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ESTIMATION OF INERTIAL PLATFORM ERRORS

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Prepared by

John V. Brown

and

James S. Tyler, Jr.

PHILCO-FORD CORPORATION
Space & Re-Entry Systems Division
Palo Alto, California

for

National Aeronautics and Space Administration
Marshall Space Flight Center
Huntsville, Alabama

PHILCO
PHILCO-FORD CORPORATION

Space & Re-entry
Systems Division

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Cont)

LIST OF ILLUSTRATIONS (Cont)

SECTION 1

INTRODUCTION

This report presents the results of one of the three parts of Contract NAS8-20358, entitled Advanced Spaceborne Tracking, Detection, and Navigation Study. The objective of this part of the contract was to evaluate the feasibility of estimating the inertial guidance platform errors from data obtained during a powered flight ascent.

One method of estimating the inertial platform errors is to combine the telemetry data, which contains information on the vehicle position-velocity state plus deviations in the trajectory caused by the platform errors, with the tracking data, which also contains information on the vehicle state, but includes errors from the tracking system.

There are three potential uses for the information contained in this combined platform-tracking system. If it is possible to estimate the platform errors during a flight, then the guidance system could be updated. Also the ability to determine platform errors would be useful in a postflight analysis where the objective is to determine whether the guidance components performed according to specification. Although the primary interest in this study is to evaluate the ability to estimate the guidance errors, a third use of this data would be to obtain a better estimate of the vehicle than could be obtained with tracking data alone.

The feasibility of estimating the platform errors has been evaluated in this study by simulating an orbit determination process that would combine the telemetry and tracking data. It is the ensemble behavior of this combined system and in particular the behavior of the covariance matrix of the errors in estimate of the position-velocity state plus the platform and tracking errors, that has been investigated. A Kalman or minimum variance filter has been assumed for the estimation process.

Platform errors as well as a nominal trajectory that are similar to a Saturn V mission have been used in the study. The equations for the platform model, the tracking model, and the minimum variance estimation are presented in Section 2. A discussion of the computer program used to generate the covariance matrix of errors,and the nominal ascent trajectory along which the covariance matrix is propagated, are included in Section 3 and Section 4, respectively. Section 5 discusses the effect of individual error sources in the platform and tracking systems on the trajectory. These results are used to define the significant error sources which should be included in the combined platform-tracking system model.

The principal results of the study are presented in Section 6. In this section the feasibility of estimating the platform errors is evaluated as a parametric function of the system error sources and tracking parameters.

SECTION 2

DERIVATION OF EQUATIONS

2.1 INERTIAL PLATFORM ERROR MODEL

The inertial platform model that has been used for this study consists of up to 30 error sources and is similar to that used by Daniels, Neighbors, and Cole, in Reference 1. The four types of accelerometer errors and the four types of gyro errors considered are:

α_i the i^{th} accelerometer bias, km/sec^2

β_{ij} the misalignment of the i^{th} accelerometer into the j^{th} axis, radians

ϵ_i the scale factor error of the i^{th} accelerometer, parts/part

s_i the threshold of the i^{th} accelerometer, km/sec^2

ϕ_{oi} the initial platform misalignment about the i^{th} axis, radians

$\dot{\phi}_{oi}$ the steady-state drift rate of the i^{th} gyro, rad/sec

$\left.\begin{matrix}\mu_{Ii}\\ \mu_{si}\end{matrix}\right\}$ the mass unbalance drift about the input and spin axes, (rad/sec)/(km/sec^2)

c_i the anisoelastic drift of the i^{th} gyro, (rad/sec)/(km/sec^2)^2

The platform is oriented in an inertial frame fixed at launch as shown in Figure 2-1.

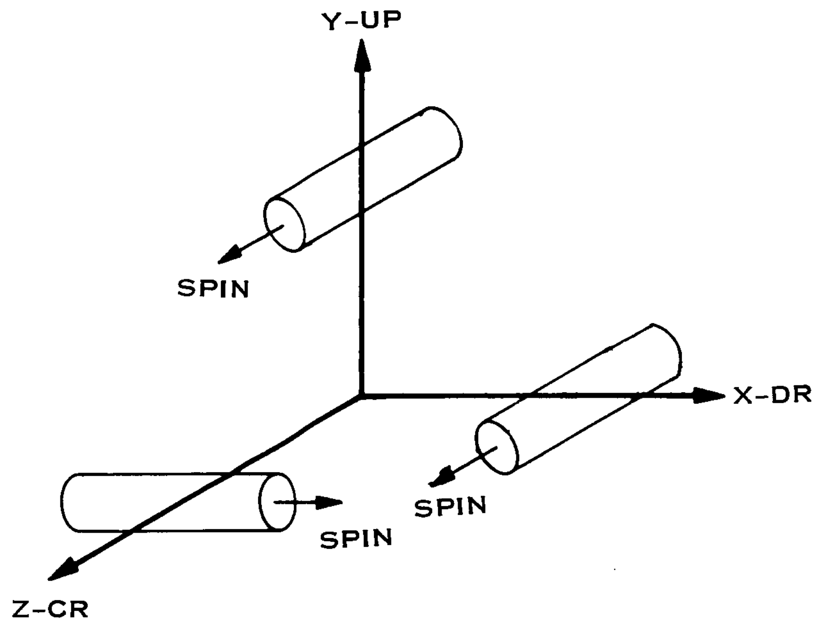

Figure 2-1 Platform Orientation

The gyro input axes are along the X, Y, and Z axes and the three accelerometers are also assumed to be mounted along these axes.

The platform error model used in this study includes the total error in acceleration caused by (1) the misalignment of the platform axes resulting from the gyro drifts and initial misalignment, and (2) the errors in the accelerometers themselves which are mounted on the drifting platform axes, for a given nominal acceleration time history. The drift rate for the i^{th} gyro is given by

$$\dot{\Phi}_i = \dot{\phi}_{oi} + \begin{bmatrix} \mu_{si} & \mu_{Ii} & 0 \end{bmatrix} \begin{bmatrix} a_{Gis} \\ a_{GiI} \\ a_{Gio} \end{bmatrix} + c_i (a_{Gis})^2, \tag{2-1}$$

where a_{Gis}, a_{GiI}, and a_{Gio} are the accelerations along the spin, input, and output axes of the i^{th} gyro, respectively. The total platform drift, which is found by integrating (2-1), is

$$\Phi_i = \phi_{oi} + \dot{\phi}_{oi} t + \begin{bmatrix} \mu_{si} \mu_{Ii} & 0 \end{bmatrix} T_{PI2Gi} \int_o^t a_{PIi}(\tau) d\tau + c_i \int_o^t \left(q_i \, a_{PIi}(\tau) \right)^2 d\tau, \tag{2-2}$$

where a_{PIi} is the acceleration along the i^{th} ideal platform axis, T_{PI2Gi} is the transformation from the ideal platform axes to the i^{th} gyro axis, and q_i, is the first row of T_{PI2Gi}. For the gyro orientation in Figure 2-1, T_{PI2G1}, the transformation from the ideal platform axes to the X gyro axes, is

$$T_{PI2G1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2-3}$$

where the order of the gyro axes is $\begin{bmatrix} s \\ I \\ o \end{bmatrix}$.

The accelerometer errors are found from the sensed acceleration along the accelerometer axes, a_{si}', which is

$$a_{si}' = \begin{bmatrix} \alpha_x + s_x \\ \alpha_y + s_y \\ \alpha_z + s_z \end{bmatrix} + \left\{ I + \begin{bmatrix} \varepsilon_x & \beta_{xy} & \beta_{xz} \\ \beta_{yx} & \varepsilon_y & \beta_{yz} \\ \beta_{zx} & \beta_{zy} & \varepsilon_z \end{bmatrix} \right\} \begin{Bmatrix} a_{PIx} \\ a_{PIy} \\ a_{PIz} \end{Bmatrix} \qquad (2\text{-}4)$$

The total error in acceleration is given by the sum of the accelerometer errors plus the error in acceleration caused by the platform misalignment. The total sensed acceleration can be written as the sum of the acceleration in (2-4) plus the cross product of the platform orientation angles, Φ_i in (2-2), and the true acceleration along the ideal platform axes. This expression is

$$a_{si} = \begin{bmatrix} \alpha_x + s_x \\ \alpha_y + s_y \\ \alpha_z + s_z \end{bmatrix} + \left[I + E \right] a_{PI} + \begin{bmatrix} \Phi_x \\ \Phi_y \\ \Phi_z \end{bmatrix} \otimes a_{PI}, \qquad (2\text{-}5)$$

where $\otimes$ denotes the cross product, and E is the matrix of scale factor and misalignment errors in (2-4).

2.2 DYNAMIC EQUATION OF THE VEHICLE

The dynamic equations of the vehicle include the effects of gravity as well as the thrusting accelerations and the acceleration errors. These equations can be expressed in an inertial frame as

$$\ddot{R} = G(R,t) + T_{A2I}\, a_{PI}(t), \qquad (2\text{-}6)$$

where $G(R,t)$ is the gravitational acceleration and T_{A2I} is the transformation from accelerometer axes to the inertial axes, and $a_{PI}(t)$ has been defined as the true value of the non-gravitational acceleration. Since $a_{PI}(t)$ can be expressed as the difference between the sensed acceleration and the acceleration errors, equation (2-6) can be written as

$$\ddot{R} = G(R,t) + T_{A2I}\left[a_s(t) - \delta_a(e,t)\right], \tag{2-7}$$

where $\delta_a(e,t)$ is a function of the platform errors and from (2-5) is given by

$$\delta_a(e,t) = \begin{bmatrix} \alpha_x + s_x \\ \alpha_y + s_y \\ \alpha_z + s_z \end{bmatrix} + E\ a_{PI} + \Phi \otimes a_{PI} \tag{2-8}$$

Representing (2-7) as

$$\ddot{R} = G(R,t) + f(e,t), \tag{2-9}$$

and considering perturbations only about the nominal values of R and a, results in the linear equations of motion that are used for the error analysis. These equations are

$$\Delta\ddot{R} = F\Delta R + B\ \Delta a, \tag{2-10}$$

where

$$F = \left.\frac{\partial\ G(R,t)}{\partial R}\right|_{R\ \text{Nominal}} \qquad B = \left.\frac{\partial f(e,t)}{\partial a}\right|_{a\ \text{Nominal}}$$

For the purpose of error propagation, the total state vector (z), in general, may consist of the 6 vector of position (R) and velocity (V) plus any number of the sensor error and tracking bias error sources (e), i.e.,

$$z = \begin{bmatrix} R \\ V \\ e \end{bmatrix} = \begin{bmatrix} x \\ e \end{bmatrix} \tag{2-11}$$

The state transition matrix for this expanded state vector is given by

$$\Phi(t,t_o) = \begin{bmatrix} \frac{\partial x}{\partial x_o} & \frac{\partial x}{\partial e} \\ 0 & I \end{bmatrix} = \begin{bmatrix} \phi & \phi_u \\ 0 & I \end{bmatrix} \tag{2-12}$$

The expression $\frac{\partial x}{\partial x_o}$ is found by solving the following equation for $\frac{\partial R}{\partial x_o}$:

$$\frac{\partial \ddot{R}}{\partial x_o} = \frac{d^2}{dt^2} \frac{\partial R}{\partial x_o} = \frac{\partial G}{\partial R} \frac{\partial R}{\partial x_o} , \tag{2-13}$$

where $\frac{\partial R}{\partial x_o}$ is a 3x6 matrix and

$$\frac{\partial G}{\partial R} = F$$

The sensitivities of the position and velocity state to the i^{th} platform error source are found by integrating the equation

$$\frac{d^2}{dt^2} \frac{\partial R}{\partial e_i} = \frac{\partial \ddot{R}}{\partial e_i} = \frac{\partial G}{\partial R} \frac{\partial R}{\partial e_i} - \frac{\partial \delta a}{\partial e_i} \tag{2-14}$$

The velocity partial $\frac{\partial V}{\partial e_i}$ in (2-12) is found by differentiating the solution for $\frac{\partial R}{\partial e_i}$ in (2-14). Detailed descriptions of the error source sensitivities $\left(\frac{\partial\ \delta a}{\partial e_i}\right)$ are given in Reference 2.

2.3 TRACKING EQUATIONS

The tracking model is obtained by assuming that the radar observations are of the form

$$y = f(X, W, T) + q$$

where X = 6 state of position and velocity (2-15)

W = tracking bias errors

Linearization of this equation about a nominal trajectory results in the following equations for the measurement y:

$$y = \frac{\partial f}{\partial X} x + \frac{\partial f}{\partial W} W + q(t) \tag{2-16}$$

The tracking model consists of the partials of the observation with respect to the state (H) and the partial of the observation with respect to the measurement bias parameters (G), and the random measurement error vector q(t). For each observation the model is given by

$$y = Hx + Gw + q(t), \tag{2-17}$$

where H and G are row vectors.

The tracking that can be simulated includes range, range-rate, azimuth, elevation, right ascension, declination, and direction cosines and their rates. Each of these measurements may contain a random and a bias error, in addition to the station-location errors for the station and a station clock error.

2.4 MINIMUM VARIANCE ESTIMATION EQUATIONS

Since the equations for the minimum-variance estimator are an essential part of the error analysis of the inertial platform and tracking systems, these equations are summarized here. It is the ensemble type behavior of the inertial platform-tracking system that is to be studied and therefore the variance-covariance matrix of the error in estimate of the state (P) is processed by the filter. The nominal trajectory is used throughout to evaluate the measurement, platform, and state sensitivities, H, ϕ_u, and ϕ, respectively. Residuals are not determined in this analysis. The following equations summarize the minimum variance or Kalman estimation process.

<u>Propagating in Time</u>

$$x(t) = \phi(t,t_o)\ x(t_o) \tag{2-18}$$

$$P(t) = \phi(t,t_o)\ P(t_o)\ \phi(t,t_o)^T \tag{2-19}$$

<u>After an Observation</u>

$$\hat{x}_n = \hat{x}(t) + P_n\ H^T(HP_nH^T + Q)^{-1}(y-\hat{y}) \tag{2-20}$$

or

$$\hat{x}_n = \hat{x}(t) + K(y-\hat{y}) \tag{2-21}$$

$$P_n = P(t) - P(t)\ H^T(HP(t)H^T + Q)^{-1}HP(t) \tag{2-22}$$

where

$\hat{x}_n$ = new estimate of the state after an observation

$\hat{x}(t)$ = estimate of the state at time t before an observation

H = the gradient of the observation with respect to the state

y = measurement

K = filter gain

$Q = E\left[qq^T\right]$ - the covariance matrix of the random measurement error q

$\tilde{x} = x - \hat{x}$ - the error in estimate of the state

$P_n = E\left[\tilde{x}\,\tilde{x}^T\right]$ - variance-covariance matrix of the error in estimate of the state

Since the measurements are assumed to be processed sequentially, H = h is a 1 x m vector and Q = q is a scalar. Also, if no bias errors are estimated x is a 6 x 1 vector of the position-velocity state. However, if dynamic bias errors and measurement bias errors are estimated, then the total state vector x_e is given by

$$\chi_e = \begin{bmatrix} x \\ u \\ w \end{bmatrix}, \tag{2-23}$$

where

u = d x 1 vector of dynamic bias errors

w = m x 1 vector of measurement bias errors

For this study u may include the inertial platform bias error sources and w may include (1) range, range-rate, azimuth, elevation, or direction cosine, etc., measurement bias errors, (2) station location errors in latitude, longitude, and elevation, and (3) the clock or measurement timing error. The transition matrix for the expanded state vector x_e is then given by

$$\Phi_e = \begin{bmatrix} \overset{6x6}{\phi} & \overset{6xd}{\phi_u} & \overset{6xm}{0} \\ 0 & \overset{dxd}{I} & 0 \\ 0 & 0 & \overset{mxm}{I} \end{bmatrix} \tag{2-24}$$

The covariance matrix of the error in estimate of the expanded state (P_e) will have the dimensions of Φ_e and is again propagated in time according to Section 2.4 with the Φ_e of (2-24), and updated at an observation according to (2-20) with an expanded H vector that is given by

$$H_z = \begin{bmatrix} H \\ 0 \\ G \end{bmatrix}, \tag{2-25}$$

and an expanded covariance matrix given by

$$P_e = \begin{bmatrix} \overset{6x6}{P} & C_{de} & C_{me} \\ C_{de}^T & D & C_1 \\ C_{me}^T & C_1^T & W \end{bmatrix}, \tag{2-26}$$

where

$D = E[uu^T]$ - the covariance matrix of the dynamic bias errors (including the inertial guidance errors)

$W = E[ww^T]$ - the covariance matrix of the measurement bias errors and station location errors.

A more detailed description of these equations is given in Reference 3.

2.4.1 Equations for Determining the Effect of Neglecting Error Sources

The standard equations for the Kalman filter, which have been summarized in the previous section, are applicable when all the elements in the state vector are to be estimated. A variation on this basic estimation process has been developed which enables one to determine the effect of neglecting error sources.

The problem of determining which error sources are significant arises because of the large number of errors that may exist in the combined inertial platform-tracking system. Since it is unlikely that in an actual orbit determination, all the known error sources would be estimated, it becomes necessary to determine in advance, which sources contribute significantly to the error in estimate of the state, and therefore should be modeled. Determining which error sources are significant can be accomplished by simulating the mission with the error sources of questionable importance neglected from the model, and determining the effect of neglecting these sources at the end of the simulation.

The general form of the total state covariance matrix (P_{ne}), when some parameters are estimated and other parameters are neglected, is shown in (2-27). The equations for updating the estimated parameters whose covariance matrices are P, D and W, have been given in the previous section. The variances of the neglected dynamic parameters D_o and the neglected measurement parameters W_o, are constant, and therefore not updated in time or after one observation. However, the correlations between the neglected parameters and the estimated state, C_{dn} and C_{mn}, are updated, and it is through these correlations that the effect of neglecting parameters is determined.

$$P_{ne} = \left[\begin{array}{ccc|cc} \overset{6\times 6}{P} & C_{de} & C_{me} & C_{dn} & C_{mn} \\ C_{de}^T & D & C_1 & & \\ C_{me}^T & C_1^T & W & & \\ \hline & & & D_o & 0 \\ & & & 0 & W_o \end{array}\right] \tag{2-27}$$

The correlation between the state and the measurement biases (C_{mn}) and the correlation between the state and the dynamic biases (C_{dn}) are computed as follows:

$$C_{dn}(t) = \phi(t,t_o)\ C_{dn}(t_o) + \phi_u D \qquad (2\text{-}28)$$

$$C_{mn}(t) = \phi(t,t_o)\ C_{mn}(t_o) \qquad (2\text{-}29)$$

$$C_{dn}^{+} = [I-KH]\ C_{dn}^{-} \qquad (2\text{-}30)$$

$$C_{mn}^{+} = [I-KH]\ C_{mn}^{-} - KGW_o\ , \qquad (2\text{-}31)$$

where (-) and (+) denote time before and after an observation, respectively.

The total covariance matrix P_T which gives the uncertainty of having neglected measurement and dynamic bias errors, in addition to the uncertainties given by P_n and P(t) in (2-19) and (2-22), is

$$P_T = P_e + C_{dn} D_o^{-1} C_{dn}^{T} + C_{mn} W_o^{-1} C_{mn}^{T} \qquad (2\text{-}32)$$

where P_e is the m x n matrix in (2-26) which includes P, D and W. Since the last two terms in (2-32) can be written with C_{dn} and C_{mn} as column vectors and D and W as scalars, or with C_{dn}, C_{mn}, D, and W as matrices, equation (2-32) may be used to obtain the effect of neglecting individual error sources or groups of error sources.

2.4.2 The Concept of an Equivalent Observation

One of the implications of this study is that a parametric analysis of the error sources be made in order to determine the effect of different numerical values of the error sources. A parametric study of these error sources requires that the initial standard deviations of each error parameter be changed through a wide range of values, and the effect of these changes on the uncertainty in the other error sources as well as the state, noted at the trajectory. The technique presented in this section enables the initial standard deviations of any of the estimated bias parameters, to be changed in one simulation of the mission, i.e., one computer run.

A special case of what has been defined as an equivalent observation, is where there is no random measurement error. In this case a perfect observation of the position-velocity state or of other parameters can be made. The perfect observation is useful for determining the effect of an uncertainty in one parameter on the uncertainty in another parameter, or the uncertainty in the position or velocity state.

2.4.2.1 Equations For an Equivalent Observation. In a conventional error analysis, where for example one element of P may be the variance of an inertial guidance error and a number of observations are made of the vehicle (range, range-rate, etc.), the effect of changing the initial variance on the final uncertainty of the vehicle state, would be determined by simulating the mission with a new value of this particular variance. This would correspond to changing the initial variance of the parameter (P_o) by a factor of k, i.e.,

$$P_o' = kP_o \tag{2-33}$$

However, this same value of the initial variance (P_o') can be found by taking a direct observation of the parameter whose variance is to be changed (h = 1), with a measurement error variance (q) that is specified such that the state variance after the observation is P_o'. For the scalar case the equation for an observation is

$$P = \frac{P_o q}{h^2 P_o + q} \tag{2-34}$$

Since the value of P after the observation (P_o') is given by (2-33), equating (2-33) to (2-34) and solving for q gives the result

$$q = P_o \frac{k}{1-k} \tag{2-35}$$

The second part of this derivation is to show that the order of the observations does not matter, i.e. the direct observation can be made at the end of the run as well as at the beginning. The fact that the order of the observations can be interchanged without changing the final state uncertainty can easily be seen from the least squares form of P.
The identity between the Kalman and least squares form of the observation equation is

$$P = P_o - P_o H^T [HP_o H^T + Q]^{-1} HP_o$$

$$= [P_o^{-1} - H^T Q^{-1} H]^{-1} \tag{2-36}$$

After a number of observations it can be seen that the state uncertainty P is given by

$$P_n = [P_o^{-1} + H_1^T Q_1^{-1} H_1 + H_2^T Q_2^{-1} H_2 + \dots$$
$$H_n^T Q_n^{-1} H_n]^{-1} \tag{2-37}$$

Clearly the order in which the terms in (2-37) are added, does not affect the result. Therefore it is possible to change the initial variance of a parameter by a direct observation of this parameter at the end of a run.

The use of an equivalent observation to change the initial variance can be summarized with the aid of Figure 2- 2.

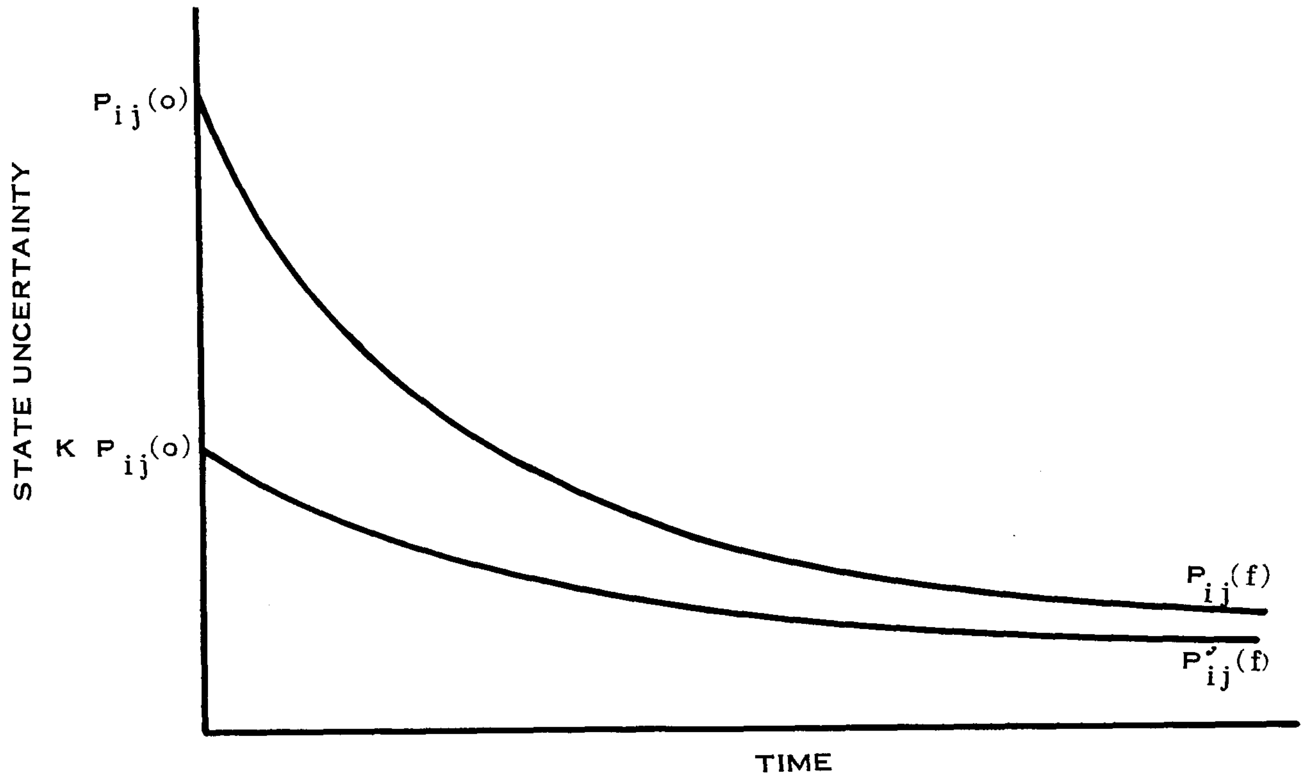

Figure 2-2 Effect of an Equivalent Observation

The standard method of studying a parameter P_{ij} is to first make a run where the initial value is $P_{ij}(0)$ and the final value is $P_{ij}(f)$. A second run is then made where the initial uncertainty is changed by a factor of k $P_{ij}(0)$ to P'_{ij} (0). The uncertainty at the end of the run is P'_{ij} (f). With the equivalent observation concept, the final value $P'_{ij}(f)$, as well as effect on other elements of the state, is obtained by the equation

$$P'(f) = P(f) - P(f)\,H^T \left[H\,P(f)H^T + P_{ij}(0)\frac{k}{1-k} \right]^{-1} HP(f) \qquad (2\text{-}38)$$

where

$$H = [0\cdots1\cdots0], \qquad (2\text{-}39)$$

and the location of the 1 in (2-39) corresponds to the parameter that is being observed.

In this manner any number of changes can be made in one or more of the initial variances by adding equivalent observations at the end of a run. Thus the results of a number of computer runs can be obtained in one run.

2.4.2.2 The Equations For a Perfect Observation. A special case of the equivalent observation concept is where a perfect observation is made of a parameter. An observation of this type has the H of (2-39), where the element in H which is 1 corresponds to the element of the state which is being observed, but there is no measurement error variance Q. The effect of a perfect observation of a measurement bias error on the covariance matrix P is given by

$$P' = P - C_m\,W^{-1}\,C_m^T \qquad (2\text{-}40)$$

where C_m is the column vector of correlations between the state and the measurement bias which is being observed. The effect on P of a perfect observation of a dynamic bias error is given by

$$P' = P - C_d\,D^{-1}C_d^T \qquad (2\text{-}41)$$

A perfect observation of a parameter may be useful for determing some of the interrelationships between the parameter uncertainties. For example, by comparing the uncertainty in a platform error source at the end of a run with the same uncertainty after a perfect observation of a measurement bias error, the effect of the uncertainty in the measurement bias error on the uncertainty in the platform error source can be determined. A more detailed discussion of these concepts are included in Reference 4.

SECTION 3

POWERED FLIGHT ERROR PROPAGATION PROGRAM

The computer program that has been used to obtain the numerical results for this study is the Powered Flight Error Propagation Program. The original version of this program was written by Philco-Ford for NASA Goddard under Contract NAS-5-9700, and is described in Reference 2.

The program simulates an actual orbit determination that would combine the tracking data from the ETR radars with the telemetry data from the vehicle, and would be used to estimate the position and velocity of the vehicle plus any additional parameters such as the inertial platform errors or the tracking bias errors. As a result, it is a useful tool for studying the ensemble behavior of the combined inertial platform-tracking system under a wide variety of conditions.

The essential output from the program is a variance-covariance matrix of errors in estimate of the position and velocity state, and the estimated error parameters. The covariance matrix is propagated along a nominal powered flight trajectory. A general flow diagram of the program functions are shown in Figure 3-1.

The program reads an input tape of the nominal trajectory position X(t) and acceleration A(t) that is generated by the Philco Ford Powered Flight Optimization Program. (Ref. 2) The gravity partials (F) are evaluated for the nominal values of X(t) and the platform error partials are evaluated for the nominal values of acceleration A(t). These partials define the differential equations for the deviations about the nominal trajectory. These equations are integrated for short time intervals in order to obtain the transition matrices ϕ and ϕ_u. The matrix ϕ_u defines the pertubation of the trajectory resulting from the guidance errors.

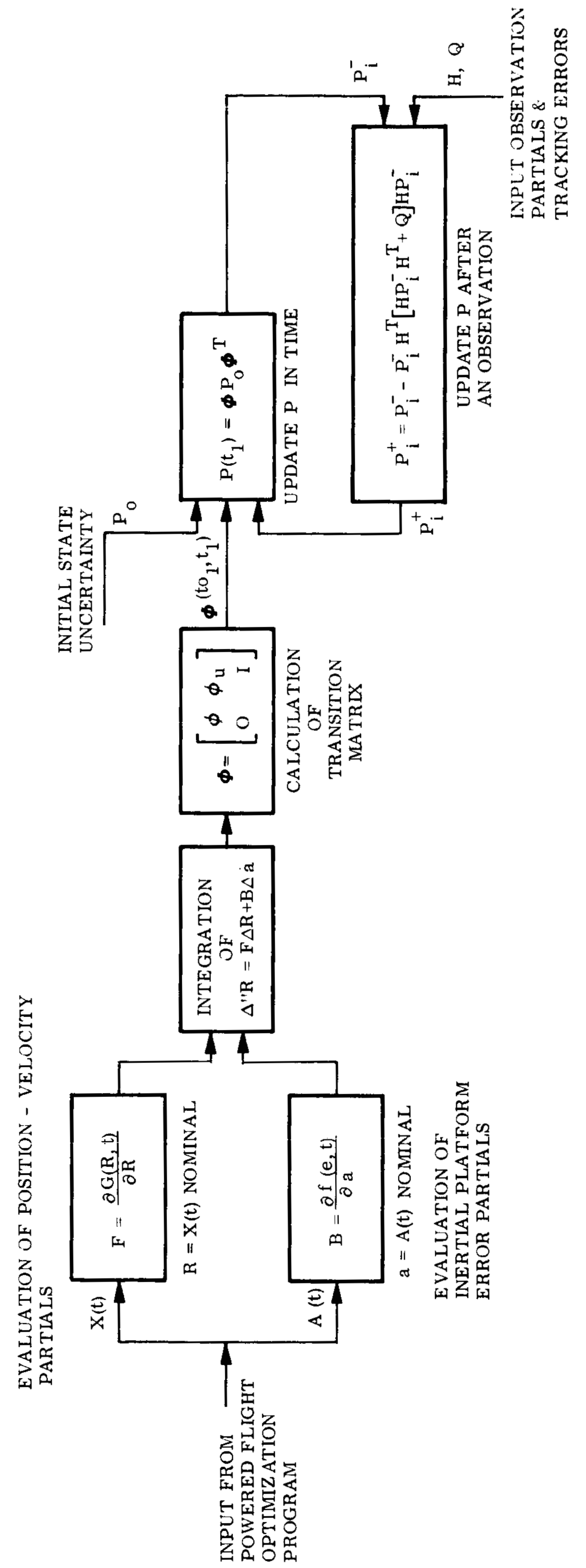

Figure 3-1 Functional Diagram of Powered Flight Error Analysis Program

The transition matrices are used to update the covariance matrix (P) in time. There is no modification made to the trajectory in the sense that the actual perturbed trajectory is calculated.

The essential part of the estimation process which simulates an orbit determination is the updating of the covariance matrix P. As shown in Figure 3-1 P is propagated between two points in time based in the previous value of P and the total transition matrix Φ. This propagation in time accounts for the fact that the guidance errors will in general increase the state uncertainty P as the time along the trajectory increases.

The data from the trackers is incorporated into the estimation by updating the state uncertainty at each observation. Observations are processed one at a time, that is the partial of the observation with respect to the state (H) is a vector and the random tracking error q is a scalar.

The program has a capability of handling a 60 x 60 covarinace matrix. It's elements include the uncertainty in the three coordinates of position and the three coordinates of velocity, which are initially zero, plus the uncertainly in any inertial guidance error, or tracking bias error. Up to 30 guidance errors may be included in the guidance model. The 30 guidance errors that may be included in the program are summarized below. The numbers associated with them are used in the program for "bookkeeping" purposes.

ERROR SOURCE	PROGRAM NUMBER
Accelerometer scale factor ~ x, y, z axes	1,5,9 respectively
Accelerometer misalignment y to x, y to z axes	2,8
Accelerometer misalignment z to x, z to y axes	3,6
Accelerometer misalignment x to y, x to z axes	4,7
Accelerometer bias x, y, z axes	10,11,12
Accelerometer thresholds x, y, z axes	13,14,15
Initial platform misalignment x, y, z axes	16,17,18
Steady state drift rate Roll,Yaw,Pitch Gyros	19,20,21

ERROR SOURCE		PROGRAM NUMBER
Input axis mass unbalance drift	Roll, Yaw, Pitch Gyros	22,23,24
Spin axis mass unbalance drift	Roll, Yaw, Pitch Gyros	25,26,27
Anisoelastic drift of	Roll, Yaw, Pitch Gyros	28,29,30

These numbers will appear in the covariance matrix output shown in Appendix A. In addition, tracking bias errors such as range, range-rate, azimuth, elevation, latitude, longitude, and altitude can be included in the covariance matrix.

The program has a number of options which have been developed for this study. There are two basic options for treating bias error sources. The standard option is where all the bias errors are estimated. In this option an initial variance is assigned to a bias error source such as a platform error or tracking bias error, and the estimation process attempts to reduce this initial uncertainty in the bias error. The bias error is part of the total covariance matrix P and the applicable equations for updating P are those discussed in Section 2.4.

The second option that can be used for bias errors is the neglect option. For this option the variance of the error source remains constant throughout the run; however, the correlations between the neglected parameters and the estimated parameters are updated according to (2-28) through (2-31). The effect on the total state uncertainty (the position-velocity state plus any estimated error sources) of neglecting the error sources is calculated at each output time according to (2-32).

Two additional options exist in the Powered Flight Error Progagation Program that enable equivalent observations to be made. The first option has been described in Section 2.4.2 and provides a way of studying bias errors parametrically in one computer run. The effect on the final covariance matrix of varying the initial value of one or more bias error sources through a wide range of values is obtained by adding equivalent observations at the end of a run.

The second equivalent observation, a direct observation of a specific parameter (h = 1) with zero error variance (q), can be used to determine the uncertainty in the position, velocity, or other error parameter, that results from another error source. The use of this option provides a means of separating the total uncertainty in one error source according the contribution from other error sources.

SECTION 4

THE ASCENT TRAJECTORY

The ascent trajectory which has been used as the nominal trajectory for this study is similar to a Saturn V trajectory. This trajectory was generated by the Philco-Ford Powered Flight Optimization Program. The output of the program is a trajectory tape which contains the vehicle position and acceleration at discrete time points. An altitude profile of the nominal trajectory is shown in Figure 4-1. The ground track for this trajectory (latitude vs. longitude) is shown in Figure 4-2. The locations of the tracking stations are also shown in this figure.

Variations of this basic trajectory have also been used. However, it was determined the results presented in the succeeding sections are not sensitive to small variations in the trajectory. In particular, one trajectory whose final altitude was about twice the value in Figure 4-1 caused no significant changes in the ability to estimate the guidance errors. Acceleration levels in the DR, CR, UP coordinate system are shown in Figure 4-3.

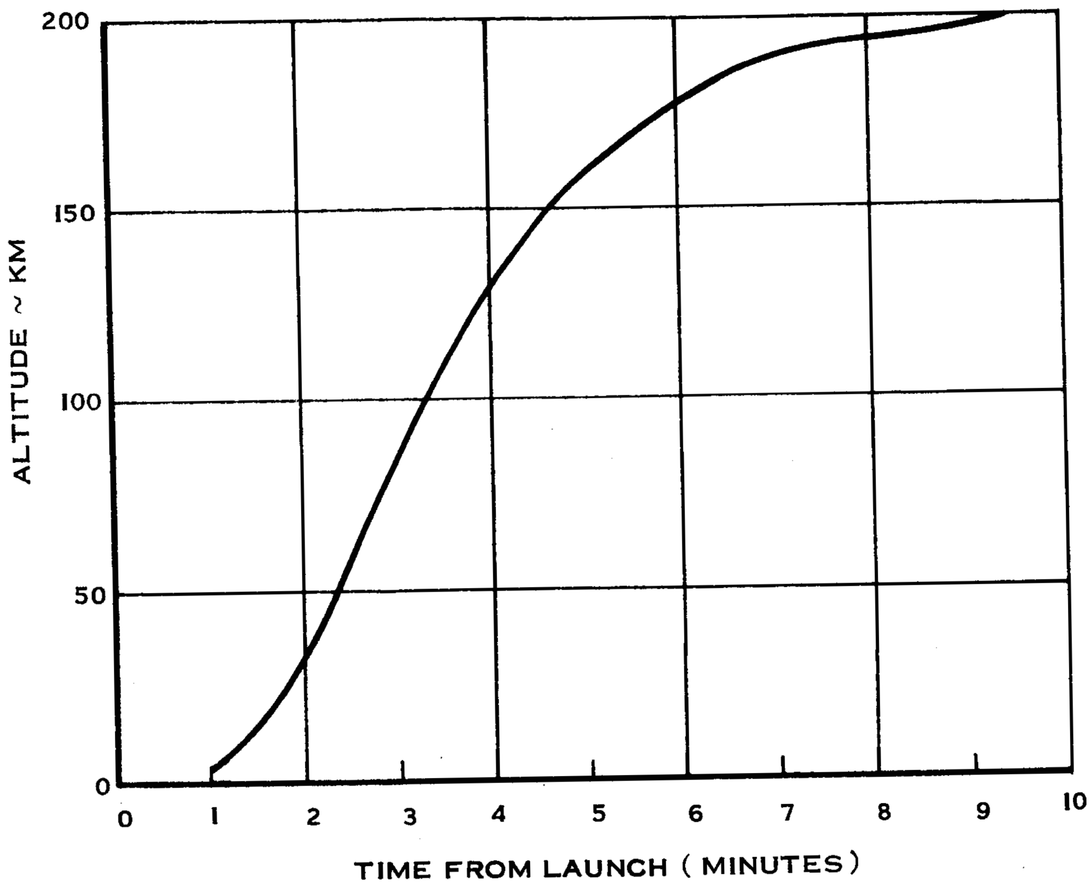

Figure 4-1 Trajectory Altitude vs Time

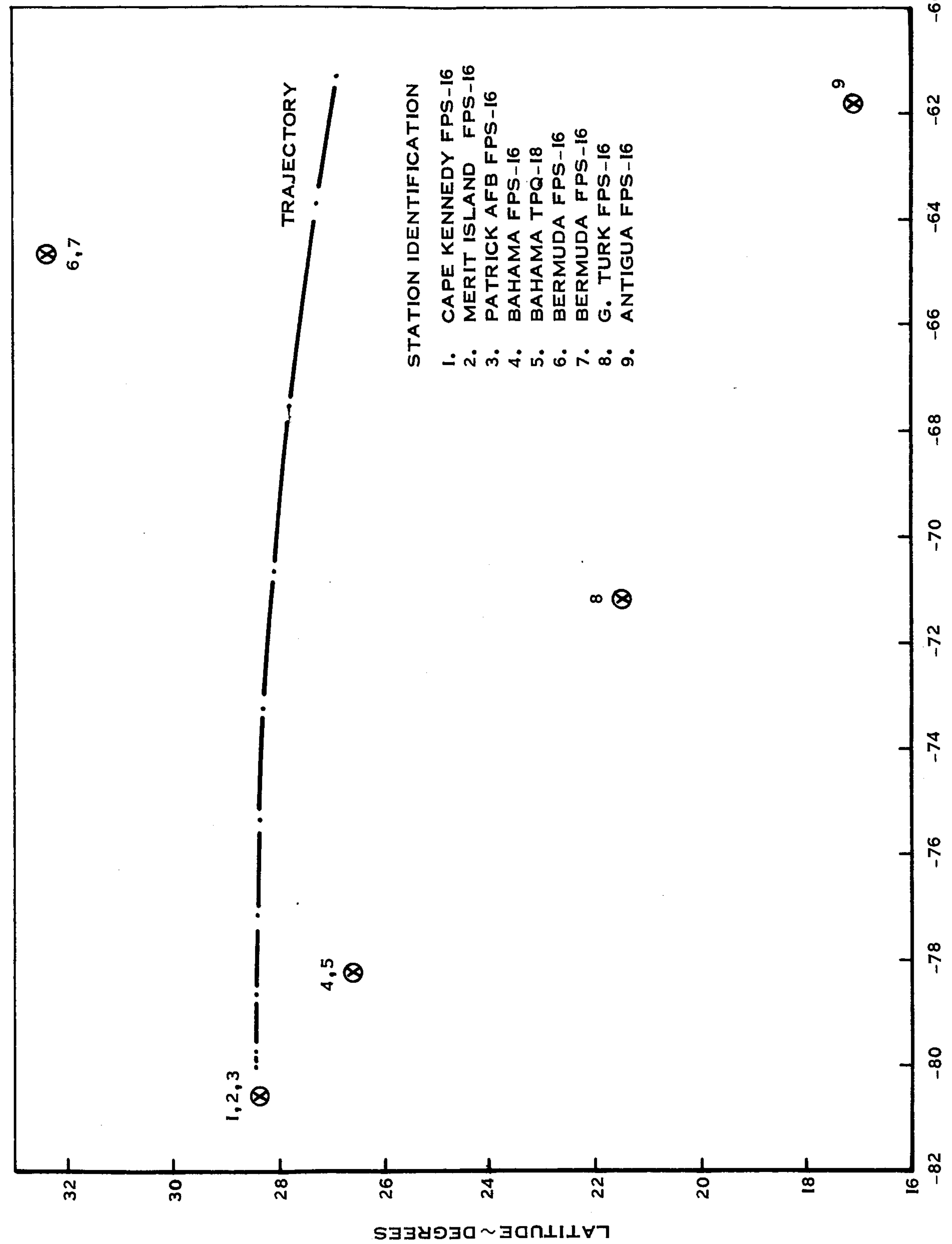

Figure 4-2 Trajectory and Tracking Station Location

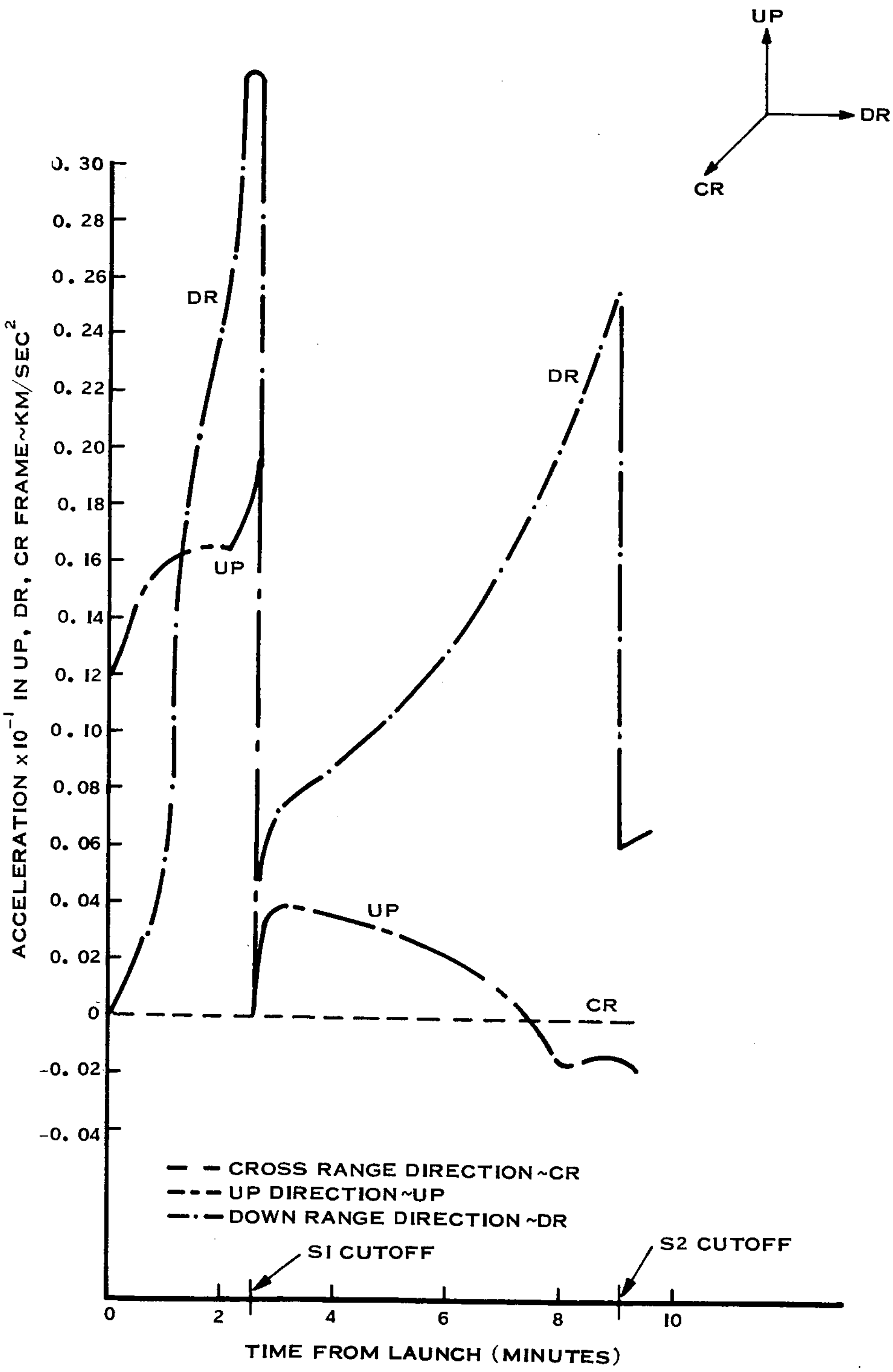

Figure 4-3 Acceleration Levels for Nominal Trajectory

SECTION 5

SELECTION OF COMBINED PLATFORM-TRACKING

Prior to determining the feasibility of estimating the inertial platform errors, an analysis was made to evaluate the effect of individual errors, both from the tracking and the inertial platform. This analysis shows (1) the effect of the inertial platform errors on the trajectory dispersions, (2) the effect of varying the numerical values of the platform errors, and (3) the effect of the tracking bias errors.

5.1 DISPERSIONS FROM THE PLATFORM ERRORS WITHOUT TRACKING

The dispersions resulting from the inertial platform errors have been determined by propagating an initial covariance matrix (P_o), consisting of the 30 platform errors, along the nominal trajectory according to (2-19). The RMSP and the RMSV of these errors are shown in Figures 5-1 and 5-2 where the RMSP and RMSV are defined as

$$\text{RMSP} = \sqrt{P_{11} + P_{22} + P_{33}}$$

$$\text{RMSV} = \sqrt{P_{44} + P_{55} + P_{66}}$$

The total uncertainty in position and velocity at the end of the trajectory is seen to be 0.5 km and 2 m/sec, respectively. A breakdown of these errors in the local tangent plane (DR, CR, UP) is shown in Figures 5-3 through 5-8. Although the three components of errors do not differ significantly, the largest error in both position and velocity is an altitude, and the smallest is in the down range direction.

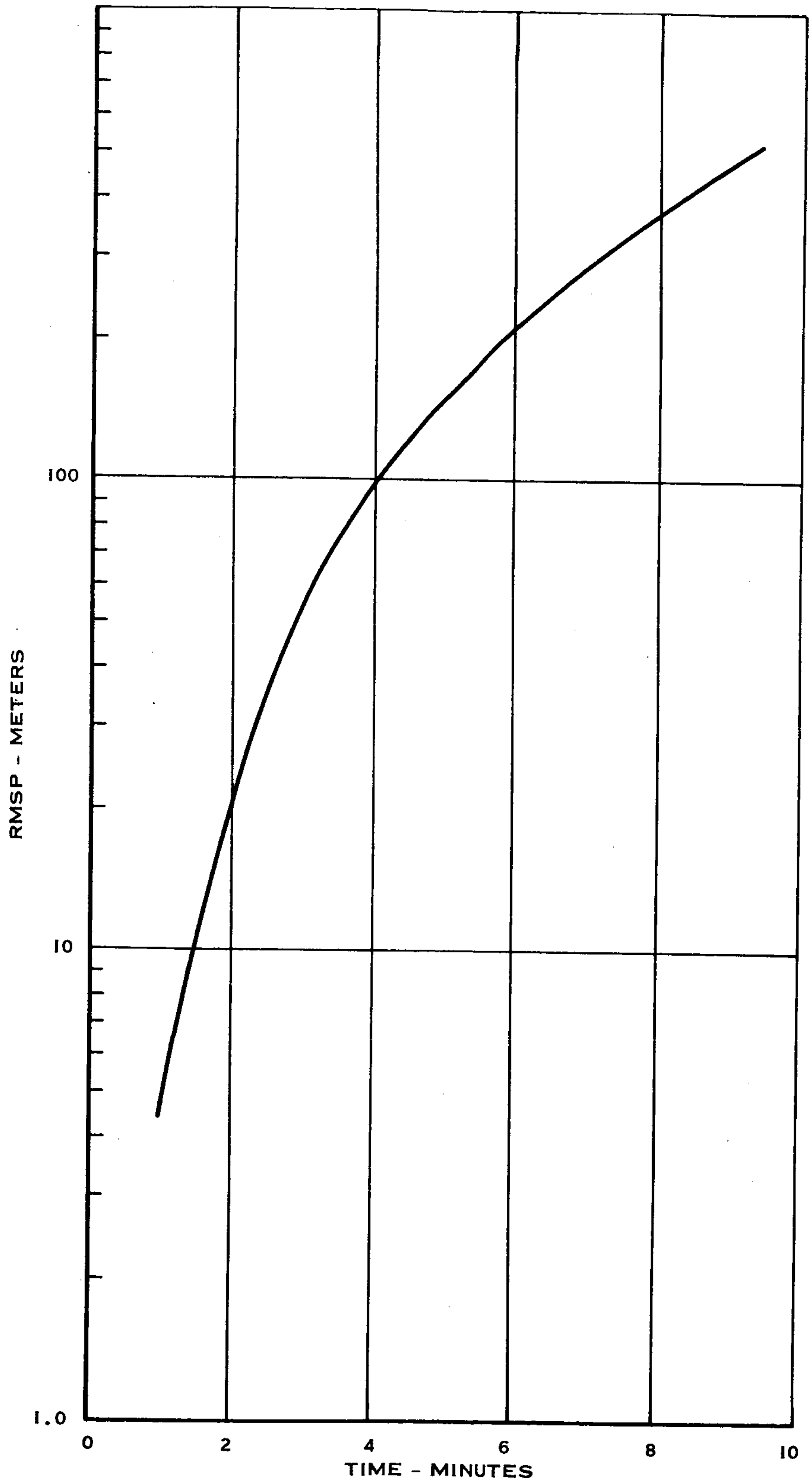

Figure 5-1 RMS Uncertainty in Position State From the 30 Platform Error Sources (No Tracking)

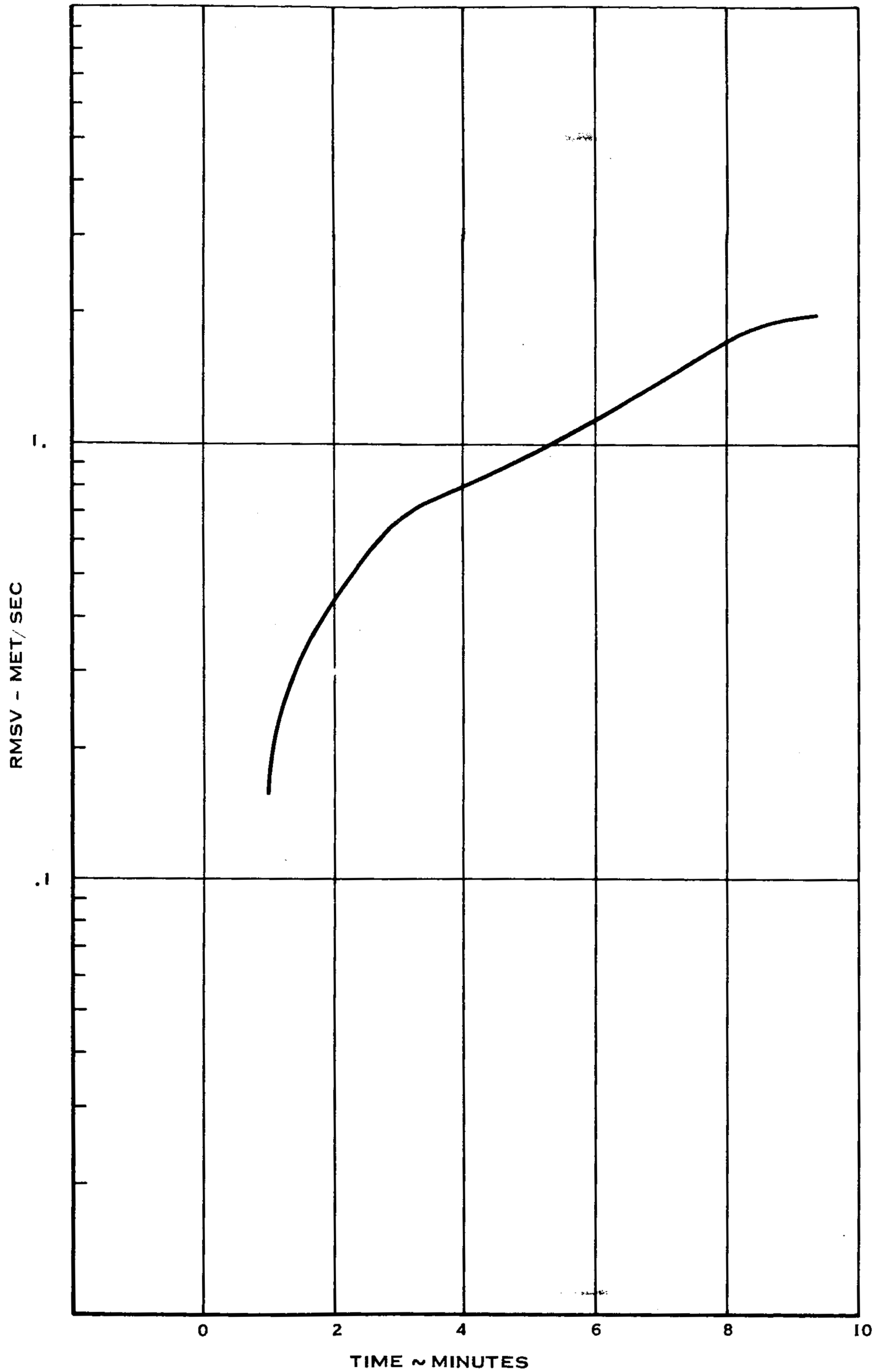

Figure 5-2 RMS Uncertainty in Velocity State from the 30 Platform Error Sources

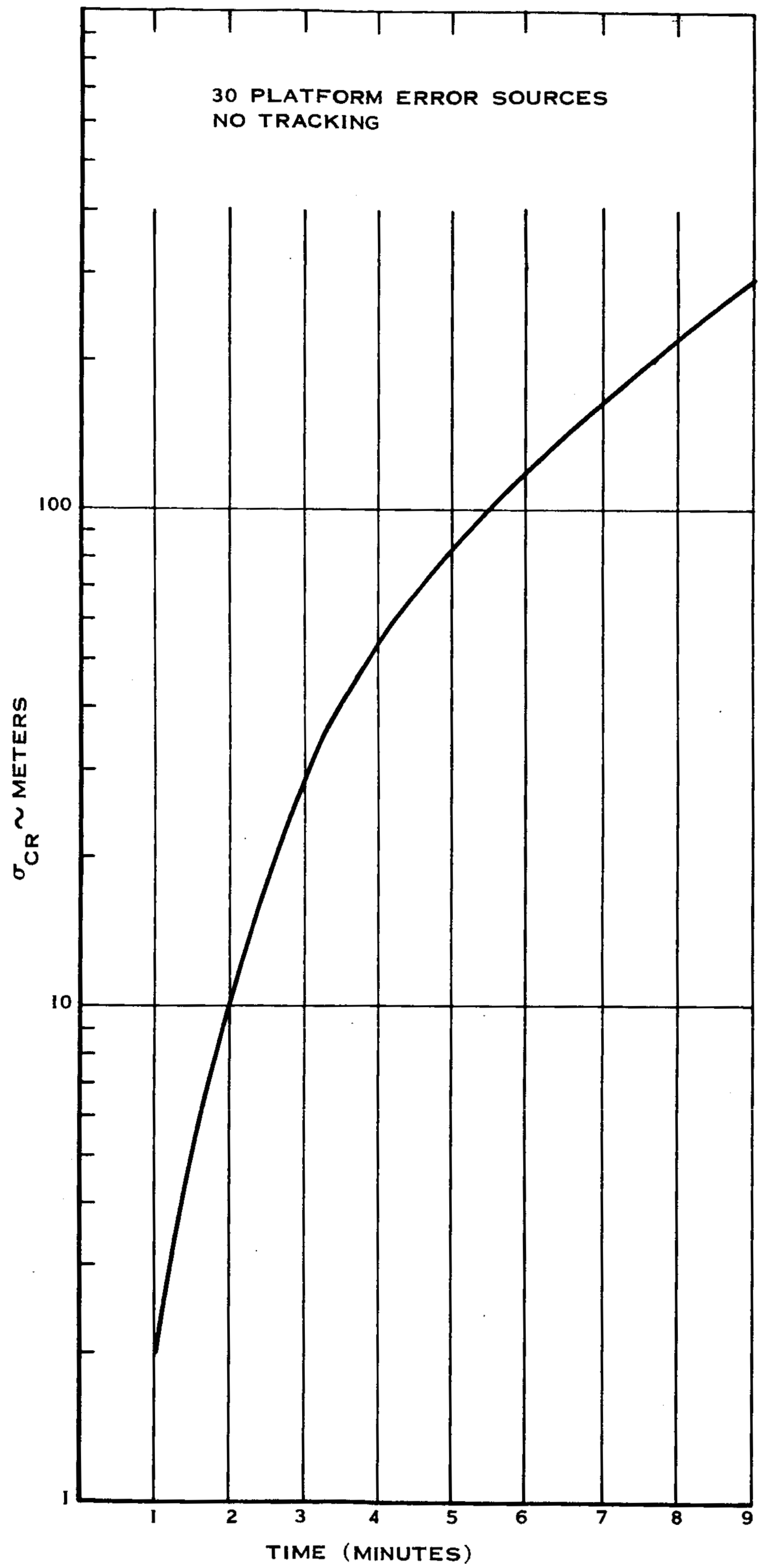

Figure 5-3 3σ Standard Deviation of Error in Estimate of the State in Cross Range Direction

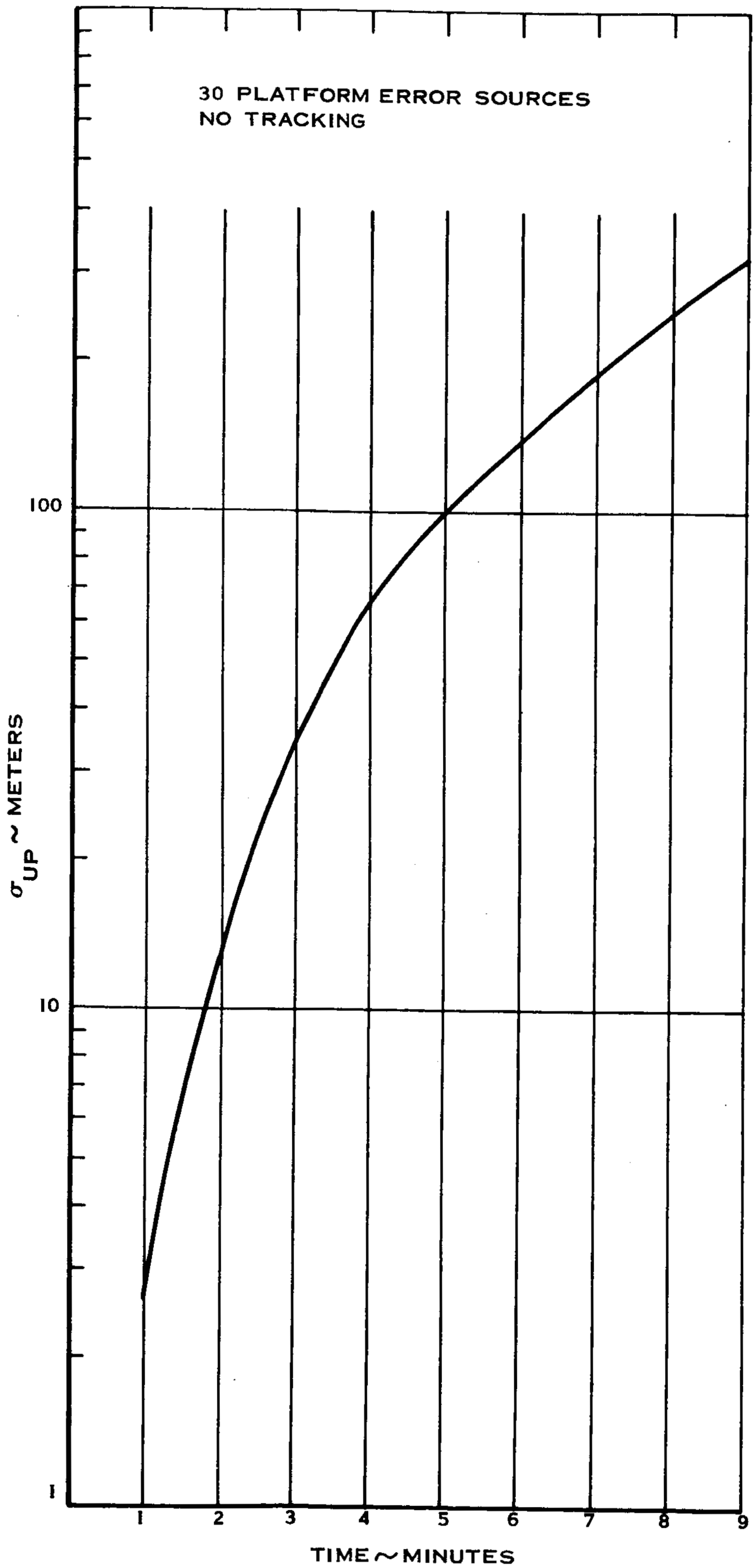

Figure 5-4 3σ Standard Deviation of Error in Estimate of the State in the Up Direction

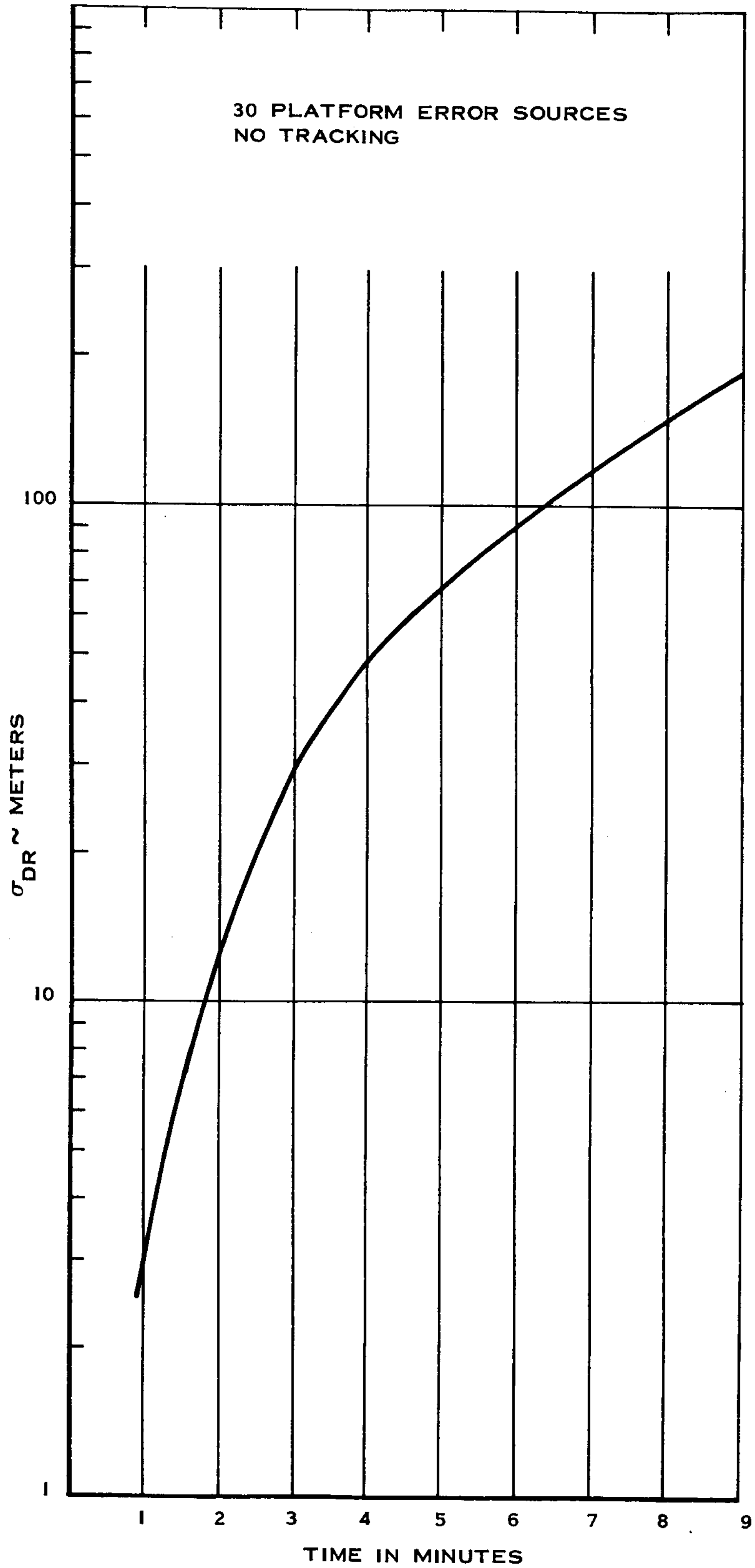

Figure 5-5 **3σ Standard Deviation of Error in Estimate of the State in the Down-Range Direction**

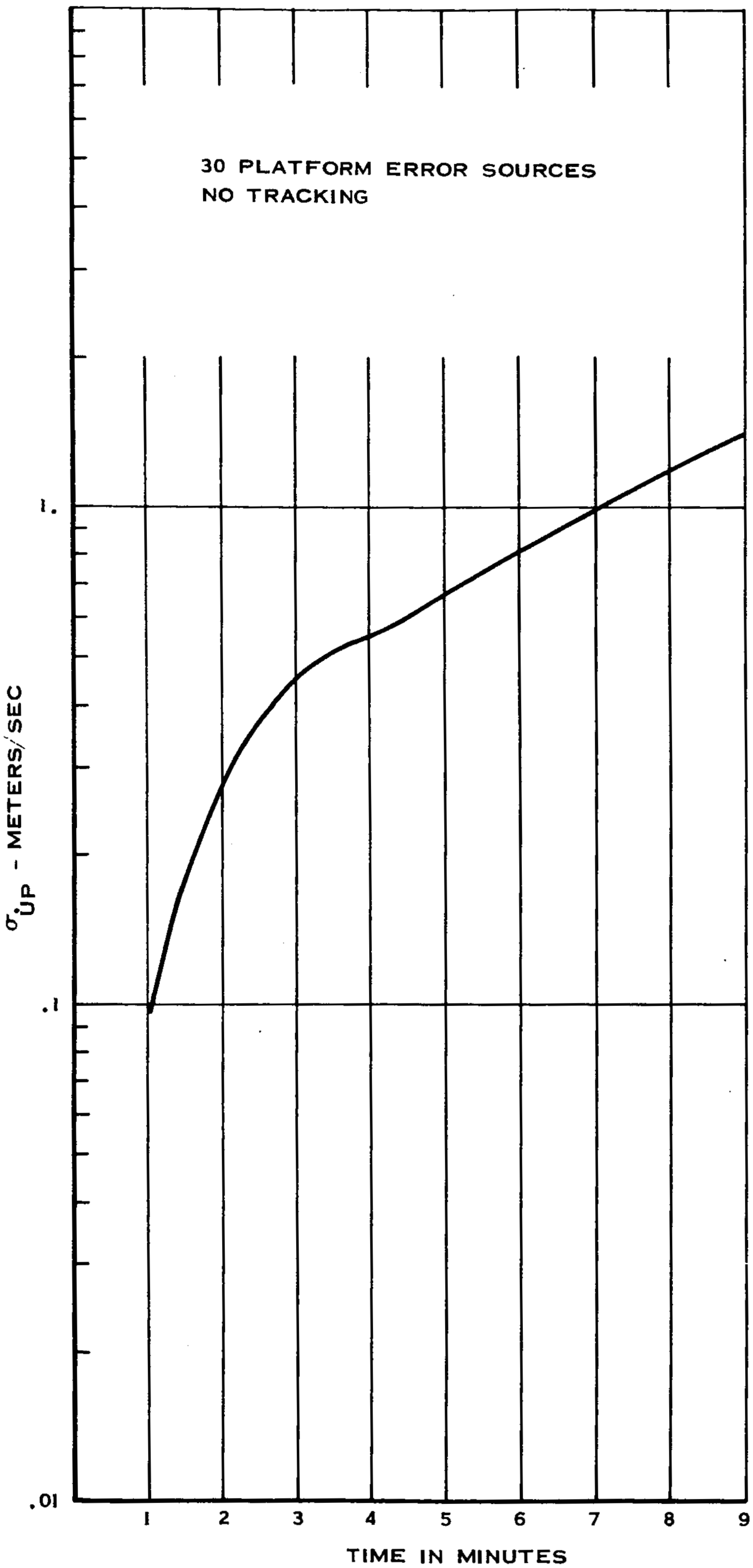

Figure 5-6 3σ Standard Deviation of Error in Estimate of the Velocity State in the Up Direction

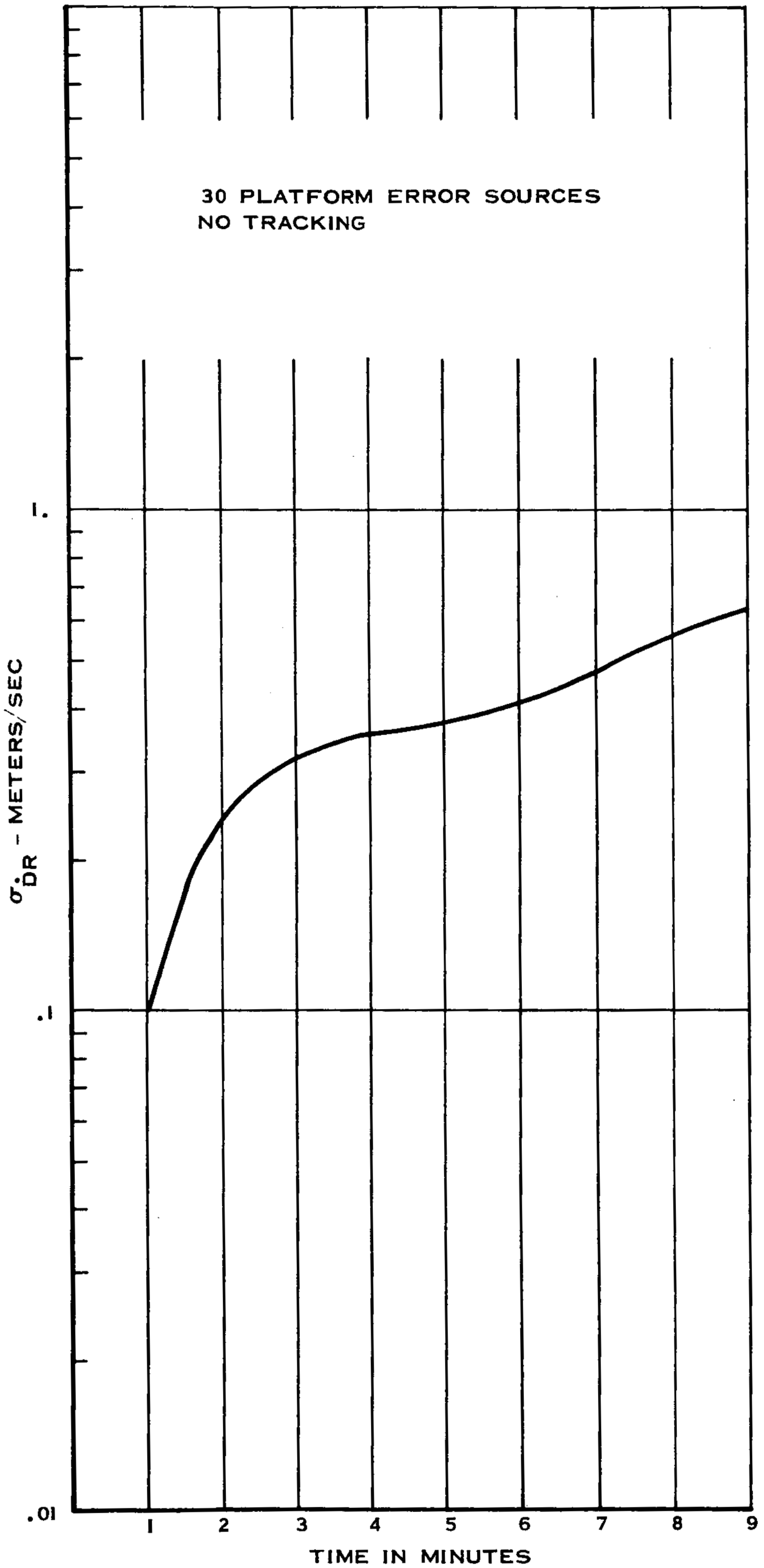

Figure 5-7 3σ Standard Deviation of Error in Estimate of the Velocity State in the Down-Range Direction

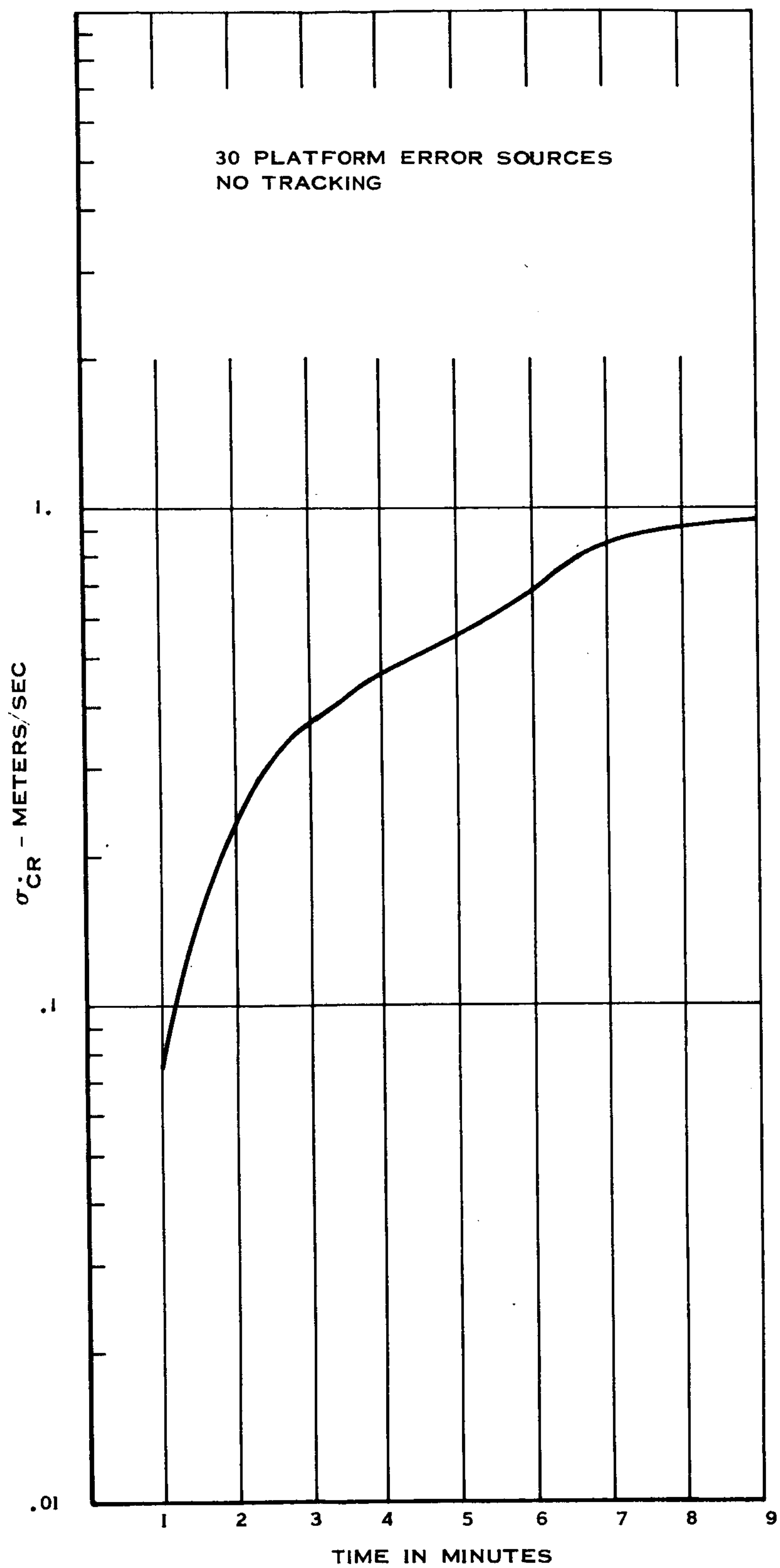

Figure 5-8 3σ Standard Deviation of Error in Estimate of the Velocity State in Cross Range Direction

In order to determine the relative effect of individual error sources on the total position and velocity dispersions, the effect of neglecting each of the 30 error sources has been evaluated. The effect of each error source on the position uncertainty is given by

$$\mathrm{RMSPP} = \sqrt{\sum_{i=1}^{3} C_i \frac{1}{d} C_i^T} \ ,$$

where each C_i is the correlation between one inertial error source and one of the three coordinates of position. Since the total position uncertainty is only from the platform error sources, the square root of the sum of the RMSPP's squared from all the error sources is equal to the 0.5 km in Figure 5-1. The relative importance of the individual error sources is shown in Table 5-1. The most significant error sources are seen to be the initial platform misalignments. The first 20 error sources listed in Table 5-1 were selected for the final error model of the inertial platform. The last 10 error sources in this table clearly have a negligible effect on the trajectory, and therefore were omitted from the final model.

If the same platform error sources are propagated by considering each source as an element of the state vector, i.e., by (2.19), then the effect of parametric changes in the platform error sources can be obtained. This is done by making an equivalent observation of the particular error source of interest, for each parametric change. The effect of changing the initial variance for each source, through a range of 10^{-2} to 10^{2} around the nominal value, has been obtained. Changes in the final RMSP for changes in the initial variances of the five most significant error sources, are shown in Figures 5-9 through 5-13. As might be expected, it takes a significant change in any one source to improve the RMSP at injection, however, a slight increase in one of the sensor variances will cause a significant change in the RMSP.

TABLE 5-1 ORDER OF IMPORTANCE OF THE INERTIAL PLATFORM ERRORS

PROGRAM NO.	RANK	SOURCE		RMSPP IN METERS
17	1.	ϕ_{oy}	Initial platform misalignment about Y-axis	316
18	2.	ϕ_{oz}	Initial platform misalignment about Z-axis	213
2	3.	β_{yx}	Y Accelerometer misalignment into X-axis	146
3	4.	β_{zx}	Z Accelerometer misalignment into X-axis	134
16	5.	ϕ_{ox}	Initial platform misalignment about X-axis	129
21	6.	$\dot{\phi}_z$	Steady-state drift of Z gyro	112
4	7.	β_{xy}	X Accelerometer misalignment into Y-axis	110
6	8.	β_{zy}	Z Accelerometer misalignment into Y-axis	110
20	9.	$\dot{\phi}_y$	Steady-state drift of Y gyro	100
11	10.	α_y	Y Accelerometer bias	85
10	11.	α_x	X Accelerometer bias	77
12	12.	α_z	Z Accelerometer bias	77
1	13.	ϵ_x	X Accelerometer scale factor	65
5	14.	ϵ_y	Y Accelerometer scale factor	62
19	15.	$\dot{\phi}_x$	Steady-state drift of X gyro	38
27	16.	μ_{sz}	Spin axis mass unbalance of Z gyro	33
30	17.	C_z	Anisoelastic drift of Z gyro	32
23	18.	μ_{Iy}	Input axis mass unbalance of Y gyro	7.9

TABLE 5-1 ORDER OF IMPORTANCE OF THE INERTIAL PLATFORM ERRORS (Cont)

PROGRAM NO.	RANK	SOURCE		RMSPP IN METERS
22	19.	μ_{Ix}	Input axis mass unbalance of X gyro	6.1
15	20.	S_z	Z Accelerometer threshold	1.7
13	21.	S_x	X Accelerometer threshold	.009
8	22.	β_{yz}	Y Accelerometer misalignment into Z axis	.006
7	23.	β_{xz}	X Accelerometer misalignment into Z axis	.005
9	24.	ϵ_z	Z Accelerometer scale factor	.003
25	25.	μ_{sx}	Spin axis mass unbalance of X gyro	.002
26	26.	μ_{sy}	Spin axis mass unbalance of Y gyro	.001
24	27.	μ_{Iz}	Input axis mass unbalance of Z gyro	.0008
29	28.	C_y	Anisoelastic drift of Y gyro	$.2 \times 10^{-9}$
28	29.	C_x	Anisoelastic drift of X gyro	$.2 \times 10^{-9}$
14	30.	S_y	Y Accelerometer threshold	0

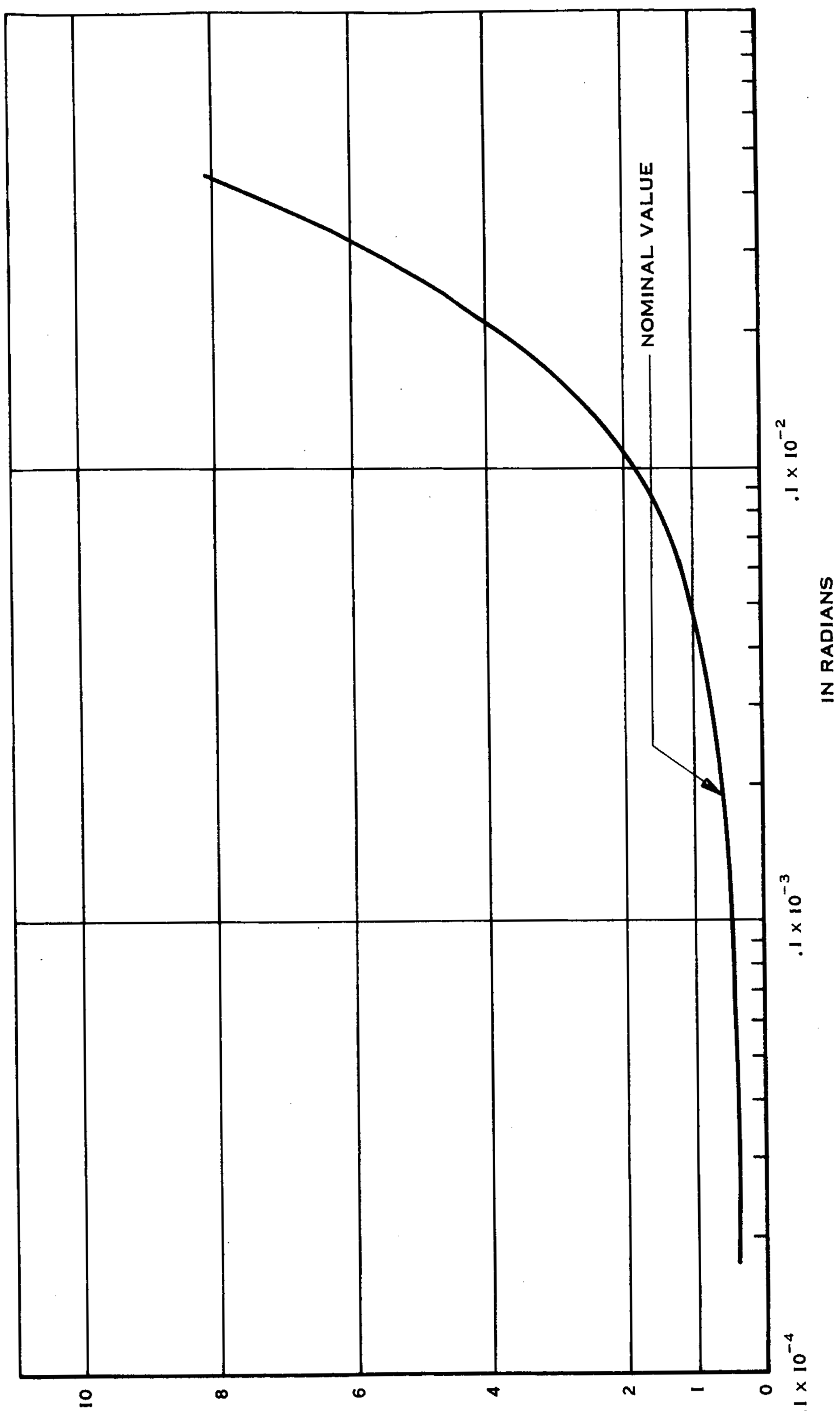

Figure 5-9 Variations in Final RMSP for Changes in Initial Standard Deviation of Initial Misalignment About Y Axis

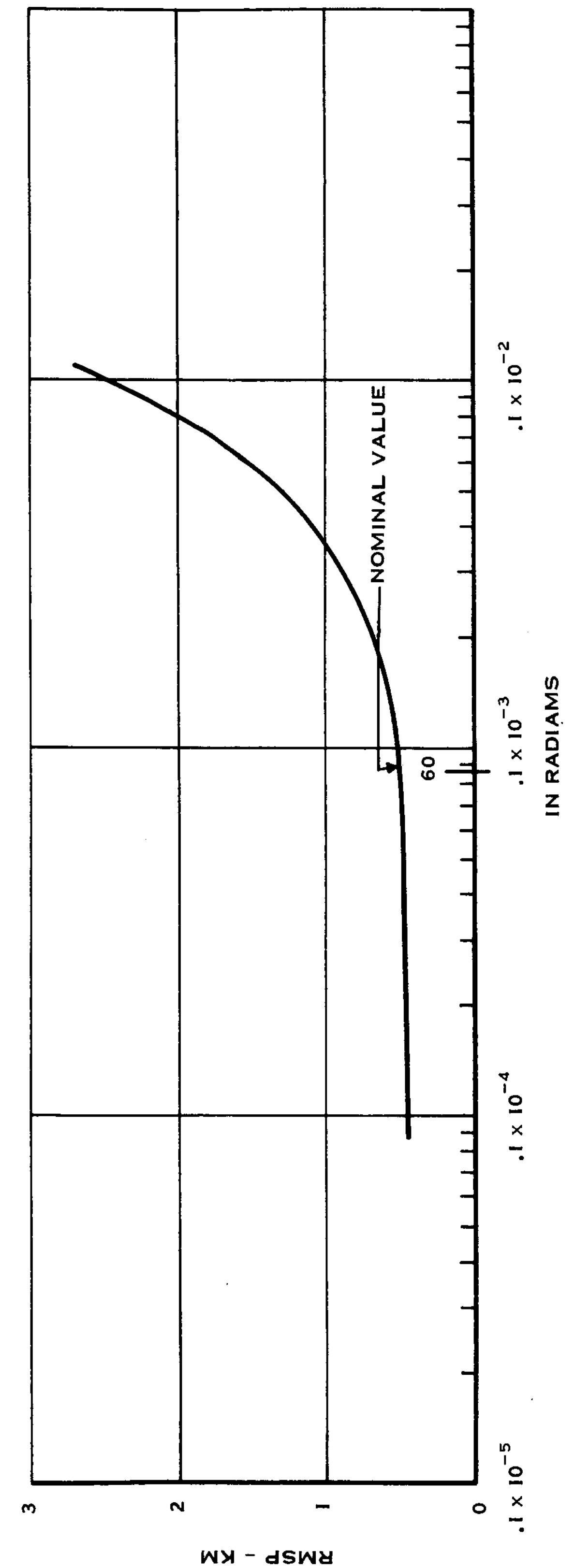

Figure 5-10 Variations in Final RMSP for Changes in Initial Standard Deviation of Initial Platform Misalignment About Z Axis

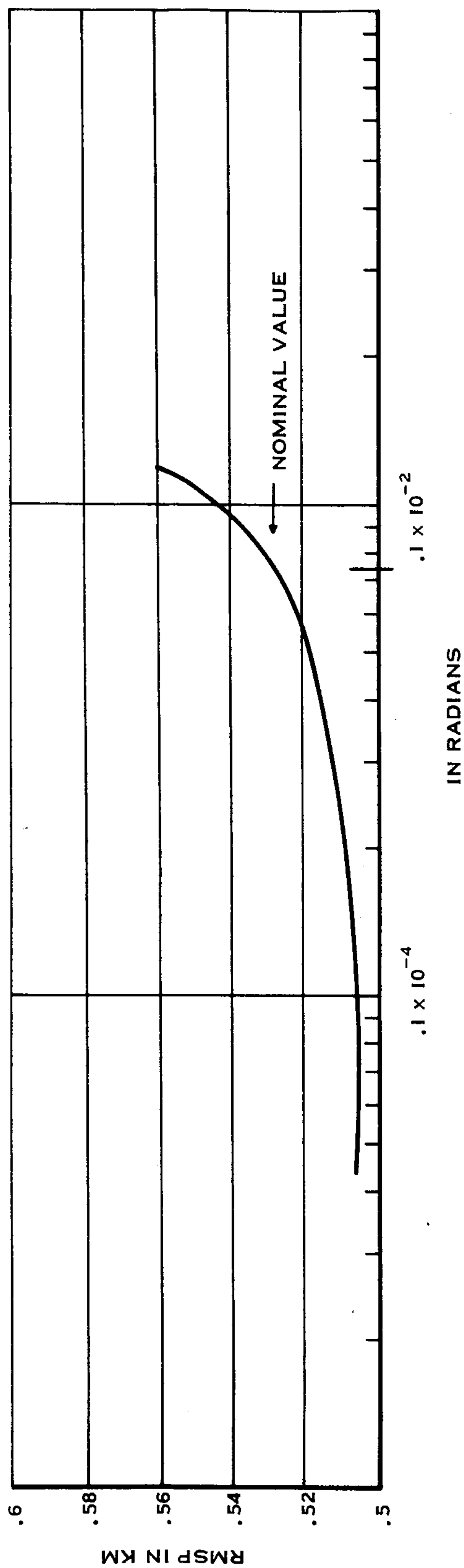

Figure 5-11 Variations in Final RMSP for Changes in Initial Standard Deviation of Y Accelerometer Into X Axis

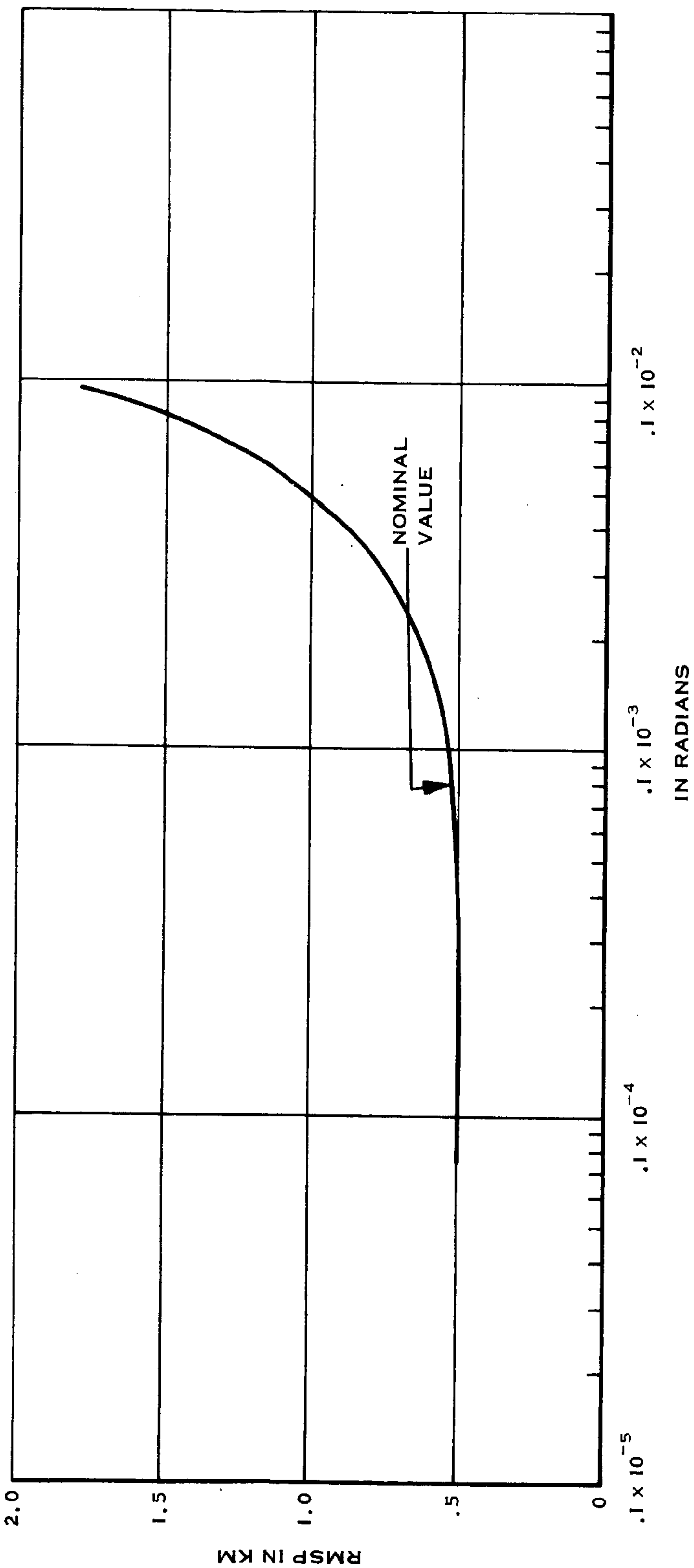

Figure 5-12 Variations in Final RMSP for Changes in Initial Standard Deviation of Z Accelerometer Into X Axis

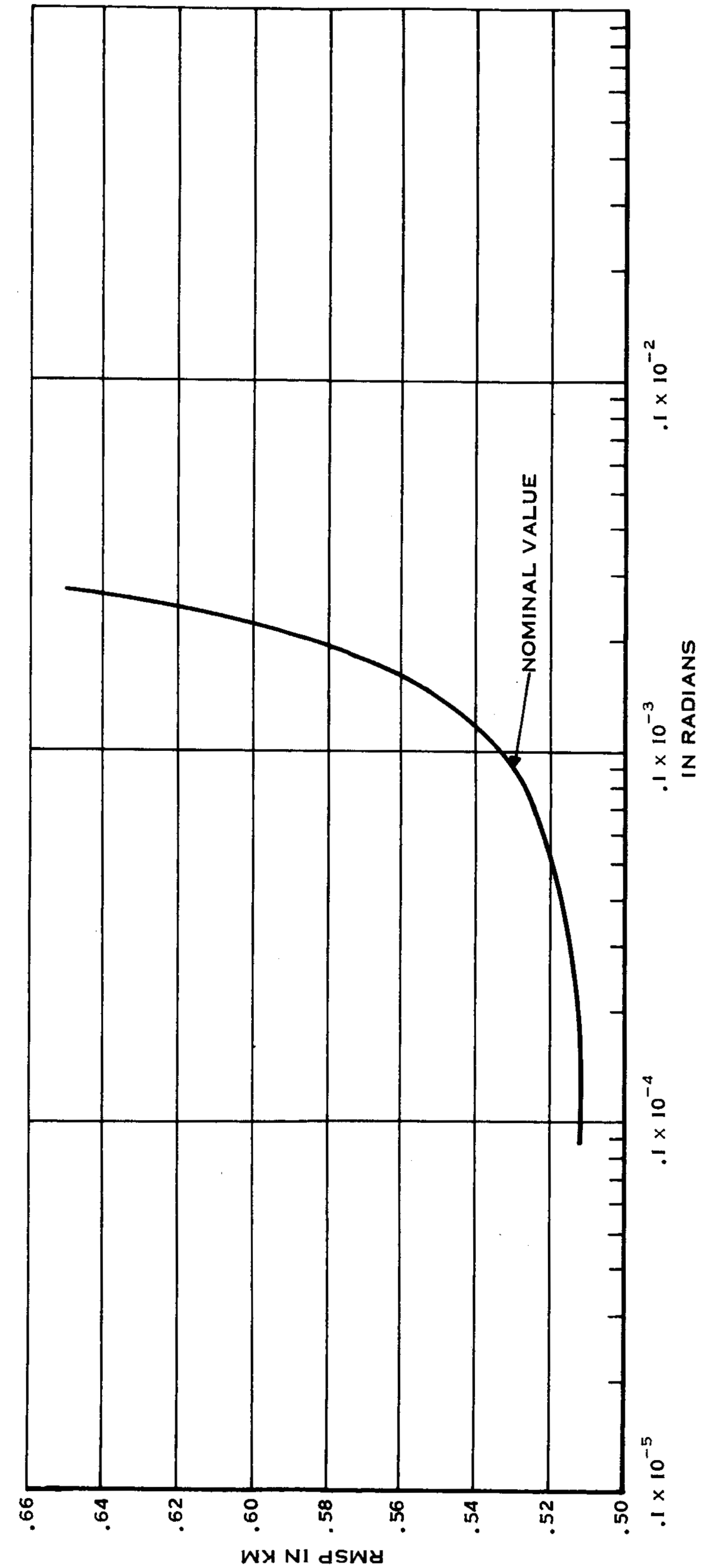

Figure 5-13 Variations in Final RMSP for Changes in Initial Standard Deviation of Initial Platform Misalignment About X Axis

5.2 THE TRACKING MODEL

The tracking model that was used for this study consisted of significant error sources from the C-band radars. Up to nine trackers were considered. Ascension Island could not observe the vehicle until near the very end of the trajectory, and therefore the two radars at this site were not considered. The error sources at each radar consisted of random and bias errors in range, azimuth and elevation. Station location errors (latitude, longitude, and altitude) were also considered.

The numerical values of the tracking errors as well as the location of the C-band radars are listed in Table 5-2. These values were obtained from Reference 5, with the exception of the station location errors and the accuracy of the NASA tracker at Bermuda.

Since the total number of bias errors from the nine trackers could be as many as 54, a study was first made to determine the importance of these errors in order to eliminate any errors that do not significantly affect the system. This was accomplished by making two runs; first, with the measurement biases neglected, and second, with the station location errors neglected from the model. In both cases the combined inertial platform-tracking system was simulated with 20 guidance errors included in the model. The effect of neglecting the station location and measurement bias is shown in Table 5-3.

The values shown represent the effect of neglecting the individual tracking errors on both the position uncertainty and the velocity uncertainty. Since the total size of the covariance matrix (P) could be up to 60 x 60, and this number must include the 20 guidance errors and the six coordinates of position and velocity, up to 34 tracking biases could be included in the model. The tracking bias errors that were omitted from the final model are indicated in Table 5-3.

TABLE 5-2 C-BAND RADAR ACCURACIES

STATION NUMBER	STATION NAME	LAT (Deg)	LON (Deg)	ALT (ft)	RANDOM ERRORS			STATION LOCATION ERRORS*			BIAS ERRORS		
					$3\sigma_R$ (ft)	$3\sigma_A$ (mr)	$3\sigma_E$ (mr)	$3\sigma_{LAT}$ (ft)	$3\sigma_{LON}$ (ft)	$3\sigma_{ALT}$ (ft)	$3\sigma_R$ (ft)	$3\sigma_A$ (mr)	$3\sigma_A$ (mr)
	TPQ-18 RADARS												
19.18	MERRITT IS.	28.417	80.667	36.9	60	0.45	0.45	4.46	4.46	0	150	0.3	0.3
3.18	GRAND BAHAMA	26.633	78.267	39.0	60	0.45	0.45	22.3	22.3	13.4	150	0.3	0.3
7.18	GRAND TURK	21.467	71.133	118.9	60	0.45	0.45	325	325	107	150	0.3	0.3
	FPQ-6 RADARS												
91.18	ANTIQUA	17.15	61.8	138.8	60	0.45	0.45	475	475	147	150	0.3	0.3
0.18	PATRICK	28.233	80.6	48.9	60	0.45	0.45	18	18	0	150	0.3	0.3
NASA**	BERMUDA	32.348	64.654	0.0984	60	0.45	0.45	1050	1050	151	150	0.3	0.3
	FPS-16 RADARS												
1.16	CAPE KENNEDY	28.483	80.583	44.8	60	0.6	0.6	8.92	8.92	0	150	0.3	0.3
3.16	GRAND BAHAMA	26.617	78.35	45.5	60	0.6	0.6	22.3	22.3	13.4	150	0.3	0.3
NASA	BERMUDA	32.348	64.654	0.0984	60	0.6	0.6	1050	1050	151	150	0.3	0.3

*Station Location Errors, except for Bermuda, obtained from: "The accuracy of AMR instrumentation," H. P. Mann, AD #432034, Systems Analysis RCA Service Co., Patrick Air Force Base, Florida, 13 December 1963.

**NASA Tracking Station Accuracies at Bermuda obtained from: APOLLO Navigation Working Group TR No. AN-1.1- NASA Goddard, Greenbelt, Md., April 4, 1966.

TABLE 5-3

STATION	ERROR	POSITION UNCERTAINTY (RMSPP) (km)	VELOCITY UNCERTAINTY (RMSVP) (km/sec)
BERMUDA 1	La	.164	$.527 \times 10^{-3}$
BERMUDA 2	La	.163	$.524 \times 10^{-3}$
ANTIGUA	La	.134	$.889 \times 10^{-3}$
BERMUDA 1	Lo	$.548 \times 10^{-1}$	$.261 \times 10^{-2}$
BERMUDA 2	Lo	$.546 \times 10^{-1}$	$.261 \times 10^{-2}$
ANTIGUA	R	$.518 \times 10^{-1}$	$.306 \times 10^{-3}$
ANTIGUA	Lo	$.461 \times 10^{-1}$	$.330 \times 10^{-3}$
G. TURK	Lo	$.451 \times 10^{-1}$	$.569 \times 10^{-3}$
G. TURK	La	$.288 \times 10^{-1}$	$.453 \times 10^{-3}$
BERMUDA 1	R	$.212 \times 10^{-1}$	$.146 \times 10^{-3}$
BERMUDA 2	R	$.212 \times 10^{-1}$	$.146 \times 10^{-3}$
BAHAMA 1	R	$.171 \times 10^{-1}$	$.282 \times 10^{-3}$
BAHAMA 2	R	$.171 \times 10^{-1}$	$.282 \times 10^{-3}$
G. TURK	R	$.163 \times 10^{-1}$	$.189 \times 10^{-3}$
BERMUDA 2	$A\ell$	$.155 \times 10^{-1}$	$.795 \times 10^{-4}$
BERMUDA 1	$A\ell$	$.154 \times 10^{-2}$	$.793 \times 10^{-4}$
CAPE 5	R	$.993 \times 10^{-2}$	$.724 \times 10^{-4}$
MERIT IS.	R	$.942 \times 10^{-2}$	$.770 \times 10^{-4}$
PATRICK	R	$.920 \times 10^{-2}$	$.108 \times 10^{-4}$
ANTIGUA	$A\ell$	$.842 \times 10^{-2}$	$.728 \times 10^{-4}$
G. TURK	$A\ell$	$.527 \times 10^{-2}$	$.377 \times 10^{-4}$
BAHAMA 1	Lo	$.110 \times 10^{-2}$	$.259 \times 10^{-4}$
BAHAMA 2	Lo	$.109 \times 10^{-2}$	$.254 \times 10^{-4}$
BAHAMA 1	La	$.982 \times 10^{-3}$	$.229 \times 10^{-4}$
PATRICK	Lo	$.973 \times 10^{-3}$	$.108 \times 10^{-4}$
BAHAMA 2	La	$.961 \times 10^{-3}$	$.227 \times 10^{-4}$
PATRICK	E	$.925 \times 10^{-3}$	$.135 \times 10^{-4}$
MERIT IS	E	$.859 \times 10^{-3}$	$.124 \times 10^{-4}$
BERMUDA 2	E	$.828 \times 10^{-3}$	$.256 \times 10^{-5}$

TABLE 5-3 (Cont)

STATION	ERROR	POSITION UNCERTAINTY (RMSPP)(km)	VELOCITY UNCERTAINTY (RMSVP)(km/sec)
MERIT IS	A	$.764 \times 10^{-3}$	$.203 \times 10^{-4}$
PATRICK	A	$.633 \times 10^{-3}$	$.195 \times 10^{-4}$
ANTIGUA	E	$.531 \times 10^{-3}$	$.272 \times 10^{-5}$
CAPE 5	E	$.516 \times 10^{-3}$	$.720 \times 10^{-4}$
G. TURK	E	$.503 \times 10^{-3}$	$.161 \times 10^{-5}$
BERMUDA 2	A	$.498 \times 10^{-3}$	$.555 \times 10^{-5}$*
CAPE 5	A	$.482 \times 10^{-3}$	$.125 \times 10^{-4}$*
CAPE 5	Lo	$.481 \times 10^{-3}$	$.373 \times 10^{-5}$*
BAHAMA 2	Aℓ	$.477 \times 10^{-3}$	$.103 \times 10^{-4}$ *
BAHAMA 2	E	$.476 \times 10^{-3}$	$.108 \times 10^{-4}$ *
BERMUDA 1	E	$.466 \times 10^{-3}$	$.144 \times 10^{-5}$ *
BAHAMA 1	Aℓ	$.462 \times 10^{-3}$	$.101 \times 10^{-4}$ *
BAHAMA 2	A	$.445 \times 10^{-3}$	$.117 \times 10^{-4}$ *
BERMUDA 1	A	$.280 \times 10^{-3}$	$.312 \times 10^{-5}$ *
BAHAMA 1	E	$.268 \times 10^{-3}$	$.611 \times 10^{-5}$ *
G. TURK	A	$.265 \times 10^{-3}$	$.154 \times 10^{-5}$ *
BAHAMA 1	A	$.250 \times 10^{-3}$	$.656 \times 10^{-5}$ *
ANTIGUA	A	$.250 \times 10^{-3}$	$.182 \times 10^{-5}$ *
MERIT IS	Lo	$.226 \times 10^{-3}$	$.191 \times 10^{-5}$ *
PATRICK	La	$.218 \times 10^{-3}$	$.629 \times 10^{-5}$ *
CAPE 5	La	$.320 \times 10^{-4}$	$.131 \times 10^{-5}$ *
MERIT IS	La	$.274 \times 10^{-4}$	$.946 \times 10^{-6}$ *

* Denotes those error sources eliminated

The tracking model therefore consists of thirty-four tracking station error sources.

Although the selection of the tracking model has been made on the basis of the position and velocity uncertainties, the effect of neglecting tracking biases on the guidance errors has also been evaluated. Table 5-4 shows these results. The initial values of the guidance error uncertainties and the final values for the nominal model are shown. Also the final values of the guidance error standard deviations are shown for the cases where (1) all the measurement biases are neglected and (2) all the station location errors are neglected. Clearly both of these classes of error sources are too important to be neglected from the model, if the guidance errors are to be estimated.

The tracking radars that have been used for this study are in general the same ones that have been used for preliminary Saturn test flights (Reference 6).

TABLE 5-4

EFFECT OF NEGLECTING TRACKING BIASES ON THE ESTIMATION OF PLATFORM ERRORS

ERROR* SOURCE	INITIAL 3σ VALUE	FINAL 3σ VALUES		
		For Nominal Model	With Measurement Biases Neglected	With Station Location Biases Neglected
17	0.174×10^{-3}	0.632×10^{-4}	0.566×10^{-3}	0.309×10^{-3}
18	0.873×10^{-4}	0.394×10^{-4}	0.218×10^{-3}	0.474×10^{-3}
16	0.873×10^{-4}	0.312×10^{-4}	0.191×10^{-3}	0.474×10^{-3}
21	0.242×10^{-6}	0.127×10^{-6}	0.107×10^{-5}	0.351×10^{-5}
4	0.739×10^{-4}	0.352×10^{-4}	0.176×10^{-3}	0.208×10^{-3}
20	0.242×10^{-6}	0.156×10^{-6}	0.139×10^{-5}	0.122×10^{-5}
11	0.5×10^{-6}	0.396×10^{-6}	0.167×10^{-5}	0.240×10^{-5}
10	0.5×10^{-6}	0.333×10^{-6}	0.271×10^{-5}	0.660×10^{-5}
12	0.5×10^{-6}	0.448×10^{-6}	0.310×10^{-5}	0.653×10^{-5}
1	0.36×10^{-4}	0.203×10^{-4}	0.141×10^{-3}	0.310×10^{-3}
5	0.36×10^{-4}	0.197×10^{-4}	0.207×10^{-3}	0.355×10^{-3}
19	0.242×10^{-6}	0.229×10^{-6}	0.560×10^{-6}	0.126×10^{-5}
27	0.371×10^{-4}	0.370×10^{-4}	0.570×10^{-4}	0.429×10^{-4}
30	0.201×10^{-2}	0.201×10^{-4}	0.386×10^{-2}	0.231×10^{-2}
23	0.148×10^{-4}	0.148×10^{-4}	0.151×10^{-4}	0.154×10^{-4}
22	0.148×10^{-4}	0.148×10^{-4}		
15	0.2×10^{-7}	0.200×10^{-7}		

*These numbers refer to program number assigned to each error source in Table 5.1

SECTION 6

THE ESTIMATION OF INERTIAL PLATFORM ERRORS

The principal results of the study are presented in this section. These results show the feasibility of estimating the inertial platform errors during a powered flight ascent by combining the telemetry and tracking data.

The general method that has been used is to examine the behavior of a covariance matrix of error dispersions along the nominal trajectory. In particular, the amount by which the standard deviations of the guidance errors reduced, was used as a criterion. Although no absolute figure of merit has been defined for the amount by which these standard deviations decrease, it has been assumed that a decrease in the 3σ value of an error source by an order of magnitude, would be significant; conversely, if the standard deviation of a particular error was reduced by a small percentage, it has been assumed that this error source could not be estimated very well in an actual fitting process using real telemetry and tracking data.

In addition to the results for the nominal platform-tracking model, the results of a number of parametric variations of the error sources are presented. The objective for studying the error sources parametrically was twofold. First, it was desired to determine whether the ability to estimate each of the error sources depended on the relative accuracies of the guidance error compared to the tracking accuracies, or whether there are certain error sources that cannot be estimated regardless of the relative accuracies. The second reason for presenting parametric data was simply to show how the results changed for changes in significant parameters of the model.

6.1 ESTIMATION OF PLATFORM ERRORS WITH NOMINAL TRACKING

A simulation of the combined platform-tracking system has been made with an error model that includes (1) the first 20 guidance errors in Table 5-1. (2) the first 34 tracking has errors in Table5-3, and (3) the uncertainties in the six components of position and velocity. Although the primary objective was to evaluate the behavior of the standard deviations of the guidance errors, the uncertainty in the vehicle position (RMSP) and velocity (RMSV) have also been included in the results.

The uncertainties in the vehicle position and velocity are about 30 meters and 0.2 meters/sec as shown in Figures 6-1 and 6-2.* A breakdown of these uncertainties into the DR, CR, and UP directions is also shown in the figures. Although the magnitude of the dispersions has been reduced considerably from the case where there was no tracking (Section 5.1), the largest uncertainty is still in the UP direction and the smallest is in the DR direction.

As a point of interest, the ability to estimate the vehicle position and velocity with the combined platform-tracking model was compared with an estimate of the vehicle state with tracking only. Figure 6-3 shows the position and velocity uncertainties with no platform errors in the model. A large initial uncertainty was assumed in order to evaluate the tracking system alone. A comparison of Figures 6-1 and 6-2 with 6-3 shows that a much better estimate of the vehicle state can be obtained with the guidance errors in the model. (60 meters in position compared to the 30 meters in Figure 6-1, and 0.06 m/sec in velocity compared to the 0.2 m/sec in Figure 6-2.)

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*Figures appear at end of Section 6.

The time histories of the first 17 guidance error standard directions are shown in Figures 6-4 through 6-20. Five error sources showed no improvement along the trajectory for the nominal case. These sources are

1. Spin axis mass unbalance of the Z gyro
2. Anisoelastic drift Z gyro
3. Input axis mass unbalance of the Y gyro
4. Input axis mass unblance of the X gyro
5. Threshold of the Z accelerometer

The last-three errors sources in this group were not plotted. The first two were plotted (Figure 6-19 through 6-20) as there was some variation for changes in the tracking. These results will be discussed in a later section.

In Figures 6-4 through 6-6, the one curve which is shown is for the nominal model. In Figures 6-7 through 6-10, the curve labeled (1) pertains to the nominal model. In Figures 6-11 through 6-20 the nominal curve has been labeled as such.

The behavior of the guidance error standard deviations for nominal tracking may be summarized with the aid of Table 6-1. The initial and final values of the standard deviations are shown as well as the percentage decrease.

In general, the greatest improvement the standard deviations occurred for those error sources which caused the largest dispersion in the trajectory as determined in Table 5-1. An additional explanation for some specific component errors may be found in terms of the acceleration levels. As shown in Figure 4-3, the acceleration in the Z direction (CR) is very small. As a result the components that cause errors in this direction have less effect and therefore their standard deviations do not improve significantly.

TABLE 6-1

PERCENTAGE DECREASE IN GUIDANCE ERROR UNCERTAINTIES FOR NOMINAL TRACKING

ERROR SOURCE*	INITIAL 3σ VALUE	FINAL 3σ VALUE	PERCENT CHANGE
17	0.174-3	0.869-4	50
18	0.873-4	0.492-4	44
2	0.739-4	0.52-4	30
3	0.739-4	0.688-4	7
16	0.873-4	0.594-4	32
21	0.242-6	0.132-6	45
4	0.739-4	0.474-4	36
6	0.739-4	0.579-4	22
20	0.242-6	0.158-6	35
11	0.5-6	0.418-6	16
10	0.5-6	0.334-6	33
12	0.5-6	0.448-6	10
1	0.36-4	0.203-4	43
5	0.36-4	0.198-4	45
19	0.242-6	0.229-6	5
27	0.371-4	0.371-4	No Change
30	0.202-2	0.202-2	No Change
23	0.148-4	0.148-4	No Change
22	0.148-4	0.148-4	No Change
15	0.2-7	0.2-7	No Change

*The numbers in this column are program numbers. See Table 5-1 for the corresponding error source.

For example, the improvement in the knowledge of the Z accelerometer misalignments in the X (No. 3), or Y (No. 6) direction is less than that of the X accelerometer misalignment in the Y direction (No. 4), or the Y accelerometer in the X direction (No. 2). Also the improvement in the Z accelerometer bias (No. 12) was smaller than the improvement in the X or Y accelerometer biases (Numbers 10 and 11). (See Table 6-1 for these comparisons.)

The results of simulating the combined platform-tracking system indicate that it would not be possible, at least for the values of the tracking accuracies that have been assumed, to significantly improve the uncertainty of the platform error sources. Therefore if the only objective is to update the guidance system during a flight, this method would not be feasible, assuming the guidance errors were equal or less than their 3σ values. However, if the objective is a postflight analysis, or if the tracking is improved, the conclusion may be considerably different.

6.2 ESTIMATION OF GUIDANCE ERRORS WITH ADDITIONAL KNOWLEDGE OF PLATFORM

In the previous section, the results were presented for the estimation of the guidance errors with nominal tracking. In succeeding sections, the estimation of these errors for improved tracking and degraded guidance errors will be presented. However, before discussing the results for parametric variations of the platform-tracking system, an important point concerning the correlation between the initial platform errors (errors in the accelerometer misalignments and the gyro misalignments) should be noted.

Preliminary results for estimating the guidance errors with improved tracking showed that there was no little improvement in the uncertainty of the gyro misalignments or the accelerometer misalignments regardless of how good the tracking was. This indicated that there was not enough information in the combined platform-tracking system to distinguish between platform misalignments and accelerometer misalignments. The complete covariance matrix for the nominal simulation is included in Appendix A, and can be used to explain this result. It shows that the normalized correlations between (1) the Y accelerometer misalignment to the X axis and the platform misalignment about the Z axis ($C_{2/18}$), (2) Z accelerometer misalignment to the X axis and the platform misalignment about the Y axis ($C_{3/17}$), and (3) the Z accelerometer misalignment about the Y axis and the platform misalignment about the X axis, ($C_{6/16}$), are almost unity. As a result a large uncertainty in these three accelerometer misalignments prevents an improvement in the knowledge of the initial platform misalignments.

In order to see the effect of additional knowledge of the platform on the estimation of the initial gyro misalignments, an additional simulation was made assuming no errors in (1) the Z accelerometer misalignment about the Y axis, (2) the Z accelerometer misalignment about the X axis and (3) the misalignment of the Y accelerometer about the X axis. For the nominal tracking with these three errors omitted the uncertainties in the three initial gyro misalignments and the misalignment of the X accelerometer about the Y axis reduces as shown in Figure 6-7 through 6-10, curve (2).

There is additional justification for omitting these three platform error sources. As stated in Reference 1 the misalignments of the Z accelerometers would be eliminated before the flight by aligning the X-Y plane. In addition some calibration on either the X accelerometer or Y accelerometer would be necessary in order to be able to predict excessive variations in the initial gyro drift about the Z axis. This was evident from runs that were made with all five accelerometer misalignments. The result was that for very large values of the initial variances of the gyro misalignments, or very good tracking, no significant improvement was found in the uncertainty of the gyro misalignments at the end of the trajectory.

For the parametric results that are presented in the following sections, the following accelerometer errors were omitted from the model:

1. Z accelerometer misalignment in the direction of the X axis
2. Z accelerometer misalignment in the direction of the Y axis
3. Y accelerometer misalignment in the direction of the X axis.

6.3 ESTIMATION OF GUIDANCE ERRORS WITH IMPROVED TRACKING

Due to the fact that the ability to estimate the guidance errors with nominal tracking has been found to be somewhat marginal, a number of studies were made to investigate the system parametrically. The first parametric study involved the tracking accuracies. Runs were made with all the tracking error standard deviations (random and bias measurement errors, and station location errors) reduced by factors of 10, 100, 1000, and finally, with random errors reduced by a factor of 1000 and no bias errors. These results are shown in Figures 6-7 through 6-20. The purpose of this study was to determine if with good tracking, the knowledge of the guidance errors would improve. As discussed in the previous section, even with perfect tracking, it is not possible to estimate the errors in the initial gyro misalignments or the accelerometer misalignments any better than for the nominal tracking run. This is because of the high correlation between uncertainties in these platform error sources. As a result the improved tracking runs assumed a perfect knowledge of three accelerometer errors. As shown by Figures 6-7 through 6-20 all of the guidance error uncertainties in these figures can be improved significantly with better tracking. Again, in general, the amount by which the guidance errors improve depends on their relative effect on the trajectory. Of the total 17 guidance error sources, the three errors which have the least effect on the trajectory were not plotted. These errors are

(1) The input mass unbalance of the Y gyro

(2) The input mass unbalance of the X gyro

(3) The Z accelerometer threshold error

The uncertainty in these error sources does not improve significantly even with good tracking.

From the results presented in this section it may be concluded that, in general, the feasibility of estimating the guidance errors does depend on the relative accuracies of the guidance errors and the tracking errors.

Approximately one order of magnitude improvement in the tracking accuracies would be required to obtain a significant improvement in the guidance error uncertainties during an actual flight. In addition this improvement would be contingent on a preflight calibration of three of the accelerometer misalignments.

6.4 ESTIMATION OF GUIDANCE ERRORS WITH PERFECT KNOWLEDGE OF THE TRAJECTORY END POINT

One method that was considered for improving the estimate of the guidance errors was to track the vehicle in a parking orbit. That is, the telemetry data and the tracking data would be combined in the same manner, only for a parking orbit rather than a powered flight ascent. The end result of this simulation would be an improvement in the knowledge of the end point of the powered flight trajectory.

Before simulating the system for a parking orbit, a check was made to determine the effect of perfect knowledge of the trajectory end point.

This would show the maximum decrease in the uncertainty of the guidance errors and would represent the results for perfect tracking. Six equivalent observations were made of the position and velocity states with zero error variance. The results are summarized in Table 6-2. As shown in the table there is very little improvement in the uncertainty of the guidance errors by a perfect knowledge of the trajectory end point. This indicates that there is even less knowledge that would be added to the guidance error uncertainties as a result of tracking the vehicle in a parking orbit. The parking orbit simulation was therefore not implemented.

6.5 ESTIMATION OF GUIDANCE ERROR WITH LARGE INITIAL UNCERTAINTIES

In addition to using the combined platform-tracking system to improve the knowledge of the vehicle state, or to update the guidance system during a flight, a third and perhaps more useful application would be in a post-flight analysis. For this latter objective it would be desirable to determine whether the guidance components performed normally, or whether there was a malfunction.

To evaluate the feasibility of estimaing the guidance errors in a post-flight analysis, the combined platform-tracking system has been simulated for parametric variations in the guidance errors themselves. Large initial values of the guidance error standard deviations have been assumed to simulate a malfunction or a component error that was larger than predicted by the 3σ nominal value.

The time histories of the guidance error standard deviations are shown in Figures 6-21 through 6-32 for initial standard deviations of 10, and 100 times that of the nominal values. These results do not include curves for the three accelerometer errors Z to X, Z to Y, and Y to X. It was found that with these error sources in the model, the standard deviations of the gyro misalignments did not decrease, even for large initial values of guidance errors. However, with the assumption of a perfect knowledge of these errors, it is possible to reduce the uncertainty in the gyro misalignments.

TABLE 6-2

THE EFFECT OF A PERFECT KNOWLEDGE OF THE TRAJECTORY END-POINT ON THE GUIDANCE ERROR UNCERTAINTIES

ITEM	NOMINAL @ t=9.5 MIN.	PERFECT OBSERVATION OF POSITION VELOCITY STATE @ t=9.5 MIN.
RMSP	0.938×10^{-2}	0.754×10^{-6}
RMSV	0.695×10^{-4}	0.697×10^{-8}
GUID. ERR. #17	0.874×10^{-4}	0.847×10^{-4}
18	0.489×10^{-4}	0.473×10^{-4}
16	0.599×10^{-4}	0.586×10^{-4}
21	0.100×10^{-6}	0.599×10^{-7}
4	0.478×10^{-4}	0.464×10^{-4}
20	0.146×10^{-6}	0.127×10^{-6}
11	0.423×10^{-6}	0.418×10^{-6}
10	0.350×10^{-6}	0.319×10^{-6}
12	0.457×10^{-6}	0.435×10^{-6}
1	0.211×10^{-4}	0.206×10^{-4}
5	0.221×10^{-4}	0.182×10^{-4}
19	0.228×10^{-6}	0.208×10^{-6}

The behavior of the 5 least important guidance errors, from Table 5-1, was not plotted. The uncertainty in these errors remain essentially constant at their initial values.

Of the twelve error sources which are plotted in Figures 6-21 through 6-32, it may be seen that the uncertainties in all these error sources decrease significantly for large initial values. It may therefore be concluded that it would be feasible to estimate these errors in a post-flight analysis. That is, a malfunction in one of the guidance components that caused large deviation in the trajectory, could be identified by combining the telemetry and tracking data.

6.6 ESTIMATION OF GUIDANCE ERRORS WITH INCREASED OBSERVATION RATE

For the nominal tracking model an observation rate of one per second was used. One additional run was made to determine the effect of a reduced observation rate on the guidance error uncertainties. Table 6-3 shows a comparison of the guidance error uncertainties for 1 observation per second and 10 observations per second at 7 minutes along the trajectory. A decrease in the observation rate by an order of magnitude is seen to reduce the guidance error uncertainties by at most a factor of 2. The RMSP and RMSV are also shown in the table for the two observation rates.

TABLE 6-3

VARIATION OF GUIDANCE ERRORS, POSITION AND VELOCITY WITH OBSERVATION RATE

	STATES AT 7 MIN.	
	NOMINAL CASE OBS ~ 1 SEC	OBS ~ 0.1 SEC
RMSP	0.193×10^{-1}	0.808×10^{-2}
RMSV	0.160×10^{-3}	0.775×10^{-4}
GUID. ER. #17	0.653×10^{-4}	0.439×10^{-4}
18	0.415×10^{-4}	0.287×10^{-4}
16	0.336×10^{-4}	0.245×10^{-4}
21	0.179×10^{-6}	0.130×10^{-6}
4	0.401×10^{-4}	0.250×10^{-4}
20	0.200×10^{-6}	0.156×10^{-6}
11	0.410×10^{-6}	0.337×10^{-6}
10	0.340×10^{-6}	0.233×10^{-6}
12	0.460×10^{-6}	0.366×10^{-6}
1	0.254×10^{-4}	0.189×10^{-4}
5	0.232×10^{-4}	0.175×10^{-4}
19	0.239×10^{-6}	0.237×10^{-6}
27	0.370×10^{-4}	0.371×10^{-4}
30	0.201×10^{-2}	0.202×10^{-2}
23	0.148×10^{-4}	0.148×10^{-4}
22	0.148×10^{-4}	0.148×10^{-4}
15	0.200×10^{-7}	0.200×10^{-4}

6.7 ESTIMATION OF GUIDANCE ERRORS WITH DIFFERENT KINDS OF TRACKING

In order to determine which type of tracking is the most effective for reducing the guidance error uncertainties, two runs were made; one with range observations only, and a second run with azimuth and elevation measurements only.

Position uncertainty (RMSP) and velocity uncertainty (RMSV) for these two cases are shown in Figures 6-33 and 6-34, respectively. The uncertainty in the guidance errors at the end of each of these runs is shown in Table 6-4.

The results of this study show very little difference between the effectiveness of range measurements and azimuth and elvation measurements.

TABLE 6-4

VARIATION OF GUIDANCE ERRORS FOR DIFFERENT TRACKING
@ t=9.5 MIN.

	RANGE ONLY	AZ&EL ONLY
RMSP	0.944×10^{-1}	0.853×10^{-1}
RMSV	0.485×10^{-3}	0.421×10^{-3}
GUID. ERR. #17	0.751×10^{-4}	0.705×10^{-4}
18	0.417×10^{-4}	0.480×10^{-4}
16	0.369×10^{-4}	0.414×10^{-4}
21	0.195×10^{-6}	0.178×10^{-6}
4	0.370×10^{-4}	0.486×10^{-4}
20	0.170×10^{-6}	0.210×10^{-6}
11	0.409×10^{-6}	0.432×10^{-6}
10	0.357×10^{-6}	0.459×10^{-6}
12	0.467×10^{-6}	0.475×10^{-6}
1	0.222×10^{-4}	0.336×10^{-4}
5	0.207×10^{-4}	0.300×10^{-4}
19	0.231×10^{-6}	0.239×10^{-6}
27	0.370×10^{-4}	0.371×10^{-4}
30	0.201×10^{-2}	0.201×10^{-2}
23	0.148×10^{-4}	0.148×10^{-4}
22	0.148×10^{-4}	0.148×10^{-4}
15	0.200×10^{-7}	0.200×10^{-7}

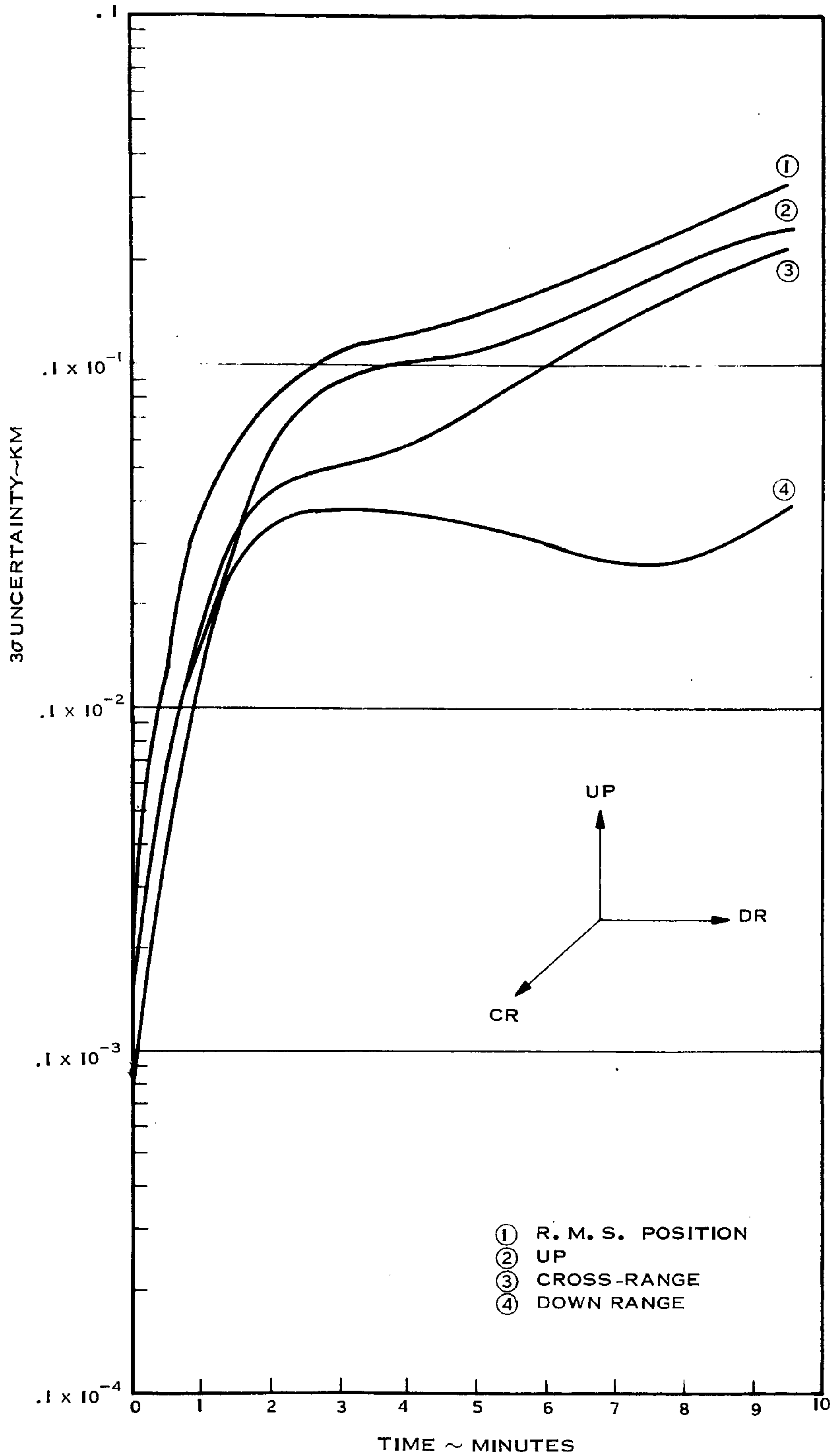


Figure 6-1 Variation in Position Uncertainties ~ Nominal Case

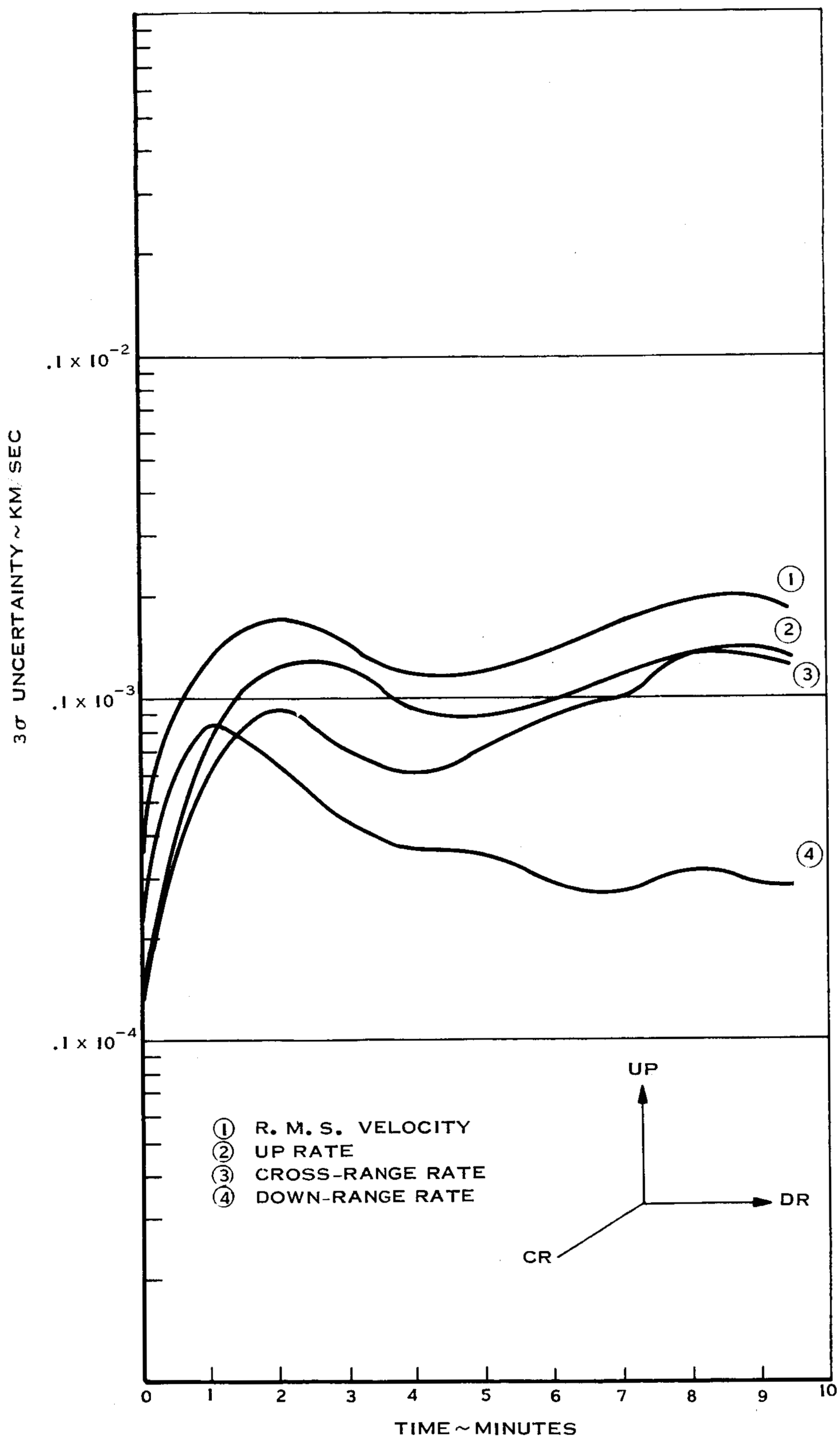


Figure 6-2 Variation in Velocity Uncertainties ~ Nominal Case

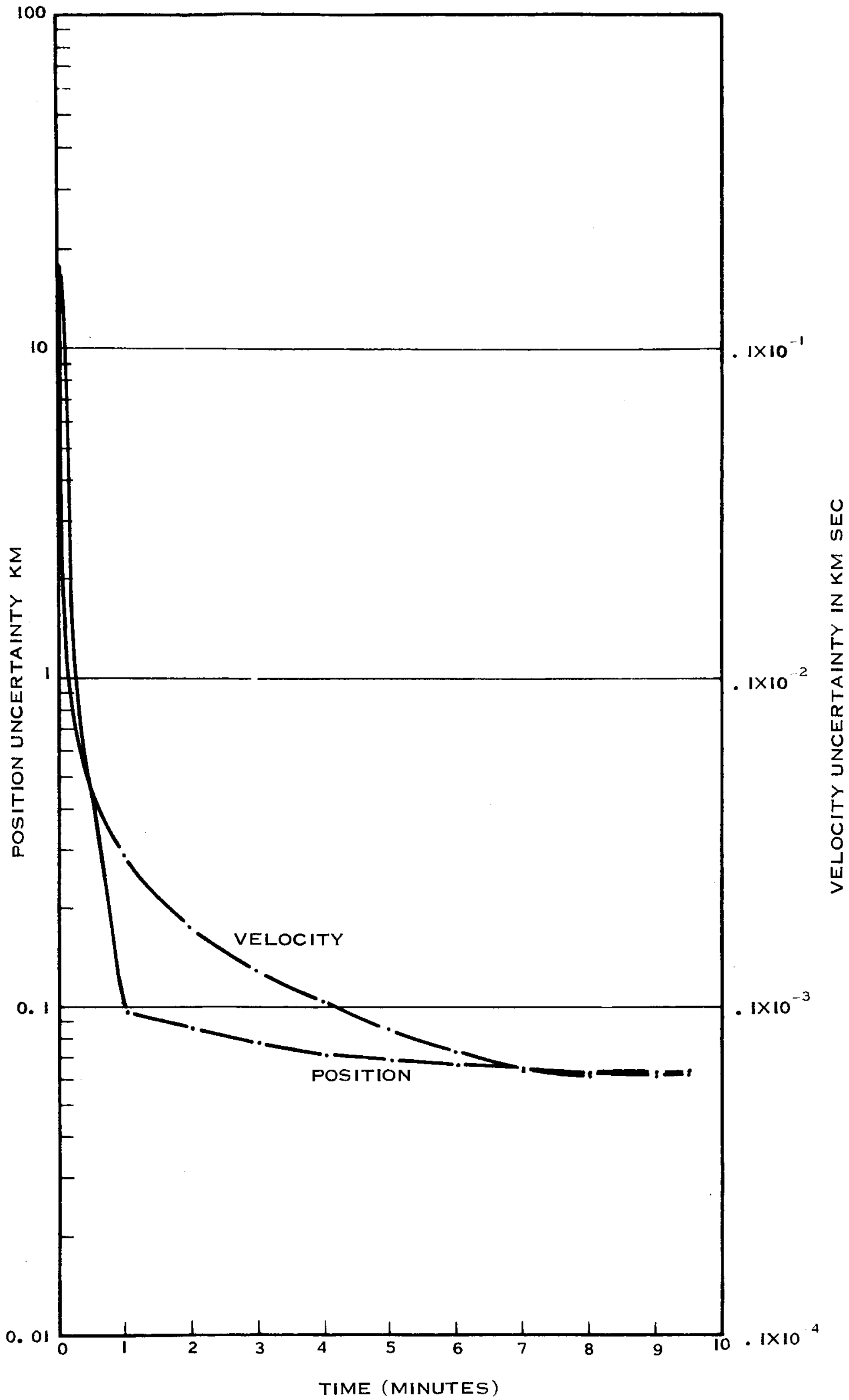


Figure 6-3 Position and Velocity Uncertainties with Tracking Only (No Telemetry Data)

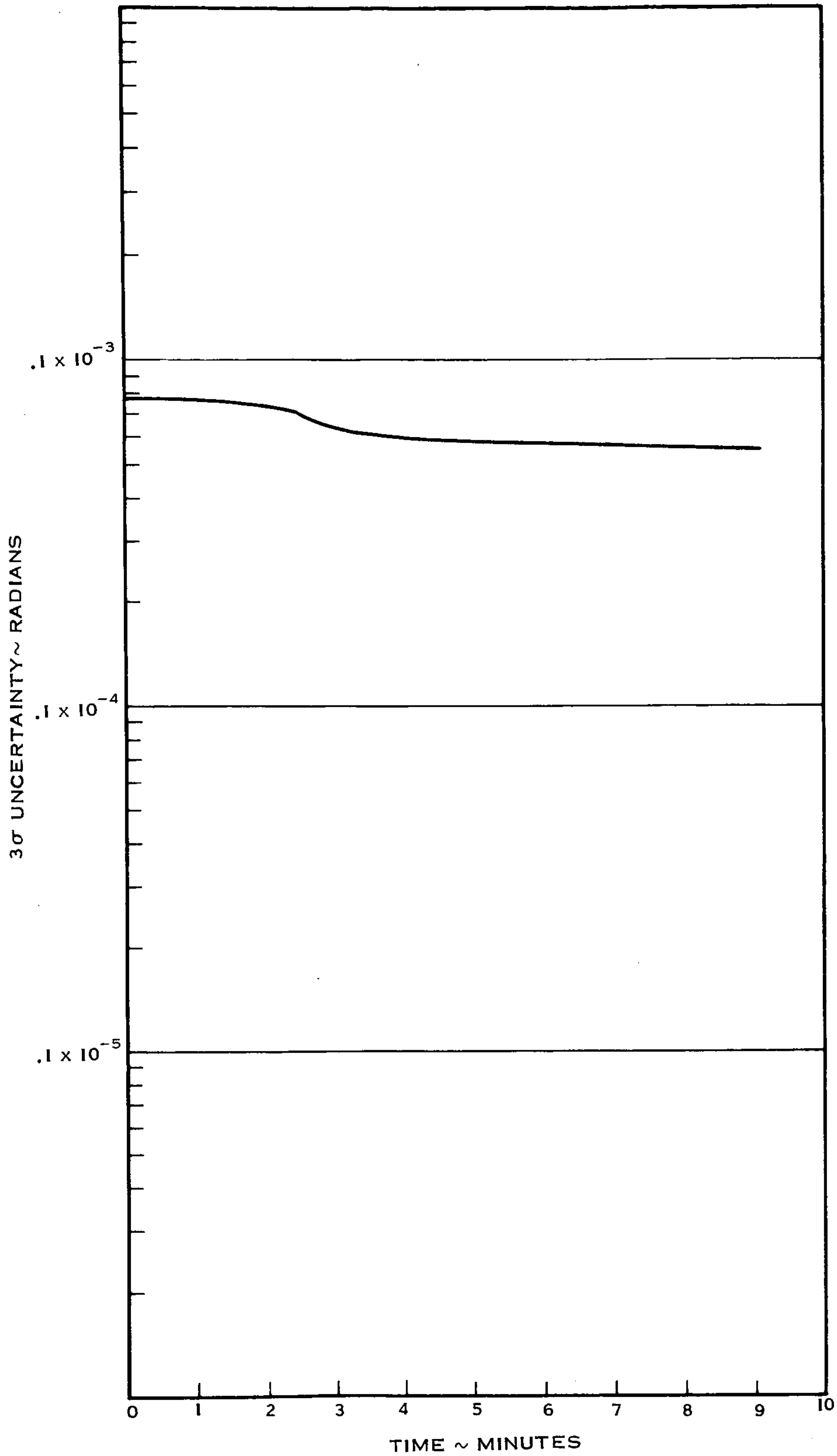


Figure 6-4 3σ Uncertainty in Y-Accelerometer Misalignment into X-Axis

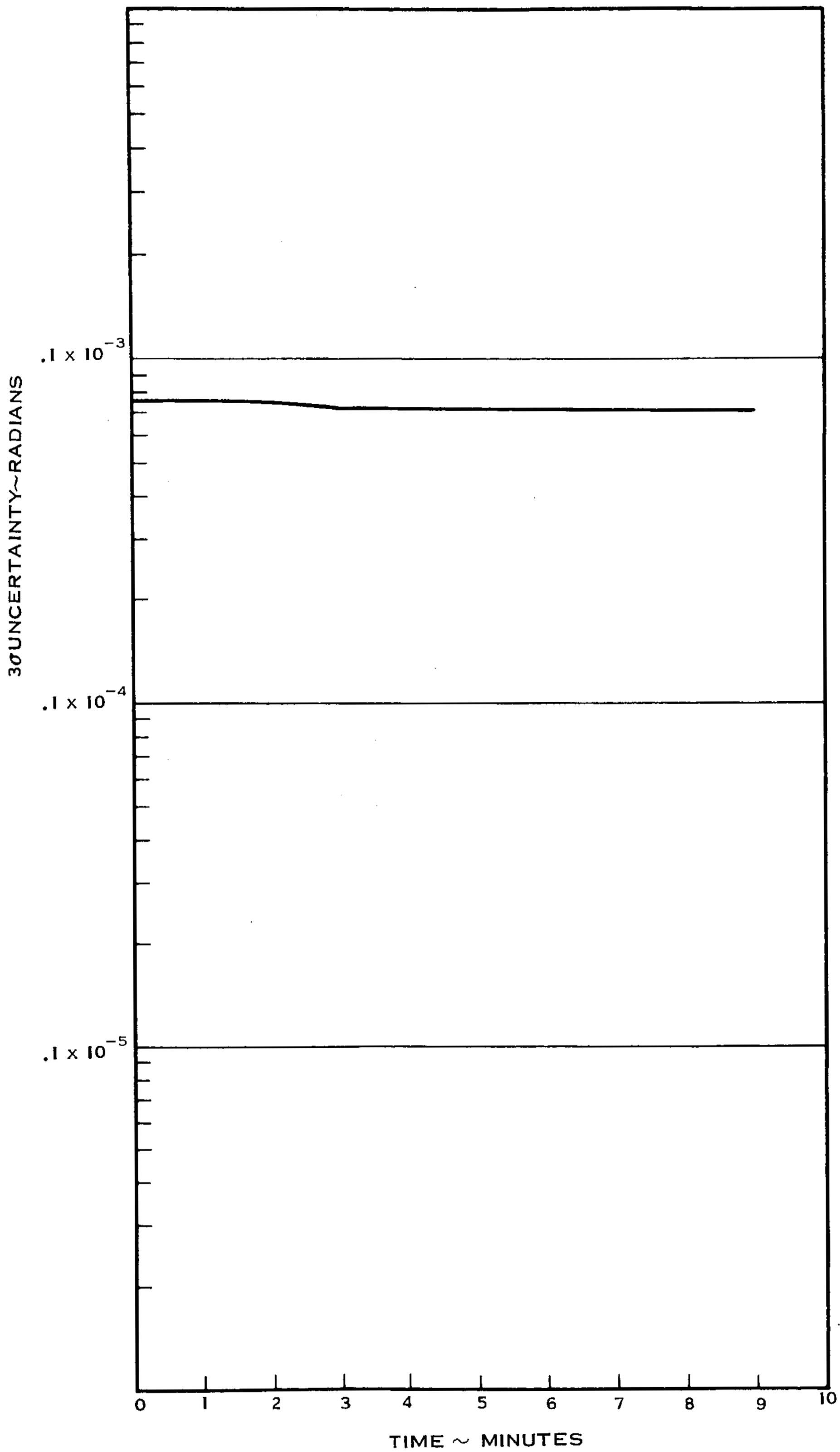


Figure 6-5 3σ Uncertainty In Z-Accelerometer Misalignment Into X-Axis

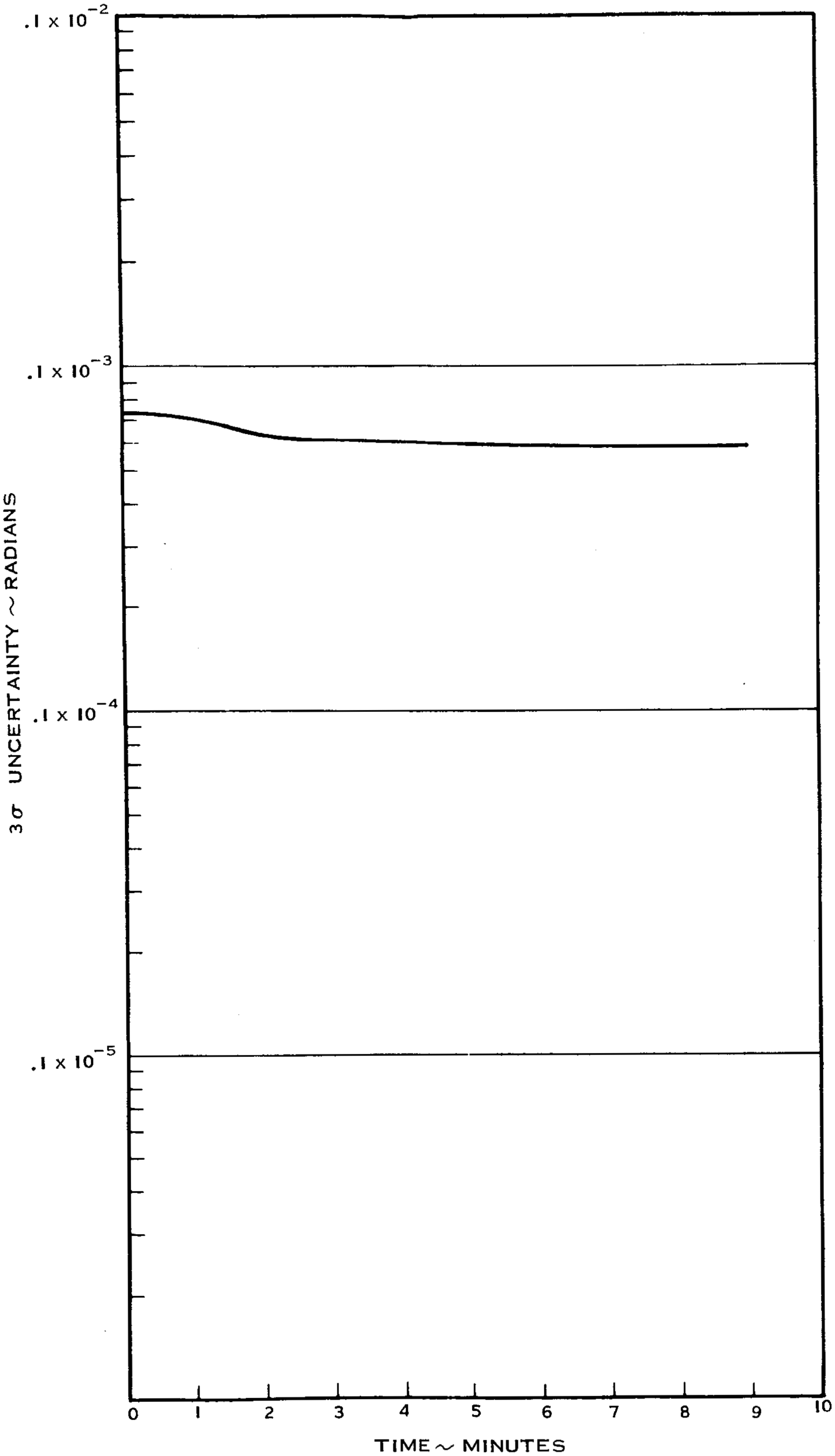


Figure 6-6 3σ Uncertainty in Z-Accelerometer Misalignment Into Y-Axis

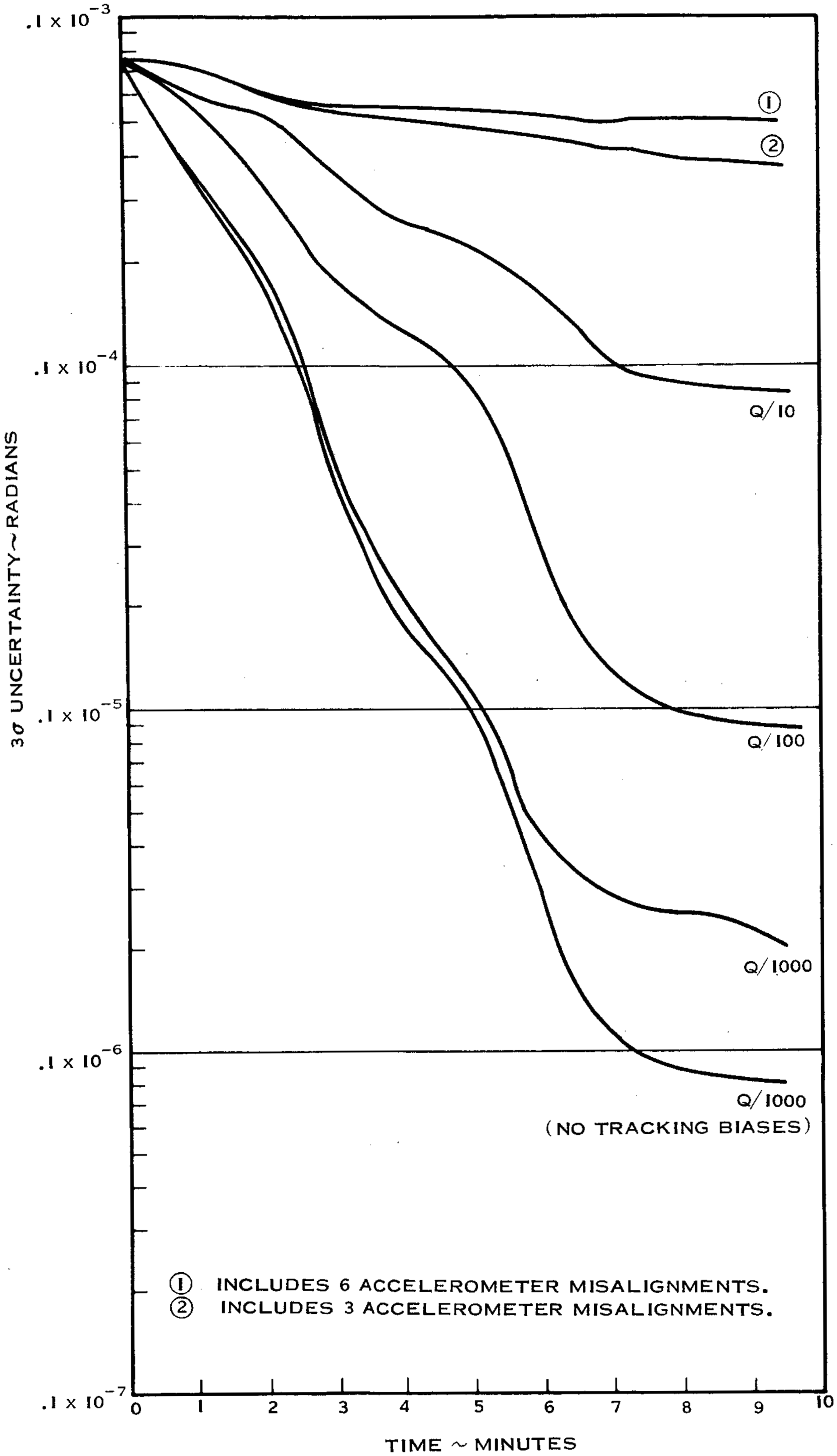


Figure 6-7 3σ Uncertainty in X-Accelerometer Misalignment Into Y-Axis

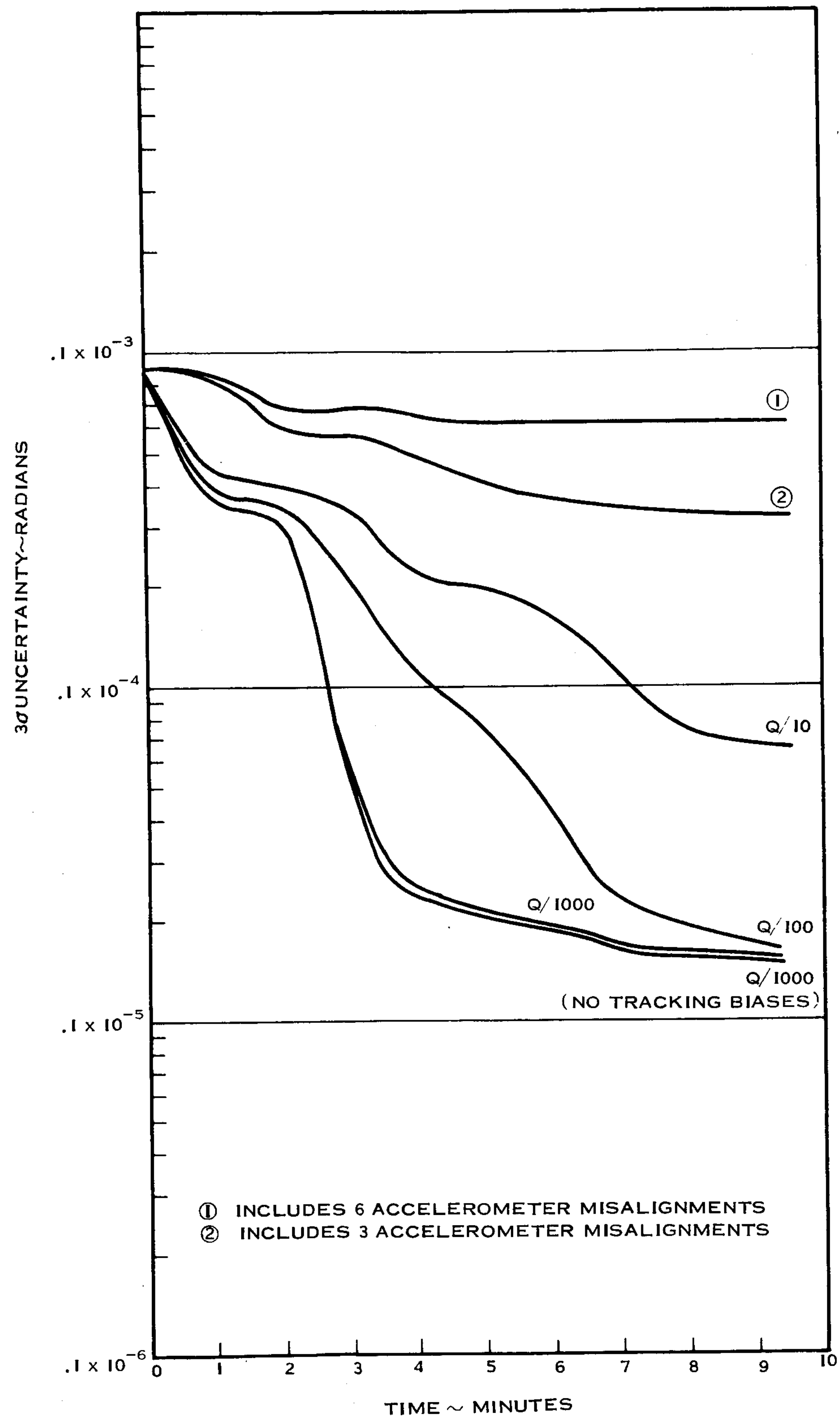


Figure 6-8 3σ Uncertainty in Initial Platform Misalignment About X-Axis

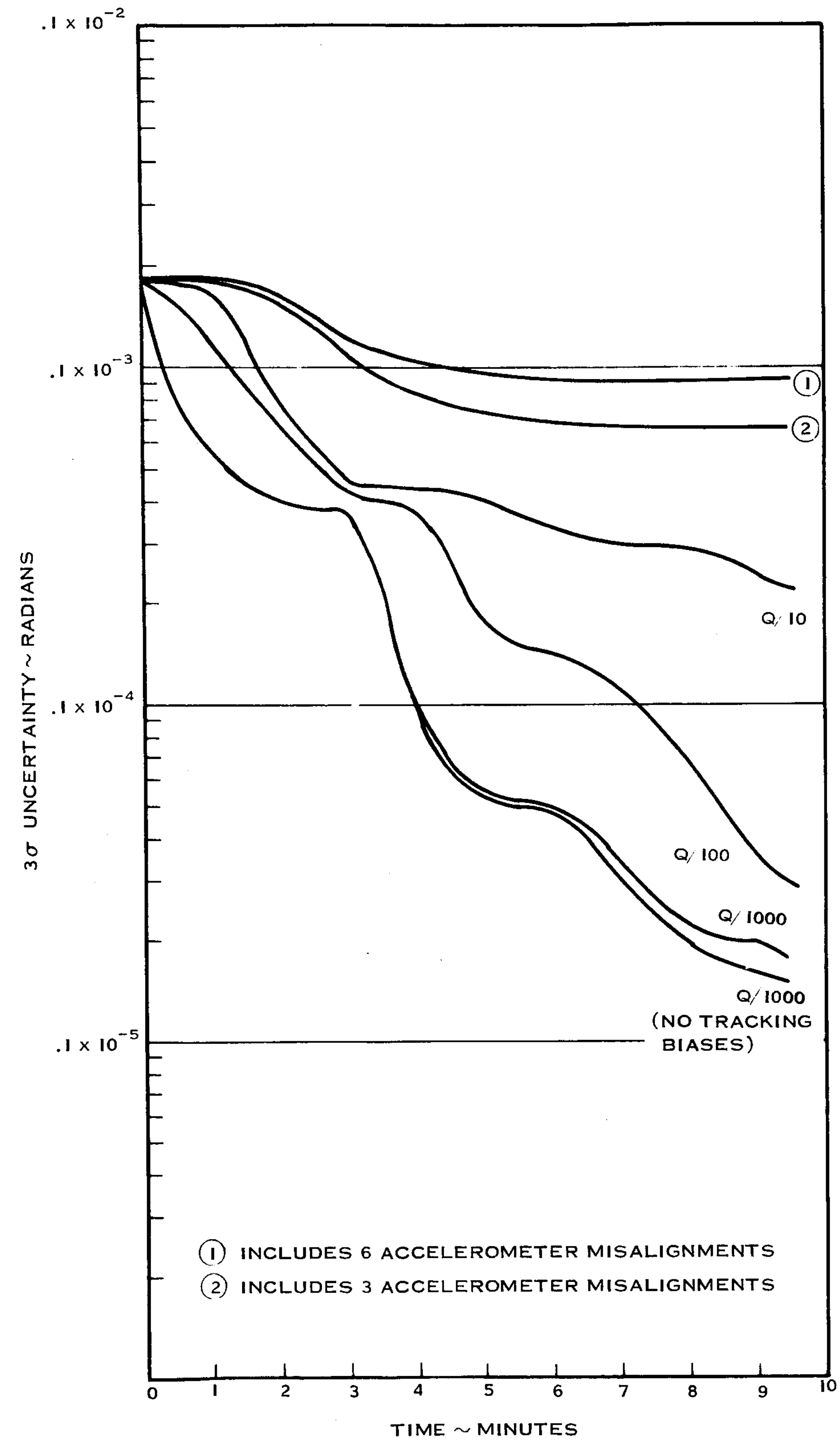


Figure 6-9 3σ Uncertainty in Initial Platform Misalignment About Y-Axis

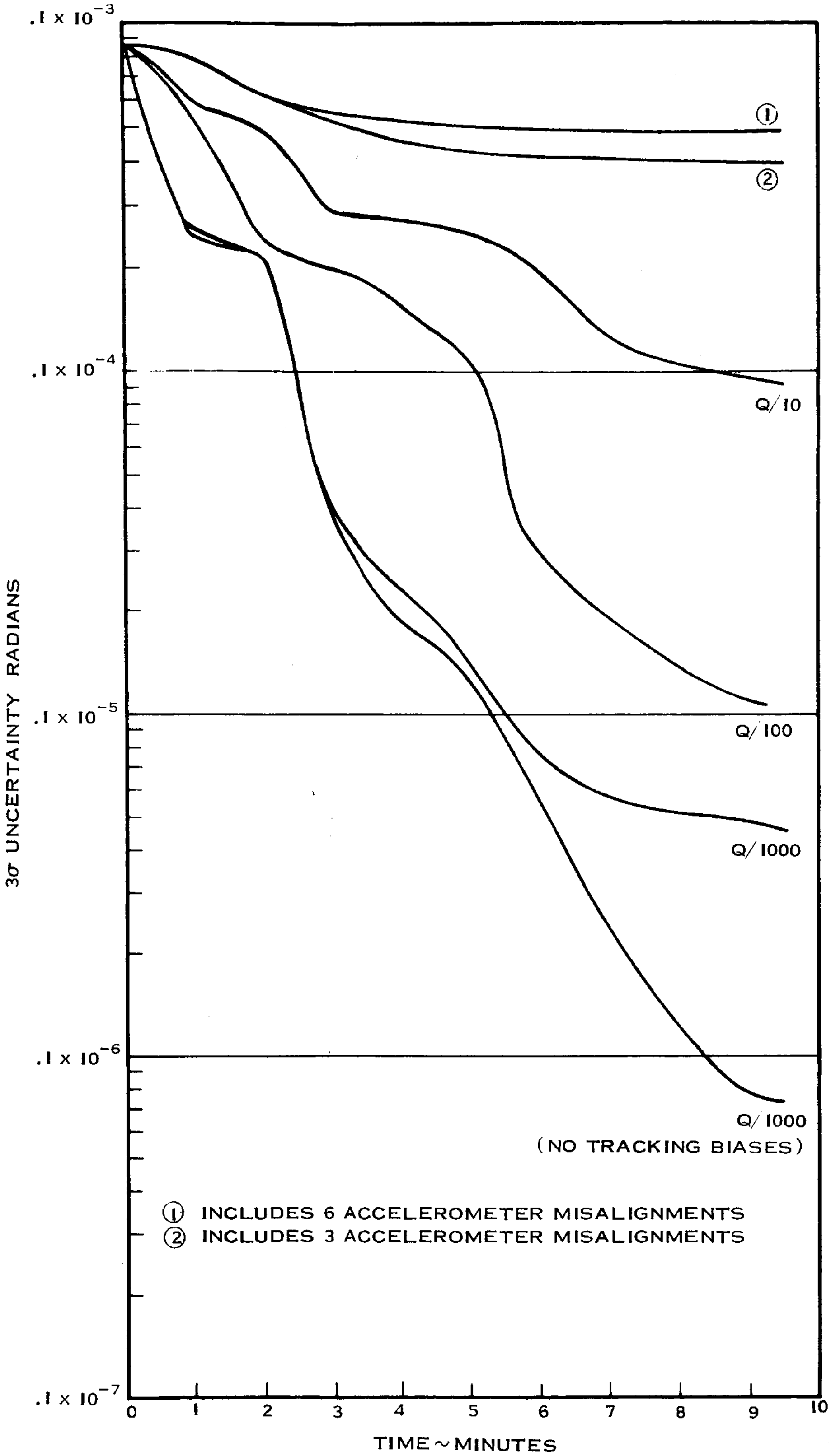


Figure 6-10 3σ Uncertainty in Initial Platform Misalignment About Z-Axis

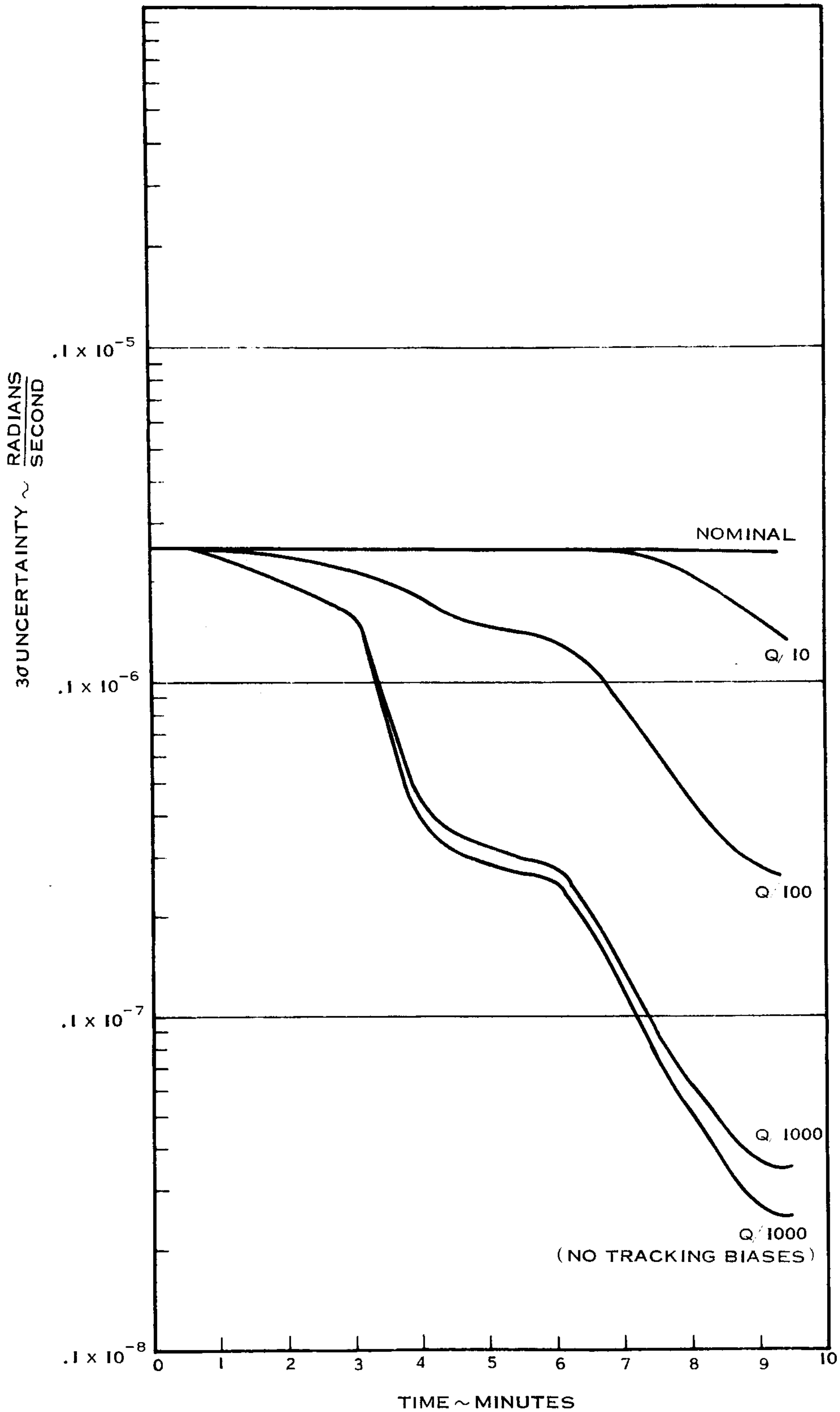


Figure 6-11 3σ Uncertainty in X-Axis Gyro Drift Rate

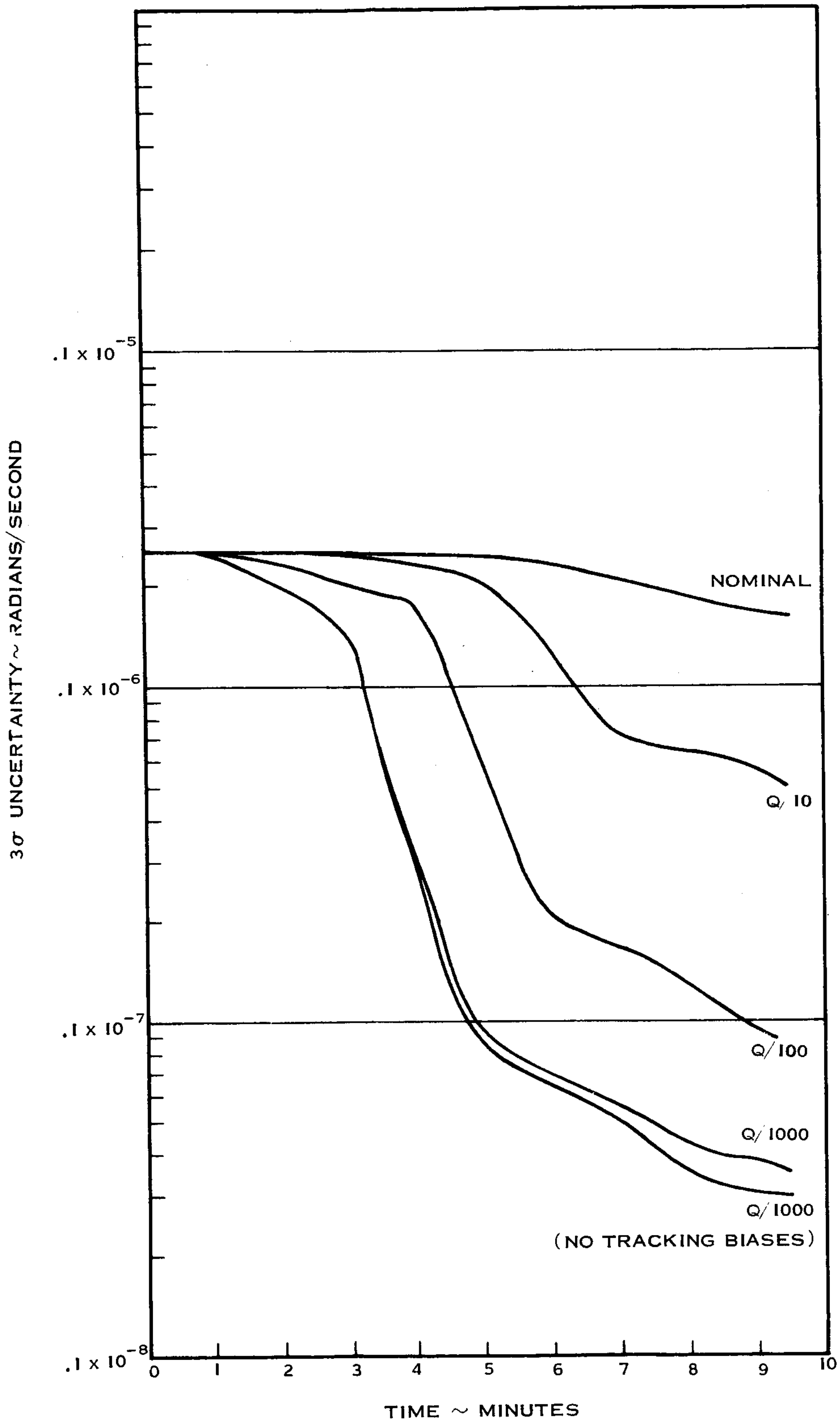


Figure 6-12 3σ Uncertainty in Y-Axis Gyro Drift Rate

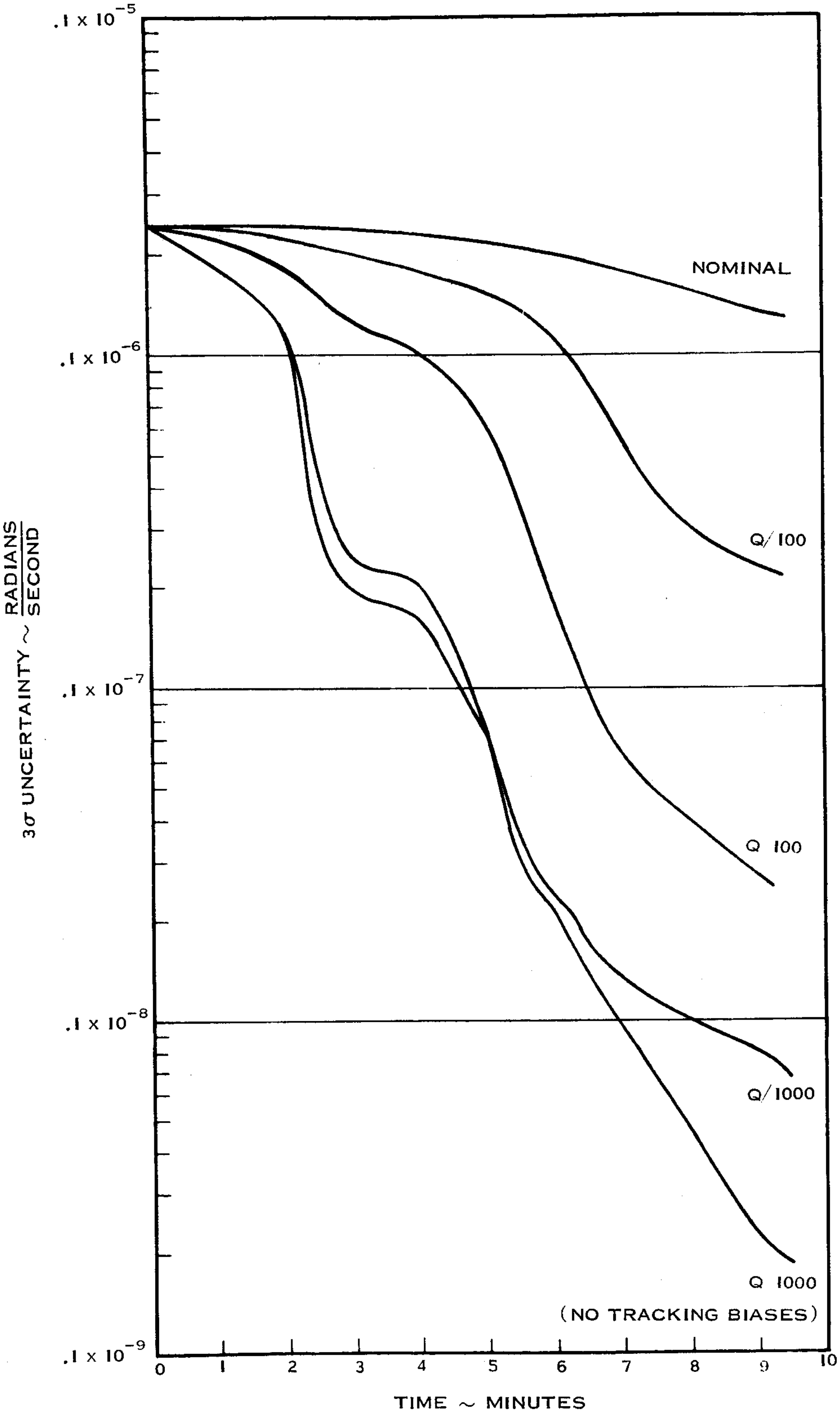


Figure 6-13 3σ Uncertainty in Z-Axis Gyro Drift Rate

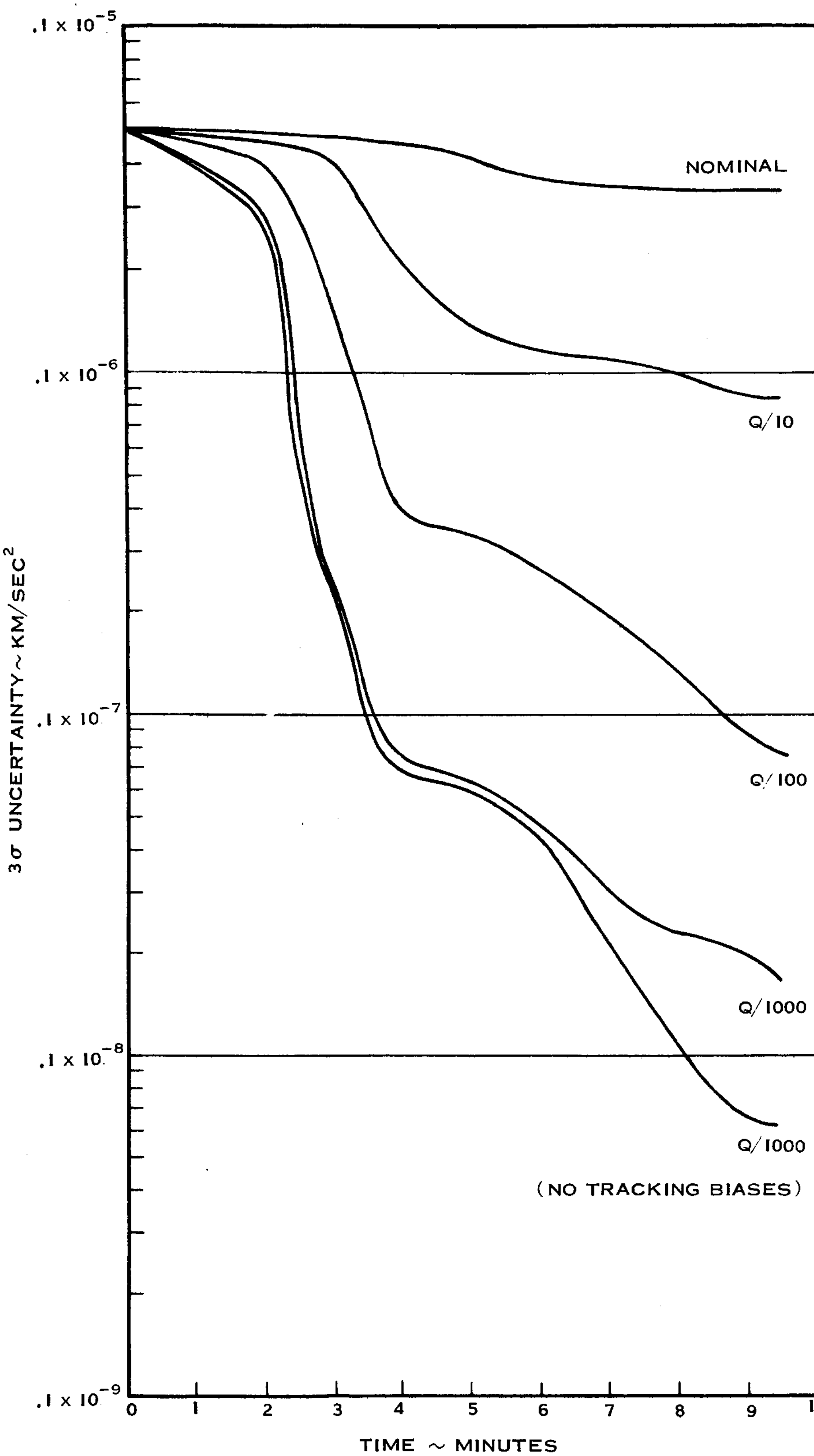


Figure 6-14 3σ Uncertainty in X-Accelerometer Bias

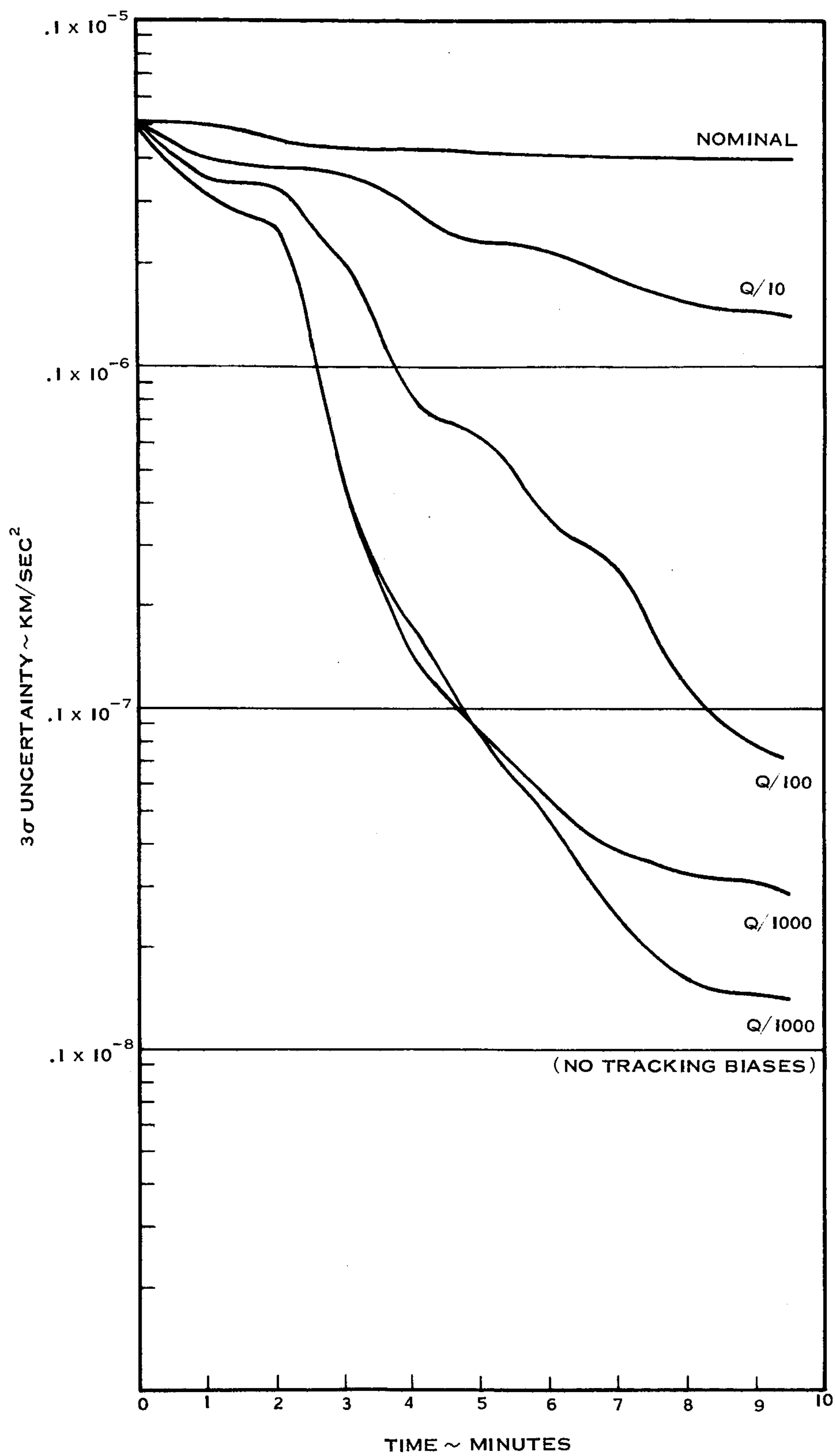


Figure 6-15 3σ Uncertainty in Y Accelerometer Bias

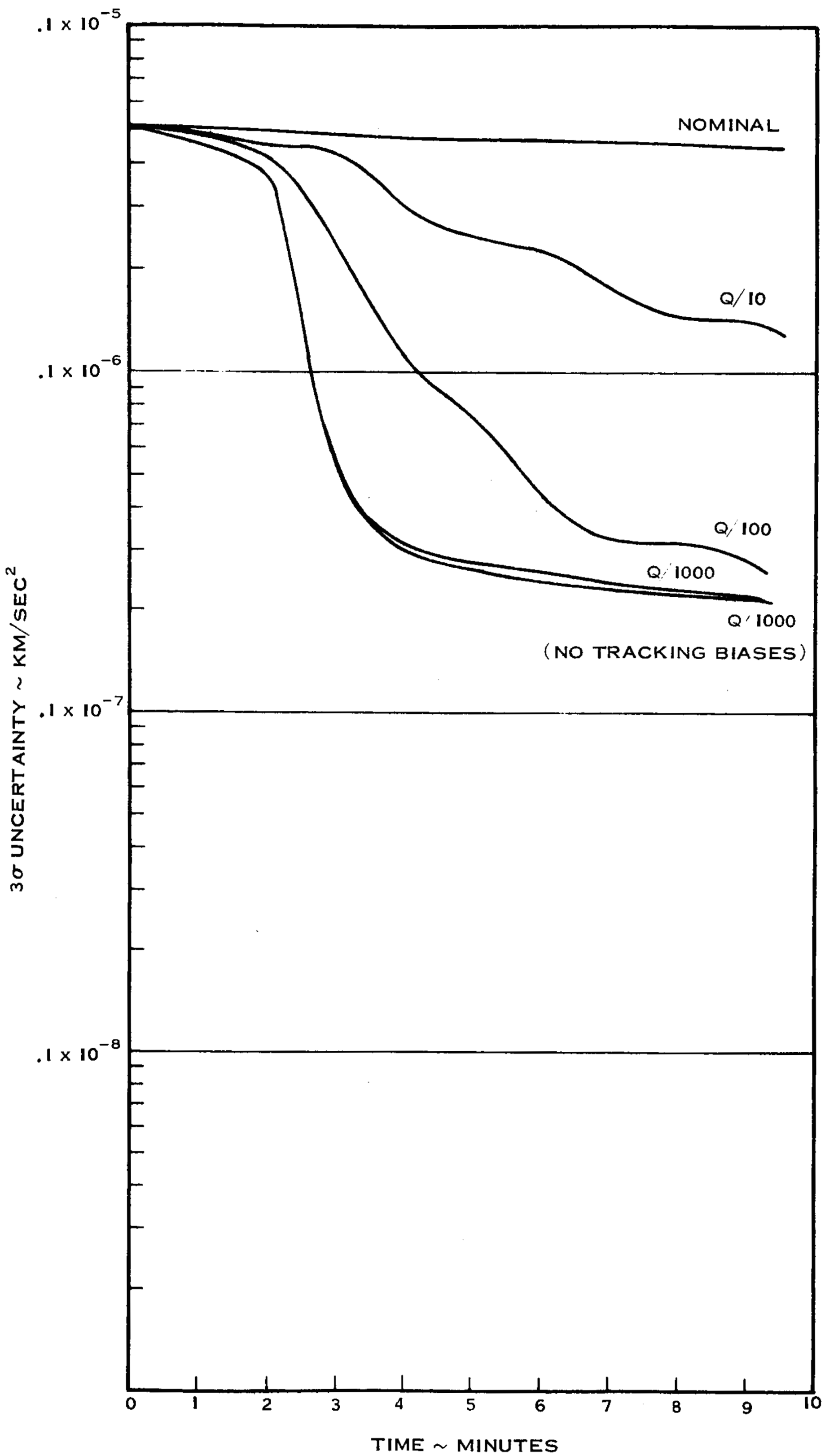


Figure 6-16 3σ Uncertainty in Z-Accelerometer Bias

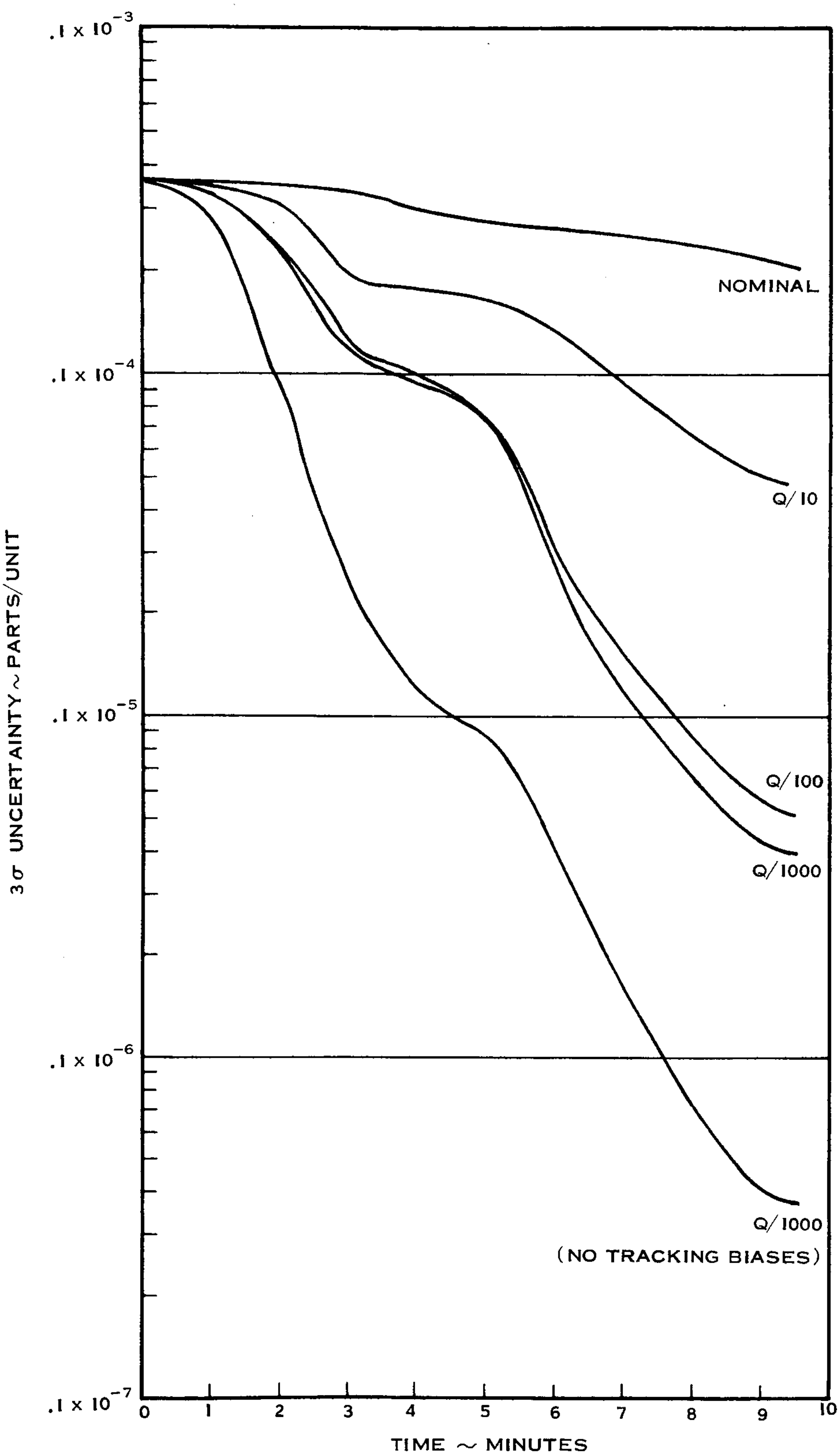


Figure 6-17 3σ Uncertainty in X-Accelerometer Scale Factor

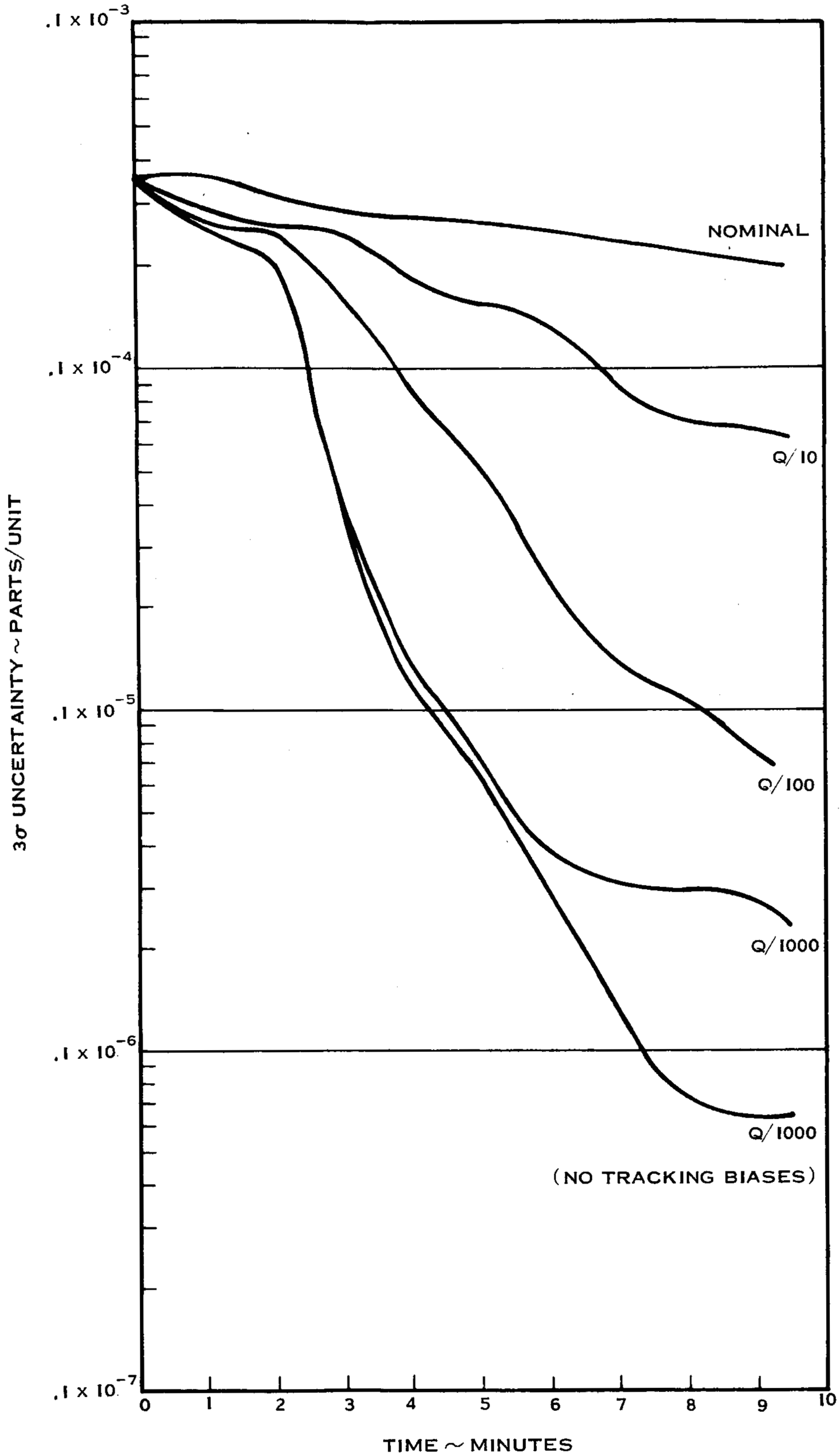


Figure 6-18 3σ Uncertainty in Y-Accelerometer Scale Factor

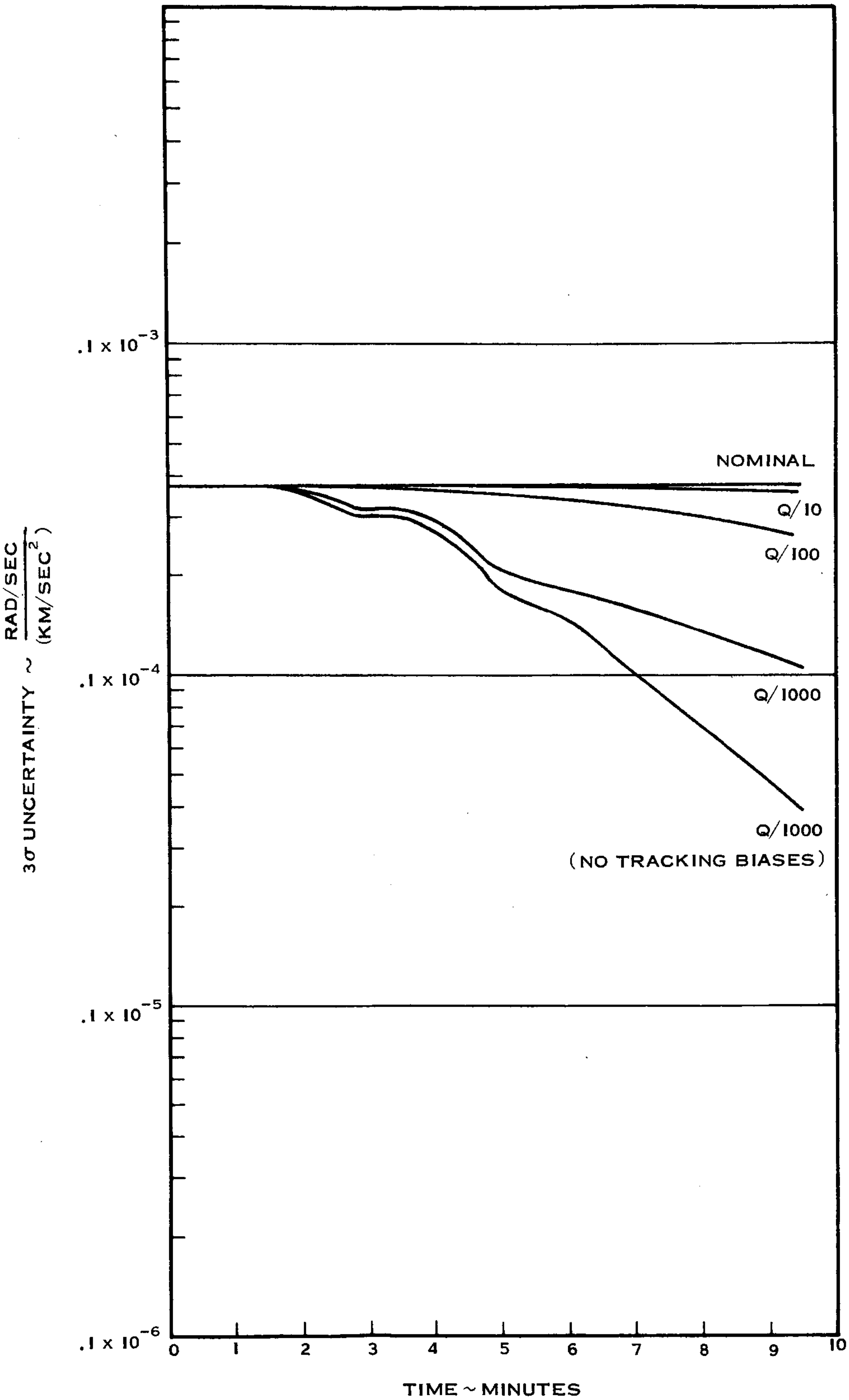


Figure 6-19 3σ Uncertainty in Z-Gyro Spin Axis Mass Unbalance

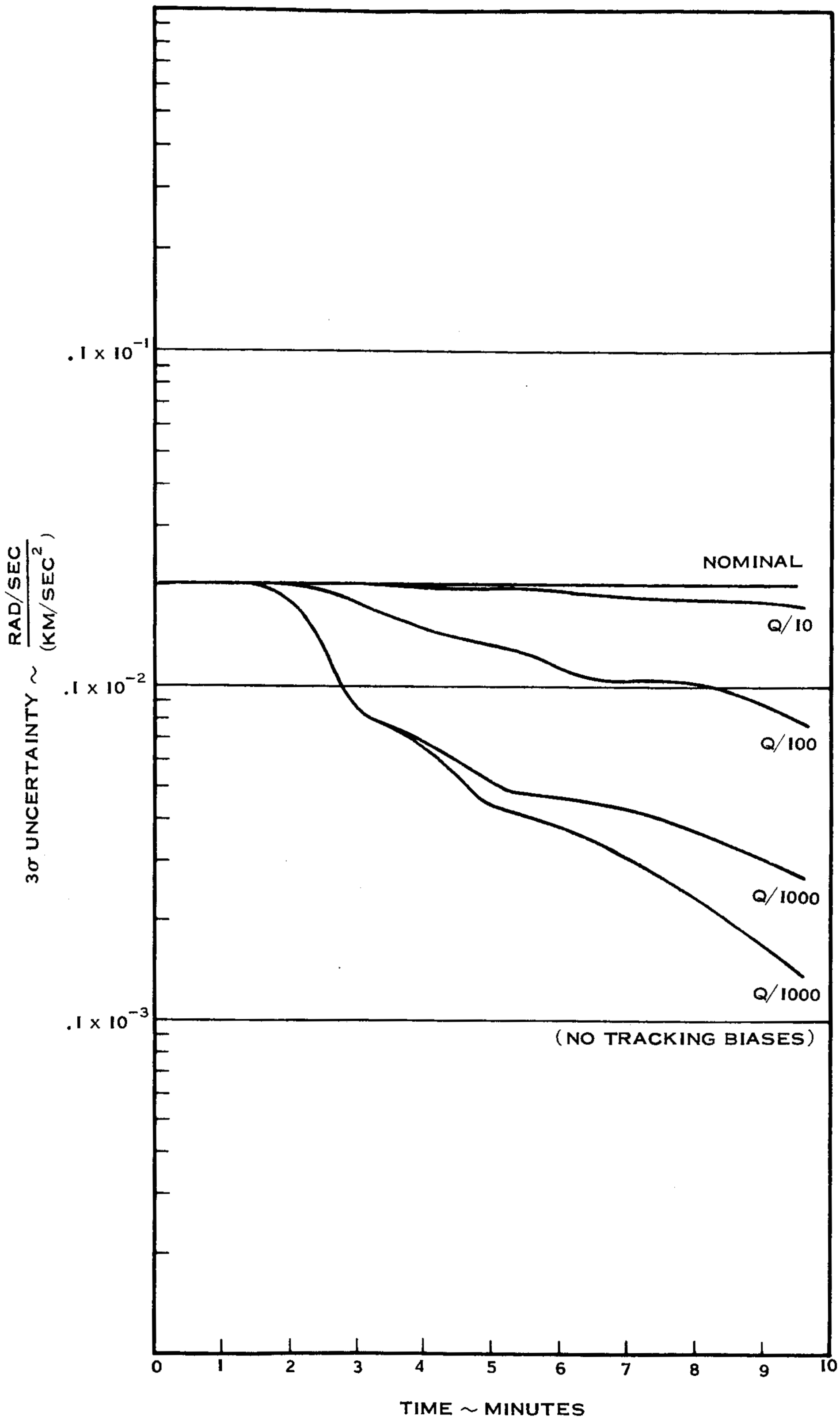


Figure 6-20 3σ Uncertainty in Z-Gyro Anisoelastic Drift

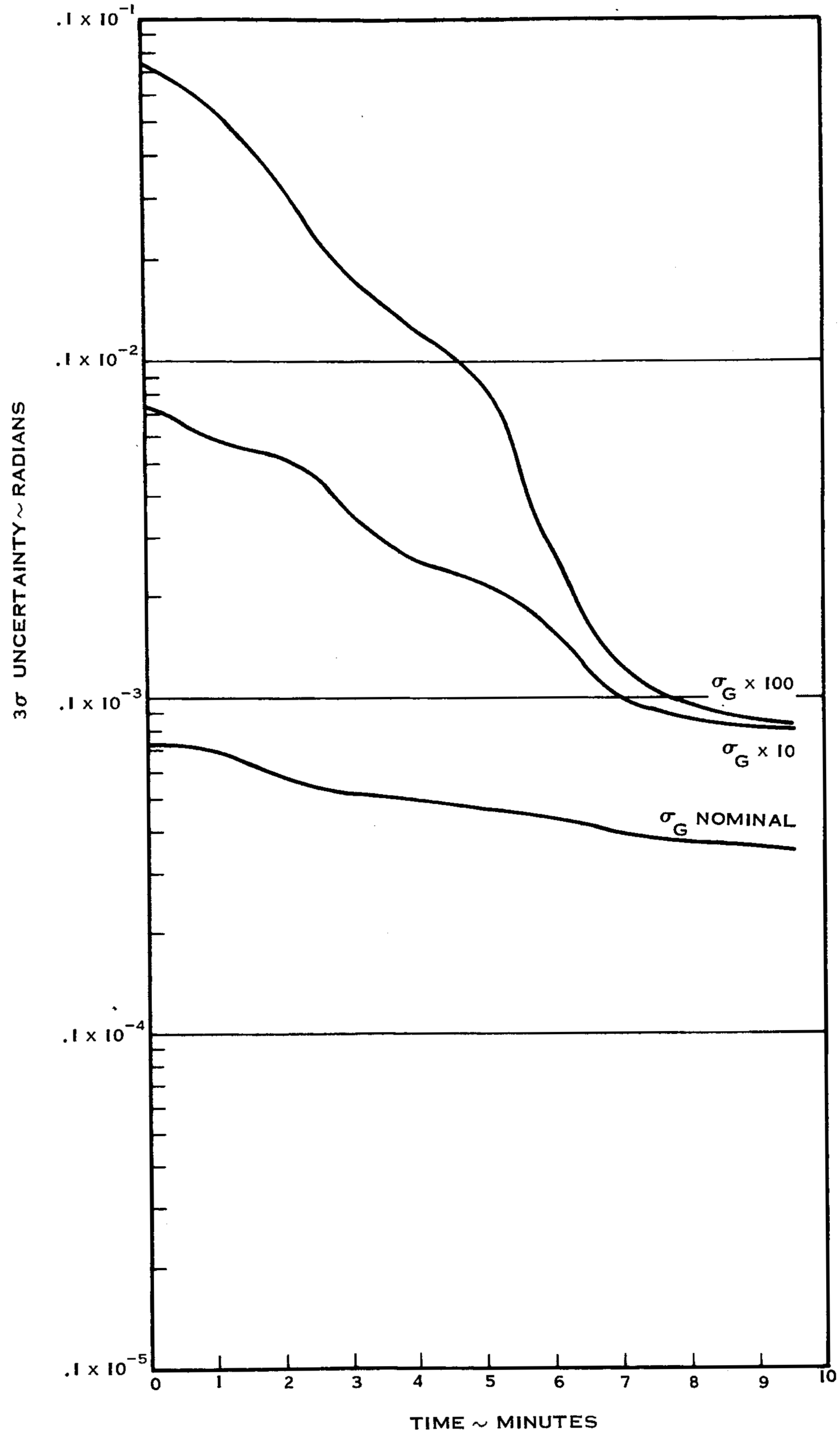


Figure 6-21 3σ Uncertainty in X-Accelerometer Misalignment Into Y-Axis for High Initial Uncertainties

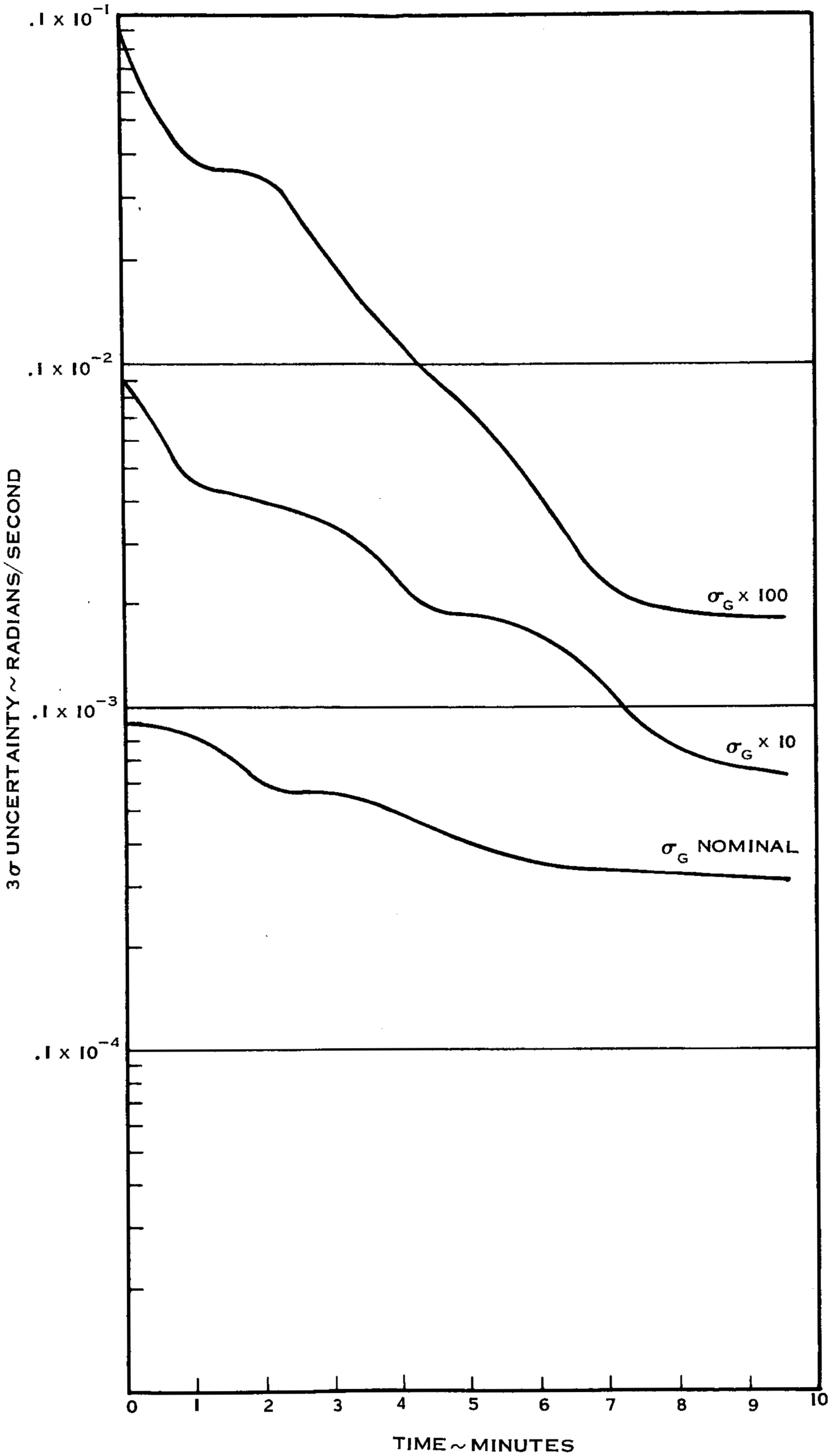


Figure 6-22 3σ Uncertainty in Initial Platform Misalignment About X-Axis for High Initial Uncertainties

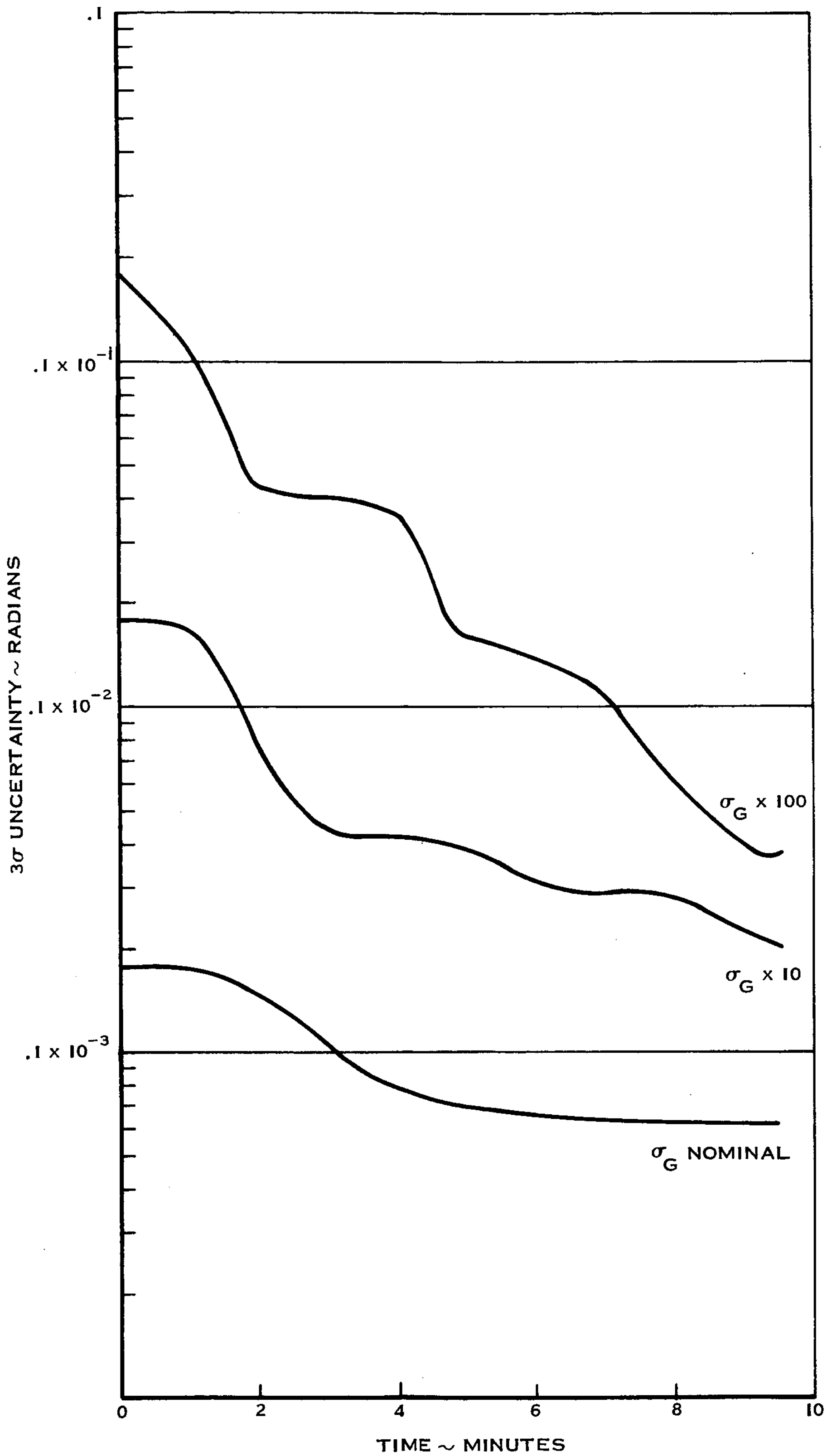


Figure 6-23 3σ Uncertainty in Initial Platform Misalignment About Y-Axis for High Initial Uncertainties

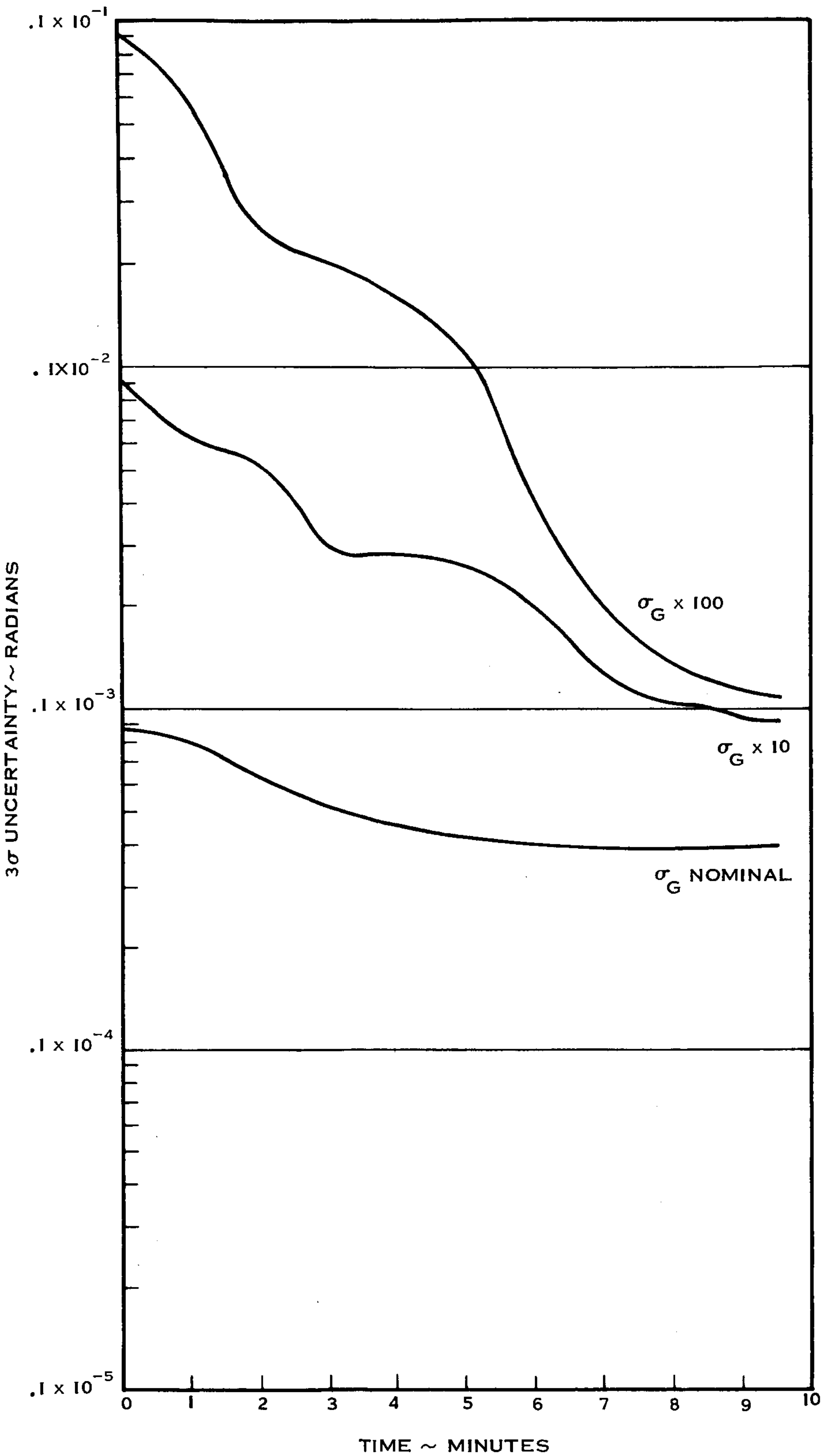


Figure 6-24 **3σ Uncertainty in Initial Platform Misalignment About Z-Axis for High Initial Uncertainties**

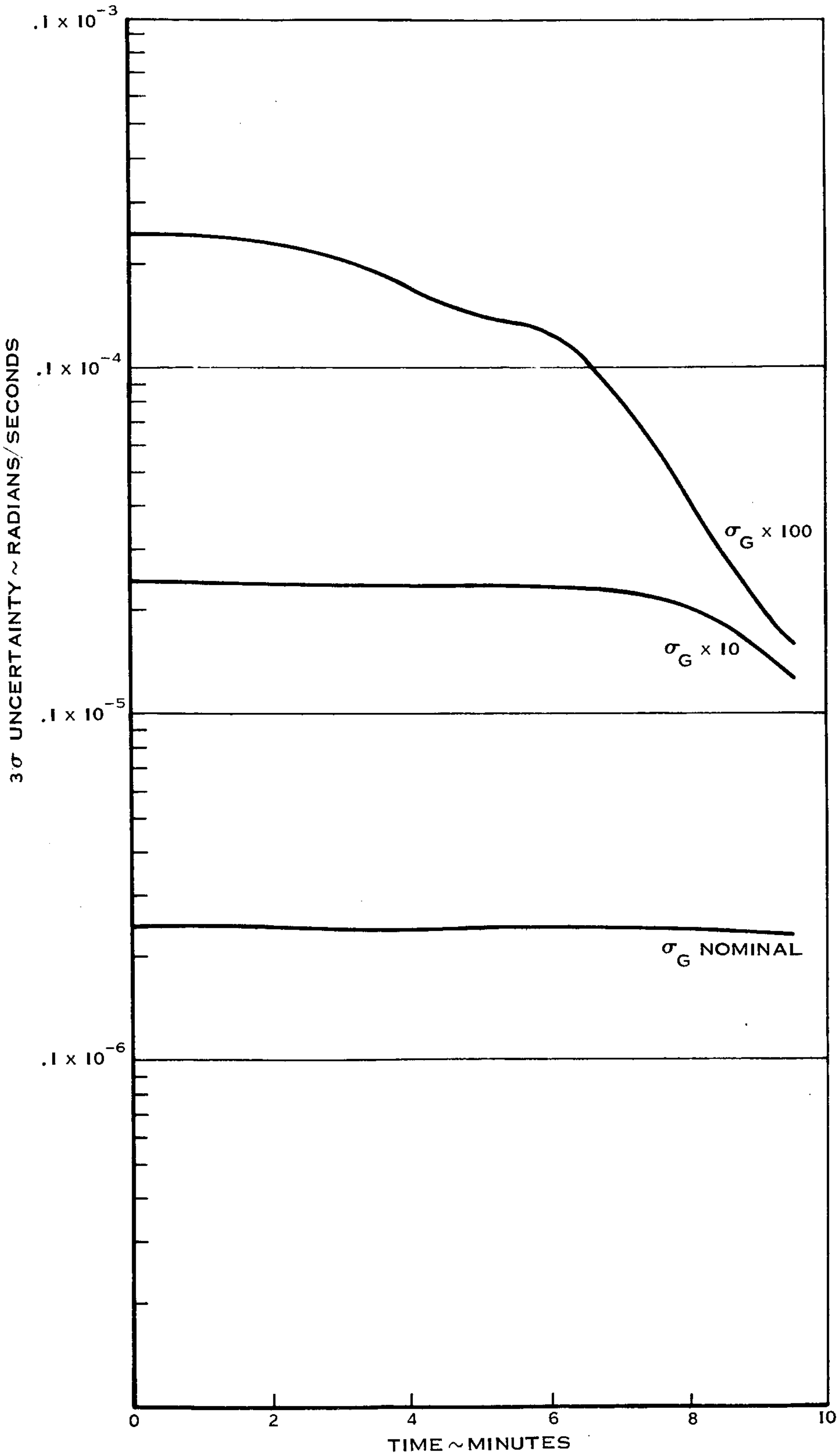


Figure 6-25 3σ Uncertainty in X-Axis Gyro Drift Rate for High Initial Uncertainties

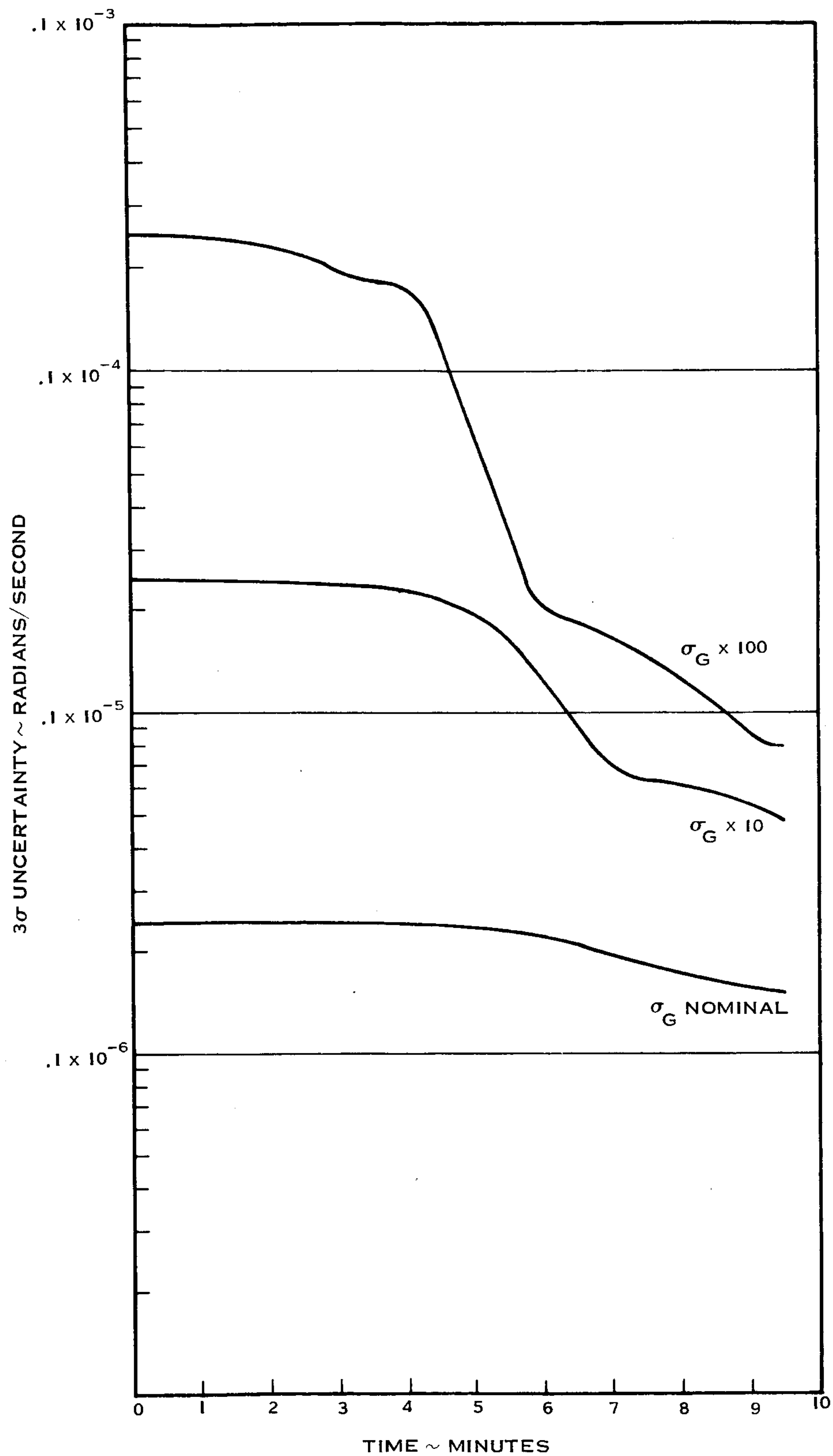


Figure 6-26 3σ Uncertainty in Y-Axis Gyro Drift Rate for High Initial Uncertainties

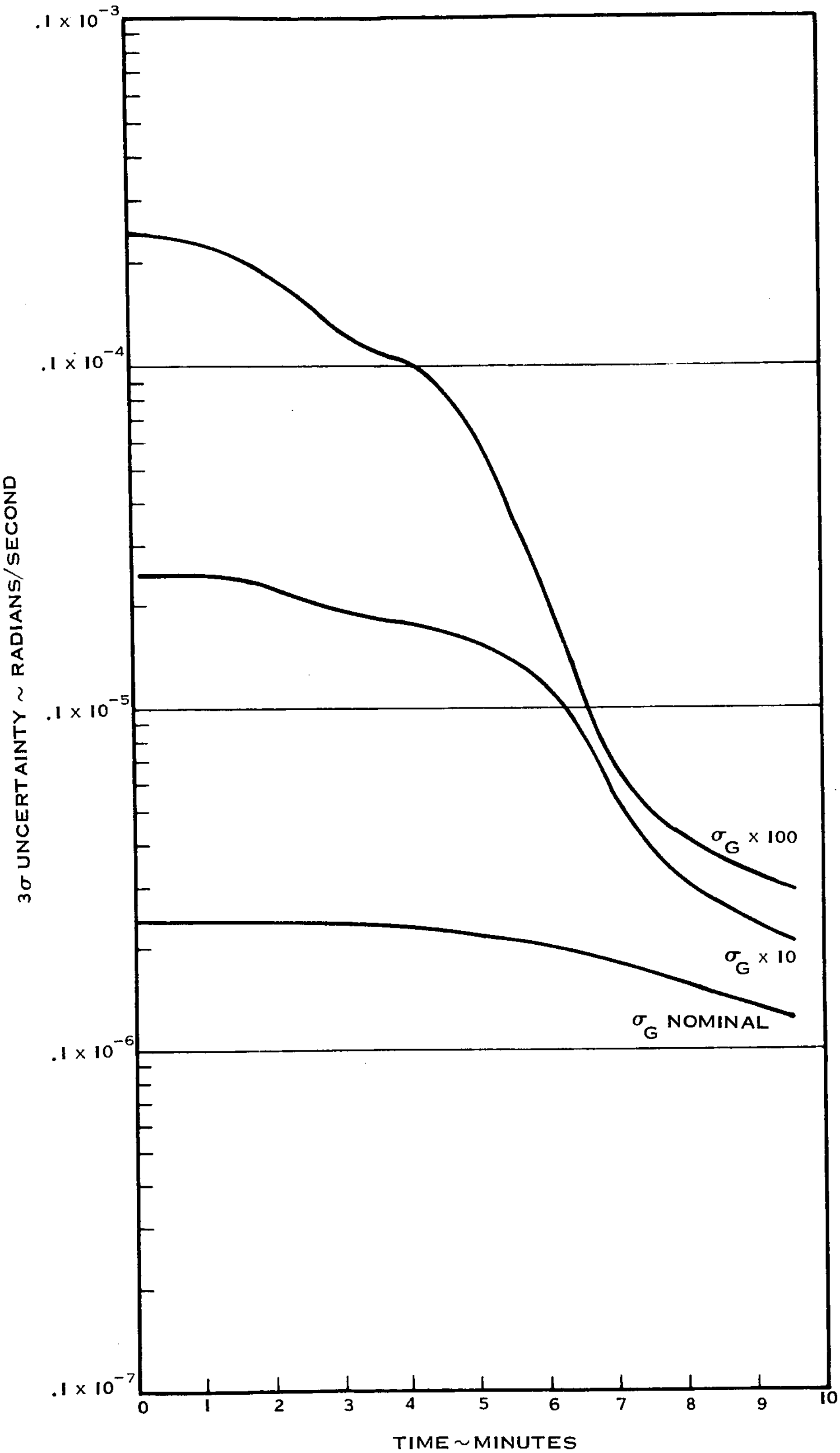


Figure 6-27 3σ Uncertainty in Z-Axis Gyro Drift Rate for High Initial Uncertainties

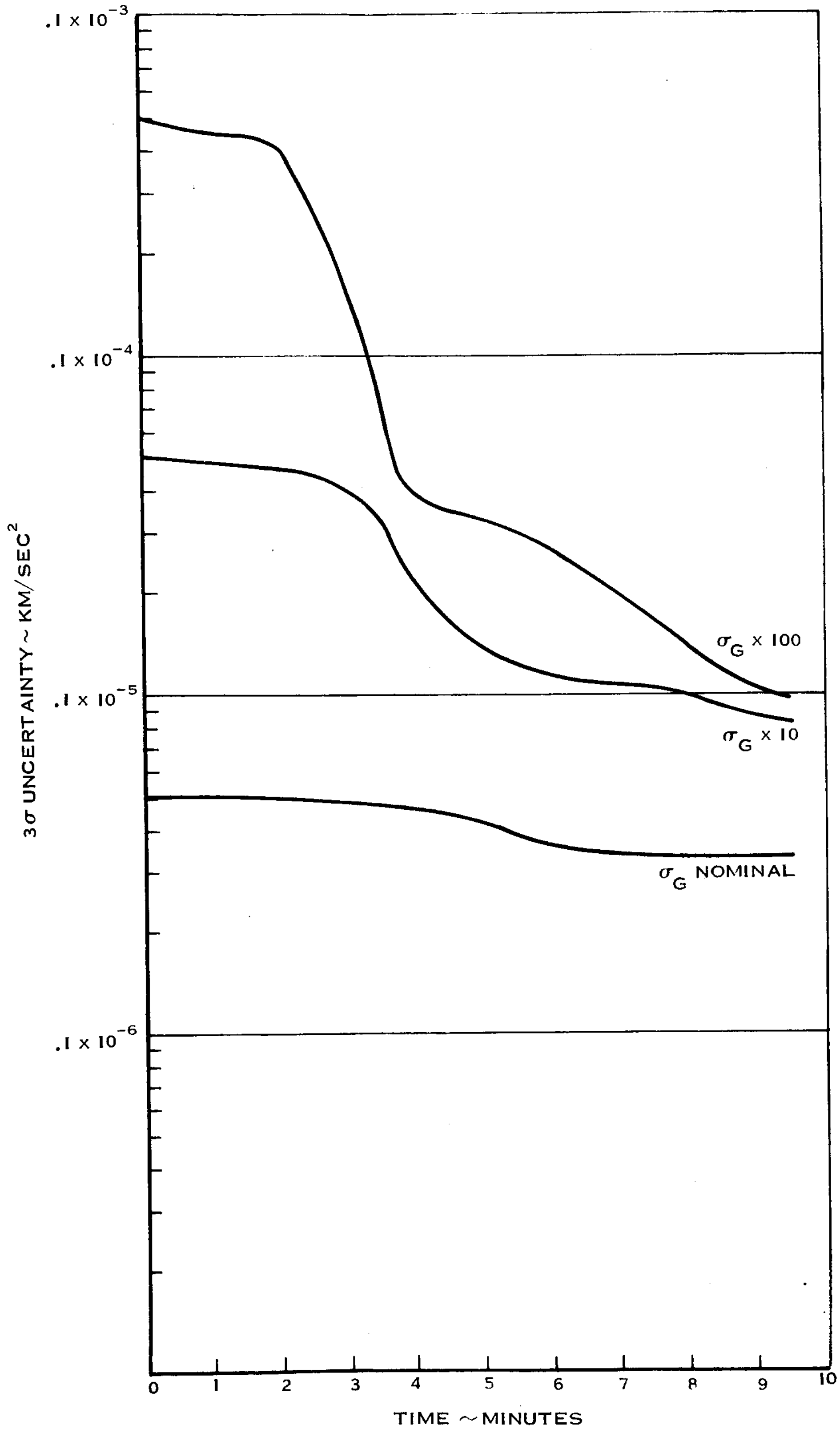


Figure 6-28 3σ Uncertainty in X-Accelerometer Bias for High Initial Uncertainties

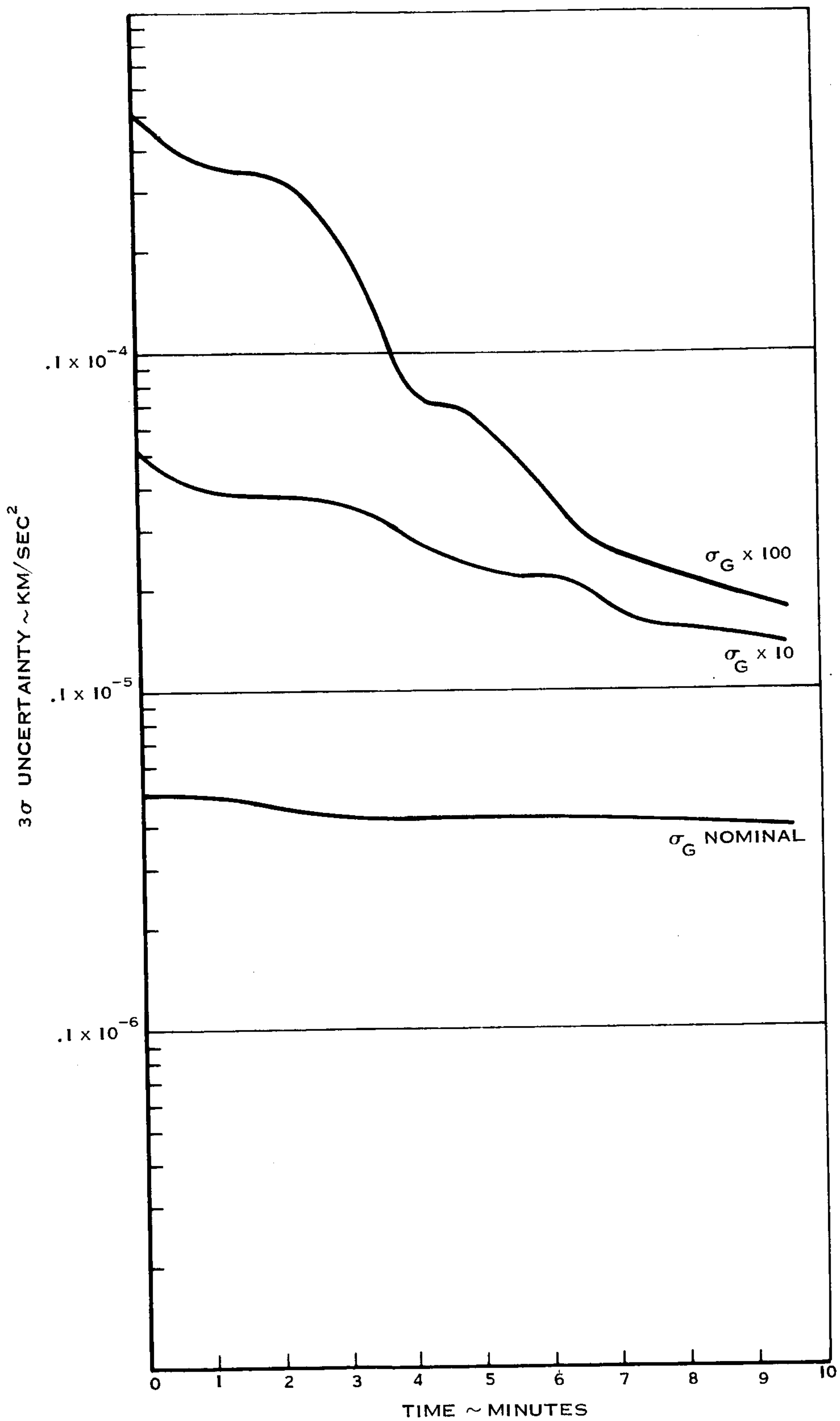


Figure 6-29 3σ Uncertainty in Y-Accelerometer Bias for High Initial Uncertainties

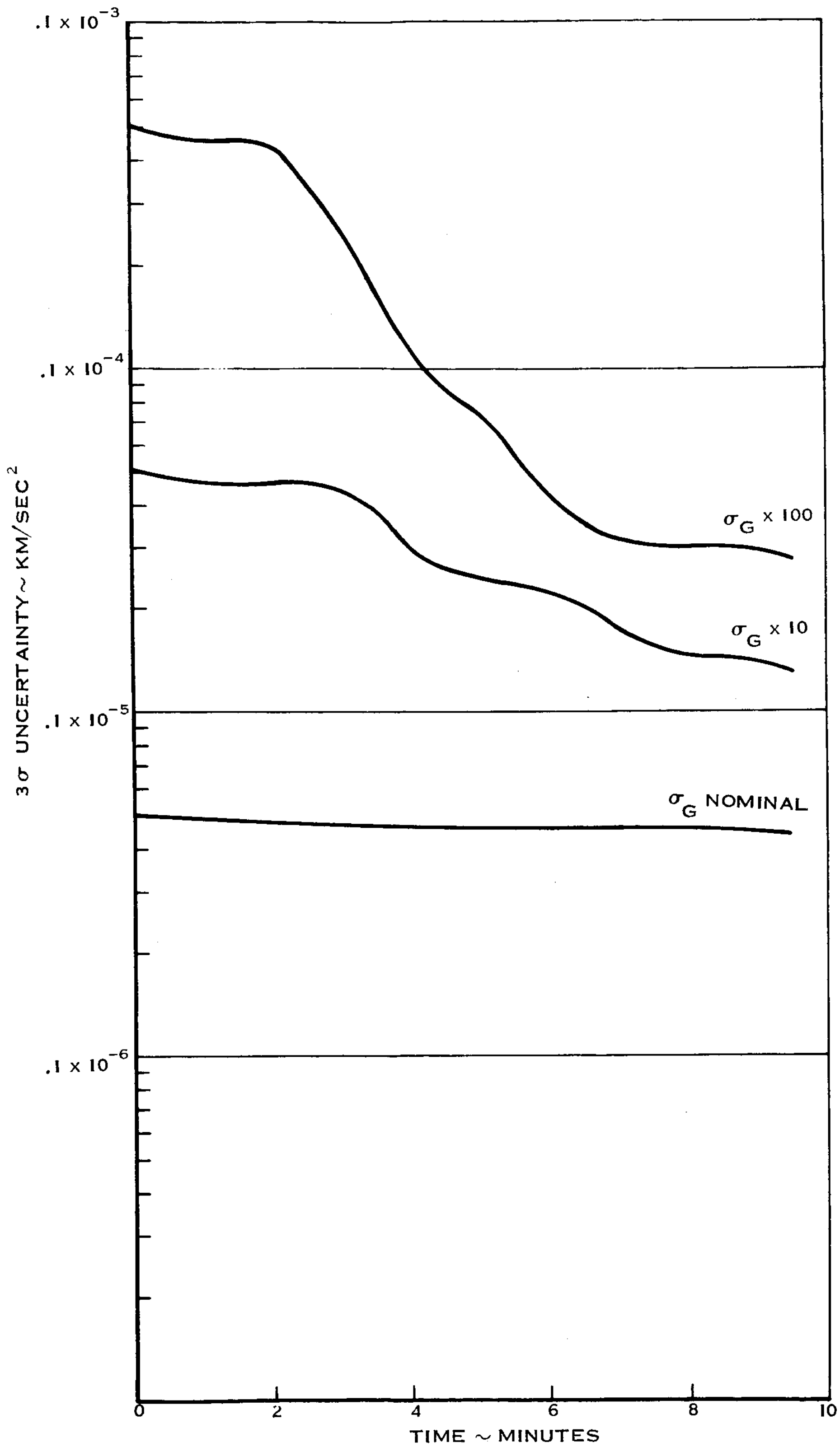


Figure 6-30 3σ Uncertainty in Z-Accelerometer Bias for High Initial Uncertainties

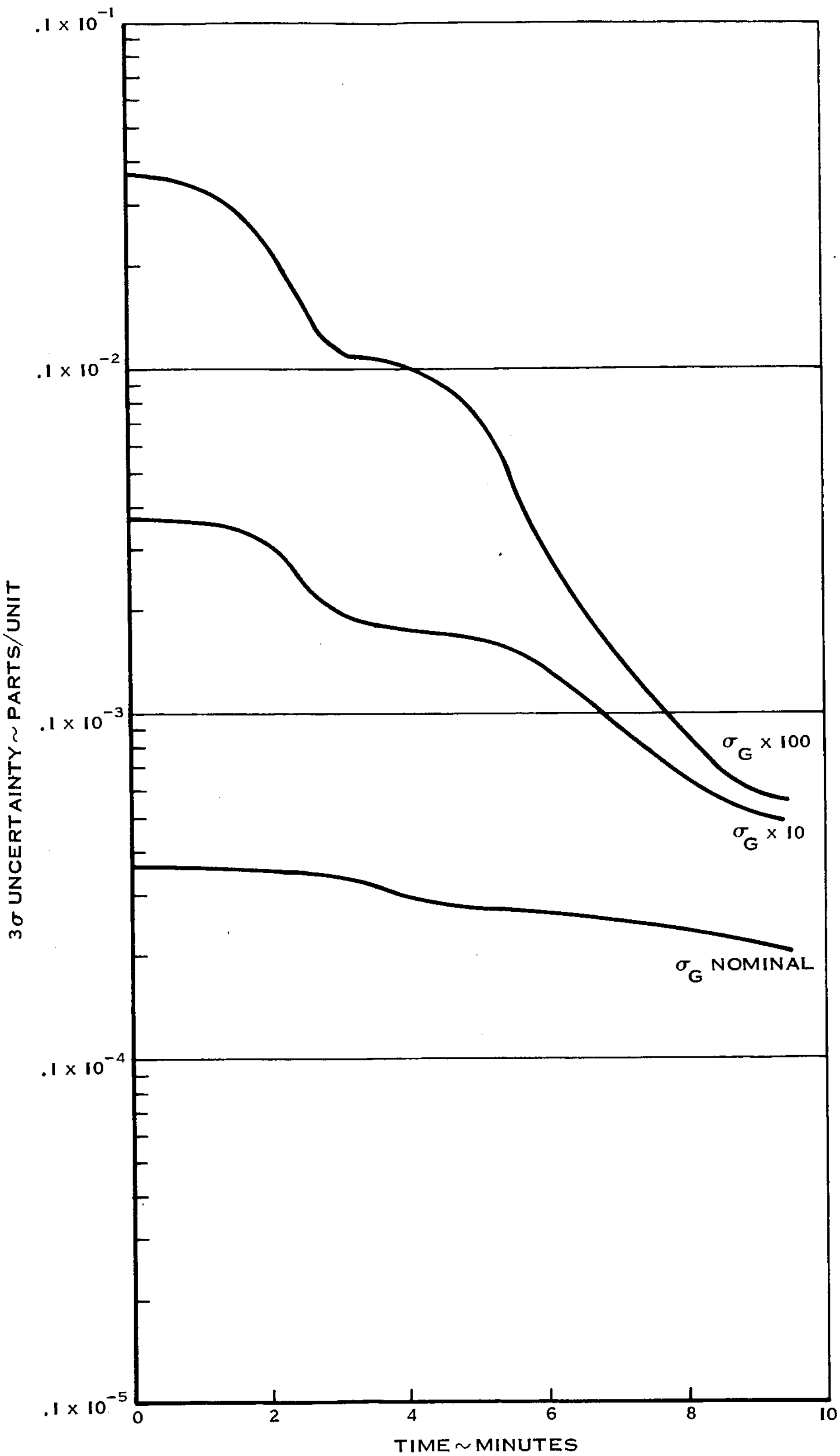


Figure 6-31 3σ Uncertainty in X-Accelerometer Scale Factor for High Initial Uncertainties

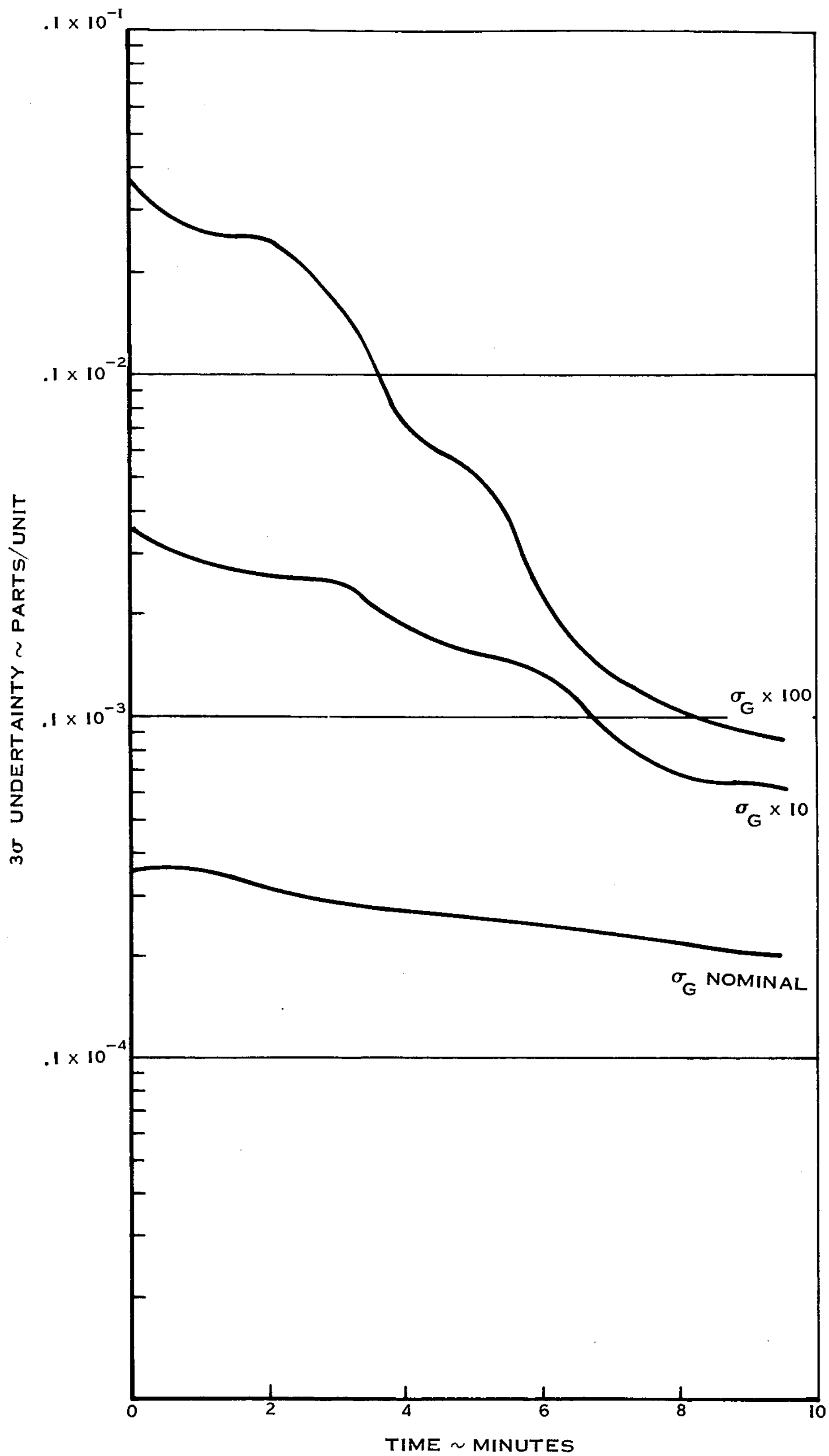


Figure 6-32 3σ Uncertainty in Y-Accelerometer Scale Factor for High Initial Uncertainties

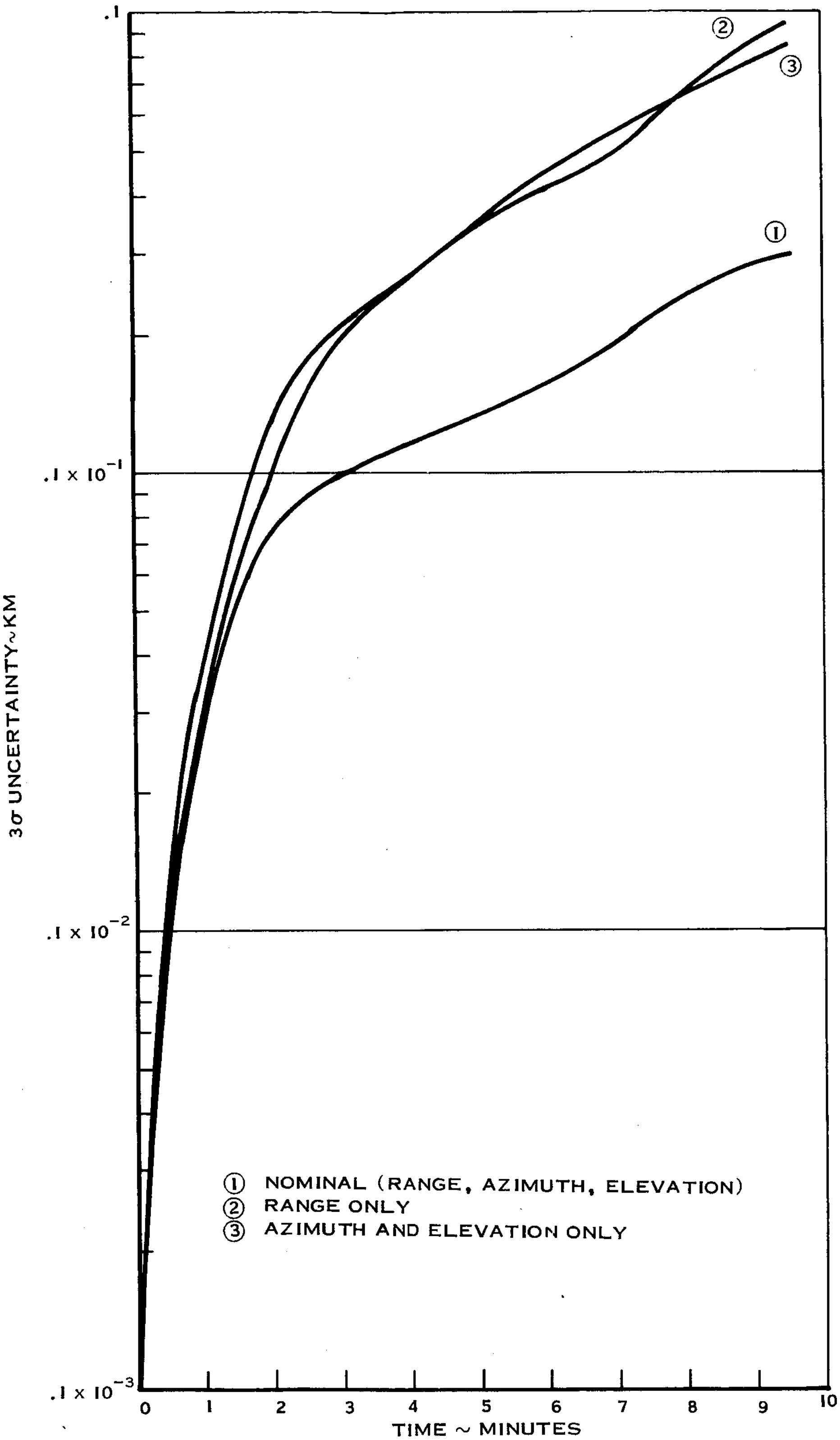


Figure 6-33 Variation in Position Uncertainty for Different Kinds of Tracking

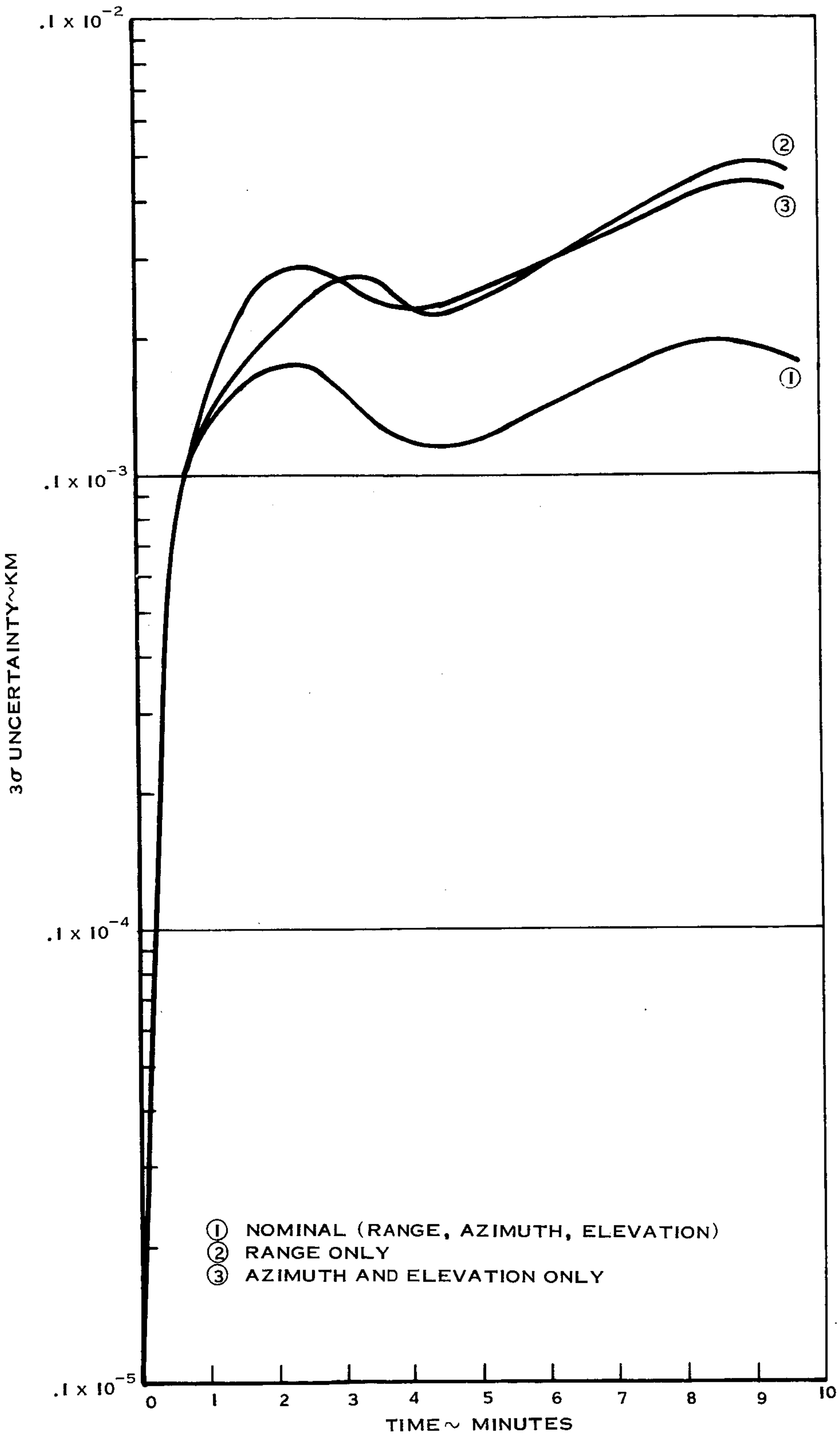


Figure 6-34 Variation in Velocity Uncertainty For Different Kinds of Tracking

SECTION 7
CONCLUSIONS

The over-all concept of combining telemetry and tracking data to estimate either the trajectory or error parameters that affect the trajectory has been found to be effective. However, the specific numerical results that can be obtained are highly dependent on the accuracy of the guidance system relative to that of the tracking system. As a result, the concept is, at the present time, more useful for some of the applications that were considered than it is for others.

The preliminary results presented in this report are mainly concerned with the importance of individual error sources in both the guidance and tracking systems. The effect of the guidance errors alone on the trajectory uncertainty is about 0.5 km in position and 2 meters/second in velocity. The most important error sources are the platform misalignments, the gyro drifts, the accelerometer biases, and some of the accelerometer misalignments and scale-factor errors. Some of the error sources, such as the Z accelerometer scale factor and the X and Y accelerometer misalignments in the Z direction, do not have a significant effect on the trajectory due to the low acceleration levels in the cross range direction. Other error sources, such as most of the mass unbalances, anisoelastic drifts, and accelerometer thresholds, do not cause any noticeable deviations in the trajectory. The final guidance model included twenty significant error sources.

The significant errors in the C-band radars included random and measurement bias errors in range, azimuth and elevation, as well as the station location errors. It was found that some of the station location errors and some of the azimuth and elevation biases could be omitted from the model. In addition to the random tracking errors, thirty-four bias errors were included in the final model.

The principal results of the report show the feasibility of estimating the platform error sources, both for the purpose of updating the guidance system during a flight and for a post-flight analysis. Although the primary interest for this investigation was in estimating the platform errors, results are also presented for estimating the trajectory.

With the combined platform-tracking model, the vehicle could be estimated to within 30 meters and 0.06 meters/second. This compared favorably to the 60 meters and 0.2 meters/second uncertainty with tracking only, i.e., no telemetry data.

For the tracking model used in this study, it may be concluded that the Saturn V type inertial platform errors are so small that it is not possible to significantly reduce the uncertainties in these error sources during a powered flight. The uncertainties in these error sources have been found to be reduced by about 20 to 50 percent. With an order of magnitude improvement in the present tracking accuracies, the significant guidance errors could be updated, i.e., these uncertainties could be reduced by about an order of magnitude. The general conclusion, therefore, is that the extent to which the guidance error uncertainties can be reduced does depend on the relative accuracies of the guidance components and the tracking system.

A notable exception to the above conclusions is that the ability to estimate the platform misalignments (orthogonal rotations), depends on a good initial calibration of the accelerometer misalignments. It was found that there is not enough information obtained from the telemetry-tracking system to distinguish between platform misalignments and accelerometer misalignments. However, if the Z accelerometer misalignments are eliminated, as well as either the X accelerometer misalignment in the Y direction, or the Y accelerometer misalignment in the X direction, then the three initial platform misalignments can be tied down.

The most effective use of the telemetry-tracking system is perhaps for a post-flight analysis. For this type of analysis, it would be possible to distinguish error sources that caused a malfunction, or more than normal error, from those components whose errors were less than the 3σ value. This result has been obtained by assuming large initial values for the guidance error standard deviations, and noting that there is a significant decrease in the uncertainty of these errors along the trajectory.

Additional results have been presented which show that:

(1) a perfect knowledge of the trajectory end point has little effect on reducing the guidance error uncertainties,

(2) the effectiveness of range tracking alone is about the same as the effectiveness of azimuth and elevation angle measurements, and

(3) the effect of increasing the tracking rate by a factor of 10 decreases the guidance error uncertainties by, at most, a factor of 2.

APPENDIX A

COVARIANCE MATRIX DESCRIPTION

A complete covariance matrix as defined in (2-26) is presented in this section. It is taken from the Powered Flight Error Propagation Program (Reference 2) and lists values from the nominal tracking-guidance model at the end of an ascent trajectory. The off-diagonal numbers are normalized in UP, DR, CR coordinates with standard deviations on the diagonal, where

$$\text{Standard Deviation} = \sqrt{P_{ij}} \qquad i=j$$

$$\text{Normalized Correlations} = \frac{P_{ij}}{\sqrt{P_{ii}\,P_{jj}}} \qquad i\neq j$$

and P_{ij} is computed by equations (2-19) and (2-22).

The output format can be broken into three sections.

1. Time of printout. In this case time is that from launch until the end of the ascent trajectory

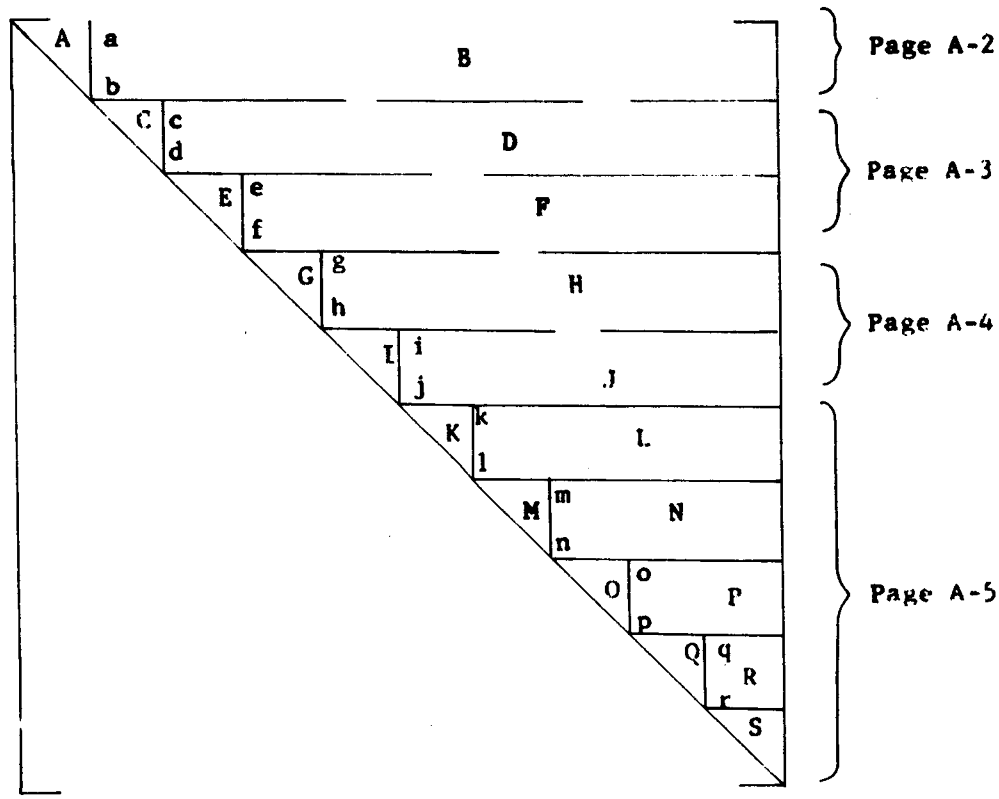
2. RMS position and velocity, where

$$RMSP = \sqrt{P_{11} + P_{22} + P_{33}}$$

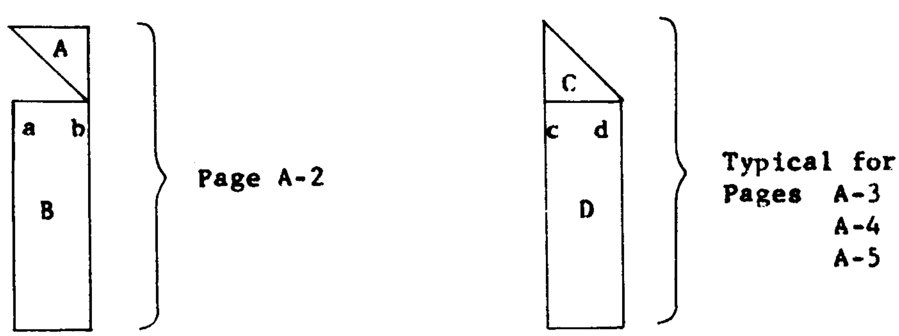
$$RMSV = \sqrt{P_{44} + P_{55} + P_{66}}$$

and standard deviations of bias errors being estimated.

3. Covariance Matrix. Since the covariance matrix is symmetrical only half of it is outputed. The format is as follows:



The format for the covariance matrix as defined above has been divided into pages as follows:



To identify the standard deviation of specific error sources (diagonal terms) or the correlation between the bias error sources from the numbers printed out on the left and top of each page, the following code is used.

Guidance Errors ~ Numbers 1 through 30 .
Only 20 errors were included in the model.
The specific guidance errors defined by these numbers are shown in page 3-3 and Table 5-1.

Tracking Errors ~ The first digit denotes the station. The second and third digits specify the type of tracking error.

Stations:

1	Cape Kennedy	FPS-16
2	Merit Island	FPS-16
3	Patrick A.F.B.	FPS-16
4	G. Bahama	FPS-16
5	G. Bahama	TPQ-18
6	Bermuda	FPS-16
7	Bermuda	FPQ-16
8	G. Turk	FPS-16
9	Antigua	FPS-16

Errors:

01 Range ~ meters
02 Range rate ~ meters/sec
03 Azimuth ~ millirad
04 Elevation ~ millirad
05 Azimuth rate ~ millirad/sec
06 Elevation rate ~ millirad/sec
07 Station latitude ~ degrees
08 Station longitude ~ degrees
09 Station altitude ~ meters
10 Station clock ~ seconds

The following examples illustrate the use of this numbering system:

1. 308 ~ Patrick AFB ~ longitude
   10 ~ Accelerometer bias X-axis

   The number in column 10 and row 308 (page A-3) is the correlation between Patrick AFB longitude error and accelerometer bias error in the X axis.

2. The value in the DR column and row 5 (page A-2) specifies the correlation between the Y-accelerometer scale factor error and the uncertainty in the downrange coordinate.

3. The number in column 3 and row 22 (page A-3) gives the correlation between Z accelerometer error into X-axis and X-gyro input axis mass unbalance.

4. The item at column 11 and row 11 is the standard deviation of accelerometer bias error on the Y-axis.

END CONDITIONS

0 DAY 0 HRS 9 MIN 25.985 SEC

CASE 3 REC. 10 EVENT 8

CURRENT RMS VALUES

RMSP= 0.2856899E-01 RMSV= 0.1747118E-03 1= 0.2033137E-04 2= 0.5203584E-04 3= 0.6876384E-04 4= 0.4735066E-04

5= 0.1983611E-04	6= 0.5794824E-04	10= 0.3340303E-06	11= 0.4181578E-06	12= 0.4485762E-06	15= 0.1999510E-07
16= 0.5936066E-04	17= 0.8694120E-04	18= 0.4917875E-04	19= 0.2294555E-06	20= 0.1577291E-06	21= 0.1315124E-06
22= 0.1482978E-04	23= 0.1482969E-04	27= 0.3701221E-04	30= 0.2006918E-02	101= 0.1293153E-02	104= 0.2660293E-04
201= 0.1290303E-02	203= 0.2203202E-04	204= 0.2064433E-04	301= 0.4830693E-02	303= 0.2261715E-04	304= 0.2158076E-04
308= 0.4959798E-02	401= 0.3155088E-02	407= 0.4882960E-02	408= 0.2713148E-02	501= 0.3007574E-02	507= 0.4666228E-02
508= 0.2639081E-02	601= 0.1197393E-01	607= 0.2311298E-01	608= 0.1061544E-01	609= 0.2156260E-01	701= 0.1239769E-01
704= 0.4026404E-04	707= 0.2460006E-01	708= 0.1059291E-01	709= 0.2821349E-01	801= 0.1204296E-01	804= 0.3909360E-04
807= 0.2049557E-01	808= 0.9650091E-02	809= 0.2542227E-01	901= 0.3463467E-01	904= 0.4252512E-04	907= 0.3890744E-01
908= 0.2497937E-01	909= 0.4055488E-01				

P MATRIX. STD DEV. ON DIAGONAL

	UP	DR	CR	UPD	DRD	CRD
UP	0.20048940E-01	-0.72755066E 00	-0.13398790E 00	0.82198008E 00	0.62302793E-01	-0.18234316E 00
DR		0.33592640E-02	-0.34594461E 00	-0.53763217E 00	0.46907973E 00	-0.25564921E 00
CR			0.20073430E-01	-0.16917249E 00	-0.73116932E 00	0.90566488E 00
UPD				0.11693026E-03	-0.81705669E-01	-0.24253006E 00
DRD					0.29480641E-04	-0.62105574E 00
CRD						0.12642160E-03
1	0.91231061E-01	-0.32064452E-01	-0.14935736E 00	0.14909848E 00	-0.25194991E-01	-0.15599953E 00
2	-0.31743747E-01	-0.10336913E 00	0.24144885E-01	-0.87517311E-01	-0.14850257E 00	0.86882852E-01
3	0.63290792E-02	-0.67216609E-02	-0.38867944E-03	-0.87547291E-02	0.24423477E-02	-0.11494868E-01
4	-0.33068204E-01	-0.52873698E-01	0.17000173E-01	-0.24118848E-01	-0.14103926E 00	0.15750782E-01
5	0.36261865E-01	0.18392624E 00	0.20936755E-01	0.11761894E-01	0.31634574E 00	-0.12132836E-01
6	0.92594163E-02	0.13775016E-02	-0.61100669E-02	0.52652875E-02	0.27914936E-01	0.23413349E-01
10	0.57764896E-01	0.54461265E-01	-0.19381994E-01	-0.21791579E-01	0.27908096E 00	0.22833622E-01
11	0.26094338E-01	0.25884537E-01	-0.21347065E-01	0.17192540E-01	0.47877736E-01	-0.16825024E-01
12	0.50592117E-01	-0.90050353E-01	0.50763714E-01	0.92941022E-01	-0.13063518E 00	0.10225735E-01
15	0.13347085E-02	-0.31883261E-02	0.14651212E-02	0.10588128E-02	-0.42325672E-02	-0.19595403E-02
16	0.12598020E-01	0.18951957E-02	-0.83067231E-02	0.71805618E-02	0.37998983E-01	0.31820817E-01
17	-0.27896034E-01	0.29614850E-01	0.17515309E-02	0.38540602E-01	-0.10968542E-01	0.50698912E-01
18	-0.24368947E-02	-0.81457169E-01	0.12789325E-01	-0.96662469E-01	-0.29722160E-01	0.10695265E 00
19	-0.38469599E-01	0.30290373E-01	-0.31353784E-01	-0.17247277E 00	0.10118328E 00	-0.11564036E 00
20	-0.91257966E-01	0.88611831E-01	-0.12528340E 00	-0.26824580E 00	0.14521143E 00	-0.23311961E 00
21	0.23858515E 00	0.14113449E 00	-0.36250129E 00	0.49898253E 00	0.29741040E 00	-0.60257231E 00
22	-0.71957074E-03	0.13321728E-02	-0.14491146E-02	-0.52062195E-03	0.10105911E-02	-0.34278661E-03
23	0.72451828E-03	-0.13734814E-02	0.14921265E-02	0.45310488E-03	-0.10186034E-02	0.28433008E-03
27	0.46699094E-02	0.11044154E-01	-0.22162350E-03	0.28959971E-02	-0.29899794E-02	-0.11657893E-02
30	0.80615882E-02	0.15524325E-01	-0.17468233E-02	0.94192744E-02	-0.50578968E-02	-0.77576293E-02
101	0.28689031E-01	0.22127054E 00	0.72170274E-01	0.16218127E-01	0.24059656E-01	-0.12559606E-01
104	0.13846909E 00	-0.14907918E-01	-0.15431532E 00	0.38457969E-01	0.20710264E 00	-0.78573697E-01
201	-0.55262423E-02	0.25125632E 00	0.67472512E-01	0.39227199E-02	0.16128408E-01	-0.19274487E-01
203	0.25918534E 00	-0.26406754E 00	0.10222542E 00	0.11232743E 00	-0.21271355E-01	0.77499805E-01
204	0.17827739E 00	-0.21501845E-01	-0.19654651E 00	0.50911494E-01	0.26204798E 00	-0.10110217E 00
301	0.20926240E-03	0.20629991E-01	0.67575242E-01	-0.16787910E-01	-0.13195222E-01	0.23985856E-01
303	0.25616890E 00	-0.26993509E 00	0.11732608E 00	0.11150236E 00	-0.30200236E-01	0.80913138E-01
304	0.17112139E 00	-0.55680295E-02	-0.20572191E 00	0.52749346E-01	0.26140123E 00	-0.10580005E 00
308	0.25327264E-01	-0.73312926E-01	0.62668495E-01	-0.78863024E-02	-0.22153402E-01	0.37946350E-01
401	-0.16629288E 00	0.31832735E 00	-0.19364304E 00	-0.15716302E 00	0.20022125E 00	-0.17315564E 00
407	0.46470044E-01	-0.39728430E-02	-0.18811714E-01	-0.41034572E-01	0.67213189E-01	-0.88960845E-01
408	0.30281117E 00	-0.44903222E 00	0.21892686E 00	0.98585642E-01	-0.11390346E 00	0.11886989E 00
501	-0.14338370E 00	0.31505682E 00	-0.21089155E 00	-0.15546971E 00	0.23086473E 00	-0.17118684E 00
507	0.80335412E-01	-0.24762694E-01	-0.25274959E-01	-0.33080983E-01	0.89228955E-01	-0.81926209E-01
508	0.33263674E 00	-0.47195243E 00	0.21511480E 00	0.10733914E 00	-0.98163224E-01	0.12834586E 00
601	0.11023329E 00	-0.33417249E-01	0.12334604E 00	0.38617338E 00	-0.17143103E 00	0.12984199E 00
607	0.55316677E 00	-0.51090577E 00	0.48640841E-01	0.28678795E 00	-0.15375272E-01	-0.25698951E-01
608	-0.35311820E 00	0.26709825E 00	-0.20606870E 00	-0.67136344E 00	0.36232410E 00	-0.86781039E-01
609	0.32326900E 00	-0.12389681E 00	-0.34525391E 00	0.11998919E 00	0.32895731E 00	-0.27896765E 00
701	0.19886422E 00	-0.10378596E 00	0.99989117E-01	0.43647229E 00	-0.15323203E 00	0.99989112E-01
704	-0.20387676E 00	0.17017608E 00	0.13737527E-01	-0.14106087E 00	-0.99071576E-02	0.23999827E-01
707	0.35481957E 00	-0.35320872E 00	0.77683095E-01	0.14836934E 00	-0.31712322E-01	0.19642737E-01
708	-0.37378172E 00	0.28361762E 00	-0.20162018E 00	-0.68267053E 00	0.35822675E 00	-0.81036389E-01
709	0.40905381E 00	-0.21908864E 00	-0.29418463E 00	0.21351676E 00	0.26649199E 00	-0.25533588E 00
801	-0.15565760E 00	0.23062857E 00	-0.17340777E 00	-0.29151884E 00	0.28601842E 00	-0.27050570E 00
804	0.15335818E 00	-0.92463849E-01	-0.68457761E-01	0.94418038E-01	0.53651639E-01	-0.81068354E-01
807	0.11852083E 00	-0.14842042E 00	0.12681832E 00	-0.71113870E-01	0.23954914E-01	-0.19299357E-02
808	0.49932774E 00	-0.68294381E 00	0.54866196E 00	0.32375256E 00	-0.39142656E 00	0.50504562E 00
809	0.93645363E-03	0.51056808E-01	-0.97273146E-01	0.23903791E-01	0.82902117E-01	-0.43833013E-02
901	-0.69598460E-01	0.10285743E 00	-0.93541984E-01	-0.12727913E 00	0.12941284E 00	-0.14161520E 00
904	0.12663237E 00	-0.89335711E-01	-0.93658289E-02	0.83975865E-01	0.21204894E-01	0.99069613E-02
907	0.14515272E 00	-0.25202802E 00	0.30908509E 00	0.29923384E-01	-0.18182857E 00	0.21492740E 00
908	0.32385404E 00	-0.49890124E 00	0.52864938E 00	0.28522425E 00	-0.42133666E 00	0.57661249E 00
909	0.15861816E-01	0.67281840E-01	-0.19059390E 00	0.39849909E-01	0.12185946E 00	-0.16244000E 00

DETERMINISTIC ERRORS

	1	2	3	4	5	6
1	0.20331372E-04					
2	0.44148847E-01	0.52035842E-04				
3	-0.21672424E-02	0.16681603E-02	0.68763840E-04			
4	0.10762037E 00	-0.67063317E 00	0.19109780E-02	0.47350657E-04		
5	-0.13041863E 00	-0.12311295E 00	-0.30164280E-02	-0.66478305E-01	0.19836109E-04	
6	-0.96963818E-03	-0.66027091E-02	-0.15713352E-01	-0.10337043E-01	0.38623158E-01	0.57948236E-04
10	-0.95026130E 00	-0.10299002E 00	-0.60572501E-03	-0.15667539E 00	0.24797986E 00	0.16148224E-01
11	0.87032021E-01	-0.31933244E 00	-0.10032831E-02	-0.20291715E 00	-0.53683719E 00	-0.23728974E-02
12	0.44975948E-02	0.13476016E-01	-0.50643934E-01	0.19649995E-01	-0.95576903E-01	-0.99763983E-01
15	0.12169724E-04	0.82176804E-03	-0.26526007E-02	0.11129258E-02	-0.50082944E-02	0.55729675E-02
16	-0.13492103E-02	-0.89851721E-02	-0.21372384E-01	-0.14061016E-01	0.52218121E-01	-0.85375673E 00
17	0.95989052E-02	-0.72818338E-02	0.68739876E 00	-0.81629292E-02	0.13524723E-01	0.69274211E-01
18	-0.79283624E-01	-0.60202479E 00	-0.10430522E-03	0.94018696E 00	-0.92313653E-01	0.41330006E-02
19	-0.11740905E-01	0.93877175E-02	-0.50734961E-01	0.69926976E-02	-0.37466879E-01	-0.30939276E-01
20	-0.29694742E-01	0.22683673E-01	0.12030392E 00	0.20963862E-01	-0.51734319E-01	-0.15914860E 00
21	0.83286678E-01	-0.32012010E 00	0.13149878E-02	-0.11563309E 00	0.47448581E 00	-0.10916508E-01
22	-0.34068913E-03	-0.39174873E-04	-0.16032746E-02	-0.12416677E-03	0.13436067E-02	-0.28191473E-02
23	0.34712108E-03	0.49687926E-04	0.16481895E-02	0.13913808E-03	-0.14447516E-02	0.30429261E-02
27	0.20384581E-02	-0.31461415E-02	0.67636988E-03	0.11866794E-01	-0.36717330E-01	0.20528529E-03
30	-0.73206585E-03	0.69225515E-04	0.11821526E-02	0.14978350E-01	-0.65387649E-01	0.32633687E-03
101	-0.22249033E 00	-0.20591723E 00	0.33146577E-02	-0.10020725E 00	0.36426173E 00	0.13368372E-02
104	0.22987319E-01	-0.28987565E-01	-0.26181834E-03	-0.42405804E-01	0.34481358E-01	0.16624886E-01
201	-0.22226564E 00	-0.20479004E 00	0.54698228E-02	-0.96533433E-01	0.35678575E 00	-0.41333025E-02
203	-0.15111883E-01	-0.50688263E-02	-0.17413181E-01	-0.16343407E-01	0.53629482E-01	0.38359239E-01
204	0.27445049E-01	-0.35125177E-01	-0.77795957E-03	-0.52773969E-01	0.44800223E-01	0.21651714E-01
301	-0.43600721E-01	-0.92098136E-01	0.11972042E-01	-0.44855956E-01	0.14132886E 00	-0.83706097E-02
303	-0.26810271E-01	-0.18836214E-01	-0.15220766E-01	-0.22542309E-01	0.72003838E-01	0.36186678E-01
304	0.22507855E-01	-0.23942212E-01	-0.47014391E-02	-0.46409800E-01	0.30615513E-01	0.24439164E-01
308	0.14042196E-01	-0.39472780E-01	0.91616673E-02	-0.21152466E-01	0.53115409E-01	-0.38878481E-02
401	0.19394848E-01	-0.22932307E-01	0.79600154E-02	-0.11085727E-01	-0.12731354E-01	0.15597419E-01
407	-0.13817852E-01	-0.53211963E-02	-0.17776586E-01	-0.48191764E-03	-0.13551067E-01	0.62512431E-01
408	0.10086498E 00	0.52032230E-01	0.21462746E-01	0.23046252E-01	-0.10911387E 00	0.58641850E-02
501	0.19298299E-01	-0.24923485E-01	0.71098447E-02	-0.17728613E-01	0.37037128E-02	0.18538662E-01
507	-0.15543900E-01	-0.66759310E-02	-0.19706314E-01	-0.64996839E-02	0.28856702E-02	0.67486029E-01
508	0.10301203E 00	0.53365273E-01	0.20898052E-01	0.19075786E-01	-0.99880809E-01	0.78095509E-02
601	0.92144925E-03	-0.53033293E-01	-0.19724887E-01	-0.13359453E-01	0.10018950E 00	0.36376788E-01
607	0.18629192E-01	0.78704526E-02	0.14670915E-01	-0.97135404E-02	-0.15478560E-01	-0.16184014E-01
608	-0.33237319E-01	0.81047426E-01	0.78557580E-02	-0.12839320E-02	-0.71154070E-01	0.48610171E-02
609	0.64089153E-01	0.11855021E-01	0.85632314E-02	-0.22591878E-01	-0.18383618E-01	0.25335854E-02
701	0.17274407E-01	-0.50335536E-01	-0.16467096E-01	-0.14052588E-01	0.87593473E-01	0.31333366E-01
704	-0.38235364E-01	-0.14447713E-02	-0.55615064E-02	0.28469179E-02	0.15969740E-01	0.69593388E-02
707	-0.11845287E-01	0.71669419E-02	0.96540597E-02	-0.70689924E-02	-0.48716352E-03	-0.92348987E-02
708	-0.36877095E-01	0.80372820E-01	0.70835656E-02	-0.10305501E-02	-0.68258323E-01	0.60473141E-02
709	0.77938135E-01	0.87766815E-02	0.10419647E-01	-0.19299319E-01	-0.26940043E-01	-0.35892440E-02
801	-0.25808310E-01	-0.21891185E-01	0.20353150E-01	-0.72779093E-02	0.89212431E-01	-0.45365115E-01
804	0.12462991E-01	-0.87349656E-02	0.57528248E-02	-0.76522044E-02	0.10042663E-01	0.27857865E-02
807	-0.42190103E-01	-0.15265569E-01	0.13740321E-01	-0.75596971E-02	0.87887845E-01	-0.26824635E-01
808	-0.39620473E-01	0.33350176E-01	0.97347648E-02	-0.34188232E-02	-0.51249518E-02	-0.18692710E-01
809	0.14106110E-01	0.13466442E-01	-0.10785928E-01	-0.79546667E-02	-0.13754833E-01	0.15529323E-01
901	-0.22415594E-01	-0.70364753E-02	0.94676890E-02	-0.72777609E-03	0.33290759E-01	-0.27636508E-01
904	0.30789238E-02	0.16907083E-02	0.72112309E-03	-0.83116903E-02	0.39364459E-02	0.30347650E-02
907	-0.56311807E-01	-0.35852308E-02	0.97478011E-02	-0.20048290E-02	0.44645865E-01	-0.23049316E-01
908	-0.11290808E-01	0.27081520E-01	-0.68646174E-02	-0.67152807E-02	-0.31271364E-01	0.25688287E-01
909	0.37814016E-01	-0.33414209E-02	-0.31461933E-02	-0.16458451E-02	-0.14342183E-01	0.11418185E-01

DETERMINISTIC ERRORS

	10	11	12	15	16	17
10	0.33403033E-06					
11	-0.67950388E-01	0.41815781E-06				
12	-0.33664738E-01	0.17203711E-01	0.44857615E-06			
15	-0.16920141E-02	0.82352698E-03	-0.85679367E-02	0.19995096E-07		
16	0.22003570E-01	-0.32688851E-02	-0.13570403E 00	0.75788262E-02	0.59360658E-04	
17	0.24976317E-02	0.44527947E-02	0.22321171E 00	0.11691035E-01	0.94222743E-01	0.86941200E-04
18	0.58346219E-01	-0.19854958E 00	-0.64933473E-02	-0.28149586E-03	0.56163375E-02	0.21538485E-03
19	-0.18390341E-02	0.11350349E-01	-0.52709797E-01	-0.35996815E-02	-0.42088108E-01	0.22362234E 00
20	-0.45536782E-02	0.14044464E-02	0.57132550E-01	-0.99522424E-03	-0.21646845E 00	-0.53027890E 00
21	0.43474130E-01	-0.60265867E-02	0.17320353E-01	0.76644037E-03	-0.14790007E-01	-0.56914284E-02
22	0.54532520E-03	-0.47574694E-03	0.25551476E-02	0.94462946E-04	-0.38340835E-02	0.70665993E-02
23	-0.57641705E-03	0.50615624E-03	-0.26947891E-02	-0.10275664E-03	0.41384236E-02	-0.72645692E-02
27	-0.62964420E-03	0.90487893E-03	-0.52201395E-02	-0.21247343E-03	0.27295406E-03	-0.29771687E-02
30	0.55102360E-03	0.46916245E-02	-0.92065607E-02	-0.37882873E-03	0.43264179E-03	-0.52008930E-02
101	0.20894830E 00	0.80832150E-01	0.23517546E-02	0.22479314E-03	0.18624721E-02	-0.14451822E-01
104	0.46989395E-01	0.84024936E-01	-0.35737423E-01	-0.17513347E-02	0.22600456E-01	0.11876276E-02
201	0.20262337E 00	0.76751014E-01	-0.58294376E-02	0.57522339E-04	-0.55780003E-02	-0.23956304E-01
203	0.39125550E-01	0.50320139E-02	0.87956867E-01	0.23453652E-02	0.52179781E-01	0.76784384E-01
204	0.61176684E-01	0.10226776E 00	-0.42993705E-01	-0.21494755E-02	0.29435756E-01	0.34717803E-02
301	0.38000748E-01	0.58315897E-01	-0.52450573E-01	-0.16296975E-02	-0.11370259E-01	-0.52739669E-01
303	0.48330395E-01	0.10588135E-01	0.79570760E-01	0.21055031E-02	0.49226953E-01	0.67128818E-01
304	0.64136127E-01	0.90075326E-01	-0.24561329E-01	-0.15641230E-02	0.33226117E-01	0.20768113E-01
308	-0.12812304E-01	0.39021031E-01	-0.43798367E-01	-0.14605073E-02	-0.52839776E-02	-0.40390373E-01
401	-0.79168828E-02	0.55695711E-01	-0.14856143E 00	-0.59991800E-02	0.21204689E-01	-0.35090139E-01
407	0.14007439E-01	0.96815411E-02	-0.26830624E-01	-0.27366383E-02	0.85018620E-01	0.78361614E-01
408	-0.96548979E-01	0.12349262E-01	-0.78810285E-01	-0.26615320E-02	0.79592090E-02	-0.94655698E-01
501	0.33254054E-02	0.62402488E-01	-0.14820190E 00	-0.60628616E-02	0.25206765E-01	-0.31333595E-01
507	0.25843069E-01	0.14040517E-01	-0.20640153E-01	-0.26355138E-02	0.91785111E-01	0.86876904E-01
508	-0.89935512E-01	0.15703361E-01	-0.75061010E-01	-0.25641468E-02	0.10606159E-01	-0.92160851E-01
601	0.27059263E-01	-0.73907318E-01	-0.78407586E-01	-0.63281370E-02	0.49509755E-01	0.86959038E-01
607	0.24400723E-01	0.41352144E-01	0.86562953E-01	0.47979602E-02	-0.22023524E-01	-0.64659777E-01
608	0.26910213E-01	0.70856011E-01	-0.12046895E-02	0.18943710E-02	0.65831287E-02	-0.34605911E-01
609	0.36559887E-01	0.62348023E-01	0.70689287E-02	0.99369301E-03	0.34312170E-02	-0.37753052E-01
701	0.22822527E-01	-0.62963787E-01	-0.63579197E-01	-0.54381441E-02	0.42646022E-01	0.72591177E-01
704	0.11068241E-01	-0.17006539E-01	-0.26927563E-01	-0.13732070E-02	0.94706889E-02	0.24533771E-01
707	0.28937076E-01	0.26025739E-01	0.61408105E-01	0.34491340E-02	-0.12566869E-01	-0.42533304E-01
708	0.27709330E-01	0.68129603E-01	-0.46750144E-02	0.16858986E-02	0.81976334E-02	-0.31200598E-01
709	0.21923295E-01	0.59423184E-01	0.24307496E-01	0.17433275E-02	-0.48969758E-02	-0.45947336E-01
801	0.34604023E-01	-0.40865392E-01	0.96774256E-01	0.70240033E-02	-0.61680857E-01	-0.89682031E-01
804	0.12156949E-01	0.15197273E-01	-0.11178334E-01	-0.36300281E-03	0.37888672E-02	-0.25344286E-01
807	0.36320993E-01	-0.36277352E-01	0.11382794E 00	0.65562900E-02	-0.36465327E-01	-0.60532378E-01
808	0.30456480E-01	-0.20390262E-02	0.75485395E-01	0.32262348E-02	-0.25424329E-01	-0.42885578E-01
809	0.19323525E-01	0.14125028E-01	-0.23428109E-01	-0.17604649E-02	0.21115703E-01	0.47529462E-01
901	0.24067353E-01	-0.24170110E-01	0.41239360E-01	0.35181582E-02	-0.37575966E-01	-0.41709360E-01
904	0.20376445E-01	0.12641403E-01	-0.18057255E-02	-0.21354142E-03	0.41259642E-02	-0.31715766E-02
907	0.24986804E-01	-0.20581882E-01	0.66241663E-01	0.38536300E-02	-0.31336493E-01	-0.42931279E-01
908	-0.55769101E-02	0.37467457E-01	-0.26557157E-01	-0.33089057E-02	0.34925103E-01	0.30257096E-01
909	-0.83386090E-02	0.10824267E-01	-0.23397668E-01	-0.13624071E-02	0.15525042E-01	0.13851198E-01

DETERMINISTIC ERRORS

	18	19	20	21	22	23
18	0.49178747E-04					
19	0.44588808E-02	0.22945548E-06				
20	0.53179587E-02	-0.24157364E 00	0.15772911E-06			
21	-0.31679782E 00	-0.14966369E-01	-0.40095346E-02	0.13151242E-06		
22	0.10881236E-03	-0.17891611E-02	-0.91192784E-02	-0.63671602E-04	0.14829781E-04	
23	-0.11339728E-03	0.17863653E-02	0.93393839E-02	0.62820530E-04	0.16391056E-03	0.14829688E-04
27	-0.20556642E-01	-0.11962564E-02	0.19626084E-02	-0.28591614E-01	0.13591201E-03	-0.14320402E-03
30	-0.19990589E-01	-0.26186033E-02	0.31797156E-02	-0.31522340E-01	0.24365586E-03	-0.25701522E-03
101	-0.16913622E 00	0.11811105E-02	0.31078967E-01	0.32289154E 00	0.38927900E-03	-0.40362724E-03
104	0.14152196E-01	-0.34072642E-02	-0.21379834E-03	-0.22571802E-01	0.55940678E-03	-0.59249602E-03
201	-0.17240272E 00	0.39773605E-02	0.43828425E-01	0.33033522E 00	0.60673687E-03	-0.62714433E-03
203	0.14451424E-01	-0.21305660E-01	-0.10398806E 00	-0.31678751E-01	-0.20787349E-02	0.21451558E-02
204	0.19012582E-01	-0.47361160E-02	-0.24504486E-02	-0.28067676E-01	0.65536731E-03	-0.69532068E-03
301	-0.75597276E-01	0.11852757E-01	0.64738858E-01	0.11909977E 00	0.13585404E-02	-0.14066419E-02
303	0.24713361E-02	-0.18964838E-01	-0.92511336E-01	-0.94344388E-02	-0.18556241E-02	0.19147723E-02
304	0.26961137E-01	-0.87053685E-02	-0.22535051E-01	-0.37354790E-01	0.20477405E-03	-0.22846730E-03
308	-0.29815107E-01	0.91093234E-02	0.44687105E-01	0.30267738E-01	0.10294807E-02	-0.10667989E-02
401	-0.18935468E-01	0.12907300E-02	0.69193666E-01	0.36495471E-01	0.28537991E-02	-0.29868276E-02
407	-0.71979293E-02	-0.26130683E-01	-0.82110941E-01	0.31747820E-01	-0.29019435E-03	0.26639238E-03
408	0.45789407E-01	0.16768443E-01	0.98378601E-01	-0.12231930E 00	0.23893328E-02	-0.24867233E-02
501	-0.12959730E-01	0.13897770E-03	0.64465873E-01	0.21792694E-01	0.28059501E-02	-0.29385875E-02
507	-0.11225325E-02	-0.28433953E-01	-0.93345589E-01	0.17576155E-01	-0.48005392E-03	0.46168032E-03
508	0.53080764E-01	0.16114242E-01	0.93872282E-01	-0.13939250E 00	0.22975969E-02	-0.23919450E-02
601	-0.60260002E-01	-0.15747116E 00	-0.35128143E 00	0.21557608E 00	-0.80806317E-03	0.71752289E-03
607	0.24632786E-01	0.68667808E-01	0.11249068E 00	-0.48645909E-02	-0.52789204E-03	0.60606422E-03
608	0.12120111E 00	0.12128846E 00	0.30189659E 00	-0.40409049E 00	0.16612523E-02	-0.16473460E-02
609	0.47782360E-01	0.34223555E-01	0.11806233E 00	-0.75114334E-02	0.87206015E-03	-0.88386992E-03
701	-0.55353342E-01	-0.14686758E 00	-0.32320721E 00	0.21987972E 00	-0.79254570E-03	0.71127103E-03
704	-0.59489508E-02	-0.15697368E-02	-0.28015480E-01	-0.18282049E-01	-0.25122033E-04	0.20299465E-04
707	0.20048164E-01	0.58986623E-01	0.84803133E-01	-0.31910749E-01	-0.45935284E-03	0.52409304E-03
708	0.11986678E 00	0.11878436E 00	0.29536283E 00	-0.40416368E 00	0.16580692E-02	-0.16463644E-02
709	0.38827629E-01	0.29972096E-01	0.10776109E 00	0.23425010E-01	0.62681612E-03	-0.62836549E-03
801	-0.22508742E-01	0.12764296E 00	0.35629819E 00	0.95749692E-01	0.10344081E-02	-0.96603946E-03
804	-0.26119117E-02	-0.10758585E-01	0.36103545E-01	0.45451042E-01	0.74889843E-03	-0.78897685E-03
807	-0.12363333E-01	0.91326128E-01	0.20669686E 00	0.28188420E-01	-0.25861097E-03	0.34998589E-03
808	0.53750376E-01	0.88648150E-02	-0.73267215E-01	-0.20531893E 00	-0.15729698E-02	0.16521694E-02
809	0.30522767E-01	-0.82488758E-02	-0.10043706E 00	-0.58170497E-01	-0.73970822E-03	0.75804233E-03
901	-0.93969752E-02	0.10161996E 00	0.16557723E 00	0.51543077E-01	-0.85514450E-04	0.16724801E-03
904	0.13642115E-01	-0.41676300E-02	-0.16385264E-01	-0.16893054E-01	-0.30719923E-04	0.27867669E-04
907	-0.25960130E-02	0.65788666E-01	0.66134796E-01	-0.50794738E-01	-0.89027757E-03	0.98051410E-03
908	0.48931493E-01	-0.11081789E 00	-0.26822856E 00	-0.24717717E 00	-0.94632697E-03	0.89899473E-03
909	-0.27180054E-02	-0.23582131E-01	-0.12861368E-01	0.63410123E-01	0.41641915E-03	-0.45356660E-03

DETERMINISTIC ERRORS

	27	30	101	104	201	203
27	0.37012210E-04					
30	-0.57288621E-02	0.20069184E-02				
101	0.33559951E-01	0.47073985E-01	0.12931531E-02			
104	-0.81563873E-02	-0.13398374E-01	0.52637889E-02	0.26602928E-04		
201	0.34100379E-01	0.47987554E-01	0.63893925E 00	-0.41290768E-02	0.12903034E-02	
203	0.33857766E-03	0.13297088E-03	0.34172095E-01	0.40653244E-01	0.12345773E-01	0.22032021E-04
204	-0.97524833E-02	-0.16080132E-01	0.66269994E-02	0.12003319E 00	-0.54784701E-02	0.53414065E-01
301	0.54935538E-02	0.70738267E-02	0.17537995E 00	0.19482914E-02	0.17566899E 00	0.16029110E-02
303	0.19146442E-02	0.22586599E-02	0.48630609E-01	0.36744180E-01	0.27225877E-01	0.17405884E 00
304	-0.87116262E-02	-0.14409230E-01	0.51706948E-02	0.11640891E 00	-0.68441314E-02	0.53905362E-01
308	-0.35335844E-02	-0.56674336E-02	0.88984530E-02	0.60253463E-02	0.69865630E-02	0.14927853E-01
401	-0.56342939E-02	-0.96333198E-02	0.10179331E 00	0.16035009E-01	0.10853327E 00	-0.65647753E-01
407	0.10533616E-02	0.10419616E-02	0.63547255E-01	-0.26030951E-02	0.49739212E-01	0.12377102E 00
408	-0.20350902E-01	-0.30582403E-01	-0.17691021E 00	0.65764979E-02	-0.19557297E 00	0.16189211E 00
501	-0.65232168E-02	-0.11128319E-01	0.10943235E 00	0.34160274E-01	0.11422437E 00	-0.56805363E-01
507	0.51637743E-03	0.10649703E-03	0.69628212E-01	0.13940168E-01	0.52877303E-01	0.14213573E 00
508	-0.21416437E-01	-0.32254133E-01	-0.18094891E 00	0.20676719E-01	-0.20174179E 00	0.17408137E 00
601	0.31277246E-02	0.10258019E-01	0.38852477E-01	-0.57412187E-01	0.46452445E-01	-0.37997976E-01
607	0.94951787E-03	-0.14257648E-02	0.30265771E-01	0.79774524E-01	0.36489800E-02	0.20904676E 00
608	-0.85749171E-02	-0.16964941E-01	-0.12082794E 00	0.10196704E 00	-0.12857097E 00	0.78405418E-02
609	0.16094788E-02	0.11706013E-02	-0.35769423E-01	0.14338487E 00	-0.52315612E-01	0.85967445E-01
701	0.37850384E-02	0.10902865E-01	0.30880908E-01	-0.40270875E-01	0.34924724E-01	-0.12681640E-01
704	-0.25951391E-02	-0.41290626E-02	0.18132950E-01	-0.30060316E-01	0.25798327E-01	-0.57577224E-01
707	-0.55762659E-03	-0.33944395E-02	0.39312909E-01	0.49996616E-01	0.19878076E-01	0.15524650E 00
708	-0.87365248E-02	-0.17122807E-01	-0.11905301E 00	0.97885047E-01	-0.12594096E 00	0.17673635E-02
709	0.27059730E-02	0.30583378E-02	-0.37774033E-01	0.13328842E 00	-0.55787574E-01	0.10558114E 00
801	-0.29361026E-05	0.11179547E-02	0.68534938E-01	0.39297216E-02	0.69408420E-01	-0.74433035E-02
804	-0.22880482E-02	-0.47686307E-02	0.26652886E-01	0.32873212E-01	0.20190350E-01	0.46128486E-01
807	0.30258293E-02	0.58586806E-02	0.75499682E-01	-0.41770063E-02	0.62652586E-01	0.11914784E 00
808	-0.19205235E-02	-0.15952530E-02	-0.40073527E-01	0.75453831E-02	-0.62847784E-01	0.20023784E 00
809	0.31689029E-02	0.60516195E-02	-0.53488567E-01	0.40550825E-01	-0.53036897E-01	-0.25308881E-01
901	0.24570184E-02	0.68716942E-02	0.27714840E-01	-0.42929743E-03	0.29641671E-01	-0.15233466E-01
904	0.37645230E-03	0.49468331E-03	0.95236457E-03	0.31144690E-01	-0.44340112E-02	0.34904290E-01
907	0.29453068E-02	0.67466211E-02	0.58719895E-01	-0.28661575E-01	0.49164988E-01	0.99889070E-01
908	-0.29642973E-02	-0.69790923E-02	-0.42291795E-01	-0.13573754E-02	-0.57494600E-01	0.13223609E 00
909	-0.89302746E-03	-0.24005877E-02	-0.20919788E-01	0.28072445E-01	-0.20031221E-01	-0.19536604E-01

DETERMINISTIC ERRORS

	204	301	303	304	308	401
204	0.20644332E-04					
301	0.16762875E-02	0.48306935E-02				
303	0.48353119E-01	0.19930514E 00	0.22617145E-04			
304	0.14755737E 00	-0.27185266E 00	-0.81741145E-02	0.21580761E-04		
308	0.70717709E-02	0.96342453E 00	0.21606643E 00	-0.27652250E 00	0.49597982E-02	
401	0.19717232E-01	0.34827173E-02	-0.70805715E-01	0.26974183E-01	-0.32858950E-01	0.31550878E-02
407	-0.10632964E-02	-0.19377489E-01	0.11880585E 00	0.10607022E-01	-0.21469989E-01	0.81586210E 00
408	0.91970861E-02	-0.21173254E-01	0.16384399E 00	-0.91866025E-03	0.47598779E-01	0.52165691E 00
501	0.42656852E-01	0.55307504E-02	-0.62740105E-01	0.49267567E-01	-0.31671097E-01	0.16046181E 00
507	0.20009556E-01	-0.18053776E-01	0.13651138E 00	0.31168983E-01	-0.19373181E-01	0.63692180E-01
508	0.27080866E-01	-0.21113845E-01	0.17551623E 00	0.16057745E-01	0.50368633E-01	-0.10882157E 00
601	-0.72662815E-01	-0.93449084E-02	-0.37736065E-01	-0.64597767E-01	-0.25560000E-01	-0.47448454E-01
607	0.10280386E 00	0.22508267E-01	0.21083590E 00	0.92621664E-01	0.41665423E-01	-0.11423416E 00
608	0.12860131E 00	-0.22846764E-01	0.56561094E-03	0.12396851E 00	0.11430154E-01	0.11637505E 00
609	0.18252858E 00	-0.17995016E-01	0.78035614E-01	0.17870968E 00	0.25315078E-02	0.17867377E-01
701	-0.50736187E-01	-0.99573176E-02	-0.12783455E-01	-0.44225511E-01	-0.20759917E-01	-0.61263516E-01
704	-0.38528484E-01	0.14416451E-02	-0.57132129E-01	-0.35385936E-01	-0.10841833E-01	0.41173748E-01
707	0.64589785E-01	0.22907451E-01	0.15748368E 00	0.57107076E-01	0.32733919E-01	-0.79052362E-01
708	0.12338467E 00	-0.22725752E-01	-0.54096984E-02	0.11911795E 00	0.10290095E-01	0.11967449E 00
709	0.16990916E 00	-0.15533659E-01	0.99002225E-01	0.16501384E 00	0.80386010E-02	-0.14466339E-01
801	0.52036570E-02	0.21371258E-01	-0.47351868E-02	0.41182243E-02	0.24120926E-02	0.10148551E 00
804	0.42176676E-01	0.70309074E-02	0.45909881E-01	0.40308697E-01	0.58620630E-02	-0.36246187E-02
807	-0.42520338E-02	0.35325767E-01	0.12526323E 00	-0.98183848E-02	0.31016669E-01	-0.28753243E-01
808	0.99629032E-02	0.26158959E-01	0.20653904E 00	-0.44389364E-02	0.63396418E-01	-0.24995926E 00
809	0.51072866E-01	-0.25056206E-01	-0.30963170E-01	0.53337114E-01	-0.14318151E-01	0.66483218E-02
901	-0.50478047E-03	0.10829040E-01	-0.13363982E-01	-0.17006815E-02	0.16484903E-02	0.42774275E-01
904	0.39641852E-01	-0.84560957E-03	0.33398651E-01	0.38322085E-01	0.33296749E-02	-0.21112423E-01
907	-0.35976257E-01	0.37896001E-01	0.10714690E 00	-0.43015013E-01	0.35620543E-01	-0.88965690E-01
908	-0.19863349E-02	0.85240884E-02	0.13379221E 00	-0.92439216E-02	0.37130675E-01	-0.19826090E 00
909	0.35823298E-01	-0.16983005E-01	-0.23030490E-01	0.38525667E-01	-0.14190413E-01	0.38187159E-01

DETERMINISTIC ERRORS

	407	408	501	507	508	601
407	0.48829602E-02					
408	0.65517434E 00	0.27131480E-02				
501	0.63712795E-01	-0.10472117E 00	0.30075738E-02			
507	0.25401746E 00	0.12116215E 00	0.79678147E 00	0.46662277E-02		
508	0.10862803E 00	0.38660315E 00	0.49280422E 00	0.63627663E 00	0.26390810E-02	
601	-0.60972943E-01	-0.90312834E-01	-0.57394538E-01	-0.71379299E-01	-0.98444702E-01	0.11973926E-01
607	0.74196056E-01	0.29065203E 00	-0.99915429E-01	0.98527077E-01	0.31137325E 00	-0.58641667E 00
608	0.76192143E-01	0.11311134E-01	0.13644785E 00	0.93031425E-01	0.24117927E-01	-0.84083000E 00
609	0.31518006E-01	0.74765906E-01	0.42458398E-01	0.55948306E-01	0.95571741E-01	0.55400575E-01
701	-0.55813468E-01	-0.50955336E-01	-0.68504879E-01	-0.62532980E-01	-0.55651254E-01	0.37255092E 00
704	-0.69954026E-02	-0.89101503E-01	0.36355407E-01	-0.14362794E-01	-0.96169306E-01	0.11597725E-01
707	0.65456387E-01	0.21142835E 00	-0.69406863E-01	0.82802081E-01	0.22562883E 00	-0.10408732E 00
708	0.75047882E-01	0.18648687E-02	0.13907991E 00	0.90989664E-01	0.13833349E-01	-0.46768695E 00
709	0.27846149E-01	0.11669880E 00	0.79129806E-02	0.51778552E-01	0.13764241E 00	-0.12974928E 00
801	0.10067967E 00	0.26086450E-01	0.97847254E-01	0.97084207E-01	0.19876181E-01	-0.26292939E 00
804	0.30231457E-01	0.51675564E-01	0.13167752E-02	0.36861106E-01	0.56636694E-01	-0.28025350E-01
807	0.11576164E 00	0.19317642E 00	-0.30720472E-01	0.12178771E 00	0.19579400E 00	-0.16596057E 00
808	-0.52083180E-01	0.33630805E 00	-0.24179763E 00	-0.32679523E-01	0.35779190E 00	0.90314162E-01
809	-0.51489218E-01	-0.71276795E-01	0.15792706E-01	-0.45901255E-01	-0.65213552E-01	0.36653608E-01
901	0.29106450E-01	0.31534759E-02	0.40621301E-01	0.26338949E-01	-0.10385303E-03	-0.12239670E 00
904	-0.56555513E-02	0.34595675E-01	-0.14589496E-01	0.15585012E-02	0.41218040E-01	0.57760999E-02
907	0.31352400E-01	0.17904869E 00	-0.92049289E-01	0.35295588E-01	0.18260366E 00	-0.28320336E-01
908	-0.75565605E-01	0.19080786E 00	-0.19023726E 00	-0.60657959E-01	0.20770994E 00	0.21449953E 00
909	0.85150853E-02	-0.51156832E-01	0.41244083E-01	0.96292155E-02	-0.50803427E-01	-0.54646243E-02

DETERMINISTIC ERRORS

	607	608	609	701	704	707
607	0.23112985E-01					
608	0.18153809E 00	0.10615442E-01				
609	-0.30369989E 00	0.17440807E 00	0.21562597E-01			
701	-0.54117403E-01	-0.46999499E 00	-0.12061781E 00	0.12397690E-01		
704	-0.13331529E 00	0.38231868E-01	-0.76817136E-01	-0.38375583E 00	0.40264042E-04	
707	0.30194765E 00	0.12788477E-01	0.14126943E 00	-0.63521548E 00	0.56645507E 00	0.24600063E-01
708	-0.38112070E-01	0.72537810E 00	0.17261595E 00	-0.83660942E 00	0.13968219E 00	0.22802103E 00
709	0.26698819E 00	0.10199019E 00	0.28573175E 00	0.24118859E 00	-0.74281954E 00	-0.49171221E 00
801	0.24074252E-01	0.29851523E 00	0.75984033E-01	-0.25559948E 00	0.11432222E-01	0.31047866E-01
804	0.10090136E 00	-0.16802906E-02	0.79540808E-01	-0.11244256E-01	-0.33024402E-01	0.67526412E-01
807	0.20530541E 00	0.10649900E 00	0.55444396E-01	-0.14066613E 00	-0.45769566E-01	0.16205020E 00
808	0.37281887E 00	-0.25829114E 00	0.98456517E-02	0.13359673E 00	-0.12015251E 00	0.26774060E 00
809	-0.49017978E-01	0.16136801E-01	0.36361946E-01	0.33444910E-01	0.61124074E-02	-0.43089582E-01
901	0.10689915E-01	0.13044783E 00	0.36372642E-01	-0.11834774E 00	0.48813185E-02	0.12795163E-01
904	0.72864835E-01	-0.20434783E-01	0.53221151E-01	0.17523296E-01	-0.26563320E-01	0.47543427E-01
907	0.17697239E 00	-0.62038746E-01	-0.28601954E-01	-0.11204912E-01	-0.43802951E-01	0.13878929E 00
908	0.18120362E 00	-0.30509626E 00	-0.74661815E-01	0.22849822E 00	-0.65786406E-01	0.12914287E 00
909	-0.26351413E-01	0.28878554E-01	0.56791823E-01	-0.38965002E-02	0.19834056E-02	-0.27302597E-01

DETERMINISTIC ERRORS

	708	709	801	804	807	808
708	0.10592908E-01					
709	0.46483937E-01	0.28213489E-01				
801	0.29691165E 00	0.47680041E-01	0.12042965E-01			
804	-0.55460755E-02	0.87205524E-01	-0.47314195E 00	0.39093596E-04		
807	0.10042004E 00	0.71151707E-01	0.86030883E 00	-0.60230610E 00	0.20495570E-01	
808	-0.26913501E 00	0.88709626E-01	0.12215208E 00	-0.38164100E 00	0.56970511E 00	0.96500909E-02
809	0.16859886E-01	0.25063304E-01	0.46815481E 00	-0.78015997E 00	0.64787725E 00	0.47190665E 00
901	0.12958419E 00	0.24105576E-01	0.15173276E 00	0.57438817E-02	0.78420653E-01	-0.94442385E-01
904	-0.23253629E-01	0.61077052E-01	-0.29589798E-01	0.18958811E-01	0.78905704E-02	0.77383096E-01
907	-0.66383328E-01	0.47181860E-02	0.17576322E-01	0.16572768E-01	0.15414604E 00	0.28305649E 00
908	-0.30907349E 00	-0.15345976E-01	-0.31802702E 00	0.45396405E-02	-0.13523384E-01	0.54707918E 00
909	0.28730079E-01	0.45886267E-01	0.99268950E-02	0.10477094E-01	-0.44494148E-01	-0.11561815E 00

DETERMINISTIC ERRORS

	809	901	904	907	908	909
809	0.25422265E-01					
901	-0.28831106E-01	0.34634672E-01				
904	0.10668612E-01	-0.27265326E 00	0.42525116E-04			
907	-0.66275773E-01	0.83161184E 00	-0.40380356E 00	0.38907436E-01		
908	0.13965372E-01	-0.61514533E 00	-0.87979196E-01	-0.10016623E 00	0.24979372E-01	
909	0.24315539E-01	0.11262220E 00	-0.66958137E 00	0.30486801E 00	0.26410571E 00	0.40554877E-01

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