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ASTRODYNAMICS, GUIDANCE AND CONTROL REVIEW # 1

By Astrodynamics and Guidance Theory Division
Aero-Astrodynamics Laboratory

NASA

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ASTRODYNAMICS, GUIDANCE AND CONTROL

Review #1

Astrodynamics & Guidance Theory Division
Aero-Astrodynamics Laboratory
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

ABSTRACT

This volume is a collection of papers designed as a summarial review of the activities of the Astrodynamics and Guidance Theory Division of the Aero-Astrodynamics Laboratory. These papers present a cross section of the studies being conducted by, and under the supervision of, the three branches and one office within the division. In addition, suggested areas for future research are presented in the context of these reports.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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ASTRODYNAMICS, GUIDANCE AND CONTROL

REVIEW #1

Astrodynamics & Guidance Theory Division

AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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REVIEW #1

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SUMMARY

This volume is a collection of papers designed as a summarial review of the activities of the Astrodynamics and Guidance Theory Division of the Aero-Astrodynamics Laboratory. These papers present a cross section of the studies being conducted by, and under the supervision of, the three branches and one office within the division. In addition, suggested areas for future research are presented in the context of these papers.

INTRODUCTION

The following material is intended to present a summary of the efforts now being expended in and under the supervision of the Astrodynamics and Guidance Theory Division of the Aero-Astrodynamics Laboratory. Although no designed to be an all-inclusive review of the division's activities, these papers do represent a cross-section of the studies being conducted by the three branches and one office that compose the sub-structure of the organization.

As the papers herein are reviewed by the reader, the interrelatedness of the duties and activites performed within the division becomes apparent. While each branch and office has a separate area of impact, each is also of necessity aware of and, to some degree, dependent upon the work conducted by other groups in the division. This close association of people and purpose is of particular benefit when the division is called on to participate in mission-oriented studies. It should be noted, however, that although often engaged in such mission-oriented activities, the division to a great extent remains a research directed organization.

Leading the research efforts of the division is the Scientific Advisory Office whose pure scientific research has a significant effect, not only on the work of the rest of the division, but on the activities of the whole scientific community as well. An excellent background study in one specific research area is presented by Mr. C. C. Dearman, Jr. in his paper on stability theory. In addition, Mr. Dearman reviews past and present study contracts in the area. Dr. R. W. Gunderson, on the other hand, dwells more heavily on the results being obtained and studies being conducted in-house as the Scientific Advisory Office pursues solutions to research problems in stability and control theory. Other areas under study by this office but not included in this review include Dr. R. K. J. Festa's work in studying the perturbations of artificial satellites of the planet Mars and the recently initiated activities in the study of stochastic processes.

In the field of astrodynamics, computer programs designed to chart the path of a spacecraft are indispensable tools when conducting parametric studies and flight analyses. It is small wonder therefore that great effort is expended by the Astrodynamics Branch in writing, perfecting, and utilizing such programs. Basically, there are two types of trajectory computation: the conic approximation used extensively for general survey studies and the integrating method used when more precise results are desired. A thorough and detailed analysis of the latter is presented by Mr. J. R. Duncan, Jr. Research in the Astrodynamics Branch is by no means limited to deck design and parametric studies. Extensive work is also being carried out in solar radiation effects, time line studies, orbit transfers, and numerical integration method studies.

A Guidance Theory Branch activities review is presented by Mr. W. E. Causey in a paper that places special emphasis on the enumeration and explanation of the characteristics inherent in a good guidance scheme. In addition, Mr. Causey sets forth a course of future study in the areas of injection, midcourse and terminal guidance. That the operation of a guidance system incorporates various divisions of effort is discussed in detail by Mr. H. L. Ingram. Mr. Ingram's paper also points out some recent research achievements in the field. Another recent development is a procedure called QUOTA which is discussed by Mr. R. R. Burrows. QUOTA, which can be used as an on-board guidance system, has multi-mission capability and holds great promise in guidance applications. Not represented in this review, but of active interest to the Guidance Theory Branch, are such areas as launch probability studies and launch window expansions.

The paper by Mr. R. E. Burns not only provides insight into the work which has been and is being carried on in the analytic solution of optimal guidance problems, but it also serves to show the association of the activities of the Guidance Theory and Optimization Theory Branches. The latter organization conducts research in a wide range of disciplines including optimization, functional analysis, stability and control. A general review of control system design efforts and the work being conducted in this area by the Optimization Theory Branch is provided by Mr. J. R. Redus. Mr. Redus also points out areas of research for future concentration. A more detailed analysis of load relief control is presented by Mr. J. M. Livingston as an example of a specific application of control systems research.

Although this summarial group of reports is intended to be primarily a review of past achievements and present activities of the Astrodynamics and Guidance Theory Division, many areas for future research are also discussed - areas which are being and will be explored in detail as present theories are extended and new techniques devised to meet the challenges set forth.

MOTIVATION FOR RESEARCH IN STABILITY THEORY

C. C. Dearman, Jr.

INTRODUCTION

The motivation for research in stability theory by the Astrodynamics and Guidance Theory Division of the Aero-Astrodynamics Laboratory was initially generated by a long-felt desire for an analytical method to determine the capability of a proposed space vehicle guidance system to guide the vehicle to a desired terminal state from a given initial state even when the vehicle was subjected to perturbative forces of extreme severity.

DISCUSSION

The non-analytical or "cut and try" method that was and is at present universally in use requires programming of the equations of motion (including the guidance equations) on a computer, adding the selected perturbative forces and computing the trajectory from some initial time at which the vehicle, considered as a dynamical system, is said to be in an initial state. If the guidance system permits the system to attain the desired terminal state (or an acceptable approximation thereof) within an acceptable time interval despite the influence of the perturbative forces assumed to be present, then another set of perturbative forces is introduced and the computation process is repeated. If this procedure shows that the proposed guidance system is not capable of guiding the vehicle to the desired terminal state under the perturbative forces assumed to be present, then a basis for redesign or modification of the guidance system would be indicated. If, on the other hand, the method indicates that the guidance system is indeed capable of guiding the vehicle to an acceptable neighborhood of the desired terminal state under all of the selected perturbative forces, then confidence in the capability of the guidance system would certainly be very high. But, because a guided space vehicle is a highly nonlinear dynamical system, there is always reasonable doubt that its guidance system will be adequate when other sets of perturbative forces are present. It is a common experience of investigators of the behavior of nonlinear systems that the practice of predicting the behavior of such systems under one set of perturbative forces on the basis of their known behavior under other sets of these forces is very dangerous indeed. This danger was well known, of course, to the investigators who had the responsibility for testing the capability of the guidance schemes developed in this laboratory, but, inasmuch as no better method appeared to be available, they were forced to use the only method at their disposal.

The broad problem toward which the research has been and is being directed may be stated as being that of finding the maximal region surrounding and containing the unperturbed trajectory from any point within which the vehicle may be guided to the desired terminal state, or acceptable approximation thereof, within a specified finite time interval.

If the time interval from the initial state were allowed to be infinite in duration and if it were required that the terminal state be approached arbitrarily near, then the problem would be one in the classical theory of the stability of motion, and finding the region containing the unperturbed trajectory such that from any point within the region it would be possible to guide the vehicle arbitrarily near to the terminal state would be equivalent to finding the domain of asymptotic stability in the classical theory.

In our problems, of course, only finite time intervals are meaningful, and we do not demand that the desired terminal state be approached arbitrarily near. Instead, we require that the state of the vehicle come within a specified tolerance of the desired terminal state at some time within a specified finite time interval. Therefore, it was necessary to modify the classical theory if it was to serve as a tool for solving our problems. Accordingly, work was begun in this problem area with in-house studies and via contracts let to various research groups in private industry and in universities.

The general problems described above required that several sub-problems be investigated. This was due primarily to the characteristics and limitations of the classical theory. This theory had its origins in the researches of Poincaré, Liapunov, and others, but it is the method of Liapunov that has the most general and far-reaching applications. The Liapunov method, which does not require that the solution of the differential equations describing the motion of the system be known, first appeared in 1892 in Liapunov's doctoral thesis. The thesis was translated from the Russian to the French in 1907 and appeared in this country as one of Princeton University's Annals of Mathematics Studies in 1949. Liapunov was primarily concerned with the stability of the solar system, as were many investigators in the nineteenth century, and he addressed himself to this problem and quite naturally wished to develop a theory that would correctly predict the stability characteristics of this system for all future time. For this reason the classical theory concerns itself with stability of motion over an infinite time interval. Also, Liapunov's method requires the generation or invention of a function called a Liapunov function, in the state variables (and possibly time) which must possess certain properties. The method, however, does not show how to generate or invent this function and the difficulty in doing this for nonlinear systems is one of its limitations. Presently, available procedures for generating Liapunov functions lean heavily on the experience, ingenuity and good fortune of the investigator. No generally applicable methods for generating these functions are known. There exist techniques that are applicable only to particular problems or to some very restricted classes of low-order, nonlinear systems.

The generation of a Liapunov function not only permits us to make positive statements about the stability of the system but it also opens the way for determining a domain or region of stability. Thus, the motivation for research in methods of generating Liapunov functions was clearly strong. But, since, as we have mentioned, our problems are problems in finite-time stability and not in infinite-time stability, it was necessary to try to modify the classical theory in order to make it applicable to our problems. It now appears that the necessary modification has been successfully made, and no further really basic theory in this area is needed. This modification has been developed principally by E. F. Infante and L. Weiss of Brown University under a supporting research contract from MSFC and a NASA grant and by R. Gunderson and J. George of this division.

The modification still required the generation of a Liapunov-like function, and this very vexing problem has not been solved. A two-year research effort by investigators at Drexel Institute under an MSFC contract has resulted in failure to find more generally applicable methods of generating Liapunov functions, but several promising avenues for further research have been opened as a result of this effort.

Because it was thought at the time the Drexel Institute contract was let that the result might be disappointing, another research contract was awarded to the research department of the Grumman Aircraft Engineering Corporation where two specialists in the problem area were assigned the task of exploiting the modern digital and analog computers to find methods of generating Liapunov functions and the stability regions associated with them, in good approximation, if not precisely. Funds were available only for a small effort, and while the results obtained were not decisive, they did show promise of being applicable to practical systems.

It is not possible to probe very deeply into the problem of the stability of motion of dynamical systems without being singularly impressed by its close relation to the theory of optimization, and in a guidance or control system what is desired is optimality as well as reliability and capability. Further, we would like for the systems to be relatively insensitive to unknown and unknowable variations in the vehicle parameters. The relations, then, among the concepts of optimality, stability and sensitivity should be an area for serious research. Accordingly, a contract was awarded to the Engineering Mechanics Department of the University of Texas for research in the possible relationships among these concepts. The results have indicated that the concept of sensitivity is an illusive one and that any definition which appears to afford practically useful applications is not mathematically tractable. This work is continuing and has been funded through December 1968.

CONCLUSION

In summary, then, it could be said that we have at our disposal all the necessary theory we need for investigating problems in finite-time stability. What is needed are practical methods for generating Liapunov-like functions in order to be able to make positive statements about the stability of a nonlinear system and to find its domain of stability. Finally, it is important that we know the relations among the concepts of optimality, stability and sensitivity so that we may have the tools for testing and developing guidance and control systems that are optimal, reliable and capable.

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RESEARCH IN STABILITY AND CONTROL THEORY

R. W. Gunderson

INTRODUCTION

Stated briefly, the objective of our research in stability and control theory is to develop analytical techniques which can be used to replace costly experimental procedures now used in the development of launch vehicle and spacecraft guidance and control systems. Our effort can be essentially divided into four areas: the theory of differential equations, linear systems theory, nonlinear systems theory, and applications. In the following paragraphs, an example of work carried out in each of these areas will be given, along with a brief summary of current effort.

THEORY OF DIFFERENTIAL EQUATIONS

During the past decade, much time and effort have been spent on studying the stability of solutions to ordinary differential equations. However, the behavior of many physical systems is best described by systems of partial differential equations, e.g., launch vehicle flexible body dynamics. We have recently started research into the theory of partial differential equations with the goals of (a) obtaining a stability theory, if possible, which is consistent with the mathematical background of the practicing control engineer; (b) determining practical stability criteria which can be used to determine the stability behavior of coupled systems of ordinary and partial differential equations.

LINEAR SYSTEM THEORY

An example of our effort in this area is the exploitation of a Russian linear stability analysis technique, known as the D-decomposition method of Neimark. This method was adapted for use in displaying the dependence of control system stability on certain system parameters. In particular, in its original form, the D-decomposition method was strictly an algebraic technique and required an algebraic form for the transfer function of the system investigated. However, we were able to modify the method to a form such that the transfer function need only be given in frequency response form. Hence, the transfer function could be obtained experimentally and used directly without the time consuming and, sometimes inaccurate, reduction of the experimental data by curve fitting routines. The results in this area lead to extensive use of parameter stability regions in a number of launch vehicle development programs.

NONLINEAR SYSTEMS THEORY

The popular concept of stability refers to solution behavior over an infinite time interval. However, the launch vehicle control system is only required to exhibit stable behavior over a rather brief finite interval. As a consequence, results obtained using infinite interval criteria could be misleading or, at least, overly restrictive. During the past year, we have made considerable progress toward developing a finite-time stability theory, to the point that the theory is essentially completed. However, much remains to be done in obtaining ways to use the theory.

APPLICATIONS

In some cases a successful analysis of system behavior results from coupling existing theory with unique features of the system to be analyzed. One such analysis was that of a lock-on oscillator, which was to serve as a component in an adaptive attitude control system. Normally, the phase plane technique of analyzing nonlinear systems can be used only for autonomous systems. However, the nature of the oscillator was such that, even though its behavior was non-autonomous, the phase plane technique could be profitably utilized. The resulting analysis showed the oscillator had serious shortcomings, not detected by experimental checks, and it was not used.

SUMMARY

As already mentioned, we are beginning to spend time on the problem of stability of solutions to partial differential equations. This is, however, a very difficult problem and requires extensive preparatory study. In the area of linear systems, work was recently completed on a problem in controllability of multi-input linear systems, and a report is being prepared on the Δs method of linear analysis. Effort will continue on determining means of applying the theory of infinite-time stability, where the main problem is that of systematic determination of Liapunov functions.

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INTERPLANETARY INTEGRATING COMPUTER PROGRAMS

J. Reynolds Duncan, Jr.

INTRODUCTION

An interplanetary computer program is one of the primary tools used in astrodynamical work. It is basically a numerically integrated program which provides a trajectory that a spacecraft may follow to achieve a desired set of end conditions. Constraints are usually imposed on the program so that the resulting trajectory is a realistic one. For instance, we may specify that the starting point be a particular launching site at Cape Kennedy, or we may choose as our initial conditions, a 100 n.m. parking orbit about the earth. Of course the trajectory must also be realistic in the sense that it accurately reflects the total effect of the forces which may act upon or interact with the spacecraft. Then too, the numerical integration of the equations of motion must be performed with acceptable accuracy.

DISCUSSION

Despite the difficulty and complexity of the problem there are many interplanetary computer programs which have been developed to produce these desired solution trajectories. Although these programs may provide comparable results for most problems, they often, upon closer examination, reveal significantly different characteristics which may produce conflicting results for some specific problems. Why are there so many different programs in use? How can they be significantly different and still produce comparable results for most problems? Why is it necessary to continuously modify these programs and/or produce new programs? To answer these and other questions we shall probe the composition and construction of these programs, not with the intention of identifying which program is the "best" or making recommendations on the use of particular routines within the program, but merely to present some idea of the tremendous amount of research and analysis necessary to produce such programs. As we go deeper into the makeup of these programs, we shall see that our imperfect but improving knowledge of the forces at work in our solar system combined with ever improving numerical techniques and advancements in computer hardware, provide a continuously changing foundation upon which these programs are built. Thus, if a program is to remain as efficient as possible and/or contain the most accurate mathematical models available, it must undergo periodic or continuous review and revision.

The necessity for continuous updating and the factors contributing to program differences may be best illustrated by describing the components of the program and the way in which these program parts fit together and work together to provide solutions to interplanetary trajectory problems. But first we shall give a brief overview of the make-up of these programs in hopes of providing a skeletal structure upon which to hang the ensuing more detailed discussion. We start with an n-body problem solution which provides a method for the numerical integration of the equations of motion of the spacecraft and spherical homogeneous planets. Then added to this are the effects of non-spherical nature of the planets, i.e., oblateness and higher order effects, along with mathematical models of non-gravitational forces such as radiation pressure and atmospheric drag. This produces what we shall call an expanded n-body solution. Lastly, we employ an isolation routine which uses the expanded n-body solution as a trajectory generater and, if possible, provides us with our desired solution trajectory

We shall now attempt a more detailed explanation of these program parts starting with what may be considered the core of the interplanetary program - the n-body problem. The solution to the n-body problem requires the ability to accurately represent the path of a spacecraft traveling in the solar system subject to the gravitational forces of the bodies in the solar system. Implied in this is the ability to represent the motion of these gravitational bodies subject to their interdependent gravitational fields. The attempts at the solution to the n-body problem may be classified under two basic approaches - general perturbation theory and special perturbation theory.

General perturbation theory encompasses the attempts at analytic or closed form solutions. To illustrate, let us take the simple example of an artificial satellite in orbit about a spherical homogeneous planet with no forces involved except the inverse square gravitational field of the planet An analytic solution is known for this problem. This solution allows us to know the exact position and velocity of the satellite at any desired time provided that we are given its position and velocity at some time which we shall call time zero. Therefore, if we desired to know its location at some future time, say 30 years, 6 days, 5 hours, 32 minutes, and 7 seconds from time zero, we could, by substitution of this time into a few equations, compute exactly where the satellite will be. This solution, being exact, is the most desirable type of solution, but, thus far, no one has produced such a solution to the general n-body problem. However, solutions have been produced using the second method of attack, that of special perturbation theory.

Special perturbation theory provides the basis for the n-body problem solution that is used in all of the numerically integrated interplanetary computer programs. The first step in the special perturbation approach is the formulation of the equations of motion of the spacecraft and planets. There are a variety of methods available but, fundamentally, they are all combinations or variations of three basic methods: the formulation by sum-totaling all of the effecting gravitational forces; the method of variation of coordinates; and the method of variation of parameters. The last two methods take advantage of known analytical solutions which closely approximate the actual trajectory (or ephemeris) of the spacecraft and planets over a relatively limited period of time. The first method mentioned, sometimes known as "Cowell's Method," does not take advantage of known approximate solution but may have the advantage of a clearer more straightforward formulation and, as indicated above, it involves the numerical integration of the total acting accelerating forces. The general method of variation of coordinates, of which the well-known Encke's method is a modification, involves not the total forces explicitly, but rather is concerned with the numerical integration of the departure of the actual trajectory from a fixed reference orbit. The advantage here is that the integrated differences are small and therefore longer integration steps may be made, with comparable accuracy, than are generally possible with a Cowell formulation. The variation of parameters method offers similar advantages by representing accelerating forces in terms of orbital elements or parameters. By selecting parameters which are slowly changing, the effect of the dominant central gravitational force may be suppressed and advantage may be taken of the known analytic solution to the two-body problem. As mentioned previously, these methods can be combined or altered to produce other formulations which may be better suited for specific problem. However, before trying to match formulations to problems, we must consider the second part of the special perturbation theory approach. That is, now that we have formulated the equations of motion, how are we going to numerically integrate these equations to produce our desired solution?

Again, there are a great number and variety of methods which may be used. We shall describe a few of the most popular numerical integration techniques in use today and try to indicate some of their advantages and disadvantages. Before the discussion of these techniques, we should point out that all of the commonly used numerical integration techniques are based on power series expansions and differ mainly in the method of treatment, approximation, and computation of the series.

One of the most well known techniques is the fourth order Runge-Kutta method which is the equivalent of a fourth order power series expansion. It is a self-contained package which requires as input the equations of motion and the desired time step to be used. Thus, it can be easily inserted into a program whenever a numerical integration technique is needed. However, in order to minimize the numerical errors, the time step must usually be small and in fact the Runge-Kutta method does not provide any indication of the size of the truncation or round-off errors produced by the numerical integration process. Of course, a small time step can mean a fantastic number of integration steps in the computation of an interplanetary trajectory. A short integration step and a lack of error control do not recommend this technique for use in interplanetary programs. Another technique, the multistep method, is better suited for interplanetary problems. Basically, it sets up a backward difference table which is used to represent the derivatives in a series expansion. By adding higher order differences to the tables, the accuracy of this method may be extended to almost any desired order. The multistep method works very well for problems in which a constant step size can be used such as with the numerical integration of the paths of the planets, but it may not be very efficient in the integration of the equations of motion of a spacecraft, since a spacecraft may vary over extremes in distance from gravitational bodies and thus require vastly different integration step sizes. Also, the multistep method is not self-starting but requires a table of known values before computation can begin and too, there still remains the problem of error control. Before continuing with the description of the last two numerical integration techniques, we might pause here to say a few words about the source of these numerical errors which we seek to control. The error due to truncation arises from the fact that our series expansion may not be of high enough order to accurately cover the integration time step which we have specified. The second error source, that of round-off, results mainly from the number of operations performed by the computer in the process of integrating the entire trajectory or orbit - the more operations performed, the greater the round-off error. One possibility for the control of these errors would be a variable step size which would allow the use of the largest step possible, while holding the truncation error to an acceptable level. Also, if it were possible to conveniently change the number of terms used in the series expansion while the trajectory is being integrated, it might help to hold down the error accumulation and, in conjunction with the variable time step, provide for faster, more efficient numerical integration. These are two important features of the next numerical technique we shall discuss, namely, the power series method. Numerical integration by the power series method is accomplished by directly computing and summing the terms in a power series expansion. Recursive formulae permit the calculation of the terms to any desired order and also provide a convenient means for error control and step size variation. While this method does lend itself very well to interplanetary problems, it does have its drawbacks. The development of the necessary recursion relationships may be difficult, and the relationships themselves are very dependent upon the form and structure of the equations of motion. This makes it difficult to develop a numerical integration package which may be used where needed in different programs. Also, if the equations of motion are very complex, the number of variables will be large and there may be a problem with the amount of computer storage that is available. However, all things considered, the advantages outweigh the disadvantages, and the additional time which is spent in the development of this method is well worth the effort. The final method to be discussed was developed by Fehlberg, and is actually a combination of the Runge-Kutta method with the power series technique. It offers the same advantages and has about the same disadvantages as the power series method except that it requires less computer time and utilizes a little less computer memory. Reduced computer running time is reflected in better round-off error control which produces more accurate numerical answers. These four numerical integration techniques are but a few of the many techniques that are in use, but they do provide an indication of the wide variety of methods that have been developed.

With this available assortment of techniques in mind, let us return to our original problem of selecting a special perturbation theory method for use in securing a solution to the n-body problem - that is, which combination of equation formulation and numerical technique is best for an interplanetary program. It should be obvious from the few samples that have been mentioned that there are a great number of possible combinations available, all with advantages and disadvantages, and all of these combinations can probably be shown to be the best one for use on some particular problem. Admittedly, there will be some combinations which can immediately be eliminated for use on interplanetary trajectories, but if such things as atmospheric booster flights and optimization of near earth burn arcs are to be considered as part of the overall interplanetary problem, then it may be necessary to consider the use of more than one formulation and possibly several different numerical techniques. The selection of the proper formulation(s) and numerical method(s) to use in an interplanetary program can best be made by careful consideration of the primary goal of the program. For example, a program designed for maximum accuracy with computer running time being of secondary consideration would probably be quite different in construction from a program in which some degradation in accuracy was acceptable if the computational speed could be increased. Decisions which reflect the relative importance of various program characteristics will need to be made at practically every step of the program building process. Thus, it behooves us to construct a hierarchy of desired characteristics before the selection process is started. Such factors as computer running time, ease of operation, amenability to program modification, versatility, accuracy of the numerical integration technique, formulation of the equations of motion, accuracy of the mathematical models, and dependability need to be given careful consideration and then placed in the desired order of importance.

From the n-body problem solution we shall proceed to the expanded n-body solution, that is, the inclusion of whatever additional accelerating forces may be deemed necessary for program completeness. Which forces are included is primarily a function of the overall purpose of the program. Ordinarily, most interplanetary programs are not designed for use beyond the solution of the problem of transfer from elliptical orbit about one planet to orbital insertion or impact of another planet - this in itself is problem enough. For such a program we may wish to include such forces as high and low thrust, solar radiation pressure, planetary oblateness effects and possibly electromagnetic forces and relativistic effects. Of course, mathematical models are needed to represent these forces in the program,and this presents another possibility for differences between programs since some of these forces are not well understood and others have more than one acceptable mathematical model. We shall not attempt any detailed discussion of these forces except to indicate that the increasing body of knowledge in these areas may necessitate a periodic updating of the models used in interplanetary programs. With the selection and inclusion of the desired models, the construction of our expanded n-body solution is complete. We now have a program which will compute the trajectory of a spacecraft subject to gravitational and other forces, provided, of course, that we supply a set of starting conditions for the planets and spacecraft. This brings us to our next problem, namely, how do we obtain these initial conditions? Through the use of conic, or approximate, solutions to the problem, we usually have available a set of initial conditions which, while not as accurate as we need, is at least usually in the right ball park. Combining these initial conditions with our expanded n-body solution, we then seek the aid of an isolation routine to generate an accurate set of initial conditions which, when utilized in our expanded n-body solution, will achieve our desired goals. The general method of operation of isolation routines is to start with a set of approximate initial conditions and, utilizing the expanded n-body solution, integrate out to the vicinity of the desired end conditions. Then a small change is made in one of the initial conditions and the equations of motion are again integrated. This process is continued until variations have been made in all of the independent variables, and then a check is made to see how these changes in the initial conditions have effected the cutoff conditions. Utilizing this information, an adjustment is now made in the independent variables which should provide a trajectory with end conditions that are closer to our desired values than the final values of the initial trajectory. This process, when continued, should isolate a set of initial conditions which will provide our desired cutoff conditions. All this is easier said than done, and we shall now discuss some of the difficulties which are encountered.

The basic problem confronting the isolation routine is how to deal with the generally nonlinear relationship of the dependent and independent variables which represent the final and initial conditions, respectively. Two approaches to the solution of this problem are the utilization of nonlinear functions and the substitution of more linearly dependent variables for the nonlinear ones. As was indicated in the isolation scheme described above, a number of integrations of the equations of motion are required before a correction of the approximate initial conditions can be made. The number of integrations needed increases rather rapidly when higher order relations are used and the additional machine time required for these integrations may offset the possible decrease in the number of isolation trials which are needed for convergence. The selection of a relatively linear set of independent and dependent variables permits the use of linear relationships in the isolation routine, but it is not always possible to find such nicely behaved variables over the wide range of problems encountered in interplanetary work. Improvements can sometimes be made in isolation routines when dealing with a specific class of problems, but these improvements are usually of little use when applied to other classes of problems. None of the presently available isolation routines appear to be able to provide the fast, flexible and reliable performance that is needed for use with the complex and sensitive problems associated with interplanetary trajectory studies. The development of better isolation routines would have a pronounced effect on the efficiency of interplanetary programs. Needless to say, the choice of an isolation routine for use in an interplanetary program provides yet another opportunity for program differences to grow. It is probably the isolation routine, more than any other single factor which accounts for variations in computer running times for the different interplanetary programs. With the inclusion of an isolation routine, the interplanetary program is essentially complete.

CONCLUSION

Whatever differences there may be in individual programs, it can be said that the construction of interplanetary computer programs is a complicated, difficult, and thought-provoking process, and it will be this and more, if there is a genuine interest, by the people involved, to produce a program which will perform its intended task in the most efficient way possible.

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GUIDANCE TECHNIQUES FOR INTERPLANETARY FLIGHT

Wilton E. Causey

INTRODUCTION

There are three phases of interplanetary flight which require guidance. These are the injection phase, the mid-course phase, and the terminal or orbit insertion phase. If (usually this is the case) the main objective is to deliver, as accurately as possible, the maximum useful payload into a prescribed orbit about the target planet, then there are certain characteristics which the guidance schemes must have. The object of this paper is to discuss these characteristics and to point out areas where future work will be directed.

INJECTION GUIDANCE

The injection guidance scheme for interplanetary flight should (1) be near optimal in order to take full advantage of the performance of the launch vehicle, (2) produce accurate injection conditions so that midcourse maneuvers can be kept as small as possible, (3) be flexible in order to handle in-flight perturbations. IGM possesses the characteristics described above and is quite adequate for interplanetary flight if the Saturn vehicle is used for the injection phase. However, if launch vehicles with lower accelerations are used, the IGM may not be sufficient. It is felt that new guidance schemes being developed by R-AERO-G will prove adequate for any class of launch vehicle.

Work has been initiated to develop a hypersurface that can be used for targeting for interplanetary missions. Such a hypersurface would represent a family of injection conditions all of which will produce the same arrival conditions at the target planet. Having a cutoff surface for targeting purposes rather than a fixed set of injection conditions eliminates the restriction of having to hit a fixed point in space with a specified velocity vector.

MIDCOURSE GUIDANCE

A guidance scheme for making midcourse maneuvers should stress accuracy and simplicity. The more accurate the correction maneuvers can be made, the more exact the terminal orbit about the target planet can be established. Also, additional midcourse maneuvers may be required if the correction maneuvers are not performed accurately. This would not only require more fuel, but an additional engine restart. Simplicity is desirable since each additional pound of guidance equipment results in one less pound of useful payload. Optimality is not an important factor in midcourse guidance since the Δv for midcourse correction is on the order of 5 to 10 m/sec. A Δv of this magnitude can be obtained with short burn arcs; thus, impulsive approximations can be used. The guidance problem is to determine the initial direction of the thrust — this direction is held constant during the burn — and the duration of the burn to provide the $\overrightarrow{\Delta v}$ that is needed. Ground-based tracking can be used to determine the path of the spacecraft, and the $\overrightarrow{\Delta v}$ needed to correct the trajectory can be determined on the ground and the required information relayed via radio to the spacecraft.

The target plane or critical plane at the target planet is shown in Figure 1 and is referred to as the B plane. The B plane passes through the center of the target planet and is perpendicular to the incoming asymptote. The T vector can be chosen to lie in Mars' ecliptic plane or Mars' equatorial plane. B lies in the B plane and is a vector from the center of the target planet to the point where the asumptote pierces the B plane. B and θ define the aim point, and thus, the inclination of the approach hyperbola and the radius of close approach (related to B) can be specified by B and θ. The purpose of the midcourse correction is to correct for errors in B and θ, and this can be accomplished by using two of the three available degrees of freedom (Δv_x, Δv_y, Δv_z). Guidance laws can be developed that use the remaining degree of freedom to do one (in addition to correcting B and θ) of the following three things:

1. correct time of arrival
2. minimize the magnitude of Δv
3. restrict Δv to lie in a given plane.

In a mission such as Voyager, the exact time of arrival is not important; thus, guidance law (2) or (3) would be used. If the third guidance law is used, it may be possible to perform the midcourse maneuver without losing Canopus lock. This would be desirable from a mission reliability standpoint. Manned flyby missions may require a specific time of close approach, and thus guidance law (1) would be used. All three guidance laws will be investigated.

TERMINAL GUIDANCE

Terminal guidance will be needed if the mission calls for establishing an orbit around the target planet. Accuracy is again important since capsule landing sites, orbital photography, and other scientific experiments are affected by the terminal orbit's size and orientation. Simple open-loop guidance schemes, such as a gravity turn and a constant inertial direction, have been shown to be sufficient for the Voyager mission. Ground-based tracking is used to determine the approach trajectory, and the thrust direction and duration of the insertion burn are computed on the ground and relayed to the spacecraft. The payload loss due to these simple open-loop schemes is on the order of a few pounds.

Simulation studies have shown that only navigation errors produce significant errors in the terminal orbit, and even these errors are correctable with an orbit trim Δv of 100 m/sec. An onboard approach navigation system would help reduce the injection errors; however, this information would have to be relayed to the ground and added to the tracking data, and the guidance command would have to be computed and relayed back to the spacecraft. The 20-to 30-minute time lag between the time the onboard navigation signal is sent from the spacecraft until the guidance command is received by the spacecraft limits the effectiveness of the onboard navigation system. Effective use could be made of the onboard navigation system by using onboard calculation of the guidance command. However, with the addition of the onboard navigation equipment and a guidance computer sufficiently large enough to compute the guidance commands onboard, the benefits may be over-shadowed by the loss in payload. Trade studies along these lines will have to be made.

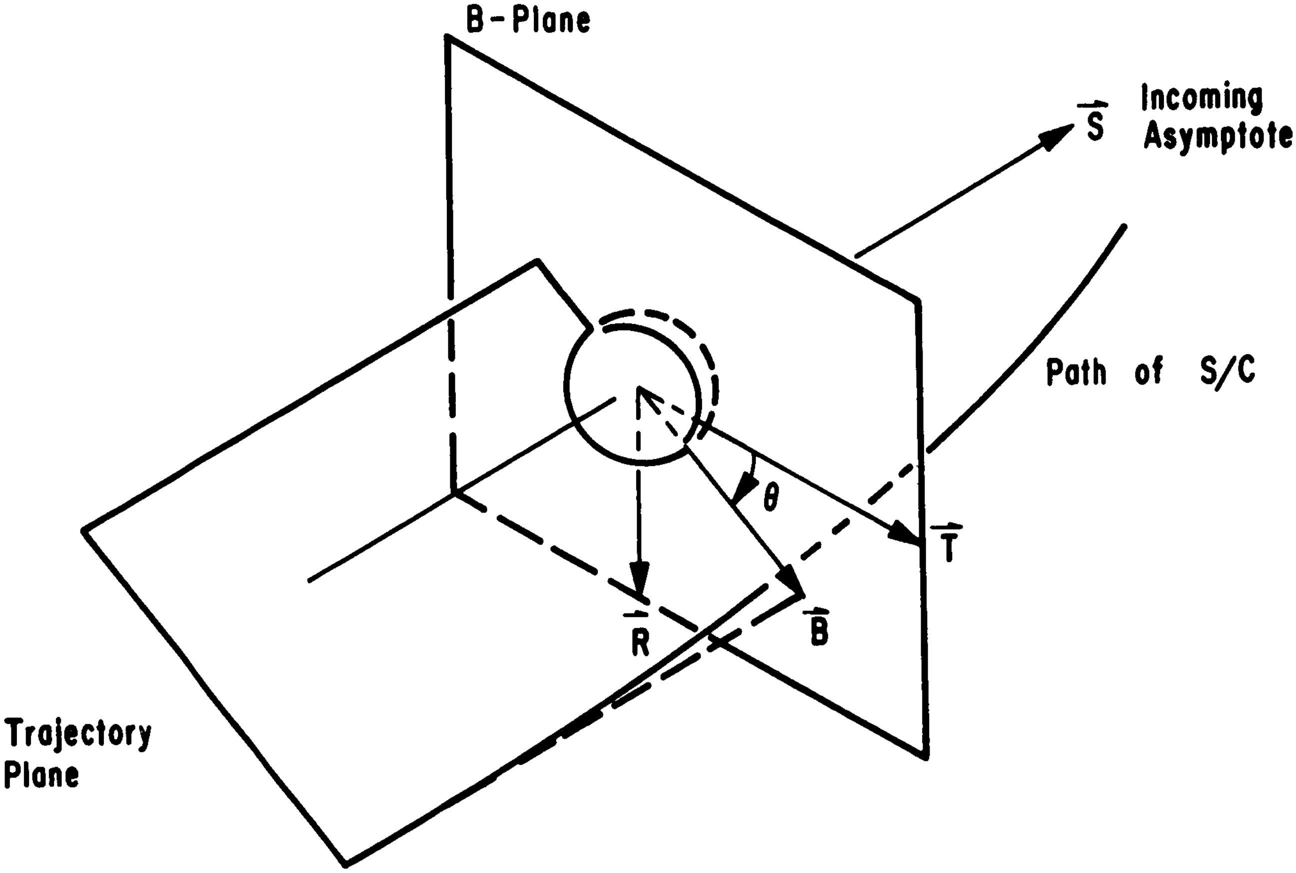
B-Plane
$\vec{S}$ Incoming Asymptote
Path of S/C
θ
$\vec{T}$
$\vec{R}$
$\vec{B}$
Trajectory Plane

FIG. 1. B PLANE

N68-18988

THE MATHEMATICAL AND NUMERICAL ANALYSIS ASSOCIATED WITH OPTIMAL GUIDANCE CONCEPTS

Hugo Ingram

INTRODUCTION

The operation of a guidance system involves three distinct divisions of effort. These divisions of effort may be called navigation, guidance signal generation, and control dynamics. The navigation part of the guidance system is concerned with determining the vehicle's present conditions or state. The guidance signal generator uses a knowledge of the vehicle's total state and a knowledge of the desired destination to determine how to steer (or where to point the engines for rocket powered vehicles). The control dynamics result from an implementation of the guidance signal which has been generated. Then this implementation of the guidance signal will affect the vehicle during the time required for the vehicle to reach the desired destination.

DISCUSSION

The navigation, generation and implementation of a guidance signal would have to be performed only once if all three parts of the guidance system could operate perfectly. That is:

1. The navigator would have to determine the present state exactly.

2. The guidance signal generator would have to know or generate an exact time history of the vehicle's path from the initial state to the desired destination. This implies that an exact time history for the steering of the vehicle is also known or determined.

3. The implementation of the guidance signal would have to be accomplished instantaneously and exactly.

In practice, none of the three conditions listed above for the perfect operation of a guidance system can be achieved. For this reason the guidance system is used repetitively with the knowledge that some of the error sources that cause the imperfect operation will decrease as the desired destination is approached. Since the operation cycle of a guidance system takes a certain amount of time, the repetitive use of the guidance system can occur no more often than the time required for one operation cycle. When the accuracy requirements for a particular mission are not very stringent or the operation of a particular guidance system is very effective, the guidance system may be employed only once or at least only a few times (instead of as often as possible).

All of the mathematical computations necessary for the proper operation of a guidance system may be performed onboard the vehicle, or they may be performed at a ground station which is in radio contact with the vehicle. Obviously, the vehicle can be much more flexible in the accomplishment of its mission if it does not have to remain in contact with a ground station. For this reason, it is desirable (although not always possible) to perform the guidance computations onboard. When the guidance computations are performed internally, a guidance computer must be carried along. Since there is not usually much space or weight aboard the vehicle reserved for a guidance computer, the guidance computations that are performed onboard must be less complex and more easily computed than guidance computations that are performed by a ground station. Unless great care is taken in the selection of onboard guidance equations, they will also be much less effective than the more complex ground-based computations. As computers that are available for onboard guidance computations become lighter, more compact, faster, and larger (with respect to the amount of information they can process), improvements that are made in guidance techniques will result in a large profit with respect to the scientific value of a particular mission. Most of the guidance computer's efforts are devoted to the generation of a guidance signal, although some computations may be needed for navigation and implementation of the guidance signal. This means that a vigorous effort to improve the mathematical approach used for guidance signal generation will have a large impact on the success and usefulness of a particular mission.

The preceding concept leads naturally to a discussion of optimal guidance. This means that some aspect of the vehicle's performance (as it is achieving a particular mission) must be optimized. For rocket-powered vehicles, optimal guidance is usually considered to be the successful delivery of the maximum amount of payload or useful weight to a particular destination. To accomplish the optimal guidance of a vehicle, a set of ordinary differential equations are formulated and used to simulate mathematically the motion of the vehicle. Appearing in these equations of motion are the so-called control variables of optimization theory. They are called control variables because they are not governed by any of the differential equations, and thus, are free to be chosen at one or every instant of time in the flight to optimize some specified aspect of the vehicle's performance. The different mathematical techniques used to accomplish the control variable selection are usually grouped into a body of knowledge known as optimization theory or calculus-of-variation theory. Since this theory is rather complicated, a detailed explanation will not be attempted at this point. Instead, some general ideas about the mathematical and numerical analysis involved in optimization problems will be discussed.

When some type of optimization theory is applied to the differential equations simulating the motion of a rocket powered vehicle, a split boundary value problem is produced. This means that all of the initial conditions for the differential equations involved are not known at the initial time. These unknown initial conditions must be determined so that they cause the differential equations to satisfy some conditions specified at the final time or destination. In order to do this, the differential equations must be integrated to yield relations which connect the initial state with the final state. Then, the desired conditions at the destination are placed in these relations, and the resulting equations solved to yield values for the missing initial conditions. These initial conditions and the relations connecting the initial conditions with the final conditions yeild the control variables' time histories that optimize the specified aspect of the vehicle's performance. For a rocket-powered vehicle, the single control variable is usually the thrust direction, and a time history of this thrust direction is determined (as described previously) to maximize the payload delivered to a particular destination.

CONCLUSION

Realistic mathematical simulations of space flight are of necessity rather complex, and for this reason, optimization theory will not often yield a rapid solution which can be used as a guidance signal. The present state-of-the-art in guidance usually relaxes the reality of the mathematical simulation or degrades the optimality of the solution in order to be able to solve the split-boundary value problem rapidly enough and use the solution as a guidance signal. With the advent of better guidance computers and improvements in methods of solution of optimization problems, some of the simplifications now used will not always be necessary. In fact, IBM under contract to MSFC has already demonstrated that a realistic mathematical simulation of the booster cutoff to orbit problem can be solved completely and rapidly enough to be used as a guidance signal. This general approach can obviously be extended to more complicated problems more effectively than a guidance procedure which makes simplifying assumptions that are not compatible with the more complicated problem. In order to extend this general solution approach to optimal guidance, a vigorous research effort is being conducted both in-house and with the aid of contracts. This research involves both the mathematical and numerical aspects of the following subjects which are needed in the solution of optimization problems:

1. Necessary and sufficient condition formulation for optimization problems.

2. Numerical integration and series expansion solution of differential equations.

3. The maximization and minimization of functions.

4. The solution of systems of simultaneous nonlinear equations.

As was mentioned, the intelligent application of this type of research has already indicated that the fear of general numerical solutions of optimization problems as a guidance procedure is unjustified. As a result, further effort is now being expended so that orbital transfer, rendezvous, and planetary landings can be attempted with optimal or near optimal guidance rather than with some less effective technique.

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QUASI-OPTIMAL TRAJECTORY ANALYSIS WITH MULTI-MISSION CAPABILITY AND GUIDANCE APPLICATION

Roger R. Burrows

INTRODUCTION

The basic idea for the method of trajectory analysis to be discussed originated at Lockheed and has been developed jointly by Lockheed and Marshall. The computer program resulting from this analysis has been nicknamed QUOTA - short for Quasi-Optimal Trajectory Analysis. We will use this acronym to refer to the technique as well. Briefly, we will indicate how QUOTA achieves its multi-mission capability. The control law used by the current vehicle guidance system, the IGM, will be compared with that used by QUOTA and the theoretical conditions for which they are optimal control laws will be discussed. The guidance application of QUOTA and some future missions for which QUOTA would be an apt tool will be presented.

DEVELOPMENT

In most of our work at Marshall, the ideal trajectory for a space vehicle to follow is the one that satisfies all the mission constraints while simultaneously maximizing the useful scientific payload at mission fulfillment. This trajectory which utilizes propellant most efficiently under the imposed constraints is called the optimal trajectory. Unfortunately, the mathematical tools which classical mathematics has developed to find such optimals are complex in nature and usually involve numerical solutions in which numerical integration plays a large role. One of the distinct advantages of both the IGM and QUOTA is that while each defines near optimal trajectories, neither one has to perform numerical integration since each scheme uses a control law allowing closed form solutions of the differential equations of motion. However, each scheme must solve a system of nonlinear equations. The system the IGM solves in essentially explicit form derives from the mission terminal conditions. QUOTA solves a larger system of equations which includes the mission terminal conditions as well as equations representing necessary optimality conditions. QUOTA solves these equations numerically using an iterative technique. The numerical iteration accounts for QUOTA's mission flexibility, and the inclusion of necessary conditions for an optimum trajectory helps insure QUOTA's optimality in performing these missions.

The control law used by QUOTA is $\tan \chi_p = \frac{a + bt}{c + dt}$ and the control law used by the IGM is $\chi_p = a' + b't$ where a, b, c, d, a', b' are constants, t is time along the trajectory and χ_p is the angle between the local horizontal and the thrust vector. The following assumption is common to finding the two dimensional conditions for which these control laws are optimal: A constant thrust, point mass vehicle is moving in a vacuum through a uniform gravitational field, i.e., gravity is independent of the position coordinates. It follows from this assumption alone that the optimal control law for a minimum fuel trajectory is

$$\tan \chi_p = \frac{a + bt}{c + dt}$$

independent of the mission terminal conditions. If it is further assumed that (1) the final altitude is specified and (2) the final velocity components are specified, then the control law for a minimum fuel trajectory is $\tan \chi_p = a'' + b''t$. The IGM approximates this by using $\chi_p = a' + b't$. Ideally then, we observe that the form of QUOTA's control law does not depend on the terminal mission conditions and that practically this form contributes to QUOTA's optimality. QUOTA also uses a truncated series for its gravity representation while the IGM uses a weighted averaging process.

UTILIZATION

Fortunately, the numerical algorithm developed for QUOTA is rapid enough to allow the possibility of real time on-board guidance. Entire trajectory problems have been solved in less than 3 iterations and less than .2 sec. of IBM 7094 computer time. These figures are the results of several vehicle performance studies and a study of QUOTA's capabilities in the presence of vehicle perturbations. A 4.5 second real time guidance cycle seems easily attainable using the present Saturn V on-board guidance computer, and with more optimal computer programming and analysis, this time should be capable of reduction by a factor of one-half or more.

EXTENSIONS

The missions so far considered at Marshall involve an increase in vehicle velocity. There are missions, for example, orbital transfer from an inner circular orbit to an outer coplanar circular orbit, which involve a velocity reduction. A continuous thrust during transfer is not the most fuel conservative, but it does minimize transfer time and would probably be the mode of operation for emergency rescue use. Initially, the vehicle thrust has a component in the direction of the velocity vector placing the vehicle on a higher energy ellipse. About half-way through the entire maneuver, the vehicle turns around causing the thrust vector to have a component opposing the velocity vector, and the vehicle is slowed until its velocity is that proper to the desired terminal orbit. At this time, no one has demonstrated that IGM can fly this mission, while QUOTA has been able to handle the cases tried.

Another use to which QUOTA has been applied is an abort in flight. In this case, the simulation was a 5-10% decrease in thrust at some time along the trajectory of an S-IV stage being inserted into a 200 nautical mile circular orbit at a specified inclination. To salvage the mission, the boundary conditions were changed in flight to achieve a 100 x 200 n.m. ellipse at the specified inclination. Again, QUOTA experienced no difficulties whatever for the cases examined.

Another extension we are currently making is a minimum fuel orbital transfer. This is a burn-coast-burn problem that so far we have not handled optimally due to inadequate criteria for beginning and ending the coast. This problem appears solvable in the near future enabling a full rendezvous problem to be considered then.

CONCLUSIONS

In conclusion, a numerical procedure for generating quasi-optimal trajectories has been developed which incorporates the ability to perform a wide variety of missions. The development of a rapid numerical isolation tool appears to make this procedure capable of being used as an on-board guidance system. In at least one class of missions, namely, continuous thrust orbital transfer, it fulfills a need that so far the current IGM has not been demonstrated to meet.

ANALYTICAL SOLUTION OF OPTIMAL GUIDANCE PROBLEMS

Rowland E. Burns

INTRODUCTION

One of the early problems of flight mechanics was the determination of a steering program by which a rocket vehicle could achieve some specified mission. The early guidance schemes, admittedly utilitarian, have since been supplanted by more sophisticated optimal trajectory schemes which maximize some quantity (such as payload) in addition to achieving other mission objectives.

The study of these optimal trajectories has produced a number of significant results. One of the more far-reaching conclusions of these studies is the solution of a set of differential equations which govern the motion of a vehicle assumed to be flying in vacuuo in a uniform gravitational field (the "flat earth" approximation). This solution - one of the few yet found - forms the basis of the guidance systems currently used in large upper stage boosters.

DISCUSSION

In this connection, it may be well to define what is meant by a solution. From Newton's laws it is possible to write equations which relate the radius vector (describing the position of the vehicle in space) to the velocity and acceleration of the rocket. The information necessary for guidance commands is not the acceleration along the trajectory but rather the angular deflection of the thrust vector as a function of time. This information would then predict the velocity and position of the rocket as a function of time. By an analytic solution, we shall mean an expression which relates the velocity or position with the time, steering angle, and initial conditions by means of elementary functions.

The reason that such analytic solutions are desirable is that algebraic equations are more readily handled by computers than are differential equations. Thus, a given guidance scheme can be implemented by a much less sophisticated computer onboard a vehicle if such a solution is known.

The solution of the system of differential equations which forms the basis of present day guidance schemes has limited applicability due to the assumptions of a drag free environment and a uniform gravitational field. The first of these assumptions is less restrictive than the second since the guidance scheme is usually applied to the upper stages of the vehicle. The assumption of a flat earth is, however, much less realistic. Certain missions which are flown with such a guidance law cannot be expected to even approximate a minimum fuel trajectory.

Current work on this problem is an attempt to relax the approximation of a flat earth and solve the problem with a better model that assumes a spherical earth.

For the sake of generality, the thrust is assumed to be continuously variable between upper and lower limits. (These limits are specified as initial input to the problem.) The mathematics then shows that some portions of the trajectory may be flown with maximum thrust, some portions with minimum thrust (which could be coasting arcs), and some portions with intermediate thrust.

The case of intermediate-thrust arcs intuitively appears to be more difficult to handle than arcs which possess a constant thrust level. While this is true from the viewpoint of hardware implementation, it is not true mathematically. For the intermediate-thrust case, a complete solution has been obtained and published in NASA TN D-4119.*

The second case to be considered is the equations of rocket motion in the case of constant thrust. This difficult problem has not been solved, but some progress has been made. Specifically, a system of three algebraic equations is known which, in theory, reduce the problem of specifying the guidance system from a set of six differential equations to a set of three differential equations. In practice, the form of the

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*The question of the payload penalty incurred by using non-throttleable engines naturally presents itself. This question is not handled in the above reference which is a theoretical discussion. The answer is apparently not available in the literature for launch from earth to orbit. One indicative comment comes from a study of the maximum altitude problem. It has been found that the inclusion of arcs of intermediate-thrust increases the final altitude by approximately 17%. The use of a "two-stage" thrust which approximates the intermediate thrust arc achieves an altitude within 1% of this value.

algebraic equations is so intractable that full potential cannot yet be realized. Present work is concerned with a transformation of the variables appearing in these algebraic equations to new variables in such a way that the reduction from six differential equations to three differential equations will be possible.

Once such a transformation is obtained, future work will center upon the derivation of three new independent algebraic integrals. If this is successful, a closed form expression for the optimal guidance law over an airless planet will be available.

A special case of rocket motion under a constant thrust is the case of no thrust, i.e., a coasting arc, which was mentioned above. This special case is of interest from the mathematical viewpoint for reasons of continuity; but, physically, it is equivalent to asking the direction of thrust when the rocket engines are not operative. This case was completely solved and published, originally, as Aeroballistics Internal Note 20-63. As would be expected, the trajectory of the vehicle is a Keplerian orbit.

REDUCTION OF CONTROL SYSTEM DESIGN EFFORT

Jerome R. Redus

INTRODUCTION

The design of a launch vehicle flight control system is currently a time-consuming process that frequently requires major redesign effort when changes are made in the vehicle characteristics or mission profile. For example, as Dr. von Braun has noted, it takes ten and one-half months to effect a change in the uprated Saturn I control system.

In addition to the long time and large effort involved, current design practices involve a large amount of iteration by trial-and-error requiring much routine effort by the design engineer, and the system evolved depends quite heavily on the designer's experience. The more difficult it is to meet the design specifications, the more time will be required for trial-and-error and the less likely it will be that a designer without related experience will achieve an entirely satisfactory design in a limited amount of time.

The reduction of control system design effort appears to be even more important in the future than it has been in the past, due to the number of different payloads being proposed for existing launch vehicles and the relatively limited number of flights planned for each payload. Not only will the number of flight control systems designed be increasing, but the cost of design will have to be divided among a smaller number of flights. In addition, the fact that the launch vehicle was not designed for the dynamic loads created by these alternate payloads will frequently increase the effort required in successful design.

Aero-Astrodynamics Laboratory is attempting to develop flight control system design techniques which will permit more rapid design of effective, reliable systems. The goal of the studies is the development of a semi-automatic, computer-oriented design procedure which would achieve as good performance as is possible within specified restrictions on the characteristics of the system.

DESIGN CONSIDERATIONS

Before summarizing the current status of the investigations, it would be useful to point out the three major problems in launch vehicle control system design.

These three aspects of control system design are

A. achieving satisfactory dynamic performance

B. signal processing or filtering

C. accommodations of uncertainties in vehicle characteristics.

First of all, of course, the flight control system must control the direction of thrust in the presence of the expected winds aloft in a manner which will meet the cutoff conditions on trajectory deviations, not exceed the vehicle structural load limits, achieve cutoff conditions on angle-of-attack and attitude rate such that stage separation can be effected, and meet other restrictions on performance. The engineer attempts to accomplish this by proper choice of vehicle sensors and sensor locations and proper choice of the gains on the sensor signals which are fed back to the gimbal command signal.

Once the sensors and gains are chosen, the sensors must be filtered to insure that additional motion sensed by the measuring devices, such as fuel slosh motion and structural vibrations, are not fed back in such a way as to pump energy into these additional degrees of freedom and destabilize them. That is, the engineer must choose filters in the control system which insure that the total motion of the vehicle is stable and that the dynamic performance remains satisfactory.

Last, the engineer must insure that satisfactory performance and stability are still achieved for the expected range of uncertain vehicle characteristics, such as variations in the aerodynamics from the nominal aerodynamics for which the system was designed.

DESIGN FOR SATISFACTORY DYNAMIC PERFORMANCE

The aspect of control system design in which the greatest development to date has occurred is that of designing a system to have satisfactory dynamic response; this is due not only to advancements in control theory but also to the recent development of accurate wind statistics.

The development of an accurate set of statistics describing the winds aloft in the vicinity of Cape Kennedy has first of all permitted the calculation of the probability of failure of the missile with the candidate control system to meet the performance requirements. In the past, the control system has been required to achieve adequate performance in the presence of a synthetic design wind profile (say, the "95% worst" profile), but there has been no further measure of comparison between competing systems, all of which meet this performance specification. A statistical comparison is now possible, and one can determine which of the competing systems is best in terms of its dynamic performance.

Indeed, using certain generally employed simplifying assumptions about the dynamic behavior of the launch vehicle, there has been developed a technique to determine the optimal linear control for the statistically described winds aloft. This control system has a lower probability of failure to meet dynamic specifications than any other linear flight control system for a given launch vehicle. Unfortunately, this system is extremely complex and would be exceedingly difficult to implement. The calculation of the optimal control would be useful, however, not only to determine the best possible performance against which to compare the performance of a realizable linear system but also to determine what the general characteristics of the optimal control are (e.g., what it measures and feeds back) in order to give the designer a sound basis on which to begin his iterative effort. In essence, the engineer can augment his experience by observing what optimization theory says is the best linear flight control system and can spend his efforts determining how to obtain a realizable system that approximates the behavior of the optimum system.

In the design of a realizable system approximating the behavior of the optimal system, techniques have been developed recently which permit the rapid assessment of the statistical performance of the vehicle. This rapid statistical analysis when employed with parameter optimization techniques allows iterative efforts to be on a sounder basis than merely trial-and-error. We are now developing design techniques based on this approach and expect to see a substantial improvement in the speed with which this phase of the design can be completed; a simultaneous improvement in performance is expected to be a side benefit of this approach.

FILTER DESIGN

In the design of stabilizing filters, new techniques have been developed which are promising, but it must be admitted that we do not yet know how to use these techniques entirely effectively. The conventional approach used in the past has been for the engineer to arbitrarily specify the filter characteristics based on his experience. He then mathematically analyzes the performance of the resulting system to determine if it is stable and otherwise satisfactory. If not, he adjusts the filter characteristics and examines the resulting system, makes further adjustments, and continues repeating the process until satisfactory response is obtained.

The filter design technique recently developed reverses the process. The engineer specifies the performance (in terms of the eigenvalues of the system) and partially specifies the filter; the computer completes the design of a filter which will attain the desired performance. There are currently problems with the technique because the filter designed is not always "realizable" (i.e., able to be implemented by passive electronic components, as would be desirable), but it is felt that further experience in using the technique and particularly a better understanding of how to specify the eigenvalues so as to obtain both satisfactory performance and realizable filters will make the technique very useful to the design engineer.

PARAMETER SENSITIVITY

Any system, no matter how designed, will have to be examined to insure that its performance is adequate for the expected range of off-nominal conditions or parameter uncertainties. If one could consciously design the system to be insensitive to parameter variations, one would have more assurance of adequate performance over the expected range of parameters. The design process would not proceed on the conventional assumption of a nominal set of parameters, but rather on the assumption that each of the parameters of motion can take on a range of values. Investigations have shown that control systems can be designed with this assumption. However, the systems evolved to date have been more complex than those designed for a nominal set of parameters and also have degraded performance for the nominal set. Design of systems consciously insensitive to parameter variation does not appear to be currently necessary in launch vehicle system design. The technique may be useful for some future large vehicle/payload combinations for which the range of possible parameters could be very large and testing to better identify the parameters quite expensive. This technique may also prove to be useful in some spacecraft applications where the range of parameters may be too large to be handled by conventional means.

SUMMARY

Continued effort to reduce the effort involved in flight control system design for launch vehicles is necessary because of the variety of missions to be accomplished in the future by existing vehicles. The most promising advances to date have been made in the area of system design to meet performance specifications.

N68-18992

LOAD RELIEF SYSTEMS FOR LAUNCH VEHICLES

John M. Livingston

INTRODUCTION

The Optimization Theory Branch is conducting several studies of load relief control systems. The objective of these studies is to develop control systems which will reduce the contribution of body dynamics to the forces acting on the structure of the launch vehicle. To fully understand the problems involved in designing a load relief control system, one must review the function of a control system and the basis of its design.

DISCUSSION

The primary purpose of the control system during the first stage flight is to keep the vehicle following a predetermined path angle until staging takes place. If a disturbance such as in-flight winds turns the vehicle from the reference angle, the control system will force the vehicle back to the proper orientation. This action causes a dynamic structural load on the vehicle which can be excessive; therefore, in selecting a control system, careful analysis must be made to determine the vehicle dynamic responses to any such disturbance for each candidate control system to insure that neither the structural load nor the deviation from reference exceeds applicable limits.

There is a definite interplay between the vehicle, the disturbance, and the control law. Changes in any one of these three will produce a new relationship among all of them. For example, the attitude control mode which consists of a measure of path error and path error rate feedback, will force the vehicle to follow the reference path closely; thus, it fulfills the basic requirements of the launch vehicle control system. However, when the vehicle encounters the high wind speed levels of the "jet stream," the attitude control mode will impose large structural loads on the vehicle, even larger than other modes of control. If the vehicle is being designed for this control system, the structure must be made strong enough to withstand these loads. Therefore, the vehicle structure is directly affected by the relationship of the disturbance and the control law.

Thus, control system design is an iterative process. The control system designer can change the mode of control or recommend change in the vehicle to meet a given law. What of the disturbance? In the case of the in-flight winds, they are, like Mr. Kilmer's "tree," out of the designer's league.

What if the launch vehicle is already built, and new mission payloads are desired? Here the structure of our launch vehicle is set; and rebuilding, if possible, would be very expensive. So, the control system designer has an opportunity to use his experience and knowledge to design a load relief control system that will allow the vehicle to meet the mission requirements without structural modifications. In approaching the design of a load relief system, it is important to recognize that there is no "universal" load relief system that works for all cases. Each vehicle and mission calls for different requirements for the system.

These requirements are determined by the structural loading in the vehicle. The structural loading, in turn, is a function of the bending moment (M.B.) where

$$\text{M.B.} = M'_\alpha \alpha + M'_\beta \beta + \sum_{i=1}^{n} M'_{\ddot{\eta}_i} \ddot{\eta}_i$$

with alpha (α) being the angle-of-attack, beta (β) being the gimbal angle of the engines, and the Σ term representing the flexible body effects. The coefficients M'_α, M'_β and M'_η are primarily functions of the vehicle characteristics at the given flight conditions and station along the vehicle.

If the flexible body effects are set aside for the moment (an admittedly large "if"), an interesting trade-off can be developed between the coefficients and the vehicle responses. The ratio of M'_α to M'_β varies along a Saturn class vehicle as shown in figure 1. The ratio of α to β, however, varies with the control mode (typical examples are shown in figure 2) and is independent of the vehicle station. Both conditions are useful in determining the proper method of reducing the bending moment. For example, the Saturn V-Voyager has a critical loads station which is located forward on the vehicle with an M'_α/M'_β ratio of 5. This means a control law that reduces the angle-of-attack during the critical time of flight will provide the greatest load relief. A system based on this premise has been under development by the Optimization Theory Branch and offers a solution to the excessive structural loading on the Saturn V/Voyager vehicles.

Conversely, if the vehicle has the critical station aft, the ratio of M'_α to M'_β will be less than 1.0, and it will be desirable to reduce β to effect the greatest decrease in bending moment. Indeed, this is the case for the Saturn V-Apollo. It had been suggested that the Saturn V/Voyager load relief control system might be applied to the Saturn V/Apollo, but the difference in the effects of α and β terms on the bending moments made this application impractical. The general design technique did, however, indicate a promising system for the Saturn V/Apollo. The control law has a different form because of the different critical station.

It was noted earlier that the control system designer has no influence over the major booster trajectory disturbance, in-flight wind. While the control system designer cannot influence the wind disturbance, he would be aided in his analysis if he had detailed knowledge about its nature. Reliable statistics on the wind structure are now becoming available from Jimsphere measurements. As wind statistics knowledge is acquired, the control problem will be solvable through stochastic system analysis. It is current practice to use "gross" wind statistics as a basis for altering the tilt program in cases where wind-biasing the trajectory will provide a solution if the launch dates are such that the wind has a predominant direction.

CONCLUSION

In summary, the control designer must use his knowledge of the vehicle, the control laws, and the disturbances to design the best system. The control designer's experience - along with a little inspiration - is invaluable in attacking the load relief problem.

FIGURE 1

TYPICAL MAGNITUDE RATIO M'_α / M'_β

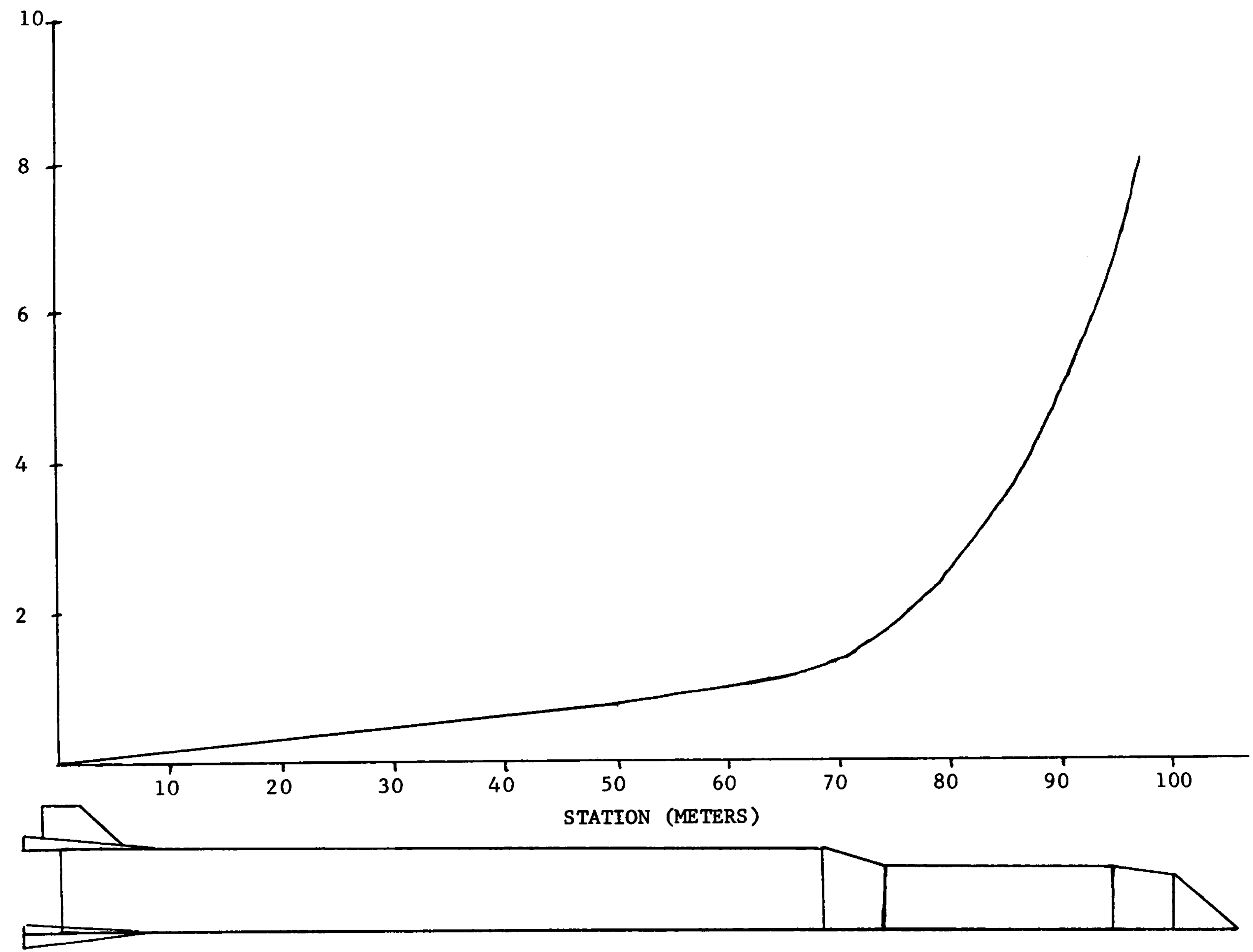
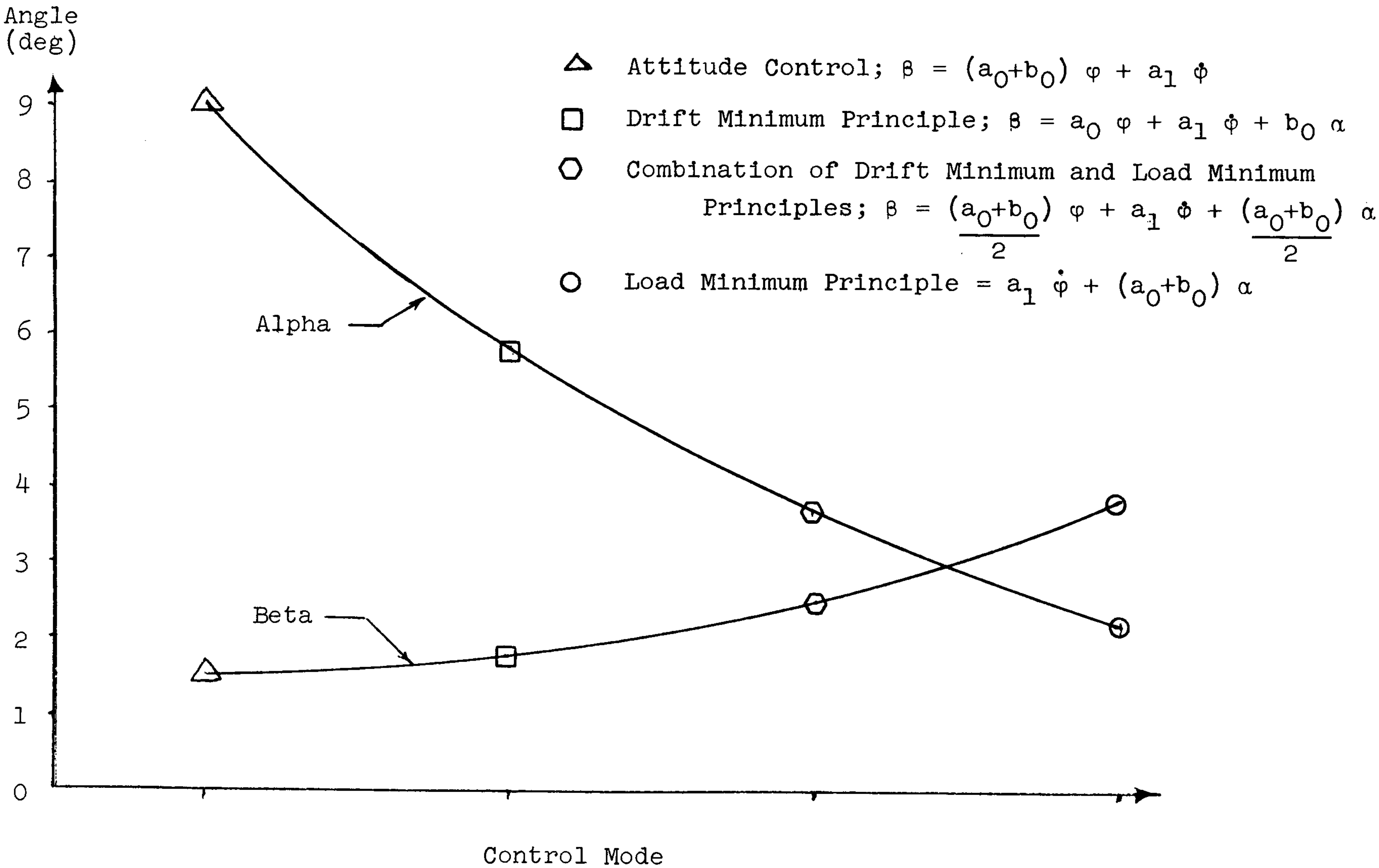


FIGURE 2

TYPICAL ANGULAR RESPONSES FOR
SELECTED CONTROL LAWS



ASTRODYNAMICS, GUIDANCE AND CONTROL

REVIEW #1

Astrodynamics & Guidance Theory Division
Aero-Astrodynamics Laboratory

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officers. This report, in its entirety, has been determined to be Unclassified.

This document has also been reviewed and approved for technical accuracy.

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