X-207-67-434
Revised February 1968

WORKING WITH ORBITS

Dale E. Gerster
Physics Teacher
Bladensburg Senior High School
Prince George's County, Maryland

September 1967

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

CONTENTS

ILLUSTRATIONS

TABLES

WORKING WITH ORBITS

INTRODUCTION

What keeps a satellite in orbit? Everyone knows that even the fastest rifle bullet eventually falls to the earth, so why doesn't a satellite do the same since it also must be acted on by gravity? The answers to these questions were discovered over 300 years ago by Sir Isaac Newton, who in turn based his investigations of the laws of moving objects and the force of gravitation on the work of Galileo Galilei.

Newton explained the situation somewhat as follows: Imagine you are on top of a very high mountain - above all other mountains, and even above most of the atmosphere. Now if you throw a stone as hard as you can, it will go only a short distance before hitting the earth at the base of the mountain. If the mountain were 100 miles high, and you were to shoot a rifle bullet horizontally at 5,000 miles per hour, it would travel about 300 miles before coming to earth. At 10,000 miles per hour it would go nearly 4,000 miles before striking the earth. However, at 17,500 miles per hour it would never reach the earth. As it moves along, gravity would constantly pull it down, but the earth's surface is curved (spherical), and the bullet's path produced by gravity would just match the curvature of the earth. Thus the bullet would go around the earth in a circle 100 miles above the surface.

Because the force of gravity is always toward the center of the earth, it is acting on the bullet at an angle of 90° to the bullet's direction of flight. Thus, because no other force acts to speed up or slow down the bullet (remember it is above most of the atmosphere so we can neglect air resistance) it will continue to travel in its circular orbit.

ORBITS THAT ARE NOT CIRCULAR

Newton's thinking went further than this circular orbit. Suppose that from this same high mountain we shot a bullet slightly <u>faster</u> than the 17,500 miles per hour needed to keep it in circular orbit. It would then fall in a curve more gradual than that of the earth's surface and so would move farther from the earth. But this would cause the pull of gravity to be at an angle of less than 90° with the bullet's path. Gravity now pulls slightly <u>backward</u> on the bullet, slowing it down. As it slows down it curves more sharply, until at its farthest point from earth it is again moving parallel to the earth's surface. Because the bullet is now traveling at less than orbital speed for a circular path at this altitude, it begins to curve back toward the earth. The force of gravity now begins to act slightly <u>forward</u> along the path of the bullet, speeding it up along its path as it swings in toward the earth. It continues to speed up until it has reached its original velocity, at which time it is back to its starting point just above the top of the mountain and again moving parallel to the earth's surface. (For the purpose of this explanation we have assumed that the earth did not rotate during the period of the orbit.) This process, repeated each time around, produces an orbit in the shape of an ellipse. The center of the earth is near one "end" of the ellipse, at a point called its focus.

If the speed of the bullet were increased further, the ellipse would lengthen until, at a speed of about 25,000 miles per hour, the bullet would never curve back toward earth but continue into space. This speed is called the escape velocity, since the bullet escapes earth's gravity completely; and this path of the bullet is called a parabola. At still higher speeds, the path would curve less and less, and would assume a shape called a hyperbola, which is similar to a parabola, but less curved.

All this is fine as a general description of satellite motion, but it hardly serves to tell a scientist or engineer the things he needs to know before placing a satellite into orbit. Such things include the size of the orbit, its position relative to the earth, and the velocity of the satellite at various points in the orbit. To understand orbital motion better we must use some mathematics. There are many steps in the calculations, but the processes are fairly simple.

INTRODUCTION TO CONCEPTS AND FORMULAS

What do we plan to accomplish by these calculations? First, knowing that all closed orbits are ellipses or circles when affected only by the force of gravity, we will decide on an orbit and define its shape and size. Then we will calculate the required velocity at perigee, apogee, and some intermediate points. We will note that the perigee and apogee velocities form a 90° angle with a line through the center of the earth, but the angles at other points in the orbit will not be 90°. Using these angles and the calculated force of gravity at these various points we will predict the velocity change between points, showing that only gravity is necessary to determine the elliptical path of a satellite. Finally, using energy considerations, we will calculate the velocity needed for an object to escape from the influence of gravity and leave the earth.

Along the way to these solutions we will find it necessary to use some of the skills of mechanical drawing, and some elementary knowledge of arithmetic, algebra, geometry, trigonometry, physics, and astronomy.

Before beginning our calculations we must understand the quantities that are involved. A satellite in orbit has energy, which is in two parts - kinetic and potential. Its kinetic energy (E_k), due to its mass and motion, is calculated by multiplying its mass by the square of its velocity and dividing this product by two. In equation form this is:

$$E_k = \frac{mv^2}{2}.$$

Its potential energy (U) is due to the work needed to move it away from the earth against the pull of gravity. This energy is calculated by dividing the product of its mass and the mass of the earth by the distance of the satellite from the center of the earth, then multiplying by a constant (G) to change the units of mass and distance to units of energy. In equation form this is:

$$U = -G\frac{Mm}{R}.$$

The negative sign is due to the fact that the <u>zero</u> value for potential energy is, for mathematical convenience, defined to be at an infinite distance from the earth. Then because, for an attractive force like gravity, potential energy is less at closer distances, it must be expressed as a negative quantity to show that it is less than zero. The total energy ($E_k + U$) of a satellite is of course given to it by the burning of fuel in the rocket used to lift it from the earth and accelerate it to orbital velocity. When a satellite is in an elliptical orbit, potential energy increases and kinetic energy decreases as it goes farther from the earth. As it approaches the earth the reverse process takes place.

Another quantity associated with any object moving in a curved path is angular momentum (L). This is calculated by first multiplying the mass of the object by its distance (R) from the center about which it is moving, and then multiplying this product by its velocity ($v_\perp$)* in a direction 90° with the line from object to center of revolution. The equation is expressed:

$$L = mRv_\perp.$$

The predominant force acting on a satellite in orbit is gravity. This is not constant, but depends on distance of the satellite from the center of the earth. To calculate the magnitude of this force we must divide the product of the satellite's mass and the mass of the earth by the square of the distance from satellite to the center of the earth, then multiply this expression previously mentioned constant, G.

*$v_\perp$ is read "vee perpendicular".

The equation for this is:

$$F = G\frac{Mm}{R^2}.$$

The direction of this gravitational force on the satellite is, of course, always toward the center of the earth.

Finally, we need to know four measurements of the orbit. These are the perigee and apogee radii (R_P and R_A) and the semi-major and semi-minor axes of the ellipse (a and b). The perigee radius is the distance of the closest point on the orbit from the center of the earth, and the apogee radius is the distance of the farthest point. The semi-major axis is half the greatest distance across the ellipse, and the semi-minor axis is half the shortest distance. These measurements are labeled in the diagram of an elliptical orbit (Figure 1).

MAKING THE CALCULATIONS

Now we are ready to begin calculating orbital quantities. It is first necessary to draw an accurate representation of our desired orbit, so get a large sheet of paper (say about 18" by 24") and fasten it on a drawing board or a large piece of heavy cardboard. Draw two lines through the center of the paper at a 90° angle to each other as shown in Figure 1. Measure an equal distance from the intersection of these lines each way from the center and along the longer dimension of the paper, and push a pin into the paper at each point thus located. These, labeled f_1 and f_2 in Figure 1, are the two foci of the ellipse. Now tie a loop of string such that, when placed over both pins and drawn taut along the line between the pins, it will extend from one pin to a point about the same distance beyond the other as that pin is from the intersection of the two lines first drawn (see Figure 2). Insert the point of a well-sharpened pencil inside the loop and, keeping the loop taut, move the pencil around the two pins so that it traces a line on the paper. This line will be an ellipse whose center is the intersection of the two original lines. Each of the pins marks a focus of the ellipse. This line must be a smooth curve if the rest of your work is to be accurate, so if you don't succeed at first, erase and try again.

Next, pick one focus and draw a circle around it to represent the earth. Do not let this circle extend beyond the ellipse, and pick a size such that the radius of the circle conveniently represents the 4,000 mile radius of the earth. Accurate measurement here, and from now on, is important. The scale of the remainder of your drawing is determined by the size of this circle. (e.g., if 2 inches represent 4,000 miles, then 1/2 inch is equivalent to 1,000 miles,

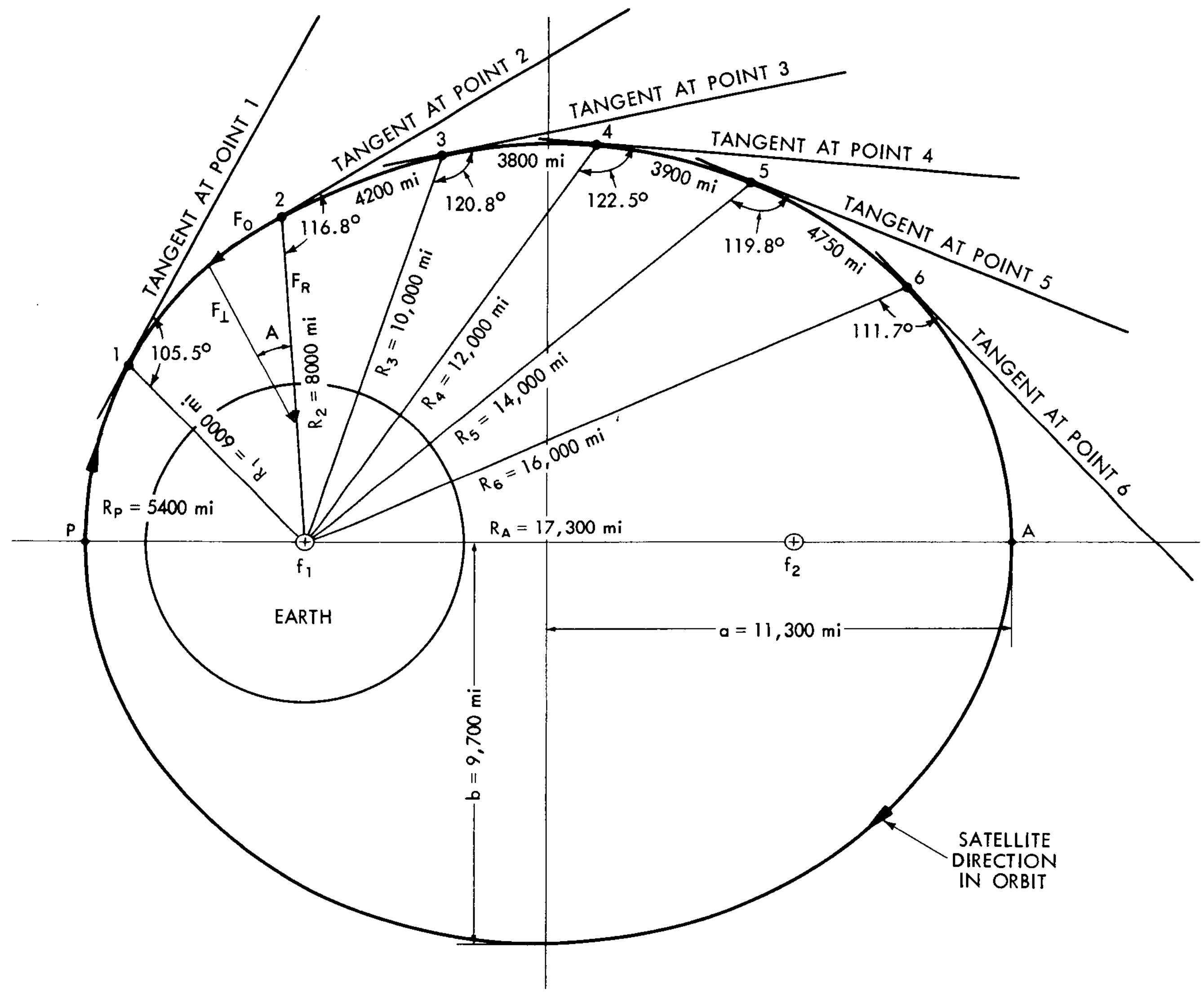

Figure 1. Example of An Elliptical Orbit

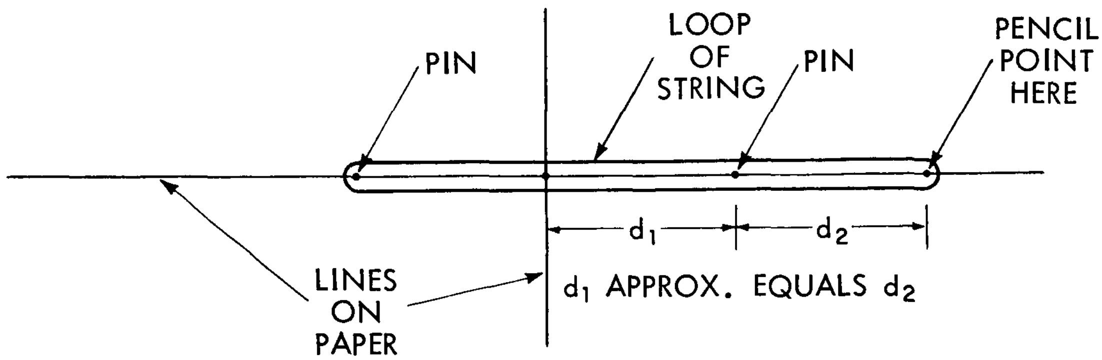

Figure 2. Construction of An Ellipse

1/4 inch represents 500 miles, etc.) Draw several radii to points on the ellipse as shown in Figure 1 and, using your distance scale, measure them in terms of miles. Also measure the perigee and apogee radii using the same scale. (NOTE: It is more convenient to select radii that measure to exact fractions of an inch than to try to guess between marks on the ruler.) Finally, measure the semi-major and semi-minor axes of your ellipse, again using the distance scale in miles. Other quantities which must be known are mass of the satellite (m), mass of the earth (M), and the value of the constant G.

For convenience we will assume the mass of the satellite to be one kilogram. The mass of the earth is 5.98×10^{24} kilograms, and the value of G is 6.67×10^{-11} meters cubed per kilogram-second squared; that is

$$G = 6.67 \times 10^{-11} \left(\frac{\text{m.}^3}{\text{kg.-sec.}^2} \right).$$

We can now begin computation.

DETAILS OF THE CALCULATIONS

First, because the calculating process is somewhat easier if we use the metric system, we must convert the measured distances from miles to meters. This is done by multiplying the number of miles by 1609, which is the number of meters equivalent to one mile. We have tabulated our data for these distances in Table 1, columns 2 and 3.

Table 1
Geometric Data from Figure 1

1.	2.	3.	4.	5.	6.
Item	Radius in Miles	Radius in Meters	Angle Between Orbit & Radius	Angle Between Orbit & to Rad.	Sine of Angle in Col. 5
R_p	5,400	8,680,000	90.0°	0.0°	0.000
R_1	6,000	9,650,000	105.5°	15.5°	0.268
R_2	8,000	12,890,000	116.8°	26.8°	0.451
R_3	10,000	16,090,000	120.8°	30.8°	0.512
R_4	12,000	19,300,000	122.5°	32.5°	0.538
R_5	14,000	22,500,000	119.8°	29.8°	0.497
R_6	16,000	25,700,000	111.7°	21.7°	0.370
R_A	17,300	27,800,000	90.0°	0.0°	0.000
a	11,300	18,200,000	—	—	—
b	9,700	15,600,000	—	—	—

Two equations relate the total energy (E) and angular momentum (L) of a satellite to the semi-major and semi-minor axes of its orbit. These are:

$$a = -G\frac{Mm}{2E} \quad (1)$$

$$b = \frac{L}{\sqrt{-2mE}} \quad (2)$$

These equations must be solved for E and L.

$$\text{From (1):}\quad E = -G\frac{Mm}{2a} \quad (1a)$$

$$\text{From (2):}\quad L = b\sqrt{-2mE} \quad (2a)$$

Using the measured values of a and b from Table 1, together with the previously given values for M, m, and G, we can calculate the values of E and L by substituting in Equations 1a and 2a as follows:

$$E = (-6.67 \times 10^{-11}) \frac{(5.98 \times 10^{24})\ (1)}{(2)\ (1.82 \times 10^{7})} = -1.095 \times 10^{7} \text{ joules}$$

Substituting this value of E in Equation 2a we get:

$$L = (1.56 \times 10^{7}) \sqrt{-(2)\ (1)\ (-1.095 \times 10^{7})},$$

$$= 7.30 \times 10^{10} \frac{\text{kg.-m.}^2}{\text{sec.}}.$$

The arithmetic was done on a slide rule. (Note that m. stands for meters, while m without a period stands for mass of the satellite.) You should make the same calculations as above, but using the values of a and b taken from the ellipse which you have drawn.

Now it follows from the laws of conservation of energy and momentum that neither quantity will change during an orbit. We can use either quantity to determine the velocity of the satellite at perigee and at apogee. Using the equation, $L = mRv_{\perp}$, we can substitute the values for L, m, and R and solve for $v_{\perp}$, which is the orbital velocity at these points because at both perigee and apogee the orbital path is at 90° to the radius R. The velocity calculation for perigee is as follows:

$$v_P = \frac{L}{mRp},$$

$$= \frac{(7.30 \times 10^{10}) \dfrac{\text{kg.-m.}^2}{\text{sec.}}}{(1\text{kg.})\ (8.68 \times 10^{6} \text{ m.})},$$

$$= 8.40 \times 10^{3} \text{ m./sec.} = 8400 \text{ m./sec.}$$

For apogee we have:

$$v_A = \frac{(7.30 \times 10^{10}) \dfrac{\text{kg.-m.}^2}{\text{sec.}}}{(1 \text{ kg.})\ (27.8 \times 10^{6}\text{m.})},$$

$$= 2.62 \times 10^{3} \text{ m./sec.} = 2620 \text{ m./sec.}$$

The calculations are more complicated when the energy value is used. We must first separate the total energy (E) into kinetic energy (E_k) and potential energy (U), and then use the value of E_k to determine velocities. The first step is to calculate the potential energy U at both perigee and apogee. At perigee:

$$U_P = -G\frac{Mm}{R_P} = (-6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})(1)}{(8.68 \times 10^{6})},$$

$$= -4.60 \times 10^{7} \text{ joules.}$$

At apogee:

$$U_A = (-6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})(1)}{(27.8 \times 10^{6})},$$

$$= -1.43 \times 10^{7} \text{ joules.}$$

The second step is to solve the equation $E = E_k + U$ for E_k. This must be done for both perigee and apogee. For perigee:

$$E_{k_P} = E - U_P = -1.095 \times 10^{7} + 4.60 \times 10^{7},$$

$$= 3.51 \times 10^{7} \text{ joules.}$$

For apogee:

$$E_{k_A} = -1.095 \times 10^{7} + 1.43 \times 10^{7}$$

$$= 0.335 \times 10^{7} \text{ joules.}$$

Now to find the perigee and apogee velocities, we solve the kinetic energy equation, $E_k = \frac{mv^2}{2}$, for v.

For perigee this is:

$$v_P = \sqrt{\frac{2E_{k_P}}{m}} = \sqrt{\frac{2 \times 3.51 \times 10^{7}}{1}},$$

$$= 7.02 \times 10^{7} = 70.2 \times 10^{6}$$

$$= 8.37 \times 10^{3} \text{ m./sec.} = 8370 \text{ m./sec.}$$

At apogee:

$$v_A = \sqrt{\frac{2 \times 0.335 \times 10^7}{1}} = \sqrt{0.670 \times 10^7}$$

$$= \sqrt{6.70 \times 10^6} = 2.59 \times 10^3 = 2590 \text{ m./sec.}$$

The slight differences in the values of v_P and v_A from those values computed using the angular momentum L arise primarily from the restricting of values to three-digit numbers on the slide rule.

Now comes the tedious part of our work. We must make calculations of energy and velocity* for all other radii (R_1 through R_6) like those calculations made for perigee (R_P) and apogee (R_A). When this is done, using values from Figure 1, we get the results shown in columns 2, 3, 4, and 5 of Table 2. You should make similar calculations based on the measurements of the ellipse you have constructed.

We will now examine how gravity acts on the satellite to change its velocity around the orbit. Look again at Figure 1 and examine the right triangle whose hypotenuse is labeled F_R, and whose sides are F_O and $F_\perp$. F_R represents the force of gravity on the satellite, which is always directed, as shown, toward the center of the earth. F_O is the portion of the gravity force that acts along the orbital path, and $F_\perp$ is the portion perpendicular to F_O. ($F_\perp$ and F_O are called rectangular components of F_R.) Because $F_\perp$ is always at a 90° angle to the direction in which the satellite is moving, it can change only the direction, not the speed. F_O, however, acts to change speed. In the case illustrated in Figure 1, it will slow down the satellite. (At a corresponding point on the other half of the orbit it would speed up the satellite.) If we know the amount of the gravity force, F_R, and the angle A, we can determine the magnitude of F_O. There are two ways to do this.

For each method we need first to know the values of F_R. These are calculated from the equation, $F = G\frac{Mm}{R^2}$, given previously. We have done these calculations for our drawing and have tabulated the results in column 6 of Table 2. For example, the calculation for position 2 in the orbit of Figure 1 is:

*Note that velocities must be calculated from E_k rather than from L, because by using L we can get only that part of the velocity that is in a direction perpendicular to the radius.

$$F_{R_2} = G\frac{Mm}{R_2^{\,2}} = (6.67 \times 10^{-11})\,\frac{(5.98 \times 10^{24})\,(1)}{(12.89 \times 10^{6})^2}$$

$$= \frac{39.8 \times 10^{13}}{166 \times 10^{12}} = 0.240 \times 10^{1} = 2.40 \text{ newtons}$$

You should make these calculations for the radii you have measured in your ellipse.

It is next necessary to measure carefully the angle formed by the orbital path and the radius at each point. To do this, we must draw a line tangent to the orbit at each point where a radius ends, and measure the angle thus produced. These are shown and labeled in Figure 1, and tabulated in column 4 of Table 1. When you draw them on your ellipse, be very careful and use a sharp pencil. These tangents must just touch the curve at the right point or the angles will be incorrect. Measure the angles with a protractor to an accuracy of 0.5° or less. To get the value for angle A at each radius, subtract 90° from each measured angle. (The reason for doing this can be found in any plane geometry textbook.) We have tabulated these values for our ellipse in column 5 of Table 1.

In column 6 of Table 1 are numbers which are called sines of the angles in column 5. In a right triangle (one having a 90° angle), the sine of one of the smaller angles is a number obtained by dividing the length of the side opposite that angle by the length of the hypotenuse (the side opposite the 90° angle). In the triangle in our ellipse this is: sine A $= \frac{F_O}{F_R}$. The size of the triangle does not matter. The value of the sine of a given angle is always the same. We do not have to construct triangles and measure them to get these values; they are found in prepared tables in all trigonometry textbooks. We can get the value for any angle from the table.

Now we can solve the equation, sine A $= F_O / F_R$, for F_O, and then knowing F_R and sine A, we can calculate F_O. For example, the triangle at point 2 in Figure 1 is solved as follows:

$$F_{O_2} = F_{R_2} \times \text{sine A} = 2.40 \times 0.451 = 1.08 \text{ newtons}$$

The other values have been calculated in like manner, and values are shown in column 7 of Table 2. You should make the same kind of calculations for the angles in your drawing.

The second way of finding values for F_O is to make proportionate drawings of the triangles, first scaling off F_R, then drawing a line from one end of F_R at

Table 2
Calculated Data from Figure 1

1	2	3	4	5	6	7	8	9
Position of Satellite in Orbit	Potential Energy, U (joules)	Total Energy, E (joules)	Kinetic Energy, E_k (joules)	Velocity in Orbit, v (m./sec.)	Force of Gravity on Satellite, F_R (newt.)	Component of Gravity along Orbit, F_O (newt.)	Distance along Orbit from Point to Point	Velocity, calc. from Previous v and accel. (m/sec.)
P	-4.60×10^7	-1.095×10^7	3.51×10^7	8,370	5.29	0.00	—	—
1	-4.13×10^7	-1.095×10^7	3.04×10^7	7,780	4.28	1.15	—	—
2	-3.09×10^7	-1.095×10^7	2.00×10^7	6,320	2.40	1.08	—	—
3	-2.48×10^7	-1.095×10^7	1.38×10^7	5,250	1.54	0.788	4,200 miles 6,750,000 m.	5,230
4	-2.06×10^7	-1.095×10^7	0.965×10^7	4,390	1.07	0.575	3,800 miles 6,110,000 m.	4,390
5	-1.77×10^7	-1.095×10^7	0.675×10^7	3,670	0.785	0.390	3,900 miles 6,280,000 m.	3,640
6	-1.55×10^7	-1.095×10^7	0.455×10^7	3,010	0.603	0.223	4,750 miles 7,640,000 m.	2,970
A	-1.43×10^7	-1.095×10^7	0.335×10^7	2,590	0.515	0.00	—	—

the proper angle A to it, and finally making F_O from the other end of F_R in such a direction that it meets the second line at a 90° angle. Measuring the length of F_O to the same scale as that used for F_R will give the magnitude of F_O. This method will not be as accurate as using the sines unless the triangles are made quite large (e.g., on a full sheet of typing paper) and the angle A is made very accurately.

Again referring to Figure 1, and to column 8 of Table 2, you will see that we have measured distances between points on the orbit. We did not include distances from perigee to point 1, from point 1 to point 2, and from point 6 to apogee because of the sharp curvature of the path in these regions. Where we measured, the curvature is slight, and hence the straight-line distances are nearly equal to the actual path distances. We will use these distances and the forces F_O to calculate the slowing-down effect of gravitational forces on the satellite.

To do this we use two more equations developed by Newton. One equation says that the acceleration (a) of an object is equal to the force (F) applied to it divided by its mass (m). That is: $a = F/m$. The other says that, if the force on an object is constant, the square of the velocity achieved (v^2_{final}) is equal to the square of the velocity at the time the force is applied ($v^2_{initial}$) plus twice the product of the resulting acceleration (a) and the distance covered (d). That is:

$$v^2_{final} = v^2_{initial} + 2(a)\ (d).$$

Putting F/m in place of a in this equation gives us:

$$v^2_f = v^2_i + 2(F/m)\ (d).$$

There is, as you may already have noticed, one difficulty. Our force (F_O) is not constant, but continuously decreases from perigee to apogee. We get around this problem in the following way:

If the force changes uniformly with distance, we can use the average force acting over the distance between points on the orbit in our equation. This condition is very nearly true between points 2 and 6. We therefore use half the sum of the force at the beginning of the interval and the force at the end of the interval. For example, the calculation of the expected velocity at point 3, from the known velocity at point 2, is as follows:

$$v_3{}^2 = v_2{}^2 - 2\frac{(F_{o_2} + F_{o_3})}{2}(6.75 \times 10^6),$$

$$= 6320^2 - (1.08 + 0.788)(6.75 \times 10^6),$$

$$= 40.0 \times 10^6 - 12.6 \times 10^6 = 27.4 \times 10^6$$

$$v_3 = \sqrt{27.4 \times 10^6} = 5.23 \times 10^3 = 5320 \text{ m./sec.}$$

Note that a negative sign is used for the acceleration because it is in the opposite direction to the velocity of the satellite. That is, F_0 produces a slowing-down, or deceleration, of the satellite. We have made the calculations for the other distances measured in our ellipse, and we have tabulated the results in the column 9 of Table 2. Note how closely they agree with the velocities in column 5, which were calculated from kinetic energy of the satellite.

In these calculations we have used as the initial velocity for each interval of distance the velocity which was calculated for that point of the orbit from the energy of the satellite. However, calculating by the same method as above, but using as the initial velocity for each interval the calculated final velocity of the previous interval, we arrive at a figure for v_6 of 2860 m./sec. This value is only 5% different from the 3010 m./sec. calculated for the energy of the satellite at point 6.

CALCULATION OF ESCAPE VELOCITY

Early in the discussion it was mentioned that if an object were to be given a velocity of 25,000 miles per hour, directed away from the earth, it would not return to earth. Let us see how this escape velocity (v_e) is determined.

We have indicated that the potential energy at an infinite distance from the earth is defined to be zero, and that therefore all potential energies at lesser distances are negative. The equation for potential energy at a distance R from the center of the earth is:

$$U = -G\frac{Mm}{R}$$

Thus at the surface of the earth the potential energy of our 1 kg. satellite is:

$$U_{surf} = -G\frac{M\ (1\text{ kg.})}{R_E}$$

The radius of the earth (R_E) is 6.38×10^6 meters. The change in potential energy in going from the surface of the earth to infinity would be:

$$U_{inf} - U_{surf} = 0 - (-G\frac{Mm}{R_E}) ,$$

$$= +G\frac{Mm}{R_E} = (6.67 \times 10^{-11}) \frac{(5.98 \times 10^{24})(1)}{(6.38 \times 10^6)} ,$$

$$= 6.25 \times 10^7 \text{ joules.}$$

This amount of energy must be supplied by the launch vehicle to the spacecraft as kinetic energy within a few miles of the surface. Thus:

$$E_k = \frac{mv_e^2}{2} = 6.25 \times 10^7 \text{ joules}$$

$$v_e^2 = 2E_k = 2 \times 6.25 \times 10^7 , \ (m = 1 \text{kg.})$$

$$v_e^2 = 12.5 \times 10^7 = 125 \times 10^6 ,$$

$$v_e = 11.17 \times 10^3 \text{ m./sec.}$$

To convert meters per second to miles per hour, multiply by 2.237. This gives almost exactly 25,000 miles/hour.

EPILOGUE

Now here is a final problem for you to solve by yourself. Several hundred years ago an astronomer, Johannes Keppler, discovered that if the cube of the radius of a planet's orbit was divided by the square of the time required for one trip around the sun, then the resulting quotient was the same for all planets. Later, Newton proved that this was true for any satellite in orbit around a central object. The only difference was that the value of the quotient depended on the mass of the central object. For satellites in elliptical orbits, the value of the radius used is the average of the minimum and maximum radii, i.e., the average of the radii at perigee and at apogee. Because the earth has one natural satellite, the moon, whose distance and period of revolution are well known, we can obtain a value for R^3/T^2. Can you determine this value and use it to calculate the period of revolution of a satellite in the orbit which you have drawn? The only information you need outside of this pamphlet is the information about the moon, which you can find in many astronomy books. Good luck!

APPENDIX

Throughout our discussion we have used the "powers-of-ten" notation for very large and very small numbers. This is not difficult to understand, and is a great convenience in handling such numbers. For example: The radius of the earth is 6,380,000 meters. This number can be obtained by multiplying the number 6.38 by 1,000,000. In mathematical notation this is 6.38 x 1,000,000. But 1,000,000 can be obtained by multiplying six tens together like this: 10 x 10 x 10 x 10 x 10 x 10. A "shorthand" way of indicating this multiplication is 10^6, where the six is called an exponent. This is read as "ten to the sixth power," or simply, "ten to the sixth." We can therefore indicate the number 6,380,000 as:

$$6.38 \times 10^6.$$

We can tell where the decimal point belongs by the indicated power of ten (exponent). In this case, the six indicates that the decimal point really should be six places to the right of where it is written in 6.38.

A minus sign in front of the exponent shows that the decimal point really belongs that many places to the left of where it is written. For example, the value of "G" is written: 6.67×10^{-11}. This means that the decimal point belongs eleven places to the left of its location in 6.67. To put the decimal point there, we must fill ten of these places with zeros, and write the number in the form: .0000000000667.

Multiplying two numbers written in powers-of-ten notation requires two steps. First, multiply the ordinary parts, then separately multiply the powers of ten. We do the latter multiplication by adding the exponents. For example:

$$(3 \times 10^4)\ (5 \times 10^8) = (3 \times 5)\ (10^4 \times 10^8).$$

$$3 \times 5 = 15 \quad \text{and} \quad 10^4 \times 10^8 = 10^{(4+8)} = 10^{12}$$

$$\text{thus:}\ (3 \times 10^4)\ (5 \times 10^8) = 15 \times 10^{12}$$

This answer can also be written as 1.5×10^{13}. Do you understand why?

Division is accomplished in a similar fashion, but the exponent of the divisor is subtracted from the exponent of the dividend. For example:

$$\frac{6 \times 10^9}{2 \times 10^4} = \frac{6}{2} \times \frac{10^9}{10^4} = 3 \times 10^5$$

If one exponent is negative, it is handled the same as when adding or subtracting signed numbers in ordinary work. For example:

$$\frac{6 \times 10^9}{2 \times 10^{-4}} = \frac{6}{2} \times \frac{10^9}{10^{-4}} = 3 \times 10^{13}.$$