\, v}{R^2}\right) \qquad \text{where } \left(\frac{m}{g_c}\right) = \left(\frac{w}{g}\right)$$

$$\left[\left(\frac{w}{V}\right)_{\text{sphere}} - \left(\frac{w}{V}\right)_{\text{fluid}}\right] = 18\left(\frac{\mu_f\, v}{R^2}\right) \qquad \text{which is Stokes law}$$

Stokes law is generally assumed applicable from Re = 0 to Re = 2.

Flow region 2. For drag on a ship the surface tension T is negligible. Thus $(T)^a = (T)^o = 1$, or a = 0. Customarily $(C_D/2)$, where C_D is a drag coefficient, has been used instead of Eu. The general equation becomes:

$$(Eu) = C(We)^{-o}(Fr)^{-b}(Re)^{-c}\left(\frac{L}{D}\right)^d = f\left(Fr,\ Re,\ \frac{L}{D}\right)$$

$$\left(\frac{Fg_c}{\rho A v^2}\right) = \frac{C_D}{2} = f\left(Fr,\ Re,\ \frac{L}{D}\right) \qquad \text{where } A = L^2$$

$$F = C_D A\left(\frac{\rho v^2}{2g_c}\right) = \left[f\left(Fr,\ Re,\ \frac{L}{D}\right)\right] A\left(\frac{\rho v^2}{2g_c}\right)$$

Flow region 3A. If surface tension T and gravity g are unimportant, then $(T)^a = (T)^o = 1$ or a = 0, and $(g)^b = (g)^o = 1$ or b = 0. The general equation becomes with $(C_D/2)$ written for Eu

$$(Eu) = \left(\frac{C_D}{2}\right) = C(We)^{-o}(Fr)^{-o}(Re)^{-c}\left(\frac{L}{D}\right)^d$$

$$C_D = 2C(Re)^{-c}\left(\frac{L}{D}\right)^d$$

Experimental evidence by Lapple and Shephard (16, 18) for solid bodies of $\left(\frac{L}{D}\right)^d = 1^d = 1$ in the region having Reynolds numbers greater than that for which Stokes' law applies (above Re = 2), indicates

$$C_D = 18.7(Re)^{-0.68}$$

There is some indication that this region may extend to Re = 700.

Flow region 3B. If surface tension T and gravity g are unimportant, then $(T)^a = (T)^o = 1$ or a = 0, and $(g)^b = (g)^o = 1$ or b = 0. The general equation becomes:

$$(Eu) = C(We)^{-o}(Froude)^{-o}(Re)^{-c}\left(\frac{L}{D}\right)^d$$

$$\left(\frac{Fg_c}{\rho A v^2}\right) = C(Re)^{-c}\left(\frac{L}{D}\right)^d$$

$$\frac{\left(\frac{F}{A}\right)}{\left(\frac{\rho}{g_c}\right)} = C(Re)^{-c}\left(\frac{L}{D}\right)^d v^2$$

$$\left(\frac{F}{A}\right) = \Delta P = C(Re)^{-c}\left(\frac{L}{D}\right)^d \frac{v^2 \rho}{g_c}$$

Using $\left(\frac{\rho}{g_c}\right) = \left(\frac{m}{Vg_c}\right) = \left(\frac{w}{Vg}\right)$ and if $d = 1$

$$\frac{\Delta P}{\left(\frac{w}{V}\right)} = C(Re)^{-c}\left(\frac{L}{D}\right)\frac{v^2}{g}$$

where

$$\Delta P = \left(\frac{w}{V}\right)H$$

or

$$\frac{\Delta P}{\left(\frac{w}{V}\right)} = H = \text{head, ft}$$

$$H = [f(Re)]\left(\frac{L}{D}\right)\frac{v^2}{2g} \qquad \text{[Darcy Equation]}$$

This is the Darcy equation for head loss for turbulent flow in horizontal pipe, where f the friction factor is a function of $Re = 2C(Re)^{-c}$. Multiply both sides by (w/m)

$$\frac{wH}{m} = [f(Re)]\left(\frac{L}{D}\right)\frac{v^2}{2g}\left(\frac{w}{m}\right)$$

where

$$\left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right) \quad \text{or} \quad \frac{w}{mg} = \frac{1}{g_c}\text{, thus}$$

$$\left(\frac{\text{Energy drop}}{\text{Mass}}\ \frac{\text{ft lbf}}{\text{lbm}}\right) = [f(Re)]\left(\frac{L}{D}\right)\frac{v^2}{2g_c}$$

Flow region 4. If the effect of viscosity is negligible $(\mu_f)^c = (\mu_f)^o = 1$ or $c = 0$. The general equation becomes

$$(Eu) = \left(\frac{C_D}{2}\right) = C(We)^{-a}(Fr)^{-b}(Re)^o\left(\frac{L}{D}\right)^d$$

If $-a = 3$ and $b = 1$, this becomes

$$\left(\frac{C_D}{2}\right) = C\,\frac{(We)^3}{(Fr)} = C\,\frac{(We)^3}{(Fr)}\,\frac{(Re)^4}{(Re)^4}$$

$$= C\,\frac{\left(\dfrac{\rho v^2 D}{T g_c}\right)^3 (Re)^4}{\left(\dfrac{v^2}{Dg}\right)\left(\dfrac{\rho v D}{\mu_f g_c}\right)^4}$$

$$= C\left(\frac{\rho^3 v^6 D^3}{T^3 {g_c}^3}\right)\left(\frac{Dg}{v^2}\right)\left(\frac{\mu_f^4 g_c^4}{\rho^4 v^4 D^4}\right)(Re)^4$$

$$= C\left(\frac{g\,\mu_f^4\,g_c}{\rho T^3}\right)(Re)^4$$

$$= C(G)(Re)^4$$

where

G = dimensionless number, unnamed

$$= \frac{(We)^3}{(Fr)(Re)^4} \qquad \left[\begin{array}{c}\text{Comparing last and first}\\ \text{equation for } C_D/2\end{array}\right]$$

The dimensionless number G has appeared in the literature in discussions by Rosenberg (17), Peebles and Barger(18), and Fritz (10), with some evidence to show that the relation $(Eu) = C(G)(Re)^4$ is applicable for Re 700 to 1300.

Flow region 5. 'If surface tension T and gravity g are important, the dimensionless numbers We and Fr containing these properties are important. It may then be assumed that the other dimensionless numbers, Eu (containing F) and Re (containing μ_f) and $(L/D)^d$ are unimportant, or $Eu = 1$ and $(Re)^c = (Re)^o = 1$, and $\left(\frac{L}{D}\right)^d = \left(\frac{L}{D}\right)^o = 1$. The general equation then becomes

$$Eu = 1 = \frac{C}{(We)^a (Fr)^b (Re)^o} = \frac{C}{(We)^a (Fr)^b}$$

This appears to be a region above Re = 1300, for liquid N_2 at 14.7 psia 139 F for which Fritze (10) gives

$$1 = 1.20\left[\frac{gT\,g_c}{\rho v^4}\right]^{0.25}$$

$$= \frac{1.20}{\left[\left(\frac{\rho v^2 D}{T g_c}\right)\left(\frac{v^2}{Dg}\right)\right]^{0.25}}$$

$$1 = \frac{(1.20)^4}{[(We)(Fr)]}$$

or

$$(1.20)^4 = 2.08 = (We)(Fr)$$

Evidently a = b = 1.

Flow region 6. If surface tension T is dominant, the effects of velocity v are less dominant. Terms with v^2 have negligible effect or

$$Eu = \left(\frac{F g_c}{\rho A v^2}\right) = (Eu)^o = 1 \quad \text{and} \quad (Fr)^b = \left(\frac{v^2}{Dg}\right)^o = 1$$

The general equation becomes

$$(Eu) = 1 = C(We)^{-a}(Fr)^{-o}(Re)^{-c}\left(\frac{L}{D}\right)^d$$

Experimental evidence of Schmidt, Kaissling and Rosenberg correlated by Peebles (18) and Fritz (10a) as their region 3 indicates

$$Re = 1.91\left[\frac{D\rho T g_c}{\mu_f^2\, g_c^2}\right]^{\frac{1}{2}} \qquad \text{Where Re is based on D}$$

$$= 1.91\left[\left(\frac{T g_c}{\rho v^2 D}\right)\left(\frac{\rho^2 v^2 D^2}{\mu_f^2\, g_c^2}\right)\right]^{\frac{1}{2}}$$

$$= 1.91[(We)(Re)^2]^{\frac{1}{2}} \qquad \text{Where We and Re are based on D}$$

$$(Re)^2 = (1.91)^2 [(We)(Re)^2]$$

$$1 = 1.91^2 (We) = 3.65\left(\frac{Tg_c}{\rho v^2 D}\right)$$

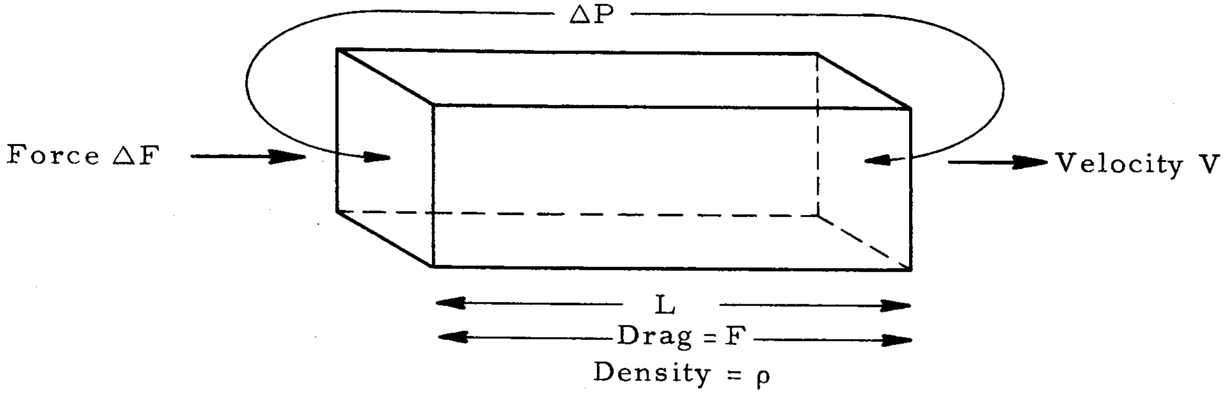
It appears that $(Re)^{-c} = 1 = Re^{-o}$ or $c = 0$ and $\mu^c = \mu^o = 1$ or viscosity has little effect.

Summary. Results are presented in the tabulation. The Re ranges of the various regions are approximate and the regions themselves are tentative as there may be more regions in which certain properties are dominant, these regions merging one into another. More study of the literature is desirable. Sufficient has been given to demonstrate that the ARDA dimensional analysis method and the resultant general drag equation can be applied to data in the literature to present an overall correlation and perhaps better visualization of flow drag phenomena in general.

DRAG ON BODIES IN FLUIDS BY ARDA

Region — Extent of these regions is defined with Re based on D	Properties: Dominant	Properties: Negligible	Equation
			$(Eu) = \left(\frac{C_D}{2}\right) = C(We)^{-a} \quad (Fr)^{-b} \quad (Re)^{-c} \quad \left(\frac{L}{D}\right)^d$ $(\text{Euler}) = C(\text{Weber})^{-a} \quad (\text{Froude})^{-b} \quad (\text{Reynolds})^{-c} \quad (\text{Shape Factor})$ $\left(\frac{F}{\rho A}\frac{g_c}{v^2}\right) = C\left(\frac{\rho v^2 D}{T g_c}\right)^{-a} \quad \left(\frac{v^2}{Dg}\right)^{-b} \quad \left(\frac{\rho v D}{\mu_f g_c}\right)^{-c} \quad \left(\frac{L}{D}\right)^d$ $= C\left(\frac{T g_c}{\rho v^2 D}\right)^{a} \quad \left(\frac{Dg}{v^2}\right)^{b} \quad \left(\frac{\mu_f g_c}{\rho v D}\right)^{c} \quad \left(\frac{L}{D}\right)^d$
1 Stokes Law rigid sphere in laminar fluid Re < 2	$\Delta\left(\frac{w}{V}\right)$ μ_f	T Eu	$[(Eu) = 1] = 18[(We)^o = 1] \left(\frac{v^2}{Dg}\right)^1 \quad \left(\frac{\mu_f g_c}{\rho v D}\right) \quad \left(\frac{D}{D}\right)^d$ $\Delta\left(\frac{w}{V}\right) = \Delta\left(\frac{m}{V}\right)\frac{g}{g_c} = 18\left(\frac{\mu_f v}{R^2}\right)$ Stokes Law
2 Drag F on Ship of Cross Section A	ρ v	T	$\left(\frac{F g_c}{\rho A v^2}\right) = C[(We)^o = 1] \quad (Fr)^{-b} \quad (Re)^{-c} \quad \left(\frac{L}{D}\right)^d$ $= C f\left(Fr, Re, \frac{L}{D}\right) = \frac{C_D}{2}$ Ship Drag
3	μ v	T g	$\left(\frac{F g_c}{\rho A v^2}\right) = \left(\frac{C_D}{2}\right) = C[(We)^o = 1] \quad [(Fr)^o = 1] \quad (Re)^{-c} \quad \left(\frac{D}{D}\right)^d$
3. a. Solid body in laminar fluid Re 2 to 700			$C_D = 18.7(Re)^{-0.68}$ Lapple and Shepherd
3. b. Pressure drop turbulent flow horizontal pipe			$H = [f(Re)] \left(\frac{L}{D}\right)\frac{v^2}{2g}$ Darcy Equation
4 Surface tension dominant Re 700 to 1300	T	μ	$\left(\frac{C_D}{2}\right) = C(We)^{-a} \quad (Fr)^{-b} \quad [(Re)^o = 1]$ $= CG(Re)^4$ where $G = \left(\frac{g \mu_f^4 g_c}{\rho T^3}\right)$
5 T and g dominant Re > 1300	T g	μ F	$[Eu = 1] = C(We)^{-1} \quad (Fr)^{-1} \quad [(Re)^o = 1]$ $2.08 = (We)(Fr)$
6 Re > 700	T	v	$[Eu = 1] = C(We)^a \quad [(Fr)^o = 1] \quad Re^{-c} \quad \left(\frac{D}{D}\right)^d$ $1 = (1.91)^2 (We)$ Schmidt, Kaissling, Rosenberg via Peebles

<u>DRAG ENERGY</u>. Consider the fluid flow element.



$$(\text{Drag Energy}) = (\text{Drag})\, L$$

$$= F\,L$$

$$= (\Delta F)L$$

$$= A(\Delta P)L$$

$$= V\,\Delta P$$

$$= \Delta(PV) \quad \text{for incompressible fluids}$$

Thus for incompressible fluids any decrease in flow energy is used to overcome drag.

<u>ECKERT NUMBER</u>. The Eckert Number occurring in heat transfer is a conversion of kinetic energy to heat capacity.

$$\frac{KE}{J} = (\text{Heat Capacity})$$

$$Ec = \begin{pmatrix} \text{Eckert Number} \\ \text{Dimensionless} \end{pmatrix}$$

$$= \frac{\left(\frac{KE}{J}\right)}{(\text{Heat Capacity})}$$

$$= \frac{\frac{1}{2}\frac{mv^2}{g_c J}}{m\,C_p\,\Delta T}$$

$$= \frac{1}{2}\left(\frac{v^2}{C_p \Delta T\, g_c\, J}\right)$$

$$= \frac{1}{2}\,\frac{\left(v^2\ \frac{ft^2}{sec^2}\right)}{\left(C_p\ \frac{Btu}{lbm\ F}\right)\left(\Delta T\ F\right)\left(32.2\ \frac{lbm\ ft}{lbf\ sec^2}\right)\left(778\ \frac{ft\ lbf}{Btu}\right)}$$

ELASTICITY DOMAIN. ARDA analysis gives

$$Ca = fcn\,(Fr, We, Eu, Re)$$

Derivation of elasticity domain. Elasticity is usually defined by the modulus of elasticity E.

$$E = \frac{(Stress)}{(Strain)}$$

$$= \frac{\left(Pressure\ \frac{lbf}{ft^2}\right)}{\left(\frac{change\ in\ length\ \ ft}{original\ length\ \ ft}\right)} = \frac{\left(P\ \frac{lbf}{ft^2}\right)}{\left(\frac{\Delta L\ ft}{L\ ft}\right)} = \left(\frac{PL}{\Delta L}\ \frac{lbf}{ft^2}\right)$$

$$E = C(D)^a\,(\rho)^b\,(g_c)^c\,(v)^d\,(g)^e\,(T)^f\,(P)^g\,(\mu_f)^h$$

$$\left(E\ \frac{lbf}{ft^2}\right) = C\ (D\ ft)^a\left(\rho\ \frac{lbm}{ft^3}\right)^b\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)^c\left(v\ \frac{ft}{sec}\right)^d\left(g\ \frac{ft}{sec^2}\right)^e$$

$$\left(T\ \frac{lbf}{ft}\right)^f\left(P\frac{lbf}{ft^2}\right)^g\left(\mu_f\,\frac{lbf\ sec}{ft^2}\right)^h$$

lbf $\quad 1 = -c + f + g + h \qquad\qquad c = f + g - 1 + h$

ft $\quad -2 = a - 3b + c + d + e - f - 2g - 2h$

$$-2 = a + (-3 + 3f + 3g + 3h) + (f + g - 1 + h)$$

$$+ (-2f - 2g - 2e + 2 - h) + e - f - 2g - 2h$$

$$= a + f - e + h \qquad\qquad a = -f - 2 + e - h$$

lbm $\quad 0 = b + c$

$$= b + f + g - 1 + h \qquad\qquad b = 1 - f - g - h$$

$\underline{\text{sec}}$ $\quad 0 = -2c - d - 2e + h$

$$= -2f - 2g + 2 - 2h - d - 2e + h \qquad\qquad d = -2f - 2g - 2e + 2 - h$$

$$E = C(D)^{-f - 2 + e - h} (\rho)^{1 - f - g - h} (g_c)^{f + g - 1 + h}$$

$$(v)^{-2f - 2g - 2e + 2 - h} g^e T^f P^g (\mu_f)^h$$

$$\left(\frac{E g_c}{\rho v^2}\right) = C\left(\frac{gD}{v^2}\right)^e \left(\frac{T g_c}{v^2 \rho D}\right)^f \left(\frac{P g_c}{\rho v^2}\right)^g \left(\frac{\mu_f g_c}{\rho v D}\right)^h$$

$$(Ca) = C(Fr)^e (We)^f (Eu)^g (Re)^{-h}$$

This is also obtainable as a special case under Flow Domain.

<u>ELECTROMAGNETIC DOMAIN</u>. ARDA analysis gives

$$Rm = fcn\,(Rm, Em1, Em2) \quad .$$

<u>Derivation of electromagnetic domain</u>. For electromagnetic phenomena occurring in radio antennae, cavity resonators, eddy currents, skin-effect in bus-bars, transformers, etc. (24, p. 116):

$$\left(\frac{I}{L^2}\right) = fcn\,(E, \epsilon, \mu_p, \sigma, H, L, t, J_m)$$

where

$\left(\frac{I}{L^2}\right)$ = current density, $\frac{\text{amp}}{\text{ft}^2}$

E = electric field strength, $\frac{\text{volt}}{\text{ft}}$

ϵ = electrical permittivity, $\frac{\text{amp}^2\ \text{sec}^2}{\text{lbf ft}^2}$

μ_p = magnetic permeability, $\frac{\text{lbf}}{\text{amp}^2}$

σ = electrical conductivity, $\frac{\text{amp}}{\text{volt ft}}$

H = magnetic field strength, $\frac{\text{amp}}{\text{ft}}$

L = length, ft

t = time, sec

J_m = conversion-constant factor, work to joule electrical units

$$= 0.738 \frac{\text{ft lbf}}{\text{joule}} = 0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}$$

These units are more completely defined under the section on Electromagnetic Units.

$$\frac{I \text{ amp}}{L^2 \text{ ft}^2} = C \left(E \frac{\text{volt}}{\text{ft}}\right)^a \left(\epsilon \frac{\text{amp}^2 \text{ sec}^2}{\text{lbf ft}^2}\right)^b \left(\mu_P \frac{\text{lbf}}{\text{amp}^2}\right)^c \left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)^d \left(H \frac{\text{amp}}{\text{ft}}\right)^e$$

$$(L \text{ ft})^f (t \text{ sec})^g \left(J_m \frac{\text{ft lbf}}{\text{amp volt sec}}\right)^h$$

amp	$1 = 2b - 2c + d + e - h$	
lbf	$0 = b + c + h$	
adding	$1 = b - c + d + e$	$c = -1 + b + d + e$
From lbf	$0 = -b - 1 + b + d + e + h$	$h = 1 - d - e$
volts	$0 = a - d - h$	
	$0 = a - d - 1 + d + e$	$a = 1 - e$
sec	$0 = 2b + g - h$	
	$0 = 2b + g - 1 + d + e$	$g = 1 - 2b - d - e$
ft	$-2 = -a - 2b - d - e + f + h$	
	$-2 = -1 + e - 2b - d - e + f + 1 - d - e$	
		$f = -2 + 2b + 2d + e$

$$\left(\frac{I}{L^2}\right) = C (E)^{1-e} (\epsilon)^b (\mu_P)^{-1+b+d+e} (\sigma)^d (H)^e (L)^{-2+2b+2d+e}$$

$$(t)^{1-2b-d-e} (J_m)^{1-d-e}$$

$$\left(\frac{\mu_p L^2\left(\frac{I}{L^2}\right)}{J_m t E}\right) = \left(\frac{\mu_p \sigma L^2}{J_m t}\right)^d \left(\frac{\mu_p \epsilon L^2}{t^2}\right)^b \left(\frac{J_m E t}{\mu_p H L}\right)^{-e}$$

$$(Rm) = (Rm)^d (Em1)^b (Em2)^{-e}$$

$$1 = (Rm)^a (Em1)^b (Em2)^c$$

This equation is really a special case of the magnetohydrodynamics domain equation in which the force parameter $Eu = 1$.

where

$d - 1 = a, \; -e = c$

Rm = magnetic Reynolds number

Em1 = electromagnetic dimensionless number 1

Em2 = electromagnetic dimensionless number 2

The dimensionless numbers Em1 and Em2 do not seem to have accepted names.

ELECTROMAGNETIC FLUID PARAMETERS. Analysis in this domain has indicated the existence of a number of parameters some of which have found names and others which will be designated Em1, etc. Also see under Electromagnetic and Magnetohydrodynamics Domains.

$$Rm = \left(\begin{matrix}\text{Magnetic Reynolds Number}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \left(\frac{\mu_p \sigma L^2}{J_m t}\right) = \left(\frac{\mu_p \sigma L v}{J_m}\right)$$

$$= \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)\left(L^2 \text{ft}^2\right)}{\left(0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}\right)\left(t \text{ sec}\right)}$$

$$Rm = (Re)^{-1}(Em3)(Ha)$$ [Also see under Magnetic Reynolds Number]

Also, (see under Magnetic Reynolds Number)

$$Rm = \frac{\mu_p L^2\left(\frac{I}{L^2}\right)}{J_m t E} = \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(L^2 \text{ft}^2\right)\left(\frac{I \text{ amp}}{L \text{ ft}^2}\right)}{\left(0.738 \frac{\text{ft lbf}}{\text{volt amp sec}}\right)\left(t \text{ sec}\right)\left(E \frac{\text{volt}}{\text{ft}}\right)}$$

$$\mathrm{Em1} = \left(\begin{matrix}\text{Electromagnetic Number 1}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \left(\frac{\mu_p \, \epsilon \, L^2}{t^2}\right) = (\mu_p \epsilon \; v^2)$$

$$= \left(\mu_p \frac{\mathrm{lbf}}{\mathrm{amp}^2}\right)\left(\epsilon \; \frac{\mathrm{amp}^2\,\mathrm{sec}^2}{\mathrm{lbf}\;\mathrm{ft}^2}\right)\left(\frac{L^2\,\mathrm{ft}^2}{t^2\,\mathrm{sec}^2}\right)$$

$$= \frac{v^2}{\left(\dfrac{1}{\mu_p \epsilon}\right)} = \frac{(\text{Fluid Velocity})^2}{(\text{Electromagnetic Wave Velocity})^2}$$

This parameter is of interest because in a vacuum the velocity corresponding to μ_p and ϵ in the vacuum is the velocity of light (31, p. 27-5).

$$\mathrm{Em2} = \left(\begin{matrix}\text{Electromagnetic Number 2}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \left(\frac{J_m E\, t}{\mu_p H\, L}\right) = \frac{J_m E}{\mu_p H\, v}$$

$$= \frac{\left(0.738 \; \dfrac{\mathrm{ft\;lbf}}{\mathrm{amp\;volt\;sec}}\right)\left(E \; \dfrac{\mathrm{volt}}{\mathrm{ft}}\right)}{\left(\mu_p \dfrac{\mathrm{lbf}}{\mathrm{amp}^2}\right)\left(H \; \dfrac{\mathrm{amp}}{\mathrm{ft}}\right)\left(v \; \dfrac{\mathrm{ft}}{\mathrm{sec}}\right)}$$

$$\mathrm{Em3} = \left(\begin{matrix}\text{Electromagnetic Number 3}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \left(\frac{\rho v^2}{\mu_p H^2 g_c}\right)$$

$$= \frac{\left(\dfrac{\mathrm{m\,lbm}}{g_c \, \dfrac{\mathrm{lbm}}{\mathrm{lbf}} \, \dfrac{\mathrm{ft}}{\mathrm{sec}^2} \; V \; \mathrm{ft}^3}\right)\left(V^2 \; \dfrac{\mathrm{ft}^2}{\mathrm{sec}^2}\right)}{\left(\mu_p \dfrac{\mathrm{lbf}}{\mathrm{amp}^2}\right)\left(H^2 \; \dfrac{\mathrm{amp}^2}{\mathrm{ft}^2}\right)} = \text{Ratio} \; \frac{\left(\text{Inertial Stress} \; \dfrac{\mathrm{lbf}}{\mathrm{ft}^2}\right)}{\left(\text{Magnetic Stress} \; \dfrac{\mathrm{lbf}}{\mathrm{ft}^2}\right)}$$

$$= \frac{2\left(\dfrac{\text{Kinetic Energy lbf ft}}{V \; \mathrm{ft}^3}\right)}{\left(\dfrac{\text{Magnetic Energy lbf ft}}{V \; \mathrm{ft}^3}\right)}$$

$$Ha = \left(\begin{array}{c}\text{Hartmann Number}\\ \text{Dimensionless}\end{array}\right)$$

$$= \left(\frac{\mu_p{}^2 \sigma H^2 L^2}{\mu_f J_m}\right)$$

$$= \frac{\left[\dfrac{\left(\mu^2 \dfrac{\text{lbf}^2}{\text{amp}^4}\right)\left(\sigma \dfrac{\text{amp}}{\text{volt ft}}\right)\left(H^2 \dfrac{\text{amp}^2}{\text{ft}^2}\right)\left(L^2 \text{ft}^2\right)}{\left(J_m \dfrac{\text{ft lbf}}{\text{amp volt sec}}\right)}\right]}{\left(\mu_f \dfrac{\text{lbf sec}}{\text{ft}^2}\right)}$$

$$= \frac{(\text{Magnetic Viscous Stress})}{(\text{Ordinary Viscous Stress})}$$

$$Pm = \left(\frac{\text{Magnetic Prandtl Number}}{\text{Dimensionless}}\right)$$

$$= \frac{(Rm)}{(Re)} = \frac{(\text{Magnetic Reynolds Number})}{(\text{Reynolds Number})}$$

$$= \frac{\left(\dfrac{\mu_p \sigma L v}{J_m}\right)}{\left(\dfrac{\rho v L}{\mu_f g_c}\right)} \quad \frac{\mu_p \sigma \mu_f g_c}{J_m P}$$

$$= \frac{\left(\mu_p \dfrac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \dfrac{\text{amp}}{\text{volt ft}}\right)\left(\mu_f \dfrac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \dfrac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(0.738 \dfrac{\text{ft lbf}}{\text{amp volt sec}}\right)\left(\rho \dfrac{\text{lbm}}{\text{ft}^3}\right)}$$

where

μ_p = magentic permeability, lbf/amp^2

ϵ = electrical permittivity, amp^2 sec^2/lbf ft^2

H = magnetic field strength, amp/ft

E = electric field strength, volt/ft

σ = electrical conductivity, amp/volt ft

L = length, ft

t = time, sec

v = velocity, ft/sec

J_m = conversion constant factor, work to joules

= 0.738 ft lbf/joule = 0.738 ft lbf/amp volt sec

g_c = conversion constant = 32.2 lbm ft/lbf sec^2

ELECTROMAGNETIC UNITS. Electromagnetic field quantities are established by the following laws, expressed by equations in engineering units of lbm, lbf, ft, sec, amp and volt.

Conversion factors to other systems of units. If it is desired to express these quantities in other units such as MLT units, a conversion factor such as $\left(g_c \frac{\text{lbm}}{\text{lb}} \frac{\text{ft}}{\text{sec}}\right)$ may be used to eliminate force by expressing it in mass units. In the case of electrical phenomena a fourth unit besides mass, length and time is required as a minimum (23, p. 87; 24, p. 43). This unit can be an ampere, a unit of μ, ϵ, etc. In the MLT amp system a conversion factor of

$$\left(g_c \frac{\text{lbm ft}}{\text{lbf sec}}\right)\left(J_m \frac{\text{ft lbf}}{\text{amp volt sec}}\right)$$

would be required to eliminate volts by expressing it in MLTA units.

Force between two magnetic poles R ft apart.

$$F \text{ lbf} = \frac{(m \text{ magnetic pole})(m \text{ magnetic pole})}{\left(\mu_p \frac{\text{poles}^2}{\text{lbf ft}^2}\right)\left(R^2 \text{ft}^2\right)}$$

Also

$$\frac{1}{\mu R^2} = \frac{F}{m^2}$$

$$\frac{F}{\mu R^2} = \left(\frac{F}{m}\right)^2 = H^2 \qquad \text{[Definition of H]}$$

$$F = \mu_p H^2 R^2 \qquad \text{known as magnetic force}$$

$$= \left(\mu_p \frac{\text{poles}^2}{\text{lbf ft}^2}\right)\left(H^2 \frac{\text{lbf}^2}{\text{poles}^2}\right)\left(R^2 \text{ft}^2\right) \quad \text{, alternately}$$

$$= \left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(H^2 \frac{\text{amp}^2}{\text{ft}^2}\right)\left(R^2 \text{ft}^2\right)$$

Force between two electric charges R ft apart.

$$F \text{ lbf} = \frac{(q \text{ coulomb})(q \text{ coulomb})}{\left(\epsilon \dfrac{\text{coulomb}^2}{\text{lbf ft}^2}\right)(R^2 \text{ft}^2)}$$

$$= \frac{q^2 \text{amp}^2 \text{sec}^2}{\left(\epsilon \dfrac{\text{amp}^2 \text{ sec}^2}{\text{lbf ft}^2}\right)(R \text{ ft}^2)}$$

Also,

$$\frac{1}{\epsilon R^2} = \frac{F}{q^2}$$

$$\frac{F}{\epsilon R^2} = \left(\frac{F}{q}\right)^2 = (E\, J_m)^2 \qquad \text{[Definition E]}$$

$$F = \epsilon E^2 R^2 J_m{}^2 \qquad \text{[Force of Electric Field]}$$

$$= \left(\epsilon \frac{\text{amp}^2 \text{sec}^2}{\text{lbf ft}^2}\right)\left(E^2 \frac{\text{volt}^2}{\text{ft}^2}\right)(R^2 \text{ft}^2)\left(0.738^2 \frac{\text{ft}^2 \text{lbf}^2}{\text{amp}^2 \text{volt}^2 \text{sec}^2}\right)$$

Biot law for electric field strength H.

$$\left(H \frac{\text{lbf}}{\text{pole}}\right) = \left(\frac{I \text{ amp } L \text{ ft}}{R^2 \text{ft}^2}\right)$$

Evidently,

$$\left(H \frac{\text{lbf}}{\text{pole}}\right) = \left(H \frac{\text{amp}}{\text{ft}}\right) = \text{magnetic field strength}$$

E electric field units.

$$E = \frac{F}{J_m q} = \frac{F \text{ lbf}}{\left(J_m \dfrac{\text{ft lbf}}{\text{amp volt sec}}\right)(q \text{ amp sec})}$$

$$= \left(\frac{F}{J_m q} \frac{\text{volt}}{\text{ft}}\right) = E \frac{\text{volt}}{\text{ft}}$$

ϵ electrical permittivity units.

$$\epsilon = \frac{q^2}{FR^2} \qquad [\text{From } F = \frac{q^2}{\epsilon R^2}]$$

$$\left(\frac{q^2 \text{ amp}^2 \text{ sec}^2}{F \text{ lbf } R^2 \text{ ft}^2}\right) = \epsilon \frac{\text{amp}^2 \text{ sec}^2}{\text{lbf ft}^2}$$

To eliminate force units

$$(\epsilon J_m) = \left(\epsilon \frac{\text{amp}^2 \text{ sec}^2}{\text{lbf ft}^2}\right)\left(0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}\right)$$

$$= (0.738\,\epsilon) \frac{\text{amp sec}}{\text{volt ft}}$$

H magnetic field strength units.

By definition

$$H = \left(\frac{F}{m}\right) = \frac{m}{\mu_p R^2} \qquad [\text{from } F = \frac{mm}{\mu_p R^2}]$$

$$H^2 = \left(\frac{F}{m}\right)^2 = \left(\frac{F}{\mu_p R^2}\right) = \left(\frac{J^2 L^2}{R^4}\right) \qquad [\text{Biot Law}]$$

$$H = \frac{I \text{ amp}}{L \text{ ft}} \qquad [\text{Assuming } L \text{ ft} = R \text{ ft}]$$

μ_p magnetic permeability units.

$$\left(\frac{F}{\mu_p R^2}\right) = \left(\frac{I^2 L^2}{R^4}\right) \qquad (\text{Biot Law above})$$

$$\frac{F}{\mu_p} = I^2 \qquad [\text{Assuming } L \text{ ft} = R \text{ ft}]$$

$$\left(\frac{F \text{ lbf}}{I^2 \text{ amp}^2}\right) = \mu_p \frac{\text{lbf}}{\text{amp}^2}$$

To eliminate force units

$$\left(\frac{\mu_p}{J_m}\right) = \left(\frac{\mu_p \frac{\text{lbf}}{\text{amp}^2}}{0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}}\right) = \left(\frac{\mu_p}{0.738}\right) \frac{\text{volt sec}}{\text{amp ft}}$$

σ electrical conductivity.

$$\sigma = \frac{1}{(\text{Resistivity})} = \frac{1}{R\left(\frac{A}{L}\right)} \qquad [\text{where } R = \text{resistance}]$$

$$= \frac{IL}{VA} \qquad [\text{where } I = \frac{V}{R} \text{ volts}]$$

$$\left(\frac{I \text{ amps } L \text{ ft}}{V \text{ volts } A \text{ ft}^2}\right) = \sigma \, \frac{\text{amp}}{\text{volt ft}}$$

ENERGY RATIOS. Dimensionless numbers may be frequently interpreted as force ratios (see Force Ratios). If both numerator and denominator are multiplied by distance L they are also FL or energy ratios. A typical example follows dependent on its nature either a force or energy ratio or some combination may be taken as most descriptive.

$$Eu = \frac{(\text{Flow Energy})}{(\text{Kinetic Energy})} = \frac{2Pg_c}{\rho v^2} = \frac{(PV)}{(KE)}$$

EQUATIONS, DIMENSIONALLY CONSISTENT. The ARDA concept is based on the premise that every equation must be dimensionally consistent. The units and exponents on the left-hand side must equal the units and exponents on the right-hand side of the equation. This principle is applicable to the equation as a whole and is also applicable to each one of the unit-properties.

EULER NUMBER. This number expressing a force F per length squared L^2 is encountered in fluid flow and is sometimes called a force coefficient. The length L is a significant length or distance in the process or L^2 may be an area so that $F/L^2 = F/A$ may be a force or drag per unit area or a pressure P or pressure difference ΔP. The Euler number is encountered as a drag coefficient $C_D = 2$ Eu, a pressure coefficient (24, p. 88) and even a wall boundary shear stress varying with the nature of the problem. Dimensional analysis does not yield a physical picture so that additional consideration of the physical phenomena is necessary for proper interpretations of Eu.

The velocity in Eu is the flow velocity.

Units

$$Eu = \begin{pmatrix}\text{Euler Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{Pg_c}{\rho v^2} = \frac{\left(P\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(32.2\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(v^2\ \frac{\text{ft}}{\text{sec}^2}\right)}$$

$$Eu = \frac{Fg_c}{\rho A v^2} = \frac{\left(F\ \text{lbf}\right)\left(32.2\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(A\ \text{ft}^2\right)\left(v^2\ \frac{\text{ft}^2}{\text{sec}^2}\right)}$$

$$Eu = \left(\frac{C_D}{2}\right)$$

where

P may be ΔP
A may be L^2
P may also be replaced by the shear stress S (lbf/ft^2) since from a dimensional analysis standpoint it has the same dimensions. When S has been used it has been called Fanning Number (24, p. 135), but the introduction of another name for the same kind of dimensionless number is probably unnecessary.

Euler Number in terms of pressure head. Pressure P may be expressed in terms of head H.

$$P = H\left(\frac{w}{V}\right) \qquad \text{where} \left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right) \text{or } w = \frac{mg}{g_c}$$

$$= H\frac{m}{V}\frac{g}{g_c}$$

$$= H\rho\left(\frac{g}{g_c}\right)$$

Thus,

$$Eu = \frac{Pg_c}{\rho v^2} = \frac{H\rho\left(\frac{g}{g_c}\right)g_c}{\rho v^2}$$

$$= \frac{Hg}{v^2} = \frac{\left(H\ ft\right)\left(g\ \frac{ft}{sec^2}\right)}{\left(v^2\ \frac{ft^2}{sec^2}\right)}$$

The Euler number expressed as (Hg/v^2) resembles the Froude number as (Lg/v^2) but H is a pressure head, whereas the L or D in the Froude number is related to object size.

<u>Euler number in terms $\dot{V}$ cu ft per second.</u>

$$Eu = \frac{Pg_c}{\rho v^2} = \frac{Pg_c L^4}{\rho(L^2 v)^2}$$

$$= \frac{Pg_c L^4}{\rho \dot{V}^2} = \frac{\left(P\ \frac{lbf}{ft^2}\right)\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)\left(L\ ft\right)^4}{\left(\rho\ \frac{lbm}{ft^3}\right)\left(\dot{V}\ \frac{ft^3}{sec}\right)^2}$$

<u>Euler number in terms $\dot{W}$.</u>

$$Eu = \frac{\dot{W}g_c}{\rho L^2 v^3} = \frac{Fv\,g_c}{\rho L^2 v^3} = \frac{Fg_c}{\rho L^2 v^2}$$

<u>Power $\dot{W}$ in terms Eu.</u>

$$(Eu)(Sh) = \left(\frac{Fg_c}{\rho D^2 v^2}\right)\left(\frac{ND}{v}\right)$$

$$= \frac{(FND)\,g_c}{\rho D^2 v^3}$$

$$= \frac{\dot{W}g_c}{\rho D^2 v^3} = \frac{\left(\dot{W}\ \frac{ft\ lbf}{sec}\right)\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)}{\left(\rho\ \frac{lbm}{ft^3}\right)\left(D\ ft\right)^2\left(v\ \frac{ft}{sec}\right)^3}$$

$$\frac{Eu}{(Sh)^2} = \frac{(Eu)(St)}{(Sh)^3} = \frac{\left(\frac{\dot{W} g_c}{\rho D^2 v^3}\right)}{\left(\frac{N^3 D^3}{v^3}\right)}$$

$$= \left(\frac{\dot{W} g_c}{\rho N^3 D^5}\right) = \frac{\left(\dot{W}\,\frac{\text{ft lbf}}{\text{sec}}\right)\left(g_c\,\frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(N\,\frac{1}{\text{sec}}\right)^3 (D\ \text{ft})^5}.$$

Eu as a number of velocity heads.

$$Eu = \frac{P g_c}{\rho v^2} = \frac{P}{\left(\frac{m}{g_c V}\right) v^2} = \frac{P}{\left(\frac{w}{g V}\right) v^2} = \frac{\left[\frac{P}{\left(\frac{w}{g}\right)}\right]}{2\left(\frac{v^2}{2g}\right)} = \frac{\left[\begin{matrix}\text{Number of}\\ \text{Velocity}\\ \text{Heads}\end{matrix}\right]}{2} = \frac{C_D}{2}$$

Eu as a pressure force parameter. Euler number may be defined as a ratio of forces. Force of acceleration is also known as inertia force.

$$Eu = \frac{(\text{Force of Pressure})}{(\text{Force of Acceleration})} = \frac{PA}{\left(\frac{m}{g_c}\right) a}$$

$$= \frac{PA g_c}{m\left(\frac{L}{t^2}\right)} = \frac{PA g_c}{\left(\frac{m}{AL}\right) A \left(\frac{L^2}{t^2}\right)} = \frac{P g_c}{\rho v^2}$$

The pressure P may also be a pressure difference ΔP.

Eu as a drag force parameter. Euler number may also be defined as in terms of a drag force on the wall.

$$Eu = \frac{(\text{Drag Force})}{(\text{Force of Acceleration})} = \frac{F}{\left(\frac{m}{g_c}\right) a}$$

$$= \frac{F g_c}{m\left(\frac{v}{t}\right)} = \frac{F g_c}{\left(\frac{m}{AL}\right) A \left(\frac{L}{t}\right) v} = \frac{F g_c}{\rho A v^2}$$

Eu as a energy ratio. Euler number visualized as a $\frac{\text{(Force of Pressure)}}{\text{(Force of Acceleration)}}$ may have both numerator and denominator multiplied by (V/A) to obtain a ratio of $\frac{\text{(Flow Energy)}}{\text{(Kinetic Energy)}}$.

$$\mathrm{Eu} = \frac{F g_c}{\rho A v^2} = \frac{F g_c\left(\frac{V}{A}\right)}{\left(\frac{m}{V}\right) A v^2 \left(\frac{V}{A}\right)} = \frac{\left(\frac{F}{A}\right) V g_c}{m v^2} = \frac{PV}{\left(\frac{m v^2}{g_c}\right)}$$

$$= \frac{\text{(Flow Energy)}}{2\text{(Kinetic Energy)}}$$

Eu in terms of Ma.

$$\mathrm{Eu} = \frac{P g_c}{\rho v^2} = \frac{1}{\left[\frac{v^2}{\left(\frac{P g_c}{\rho}\right)}\right]}$$

$$= \frac{1}{\left[\frac{v^2}{\left(\frac{PV}{m} g_c\right)}\right]}$$

$$= \frac{1}{\left(\frac{v^2}{R T g_c}\right)}$$

$$= \frac{1}{k\left(\frac{v^2}{g_c k R T}\right)}$$

$$= \frac{1}{k(\mathrm{Ma})^2}$$

$$\frac{1}{(\mathrm{Eu})k} = (\mathrm{Ma})^2$$

Fluid flow element. It is convenient to consider an element of flowing fluid.

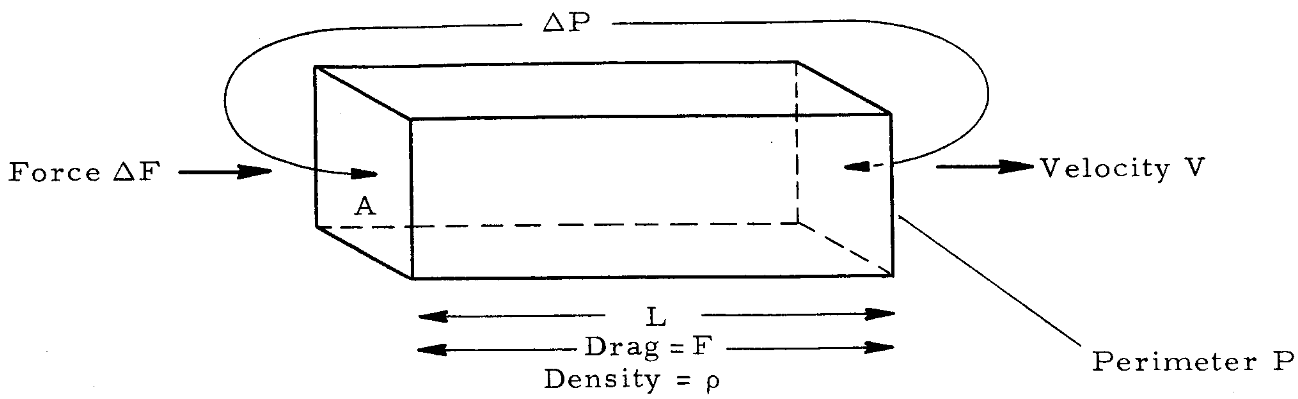


FIG. FLUID FLOW ELEMENT

Fluid flow may be visualized as a continuous variation of forces on a fluid particle to change the particle velocity.

If the overall fluid velocity does not change these forces and velocites are internal (or microscopic).

Kinetic energy is the work (ft lbf) energy used to produce an overall (macroscopic) velocity.

Euler number is a ratio of the flow energy (PV) used to produce an external kinetic energy of an overall velocity in a given direction.

When flow energy (and potential energy) are not completely converted into external kinetic energy, the remaining part results in internal kinetic energy in which the fluid particles are each moving with varying fluid velocities in various directions (disorder) in turbulent flow. This movement may be frictionless or some of the energy may appear as heat tending to increase internal energy unless removed.

FALLING BODY. A freely falling body in mechanics falls a distance S which may be presumed to be a function of weight, gravity and time.

$$S\,ft = C(w\ lb)^a \left(g\ \frac{ft}{sec^2}\right)^b (t\ sec)^c$$

lb	$0 = a$	$a = 0$
ft	$1 = b$	$b = 1$
sec	$0 = -2b + C$	$c = 2b = 2$

$$S = c(w)^0 (g)^1 (t)^2$$

$$= Cgt^2$$

[weight has no effect]

FANNING NUMBER. See Euler number.

FLOW CONCEPTS. One concept of fluid flow has been given under Reynolds number in which semi-microscopic particles of fluid are conceived of as varying continuously in velocity in one direction from zero to v where v is the mean fluid velocity in feet per second. One hypothesis is that turbulent flow exists in which small elemental volumes V of fluid, as a result of viscosity forces, are continually varying in velocity from 0 to v in the direction of main stream macroscopic velocity, this 0 to v acceleration change of velocity being superimposed on the macroscopic velocity. This variation of 0 to v velocity in the direction of flow can exist only if the fluid elements are rotating with a peripheral velocity v.

The picture of turbulent flow with viscosity then emerges on a semi-microscopic scale as that of Fig. .

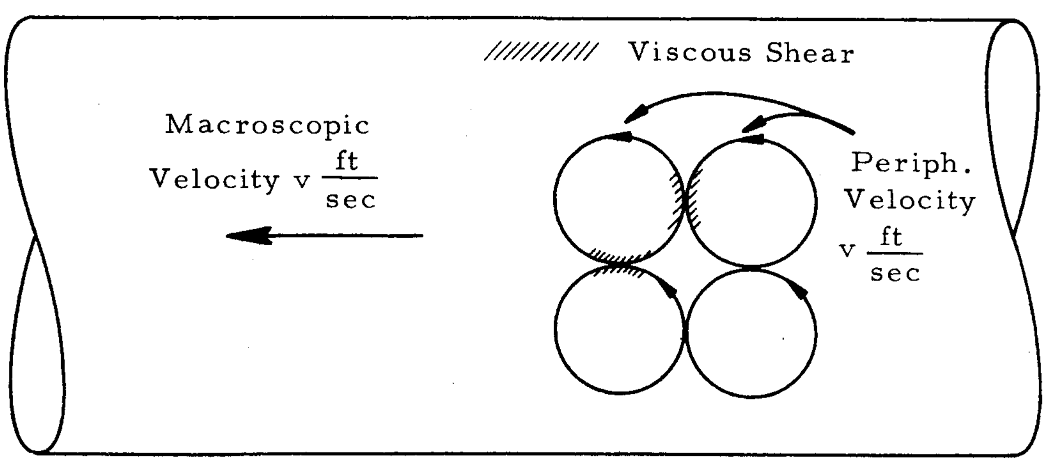


FIG. SEMI-MICROSCOPIC TURBULENT FLOW

Here the fluid is pictured as semi-microscopic elemental cubes or spheres of volume V all rotating in one direction with the viscosity shear resistance forces between elemental volumes requiring the expenditure of work. The rotating cubes or spheres are moving with an overall velocity v.

FLOW DOMAIN. For the flow of fluids ARDA analysis gives

$$Eu = fcn\left(\frac{\theta}{\phi},\ Fr,\ Sh,\ \frac{L}{D},\ Re,\ Ca,\ Fa,\ We\right)$$

Derivation of fluid flow domain. A general equation may be developed for the interaction of an elastic vibrating solid with a fluid. Usual symbols are applicable with the addition of the following special symbols.

θ = angle presented by force of solid to moving fluid

ϕ = angle formed by movement of center of gravity of solid with respect to moving fluid

E = modulus of elasticity

S = shear modulus

f = frequency of vibration or rotation

$$F\ lbf = C(\theta\ deg)^a(\phi\ deg)^b\left(v\ \frac{ft}{sec}\right)^c(m\ lbm)^d(D\ ft)^e(L\ ft)^f$$

$$\left(\rho\ \frac{lbm}{ft^3}\right)^g\left(g\ \frac{ft}{sec^2}\right)^h\left(g_c\ \frac{lbm}{lbf}\ \frac{ft}{sec^2}\right)^i\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)^j$$

$$\left(f\ \frac{1}{sec}\right)^k\left(E\ \frac{lbf}{ft^2}\right)^m\left(S\ \frac{lbf}{ft^2}\right)^n\left(T\ \frac{lbf}{ft}\right)^p$$

deg	$0 = a + b$	$b = -a$
lbf	$1 = -i + j + m + n + p$	$i = j + m + n + p - 1$
sec	$0 = -c - 2h - 2i + j - k$	
	$= -c - 2h - 2j - 2m - 2n - 2p + 2$	
	$+ j - k$	$c = -2h - j - 2m - 2n$
		$-2p + 2 - k$
lbm	$0 = d + g + i$	
	$= d + g + j + m + n + p - 1$	$g = -d - j - m - n - p + 1$

$\underline{ft}$ $\qquad 0 = c + e + f - 3g + h + i - 2j - 2m - 2n - p$

$$0 = (-2h - j - 2m - 2n - 2p + 2 - k) + e + f$$

$$+ (3d + 3j + 3m + 3n + 3p - 3) + h$$

$$+ (j + m + n + p - 1) - 2j - 2m - 2n - p$$

$$= -h + j + 3d - 2 - k + e + f + p \qquad f = h - j - 3d + 2 + k - e - p$$

$$F = c(\theta)^a (\phi)^{-a} (v)^{-2h - j - 2m - 2n - 2p + 2 - k} (m)^d (D)^e$$

$$(L)^{h - j - 3d + 2 + k - e - p} (\rho)^{-d - j - m - n - p + 1}$$

$$(g)^h (g_c)^{j + m + n + p - 1} (\mu)^j (f)^k (E)^m (S)^n (T)^p$$

$$\frac{F g_c}{v^2 \rho L^2} = C\left(\frac{\theta}{\phi}\right)^a \left(\frac{Lg}{v^2}\right)^{-h} \left(\frac{m}{L^3 \rho}\right)^d \left(\frac{LN_s}{v}\right)^k \left(\frac{D}{L}\right)^e \left(\frac{g_c \mu}{\rho L v}\right)^j \left(\frac{g_c E}{v^2 \rho}\right)^m$$

$$\left(\frac{g_c S}{v^2 \rho}\right)^n \left(\frac{T g_c}{\rho v^2 L}\right)^p$$

$$(Eu)^q = C\left(\frac{\theta}{\phi}\right)^a (Fr)^h \left(\frac{m}{\rho L^3}\right)^d (Sh)^k \left(\frac{D}{L}\right)^e (Re)^j (Ca)^m (Fa)^n (We)^p$$

where

(Eu), (Re), etc. are dimensionless numbers.

(Eu) has been replaced by $(Eu)^q$.

The expression $\frac{m}{\rho L^3}$ is obviously a statement that $\frac{\left(\frac{m}{L^3}\right)}{\rho} = 1$ or $\rho = \frac{m}{L^3}$ which can be dropped as unnecessary.

The preceding equation contains a number of terms because of its comprehensive general nature. In any specific application, dimensions not applicable are omitted by considering that the exponent is zero or mathematically: $(\text{property})^0 = 1$. For example: for incompressible fluids the effects of elasticity are omitted by setting $(E)^m = (E)^0 = 1$, also $(Ca)^m = (Ca)^0 = 1$.

In a similar manner any property not included in the preceding general equation may be included by including its proper dimensionless number as discussed under the associative ARDA procedure.

One term frequently added is roughness, included as a dimensionless length number $\frac{e}{D}$ (28, p. 128).

Fluid domain equation by force summation. Fluid flow may be visualized as a macroscopic flow of a vast number of microscopic particles. The motion of the microscopic particles is constantly varying in velocity, the varying force of acceleration or deceleration F_m being provided by the algebraic sum of all forces acting on the fluid particle.

Some of the forces that may act on the fluid particle may be viscous drag force F_μ, force due to pressure F_P, gravity force F_g, surface tension force F_T, elasticity force F_E, shear force F_s, etc.

It was shown under Dimensionless Numbers as Ratios of Forces that each one of these forces may be expressed in a dimensionless number, thus

$$\text{Flow Motion} = f(F_m, F_\mu, F_P, F_g, F_T, F_E, F_s)$$

$$= f\left(\frac{F_m}{F_\mu}, \frac{F_P}{F_m}, \frac{F_m}{F_g}, \frac{F_m}{F_T}, \frac{F_m}{F_E}, \frac{F_m}{F_s}\right)$$

$$= f(\text{Re, Eu, Fr, We, Ca, Fa})$$

If any of these forces is absent, the particular dimensionless parameter is absent. If any other property is present that may affect the motion of the fluid particle, it should be added to the functional expression for properties and its corresponding dimensionless number should be added to the functional expression of dimensionless numbers.

Thus, if angles such as θ and ϕ, frequency N, and significant length parameters such as D are to be included they, along with their corresponding dimensionless numbers are added to form the complete fluid domain equation and the preceding relations become

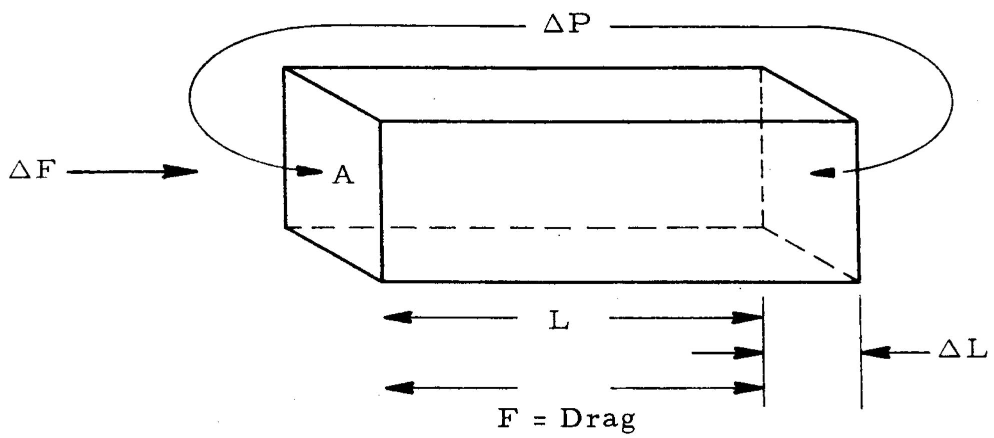
$$\text{Flow Motion} = f(F_m, F_\mu, F_P, F_g, F_T, F_E, F_s, \theta, \phi, N, D)$$

$$\text{Eu} = f\left(\text{Re, Fr, We, Ca, Fa}, \frac{\theta}{\phi}, \text{Sh}, \frac{L}{D}\right)$$

FLOW ENERGY. This energy in ft lbf possessed by a flowing fluid is the work required to push the preceding fluid.

$$FE = FL = (PA)L = P(AL) = PV = \left(P\ \frac{lbf}{ft^2}\right)(V\ ft^3) = (PV)\ ft\ lbf$$

Flow element. Consider the element.



Flow energy change.

$$\Delta FE = \Delta(\text{Flow Energy})$$

$$= \Delta(PV)$$

$$= P\Delta V + V\Delta P$$

$$= PA\Delta L + LA\Delta P$$

$$= F\Delta L + L\Delta F$$

$$= \Delta W + LF$$

$$= \Delta W + \text{Drag Work}$$

where $\Delta W = \Delta$Work to change value of fluid.

Thus, any flow energy decrease is used to overcome drag.

FLUID DRAG. This is also treated under Drag Domain and Fluid Flow Domain. For drag of a viscous fluid only

$$F\ \mathrm{lbf} = C\left(\rho\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)^a \left(\mu_f\ \frac{\mathrm{lbf\ sec}}{\mathrm{ft}^2}\right)^b (L\ \mathrm{ft})^c \left(v\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)^d \left(g_c\ \frac{\mathrm{lbm\ ft}}{\mathrm{lbf\ sec}^2}\right)^e$$

lbf	$1 = b - e$	$e = b - 1$
lbm	$0 = a + e$	
	$0 = a + b - 1$	$a = 1 - b$
sec	$0 = b - d - 2e$	
	$= b - d - 2b + 2 = -b - d + 2$	$d = 2 - b$
ft	$0 = -3a - 2b + c + d - e$	
	$= -3 + 3b - 2b + c + 2 - b - b - 1$	
	$= -2 - b + c$	$c = 2 + b$

$$F = C(\rho)^{1-b} (\mu_f)^b (L)^{2+b} (v)^{2-b} (g_c)^{b-1}$$

$$\frac{F g_c}{\rho L^2 v^2} = C\left(\frac{\mu_f g_c}{\rho v L}\right)^b$$

This is a special case of $\begin{cases}\text{Drag Domain}\\ \text{Fluid Flow Domain}\end{cases}$

$$(Eu) = f(Re)$$

FLUID DYNAMICS NUMBER. This unnamed dimensionless number is encountered in the literature (10).

$$Fdl = \left[\frac{g \mu_f^4 g_c^4}{\rho T^3 g_c^3}\right] = \frac{(We)^3}{(Fr)(Re)^4} = \frac{(Bo)(We)}{(Re)^4}$$

$$\left[\frac{g \mu^4}{\rho T^3 g_c^3}\right] = \frac{(We)^3}{(Fr)(Re)^4 (g_c)^4}$$

$$= \frac{\left[\dfrac{\rho v^2 L}{T g_c}\right]^3}{\left[\dfrac{v^2}{2gL}\right]\left[\dfrac{Dv\rho}{\mu_f g_c}\right]^4 (g_c)^4}$$

$$= \frac{\left[\dfrac{\rho^3 v^6 L^3}{T^3 {g_c}^3}\right]}{\left[\dfrac{v^2}{2gL}\right]\left[\dfrac{D^4 v^4 \rho^4}{{\mu_f}^4 {g_c}^4}\right]{g_c}^4}$$

$$= \left(\frac{\rho^3 v^6 L^3}{T^3 {g_c}^3}\right)\left(\frac{2gL}{v^2}\right)\left(\frac{{\mu_f}^4 {g_c}^4}{D^4 v^4 \rho^4}\right)\left(\frac{1}{{g_c}^4}\right)$$

$$= \left[\frac{g{\mu_f}^4}{\rho T^3 {g_c}^3}\right]$$

FORCE. A force is a "push." Force is required to accelerate or decelerate a mass in accordance with the Newton acceleration law written as a unit-consistent equation in engineering units as:

$$F = \left(\frac{m}{g_c}\right)a$$

$$F \text{ lbf} = \left(\frac{m \text{ lbm}}{32.2 \dfrac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(a \frac{\text{ft}}{\text{sec}^2}\right)$$

The previous equation does not contain weight w or gravity g. For a discussion of weight see under Weight.

FORCE RATIOS. Dimensionless numbers may frequently be interpreted as force ratios. Typical examples follow. If both numerator and denominator are multiplied by distance L they are also FL or energy ratios (see Energy Ratios). Dependent on its nature either a force or energy ratio or some combination may be taken as most descriptive.

Number	Ratio	Formula	Use
Nc	$= \dfrac{\text{(Buoyancy Force)}}{\text{(Viscous Drag Force)}}$	$= \dfrac{L^2 wB\Delta T}{\mu_f v V} = \dfrac{\dfrac{wB\Delta T}{\left(\dfrac{V}{L}\right)\left(\dfrac{v}{L}\right)}}{\mu_f} = \dfrac{\dfrac{F}{A\dfrac{v}{L}}}{\mu_f}$	$\dfrac{\text{Buoyancy Due to } \Delta T}{\text{Drag}}$
Bo	$= \dfrac{\text{(Gravity Force)}}{\text{(Surface Tension Force)}}$	$= \dfrac{w}{(TL)}$	Bubbles

Number		Ratio		Formula	Use
C_D	=	$\frac{(\text{Drag Force})}{(\text{Inertia Force})}$	=	$\frac{2Pg_c}{\rho v^2} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{KE}{V}\right)}$	
Fr	=	$\frac{(\text{Inertia Force})}{(\text{Gravity Force})}$	=	$\frac{v^2}{gL} = \frac{\frac{1}{2}\left(\frac{m}{g_c}\right)\frac{v^2}{L}}{g\left(\frac{m}{g_c}\right)} = \frac{\frac{KE}{L}}{w}$	
Re	=	$\frac{(\text{Inertia Force})}{(\text{Viscous Drag Force})}$	=	$\frac{\rho vL}{\mu_f} = \frac{\left[\frac{F}{A\left(\frac{v}{L}\right)}\right]}{\mu_f}$	Flow Similarity
St	=	$\left(\frac{Fr}{Re}\right) = \frac{(\mu v g_c)}{(\rho R^2 g)}$	=	$\frac{(\text{Viscous Drag Force})}{(\text{Gravity Force})}$	Stokes Law
We	=	$\frac{(\text{Inertia Force})}{(\text{Surface Tension})}$	=	$\frac{(\rho v^2 L)}{(2Tg_c)} = \frac{\left[\frac{\frac{1}{2}\left(\frac{m}{g_c}\right)v^2}{L}\right]}{TL} = \frac{\frac{KE}{L}}{TL}$	

FOURIER NUMBER. This number occurs in transient heat transfer involving conduction and heat capacity.

$$Fo = \left(\begin{array}{c}\text{Fourier Number}\\ \text{Dimensionless}\end{array}\right)$$

$$= \frac{at}{L^2}$$

$$= \left(\frac{k}{\rho C_p}\right)\left(\frac{t}{L^2}\right) = \frac{\left(k\,\frac{\text{Btu ft}}{\text{hr ft}^2\text{ F}}\right)\left(\frac{t\text{ hr}}{L^2\text{ ft}^2}\right)}{\left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(C_p\,\frac{\text{Btu}}{\text{lbm F}}\right)}$$

where a = thermal diffusivity ft^2 /hr.

FROUDE NUMBER. This number expresses the effect of gravity g.

$$Fr = \begin{pmatrix}\text{Froude Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{v^2}{gL} = \frac{v^2 \frac{ft^2}{sec^2}}{\left(g \frac{ft}{sec^2}\right)(L\ ft)}$$

Froude Number has been frequently defined as $\frac{v}{\sqrt{Lg}}$ but this form seems inconsistent with other dimensionless numbers which do not have 1/2 powers, thus, is not a preferred form.

Gravity affects flow in general. Surface waves are a gravity effect.

Froude Number in terms of $\dot{V}$. For flow = $\dot{V}\ \frac{ft^3}{sec}$

$$Fr = \left(\frac{v^2}{gL}\right)\left(\frac{A^2}{A^2}\right) = \frac{(vA)^2}{gL(L^2)^2}$$

$$= \frac{\dot{V}^2}{gL^5} = \frac{\left(\dot{V}\ \frac{ft^3}{sec}\right)^2}{\left(g \frac{ft}{sec^2}\right)(L\ ft)^5}$$

Froude Number in terms of (power). For Power = Fv

$$(Fr)^{3/2}(Eu) = \left(\frac{v^3}{g^{3/2}\ L^{3/2}}\right)\left(\frac{Fg_c}{\rho L^2 v^2}\right)$$

$$= \frac{(Fv)g_c}{\rho L^{7/2} g^{3/2}} = \frac{(\text{Power})\ g_c}{\rho L^{7/2}\ g^{3/2}}$$

Physical significance. Froude Number may be visualized as a measure of the kinetic energy indicated by a velocity resulting from a decrease in potential energy. Potential energy is dependent on gravity g. When this aspect is considered L may be a H so that Froude number may be written as:

$$Fr = \frac{v^2}{gH}$$

Froude Number as an energy ratio.

$$(Fr) = \frac{KE}{\frac{1}{2}(PE)}$$

$$= \frac{\frac{1}{2}\left(\frac{w}{g}\right)v^2}{\frac{1}{2}wH}$$

$$= \frac{v^2}{gH}$$

Froude Number as a force ratio.

$$Fr = \frac{\text{(Force of Acceleration)}}{\text{(Force of Gravity)}} = \frac{\left(\frac{m}{g_c}\right)a}{\left(\frac{m}{g_c}\right)g}$$

$$= \frac{\left(\frac{v}{t}\right)}{g} = \frac{v\left(\frac{L}{t}\right)}{gL} = \frac{v^2}{gL}$$

Froude Number as (We)/(Bo).

$$Fr = \left(\frac{We}{Bo}\right) = \frac{\left(\frac{\rho v^2 L}{Tg_c}\right)}{\left(\frac{w}{TL}\right)} = \frac{\left(\frac{m}{g_c}\right)\frac{v^2 L}{L^3 TL}}{\left(\frac{w}{g}\right)\frac{g}{TL}} = \frac{v^2}{gL}$$

(Eu)(Fr) relationship. For a complete conversion of potential energy to flow energy without other energies involved.

$$\Delta FE = \Delta PE$$

$$\frac{\Delta FE}{\Delta PE} = 1$$

$$\left(\frac{\frac{1}{2}\Delta FE}{KE}\right)\left(\frac{KE}{\frac{1}{2}PE}\right) = 1$$

$$(Eu)(Fr) = 1$$

$$\left(\frac{(\Delta P)\, g_c}{\rho v^2}\right)\left(\frac{v^2}{gH}\right) = 1$$

$$\frac{(\Delta P)}{\left(\frac{m}{V}\right)}\, \frac{g_c}{g}\left(\frac{1}{H}\right) = 1$$

$$\frac{V\Delta P}{wH} = 1$$

[where $P\Delta V = 0$ for incompressible fluid]

Eu/Fr relationship. In the fluid flow equation.

$$(Eu) = C(\quad)(Fr)^b\,(Re)^{-j}(\quad)$$

$$(Eu)\,\frac{1}{(Fr)} = \text{fcn}\,(Re)$$

$$\left(\frac{\frac{1}{2}\,\Delta FE}{KE}\right)\left(\frac{\frac{1}{2}\,PE}{KE}\right) = \text{fcn}\,(Re)$$

Eu as a measure of ΔFE and Fr as a measure of PE produce fluid flow as a function of Re.

GAS LAW NUMBER. The perfect gas law is one of the most used and familiar laws and is expressible in dimensionless number form. A numerical constant is included.

One formulation:

$$\left(P\,\frac{\text{lbf}}{\text{ft}^2}\right)(V\ \text{ft}^3) = (m\ \text{lbm})\ R\left(\frac{\text{ft lb}}{\text{lbm F abs}}\right)(T\ \text{F abs})$$

For M mole (equivalent to dividing each side by moles):

$$\left(P\,\frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{V}{M}\,\frac{\text{ft}^3}{\text{mole}}\right) = \left(\frac{m}{M}\,\frac{\text{lbm}}{\text{mole}}\right)\left(R\ \frac{\text{ft lbf}}{\text{lbm F abs}}\right)(T\ \text{F abs})$$

$$= \left(1545\ \frac{\text{ft lbf}}{\text{mole F abs}}\right)(T\ \text{F abs})$$

where

$$\left(\frac{m \text{ lbm}}{M \text{ mole}}\right) = \text{"molecular weight"}$$

$$M = \text{moles}$$

Thus,

$$Gc = \begin{pmatrix} \text{Dimensionless Number} \\ \text{Expressing Universal} \\ \text{Gas Constant} \end{pmatrix} = 1 = \frac{\left(\frac{m}{M}\right)R}{1545}$$

$$Ga = \begin{pmatrix} \text{Dimensionless Number} \\ \text{Expressing Perfect} \\ \text{Gas Law} \end{pmatrix} = 1 = \left(\frac{mRT}{PV}\right) = \left(1545 \frac{TM}{PV}\right)$$

GRAETZ NUMBER. This dimensionless number occurs in convection heat transfer. It appears redundant in that it is a product of the more basic dimensionless numbers (Re)(Pr) and (D/L).

$$Gz = \begin{bmatrix} \text{Graetz Number} \\ \text{Dimensionless} \end{bmatrix}$$

$$= \left[\frac{\pi}{4}(Pe)\frac{D}{L}\right]$$

$$= \left[\frac{\pi}{4}(Re)(Pr)\frac{D}{L}\right]$$

$$= \left[\frac{\pi}{4}\left(\frac{Dv\frac{m}{V}}{\mu_f g_c}\right)\left(\frac{3600\, C_p \mu_f g_c}{k}\right)\frac{D}{L}\right]$$

$$= \left[\left(\frac{\pi}{4} D^2 v \frac{m}{V}\right)\frac{C_p}{kL}\right]$$

$$= \left(\frac{\dot{m} C_p}{kL}\right) = \left[\frac{\left(\dot{m}\frac{\text{lbm}}{\text{hr}}\right)\left(C_p \frac{\text{Btu}}{\text{lbm F}}\right)}{\left(k \frac{\text{Btu ft}}{\text{hr ft}^2 \text{ F}}\right)(L \text{ ft})}\right]$$

GRASHOF NUMBER. This dimensionless number used in natural convection equations appears to be redundant in that it involves Reynolds Number and another dimensionless number $(B\Delta T) = \left(\frac{\Delta\rho}{\rho}\right)$.

$$Gr = \begin{pmatrix}\text{Grashof Number} \\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{(Re)^2\left(\frac{\Delta\rho}{\rho}\right)}{(Fr)} = \frac{(Re^2)B\,\Delta T}{(Fr)} = \frac{(Re)(Bu)B\Delta T}{(Fr)}$$

$$= \frac{\left(\frac{\rho^2\, v^2\, L^2}{\mu_f^2\, g_c^2}\right)B\Delta T}{\left(\frac{v^2}{gL}\right)}$$

$$= \frac{\rho^2\, L^3\, gB\Delta T}{\mu_f^2\, g_c^2} \qquad \left[\text{where } \mu_f\, g_c = \mu_s = \frac{\mu_m}{3600}\right]$$

$$= \frac{\left(\rho^2\,\frac{lbm^2}{ft^6}\right)(L^3\ ft^3)\left(g\,\frac{ft}{sec^2}\right)\left(B\,\frac{1}{F\ abs}\right)(\Delta T\ F\ abs)}{\left(\mu_f^2\,\frac{lbf^2 sec^2}{ft^4}\right)\left(32.2^2\,\frac{lbm^2\, ft^2}{lbf^2\, sec^4}\right)}$$

GRAVITY CONSTANT. This or similar terms have been applied to several constants which should be clearly distinguished from each other, best done by examining the nature of their units. These constants are:

g = acceleration of gravity, $\frac{ft}{sec^2}$

g_c = accleration constant in $F = \left(\frac{m}{g_c}\right)a$ eq.

$= 32.2\ \frac{lbm\ ft}{lbf\ sec^2}$

G = gravitational attraction constant in $F = \frac{mm}{GR^2}$ eq.

$= 3.01 \times 10^{10}\ \frac{lbm^2}{lbf\ ft^2}$

Standard gravity acceleration g. The International Committee on Weights and Measures (32, p. xvi) has adopted a standard value for the acceleration of gravity on the surface of the earth.

$$g = 32.1740 \frac{ft}{sec^2}$$

$$g = 980.665 \frac{cm}{sec^2}$$

Universal acceleration constant g_c. If the Newton acceleration law is considered to be dimensionally unit-consistent:

$$F = \left(\frac{m}{g_c}\right) a$$

$$F\ lbf = \left(\frac{m\ lbm}{g_c \frac{lbm\ ft}{lbf\ sec^2}}\right)\left(a \frac{ft}{sec^2}\right)$$

In this equation g_c is the acceleration constant with numerical and unit parts. In many systems of units the numerical value of g_c is taken as unity. In the engineering system g_c has a numerical value such that 1 lbm has a weight of 1 lbf. Although no international group has adopted such value it is universally customary to take the numerical value of g_c the same as the numerical value of the standard acceleration of gravity g on the surface of the earth.

Thus,

$$g_c = 32.1740 \frac{lbm\ ft}{lbf\ sec^2}$$

$$g_c = 980.665 \frac{kgm}{kgf} \frac{m}{sec^2}$$

This numerical value of g_c thus selected is thus an unvarying constant throughout the universe.

The Newton law and constant g_c is used in a so called system of units to relate force and mass, primarily because acceleration and deceleration of masses involving forces occur in any process that involves motion, even in uniform fluid flow in which the individual particles are moving with varying velocities.

The Newton law not only relates force to mass and acceleration but it also serves to express mathematically force in terms of mass, length and time units.

Gravity attraction constant. The constant C in the gravity attraction law of two masses m_1 and m_2 here conveniently designated by the same symbol m and m which would be the case if they were of the same or unit size.

$$F = \frac{mm}{GL^2}$$

$$F\ \text{lbf} = \frac{(m\ \text{lbm})(m\ \text{lbm})}{\left(G\ \frac{\text{lbm}^2}{\text{lbf ft}^2}\right)(L^2\ \text{ft}^2)}$$

where

G = gravity attraction constant. Units are selected such as to make the equation dimensionally consistent.

$$= \frac{10^8}{6.670}\ \frac{\text{g mass}^2}{\text{dyne cm}^2} \qquad \text{(Ref. 33, p. 5)}$$

$$= \frac{10^{11}}{6.670}\ \frac{\text{kg mass}^2}{\text{Nm}^2} \qquad \text{(Ref. 33, p. 5)}$$

$$= \frac{\left(\frac{10^{11}}{6.670}\ \frac{\text{kg}^2}{\text{Nm}^2}\right)\left(4.4482216152605\ \frac{\text{N}}{\text{lbf}}\right)\left(0.3048^2\ \frac{\text{m}^2}{\text{ft}^2}\right)}{\left(0.45359237^2\ \frac{\text{kg}^2}{\text{lbm}^2}\right)}$$

$$= 3.01 \times 10^{10}\ \frac{\text{lbm}^2}{\text{lbf ft}^2}$$

The question may arise as to why this very fundamental law was not selected to define force in terms of mass instead of using the Newton F = ma law to establish the various systems of units. The reason is probably historical in that F = ma phenomena are more observable on the surface of the earth than the $F = mm/CL^2$ phenomena observable in the motion of heavenly bodies. There is some inconsistency in not using this mass attraction law because the laws for magnetic pole m and electric charge q repulsion have the same form.

$$F = \frac{mm}{\mu L^2} \text{ and } F = \frac{qq}{\Sigma L^2}$$

These laws are used for the definition of electromagnetic units (see Electromagnetic Units).

HEAT VALUE NUMBER. It is desired to develop a dimensionless number containing the heat release or heat value q.

$$Hv = \begin{pmatrix}\text{Dimensionless}\\ \text{Number}\end{pmatrix}$$

$$= \frac{\left(q \frac{Btu}{lbm}\right)}{(\text{Denominator})}$$

$$= \frac{q \frac{Btu}{lbm}}{\left(C_p \frac{Btu}{lbm\ F}\right)(\Delta T\ F)}$$

$$= \frac{q}{C_p \Delta T}$$

where the denominator has been completed by inspection by inserting the two best known dimensions having the proper units.

HYDRAULIC FLOW. Equations such as that of Poiseuille, D'Arcy and Chezy are obtainable from the ARDA drag equation or flow equation (see Fluid Flow) by retaining the basic parameters likely to be involved, namely Euler for drag, Froude for head and Reynolds for flow.

$$Eu = C(Fr)^a (Re)^{-b} \left(\frac{D}{L}\right)^{-c}$$

Poiseuille laminar flow. Omitting the Fr term as surface is not involved

$$Eu = 32(Re)^{-1} \left(\frac{D}{L}\right)^{-1}$$

$$\left(\frac{\Delta P g_c}{\rho v^2}\right) = 32 \left(\frac{\mu_f g_c}{\rho v D}\right)\left(\frac{L}{D}\right)$$

$$\left(\frac{\Delta P\, D^2}{128\ \mu_f\, L}\right) = \frac{v}{4}$$

$$\left(\frac{\pi\, \Delta P\, D^4}{128\, \mu_f\, L}\right) = \frac{\pi D^2\, v}{4} = \dot{V}$$

$$\dot{V} = \frac{\pi\, \Delta P\, 8R^3}{128\ \mu_f}\left(\frac{D}{L}\right) = \frac{\pi}{128}\ \frac{\left(\Delta P\ \frac{\text{lbf}}{\text{ft}^2}\right) 8R^3\ \text{ft}^3}{\left(\mu_f\ \frac{\text{lbf sec}}{\text{ft}^2}\right)}\left(\frac{D\ \text{ft}}{L\ \text{ft}}\right) \qquad \text{(Ref. 24, p. 16)}$$

also,

$$\frac{\Delta P}{\left(\frac{w}{V}\right)} = \frac{32(\mu_f\, Lv)}{\left(\frac{w}{V}\right)D^2} \qquad \text{(Ref. 24, p. 100)}$$

$$\Delta H = \frac{32\ \ \mu_f\, Lv}{\left(\frac{w}{V}\right) D^2} = \frac{32\left(\mu_f\ \frac{\text{lbf sec}}{\text{ft}^2}\right)(L\ \text{ft})\left(v\ \frac{\text{ft}}{\text{sec}}\right)}{\left(\frac{w}{V}\ \frac{\text{lbf}}{\text{ft}^3}\right)(D^2\ \text{ft}^2)} \qquad \text{(Ref. 30, p. 98)}$$

where

$\dot{V}$ = volume flow rate, ft^3/sec

ΔH = head loss, ft.

Friction factor f for laminar flow. To obtain the D'Arcy equation in terms of a head H the flow equation is used with the Froude number containing H rather than the Eu containing ΔP. The constant C is the same as for the Poiseuille equation.

$$1 = 32(Fr)^h\,(Re)^{-1}\left(\frac{L}{D}\right)$$

$$\frac{1}{Fr} = \frac{32}{Re}\left(\frac{L}{D}\right) \qquad \text{(A)}$$

$$\frac{gH}{v^2} = \frac{32}{Re}\left(\frac{L}{D}\right)$$

$$H = \left(\frac{64}{Re}\right)\left(\frac{L}{D}\right)\frac{v^2}{2g} \qquad \text{(Ref. 30, p. 98)}$$

$$= f\left(\frac{L}{D}\right)\left(\frac{v^2}{2g}\right) = f\left(\frac{L\ \text{ft}}{D\ \text{ft}}\right)\left(\frac{v^2\ \frac{\text{ft}^2}{\text{sec}^2}}{64.4\ \frac{\text{ft}}{\text{sec}^2}}\right)$$

where

$$f = \text{friction factor} \qquad \left[\begin{array}{c}\text{By definition}\\ \text{of } f\end{array}\right] \qquad \text{(Ref. 30, p. 99)}$$

$$= \frac{64}{Re} = \frac{2L}{D}(Fr) \qquad \text{(From Eq. A)}$$

<u>Chezy laminar channel flow</u>. Using the same equation as was used in the derivation of f:

$$1 = 32\,(Fr)(Re)^{-1}\left(\frac{D}{L}\right)^{-1}$$

$$\left(\frac{Re}{32}\right)\left(\frac{1}{Fr}\right)\left(\frac{D}{L}\right) = 1$$

$$1 = \frac{Re}{32}\left(\frac{gH}{v^2}\right)\frac{D}{L}$$

$$= \frac{Re}{8}\left(\frac{gD}{v^2 4}\right)\frac{H}{L}$$

$$= \frac{Re}{8}\left(\frac{g}{v^2}\,\frac{\pi D^2}{4\pi D}\right)\frac{H}{L}$$

$$= \frac{Re}{8}\left(\frac{g}{v^2}\right)\left(\frac{A}{P}\right)\frac{H}{L} \qquad \text{(where P = perimeter)}$$

$$v^2 = \frac{g}{8}\,Re\left(\frac{A}{P}\right)\frac{H}{L}$$

$$v = \sqrt{\frac{8g}{\left(\frac{64}{Re}\right)}\left(\frac{A}{P}\right)\frac{H}{L}} \qquad \text{(Ref. 30, p. 163)}$$

$$= \sqrt{\frac{8g}{f}\left(\frac{A}{P}\right)\frac{H}{L}} \qquad \text{(ref. 24, p. 101)}$$

$$= C\sqrt{R\,\frac{H}{L}}$$

where

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8g}{\left(\frac{64}{Re}\right)}} = \sqrt{\frac{g}{8}\,Re} = 2\sqrt{Re}$$

$$R = \text{hydraulic radius} = \text{hydraulic depth} = \frac{\text{flow area}}{\text{wetted perimeter}}$$

$$\frac{H}{L} = \text{channel slope (ft drop per ft length)}$$

<u>Friction factor f turbulent flow</u>. Experiment (30, p. 99) indicates

at moderate Re: $f = fcn\left(\frac{e}{D},\ Re\right)$ for rough tubes

$f = fcn\,(Re)$ for smooth tubes

at high Re: $f = fcn\left(\frac{e}{D}\right)$

where $\frac{e}{D}$ = roughness, ft height per ft tube diameter.

<u>INERTIA</u>. This is a property of mass such that it tends to resist change of velocity.

<u>Inertia force</u>. The force required to change velocity 0 to v or v to 0 depends on the mass, the velocity and the distance L.

$$F = \frac{m}{g_c}\,a = \frac{m}{g_c}\left(\frac{v - 0}{t}\right) \quad \text{where } L = v_{avg}\,t = \frac{v}{2}\,t \text{ or } t = \frac{2L}{v}$$

$$= \frac{m}{g_c}\,\frac{v}{\left(\frac{2L}{v}\right)}$$

$$= \frac{1}{2}\left(\frac{m}{g_c}\right)\frac{v^2}{L} = \frac{1}{2}\,\frac{(m\ \text{lbm})\left(v^2\ \frac{\text{ft}^2}{\text{sec}^2}\right)}{\left(32.2\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)(L\ \text{ft})}$$

$$= \frac{1}{2}\left(\frac{w}{g}\right)\frac{v^2}{L} = \frac{1}{2}\,\frac{(w\ \text{lbm})\left(v^2\ \frac{\text{ft}^2}{\text{sec}^2}\right)}{\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)(L\ \text{ft})}$$

Inertia or kinetic energy. The work energy required to accelerate a mass from 0 to v over a distance is:

$$KE = FL$$

$$= \frac{m}{g_c} aL = \frac{m}{g_c}\left(\frac{v - 0}{t}\right)L = \frac{m}{g_c} \frac{v}{\left(\frac{2L}{v}\right)} L$$

$$= \frac{1}{2}\left(\frac{m}{g_c}\right)v^2 = \frac{1}{2}\left(\frac{m \text{ lbm}}{32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)$$

$$= \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(\frac{w \text{ lbf}}{32.2 \frac{\text{ft}}{\text{sec}^2}}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)$$

Inertia or velocity pressure. The force per unit area or pressure produced by a fluid changing in velocity from 0 to v depends on the density and the velocity.

$$P = \frac{F}{A} = \frac{\left(\frac{m}{g_c}\right)a}{A} = \frac{\left(\frac{m}{V}\right) AL\left(\frac{v - 0}{t}\right)}{g_c A} = \frac{\rho L v}{g_c \left(\frac{2L}{v}\right)}$$

$$= \frac{1}{2}\left(\frac{\rho}{g_c}\right)v^2 = \frac{1}{2}\,\frac{\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)}{\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}$$

$$= \frac{1}{2}\,\frac{\left(\frac{w}{V}\right)}{g}\,v^2 = \frac{1}{2}\,\frac{\left(\frac{w}{V}\frac{\text{lbf}}{\text{ft}^3}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)}{\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)}$$

Velocity head. The inertia pressure produced by a fluid in being brought to rest may be equated to the pressure produced by a column of fluid of density ρ under gravity action.

$$\frac{1}{2}\left(\frac{\rho}{g_c}\right)v^2 = \frac{1}{2}\left(\frac{w}{V}\right)\frac{v^2}{g} = \left(\frac{w}{V}\right)h \qquad \left[\left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right)\right]$$

$$h = \frac{v^2}{2g} = \frac{v^2 \frac{\text{ft}^2}{\text{sec}^2}}{2\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)}$$

This head produced depends only on the velocity and gravity. It could be termed an inertia head.

INERTIA FORCE. This is the force F required to accelerate a mass m from 0 to v over a distance L during time t. It may also be defined as a deceleration force or drag.

$$F = \frac{m}{g_c} a \qquad \text{where } a = \left(\frac{v - 0}{t}\right)$$

$$= \frac{m}{g_c}\left(\frac{v - 0}{t}\right) \qquad \text{where } L = \left(\frac{v + 0}{2}\right) t$$

$$\text{or } t = \frac{2L}{v}$$

$$= \frac{m}{g_c} \frac{v}{\left(\frac{2L}{v}\right)}$$

$$= \frac{1}{2}\left(\frac{m}{g_c}\right) \frac{v^2}{L} = \frac{KE}{L}$$

$$= \frac{1}{2}\left(\frac{w}{g}\right) \frac{v^2}{L} = \frac{KE}{L}$$

$$= \frac{1}{2}\left(\frac{m}{V\, g_c}\right) V \frac{v^2}{L}$$

$$= \frac{1}{2} \rho v^2 A = \frac{1}{2}\left(\frac{w}{gV}\right) v^2 A$$

$$KE = FL$$

$$= \left[\frac{1}{2}\left(\frac{m}{g_c}\right)\frac{v^2}{L}\right] L$$

$$= \frac{1}{2}\left(\frac{m}{g_c}\right) v^2 = \frac{1}{2}\left(\frac{w}{g}\right) v^2$$

JOULE. A joule is a metric energy unit having aspects defining heat, work and energy.

Conversion factors. (The symbol J_m will be used to define all forms involving joule or metric units.)

$$J_m = \left(1 \frac{\text{joule}}{\text{Nm}}\right) = \left(1 \frac{\text{joule}}{\text{amp volt sec}}\right) = \left(1 \frac{\text{Nm}}{\text{amp volt sec}}\right)$$

$$1 = \left(1 \frac{\text{joule}}{\text{watt sec}}\right) \qquad 1 = \left(1055 \frac{\text{joule}}{\text{Btu}}\right)$$

Joule as a heat unit. The metric system defines the numerical value of Jm as 1 compared to a numerical value of J in the engineering system of 778 (see Mechanical Equivalent of Heat).

$$J_m = \left(1 \frac{\text{Nm}}{\text{joule}}\right) \qquad J = \left(778 \frac{\text{ft lbf}}{\text{Btu}}\right)$$

$$J_m = \frac{1}{1.3558179 \dfrac{\text{joule}}{\text{ft lbf}}} = 0.738 \frac{\text{ft lbf}}{\text{joule}} \qquad \text{(Ref. 33, p. 14)}$$

Joule as a work unit. A joule is the work of 1 newton N acting through a distance of 1 meter m.

$$(1 \text{ joule}) = 1 \text{ Nm}$$

Joule as an electrical unit. A joule is used to define electrical units such that one joule of energy is required for the electrical work of moving one coulomb (= volt sec) of charge through a potential difference of 1 volt.

$$1 \text{ joule} = 1 \text{ coulomb sec}$$

$$= 1 \text{ amp volt sec}$$

Joule per sec as a watt. The watt is a joule per sec (23, p. 19).

$$1 \text{ watt} = 1 \frac{\text{joule}}{\text{sec}}$$

The following conversion factors are applicable.

$$1 = 1 \frac{(\text{watt})}{\left(\dfrac{\text{joule}}{\text{sec}}\right)} = \left(1 \frac{\text{watt sec}}{\text{joule}}\right) = \left(1 \frac{\text{watt}}{\text{amp volt}}\right)$$

$$1 = \left(3413 \frac{\text{Btu}}{\text{kw hr}}\right) = \left(3.413 \frac{\text{Btu}}{\text{watt hr}}\right)$$

$$1 = \left(\frac{3600 \frac{\text{sec}}{\text{hr}}}{3.413 \frac{\text{Btu}}{\text{watt hr}}}\right) = \left(1053 \frac{\text{watt sec}}{\text{Btu}}\right)$$

$$1 = \left(3.413 \frac{\text{Btu}}{\text{watt hr}}\right)\left(778 \frac{\text{ft lbf}}{\text{Btu}}\right) = \left(3660 \frac{\text{ft lbf}}{\text{watt hr}}\right)$$

KINETIC ENERGY. The kinetic energy of a mass moving at a final velocity v is obtained as the work equal force times distance to uniformly accelerate the mass.

Proof:

$$KE = W = FL$$

$$= \left(\frac{ma}{g_c}\right) L$$

$$= \frac{m}{g_c}\left(\frac{v - 0}{t}\right) L \qquad \text{where } L = v_{avg}\, t = \left(\frac{v}{2}\right) t$$

$$\text{or } t = 2\,\frac{L}{v}$$

$$= \frac{m}{g_c} \frac{vL}{\left(\frac{2L}{v}\right)}$$

$$= \frac{1}{2}\left(\frac{m}{g_c}\right) v^2 \text{ ft lbf} = \frac{(m \text{ lbm})\left(v \frac{\text{ft}}{\text{sec}}\right)^2}{2\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}$$

$$= \frac{1}{2}\left(\frac{w}{g}\right) v^2 \text{ ft lbf} = \frac{(w \text{ lbf})\left(v \frac{\text{ft}}{\text{sec}}\right)^2}{2\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)}$$

where

$$\left(\frac{m}{g_c}\right) = \left(\frac{w}{g}\right)$$

Alternate proof.

$$KE = W = FL$$

$$= \frac{ma}{g_c} L$$

$$= \frac{m}{g_c}\left(\frac{v - 0}{t}\right) L$$

$$= \frac{m}{g_c} v \left(\frac{L}{t}\right)$$

$$= \frac{m}{g_c} v \, (v_{avg})$$

$$= \frac{m}{g_c} v \left(\frac{v}{2}\right)$$

$$= \frac{m}{g_c} v^2$$

KNUDSEN NUMBER. In rarefied-gas dynamics, flow patterns are determined primarily by the Knudsen Number (4, p. 199; 24, p. 83).

$$Kn = \left(\begin{matrix}\text{Knudsen Number} \\ \text{Dimensionless}\end{matrix}\right)$$

$$= \frac{\text{(Ratio Mean Free Path Distance of Molecular)}}{\text{(Length Dimension of System)}}$$

$$= \left(\frac{\lambda}{L}\right) = \left(\frac{\lambda \text{ ft}}{L \text{ ft}}\right)$$

LEWIS NUMBER. This dimensionless parameter involving diffusion is encountered in mass transfer. It is also known as the Semenov Number. It is a redundant dimensionless number in that it is equal to the basic number ratio Prandtl/Schmidt.

$$Le = \left(\begin{matrix}\text{Lewis Number} \\ \text{Dimensionless}\end{matrix}\right)$$

$$= \frac{(Pr)}{(Sc)} = \frac{\left(\frac{C_p \mu_m}{k}\right)}{\left(\frac{\mu_m}{\rho D_m}\right)}$$

$$= \frac{\rho C_p D_m}{k} = \frac{\left(\rho \frac{lbm}{ft^3}\right)\left(C_p \frac{Btu}{lbm\ F}\right)\left(D_m \frac{ft^2}{hr}\right)}{\left(k \frac{Btu\ ft}{ft^2\ hr\ F}\right)}$$

$$= \frac{D_m}{\left(\frac{k}{\rho C_p}\right)} = \frac{D_m}{a} = \frac{\text{(Mass Diffusivity)}}{\text{(Thermal Diffusivity)}}$$

LIMITATIONS. See Sufficiency.

MACH NUMBER. This dimensionless number refers an actual velocity to a base or reference velocity usually a sound velocity.

$$Ma = \left(\begin{array}{c}\text{Mach Number}\\ \text{Dimensionless}\end{array}\right)$$

$$= \left(\frac{v \frac{ft}{sec}}{v_s \frac{ft}{sec}}\right)$$

where v_s frequently equals sound or acoustic velocity, ft/sec.

$$= \sqrt{g_c kRT}$$

$$= \sqrt{g_c k \left(\frac{PV}{m}\right)} \qquad \text{for a perfect gas}$$

$$= \sqrt{\frac{kP}{\left(\frac{m}{Vg_c}\right)}}$$

$$= \sqrt{\frac{kP}{\rho}}$$

$$k = \text{specific heat ratio, } \frac{C_p}{C_v}.$$

Referred to the last equation for v_s it appears that Mach Number is related to a pressure-density ratio, perhaps an elasticity aspect.

Ma as a criteria. For compressible fluid flow Mach number represents a criteria between flow regimes. Below Mach one the flow is subsonic. Near and at Mach one equal to the velocity of sound the flow is transonic. Above Mach one the flow is supersonic. Above Mach five the flow is hypersonic.

Ma in dimensional analysis. In dimensional analysis where a property depends on two velocities v_1 and v_2:

$$\text{Properties} = f(\ldots . v_1,\ v_2 \ldots .)$$

if the result is an expression of dimensionless numbers a (v_1/v_2) term will always be obtained where v_2 will be the velocity of sound if v_2 in the original expression is the velocity of sound. However, v_2 may be some other velocity such as ship or object velocity. In that case (v_1/v_2) is a velocity ratio. This will be termed Mach number. Thus in dimensional analysis Mach number is a ratio of velocities in which the reference velocity may or may not be the velocity of sound depending on the nature of the problem.

Ma in terms Eu.

$$(\text{Ma})^2 = \frac{v^2}{g_c kRT}$$

$$= \frac{1}{k\left(\frac{g_c PV}{v^2 m}\right)}$$

$$= \frac{1}{k\left(\frac{g_c P}{v^2 \rho}\right)} = \frac{\rho v^2}{k(Pg_c)} = \frac{\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)}{k\left(P \frac{\text{lbf}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}$$

$$= \frac{1}{k(\text{Eu})}$$

MAGNETIC PRANDTL NUMBER. This fluid property resembling the Prandtl Number has been mentioned in the electromagnetic literature.

$$\text{Pm} = \left(\begin{array}{c}\text{Magnetic Prandtl Number}\\ \text{Dimensionless}\end{array}\right)$$

$$\frac{(Rm)}{(Re)} = \frac{\text{(Magnetic Reynolds Number)}}{\text{(Reynolds Number)}}$$

$$= \frac{\left(\frac{\mu_p \sigma L v}{J_m}\right)}{\left(\frac{\rho v L}{\mu_f g_c}\right)} = \left(\frac{\mu_p \sigma \mu_f g_c}{J_m \rho}\right)$$

$$= \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}\right)\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)}$$

MAGNETIC REYNOLDS NUMBER. This dimensionless number expresses the properties of an electromagnetic field.

$$Rm = \left(\begin{array}{c}\text{Magnetic Reynolds Number} \\ \text{Dimensionless}\end{array}\right)$$

$$= \left(\frac{\mu_p \sigma L v}{J_m}\right)$$

$$= \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)(L \text{ ft})\left(v \frac{\text{ft}}{\text{sec}}\right)}{\left(0.738 \frac{\text{ft lbf}}{\text{amp volt sec}}\right)}$$

where

μ_p = magnetic permeability, lbf/amp^2

σ = electric conductivity, amp/volt ft

L = length, ft

v = velocity, ft/sec

J_m = conversion factor = 0.738 ft lbf/joule = 0.738 ft lbf/amp volt sec

$$= \left(778 \frac{\text{ft lbf}}{\text{Btu}}\right)\left(3.414 \frac{\text{Btu}}{\text{amp volt hr}}\right)\left(\frac{1}{3600} \frac{\text{hr}}{\text{sec}}\right)$$

where 1 watt = 1 amp volt.

Rm in terms of Re. Magnetic Reynolds Number Rm is also expressible in terms of Reynolds Number and two other dimensionless numbers (Ref. 24, p. 121).

$$Rm = (Re)^{-1}\,(Em\ 3)(Ha)$$

$$= \left[\frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)(L\ \text{ft})\left(v \frac{\text{ft}}{\text{sec}}\right)}{\left(J_m \frac{\text{ft lbf}}{\text{amp volt sec}}\right)}\right]$$

$$= \left[\frac{\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)\left(v \frac{\text{ft}}{\text{sec}}\right)(L\ \text{ft})}\right]$$

$$\left[\frac{\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)\left(v^2 \frac{\text{ft}^2}{\text{sec}^2}\right)}{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(H^2 \frac{\text{amp}^2}{\text{ft}^2}\right)\left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}\right]$$

$$\left[\frac{\left(\mu_p^2 \frac{\text{lbf}^2}{\text{amp}^4}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)\left(H^2 \frac{\text{amp}^2}{\text{ft}^2}\right)(L^2\ \text{ft}^2)}{\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(J_m \frac{\text{ft lbf}}{\text{amp volt sec}}\right)}\right]$$

where

$$Re = \text{Reynolds Number} = \left(\frac{\rho v L}{\mu_f g_c}\right)$$

$$Em3 = \text{Electromagnetic Number 3} = \left(\frac{\mu_p H^2 g_c}{\rho v^2}\right)$$

$$Ha = \text{Hartmann Number} = \left(\frac{\mu_p^2 \sigma H^2 L^2}{\mu_f J_m}\right)$$

Magnetic Re in terms of current density.

$$Rm = \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)\left(\sigma \frac{\text{amp}}{\text{volt ft}}\right)(L\ \text{ft})\left(v \frac{\text{ft}}{\text{sec}}\right)}{\left(J_m \frac{\text{ft lbf}}{\text{amp volt sec}}\right)}$$

$$= \frac{\mu_p \sigma L}{J_m}\left(\frac{L}{t}\right)\left(\frac{E}{E}\right)$$

$$\left(\frac{E}{H}\right) = \text{ratio}\ \frac{\text{(electric field strength, volt/ft)}}{\text{(magnetic field strength, amp/ft)}} = \frac{E}{H}\,\frac{\text{volt}}{\text{amp}}$$

L = length, ft

v = velocity, ft/sec

ρ = mass density, lbm/ft^3

μ_p = magnetic permeability, lbf/amp

J_m = conversion constant factor, work to joule electrical units

$$= 0.738\ \frac{\text{ft lbf}}{\text{joule}} = 0.738\ \frac{\text{ft lbf}}{\text{amp volt sec}}$$

g_c = conversion constant factor = $32.2\ \dfrac{\text{lbm ft}}{\text{lbf sec}^2}$

$$F\ \text{lbf} = C\left(\sigma\ \frac{\text{amp}}{\text{volt ft}}\right)^a \left(\epsilon\ \frac{\text{amp}^2\ \text{sec}^2}{\text{lbf ft}^2}\right)^b \left(\frac{E}{H}\,\frac{\text{volt}}{\text{amp}}\right)^c (L\ \text{ft})^d \left(v\ \frac{\text{ft}}{\text{sec}}\right)^e$$

$$\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)^f \left(\mu_p\ \frac{\text{lbf}}{\text{amp}^2}\right)^g \left(J_m\ \frac{\text{ft lbf}}{\text{amp volt sec}}\right)^h \left(g_c\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^i$$

<u>volt</u> $0 = -a + c - h$ $\qquad h = c - a$

<u>amp</u> $0 = a + 2b - c - 2g - h$

$= a + 2b - c - 2g - c + a$

$= 2a + 2b - 2c - 2g$ $\qquad g = a + b - c$

<u>lbf</u> $1 = -b + g + h - i$

$= -b + a + b - c + c - a - i$ $\qquad i = -1$

<u>lbm</u> $0 = f + i = f - 1$ $\qquad f = 1$

<u>sec</u> $0 = 2b - e - h - 2i$

$= 2b - e - c + a + 2$ $\qquad e = 2b + a - c + 2$

$$= \left(\frac{\mu_p L^2}{J_m t E}\right)\left(\frac{E}{\sigma}\right)$$

$$= \left(\frac{\mu_p L^2}{J_m t E}\right)\left(\frac{I}{L^2}\right)$$

$$= \frac{\left(\mu_p \frac{\text{lbf}}{\text{amp}^2}\right)(L^2\ \text{ft}^2)\left(\frac{I\ \text{amp}}{L^2\ \text{ft}^2}\right)}{\left(0.738\ \frac{\text{ft lbf}}{\text{volt amp sec}}\right)(t\ \text{sec})\left(E\ \frac{\text{volt}}{\text{ft}}\right)}$$

where

E = electric field intensity, volt/ft

σ = electric conductivity, amp/volt ft

$\left(\frac{I}{L^2}\right)$ = electric current density, amp/ft^2

$$= \left(E\ \frac{\text{volt}}{\text{ft}}\right)\left(\sigma\ \frac{\text{amp}}{\text{volt ft}}\right)$$

<u>MAGNETOHYDRODYNAMIC DOMAIN</u>. ARDA analysis gives for MHD

$$Eu = \text{fcn}\,(Rm, Em\ 1, Em\ 2) \quad \text{where } Rm = \text{fcn}\,(Re, Em, Ha)$$

<u>Derivation of MHD domain</u>. For the MHD domain occurring with the motion of an electrically conducting fluid in an electromagnetic field (24, p. 119).

$$F = \text{fcn}\,\left(\sigma, \epsilon, \frac{E}{H}, L, v, \rho, \mu_p, J_m, g_c\right)$$

where

F = force on fluid, gas or liquid, lbf

ϵ = electrical permittivity, $\text{amp}^2\ \text{sec}^2/\text{lbf ft}^2$

σ = electrical conductivity, amp/volt ft

$$\underline{ft} \qquad 0 = -a - 2b + d + e - 3f + h + i$$

$$= -a - 2b + d + 2b + a - c - 2 - 3 + c - a - 1$$

$$= -a + d - 2 \qquad\qquad d = a + 2$$

$$F \text{ lbf} = C(\sigma)^a(\epsilon)^b\left(\frac{E}{H}\right)^c (L)^{a+2}(v)^{2b+a-c+2}(\rho)^1$$

$$(\mu_p)^{a+b-c}(J_m)^{c-a}(g_c)^{-1}$$

$$\left(\frac{Fg_c}{\rho L^2 v^2}\right) = \left(\frac{\mu_p \sigma L v}{J_m}\right)^a (\mu_p \epsilon v^2)^b \left(\frac{J_m E}{\mu_p H v}\right)^c$$

$$Eu = (Rm)^a (Em\ 1)^b (Em\ 2)^c$$

This equation applies also to the electromagnetic domain of Eu = 1. This is transformable into an alternate form.

$$Eu = (Re)^{-1}(Em\ 3)(Ha)(Em\ 2)(Em\ 3)$$

where

Eu = Euler Number dimensionless

Rm = Magnetic Reynolds Number, dimensionless

Re = Reynolds Number = $\left(\frac{\rho v L}{\mu_f g_c}\right)$

Em 3 = Electromagnetic Number 3 = $\left(\frac{\mu_p H^2 g_c}{\rho v^2}\right)$

Ha = Hartmann Number = $\left(\frac{\mu_p^2 \sigma H^2 L^2}{\mu_f g_c J_m}\right)$

μ_f = viscosity, $\frac{\text{lbf sec}}{\text{ft}^2}$

MASS DIFFUSIVITY. See under Diffusivity.

MECHANICAL EQUIVALENT OF HEAT. This is a conversion factor designated as J. An exact numerical value has not been adopted in the engineering system of units, primarily because of the existence of several definitions of Btu such as the International steam table IT Btu, the thermochemical Btu, etc. The numerical value 778 is customarily used as equivalent to the approximate numerical value obtained by the conversion factors indicated below (Ref. 33).

$$J = \frac{\left(1055.04 \frac{\text{joule}}{\text{IT Btu}}\right)}{\left(1.3558179 \frac{\text{joule}}{\text{ft lbf}}\right)}$$

$$= \left(778 \frac{\text{ft lbf}}{\text{Btu}}\right)$$

$$J = \frac{\left(1 \frac{\text{Newton m}}{\text{joule}}\right)\left(1054.350264488888 \frac{\text{joule}}{\text{thermochemical Btu}}\right)}{\left(4.4482216152605 \frac{\text{Newton}}{\text{lbf}}\right)\left(0.3048 \frac{\text{m}}{\text{ft}}\right)}$$

$$= \left(778 \frac{\text{ft lbf}}{\text{Btu}}\right)$$

The numerical values are given to many places in the preceding to emphasize that while the numerical value is not as exact as 1.00 would be, it is determinable to considerable accuracy.

Thermodynamically J is not reversible, in that mechanical work in ft lbf can be fully transformed into heat but heat cannot be fully transformed into mechanical energy. For this reason J does not usually enter into dimensional analysis. On the other hand the acceleration law as expressed in g_c is reversible so that g_c does enter into dimensional analysis particularly in fluid flow problems.

The symbol J_m will be used for the metric conversion factor (33, p. 14). See Joule.

$$J_m = \left(\frac{1}{1.3558179 \frac{\text{joule}}{\text{ft lbf}}}\right) = 0.738 \frac{\text{ft lbf}}{\text{joule}}$$

METRIC SYSTEM. The Systeme International or SI metric system adopted by the U. S. Bureau of Standards 1964 as a preferred system, is outlined as follows, in comparison to the U.S. engineering system. The SI metric system is also used by other agencies such as the NASA Marshall Space Flight Center (Ref. 33) which suggests that if other units are used in reports, the equivalent SI units shall follow in parenthesis.

Metric prefix conversion factors. Those in typical use are:

$$\left(\frac{10^6 \text{ meters}}{\text{megameter}}\right) = \left(\frac{10^6 \text{ m}}{\text{Mm}}\right)$$

$$\left(\frac{1000 \text{ meters}}{\text{kilometer}}\right) = \left(\frac{10^3 \text{ m}}{\text{km}}\right)$$

$$\left(\frac{10 \text{ decimeters}}{\text{meter}}\right) = \left(\frac{10 \text{ dm}}{\text{m}}\right)$$

$$\left(\frac{100 \text{ centimeters}}{\text{meter}}\right) = \left(\frac{100 \text{ cm}}{\text{m}}\right)$$

$$\left(\frac{1000 \text{ millimeters}}{\text{meter}}\right) = \left(\frac{1000 \text{ mm}}{\text{m}}\right)$$

$$\left(\frac{10^6 \text{ microseconds}}{\text{second}}\right) = \left(\frac{\mu \text{ sec}}{\text{m}}\right)$$

$$\left(\frac{10^9 \text{ nanoseconds}}{\text{second}}\right) = \left(\frac{\text{n sec}}{\text{m}}\right)$$

Metric system advantages. The SI metric system possesses advantages in that the numerical values of g_c and J are unity rather than larger numerals as in the equivalent engineering system. Thus

$$g_c = \left(1 \, \frac{\text{kgm m}}{\text{newton sec}^2}\right) \quad \text{Versus} \quad g_c = \left(32.2 \, \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)$$

$$J = \left(1 \, \frac{\text{newton m}}{\text{joule}}\right) \quad \text{Versus} \quad J = \left(778 \, \frac{\text{ft lbf}}{\text{Btu}}\right)$$

Also the SI metric system uses multiples of ten rather than varied multipliers. For example:

$$\left(10 \, \frac{\text{mm}}{\text{cm}}\right) \quad \text{Versus} \quad \left(12 \, \frac{\text{in.}}{\text{ft}}\right)$$

$$\left(1000 \, \frac{\text{km}}{\text{m}}\right) \quad \text{Versus} \quad \left(5280 \, \frac{\text{ft}}{\text{mile}}\right)$$

Thus, some calculations will be numerically simpler in the SI metric system. No simplification is achieved in equations using the acceleration of gravity. Compare:

$$g = \left(9.81 \frac{m}{sec^2}\right) \qquad \text{Versus} \qquad \left(32.2 \frac{ft}{sec^2}\right)$$

Equations such as V_e = (escape velocity) = $\sqrt{2g\ R_o}$ involve a similar amount of calculation in either system.

Metric disadvantage of newton as a weight or force unit. The SI metric system possesses some disadvantages in specifying the newton as the force or weight unit as compared to the lbf downward (equal to lbweight) in the engineering system. The SI metric system recognizes the newton as the only force unit and does not use kgf as a force unit. In ordinary experience 1 lbf is measured as the gravity force on 1 lbm on the surface of the earth. Ordinarily the newton is not actually measured by accelerating 1 kg at a rate of 1 meter per sec in accordance with its definition. Thus, it is more difficult to recognize that 1 kg of mass exerts a force downward of 9.81 newtons weight.

Thus, the SI system is disadvantageous as compared to the engineering system in specifying the weight of a mass, in the decimal numbers are required as compared to the unity relation between lbweight and lbmass; that is

1 kg (mass) weighs 9.81 newtons force downward

1 lbm weighs 1 lbf downward = 1 lbweight

Thus, a conversion factor, not a multiple of 10, is required in the SI metric system in any problem involving weights.

The conversion factors to newtons from engineering units are:

$$1 = \left(\frac{4.45 \text{ newton}}{\text{lbf}}\right)$$

$$g_c = \left(\frac{32.2 \text{ lbm ft}}{4.45 \text{ newton sec}^2}\right) = \left(\frac{7.83 \text{ lbm ft}}{\text{newton sec}^2}\right)$$

Metric system weight. Force and weight relations may be summarized in the table.

Table Force and Weight in Metric and Engineering System

	Metric SI System	Engineering
Force to accelerate $F = \left(\frac{m}{g_c}\right) a$	$F \text{ newtons} = \dfrac{(m \text{ kgm})\left(a \frac{m}{sec^2}\right)}{\left(1 \frac{1 \text{ kgm m}}{\text{newton sec}^2}\right)}$	$F \text{ lbf} = \dfrac{(m \text{ lbm})\left(a \frac{ft}{sec^2}\right)}{32.2\left(\frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}$
Weight on surface of earth $w = \left(\frac{m}{g_c}\right) g$	$w \text{ newtons} = \dfrac{(m \text{ kgm})\left(9.81 \frac{m}{sec^2}\right)}{\left(1 \frac{\text{kgm m}}{\text{newton sec}^2}\right)}$	$w \text{ lbf} = \dfrac{(m \text{ lbm})\left(32.2 \frac{ft}{sec^2}\right)}{\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}$
Summarized force and mass relationships on surface of earth	1 kgm (= 2.21 lbm) exerts 9.81 newtons force downward or 1 kg weighs 9.81 newtons	1 lbm exerts 1 lbf (= 4.45 newtons) downward, or 1 lbm weighs 1 lbf

As an example, for the Saturn I space vehicle rocket-booster, the information shown in Table would appear simpler expressed in engineering units.

Table Saturn I Launch Vehicle

Mass of Vehicle	454,000 kg	1,000,000 lbm
Liftoff Weight	4,450,000 newtons = 4.45 meganewtons = 4.45 MN	1,000,000 lbw = 1,000,000 lbf
Thrust	6.70 MN	1,500,000 lbf
Available Acceleration Force	2.25 MN	500,000 lbf

Metric system potential energy. As another example in which calculation in the SI metric system appears more complex is in the work required to raise a mass a height.

Example: Calculate the work required to raise 100 lbw (45.4 kg) a height of 20 ft (6.10 m).

Engineering Units: $PE = wh = (100 \text{ lbf})(20 \text{ ft}) = 2,000 \text{ ft lbf}$ Ans.

Metric Units: $PE = wh = (mh)\left(\frac{g}{g_c}\right)$

$$= (45.4 \text{ kg})(6.10 \text{ m}) \frac{\left(9.81 \frac{\text{m}}{\text{sec}^2}\right)}{\left(1 \frac{\text{kg m}}{\text{newton sec}^2}\right)}$$

$$= 2720 \text{ newton meters} \qquad \text{Ans.}$$

Check:

$$(2720 \text{ newton meters}) = (2720 \text{ nm})\left(\frac{\text{ft lbf}}{1.36 \text{ Nm}}\right) = 2000 \text{ ft lbf}$$

The conversion factor $\left(1.36 \frac{\text{newton meter}}{\text{ft lbf}}\right)$ has been taken from the table of conversion factors following.

Metric versus engineering units. The scientist (physicist) uses the metric system whereas the practicing engineer uses the engineering system. As with languages such as English and German, the medium of communication that is most familiar is used. Although one system may be preferred by an individual, it would appear desirable to understand and work in either system. Important conversion factors are summarized in the following section.

Conversion factors metric to English. Factors in common use in heat and mechanics are given in the following table.

Table Metric and Engineering Unit Conversion Factors

Physical Property	Preferred Metric SI Unit	Other Metric Units of Conversion Factor	Engineering Units	Conversion Factors
length	m = meters	$\left(\frac{10^6 \text{ microns}}{\text{meter}}\right)$ $\left(\frac{10^{10} \text{ angstroms}}{\text{meter}}\right)$ $\left(\frac{1000 \text{ mm}}{\text{m}}\right)$ $\left(\frac{100 \text{ cm}}{\text{m}}\right)$	ft	$\left(\frac{3.28 \text{ ft}}{\text{meter}}\right)$ $\left(\frac{1.609 \text{ km}}{\text{mile}}\right)$
volume	m^3	$\left(\frac{1000 \text{ liters}}{\text{m}^3}\right)$ $\left(\frac{1000 \text{ cc}}{\text{liter}}\right)$		$\left(\frac{3.79 \text{ m}^3}{1000 \text{ gal}}\right)$ $\left(\frac{0.134 \text{ ft}^3}{\text{gal}}\right)$
velocity	$\frac{\text{meter}}{\text{sec}}$	$\left(\frac{3.00 \times 10^8 \frac{\text{meter}}{\text{sec}}}{\text{velocity of light}}\right)$	$\frac{\text{ft}}{\text{sec}}$ $\left(\frac{9.836 \times 10^8 \frac{\text{ft}}{\text{sec}}}{\text{velocity of light}}\right)$	$\left[\frac{3.28 \frac{\text{ft}}{\text{sec}}}{\left(\frac{\text{meter}}{\text{sec}}\right)}\right]$ $\left[\frac{0.447 \frac{\text{m}}{\text{sec}}}{\left(\frac{\text{miles}}{\text{hr}}\right)}\right]$
mass	kg = kilograms = kgm	$\left(\frac{1000 \text{ grams}}{\text{kg}}\right)$	lbm	$\left(\frac{2.21 \text{ lbm}}{\text{kgm}}\right)$ $\left(\frac{32.2 \text{ lbm}}{\text{slugmass}}\right)$
$\left(\frac{\text{mass}}{\text{time}}\right)$	$\frac{\text{kg}}{\text{sec}}$		$\frac{\text{lbm}}{\text{sec}}$	
time	s = seconds		sec	$\left(60 \frac{\text{sec}}{\text{min}}\right)$

Table Metric and Engineering Unit Conversion Factors (Cont.)

Physical Property	Preferred Metric SI Unit	Other Metric Units of Conversion Factor	Engineering Units	Conversion Factors
temp	K = Kelvin	(K = C + 273)	F	(F abs = R = F + 460)
force	N = newton	$\left(\frac{10^5 \text{ dyne}}{\text{newton}}\right)$ $\left(\frac{9.81 \text{ newton}}{\text{kg force}}\right)$	lbf	$\left(\frac{4.45 \text{ newton}}{\text{lbf}}\right) = \left(\frac{4.45 \text{ N}}{\text{lbf}}\right)$
power	W = watt	$\left(\frac{1000 \text{ watts}}{\text{kw}}\right)$ $\left(\frac{1 \frac{\text{joule}}{\text{sec}}}{\text{watt}}\right)$	hp for (mechanical work) watt for elec power Also see $\left(\frac{\text{heat}}{\text{time}}\right)$	$\frac{746 \text{ watt}}{\text{hp}}$ $\left[\frac{0.738 \frac{\text{ft lbf}}{\text{sec}}}{\text{watt}}\right]$
heat	J = joule	$\left(\frac{4187 \text{ joule}}{\text{k cal}}\right)$ $\left(\frac{10^7 \text{ erg}}{\text{joule}}\right)$	Btu	$\left(\frac{1055 \text{ joule}}{\text{Btu}}\right)$
$\left(\frac{\text{heat}}{\text{time}}\right)$	$\frac{\text{joule}}{\text{sec}}$	$\left(\frac{1 \frac{\text{joule}}{\text{sec}}}{\text{watt}}\right)$	heat rate	$\left[\frac{17.6 \text{ watt}}{\left(\frac{\text{Btu}}{\text{min}}\right)}\right]$ $\left(\frac{3413 \frac{\text{Btu}}{\text{hr}}}{\text{kw}}\right)$
$\left(\frac{\text{heat}}{\text{mass}}\right)$	$\frac{\text{joule}}{\text{kgm}}$			$\left(\frac{2324 \frac{\text{joule}}{\text{kg}}}{1 \frac{\text{Btu}}{\text{lbm}}}\right)$
$\left(\frac{\text{heat}}{\text{area}}\right)$	$\frac{\text{joule}}{\text{m}^2}$		heat flux	

Table Metric and Engineering Unit Conversion Factors (Cont.)

Physical Property	Preferred Metric SI Unit	Other Metric Units of Conversion Factor	Engineering Units	Conversion Factors
$\left(\frac{\text{heat}}{\text{mass time}}\right)$	$\frac{\text{joule}}{\text{kgm K}}$	$\left(\frac{1\ \frac{\text{joule}}{\text{kg K}}}{1\ \frac{\text{k cal}}{\text{kg C}}}\right)$	specific heat	$\left(\frac{4184\ \frac{\text{joule}}{\text{kg C}}}{1\ \frac{\text{Btu}}{\text{lbm F}}}\right)$
$\left(\frac{\text{heat}}{\text{area time}}\right)$	$\frac{\text{joule}}{\text{m}^2\ \text{sec}}$	$\left[\frac{1\ \frac{\text{watt}}{\text{m}^2}}{1\ \frac{\text{joule}}{\text{m}^2\ \text{sec}}}\right]$	heat flux rate	$\left(\frac{3.16\ \frac{\text{watt}}{\text{m}^2}}{1\ \frac{\text{Btu}}{\text{ft}^2\ \text{hr}}}\right)$ $\left(\frac{518.000\ \frac{\text{Btu}}{\text{ft}^2\ \text{hr}}}{1\ \frac{\text{Btu}}{\text{in.}^2\ \text{sec}}}\right)$
$\left(\frac{\text{heat}}{\text{area time temp}}\right)$	$\frac{\text{joule}}{\text{m sec K}}$		conductivity	$\left[\frac{519\ \frac{\text{joule}}{\text{m sec K}}}{\left(\frac{\text{Btu in.}}{\text{ft}^2\ \text{sec F}}\right)}\right]$
acceleration	$\frac{\text{m}}{\text{sec}^2}$		$\frac{\text{ft}}{\text{sec}}$	$\left(\frac{0.3048\ \frac{\text{m}}{\text{sec}}}{1\ \frac{\text{ft}}{\text{sec}}}\right)$
pressure	$\frac{\text{kilonewton}}{\text{m}^2}$ $= \frac{\text{kN}}{\text{m}^2}$	$\left(\frac{9.81\ \frac{\text{kgf}}{\text{m}^2}}{1\ \frac{\text{newton}}{\text{m}^2}}\right)$	$\text{psi} = \frac{\text{lbf}}{\text{in.}^2}$	$\left(\frac{6.895\ \frac{\text{kN}}{\text{m}^2}}{\text{psi}}\right)$
atm press	$\left(\frac{101.3\ \frac{\text{kN}}{\text{m}^2}}{\text{atm}}\right)$	$\left(\frac{101,325\ \frac{\text{newtons}}{\text{meter}^2}}{\text{atm}}\right)$ $\left(\frac{1,013,246\ \frac{\text{dyne}}{\text{cm}^2}}{\text{atm}}\right)$ $\left(760\ \frac{\text{mm Hg}}{\text{atm}}\right)$	$\text{psi} = \frac{\text{lbf}}{\text{in.}^2}$	$\left(\frac{14.7\ \text{psi}}{\text{atm}}\right)$ $\left(\frac{760\ \text{ton}}{\text{atm}}\right)$ $\left(\frac{30\ \text{in. Hg}}{\text{atm}}\right)$

Table Metric and Engineering Unit Conversion Factors (Cont.)

Physical Property	Preferred Metric SI Unit	Other Metric Units of Conversion Factor	Engineering Units	Conversion Factors
viscosity	$\frac{\text{newton sec}}{\text{meter}^2}$	$\left[\frac{10 \text{ poise}}{\left(1 \frac{\text{newton sec}}{\text{meter}^2}\right)}\right]$		$\left[\frac{47.9 \frac{\text{lbf sec}}{\text{ft}^2}}{\left(\frac{\text{newton sec}}{\text{meter}^2}\right)}\right]$
work	newton meter = Nm	$\left(\frac{1 \text{ newton meter}}{\text{joule}}\right)$ $\left(\frac{10^7 \text{ dyne cm}}{\text{joule}}\right)$ $\left(1 \frac{\text{dyne cm}}{\text{erg}}\right)$	force x distance	$\left(\frac{1.36 \text{ newton meter}}{\text{ft lbf}}\right)$ accurately 1.3558179
mass density	$\left(\frac{\text{kgm}}{\text{m}^3}\right)$	$\left[\frac{1008 \frac{\text{kgm}}{\text{m}^3}}{\left(1 \frac{\text{gm mass}}{\text{cm}^3}\right)}\right]$		$\left[\frac{16.02 \frac{\text{kgm}}{\text{m}^3}}{\left(\frac{\text{lbm}}{\text{ft}^3}\right)}\right]$
g_c	$1 \frac{\text{kg m}}{\text{newton sec}^2}$	$\left(9.81 \frac{\text{kgm m}}{\text{kgf sec}^2}\right)$	g_c conversion factor	$32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}$
g	$9.81 \frac{\text{m}}{\text{sec}^2}$ accurately 9.80665		gravity acceleration	$32.2 \frac{\text{ft}}{\text{sec}^2}$
J	$1 \frac{\text{newton m}}{\text{joule}}$		mech equiv heat	$778 \frac{\text{ft lbf}}{\text{Btu}}$
mole volume	$\frac{22.4 \text{ meter}^3}{\text{mole}}$ at OC and 1 atm with molecular weight in $\frac{\text{kgm}}{\text{mole}}$ known as kgm mole	$\frac{22.4 \text{ liters}}{\text{mole}}$ at OC and 1 atm with molecular weight in $\frac{\text{grams mass}}{\text{mole}}$ known as gm mole		$\frac{358 \text{ ft}^3}{\text{mole}}$ at 32 F and 14.7 psia with molecular weight in $\frac{\text{lbm}}{\text{mole}}$ known as lbm mole

Table Metric and Engineering Unit Conversion Factors (Cont.)

Physical Property	Preferred Metric SI Unit	Other Metric Units of Conversion Factor	Engineering Units	Conversion Factors
Universal Gas Constant	$\left(\frac{8314 \text{ joule}}{\text{mole K}}\right)$ $\left(\frac{8314 \text{ newton m}}{\text{mole K}}\right)$ with molecular weight $\frac{\text{kgm}}{\text{mole}}$	$\left(\frac{1.986 \text{ cal}}{\text{mole K}}\right)$ $\left(\frac{82.1 \text{ atm cm}^3}{\text{mole K}}\right)$ with molecular weight $\frac{\text{gr mass}}{\text{mole}}$	(w_m) R	$\left(\frac{1545 \text{ ft lbf}}{\text{mole R}}\right)$ with molecular weight $\frac{\text{lbm}}{\text{mole}}$

MODEL THEORY. See Similarity

MODULUS OF ELASTICITY. Murphy (28, p. 144) defines the resistance of any substance to compressibility as the change in pressure divided by an index to the corresponding change in size. Thus for solids under axial loading the measure of the change in size is $(\Delta L/L)$.

For solids

E = Young's modulus of elasticity

$$= \frac{\Delta P}{\left(\frac{\Delta L}{L}\right)}$$

For gases and liquids

The measure of the change in size is $\left(\frac{\Delta V}{V}\right)$

E = bulk modulus of elasticity

$$= \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

In dimensional analysis problems E appears in the Cauchy Number.

NEWTON FORCE. The newton N is a metric force unit that can be expressed in mass units or in heat-work units.

Conversion factors.

$$g_c = 1 \frac{\text{kgm m}}{\text{N sec}^2}$$ [Expresses N in mass units]

$$J_m = 1 \frac{\text{Nm}}{\text{joule}} = 1 \frac{\text{Nm}}{\text{amp volt sec}}$$

$$1 = 4.4482216152605 \frac{\text{N}}{\text{lbf}}$$ (Ref. 33, p. 9)

$$J = 778 \frac{\text{ft lbf}}{\text{Btu}}$$

$$1 = 1.3558179 \frac{\text{Nm}}{\text{ft lbf}}$$ (Ref. 33, p. 14)

Newton in acceleration constant. The newton N is a force unit expressible in terms of mass by the newton law.

$$F = \frac{m}{g_c} a$$

$$F \text{ newtons} = \frac{(m \text{ kgmass})}{\left(g_c \dfrac{\text{kgmass m}}{\text{newton sec}^2}\right)} \left(a \frac{\text{m}}{\text{sec}^2}\right)$$

where g_c = a conversion constant or factor with numerical and units value.

$$= 1 \frac{\text{kgm m}}{\text{N sec}^2}$$ [Numerical value selected as 1 in metric system]

Newtons expressed in joules. A joule is a metric energy unit defined as the work of 1 newton N acting through a distance of 1 meter m

$$(1 \text{ joule}) = 1 \text{ Nm}$$

$$1 \text{ N} = 1 \frac{\text{joule}}{\text{m}}$$

Newton force expressed in mass units. The newton law is written in the preceding form as a unit-consistent equation in which the units on both sides of the equation must be the same which is the practice followed in the ARDA dimensional analysis procedure of this text. This equation reduces to the units of newtons on both sides of the equation which indicates that a newton is the fundamental force unit. It is expressible in other terms as is usually done by application of the F = ma equation but if force is a fundamental property it is physically not mass, length or time and any expression in such terms of mass, length or time is a mathematical procedure rather than a physical concept. Simply stated force is a "push" and is not a quantity of matter, length or time.

Newton as a derived force unit. If the newton is treated as a derived unit expressible in terms of mass, length and time units which can be done only by the mathematical law F = ma, the g_c conversion factor is omitted to give:

$$F \text{ newtons} = (m \text{ kgmass})\left(a \frac{\text{ft}}{\text{sec}^2}\right)$$

$$N = (m \text{ kg})\left(\frac{L \text{ meter}}{T^2 \text{ sec}^2}\right)$$

$$\underbrace{= MLT^{-2}}_{\substack{\text{numerical}\\ \text{value}}} \quad \underbrace{\text{kg m sec}^{-1}}_{\substack{\text{units}\\ \text{description}}}$$

The use of the newton thus defined as a derived mathematical force unit having the numerical value and units of one kg m sec^{-1} is the heart of the metric system of units. From a dimensional analysis standpoint it is mathematically excellent but to treat a force not fundamentally as a push but in terms of a qunatity of matter renders the interpretation of physical phenomena in dimensional analysis difficult. To avoid this and thus simplify concepts the ARDA dimensional analysis procedure is to consider force as fundamental. If so force must be clearly designated as such which requires some such notation as lbf (to distinguish it from lbm). If this notation is adopted there is little advantage in selecting some designation other than lbf simply because one lbf has a very readily understood physical concept as the force exerted by one lbm where the acceleration field has a value of 32.2 ft/sec^2. On the surface of the earth where numerical value of g is essentially equal to the numerical value of g_c. this lbforce is the downward push or lbweight exerted by 1 lbmass of matter.

NEWTON LAW. This is a universally true observed experimental law not requiring proof.

$$F = Kma$$

The constant may be written with the g_c symbol to give

$$F = \left(\frac{m}{g_c}\right)a$$

If the ARDA principle is accepted that every equation must be unit consistent so that the units on the left-hand side of the equation must equal the units on the right-hand side of the equation g_c must have numerical and dimensional value to make this true.

$$F\,\text{lbm} = \frac{(m\ \text{lbm})}{\left(g_c\ \dfrac{\text{lbm ft}}{\text{lbf sec}^2}\right)}\left(a\ \frac{\text{ft}}{\text{sec}^2}\right)$$

In the engineering system g_c has the numerical value of 32.2. In the metric system the sizes of the other units are selected so that g_c has a numerical value of unity.

This law can be written as a dimensionless number.

$$1 = \left(\frac{ma}{F\ g_c}\right) = \frac{(m\ \text{lbm})\left(a\ \dfrac{\text{ft}}{\text{sec}^2}\right)}{(F\ \text{lbf})\left(g_c\ \dfrac{\text{lbf ft}}{\text{lbm sec}^2}\right)}$$

This dimensionless number may be considered to be part of every equation expressing physical phenomena involving mass acceleration. however, it need not be included as it is always true because the numerical value of g_c is a constant and thus is not a variable (like g ft/sec^2 for example) that can affect the results.

Newtons law in system of units. Newtons law is

$$F = Cma$$

$$= \frac{ma}{g_c}$$

If this equation is considered to be a relation between the properties force, mass and acceleration, (length per time^2) where the properties have no fixed size the value of C or g_c must necessarily be a numerical unity.

$$F = mLt^{-2}$$

This is done in a three unit system where F is defined in terms of mLt or m in terms of FLt.

However if unit values of properties are established, the symbol g_c has numerical and units value to make a unit consistent equation. The numerical value of g_c need not be unity.

$$F\ \text{lbm} = \left(\frac{m\ \text{lbm}}{32.2\ \frac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(a\ \frac{\text{ft}}{\text{sec}^2}\right)$$

NOTATION. In previous dimensional analysis procedures the symbol M represents the property of the entire amount of mass. In the ARDA method, m will be used to represent both the entire amount of mass m lbm and the numerical value m in m lbm. For simplicity the engineering system of units will be used with the different properties m lbm (lb mass) and F lbf (lb force) clearly distinguished.

NUCLEATION DOMAIN. ARDA analysis gives the form

$$\left(\frac{\dot{N}}{A}\right)\frac{D^3}{v} = \text{fcn}\,(Ja)(Re)(Fr)(Eu)(We)(Nu)_b\left(\frac{L}{D}\right)(Nu)_L\,(Re)_L\,(Pr)_L \left(\frac{e}{D}\right)\left(\frac{Y_m}{D}\right)(St)$$

ARDA derivation of nucleation domain. The work of Steele (12, p. 30) is of interest in defining the nucleation rate ($\dot{N}/A$) at which bubbles form on a surface in boiling heat transfer.

$$\left(\frac{\dot{N}}{A}\ \frac{1}{\text{sec ft}^2}\right) = C\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)^a\left(v\ \frac{\text{ft}}{\text{sec}}\right)^b(D\ \text{ft})^c(\Delta T\ \text{Fabs})^d\left(h_{fg}\ \frac{\text{Btu}}{\text{lbm}}\right)^e$$
$$\left(g_c\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^f\left(\mu_f\ \frac{\text{lbf sec}}{\text{ft}^2}\right)^g\left(k\ \frac{\text{Btu}}{\text{hr ft Fabs}}\right)^h\left(g\ \frac{\text{ft}}{\text{sec}^2}\right)^k$$
$$\left(P\ \frac{\text{lbf}}{\text{ft}^2}\right)^m\left(C_p\ \frac{\text{Btu}}{\text{lbm Fabs}}\right)^n\left(T\ \frac{\text{lbf}}{\text{ft}}\right)^p\left(\frac{q}{A}\ \frac{\text{Btu}}{\text{ft}^2\,\text{hr}}\right)^r(L\ \text{ft})^s$$
$$\left(h_c\ \frac{\text{Btu}}{\text{hr ft}^2\ \text{Fabs}}\right)^v\left(3600\ \frac{\text{sec}}{\text{hr}}\right)^w(e\ \text{ft})^x(Y_m\ \text{ft})^y\left(N\frac{1}{\text{sec}}\right)^z$$

where the usual notation is supplemented by:

e = height of surface roughness, ft

Y_m = amplitude of vibration of fluid, ft

N = frequency, 1/sec

h_{fg} = enthalpy of evaporation, Btu/lbm

ft
$$-2 = -3a + b + c + f - 2g - h + k - 2m - p - 2r + s - 2v + x + y$$
$$= (-3w + 3g + 3m + 3p) + (1 - g - 2m - 2p - 2k + w + z) + c$$
$$+ (g + m + p) - 2g + (+r + v + w) + k - 2m - p - 2r + s$$
$$- 2v + x + y$$
$$-3 + w - x - z - p + k + r + v - s - y + g = c$$

sec
$$-1 = -b - 2f + g - 2k + w - z$$
$$= -b + (-2g - 2m - 2p) + g - 2k + w - z \qquad b = 1 - g - 2m - 2p - 2k + w + z$$

lbm
$$0 = a - e + f - n$$
$$= a - e + g + m + p - w + e \qquad a = w - g - m - p$$

lbf
$$0 = -f + g + m + p \qquad f = g + m + p$$

Btu
$$0 = e + h + n + r + v$$
$$= e - r - v - w + n + r + v \qquad n = w - e$$

hr
$$0 = -h - r - v - w \qquad h = -r - v - w$$

Fabs
$$0 = d - h - n - v$$
$$= d + r + v + w - w + e - v \qquad d = -r - e$$

$$\left(\frac{\dot{N}}{A}\right) = C\Big[(\rho)^{w - g - m - p}(v)^{1 - g - 2m - 2p - 2k + w + z}$$
$$D^{-3 + w - x - z - p + k + r + v - s - y + g}(\Delta T)^{-r - e}$$
$$(h_{fg})^{e}(g_c)^{g + m + p}(\mu_f)^{g}(k)^{-i - v - w}(g)^{k}(P)^{m}(C_p)^{w - e}$$
$$(T)^{p}(q)^{r}(L)^{s}(h_c)^{v}(3600)^{w}(e)^{x}(Y_m)^{y}(N)^{z}\Big]$$

$$\left(\frac{\dot{N}}{A}\right)\frac{D^3}{v} = C\left(\frac{h_{fg}}{C_p\Delta T}\right)^e\left(\frac{\mu_f g_c}{\rho v D}\right)^g\left(\frac{gD}{v^2}\right)^k\left(\frac{Pg_c}{\rho v^2}\right)^m\left(\frac{Tg_c}{v^2\rho D}\right)^p\left(\frac{\frac{q}{A}D}{k\,\Delta T}\right)^r\left(\frac{L}{D}\right)^s$$

$$\left(\frac{h_c D}{k}\right)^v\left(\frac{3600\ C_p\rho v D}{k}\right)^w\left(\frac{e}{D}\right)^x\left(\frac{Y_m}{D}\right)^y\left(\frac{ND}{v}\right)^z$$

$$= (Ja)^e\,(Re)_b^{-g}(Fr)_b^{-k}\,(Eu)^m\,(We)_b^p\,(Nu)_b^r\left(\frac{L}{D}\right)^s(Nu)_L^v$$

$$(Re)_L^w\,(Pr)_L^w\left(\frac{e}{D}\right)^x\left(\frac{Y_m}{D}\right)^y(Sh)^z$$

where $(Pe)_L^w = (Re)_L^w\,(Pr)_L^w$.

The less frequently appearing terms are:

$\left(\frac{Y_m}{D_c}\right)$ = vibration number

$Sh = \left(\frac{ND}{v}\right)$ = Strouhal Number

$(Re_L) = \left(\frac{vD\rho}{\mu_f g_c}\right)$ = Reynolds Number flowing liquid

$\left(\frac{L}{D_c}\right)$ = shape factor

Omitting the less frequent terms:

$$\left(\frac{\dot{N}}{A}\right)\frac{D^3}{v} = C(Ja)^e\,(Re)_b^g\,(Fr)_b^k\,(Eu)^m\,(We)^p\,(Nu)^r\,(Nu)^v\,(Pr)^w\left(\frac{e}{D}\right)^x$$

NUCLEATION RATE. The nucleation rate ($\dot{N}/A$) (12, p. 30) used in the Nucleation Domain, is the number of bubbles formed per sec per sq ft of surface, in boiling heat transfer. Its units are therefore,

$$\left(\frac{\dot{N}}{A}\right)\frac{1}{\text{sec ft}^2}$$

It would be possible to use a single symbol for this quantity but to avoid proliferation of symbols and to use a symbol more easily recognized the N is used to signify a number, the dot a rate per sec in accordance to an increasingly accepted modern usage and A represents area.

NUSSELT NUMBER. This dimensionless parameter introduces the effect of surface conductance in heat transfer.

Nu Units

$$Nu = \begin{pmatrix} \text{Nusselt Number} \\ \text{Dimensionless} \end{pmatrix}$$

$$= \left(\frac{hD}{k}\right)$$

$$Nu = \frac{\left(h \frac{\text{Btu}}{\text{hr ft}^2\ \text{F}}\right)(D\ \text{ft})}{\left(k \frac{\text{Btu ft}}{\text{hr ft}^2\ \text{F}}\right)}$$

Redundant Forms

$$Nu = (St)(Pe) = (St)(Pr)(Re)$$

Nusselt Number as energy ratio. The Nusselt and Stanton Numbers can be interpreted as similar energy ratios by multiplying numerator and denominator by ΔT (4, p. 201). This is dimensionally valid, although the ΔT are different.

$$St = \frac{h}{3600\ \rho C_p V} \qquad Nu = \frac{hD}{k}$$

$$= \frac{h\Delta T}{(\rho C_p \Delta T)\ 3600\ v} \qquad = \frac{h\Delta T}{\left(\frac{k\Delta T}{D}\right)}$$

$$= \frac{(\text{total heat transfer})}{(\text{convective heat transfer})} \qquad = \frac{(\text{total heat transfer})}{(\text{conductive heat transfer})}$$

Nu as a heat ratio. For fluid flowing in a tube:

$$Nu = \frac{(\text{overall heat transfer fluid to wall})}{(\text{conductance through boundary layer})}$$

$$= \frac{\left(h \frac{\text{Btu}}{\text{hr ft}^2\ \text{F}}\right)(A\ \text{ft}^2\ \text{wall surface})(\Delta T\ \text{F fluid temperature drop})}{\left(k \frac{\text{Btu ft}}{\text{hr ft}^2\ \text{F}}\right)(A\ \text{ft}^2\ \text{wall})\left(\frac{\Delta T\ \text{F fluid to wall}}{L\ \text{ft boundary layer}}\right)}$$

$$= \frac{hL\,(\Delta T\ \text{temperature drop fluid})}{k\ (\Delta T\ \text{fluid to wall})}$$

In practice D is used instead of L.

Also,

$$\dot{Q}\,\frac{\text{Btu}}{\text{hr}} = hA\Delta T \text{ or } h = \frac{Q}{A\Delta T}$$

Thus,

$$Nu = \frac{hL}{k} = \frac{\dot{Q}L}{kA\Delta T} = \frac{\left(Q\,\frac{\text{Btu}}{\text{hr}}\right)(L\text{ ft})}{\left(k\,\frac{\text{Btu ft}}{\text{hr ft}^2\text{ F}}\right)(A\text{ ft})(\Delta T\text{ fluid temperature drop})}$$

Nu from dimensional analysis. Nusselt Number is a measure of heat transfer properties and depends on the heat transfer coefficient h, which is related to the boundary layer thickness L (related to the diameter D if flow is in a tube) and the thermal conductivity k across the boundary layer. It is independent of main stream velocity.

$$Nu = C\left(h\,\frac{\text{Btu}}{\text{hr ft}^2\text{ F}}\right)^a (L\text{ ft})^b \left(k\,\frac{\text{Btu ft}}{\text{hr ft}^2\text{ F}}\right)^c$$

$$\left.\begin{array}{ll} \underline{F} & 0 = -a - c \\ \underline{Btu} & 0 = a + c \\ \underline{hr} & 0 = -a - c \end{array}\right\} \quad c = -a$$

$$\underline{ft} \quad 0 = -2a + b - c \qquad b = 2a + c = 2a - a = a$$

$$Nu = C(h)^a (L)^a (k)^{-a}$$

$$= C\left[\frac{h_c L}{k}\right]^a = \frac{hD}{k}$$

where both C and a can be unity. Also D is usually used for L.

ORIFICE FLOW. Although related, it is possible to consider flow through an orifice as produced by gravity and as produced by pressure.

Flow through orifice produced by gravity. By dimensional analysis, for orifice of diameter D = L:

$$\left(\dot{V}\,\frac{\text{ft}^3}{\text{sec}}\right) = C\,(D\text{ ft})^a \left(g\,\frac{\text{ft}}{\text{sec}^2}\right)^b$$

$\underline{\text{sec}}$ $\quad -1 = -2b \qquad\qquad b = \frac{1}{2}$

$\underline{\text{ft}}$ $\quad 3 = a + b = a + \frac{1}{2} \qquad\qquad a = \frac{5}{2}$

$$\dot{V} = C\,(D\ \text{ft})^{\frac{5}{2}}\ (g)^{\frac{1}{2}}$$

$$C = \frac{\dot{V}}{D^{5/2}\,g^{1/2}} = \frac{\dot{V}}{L^{5/2}\,g^{1/2}}$$

$$C^2 = \frac{\dot{V}^2}{D^5 g} = \frac{(Av)^2}{D^5 g} = \frac{L^4 v^2}{L^5 g} = \frac{v^2}{gL} = Fr$$

By the associative method, if flow is produced by gravity, Fr is involved.

$$C = Fr = \frac{\dot{v}^2}{gL} = \frac{v^2 L^4}{gL^5} = \frac{(vA)^2}{gL^5} = \frac{\dot{V}^2}{gL^5}$$

To determine scale factors $v' = \frac{v_m}{v}$, etc.

$$\dot{V} = Cg^{\frac{1}{2}}\ L^{\frac{5}{2}}$$

For constant g, the gravity flow is proportional to $\frac{5}{2}$ power of L = D. As discussed under Scale Factors the proportionality also applies to scale factors, or

$$\dot{V}' = (L')^{\frac{5}{2}}$$

$$\dot{V} = Av = L^2 v = Cg^{\frac{1}{2}}\ L^{\frac{5}{2}}$$

$$v = Cg^{\frac{1}{2}}\ L^{\frac{1}{2}}$$

For constant g, the velocity v is proportional to $\frac{1}{2}$ power of L, or $v' = (L')^{\frac{1}{2}}$

$$(Fr) = C = \frac{Pg_c}{\rho v^2}$$

$$P = C\left(\frac{\rho}{g_c}\right)v^2 = C\left(\frac{w}{Vg}\right)v^2 = C\left(\frac{w}{Vg}\right)CgL$$

For constant g and densities $\left(\frac{w}{V}\right)$, P is proportional to L, or $P' = (L')$.

$$(Fr) = C = \frac{F g_c}{\rho L^2 v^2}$$

$$F = C\left(\frac{\rho}{g_c}\right) L^2 v^2 = C \frac{w}{g} L^2 C g L$$

For constant g and densities $\left(\frac{w}{V}\right)$, F is proportional to L, or $F' = (L')^3$.

A more formalized general treatment is given under Similarity Dimensionless Number Criteria.

Flow through orifice produced by pressure. By dimensional analysis:

$$\dot{V}\,\frac{ft^3}{sec} = C\left(\rho\,\frac{lbm}{ft^3}\right)^a \left(\bar{P}\,\frac{lbf}{ft^2}\right)^b (D\ ft)^c \left(g_c\,\frac{lbm\ ft}{lbf\ sec^2}\right)^d$$

sec $\quad -1 = -2d \qquad\qquad d = \frac{1}{2}$

lbf $\quad 0 = b - d = b - \frac{1}{2} \qquad\qquad b = \frac{1}{2}$

lbm $\quad 0 = a + d = a + \frac{1}{2} \qquad\qquad a = -\frac{1}{2}$

ft $\quad 3 = -3a - 2b + c + d$

$\quad = \frac{3}{2} - 1 + c + \frac{1}{2} \qquad\qquad c = 2$

$$\dot{V} = C(\rho)^{-\frac{1}{2}} (P)^{\frac{1}{2}} (D)^2 (g_c)^{\frac{1}{2}} = CD^2\sqrt{\frac{P g_c}{\rho}}$$

By association, if flow is produced by a pressure, Eu is involved.

$$1 = C(Eu) = C\left(\frac{P g_c D^4}{\rho \dot{V}^2}\right)$$

Eu is in terms $\dot{V}$ where D = L.

$$\dot{V}^2 = C\left(\frac{P g_c D^4}{\rho}\right)$$

$$\dot{V} = CD^2\sqrt{\frac{P g_c}{\rho}}$$

A general treatment is given under Similarity Dimensionless Number Criteria.

<u>OSCILLATING ELASTIC WING IN MOVING FLUID</u>. Sedov (3, p. 60) gives

$$\left(N\frac{1}{\text{sec}}\right) = C(L\ \text{ft})^a \left(E\ \frac{\text{lbf}}{\text{ft}^2}\right)^b \left(S\ \frac{\text{lbf}}{\text{ft}^2}\right)^c (m\ \text{lbm})^d \left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)^e \left(v\ \frac{\text{ft}}{\text{sec}}\right)^f$$

$$\left(g_c\ \frac{\text{lbm}}{\text{lbf}}\ \frac{\text{ft}}{\text{sec}^2}\right)^g$$

where E and S are moduli of elasticity and shear.

<u>lbf</u>	$0 = b + c - g$	$b = g - c$
<u>lbm</u>	$0 = d + e + g$	$e = -d - g$
<u>sec</u>	$-1 = -f - 2g$	$f = 1 - 2g$
<u>ft</u>	$0 = a - 2b - 2c - 3e + f + g$	
	$= a - 2g + 2c - 2c + 3d + 3g + 1 - 2g + g$	
		$a = -3d - 1$

$$N = CL^{-3d - 1} E^{g - c} G^c m^d \rho^{-d - g} v^{1 - 2g} \left(g_c\right)^g$$

$$\left(\frac{NL}{v}\right) = C\left(\frac{Eg_c}{\rho v^2}\right)^g \left(\frac{m}{\rho L^3}\right)^d \left(\frac{G}{E}\right)^c$$

$$(Sh) = C(Fa)^g \left(\frac{m}{\rho L^3}\right)^d \left(\frac{Fa}{Ca}\right)^{-c}$$

This result was also obtained as a special case under Fluids.

PECLET NUMBER. This dimensionless number occurs in convection heat transfer. It appears redundant in that it is a product of the more basic dimensionless numbers Re and Pr.

Units

$$(\text{Pe}) = \text{Peclet Number, dimensionless}$$

$$= \frac{3600\ C_p \rho v D}{k}$$

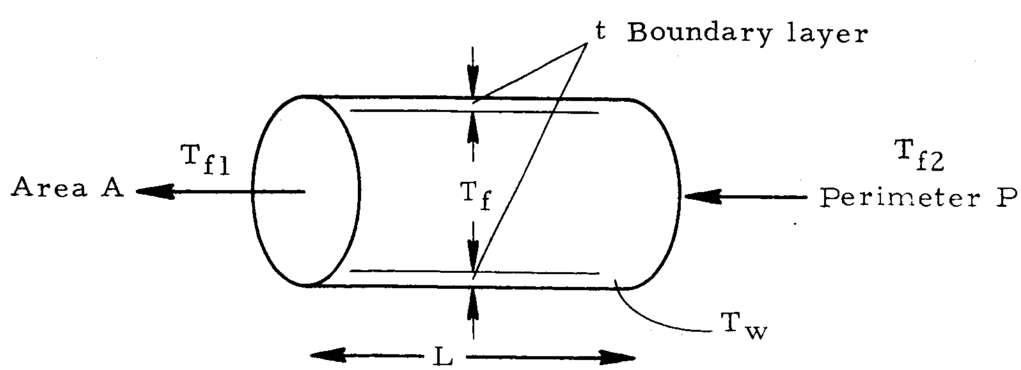
$$= \frac{\left(3600\ \frac{\text{sec}}{\text{hr}}\right)\left(C_p\ \frac{\text{Btu}}{\text{lbm F}}\right)\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(v\ \frac{\text{ft}}{\text{sec}}\right)(D\ \text{ft})}{\left(k\ \frac{\text{Btu}}{\text{hr ft F}}\right)}$$

$$= (\text{Re})(\text{Pr})$$

$$= \left(\frac{\rho v D}{\mu_f g_c}\right)\left(\frac{3600\ C_p \mu_f g_c}{k}\right)$$

$$\text{Pe} = (\text{Re})(\text{Pr})$$

Pe as ratio of heats. Some authors (28, p. 194) have defined Peclet Number as a ratio of heat available to heat transfer. If so, Pe is a more complex ratio. Consider fluid flowing in a tube of length L having a boundary layer of thickness t.



$$(\mathrm{Pe}) = \frac{(\text{heat given up by fluid in section L, Btu/sec})}{(\text{heat transfer across boundary layer in section L, Btu/sec})}$$

$$= \frac{C_p \rho v A\,(\Delta T \text{ fluid})}{PLk\,(\Delta T \text{ fluid to wall})}$$

$$= \frac{\left(C_p \frac{\text{Btu}}{\text{lbm F}}\right)\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)\left(v \frac{\text{ft}}{\text{sec}}\right)\left(\frac{\pi D^2}{4}\text{ ft}^2\right) \quad (\Delta T \text{ fluid})}{\left[\dfrac{(\pi D \text{ ft})(L \text{ ft})\left(k \frac{\text{Btu ft}}{\text{hr ft}^2 \text{ F}}\right)}{\left(3600 \frac{\text{sec}}{\text{hr}}\right)(t \text{ ft})}\right] (\Delta T \text{ fluid to wall})}$$

$$= \left(\frac{C_p \rho v D}{k}\right) \frac{3600\left(\dfrac{\Delta T \text{ temperature drop fluid}}{L \text{ length pipe}}\right)}{4\left(\dfrac{\Delta T \text{ fluid to wall}}{t \text{ thickness boundary layer}}\right)}$$

<u>Nu as Pe/Re</u>. If Pe is a measure of heat transfer as a function of flow turbulence Re, the ratio is of significance.

$$\mathrm{Nu} = \frac{\mathrm{Pe}}{\mathrm{Re}}$$

$$= \frac{\left(\dfrac{C_p \rho v D\,(3600)}{k}\right)}{\left(\dfrac{\rho v D}{\mu_f\, g_c}\right)} = \left(\frac{C_p \mu_f g_c\, 3600}{k}\right)$$

Pi theorem details. The Buckingham pi theorem and procedure is classical in dimensional analysis (30, p. 57). The pi procedure is somewhat obscure and indirect and has retarded the free use of dimensional analysis. For historical purposes it is given here and illustrated by an example. (The modernized ARDA dimensional analysis procedure used in the present text differs in several respects and the points of difference are pointed out in parenthesis).

Pi theorem procedure.

1. Any physical problem where expressed in terms of n physical quantities q such as $0 = f(q_1\ q_2\ q_3\ q_4 \ldots)$ may be replaced by an equation $0 = f(\pi_1\ \pi_2\ \pi_3 \ldots)$ or $C(\pi^a)(\pi^b)(\pi^c)\ldots$ where π represents a dimensionless number.

2. Usual pi procedure is to express each physical quantity in terms of three basic dimensions with a fourth defined by newtons law $F = MA = MLT^{-2}$ where A is acceleration LT^{-2} and the constant before the M is taken as unity. Two systems are in vogue, one is the technical system using FLT with $M = FL^{-1}T^2$, the second is the absolute system with MLT and $F = MLT^{-2}$. (In ARDA procedure an indefinite number of basic physical properties is used as may be convenient such as lbm, lbf, ft, sec, Btu, amp. Where the physical process involves a free acceleration or deceleration of mases as a result of action of forces the g_c conversion factor is included. If this is a free conversion of work energy to heat energy inclusive of the conversion factor, J may be required.)

3. For n physical quantities and usually a number k of basic properties of k = 3 for M, L and T or k = 4 if an additional basic electrical property is included to take care of electrical properties, there will be a number of π terms equal to (n - k). (In ARDA procedures the result of the procedure determines the number of dimensionless numbers.)

4. Select a number k of the physical quantities, none dimensionless and no two having the same basic dimensions, such that all of fundamental basic dimensions are included in at least one of the physical quantities.

5. The first π term is expressed as the product of the chosen quantities, each to an unknown exponent, and one other quantity to a known power, usually taken as one.

6. Retain the quantities selected in (4) as repeating variables and choose one of the remaining variables to establish the next π term. Repeat this procedure for successive π terms.

PI THEOREM. Use of the pi theorem or system limits the value of the dimensional analysis attack and it is suggested that it is not needed.

The pi theorem postulates a definite number of basic dimensions, which is an undesirable limitation.

The use of pi symbols to represent definite dimensionless numbers, most of which have well-known names, such as Re, Pr, Nu, is also an unnecessary duplication.

Where there are many variables, use of pi-theorem procedures results in many pi values (dimensionless numbers) which are redundant in that basic dimensionless numbers may occur in many pi values. It would seem to be desirable that a basic dimensionless number should appear only once in a given equation expressing phenomena of a given configuration.

In the many examples of dimensional analysis given in this book the pi theorem is not used.

Buckingham pi theorem. The theorem states (29, 28. p. 36) that the number of dimensionless and independent terms required to express a relationship is equal to the number of quantities (physical properties such as μ, ρ, etc. involved minus the number of dimensions (lbm, ft, etc.) in which those quantities may be measured. The term or π is applied to a dimensionless number. For example if $F = f(g,v,t,m,D,\rho,\mu)$ there are eight physical properties. If these are expressed in terms of L,m and t (lbf, lbm and sec) there are three dimensions. There must be $8 - 3 = 5$ dimensionless properties involved.

In the ARDA procedures g_c must be also included, or $L = f(g,v,t,m,D,\rho,\mu,g_c)$ to give nine physical properties or relationships in terms of L, m, F and t (lbf, lbm, lbf and sec) to give $9 - 4 = 5$ dimensionless properties.

A more detailed discussion of the formulation and use of the pi theorem follows.

7. For each π term solve for the unknown exponents by dimensional analysis. (In ARDA procedure omit previous steps and solve directly.)

8. There are a number of helpful relationships that are true for both the pi and ARDA procedures.

(a) If a quantity is dimensionless, it may be written as a π term (or a dimensionless number) without going through the foregoing procedure.

(b) If any two physical quantities have the same dimensions, their ratio will be one of the π terms (or a dimensionless number). For example (L/D) is dimensionless.

(c) Any π term may be replaced by any + or - power of that term. Example π^{-m} may be replaced by π^{a}, etc.

(d) Any π term may be multiplied by a numerical constant because the C term preceding the π expression represents any unknown numerical constant.

(e) Any π term may be expressed as a function of other π terms. (This should be done with caution as some dimensionless numbers are basic and if replaced may be redundant in that the same basic dimensionless number may be used more than once.)

As an example consider the drag domain for drag F per unit area A on a body.

$$f\left(\frac{F}{A}, T, g, \mu, L, v, D, \rho\right)$$

The physical quantities with their dimensions in FLT units are

$\left(\frac{F}{A}\right)$ = drag per unit area = FL^{-2}

T = surface tension = FL^{-1}

g = gravity = LT^{-2}

μ_f = viscosity = $FL^{-2}\ T$

L = length = L

v = velocity = LT^{-1}

D = diameter = L

ρ = density = ML^{-3} = $FL^{-4}\ T^{2}$

There are 8 physical quantities and 3 basic units thus (8 - 3) or 5 π-terms. Choosing diameter D, velocity v, and density ρ as the repeating variables with unknown exponents the π_1 contains D, v, ρ and F/A, π_2 contains D, v, ρ and μ etc. The π terms are, therefore,

$$\pi_1 = (L^a)(L^b T^{-b})(F^c L^{-4c} T^{2c})(FL^{-2})$$

$$\pi_2 = (L^a)(L^b T^{-b})(F^c L^{-4c} T^{2c})(FL^{-1})$$

$$\pi_3 = (L^a)(L^b T^{-b})(F^c L^{-4c} T^{2c})(LT^{-2})$$

$$\pi_4 = (L^a)(L^b T^{-b})(F^c L^{-4c} T^{2c})(FL^{-2} T)$$

$$\pi_5 = \left(\frac{L}{D}\right) \text{ written directly.}$$

Evaluating exponents for $\pi 1$

<u>For F</u>	$0 = c + 1$	$c = -1$
<u>For T</u>	$0 = -b + 2c = -b - 2$	$b = -2$
<u>For L</u>	$0 = a + b - 4c - 2 = a - 2 + 4 - 2$	$a = 0$

$$\pi 1 = D^0 \, v^{-2} \rho^{-1}\left(\frac{F}{A}\right) = \frac{F}{\rho v^2} = \text{Euler Number Eu}$$

(ARDA notation requires a g_c in these dimensionless numbers)

Similarly after some work.

$$\pi 2 = \left(\frac{T}{\rho v^2 D}\right) = \text{Weber Number}^{-1} = We^{-1}$$

$$\pi 3 = \left(\frac{Dg}{v^2}\right) = \text{Froude Number}^{-1} = Fr^{-1}$$

$$\pi 4 = \left(\frac{\mu}{\rho v D}\right) = \text{Reynolds Number}^{-1} = Re^{-1}$$

$$\pi 5 = \left(\frac{L}{D}\right) = \text{Shape Number}$$

The result is

$$f\left(Eu, We, Fr, Re, \frac{L}{D}\right) = 0$$

ARDA Procedure. The same problem (including g_c) is

$$\left(\frac{F}{A}\frac{lbf}{ft^2}\right) = C\left(T\,\frac{lbf}{ft}\right)^a \left(g\,\frac{ft}{sec^2}\right)^b \left(\mu_f\,\frac{lbf\ sec}{ft^2}\right)^c \left(L\ ft\right)^d \left(v\,\frac{ft}{sec}\right)^e \left(D\ ft\right)^f$$

$$\left(g_c\,\frac{lbm\ ft}{lbf\ sec^2}\right)^g \left(\rho\,\frac{lbm}{ft^3}\right)^h$$

lbf $\quad 1 = a + c - g \qquad\qquad g = a + c - 1$

lbm $\quad 0 = g + h \qquad h = -g \qquad h = -a - c + 1$

sec $\quad 0 = -2b + c - e - 2g$

$\quad = -2b + c - e - 2a - 2c + 2 \qquad e = -2b - c - 2a + 2$

ft $\quad -2 = -a + b - 2c + d + e + f + g - 3h$

$\quad = -a + b - 2c + d - 2b - c - 2a + 2 + f + a + c - 1 + 3a + 3c - 3$

$\quad = a - b + c + d - 2 + f \qquad f = -a + b - c - d$

$$\left(\frac{F}{A}\right) = C(T)^a (g)^b (\mu_f)^c (L)^d (v)^{-2b - c - 2a + 2} (D)^{-a + b - c - d}$$

$$(g_c)^{a + c - 1} (\rho)^{-a - c + 1}$$

$$\left(\frac{F g_c}{\rho A v^2}\right) = C\left(\frac{T g_c}{\rho v^2 D}\right)^a \left(\frac{Dg}{v^2}\right)^b \left(\frac{\mu_f g_c}{\rho v D}\right)^c \left(\frac{L}{D}\right)^d$$

$$Eu = f\left(We, Fr, Re, \frac{L}{D}\right)$$

POISEUILLE EQUATION. See under Hydraulic Formulas.

POTENTIAL ENERGY. This energy in ft lbf is possessed by a weight w at an elevation H.

$$PE = w\ lbf\ H\ ft = (wH) ft\ lbf = (mH)\frac{g}{g_c}\ ft\ lbf = (m\ lbm\ h\ ft)\left(\frac{g\ \dfrac{ft}{sec^2}}{g_c\ \dfrac{lbm\ ft}{lbf\ sec^2}}\right)$$

PRANDTL NUMBER. Prandtl number contains $C_p\mu_f$ and k, thus is known as the physical properties number.

$$(Pr) = \left(\begin{matrix}\text{Prandtl Number}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \frac{C_p \mu_f g_c (3600)}{k}$$

$$(Pr) = \frac{\left(C_p \frac{\text{Btu}}{\text{lbm F}}\right)\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)\left(3600 \frac{\text{sec}}{\text{hr}}\right)}{\left(k \frac{\text{Btu ft}}{\text{hr ft}^2 \text{ F}}\right)}$$

$$= \frac{C_p \mu_m}{k}$$

$$= \frac{\left(C_p \frac{\text{Btu}}{\text{lbm F}}\right)\left(\mu_m \frac{\text{lbm}}{\text{ft hr}}\right)}{\left(k \frac{\text{Btu ft}}{\text{hr ft}^2 \text{ F}}\right)}$$

Pr = Approximately 0.72 for air. This value may also be used for flue- and exhaust-gases resulting from combustion of fuels with air. This value may also be used for diatomic-gases such as N_2, O_2, H_2, CO.

= Approximately 1.02 for low-pressure steam. For effect of pressure and temperature see values for high pressure steam.

= Approximately 0.78 for ammonia.

= Approximately $\left[\frac{4}{9 - \frac{5 c_v}{c_p}}\right]$ for gases. (Proof not given here.)

PRESSURE. See Surface Tension Pressure. See Velocity Pressure.

PRESSURIZATION DOMAIN. ARDA analysis gives:

$$\frac{T}{Tg} = \text{fcn}\,(\text{Pn 1, Pn 2, Pn 3, Re, Fr, Pr, Nu})$$

Derivation of pressurization domain. The pressurization of liquid propellant tanks by a gas has been analyzed (Nein, 7 and 8) by Mower and Hanson (26) whose analysis follows. The following specialized notation is employed in addition to the usual notation.

$T = (T_m - T_L)$ = final gas temperature above liquid temperature, F

$T_g = (T_o - T_L)$ = pressurant gas temperature above liquid temperature

T_m = final mean gas temperature, F

T_o = initial pressurant gas temperature. F

T_L = liquid temperature, F

$\Delta V = (V_2 - V_1)$ = volume increase of tank during pressurization, ft^3

v_o = initial pressurant gas velocity, ft/hr

$$(T\ ^\circ F) = C\ (T_g\ F\ abs)^a (D\ ft)^b (\Delta V\ ft^3)^c \left(v_o\ \frac{ft}{sec}\right)^d \left(\rho\ \frac{lbm}{ft^3}\right)^e (t\ sec)^f$$

$$\left(\frac{m}{M}\ R\ \frac{ft\ lbf}{mole\ F\ abs}\right)^g \left(g_c\ \frac{lbm}{lbf}\ \frac{ft}{sec^2}\right)^h \left(\frac{m\ lbm}{M\ mole}\right)^i \left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)^j$$

$$\left(k\ \frac{Btu}{sec\ ft\ F\ abs}\right)^k \left(C_p\ \frac{Btu}{lbm\ F\ abs}\right)^\ell \left(g\ \frac{ft}{sec^2}\right)^m$$

$$\left(h_c\ \frac{Btu}{sec\ ft^2\ F\ abs}\right)^n$$

mole: $0 = -g - i$ $\qquad g = -i$

F abs: $1 = a - g - k - \ell - n$

$= a + i(+\ell + n) - \ell - n$ $\qquad a = 1 - i$

ft: $0 = b + 3c + d - 3e + g + h - 2j - k + m - 2n$

$= b + 3c + (f - j - 2m + 2i + \ell) + (-3\ell + 3j)$

$- i + (j - i) - 2j + (\ell + n) + m - 2n$

$= b + 3c + f + j - m - \ell - n$ $\qquad b = -3c - f - j + m + \ell + n$

sec: $0 = -d + f - 2h + j - k - 2m - n$

$= -d + f + (-2j + 2i) + j + (\ell + n)$

$- 2m - n$ $\qquad d = f - j - 2m + 2i + \ell$

lbm: $0 = e + h + i - \ell$

$= e + (j - i) + i - \ell$ $\qquad e = \ell - j$

lbf: $0 = g - h + j$

$= -i - h + j$ $\qquad h = j - i$

Btu: $0 = k + \ell + n$ $\qquad k = -\ell - n$

$$T = C(T_g)^{1 - i}(D)^{-3c - f - j + m + \ell + n}(\Delta V)^c (v)^{f - j - 2m + 2i + \ell}$$

$$(\rho)^{\ell - j}(t)^f\left(\frac{m}{M}R\right)^{-i}(g_c)^{j - i}\left(\frac{m}{M}\right)^i (\mu_f)^j (k)^{-\ell - n}(C_p)^{\ell}(g)^m (h_c)^n$$

$$\left(\frac{T}{T_g}\right) = C\left(\frac{v^2}{g_c T_g R}\right)^i \left(\frac{\Delta V}{D^3}\right)^c \left(\frac{vt}{D}\right)^f \left(\frac{g_c \mu_f}{\rho D v}\right)^j \left(\frac{D}{v^2 g}\right)^m \left(\frac{D v \rho C_p}{k}\right)^{\ell} \left(\frac{D h_c}{k}\right)^n$$

$$= C(Pn\,1)^i (Pn\,2)^c (Pn\,3)^f (Re)^{-j} (Fr)^{-m} (St)^{-\ell} (Nu)^n$$

where $(St)^{-1} = \dfrac{(Pr)(Re)}{Nu}$

$$\frac{T}{T_g} = C(Pn\,1)^a (Pn\,2)^b (Pn\,3)^c (Re)^d (Fr)^e (Pr)^f (Nu)^g$$

where exponents have been redesignated. The symbols Pn 1, Pn 2, Pn 3 designate pressurization numbers 1, 2, 3; in that these dimensionless numbers do not have formal names.

Nein and Thompson (8) using

$$T = f\left(J,\ g_c,\ \frac{m}{M},\ k,\ \mu_f g_c,\ C_p,\ R,\ T_o,\ T_L,\ t,\ V,\ A,\ h,\ T_a,\right.$$

$$\left. C_{pw},\ \rho_w,\ L_w,\ P,\ V_1,\ \dot{V},\ A_o\right)$$

where

$\mu = (\mu_f g_c)$

$\left(\frac{m}{M}\right)$ = molecular weight, $\frac{lb}{mole}$

T_a = ambient temperature outside tank, F

$\dot{V}$ = liquid drain rate, $\frac{ft^3}{sec}$

A_o = inlet pipe area, ft^2

V_1 = initial volume

w signifies wall

Obtain (8, p. 10):

$$\left(\frac{T}{T_g}\right) = C\left(\frac{Jg_c\left(\frac{m}{M}\right)^2 k\, T_g}{\mu_f^3 g_c^3\, r^4}\right)^a \left(\frac{T_g}{T_L}\right)^b \left(\frac{\mu_f g_c rt}{\frac{m}{M}}\right)^c .$$

$$\left(\frac{\frac{m}{M} k}{\mu_f g_c r^2 C_{pw}\rho_w d_w}\right)^d \left(\frac{g_c\left(\frac{m}{w}\right) PV_1}{\mu_f^2 g_c^2\, r^4}\right)^e \left(\frac{g_c\left(\frac{m}{M}\right) P}{\mu_f^2 g_c^2\, r}\right)^f \left(\frac{hr}{k}\right)^g$$

$$\left(\frac{T_a - T_L}{T_g}\right)^h \left(\frac{\frac{m}{M}\dot{V}}{A_D \mu_f g_c r^2}\right)^i$$

Using equations (not given) for properties μ, k and $\dot{V}$ for liquid oxygen or nitrogen the preceding may be modified (8, p. 13, 7, p. 2) to what must be viewed as a rather complex empirical relation.

$$\frac{T}{T_o} = f(T_o, r, V_1, C_{pw}, \rho_w, L_w, P, t, h, T_a, A_D)$$

PROPELLOR DOMAIN. Consider a propellor on a ship (2, p. 65, 30, p. 57). The propulsion power $\dot{W}$ (ft lbf/sec) to move the ship is the product of ship thrust or drag F ft and ship velocity v_s. This is equal to the power delivered by the propellor which depends on propellor velocity v_p, diameter D, speed N, and fluid density ρ, viscosity μ_f. For a ship on the surface gravity, g is involved in wave action. The conversion factor g_c is needed for lbf and lbm present.

Ship Thrust

$$F \text{ lbf} = C\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)^a (D \text{ ft})^b \left(v_p \frac{\text{ft}}{\text{sec}}\right)^c \left(N_s \frac{1}{\text{sec}}\right)^d \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)^e \left(v_s \frac{\text{ft}}{\text{sec}}\right)^f$$

$$\left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^g \left(g \frac{\text{ft}}{\text{sec}^2}\right)^h$$

lbf	$1 = e - g$	$g = e - 1$
lbm	$0 = a + g = a + e - 1$	$a = 1 - e$
sec	$0 = -c - d + e - f - 2g - 2h$	
	$0 = -c - d + e - f - 2e + 2 - 2h$	
	$0 = -c - d - e - f + 2 - 2h$	$c = -d - e - f + 2 - 2h$

ft $\quad 0 = -3a + b + c - 2e + f + g + h$

$$0 = -3 + 3e + b - d - e - f + 2 - 2h - 2e + f + e - 1 + h$$

$$0 = -2 + e + b - d - h \qquad b = 2 - e + d + h$$

$$F = C(\rho)^{1-e}(D)^{2-e+d+h}(v_p)^{-d-e-f+2-2h}(N_s)^d(\mu_f)^e (v_s)^f(g_c)^{e-1}(g)^h$$

$$\left(\frac{Fg_c}{\rho D^2 v_p^2}\right) = C\left(\frac{\mu_f g_c}{\rho v_p D}\right)^e\left(\frac{N_s D}{v_p}\right)^d\left(\frac{Dg}{v_p^2}\right)^h\left(\frac{v_s}{v_p}\right)^f$$

$$Eu = C(Re)^{-e}(Sh)^{-d}(Fr)^{-h}(Ma)^{-f}$$

Propellor power with non-viscous fluid. For a propellor of diameter D and speed N moving a fluid of density ρ:

$$\left(\dot{W}\frac{\text{ft lbf}}{\text{sec}}\right) = C\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)^a (D\ \text{ft})^b\left(N_s\ \frac{1}{\text{sec}}\right)^d\left(g_c\ \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^g$$

lbf	$1 = -g$	$g = -1$
lbm	$0 = a + g = a - 1$	$a = 1$
sec	$-1 = -d - 2g = -d + 2$	$d = 3$
ft	$1 = -3a + b + g = -3 + b - 1$	$b = 5$

$$\dot{W} = C(\rho)^1 (D)^5(N_s)^3 (g_c)^{-1}$$

$$\left.\begin{aligned}
\frac{\dot{W}g_c}{\rho D^5 N_s^3} &= \\
\left(\frac{\dot{W}g_c}{\rho D^2 v_t^3}\right)\left(\frac{v_t}{DN_s}\right)^3 &= \\
\left(\frac{FN_sDg_c}{\rho D^2 v_t^3}\right)\left(\frac{v_t}{DN_s}\right)^3 &= \\
\left(\frac{Fg_c}{\rho D^2 v_t^2}\right)\left(\frac{v_t}{DN_s}\right)^2 &= \\
(Eu)(Sh)^{-2} &=
\end{aligned}\right\} C$$

The final equation is the propellor domain equation with Re omitted for μ_f absent, Fr omitted for g of wave action absent and Ma for velocities absent.

Propellor power with viscous fluid. For a propellor of diameter D, tip velocity v_t and speed N moving a fluid of density ρ, viscosity μ_f and velocity v:

$$\left(\dot{W}\,\frac{\text{ft lb}}{\text{sec}}\right) = C\left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)^a (D\ \text{ft})^b \left(v_t\,\frac{\text{ft}}{\text{sec}}\right)^c \left(N_s\,\frac{1}{\text{sec}}\right)^d \left(\mu_f\,\frac{\text{lbf sec}}{\text{ft}^2}\right)^e$$

$$\left(v\,\frac{\text{ft}}{\text{sec}}\right)^f \left(g_c\,\frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^g$$

lbf	$1 = e - g$	$g = e - 1$
lbm	$0 = a + g = a + e - 1$	$a = 1 - e$
sec	$-1 = -c - d + e - f - 2g$	
	$-1 = -c - d + e - f - 2e + 2$	$c = 3 - d - e - f$
ft	$1 = -3a + b + c - 2e + f + g$	
	$= -3 + 3e + b + 3 - d - e - f - 2e + f + e - 1$	
	$0 = -2 + e + b - d$	$b = d + 2 - e$

$$\dot{W} = C(\rho)^{1-e}(D)^{d+2-e}(v_t)^{3-d-e-f}(N_s)^d(\mu_f)^e(v)^f(g_c)^{e-1}$$

$$\left(\frac{\dot{W}g_c}{\rho D^2 v_t^3}\right) = C\left(\frac{\mu_f g_c}{\rho v_t D}\right)^e \left(\frac{DN_s}{v_t}\right)^d \left(\frac{v}{v_t}\right)^f$$

$$\left.\begin{array}{l} \dfrac{FN_sDg_c}{\rho D^2 v_t^3} = \\ \left(\dfrac{Fg_c}{\rho D^2 v_t^2}\right)\left(\dfrac{N_sD}{v_t}\right) = \\ (Eu)(Sh) = \end{array}\right\} C(Re)^c(Sh)^d(Ma)^{-f} \qquad \text{where } Ma = \left(\frac{\text{tip velocity}}{\text{velocity}}\right)$$

But $v_t = \pi DN_s$ or $Sh = \left(\frac{DN_s}{v_t}\right)^d$ for propellor is a constant. Adjust C so that $d = 3$.

$$\left.\begin{array}{ll}\left(\frac{\dot{W} g_c}{\rho D^2 v_t^3}\right)\left(\frac{v_t^3}{D^3 N_s^3}\right) & = \\ \left(\frac{\dot{W} g_c}{\rho D^5 N_s^3}\right) & = \\ \left(\frac{F N_s D g_c}{\rho D^2 v^2}\right)\left(\frac{v^2}{D^3 N_s^3}\right) & = \\ \left(\frac{F g_c}{\rho D^2 v^2}\right)\left(\frac{v^2}{D^2 N_s^2}\right) & = \end{array}\right\} C(Re)^c (Ma)^{-f}$$

Comparing to propellor thrust power with non-viscous fluid, the effect of adding viscosity is to add Re term, the effect of adding velocity is to add Ma term.

$$(Eu)(Sh)^{-2} =$$

$$Eu = f(Re, Sh, Ma)$$

This is the propellor domain equation with Fr absent for g of wave action absent

PUMP DOMAIN. The pressure P developed by a rotary pump of diameter D and speed N_s depends on gravity g, the velocity v leaving the pump, the fluid density ρ and viscosity μ_f and the conversion factor g_c.

$$\left(P \frac{lb}{ft^2}\right) = C\left(\mu_f \frac{lbf\ sec}{ft^2}\right)^a \left(g \frac{ft}{sec^2}\right)^b \left(N_s \frac{1}{sec}\right)^c \left(\rho \frac{lbm}{ft^3}\right)^d \left(v \frac{ft}{sec}\right)^e$$

$$(D\ ft)^f \left(g_c \frac{lbm\ ft}{lbf\ sec^2}\right)^g$$

<u>lbf</u> $1 = a - g$ $\qquad g = a - 1$

<u>lbm</u> $0 = d + g = d + a - 1$ $\qquad d = 1 - a$

<u>sec</u> $0 = a - 2b - c - e - 2g$

$0 = a - 2b - c - e - 2a + 2$

$0 = -a - 2b - c - e + 2$ $\qquad e = 2 - a - 2b - c$

<u>ft</u> $-2 = -2a + b - 3d + e + f + a$

$-2 = -2a + b - 3 + 3a - a - 2b - c + 2 + f + a - 1$

$0 = a - b - c + f$ $\qquad f = b + c - a$

$$P = C(\mu_f)^a (g)^b (N_s)^c (\rho)^{1-a} (v)^{-a-2b-c+2} (D)^{b+c-a} (g_c)^{a-1}$$

$$\left(\frac{Pg_c}{\rho v^2}\right) = C\left(\frac{\mu_f g_c}{\rho v D}\right)^a \left(\frac{gD}{v^2}\right)^b \left(\frac{N_s D}{v}\right)^c$$

$$Eu = C(Re)^{-a} (Fr)^{-b} (Sh)^c$$

$$= f(Re,\ Fr,\ Sh)$$

Pump discharge. Rearranging the pump domain equation (24, p. 102)

$$Sh^{-c} = f(Re,\ Fr,\ Eu)$$

$$\left(\frac{v}{N_s D}\right) =$$

$$\frac{\pi}{4}\left(\frac{vD^2}{N_s D^3}\right) =$$

$$\left(\frac{\dot{V}}{N_s D^3}\right) = f(Re,\ Fr,\ Eu)$$

where $\dot{V} = v \frac{\pi}{4} D^2$ = pump discharge, ft^3/sec

Pump head. The pump domain equation may be rearranged.

$$\frac{(Eu)^{-1}}{(Fr)^{-1}} = f(Re,\ Sh)$$

$$\frac{\left(\frac{gH}{v^2}\right)}{\left(\frac{gD}{v^2}\right)} = f(Re,\ Sh)$$

$$\left(\frac{H}{D}\right) = f(Re,\ Sh)$$

where H = head developed by pump, ft

D = pump diameter, ft

Specific speed. This dimensionless number (30, p. 227) is of interest in pump performance. The pump domain equation may be rearranged and specific exponents assigned to Eu and Sh which may be rewritten

$$(Eu)^{-1} = \left(\frac{v^2}{gH}\right) = \frac{(N_S \pi D)^2}{gH} = \left(\frac{N_S{}^2 D^2}{gH}\right) \quad \text{where } \pi \text{ is in C}$$

$$(Sh)^{-1} = \left(\frac{v}{N_S D}\right) = \left(\frac{v\pi D^2}{4 N_S D^3}\right) = \left(\frac{\dot{Q}}{N_S D^3}\right) \quad \text{where } \frac{\pi}{4} \text{ is in C}$$

$$(\text{Specific Speed})^4 = (Eu)^{-3} (Sh)^{-2} = f(Re,\ Eu)$$

$$= \left(\frac{N_S{}^2 D^2}{gH}\right)^3 \left(\frac{\dot{V}}{N_S D^3}\right)^2$$

$$= \frac{N_S{}^6 D^6 \dot{V}^2}{(gH)^3 (N_S{}^2 D^6)}$$

$$= \frac{N_S{}^4 \dot{V}^2}{(gH)^3} = \frac{\left(N_S \frac{1}{\sec^4}\right)\left(\dot{V} \frac{ft^3}{\sec}\right)^2}{\left(g \frac{ft}{\sec^2}\right)^3 (H\ ft)^3}$$

$$(\text{Specific Speed}) = \frac{N_S \sqrt{\dot{V}}}{(gH)^{3/4}} = f(Re,\ Eu)$$

RAYLEIGH NUMBER. This dimensionless number occurs in convection heat transfer. It appears redundant in that it is a product of the more basic dimensionless numbers Re, Pr and Bu.

$$Ra = \left(\begin{matrix}\text{Rayleigh}\\ \text{Number}\\ \text{Dimensionless}\end{matrix}\right) = (Re)(Pr)(Bu) \qquad \left[\begin{matrix}\text{For plane surfaces}\\ \text{use H ft height}\\ \text{instead of D ft dia}\end{matrix}\right]$$

$$= (Gr)(Pr)$$

$$= \left[\frac{D^3 \left(\frac{m}{V}\right)^2 (3600)^2 gB(\Delta T)}{(\mu_m)^2}\right]\left[\frac{c_p \mu_m}{k}\right] = \left[\frac{D^3 \left(\frac{m}{V}\right)^2 (3600)^2 gB(\Delta T)(c_p)}{\mu_m k}\right]$$

$$\text{or} = \left[\frac{D^3 \left(\frac{m}{V}\right)^2 gB(\Delta T)}{\mu_f{}^2 g_c{}^2}\right]\left[\frac{3600\ c_p \mu_f g_c}{k}\right]$$

$$= \frac{D^3 \left(\frac{m}{V}\right)^2 (3600) gB\,\Delta T\, c_p}{k \mu_f g_c}$$

$$= \frac{(D\ ft^3)\left(\frac{m\ lbm}{ft^3}\right)^2 \left(3600\ \frac{sec}{hr}\right)\left(32.2\ \frac{ft}{sec^2}\right)\left(B\ \frac{1}{F\ abs}\right)\left(\Delta T\ F\ abs\right)\left(c_p\ \frac{Btu}{lbm\ F}\right)}{\left(k\ \frac{Btu}{hr\ ft\ F}\right)\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)}$$

REACTION RATE. This property is a measure of the speed of chemical reaction.

$\dot{U}$ = Reaction Rate

$$= \frac{\dot{m}\,\frac{\text{lbm}}{\text{sec}}}{m\ \text{lbm}} = \left(\frac{\dot{m}}{m}\right)\left(\frac{1}{\text{sec}}\right)$$

$$= \frac{\dot{\rho}\,\frac{\text{lbm}}{\text{ft}^3\ \text{sec}}}{\rho\,\frac{\text{lbm}}{\text{ft}^3\ \text{sec}}} = \left(\frac{\dot{\rho}}{\rho}\right)\left(\frac{1}{\text{sec}}\right)$$

REDUNDANT DIMENSIONLESS NUMBERS. Certain dimensionless numbers appear basic in that they are the simplest dimensionless numbers that represent the given processes. In the literature there are many other dimensionless numbers that are less basic in that they are more complex combinations of the basic dimensionless numbers. Some of these are listed below. Because they add unnecessary complexity, it would appear better to avoid the use of these redundant dimensionless numbers. For more discussion see under name of dimensionless number and Convection Heat Transfer.

Table Dimensionless Numbers in Convection Heat Transfer

Nature	Name	Equivalent	Units
Basic	Nusselt	Nu	$\frac{hD}{k}$
	Reynolds	Re	$\frac{\rho v D}{\mu_f g_c}$
	Prandtl	Pr	$\frac{c_p \mu_f g_c (3600)}{k}$
	Buoyancy	Bu	$\frac{D^2 wB \Delta T}{\mu_f vV}$
	Shape Factor	$\left(\frac{L}{D}\right)$	$\left(\frac{L}{D}\right)$

Table Dimensionless Numbers in Convection Heat Transfer

Nature	Name	Equivalent	Units
Redundant	Rayleigh	$Ra = (Re)(Pr)(Bu)$	$\dfrac{D^3 \rho^2 g B \Delta T c_p (3600)}{k \mu_f g_c}$
	Grashof	$Gr = (Re)(Bu)$	$\dfrac{D^3 \rho^2 g B \Delta T}{\mu_f^2 g_c^2}$
	Peclet	$Pe = (Re)(Pr)$	$\dfrac{c_p \rho v D (3600)}{k}$
	Staunton	$St = \dfrac{Nu}{(Re)(Pr)}$ $= \dfrac{Nu}{Pe}$	$\dfrac{k}{c_p \rho v (3600)}$
	Graetz	$Gz = \dfrac{\pi}{4}(Re)(Pr)\dfrac{D}{L}$	$\dfrac{\pi}{4} D^3 v\rho \left(\dfrac{c_p}{kL}\right) = \dfrac{\dot{m} c_p}{kL}$

Table Convection Heat Transfer Equation

Redundant Dimensionless Numbers	Basic Convection Heat Transfer Equations in Terms of This Dimensionless Number	
	$Nu = fcn\left(Re,\ Pr,\ Bu,\ \dfrac{L}{D}\right)$	Basic Eq.
$Ra = (Re)(Pr)(Bu)$	$Nu = fcn\left(Ra,\ \dfrac{L}{D}\right)$	
$Gr = (Re)(Bu)$	$Nu = fcn\left(Gr,\ Pr,\ \dfrac{L}{D}\right)$	
$Pe = (Re)(Pr)$	$Nu = fcn\left(Pe,\ Bu,\ \dfrac{L}{D}\right)$	
$St = \dfrac{Nu}{(Re)(Pr)} = \dfrac{Nu}{Pe}$	$Nu = fcn\left(St,\ Bu,\ \dfrac{L}{D}\right)$	
	$St = fcn\left(Bu,\ \dfrac{L}{D}\right)$	Alternate form
$Gz = \dfrac{\pi}{4}(Re)(Pr)\dfrac{D}{L}$	$Nu = fcn(Gz,\ Bu)$	

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    p. 16 Stokes Law Discussion from Refs. 25, 26, 27 of Fritz paper.
    p. 16 Lapple and Shephard from Ref. 41 of Fritz paper.
    p. 18 Kaissling and Rosenberg from Ref. 31 of Fritz paper.
    p. 20 Source not stated.

(b) Study of Gas Bubble Dynamics, Part 2, May 1965.

(c) Study of Bubble Motion in Liquid Nitrogen, August 1965.

Papers (a) and (b) are Notes P-64-5 and P-65-13 for internal use only. Paper (c) will be published in "Advances in Cryogenic Engineering," Vol. II, Proceedings of the 1965 Cyrogenic Engineering Conference in Houston, Texas.

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REYNOLDS NUMBER. This dimensionless parameter introduces the effect of viscosity μ_f. The numerator contains a length term which may be diameter D ft if there is a diameter or L ft if there is no diameter.

Re preferred units.

Re = Reynolds number, dimensionless

$$= \frac{\rho v D}{\mu_f g_c} = \frac{\left(\frac{m\ \text{lbm}}{v\ \text{ft}^3}\right)\left(v\ \frac{\text{ft}}{\text{sec}}\right)\left(D\ \text{ft}\right)}{\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf sec}^2}\right)}$$

where D ft can be replaced by L ft if L ft is more significant.

Re in mass viscosity units.

$$\text{Re} = \frac{\rho v D}{\mu_f g_c} = \frac{\rho v D}{\left(\frac{\mu_m}{3600}\right)}$$

$$= \frac{\rho v D\ 3600}{(\mu_m)} = \frac{\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(v\ \frac{\text{ft}}{\text{sec}}\right)\left(D\ \text{ft}\right)\left(3600\ \frac{\text{sec}}{\text{hr}}\right)}{\left(\mu_m\ \frac{\text{lbm}}{\text{ft hr}}\right)}$$

Re in alternate mass viscosity units.

$$\text{Re} = \frac{\rho v D}{\mu_f g_c} = \frac{\rho v D}{\mu_s} = \frac{\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(v\ \frac{\text{ft}}{\text{sec}}\right)\left(D\ \text{ft}\right)}{\left(\mu_s\ \frac{\text{lbm}}{\text{ft sec}}\right)}$$

Re in alternate ft per hr velocity units.

$$\text{Re} = \frac{3600\ \rho v D}{(\mu_m)} = \frac{\rho(3600\ v)D}{\mu_m} = \frac{\rho \mu_h D}{\mu_m} = \frac{\left(\rho\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(\mu_h\ \frac{\text{ft}}{\text{hr}}\right)\left(D\ \text{ft}\right)}{\left(\mu_m\ \frac{\text{lbm}}{\text{ft hr}}\right)}$$

Re in alternate mass flow units.

$$Re = \frac{\rho v D\ 3600}{\mu_m}$$

$$= \frac{\left(\frac{m}{V}\right) v\ 3600\ D}{\mu_m}$$

$$= \frac{\left(\frac{m}{AL}\right)\left(\frac{L}{t}\ 3600\right) D}{\mu_m}$$

$$= \frac{\left(\frac{\dot{m}}{A}\right) D}{\mu_m} = \frac{\left(\frac{\dot{m}}{A}\right) D}{(3600)\, \mu_f g_c}$$

$$= \frac{\left(\frac{\dot{m}}{A}\ \frac{lbm}{ft^2\ hr}\right)(L\ ft)}{\left(\mu_m\ \frac{lbm}{ft\ hr}\right)} = \frac{\left(\frac{\dot{m}}{A}\ \frac{lbm}{lb^2\ hr}\right)\left(L\ ft\right)}{\left(3600\ \frac{sec}{hr}\right)\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(32.2\ \frac{lbm\ ft}{lbf\ sec^2}\right)}$$

Reynolds Number in terms kinematic viscosity. The properties of mass density $\rho = (m/V)$ and viscosity (μ_f or $\mu_f g_c$) entering into Reynolds Number are physical properties of the fluid and are temperature-dependent. For numerical evaluation of Re it is convenient to combine ρ and μ_f in the form of a ratio (μ_f/ρ).

$$Re = \frac{vD}{\left(\frac{\mu_f g_c}{\rho}\right)}$$

where $\left(\frac{\mu_f g_c}{\rho}\right)$ = kinematic viscosity, $\frac{ft^2}{sec}$

$$= \frac{\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(32.2\ \frac{lbf\ ft}{lbm\ sec^2}\right)}{\left(\rho\ \frac{lbm}{ft^3}\right)}$$

Reynolds Number units. The physical concept of Reynolds Number has always been a little difficult with respect to dimensions because the requirement that it be dimensionless is not completely compatible with the very clear indication that the numerator $\rho v L$ involves mass in the terms of mass density ρ whereas the denominator involves viscosity basically defined in terms of force (see Viscosity):

$$\text{Viscosity Fundamental Definition} = \mu_f \frac{\text{lb sec}}{\text{ft}^2}$$

To meet the dimensionless criteria most past procedure in the literature has been to express viscosity in mass units in the denominator in order to cancel the mass in the mass density term of the numerator. This has made necessary the intorduction of the F = ma law to express viscosity not in terms of a drag force but in terms of the mass that would be decelerated by this drag force. The resulting term involving mass is:

$$\text{Viscosity Expressed in Mass Units} = \mu_s \frac{\text{lbm}}{\text{ft sec}}$$

The relation between μ_m and μ_s is obtained by introducing conversion factors to form the dimensionally consistent relation (see Viscosity):

$$\mu_s = \mu_f g_c$$

$$\left(\mu_s \frac{\text{lbm}}{\text{ft sec}}\right) = \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)$$

In order to preserve the physical concept of viscosity as a force the preferred form of Re is:

$$Re = \frac{\rho v L}{\mu_f g_c}$$

Not Preferred:

$$Re = \frac{\rho v L}{\mu_s}$$

Most of the literature is rather loose in that precise units are not indicated for Re as is done in ARDA dimensional analysis procedures. In the literature such terms as g_c and J are often left for the reader to supply so that Reynolds Number is somewhat indefinitely designated as:

$$Re = \frac{\rho v L}{\mu}$$

where:

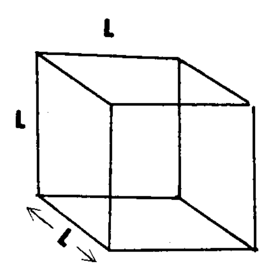
ρ = density
v = velocity
L = length
μ = viscosity

Sometimes (see Combustion Domain) where most of the concepts involve mass diffusion and mass rather than force properties are involved, a shorter solution is obtained by using μ_m in Re = $\rho vL/\mu_m$, but it should be kept in mind that this is really an alternate sometimes shorter version of Re = $(3600\ \rho vL/\mu_f g_c)$.

Re as flow parameter. A fluid in flow has the properties of velocity v, density ρ, a viscosity force and a size dimension L. Reynolds Number is a dimensionless parameter relating these properties. Several concepts and derivations are possible.

Flow. In normal turbulent flow there is a general velocity v but the individual fluid particles are in a highly variable flow pattern in which their velocity is constantly changing in magnitude and direction as a result of viscous drag forces of adjacent fluid particles.

Flow Model. It is convenient to consider a model. A fluid particle may be considered as a very small cube of length L on a side moving in a given direction. It is continuously either accelerating from 0 to a maximum velocity v in this direction or decelerating from a maximum velocity v to zero by the action of viscous forces. The maximum velocity may be considered to be the overall velocity of the fluid or at least related to the overall velocity of the fluid v. The force required to accelerate the fluid cube from 0 to v given by the F = $(m/g_c)a$ law. The viscous drag force is the viscous force due to viscosity defined as the force lbf on a plane area L^2 ft^2 to move it at a velocity v = L ft/sec past another plane area at a distance L ft from the first area.



Flow is also discussed under Flow Concepts and in the literature (28, p. 165).

Reynolds number as a force ratio. The force of deceleration is also known as inertia force.

$$Re = \frac{(\text{Force of Deceleration})}{(\text{Viscous Drag Force})} = \frac{\left(\frac{m}{g_c}\right)a}{\mu_f A\left(\frac{v}{L}\right)}$$

$$= \frac{m\left(\frac{v}{t}\right)}{\mu_f A\left(\frac{v}{L}\right)g_c} = \frac{\left(\frac{m}{AL}\right)A\left(\frac{v}{L}\ \frac{L}{t}\right)L}{\mu_f\ A\left(\frac{v}{L}\right)\ g_c} = \frac{\rho v L}{\mu_f\, g_c}$$

For a tube D is used for the size dimension L.

<u>Reynolds Number as a stress ratio</u>. The length terms in the numerator of Re may be L or D.

$$Re = \frac{\left(\frac{m}{V}\right)vL}{\mu_f\, g_c} = \frac{\left(\frac{m}{g_c}\right)v\ \frac{L}{V}}{\mu_f} = \frac{\left(\frac{F}{a}\right)v\ \frac{1}{A}}{\mu_f}$$

$$= \frac{\left(\frac{F}{A}\right)}{\mu_f\left(\frac{a}{v}\right)} = \frac{\frac{F\ \text{lbf}}{A\ \text{ft}^2}}{\mu_f\ \frac{\text{lbf}}{\text{ft}^2}\ \text{sec}\left(\frac{a\ \frac{\text{ft}}{\text{sec}^2}}{v\ \frac{\text{ft}}{\text{sec}}}\right)}$$

Thus, Re may be interpreted as a ratio of forces per area to produce acceleration of the mass of the fluid or inertial stress to the viscosity or viscous stress.

<u>Reynolds Number as an energy ratio</u>.

$$Re = \frac{(\text{KE Resulting From Acceleration of Cube from 0 to v})}{(\text{Drag Energy})}$$

$$= \frac{\frac{1}{2}\left(\frac{m}{g_c}\right)v^2}{(F)_{\text{avg drag}}\,(L)}$$

$$= \frac{\frac{1}{2}\left(\frac{m}{V}\right)v^2\,V}{g_c\left(\mu_f A\ \frac{v_a}{L}\right)(L)}$$

$$Re = \frac{\frac{1}{2}\rho\, v^2\, L^3}{g_c\, \mu_f\, L^2\, \frac{v}{2}}$$

$$= \frac{\rho v L}{\mu_f\, g_c}$$

<u>Reynolds Number as a viscosity parameter</u>. It is desired to develop Re as a dimensionless number containing viscosity μ_f. The μ_f units are written in unreduced form as they exist in the basic physical concept.

$$Re = \left[\begin{matrix}\text{Reynolds Number}\\ \text{Dimensionless}\end{matrix}\right]$$

$$= \frac{(\text{Numerator})}{\left(\mu_f\, \frac{\text{lbf ft sec}}{\text{ft}^2\ \text{ft}}\right)}$$

$$= \frac{\left(\frac{F\ \ \text{lbf}}{A\ \ \text{ft}^2}\right)\left(\frac{L\ \text{ft}}{v\ \frac{\text{ft}}{\text{sec}}}\right)}{\left(\mu_f\, \frac{\text{lbf ft sec}}{\text{ft}^2\ \text{ft}}\right)}$$

$$= \frac{\left(\frac{mv}{g_c\, t}\right)\frac{1}{A}\left(\frac{L}{v}\right)}{\mu_f}$$

$$= \frac{\left(\frac{mvL}{t\,\frac{V}{t}}\right)}{\mu_f\, g_c}$$

$$= \frac{\left(\frac{m}{V}\right)vL}{\mu_f\, g_c}$$

$$= \frac{\rho v L}{\mu_f\, g_c}$$

Reynolds Number by dimensional analysis.

$$(Re) = C(L\ ft)^a \left(v\ \frac{ft}{sec}\right)^b \left(\rho\ \frac{lbm}{ft^3}\right)^c \left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)^d \left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)^e$$

lbm	$0 = c + e$	$e = -c$
lbf	$0 = d - e$	$d = e = -c$
sec	$0 = -b + d - 2e$	$b = d - 2e$ $= -c + 2c = c$
ft	$0 = a + b - 3c - 2d + e$	$a = -b + 3c + 2d - e$ $= -c + 3c - 2c + c$ $= c$

$$Re = C(L)^c (v)^c (\rho)^c (\mu_f)^{-c} (g_c)^{-c}$$

$$= C\left[\frac{\rho v L}{\mu_f\, g_c}\right]^c = C\left[\frac{\left(\frac{m\ lbm}{V\ ft^3}\right)\left(v\ \frac{ft}{sec}\right)\left(L\ ft\right)}{\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(g_c\ \frac{lbm\ ft}{lbf\ sec^2}\right)}\right]^c$$

where both C and c can equal unity. D is used for L in tubes.

Reynolds Number as a property parameter. Viscosity is the property uniquely appearing in Reynolds Number as compared to other dimensionless parameters thus Re is considered as introducing the effect of viscosity μ_f.

SCALE FACTORS. True model similarity with a prototype is achieved if each kind of dimensional number in the law expressing the physical phenomena has the same numerical value (28, p. 170). Thus, a model theory of scale factors may be formulated.

Scale factors. Using the subscript m to indicate model as compared to the prototype value without subscript.

For some Fr

$$(Fr)_m = (Fr)$$

$$\frac{v_m^2}{g_m L_m} = \frac{v^2}{g L}$$

For same g

$$\frac{v_m^2}{L_m} = \frac{v^2}{L}$$

$$\frac{v}{v_m} = \sqrt{\frac{L}{L_m}}$$

For same Re

$$(Re)_m = (Re)$$

$$\frac{\rho_m v_m L_m}{(\mu_f)_m (g_c)_m} = \frac{\rho v L}{\mu_f g_c}$$

where $(g_c)_m = g_c$. This is always true so g_c terms will always cancel thus can always be omitted.

$$\frac{\rho_m}{(\mu_f)_m} = \left(\frac{v}{v_m} \frac{L}{L_m}\right)\frac{\rho}{\mu_f}$$

$$= \left(\sqrt{\frac{L}{L_m}} \frac{L}{L_m}\right)\frac{\rho}{\mu_f}$$

$$= \left(\frac{L}{L_m}\right)^{3/2} \left(\frac{\rho}{\mu_f}\right)$$

The relation between ρ and μ_f must be different (i.e., different fluid) for the model as compared to the prototype. Viewed in a different aspect, for same Re.

$$(Re)_m = Re$$

$$\frac{\rho_m v_m D_m}{\mu_m g_{cm}} = \frac{\rho v D}{\mu \; g_c}$$

$$\frac{\left(\frac{\rho_m}{\rho}\right)\left(\frac{v_m}{v}\right)\left(\frac{D_m}{D}\right)}{\left(\frac{\mu_m}{\mu}\right)\left(\frac{g_{cm}}{g_c}\right)} = 1$$

$$\frac{\rho' \; v' \; D'}{\mu' \; g_c'} = 1$$

where the prime is used to denote the ratio $\frac{(\text{property})_{\text{model}}}{(\text{property})_{\text{prototype}}}$.
The last relation must be true for similarity.

For same We

$$(We)_m = We$$

$$\frac{\rho_m v_m^2 L_m}{T_m (g_c)_m} = \frac{\rho v^2 L}{T g_c} \qquad \text{where } (g_c)_m = g_c$$

$$\frac{\rho_m}{T_m} = \left(\frac{v}{v_m}\right)^2 \left(\frac{L}{L_m}\right) \frac{\rho}{T}$$

$$= \left(\frac{L}{L_m}\right)^2 \left(\frac{L}{L_m}\right) \frac{\rho}{T}$$

$$= \left(\frac{L}{L_m}\right)^3 \frac{\rho}{T}$$

The relation between ρ and T should be different for model as compared to prototype.

These similarity considerations must be carried on through other dimensionless numbers if present.

Scale factor ratios. The similarity conditions that a model must satisfy as compared to a full-scale system can be expressed as the requirement of equality of appropriate dimensionless products. These result in conditions to be imposed on the various physical quantities involved can be expressed in the form of scale factor ratios.

Dimensionless number as a ratio of scale factors. For similarity the appropriate dimensionless numbers of model and prototype must be equal.

$$(Eu)_m = Eu$$

$$\frac{F_m (g_c)_m}{\rho_m L_m^2 v_m^2} = \frac{F g_c}{\rho L^2 v^2}$$

$$\frac{\left(\frac{F_m}{F}\right)\left(\frac{g_{cm}}{g_c}\right)}{\left(\frac{\rho_m}{\rho}\right)\left(\frac{L_m}{L}\right)^2\left(\frac{v_m}{v}\right)^2} = 1$$

$$\frac{F' g_c'}{\rho' (L')^2 (v')^2} = 1$$

where the prime denotes ratio $\frac{(\text{property})_{\text{model}}}{(\text{property})_{\text{prototype}}}$.

The dimensionless number for scale factors with primes is exactly similar to the original dimensionless number, thus any dimensionless number is also an expression of scale factors.

For the same Fr

$$(Fr)_m = Fr$$

$$\frac{v_m^2}{g_m L_m} = \frac{v^2}{g L} \quad \text{or} \quad \frac{\dot{v}_m^2}{g_m L^5} = \frac{v^2}{g L^5}$$

$$\frac{\left(\frac{v_m}{v}\right)^2}{\left(\frac{g_m}{g}\right)\left(\frac{L_m}{L}\right)} = 1 \qquad \frac{\left(\frac{\dot{v}_m}{\dot{v}}\right)^2}{\left(\frac{g_m}{g}\right)\left(\frac{L_m}{L}\right)^5} = 1$$

$$\frac{(v')^2}{g' L'} = 1 \qquad \frac{(v')^2}{(g')(L')^5}$$

$$(v')^2 = g' L' \qquad v' = (g')^{1/2} \, (L')^{5/2}$$

On the surface of the earth $g' = 1$.

Where several dimensionless numbers are involved, the several resulting scale factor relationships apply jointly.

Another illustration of scale factors is given under Flow Through Orifice Produced by Gravity.

SCHMIDT NUMBER. This dimensionless number expresses the diffusion of one material into another as expressed by the mass diffusivity D_m.

$$Sc = \begin{pmatrix}\text{Schmidt Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \left(\frac{\mu_m}{\rho\, D_m}\right) = \frac{\mu_m \dfrac{lbm}{ft\ hr}}{\left(\rho\ \dfrac{lbm}{ft^3}\right)\left(D_m\ \dfrac{ft^2}{hr}\right)}$$

Derivation of Schmidt Number. It is desired to develop Sc as a dimensionless number containing the mass diffusivity D_m. The D_m dimensions are written in unreduced form as they exist in the base definition (see Diffusivity)

$$Sc = \begin{pmatrix}\text{Schmidt Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{(\text{Numerator})}{D_m\ \dfrac{lbm\ ft^3\ ft}{hr\ lbm\ ft^2}}$$

(where numerator has same units as denominator)

$$= \frac{\dfrac{\left(\mu_m\ \dfrac{lbm}{ft\ hr}\right)}{\left(\rho\ \dfrac{lbm}{ft^3}\right)}}{D_m\ \dfrac{lbm\ ft^3}{hr\ lbm\ ft}}$$

$$= \frac{\left(\dfrac{\mu_m}{\rho}\right)\dfrac{ft^2}{hr}}{\left(D_m\ \dfrac{ft^2}{hr}\right)} = \frac{(\text{Momentum Diffusivity})}{(\text{Mass Diffusivity})}$$

$$= \left(\frac{\mu_m}{\rho\, D_m}\right) = \frac{\left(\mu_m\ \dfrac{lbm}{ft\ hr}\right)}{\left(\rho\ \dfrac{lbm}{ft^3}\right)\left(D\ \dfrac{ft^2}{hr}\right)}$$

SHAPE FACTOR. Dimensionless numbers of the following form may be termed shape factors.

$$SF = \left(\frac{\text{Shape Factor}}{\text{Dimensionless}}\right)$$

$$= \left(\frac{L\ ft}{D\ ft}\right)$$

Dimensional analysis makes no distinction between properties having the same units as for example length L ft, diameter D ft, width D ft, roughness e ft, etc. Where such properties enter into the functional equation

$$X = fcn\ (L, D, We \ldots\ldots)$$

the analysis will result in

$X = C\left(\frac{L}{D}\right. \ldots\ldots$ if only L and D are present or relationships

of the form

$X = C\left(\frac{L}{D}, \frac{W}{D}, \frac{e}{D}\right. \ldots\ldots$ if more properties are present.

In fact simplicity in treatment may be obtained if more than one property of the same units is omitted from the functional relationship, the ARDA analysis carried out and any pertinent term such as $\frac{L}{D}$, $\frac{W}{D}$, $\frac{\rho_p}{\rho_w}$, etc. inserted in the final relationship.

Properties having same units. For the reason previously discussed, dimensional analysis cannot distinguish between property symbols having the same units such as L ft or D ft. For this reason such dimensionless numbers as Re may be interpreted as either $\frac{\rho v D}{\mu_m}$ or $\frac{\rho v L}{\mu_m}$. A decision as to whether D or L is to be used must be based on the physical concept of the process involved.

Similar problems enter in the dimensionless numbers such as Euler Number.

$$Eu = \frac{F g_c}{\rho A v^2} \text{ or } \frac{P g_c}{\rho v^2} \text{ or } \frac{\Delta P g_c}{\rho v^2} \text{ etc.}$$

SHEAR STRESS IN PIPE. This is usually a boundary shear stress.

$$S \frac{\text{lbf}}{\text{ft}^2} = C\left(v \frac{\text{ft}}{\text{sec}}\right)^a \left(D \text{ ft}\right)^b \left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)^c \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)^d \left(e \text{ ft}\right)^e \left(g_c \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)^f$$

lbf $\quad 1 = d - f$

$\quad = d + c \qquad\qquad c = 1 - d$

lbm $\quad 0 = c + f$

$\quad f = -c = -1 - d \qquad\qquad f = d - 1$

sec $\quad 0 = -a + d - 2f$

$\quad = -a + d - 2d + 2 \qquad\qquad a = 2 - d$

ft $\quad -2 = a + b - 3c - 2d + e + f$

$\quad -2 = 2 - d + b - 3 + 3d - 2d + e + d - 1$

$\quad = b + d + e \qquad\qquad b = -d - e$

$$S = C(v)^{2-d}(D)^{-d-e}(\rho)^{1-d}(\mu_f)^d(e)^e(g_c)^{d-1}$$

$$\frac{S g_c}{v^2 \rho} = C\left(\frac{\mu_f g_c}{\rho v D}\right)^d \left(\frac{e}{D}\right)^e$$

$$Eu = f(Re)\left(\frac{e}{D}\right)$$

The shear $S \frac{lbf}{ft^2}$ is dimensionally the same as $P \frac{lbf}{ft^2}$ so that $\left(\frac{Sg_c}{v^2\rho}\right)$ dimensionally is the same as $\left(\frac{Pg_c}{v^2\rho}\right)$ = Eu. The term $\left(\frac{Sg_c}{v\rho}\right)$ has been called a Fanning number, but there seems to be little purpose in introducing another name.

SHIP DRAG. This is a classical problem in dimensional analysis (3, p. 61, 19).

The shortest solution is by Flow Region 2 of the general Drag Equation.

Alternately a solution by basic ARDA principles follows.

Drag on a ship. The drag force F on a ship is assumed to be a function of various properties as follows.

Drag = f(viscosity, gravity, velocity, density, length, gravity constant)

This may be expressed in engineering units:

$$F\ lbf = C\left(\mu_f \frac{lbf\ sec}{ft^2}\right)^a \left(g \frac{ft}{sec^2}\right)^b \left(v \frac{ft}{sec}\right)^c \left(\rho \frac{lbm}{ft^3}\right)^d \left(L\ ft\right)^e \left(g_c \frac{lbm\ ft}{lbf\ sec^2}\right)^f \tag{A}$$

where

g = local accleration of gravity, $32.2 \frac{ft}{sec^2}$, on the surface of the earth

g_c = gravity constant, $32.2 \frac{lbm\ ft}{lbf\ sec^2}$, valid at any location in the universe

The individual unit-properties and exponents on each side of the equation must be equal. For example, for lbf

$$lbf^1 = lbf^{a - f}$$

Writing the equality for the exponents alone, for each one of the unit-properties such as lbf, lbm, sec and ft:

lbf	$1 = a - f$	$f = a - 1$
lbm	$0 = d + f$	
	$d = -f = -a + 1$	$d = 1 - a$
sec	$0 = a - 2b - c - 2f$	
	$= a - 2b - c - 2a + 2$	
	$c = -a - 2b + 2$	$c = 2 - a - 2b$
ft	$0 = -2a + b + c - 3d + e + f$	
	$= -2a + b + 2 - a - 2b - 3 + 3a + e + a - 1$	
	$= a - b - 2 + e$	$e = 2 - a + b$

Substituting these values of f, d, c and e in Eq (A)

$$F = C(\mu_f)^a (g)^b (v)^{2 - a - 2b} (\rho)^{1 - a} (L)^{2 - a + b} (g_c)^{a - 1} \tag{B}$$

$$\left(\frac{F g_c}{v^2 L^2 \rho}\right) = C\left(\frac{\mu_f g_c}{L v \rho}\right)^a \left(\frac{L g}{v^2}\right)^b \tag{C}$$

$$(Eu) = C\left[\frac{1}{(Re)^a (Fr)^b}\right] \tag{D}$$

This may be variously written.

$$(Eu) = f(Re,\ Fr) \tag{E}$$

$$F = f(Re,\ Fr)\ \frac{v^2 L^2 \rho}{g_c} \tag{F}$$

$$F = (Eu)\ \frac{v^2 L^2 \rho}{g_c} \tag{G}$$

$$= \left(\frac{C_D}{2}\right)\left(\frac{v^2 A \rho}{g_c}\right) = C_D A \left(\frac{\rho v^2}{2 g_c}\right) \tag{H}$$

where:

Eu = Euler number, dimensionless = $\left(\frac{F g_c}{v^2 L^2 \rho}\right) = \left(\frac{C_D}{2}\right)$

Re = Reynolds number, dimensionless = $\left(\frac{L v \rho}{\mu_f g_c}\right)$

Fr = Froude number, dimensionless = $\left(\frac{v^2}{Lg}\right)$

A = wetted area, $ft^2 = L^2$

a, b, c, etc. = exponents which must be empirically evaluated.

SIMILARITY. As a summary it may be stated that dimensional analysis and similarity are closely related in that dimensional analysis yields dimensionless numbers and two configurations are similar if the physical phenomena pertaining to the configurations can be expressed by the same equation expressed in dimensionless number form.

As a further thought, two configurations are similar with respect to the properties expressed by a dimensionless number, if their dimensionless numbers of this property are the same. As a well-known example, two configurations having the same Reynolds number are similar with respect to the property expressed by Reynolds number.

Detailed considerations of similarity. Dimensional analysis and similarity are closely related. Two systems are similar with respect to the properties given in the physical law, if the same physical law is applicable to each system. Dimensional analysis is a procedure for formulating in a systematic simplified manner, by the use of dimensionless numbers, these physical laws to include multiple properties. For example

$$C = \left(\frac{\rho v L}{\mu_f g_c}\right) = \text{(Reynolds Number)}$$

is a formulation of $C = f(\rho, v, L, \mu_f, g_c)$ that expresses the relation between viscosity and certain other properties. If this relation is applicable to the two systems, the systems are similar with respect to these properties, but only the properties given. If properties are involved in other manners, additional relations are involved. If a drag force F is required, an additional parameter must be supplied to include its relation

$$\left(\frac{Fg_c}{\rho A v^2}\right) = C\left(\frac{\rho v L}{\mu_f g_c}\right)^a$$

$$Eu = C(Re)^a$$

would include this effect. If surface tension is involved the relation becomes

$$Eu = C(We)^a (Re)^b \text{ etc.}$$

Because there are an infinite number of properties, it may be considered that the complete defining equation would have an infinite number of terms. However, under certain conditions only certain parameters will be important and dominant. Others will be less important or negligible. Thus for convection heat transfer

$$Nu = C(Re)^a (Pr)^b (Bu)^c \left(\frac{L}{D}\right)^d$$

Thus if the dominant dimensionless number parameters are included and the effect of other parameters are negligible two systems are similar if the same equation governs.

Practical similarity constraints. As a physical law the coefficients C, a, b, c, d, etc., must usually be established empirically. A given set of numerical values is valid over limited ranges of numerical variation of the parameters such as Re, Pr involved. Use of the equation beyond these limits is subject to the usual uncertainties of an extrapolation in that the coefficients probably change. Thus similarity of two systems is restricted to the numerical range limits of the parameters involved.

For example, if two systems are known to be governed by the same physical equation involving a Reynolds number in the range between 1000 and 2000, the systems are similar in this range and not necessarily similar at Re = 4000. Thus arises the statement that true model similarity will be achieved if each kind of dimensionless number in the physical law expressing the physical phenomena has the same numerical value (or range of numerical values).

Dimensionless number limits. It follows immediately that any equation in dimensional number form should have the constraints stated as numerical ranges of the dimensionless number parameters for which the equation is valid. For each parameter these ranges also depend upon the ranges of the other parameter which means that a dimensionless parameter equation should include all dominant parameters.

Empirical evaluation. Dimensional analysis seldom establishes the numerical values of the coefficients. This must be done empirically. As a practical procedure this can be done in a given system by holding as many dimensionless numbers as possbile at a constant value and successively varying one to establish its coefficient and exponent.

In this manner dimensional analysis permits organization in the formulation of physical laws from test data. The test data may be a limited number of values in specific ranges.

Determination must be made of the physical properties governing the process from past knowledge or from tests. This is a skill developed from a study of the application of dimensional analysis to many systems. Additional thoughts are given under the discussion of the Associative Method.

Model Similarity. True model similarity with a prototype is achieved if each kind of dimensional number in the law expressing the physical phenomena has the same numerical value. Thus a model theory of scale factors may be formulated. See under Scale Factors.

SIMILARITY DIMENSIONLESS NUMBER CRITERIA. Similarity with respect to certain physical properties requires equality of corresponding related dimensionless numbers. This is because the physical property occurs only in the particular dimensionless number or because it is a dominating factor in the dimensionless number. The physical property may also occur in other dimensionless numbers which are omitted if their effect is zero, negligible or small.

This required equality of dimensionless numbers for similarity may be variously expressed. One method is utilizing scale or scaling ratios R.

$$\begin{pmatrix}\text{Dimensionless}\\ \text{Number}\end{pmatrix}_{\text{model}} = \begin{pmatrix}\text{Dimensionless}\\ \text{Number}\end{pmatrix}_{\text{prototype}}$$

$$N_m = N$$

$$\left[\frac{N_m}{N}\right] = N' = 1$$

$$\left[\frac{\left(\frac{A_m}{A}\right)\left(\frac{B_m}{B}\right)\ldots\ldots}{\left(\frac{C_m}{C}\right)\ldots\ldots}\right] = \left(\frac{A'B'\ldots\ldots}{C'\ldots\ldots}\right) = 1$$

where primes denote a scale factor of $\dfrac{(\text{Property})_{\text{model}}}{(\text{Property})_{\text{prototype}}} = \dfrac{A_m}{A}$

A, B, C denote terms in the dimensionless number.

As an example for flow:

$$Re = \left(\frac{\rho v L}{\mu_f g_c}\right)$$

For flow similarity:

$$Re' = \left(\frac{\rho' v' L'}{\mu_f' g_c'}\right) \text{ representing } \frac{Re_m}{Re} = \frac{\left(\frac{\rho_m \mu_m L_m}{\mu_{fm} g_{cm}}\right)}{\left(\frac{\rho v L}{\mu_f g_c}\right)} = 1$$

where:

$$Re' = \frac{Re_m}{Re} = \frac{(Re)_{\text{model}}}{(Re)_{\text{prototype}}}, \quad \rho' = \frac{\rho_m}{\rho} = \frac{\rho_{\text{model}}}{\rho_{\text{prototype}}}, \text{ etc.}$$

g_c' and similar ratios of constants are always = 1

At same location ratio of g = 1

Any units of property may be used because conversion factors cancel out.

Use of the preceding notation simplifies similarity problems considerably.

Similarity requirements in fluid flow. A following presentation indicates possibilities in formulating similarity requirements in the fluid flow domain. The relations are fundamentally the same whether considering a stationary body in a moving fluid (ships, piers, aerodynamic configurations or in fluids such as water or air) or moving fluids related to or contained in stationary walls (spillways, dams, pipes, pumps).

Flow of fluid particles. In general the motion of fluid particles is influenced by the result of actions of gravity and viscosity. The action of gravity tends to cause a fluid particle to tend to move downward. The action of viscosity tends to cause a fluid particle to deviate from straight-line motion in the direction of flow. Either or both these actions result in a movement (microscopic) of the individual particles with constantly changing velocities in constantly changing directions. However, an overall velocity (macroscopic) in a given direction of the fluid or object is produced which is the velocity v in the equations. This velocity v is a function of Froude Number $Fr = (v^2/gL)$ for gravity and or Reynolds Number $Re = (\rho vL/\mu_f g_c)$ for kinematic viscosity $= (\mu_f/g_c)$.

Effects establishing velocities. For flow regions where gravity g effects predominate over viscosity effects; such as partially-immersed ships on water, flow of water over dams and spillways; the Froude Number Fr establishes the law governing flow velocity v. Gravity g is not to be confused with the constant g_c.

For flow regions where kinematic viscosity (ρ/μ_f) or $(\rho/\mu_f g_c)$ effects predominate over gravity effects; such as bodies fully immersed in fluid such as aircraft, submarines, gas and fluid flow in pipes; the Reynolds Number Re establishes the law governing flow velocity v.

For flow regions where both gravity and viscosity are important both Froude Number and Reynolds Number, such as perhaps partially-immersed objects in high viscosity fluids, etc; both Fr and Re establish the flow laws.

Dimensionless number ratios. When two dimensionless numbers govern, similarity requires that numerical values of the corresponding dimensionless numbers be equal. The ratios of corresponding dimensionless numbers will always be unity. This means that when two or more dimensionless numbers govern, their ratios to any power may be combined by multiplication in any fashion desired. This is true because one raised to any exponent is always one.

The exponents of the ratios of dimensionless numbers for combining by multiplication are usually selected in such a way as to eliminate some property from the combined terms, usually velocity. Re and Fr ratios are therefore usually combined as

$$\text{Each term is ratio}\left\{\quad \frac{Re^2}{Fr} = \frac{\left(\dfrac{\rho^2 v^2 L^2}{\mu_f^2 g_c^2}\right)}{\left(\dfrac{v^2}{gL}\right)} = \left(\frac{\rho^2 q L^3}{\mu_f^2 g_c^2}\right) = 1 \qquad \text{(velocity v eliminated)}\right.$$

In the preceding the primes denoting ratios of $(\text{property})_{model}/(\text{property})_{prototype}$ have been omitted to avoid the confusion of designating every term with a prime, but with such omission the equation should be so designated so that ratios of each property will be inserted when the equation is used.

Euler Number. The Euler Number Eu is essentially a conversion factor usually used in similarity to convert velocity v to drag or force F or to pressure P (= force per unit area).

$$Eu = \left(\frac{F g_c}{\rho L^2 v^2}\right) = \left(\frac{P g_c}{\rho v^2}\right) = 1 \text{ if each term is a ratio.}$$

The Eu number has characteristics similar to g_c except that the numerical value of Eu varies with the data whereas the numerical value of g_c is a constant. For similarity the Euler Numbers must also be the same so that the Euler Number Ratio = $(Eu)_{model}/(Eu)_{prototype} = 1$. This is also true for g_c in that g_c cannot be different for model and prototype. The ratio of $g_{model}/g_{prototype}$ is usually unity also (but not in the new field of space exploration).

Thus whenever it is desired to express Fr or Re ratios (or any other dimensionless number expression such as (Re^2/Fr)) in terms of force, the dimensionless number may be multiplied by Eu to any power.

Useful combination in problems involving drag requiring a conversion of viscous effect Re to force by the introduction of Eu are

$$\left.\begin{array}{ll}
\text{Re and Eu} & Re^2\,Eu = \left(\dfrac{\rho^2 v^2 L^2}{\mu_f^2\, g_c^2}\right)\left(\dfrac{F g_c}{\rho L^2 v^2}\right) = \left(\dfrac{F\rho}{\mu_f^2 g_c}\right) = 1 \\[2ex]
\text{Fr and Eu} & Eu\,Fr = \left(\dfrac{F g_c}{\rho L^2 v^2}\right)\left(\dfrac{v^2}{gL}\right) = \left(\dfrac{F g_c}{\rho L^3 g}\right) = 1 \\[2ex]
 & Eu\,Fr^{3/2} = \left(\dfrac{F g_c}{\rho L^2 v^2}\right)\left(\dfrac{v^3}{g^{3/2}\; L^{3/2}}\right) = \left(\dfrac{(Fv) g_c}{\rho L^{7/2}\; g^{3/2}}\right) = 1
\end{array}\right\}\ \text{If Each Term Is A Ratio}$$

where (Fv) = power.

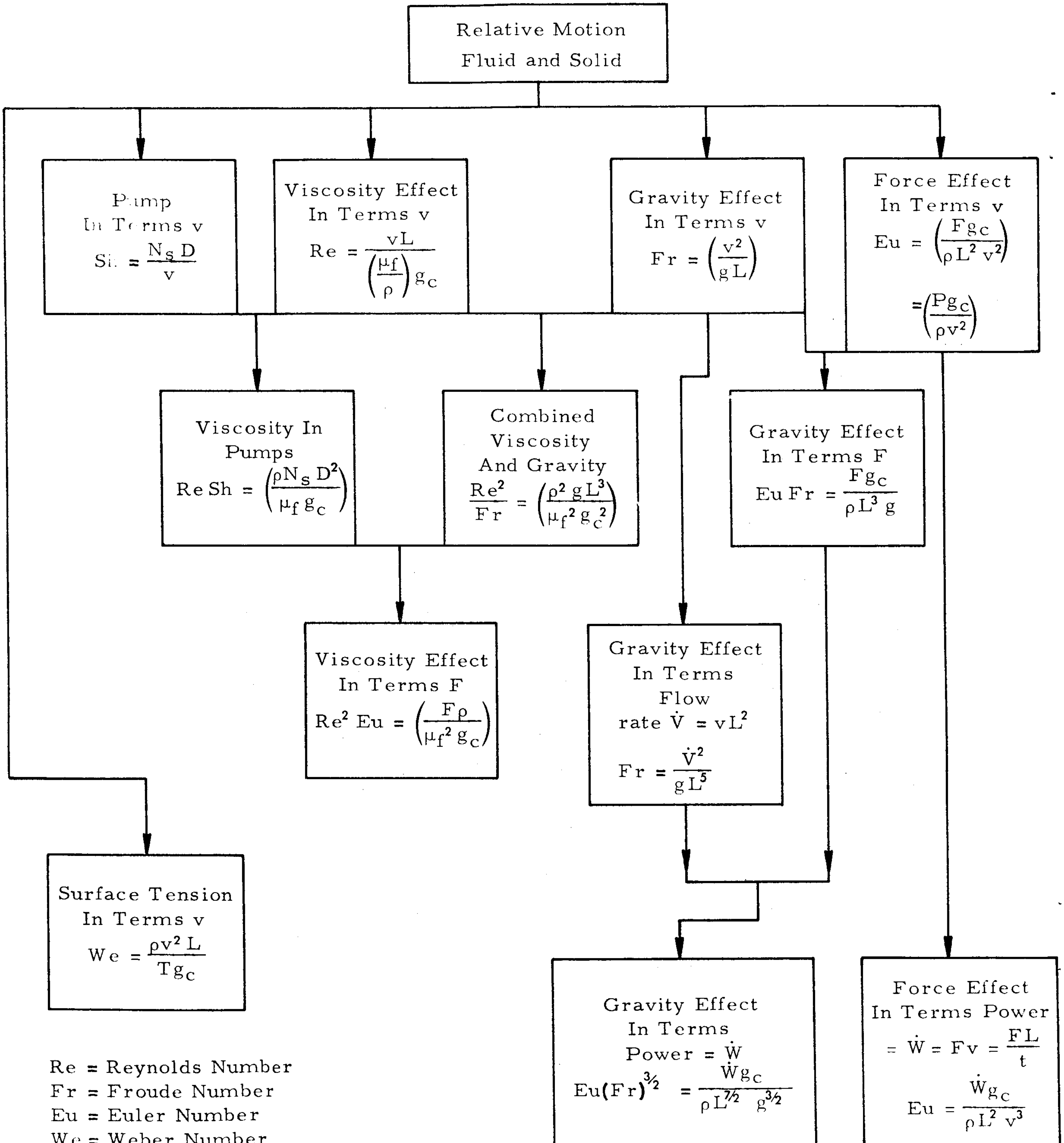
Strouhal Number. The Strouhal Number Sh is essentially a conversion factor, useful for pumps for example, for converting velocity v to pump speed that will produce that velocity. Pumps usually pump viscous fluids with negligible gravity effects, so that Re and Sh is usually the combination of interest. The length dimension of significance in a pump is rotor diameter D.

Re and Sh $$(Re)(Sh) = \left(\frac{\rho v D}{\mu_f g_c}\right)\left(\frac{N_s D}{v}\right) = \left(\frac{\rho N_s D^2}{\mu_f g_c}\right) = 1$$ if each term is a ratio

Weber Number. Under conditions of capillary flow surface tension effects appear in terms of the Weber Number.

Tabulation. The following tabulation summarizes relationships. Similar tabulations could be made up in other domains.

Fluid flow similarity.



Re = Reynolds Number
Fr = Froude Number
Eu = Euler Number
We = Weber Number
Sh = Strouhal Number

Model pump example. A model pump one and one-half times the size of the prototype pumps air to simulate with respect to viscosity effects the prototype pump pumping oil at 3600 rpm. The kinematic viscosity of air is 16.0×10^{-5} ft^2/sec compared to a value of 200×10^{-5} ft^2/sec for the oil. Determine the speed of the model pump in rpm for similarity.

$$Re'Sh' = \frac{N_s'(D')^2}{\left(\frac{\mu_f}{\rho}\right)' g_c'} = 1 \qquad \text{(where } g_c' = 1\text{)}$$

$$(N_s)_m = \frac{N_s\left[\frac{\left(\frac{\mu_f}{\rho}\right)_m}{\left(\frac{\mu_f}{\rho}\right)}\right]}{\left(\frac{D_m}{D}\right)^2} = \frac{3600\left(\frac{16.0 \times 10^{-5}}{200 \times 10^{-5}}\right)}{(1.5)^2} = 128 \text{ rpm} \qquad \text{answer}$$

Model pipe example. A pipe model in which water of kinematic viscosity 1.35×10^{-5} ft^2/sec flows at 4 ft/sec is to simulate, with respect to viscosity effects, a 20 in. pipe in which air having a kinematic viscosity (μ_f/ρ) of 16.5×10^{-5} ft^2/sec flows at 7 ft/sec. Determine the proper model pipe diameter in inches.

$$(Re') = \frac{v'D'}{\left(\frac{\mu}{\rho}\right)'} = 1 \qquad D' = \frac{\left(\frac{\mu}{\rho}\right)'}{v'}$$

$$D_m = D\left(\frac{\mu}{\rho}\right)' v' = \frac{20\left(\frac{1.35 \times 10^{-5}}{16.5 \times 10^{-5}}\right)}{\left(\frac{4}{7}\right)} = 2.86 \text{ in.} \qquad \text{answer}$$

Example of model in viscous fluid. A 1/50 size model indicates a drag of 3.15 lbf in water of viscosity 2.35 lbf sec/ft^2 and density 62.4 lbm/ft^3. For similarity with respect to viscosity effects, determine the drag of the prototype in lbf in air of viscosity 3.75 lbf sec/ft^2 and density 0.075 lbm/ft^3.

$$(Re')^2(Eu') = \frac{F'\rho'}{(\mu_f')^2 g_c'} = 1 \qquad \text{where } g_c' = 1, \quad L' \text{ is not involved}$$

$$F = \frac{F_m(\rho')}{(\mu_f')^2} = \frac{3.15\left(\frac{62.4}{0.075}\right)}{\left(\frac{2.35 \times 10^{-5}}{3.75 \times 10^{-7}}\right)} = 0.667 \text{ lbf} \qquad \text{answer}$$

Ship model example. A 1/20 scale ship model tested in water has a velocity of 8.30 ft/sec and a drag of 10 lbf. For similarity with respect to gravity effects determine for the full size ship: (a) the velocity in ft/sec and (b) the drag in lbf.

(a) $$(Fr') = \frac{(v')^2}{g'\,L'} = 1 \qquad \text{(where } g' = 1.0\text{)}$$

$$\left(\frac{v_m}{v}\right)^2 = \left(\frac{L_m}{L}\right)$$

$$v = \frac{v_m}{\sqrt{\frac{L_m}{L}}} = \frac{8.30}{\sqrt{\frac{1}{20}}} = 27.1 \text{ ft/sec} \qquad \text{answer}$$

(b) $$Eu'\,Fr' = \frac{F'\,g_c'}{\rho'\,(L')^3\,g'} = 1 \qquad \left[\begin{array}{l} \text{where } g_c' = 1 \\ \quad g' = 1 \\ \quad \rho' = 1 \end{array}\right]$$

$$\frac{F_m}{F} = \left(\frac{L_m}{L}\right)^3$$

$$F = \frac{F_m}{\left(\frac{L_m}{L}\right)^3} = \frac{10}{\left(\frac{1}{20}\right)^3} = 10\,(20)^3 = 80{,}000 \text{ lbf} \qquad \text{answer}$$

Model with gravity and viscosity similarity. A 1/4 scale model is to simulate a prototype with respect to gravity and viscosity effects. For the same gravity determine the proper ratio of kinematic viscosity (μ_f/g).

$$\frac{(Re')^2}{Fr'} = \frac{g'\,(L')^3}{\left(\frac{\mu_f'}{\rho'}\right)^2 (g_c')^2} = 1 \qquad \begin{array}{l} \text{where } g' = 1 \\ \quad g_c' = 1 \end{array}$$

$$\left(\frac{\mu_f'}{\rho'}\right)^2 = (L')^3 \text{ thus } \left(\frac{\mu_f}{\rho}\right)'_m = (L')^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8} \qquad \text{answer}$$

STEFAN NUMBER. This parameter is more in the nature of a heat transfer ratio than a dimensionless number obtained as a result of dimensional analysis. Its use should therefore probably be discouraged.

$$Sf = \begin{pmatrix}\text{Stefan Parameter}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{(\text{Radiation Heat Transfer})}{(\text{Conduction Heat Transfer})}$$

$$= \frac{\left(C\,A\,T^4\,\frac{Btu}{hr}\right)}{\left(k\,A\,\frac{\Delta T}{\Delta L}\,\frac{Btu}{hr}\right)}$$

SOUND VELOCITY. Dimensional analysis is of little assistance as the direct solution is shorter and better.

Direct solution. The general energy equation is applicable comparing conditions at a point during and after passage of a kinetic energy sound wave (15, p. 528) with PE, Q and $(W/J) = 0$.

$$KE = H$$

$$\frac{1}{2}\left(\frac{m}{g_c}\right)\frac{v^2}{J} = mh \qquad \text{where m may be cancelled.}$$

Differentiating,

$$\frac{1}{2}\,\frac{2v}{g_c\,J}\,dv = dh \qquad \text{where } \frac{2}{2} \text{ may be cancelled}$$

where

$$dv = (dV)\,\frac{v}{V} \qquad \text{from} \qquad A = \frac{V}{v} = \frac{dV}{dv}$$

$$= \left(\frac{m\,du\,J}{P}\right)\frac{v}{V} \qquad \text{from} \qquad 0 = dq = dv + \frac{P\,du}{mJ} \quad \text{for an isentropic}$$

$$dV = \frac{m\,du\,J}{P}$$

or

$$\left(\frac{v}{g_c\,J}\right)\left(\frac{m\,du\,J}{P}\right)\frac{v}{V} = dh \qquad \text{where } \frac{J}{J} \text{ may be cancelled}$$

$$v^2 = g_c\left(\frac{dh}{du}\right)\frac{PV}{m}$$

$$= g_c kRT$$

$$v = \sqrt{g_c kRT} = \text{velocity of sound in a gas}$$

Dimensional analysis solution. Applicable equations are:

$$KE = H$$

$$\frac{1}{2}\frac{m}{g_c}\frac{v^2}{J} = mh = m\,C_p\,T$$

$$PV = mRT$$

$$P = \left(\frac{m}{V}\right)RT = \rho RT$$

$$PV^k = C$$

$$k = \frac{C_p}{C_v}$$

$$v\,\frac{ft}{sec} = C\left(\rho\,\frac{lbm}{ft^3}\right)^a\left(R\,\frac{ft\ lbf}{lbm\ F\,abs}\right)^b\left(T\ F\,abs\right)^c\left(C_p\,\frac{Btu}{lbm\ F\,abs}\right)^d$$

$$\left(C_v\,\frac{Btu}{lbm\ F\,abs}\right)^e\left(g_c\,\frac{lbm\ ft}{lbf\ sec^2}\right)^f\left(J\,\frac{ft\ lbf}{Btu}\right)^g$$

sec $\quad -1 = -2f \qquad\qquad f = \frac{1}{2}$

lbf $\quad 0 = b - f + g$

$= b - \frac{1}{2} + g \qquad b + g = \frac{1}{2}$

$\frac{1}{2} + g = \frac{1}{2} \qquad\qquad g = 0$

ft $\quad 1 = -3a + b + f + g$

$= -3a + f + b + g$

$= -3a + \frac{1}{2} + \frac{1}{2} \qquad\qquad a = 0$

lbm $\quad 0 = a - b - d - e + f$

$= 0 - b - d - e + \frac{1}{2} \qquad b + d + e = \frac{1}{2}$

F abs $\quad 0 = -b + c - d - e$

$= -b - d - e + c$

$= -\frac{1}{2} + c \qquad\qquad c = \frac{1}{2}$

$\underline{\text{Btu}}$ $\quad 0 = d + e + g \qquad = d + e + 0 \qquad e = -d$

$$= -b + (b + d + e) + b + (b + g)$$

$$= -b + \frac{1}{2} - b + \frac{1}{2} \qquad b = \frac{1}{2}$$

$$v = C\rho^0\, R^{\frac{1}{2}}\, T^{\frac{1}{2}}\, (C_p)^d\, (C_v)^{-d}\, (g_c)^{\frac{1}{2}}\, (J)^0$$

$$= \sqrt{g_c\, RT}\left(\frac{C_p}{C_v}\right)^d \quad \text{where } \frac{C_p}{C_v} = k \qquad C = 1$$

$$= \sqrt{g_c\, kRT} \quad \text{where d is assumed equal to } \frac{1}{2}$$

STANTON NUMBER. This dimensionless number occurs in convection heat transfer. It appears to be redundant in that it is a product of more basic dimensionless numbers.

$$St = \begin{pmatrix}\text{Stanton Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{(Nu)}{(Pe)} = \frac{\left(\frac{hD}{k}\right)}{\left(\frac{3600\, C_p\, \rho v D}{k}\right)}$$

$$= \frac{h}{3600\, C_p\, \rho v} = \frac{h\,\frac{\text{Btu}}{\text{hr ft}^2\text{ F}}}{\left(3600\,\frac{\text{sec}}{\text{hr}}\right)\left(C_p\,\frac{\text{Btu}}{\text{lbm F}}\right)\left(\rho\,\frac{\text{lbm}}{\text{ft}^3}\right)\left(v\,\frac{\text{ft}}{\text{sec}}\right)}$$

$$= \frac{\left(\frac{\dot{Q}}{A\,\Delta T}\right)}{3600\, C_p\, \rho v}$$

$$= \frac{Nu}{Re\ Pr}$$

Redundancy. The preceding is a form of the convection heat transfer equation.

$$Nu = f(Re,\ Pr)$$

Stanton Number as energy ratio. The Stanton and Nusselt Numbers can be interpreted as similar energy ratios by multiplying numerator and denominator by ΔT (4, p. 201). This is dimensionally valid but the ΔT are different.

$$St = \frac{h}{3600\ \rho\ C_p\ v} \qquad\qquad Nu = \frac{hD}{k}$$

$$= \frac{h\,\Delta T}{(\rho\ C_p \Delta T)\ 3600\ v} \qquad\qquad = \frac{h\ \Delta T}{\left(\frac{k\,\Delta T}{D}\right)}$$

$$= \frac{\text{(Total Heat Transfer)}}{\text{(Convection Heat Transfer)}} \qquad\qquad = \frac{\text{(Total Heat Transfer)}}{\text{(Conductive Heat Transfer)}}$$

STOKES LAW. Stokes'law for the velocity of a sphere falling in a fluid is

$$\frac{2}{9}\left[\left(\frac{w}{V}\right)\text{sphere} - \left(\frac{w}{V}\right)\text{liquid}\right] = \frac{\mu v}{R^2}$$

$$\frac{2}{9}\left(\Delta\frac{w}{V}\right) = \frac{\mu v}{R^2} \quad \text{where} \quad \left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right) \text{ or } \frac{w}{V} = \frac{m}{V}\frac{g}{g_c} = \rho\left(\frac{g}{g_c}\right)$$

$$\frac{2}{9}\ \rho\ \left(\frac{g}{g_c}\right) = \frac{\mu v}{R^2} \quad \text{where } \rho \text{ is a difference of densities.}$$

$$\frac{2}{9}\left(\rho\ \frac{lbm}{ft^3}\right)\left(\frac{g\ \frac{ft}{sec^2}}{g_c\ \frac{lbm}{lbf}\ \frac{ft}{sec^2}}\right) = \frac{\left(\mu_f\ \frac{lbf\ sec}{ft^2}\right)\left(v\ \frac{ft}{sec}\right)}{(R\ ft)^2}$$

Expressed as a possible dimensionless number:

$$SL = \left(\begin{matrix}\text{Stokes Law Number}\\ \text{Dimensionless}\end{matrix}\right)$$

$$1 = \frac{9}{2}\left[\frac{\mu_f\, v\, g_c}{\rho\, R^2\, g}\right] = \frac{\text{(viscous force)}}{\text{(gravity force)}}$$

$$= 9\left[\frac{\mu g_c}{\rho v D}\right]\left[\frac{v^2}{gR}\right] \qquad \text{where } D = 2R$$

$$= 9\left[\frac{1}{Re}\right]\left[Fr\right] = 9\,\frac{(Fr)}{(Re)}$$

STROUHAL NUMBER. This parameter occurs in vibration pump and propellor problems involving frequency N_s cycles/sec or N rpm. Sedov (3, p. 58) incorrectly calls this the Strouhaille Number.

$$(Sh) = \begin{pmatrix} \text{Strouhal Number} \\ \text{Dimensionless} \end{pmatrix}$$

$$= \left(\frac{N_s L}{v}\right) = \frac{\left(N_s \frac{\text{cycles}}{\text{sec}}\right)\left(L \frac{\text{ft}}{\text{cycle}}\right)}{\left(v \frac{\text{ft}}{\text{sec}}\right)} = \frac{\left(N_s \frac{1}{\text{sec}}\right)\left(L \text{ ft}\right)}{\left(v \frac{\text{ft}}{\text{sec}}\right)}$$

$$= \frac{NL}{60\,v} = \frac{\left(N \frac{\text{rev}}{\text{min}}\right)\left(L \frac{\text{ft}}{\text{rev}}\right)}{\left(60 \frac{\text{sec}}{\text{min}}\right)\left(v \frac{\text{ft}}{\text{sec}}\right)} = \frac{\left(N \frac{1}{\text{min}}\right)\left(L \text{ ft}\right)}{\left(60 \frac{\text{sec}}{\text{min}}\right)\left(v \frac{\text{ft}}{\text{sec}}\right)}$$

Strouhal Number for propellors. The ratio of forward velocity v to tip velocity = $\pi N_s D$ is of significance and results in an advance ratio (24, p. 88).

$$\text{Advance Ratio} = \left(\frac{v}{N_s D}\right) = \frac{v \frac{\text{ft}}{\text{sec}}}{\left(N_s \frac{1}{\text{sec}}\right)\left(D \text{ ft}\right)} = \frac{1}{Sh}$$

Dimensionally the advance ratio has the units of the reciprocal of the Strouhal Number.

SUFFICIENCY. Dimensional analysis is a powerful analytical tool in determining relationships, but there are grave limitations requiring knowledge and judgement concerning utilization on a parituclar configuration.

The kind of physical properties involved in a particular procedure must be known either from experiment, analogy or perhaps a little intuition. There is no guarantee of sufficiency. If all properties are not included, the results will be faulty. If irrelevant properties are included, the results will certainly be confusing and probably faulty.

The numerical constant C and the numerical exponents a, b, c, etc., are not determined and must be found empirically.

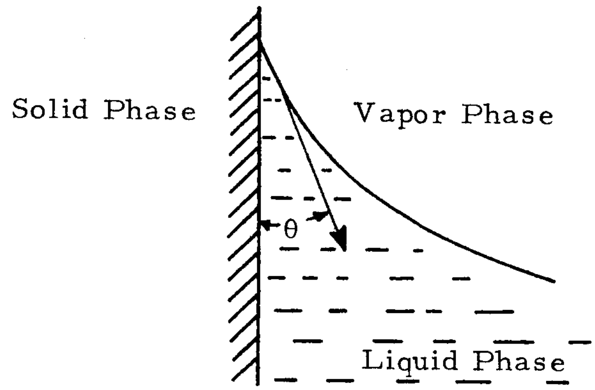
It may be expected that dimensional analysis equations will have limited ranges within which certain dimensionless numbers should be confined because of limitation of the domination of certain physical variables in certain ranges. These ranges must be determined by experiment. For example, the general convection heat-transfer equation may be divided into certain ranges in which laminar flow, turbulent flow or natural-convection physical properties and associated dimensionless numbers are dominant.

SURFACE TENSION. Consider a needle-like "line" of length L in a fluid surface interface. Surface tension (T lbf/ft) is defined as the force in the surface lbf per ft length of line that acts to pull the line apart. Of the various symbols, T, y, σ, in the literature T is preferred.

Representative surface tensions (Ref. 11) are:

Substance	Temperature	$T \frac{gmass}{sec^2}$	$T \frac{lbf}{ft}$
Water	68 °F	72.75	0.00498
Mercury	68 °F	484	0.0356
Helium	-272 °F	0.147	

When in contact with a solid the direction of the surface tension force depends on the contact or wetting angle, θ, as shown in the figure.

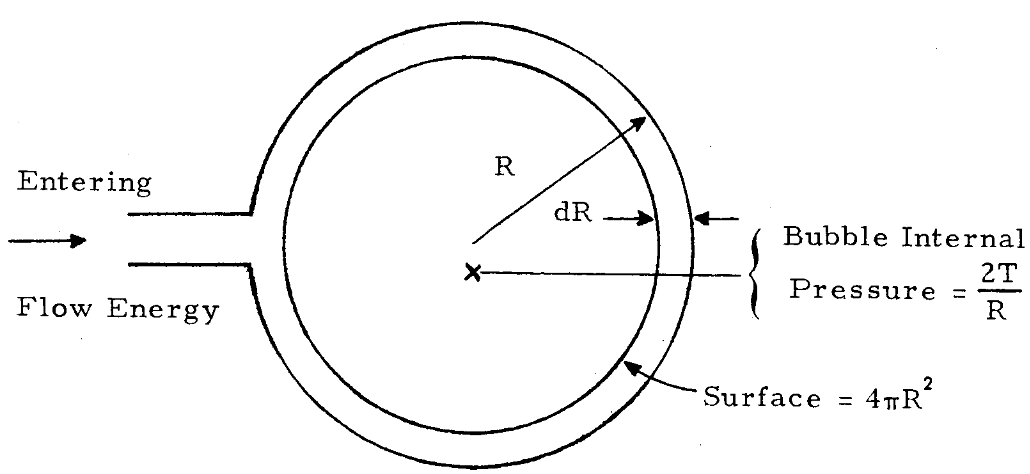


SURFACE WETTING

Surface tension as work per area. A second aspect is of significance. If this surface tension force acts through a distance, surface tension may be thought of as the work TL required to create new surface area, for instance, expanding a bubble.

$$T = \frac{F \text{ lbf } L \text{ ft}}{A \text{ ft}^2} = \left(\frac{FL}{A}\right)\frac{\text{ft lbf}}{\text{ft}^2} = \left(\frac{FL}{A}\right)\frac{\text{lbf}}{\text{ft}}$$

One proof of this bubble expansion is as in the figure.



BUBBLE EXPANSION

A bubble of radius R expands dR in radius or dV in volume requiring a work FL equal to the flow energy P dV entering through an imaginary duct. This produces new bubble surface.

$$\frac{F \text{ lbf } L \text{ ft}}{A \text{ ft}^2} = \frac{P\, dV}{4\pi(R + dR)^2 - R^2}$$

$$= \frac{\frac{2T}{R}(4\pi R^2)dR}{4\pi(2R\, dR + dR^2)}$$

where the second order dR^2 may be neglected as negligible.

$$= \frac{8T\, \pi R\, dR}{8\pi R\, dR}$$

$$= T$$

Surface tension conversion. The preferred units of surface tension are T force/length. The literature sometimes gives surface tension values in mass units T_m mass/area = T/g_c. The conversion factor is

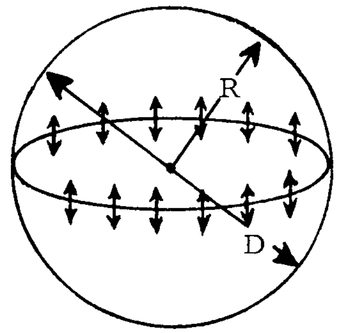
$$\frac{\left(1\ \frac{\text{grammass}}{\text{sec}}\right)}{g_c} = \frac{\left(1\ \frac{\text{grammass}}{\text{sec}^2}\right)\left(1000\ \frac{\text{gmf}}{\text{kgf}}\right)\left(2.54\ \frac{\text{cm}}{\text{in.}}\right)\left(12\ \frac{\text{in.}}{\text{ft}}\right)}{\left(453\ \frac{\text{gmf}}{\text{lbf}}\right)\left(1000\ \frac{\text{gmmass}}{\text{kgmass}}\right)\left(981\ \frac{\text{kgm cm}}{\text{kgf sec}^2}\right)}$$

$$= \frac{6.85}{10^5}\frac{\text{lbf}}{\text{ft}}$$

Expressed as a conversion factor:

$$1 = \frac{\left(\frac{T_m}{g_c}\right)}{T} = \frac{6.85\left(\frac{\text{lbf}}{\text{ft}}\right)}{10^5\left(\frac{\text{grammass}}{\text{sec}}\right)}$$

Pressure of surface tension. A stationary bubble immersed in its own fluid, such as an air bubble in air, has an internal pressure as a result of surface tension holding the bubble together as in the figure.



Volume Sphere $= \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$

Surface Sphere $= 4\pi R^2 = \frac{\pi D^2}{4}$

STATIONARY BUBBLE PRESSURE

The surface tension T around a circumference πD contains the internal pressure P_T over an area $\pi D^2/4$.

$$T(\pi D) = P_T \left(\frac{\pi D^2}{4}\right)$$

$$P_T = \frac{4T}{D} = \frac{2T}{R}$$

SYMBOLS GENERAL. Special symbols are defined at the place where they are used. General symbols are as follows:

a = acceleration

a = thermal diffusivity, $ft^2/hr = \left(\frac{k}{\rho C_p}\right)$

A = area, ft^2, frequently $A = L^2$

B = coefficient of volume expansion ft^3/ft^3F abs

C_p = specific heat and constant pressure, Btu/lbm F abs

C = any constant in general. Usually dimensionless

D = diameter, ft. In expressions not having diameter use L

= equivalent diameter for conduits = 4A/P

D_m = mass diffusivity, ft^2/hr

E = electric field strength, volt/ft

f = fcn = function of

F = force, lbf

FE = flow energy, ft lbf

g = gravity acceleration = 32.2 ft/sec^2 on surface of earth

g_c = acceleration constant = 32.2 lbm ft/lbf sec^2

h = surface heat conductance, Btu/ft^2 hr F

H = height, ft

H = magnetic field strength, amp/ft

I = electric current, amp

J = conversion factor, 778 ft lbf/Btu

J_m = metric conversion factor, 0.738 ft lbf/joule

k = specific heat ratio, cp/cv, dimensionless

KE = kinetic energy, ft lbf

L = length, ft. May also use diameter D

m = mass, lbm

N_s = revolutions or cycles, 1/sec

N = revolutions per minute, 1/min

P = perimeter, ft. Wetted perimeter or perimeter through which heat flows

P = pressure, lb/ft^2

PE = potential energy, ft lbf

q = electric charge, coulomb = amp sec

q = heat valve, Btu/lbm

Q = heat, Btu

Q = energy, joules = coulomb sec = amp volt sec

R = radius ft. Sometimes a length different from L

t = time, sec

$T = \sigma$ = surface tension, lbf/ft

T = temperature, F abs

ΔT = temperature difference, F or F abs

$\dot{U}$ = chemical reaction rate (lbm/sec)/lbm

v = velocity, ft/sec

v_s = reference velocity, frequently velocity of sound, ft/sec

V = volume, ft^3

V = electric potential, volts

w = weight, lbw

W = work, ft lbf

Δ = difference such as ΔP, ΔT, $\Delta\rho$, ΔV

$\rho = (m/V)$ = mass density, lbm/ft^3

λ = molecular free path, ft

μ = viscosity, expressible in various units. See Viscosity

μ_f = viscosity in basic units, lbf sec/ft^2

μ_m = viscosity in mass units, lbm/ft hr = $(3600\ \mu_f\, g_c)$

μ_p = magnetic permeability, lbf/amp^2

σ = electric conductivity, amp/volt ft

ϵ = electrical permittivity, $amp^2\ sec^2/lbf\ ft^2$

Dot over symbol signifies per sec, thus $\dot{V}$ ft^3/sec, $\dot{W}$ ft lbf/sec, etc.

SYSTEM OF UNITS. It is customary to adopt dimension systems in which certain dimensions are fundamental and to assume that the remaining dimensions are derived from these basic dimensions. It is thus possible to assume any basic dimension from one dimension (usually time) to an infinite number (no derived dimensions). Typical systems are:

1. mLt System

   m mass
   L length
   t time
   T temperature
   (F force, H heat, etc. expressed in terms of m, etc.

2. FLt System

   F force
   L length
   t time
   T temperature
   (m mass, H heat expressed in terms F, etc.

Basic Equations Required Are
$F = ma$
$H = FL$

3. mLtH System

   Add H heat to System 1

4. FLtH System

   Add H heat to System 2

Basic Equation Required Is $F = ma$

5. FmLtH System

   This system, in increasing present use, is compatible with the present tendency to clearly distinguish lbf from lbm with a corresponding reluctance to express lbm in lbf units.

No Basic Equation Required

6. Two Dimension Lt System

   This system is not in use but is given as an illustration that a so-called measuring system may have any number of dimensions providing certain fundamental physical laws are used. If the fundamental acceleration law $F = ma$ is used to eliminate F by expressing it in terms of m, the additional fundamental gravity law $F = mM/L^2$ may be used to eliminate mass so that both force and mass may be eliminated to express mass in terms of time and distance only.

Basic Equations Required Are
$F = ma$
$H = FL$
$$F = \frac{mM}{L^2}$$

$$ma = F = \frac{Mm}{L^2}$$

$$M = aL^2 \text{ in } \left(\frac{ft}{sec^2}\right)\left(ft^2\right) \text{ or } \frac{ft^3}{sec^2} \text{ units}$$

THERMAL DIFFUSIVITY. This number occurs in transient heat transfer involving conduction and heat capacity.

$$a = \frac{k}{\rho c_p}$$

$$a \frac{ft^2}{hr} = \frac{\left(k \frac{Btu\ ft}{hr\ ft^2\ F}\right)}{\left(\rho \frac{lbm}{ft^3}\right)\left(c_p \frac{Btu}{lbm\ F}\right)}$$

THOMA NUMBER. The Thoma number is related to pump cavitation (see Cavitation).

$$Th = \begin{pmatrix}\text{Thoma Number}\\ \text{Dimensionless}\end{pmatrix}$$

$$= \frac{P}{\Delta P} = \frac{(P_1 - P_V)\text{ psia}}{(P_2 - P_1)\text{ psia}}$$

where

$P = (P_1 - P_V)$ = allowable pressure reduction of fluid because of velocity

$\Delta P = (P_2 - P_1)$ = pressure difference across pump

psia = pounds per square inch absolute

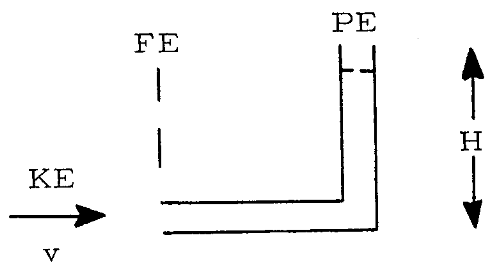
P_1 = total inlet pump pressure (or possibly centerline of pump)

P_2 = total outlet pump pressure

P_V = vapor pressure of liquid

UNIT SYSTEMS. See System of Units.

<u>VELOCITY PRESSURE</u>. A fluid moving at velocity v possesses a kinetic energy KE as shown in the following figure. When brought to rest by an impact tube, the energy becomes flow energy FE and exerts a velocity pressure P_v. Conversion is then made to potential energy PE as in the figure.



KE TO PE CONVERSION

$$FE = KE$$

$$PV = \frac{1}{2}\frac{mv^2}{g_c}$$

$$P_v = \left(\text{velocity pressure, } \frac{lb}{ft^2}\right)$$

$$= \frac{1}{2}\left(\frac{m}{V}\right)\frac{v^2}{g_c} = \frac{1}{2}\left(\frac{\rho}{g_c}\right)v^2$$

Alternately KE = PE

$$\frac{1}{2}\left(\frac{m}{g_c}\right)v^2 = \frac{1}{2}\frac{wv^2}{g} = wH = w\frac{P}{\left(\frac{w}{v}\right)}$$

because $P = H\left(\frac{w}{V}\right)$.

$$P_v = \frac{1}{2}\left(\frac{w}{V}\right)\frac{v^2}{g} = \frac{1}{2}\left(\frac{w}{V}\frac{lbf}{ft^3}\right)\frac{\left(v\,\frac{ft}{sec}\right)^2}{\left(g\,\frac{ft}{sec^2}\right)}$$

where $\rho = \left(\frac{m}{V}\right)$ = mass density, $\frac{lbm}{ft^3}$

VIBRATION. For vibration of fluids N_s, confined by walls w having radius R and length L, Chamberlain (25) gives the following. Usual notation is employed with v_f and v_w the velocity of sound in fluid and walls respectively and P the pressure of sound. The vibration effect VE is presumably equal to 1. The energy absorption E is defined as mv^2 where m is the mass and v is the spatial maximum mean square flexural velocity of the section under study.

$$VE = C\left(P\,\frac{lbf}{ft^2}\right)^a \left(N_s\,\frac{1}{sec}\right)^b \left(\rho_f\,\frac{lbm}{ft^3}\right)^c \left(\rho_w\,\frac{lbm}{ft^3}\right)^d \left(R\ ft\right)^e \left(L\ ft\right)^f$$

$$\left(v_w\,\frac{ft}{sec}\right)^g \left(v_f\,\frac{ft}{sec}\right)^h \left(E\ lbm\,\frac{ft^2}{sec^2}\right)^i \left(g_c\,\frac{lbm\ ft}{lbf\ sec^2}\right)^j$$

lbf	$0 = a - j$	$j = a$
lbm	$0 = c + d + i + j$	$c = -d - i - a$
sec	$0 = -b - g - h - 2i - 2j$	
	$= -b - g - h - 2i - 2a$	$b = -g - h - 2i - 2a$
ft	$0 = -2a - 3c - 3d + e + f + g + h + 2i + j$	
	$= -2a(+ 3d + 3i + 3a) - 3d + e + f + g + h + 2i + a$	
	$= 2a + e + f + g + h + 5i$	$e = -2a - f - g - h - 5i$

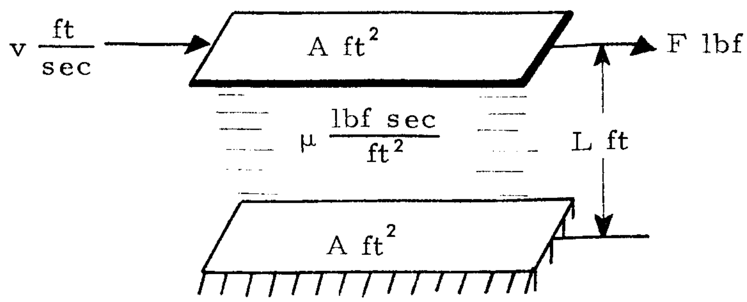
$$VE = C(P)^a (N_s)^{-g - h - 2i - 2a} (\rho_f)^{-d - i - a} (\rho_w)^d (R)^{-2a - f - g - h - 5i}$$

$$(L)^f (v_w)^g (v_f)^h (E)^i (g_c)^a$$

$$VE = C\left(\frac{Pg_c}{N_s^2\,\rho_f R^2}\right)^a \left(\frac{\rho_w}{\rho_f}\right)^d \left(\frac{L}{R}\right)^f \left(\frac{v}{N_s R}\right)^g \left(\frac{v_f}{N_s R}\right)^h \left(\frac{E}{N_s^2\,\rho_f R^5}\right)^i$$

VISCOSITY. In the literature the symbol μ is generally used to represent viscosity, regardless of the units in which it is expressed. In this text, subscripts are added to indicate viscosity expressed in particular units.

Fundamentally, viscosity is defined as a shear-force or stress required per unit area to produce a velocity-gradient or shear-rate expressed as a unit-change of velocity at a unit-distance from another shear plane as shown in the figure.



In order to avoid inertia effects, the fluid is considered to be in laminar (also known as streamline or viscous) flow. Thus the engineering units are:

$$\begin{pmatrix}\text{Viscosity}\\ \text{Of A Fluid}\end{pmatrix} = \frac{\left(\text{shear stress } \frac{\text{lbf}}{\text{ft}^2}\right)}{\left(\text{velocity gradient } \frac{v\ \frac{\text{ft}}{\text{sec}}}{\text{L ft}}\right)} = \frac{\left(\text{shear stress } \frac{\text{lbf}}{\text{ft}^2}\right)}{\left(\begin{matrix}\text{shear rate}\\ \text{per ft}\end{matrix}\ \frac{\frac{L}{t}\ \frac{\text{ft}}{\text{sec}}}{\text{L ft}}\right)}$$

$$= \frac{\begin{pmatrix}\mu_f \text{ shear force lbf}\\ \text{per}\\ \text{unit area of plane ft}^2\end{pmatrix}}{\begin{pmatrix}\text{unit velocity of plane}\\ \text{per}\\ \text{unit dist from next plane}\end{pmatrix}} \text{ per} = \frac{\left(\frac{\mu_f \text{ lbf force}}{1 \text{ ft}^2 \text{ area}}\right)}{\left[\frac{\left(1\ \frac{\text{ft}}{\text{sec}}\ \text{velocity}\right)}{(1 \text{ ft distance})}\right]}$$

$$= \left(\mu_f\ \frac{\text{lbforce}}{\text{ft}^2\,\text{area}}\right)\left(\frac{\text{ft distance}}{\frac{\text{ft}}{\text{sec}}\ \text{velocity}}\right) = \left[\mu_f \left(\frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{\text{ft sec}}{\text{ft}}\right)\right]$$

$$= \left[\mu_f\ \frac{\text{lbf sec}}{\text{ft}^2}\right]$$

A Newtonian fluid is one in which the viscosity μ is independent of the fluid velocity v.

Viscosity is fundamentally a drag or shear force and should be expressed in force units as is done in the SI International System of units. Considered as an isolated dimension or property, to express it in terms of an amount of matter tends to obscure its true nature. It therefore appears best to distinguish μ_f from μ_m and to use μ_f in force units in dimensional analysis, a procedure different from conventional procedures.

Viscosity is sometimes called absolute or dynamic viscosity to distinguish it from a so-called kinematic viscosity = (viscosity/density), a designation that should be avoided as unnecessary.

For the notation of the figure the shear force F required to pull the moving plane of A ft^2 at a velocity v ft per sec at a distance L from the fixed plane A ft^2 is given by:

$$F \text{ lbf} = \mu A \frac{v}{L}$$

$$= \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(A \text{ ft}^2\right)\frac{\left(v \dfrac{\text{ft}}{\text{sec}}\right)}{(L \text{ ft})}$$

VISCOSITY CONVERSION

Symbol And Units In This Text	Equivalent where $g_c = \left[32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right]$	Comments On Symbol Of Column 1
$\mu_f \frac{\text{lbf sec}}{\text{ft}^2}$	$= \frac{\left(\mu_m \frac{\text{lbm}}{\text{ft hr}}\right)}{\left(3600 \frac{\text{sec}}{\text{hr}}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)} = \frac{\mu_{c_p} \text{ centipoises}}{\left[47800 \frac{\text{centipoises}}{\left(\frac{\text{lbf sec}}{\text{ft}^2}\right)}\right]}$	μ_f is basic preferred unit.
$\mu_m \frac{\text{lbm}}{\text{ft hr}}$	$= \left(3600\, \mu_f g_c\right) = \left(3600 \frac{\text{sec}}{\text{hr}}\right)\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right) = (\Pr) \frac{k}{c_p}$ $= (\mu_{c_p} \text{ centipoises}) \left[2.42 \frac{\left(\frac{\text{lbm}}{\text{ft hr}}\right)}{(\text{centipoise})}\right]$	Frequently used. Undesirable force quantity expressed in mass units.
$\mu_s \frac{\text{lbm}}{\text{ft sec}^2}$	$= \left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right) = \frac{\mu_{c_p} \text{ centipoises}}{\left[1488 \frac{\text{centipoises}}{\left(\frac{\text{lbm}}{\text{ft sec}}\right)}\right]}$	Undesirable. Express as $\mu_f g_c$.
$\mu_h \frac{\text{lbf hr}}{\text{ft}^2}$	$= \frac{\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)}{\left(3600 \frac{\text{sec}}{\text{hr}}\right)}$	Undesirable. Express as $\frac{\mu_f}{\left(3600 \frac{\text{sec}}{\text{hr}}\right)}$
μ_{c_p} centipoises = (metric unit)	$= \mu_{c_p} \left(\frac{\text{centigrams of mass}}{\text{cm sec}}\right) = \left(\mu_p \text{ poises}\right)\left(100 \frac{\text{centipoise}}{\text{poise}}\right)$	Force expressed in mass units.
$\left(\frac{\mu_s}{\rho}\right) \frac{\text{ft}^2}{\text{sec}}$ = (kinematic viscosity)	$= \frac{\left(\mu_s \frac{\text{lbm}}{\text{ft sec}}\right)}{\left(\rho \frac{\text{lbm}}{\text{ft}^3}\right)} = \frac{\left(\mu_f \frac{\text{lbf sec}}{\text{ft}^2}\right)\left(32.2 \frac{\text{lbm ft}}{\text{lbf sec}^2}\right)}{\left(\frac{m}{V} \frac{\text{lbm}}{\text{ft}^3}\right)}$	Undesirable. Express as $\frac{\mu_f g_c}{\left(\frac{m}{V}\right)}$

WATT. See under Joule Per Sec.

WAVES ON SURFACE. This was treated as a special case under the Fluid Flow Domain to obtain:

$$\left(\frac{\text{L wavelength}}{\text{H height}}\right) = \text{fcn (Fr We)}$$

Waves. For waves on the surface of liquids Ipsen (4, p. 166) gives:

$$(Fr) = \left(\frac{L}{D}\right)^{e} (We)^{p}$$

where:

$$\left(\frac{L}{D}\right) = \frac{\text{(wavelength)}}{\text{(depth of liquid)}}$$

This may be written in terms of Bond Number.

$$Bo = \left(\begin{matrix}\text{Bond Number}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \left(\frac{We}{Fr}\right) = \left(\frac{D}{L}\right)$$

$$= \frac{\left(\frac{\rho v^2 L}{T g_c}\right)}{\left(\frac{v^2}{Lg}\right)} = \frac{\left(\frac{m}{L^3 g_c}\right)\left(\frac{v^2}{1}\right)\left(\frac{L}{T}\right)}{\frac{v^2}{Lg}} = \frac{\left(\frac{w}{L^3 g}\right)\frac{L^2}{T}}{\frac{1}{g}} = \frac{w}{TL}$$

$$= \left(\frac{w}{TL}\right) = \frac{\text{(gravity force)}}{\text{(surface tension force)}}$$

WEBER NUMBER. This parameter introduces the effect of surface tension T.

$$(We) = \left(\begin{matrix}\text{Weber Number}\\ \text{Dimensionless}\end{matrix}\right)$$

$$= \frac{\rho v^2 D}{T g_c}$$

$$= \frac{\left(\rho\ \frac{lbm}{ft^3}\right)\left(v^2\ \frac{ft^2}{sec^2}\right)\left(L\ ft\right)}{\left(T\ \frac{lbf}{ft}\right)\left(32.2\ \frac{lbm\ ft}{lbf\ sec^2}\right)}$$

The significant length dimension is given as L. In a particular application other lengths may be more applicable. Sometimes a diameter D is appropriate.

Surface-tension phenomena are involved in wave motion.

(We) as a force ratio. This parameter expresses the ratio of the inertial force or force of acceleration required to accelerate a particle of fluid to the surface tension force on the surface of the fluid (24, p. 93, 28, p. 168). The Weber Number is therefore applicable to moving fluids having a surface.

$$We = \frac{\text{(force of acceleration)}}{\text{(force of surface tension)}} = \frac{\left(\frac{m}{g_c}\right) a}{TL}$$

$$= \frac{m\left(\frac{v}{t}\right)}{TLg_c} = \left(\frac{\frac{m}{L^3} L \left(\frac{L}{t}\right) vL}{TLg_c}\right) = \left(\frac{\rho v^2 L}{Tg_c}\right)$$

(We) as a pressure ratio. For a bubble relation velocity of bubble and surrounding fluid produces a velocity process. Surface tension produces an internal pressure.

$$(We) = \frac{8\text{(inertia pressure)}}{\text{(surface tension pressure)}} = \frac{\text{(velocity pressure)}}{\text{(surface tension pressure)}}$$

$$= \frac{8\text{(velocity pressure at surface of bubble)}}{\text{(inside bubble pressure due to surface tension)}}$$

$$= \frac{8\left(\frac{1}{2}\frac{\rho}{g_c} v^2\right)}{\left(\frac{4T}{D}\right)} = \frac{\rho v^2 D}{Tg_c}$$

WEIGHT. A mass under force due to the attraction of gravity tends to accelerate in a downward direction. Its weight is determined by Newton acceleration law written as a unit-consistent equation in engineering units.

In general

$$F = \left(\frac{m}{g_c}\right) a$$

Under gravity

$$w = \left(\frac{m}{g_c}\right) g \qquad \text{or} \qquad \left(\frac{w}{g}\right) = \left(\frac{m}{g_c}\right)$$

$$w \text{ lbf} = \left(\frac{m \text{ lbm}}{32.2 \dfrac{\text{lbm ft}}{\text{lbf sec}^2}}\right)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)$$

where

w = weight, lbw

= force downward, lbf

On the surface of the earth where g = 32.2 ft/sec^2 an amount of mass 1 lbm exerts a force downward of 1 lbf. At other locations 1 lbm will have a different numerical value of weight lbm = lbf downward.

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DISTRIBUTION

R-DIR
R-TEST-DIR
R-TEST-C
R-TEST-C
R-TEST-S
R-TEST-SPS
R-P&VE-DIR
R-P&VE-DIR
R-P&VE-M
R-P&VE-P
R-P&VE-P
R-P&VE-P
R-P&VE-PA
R-P&VE-PE
R-P&VE-PEC
R-P&VE-PEC
R-P&VE-PM
R-P&VE-PT
K-DF
MS-H
MS-IP
MS-IPL
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