BELLCOMM, INC.
1100 Seventeenth Street, N.W.   Washington, D. C. 20036

SUBJECT: Effect of Earth Occultation on
Astronomical Observations from
Earth Orbit - Case 710.

DATE: February 15, 1968

FROM: D. B. Wood

ABSTRACT

The effect of the occultation of the celestial sphere by the earth upon astronomical observations from earth orbit is investigated as a function of orbital altitude, orbital inclination, and viewing direction. As the viewing direction approaches the orbital poles the total possible viewing time increases. Near the orbital poles there are unocculted circles which move around the celestial sphere at constant declination as the node regresses. Low earth orbits of intermediate inclination yield continuous viewing times of celestial objects within these circles on the order of 11 days, even though the orbital period is only 93 minutes. For an orbital observatory at a given inclination, the observing program is best directed toward the unocculted zones. Three astronomical observatories in low earth orbit (450km), at inclinations 15°, 45° and 75°, could cover the entire celestial sphere with their unocculted circles in about two months. Low energy orbits from Cape Kennedy of 450km altitude at an inclination of 28°5 allow good unocculted coverage of the Magellanic Clouds and other interesting southern objects. For polar (90°) or equatorial (0°) orbits the unocculted circle always views the same limited portion of the sky.

SUBJECT: Effect of Earth Occultation on Astronomical Observations from Earth Orbit - Case 710.

DATE: February 15, 1968

FROM: D. B. Wood

MEMORANDUM FOR FILE

INTRODUCTION

Any program for astronomical observations from earth orbit must take into account the effect of the occultation of the celestial sphere by the earth. This paper shows that there may be a great advantage in observing near the satellite's orbital poles, where there is no occultation. Long astronomical observations may be made without break or the necessity for superimposed exposures. Telescope pointing stability and inertia coupled with available reaction control jet fuel requirements will limit telescope slew rates, so that it may be impractical to slew to avoid earth occultation. This limits observing time to about 50% in near earth orbit unless the telescope is pointed toward the orbital polar regions.

Assuming there is no scattered light due to an optical environment problem, so that orbital astronomy may be performed in the sunlight, then ideally a very high or earth-synchronous type orbit is best, so that the earth occults but a small portion of the celestial sphere ($8°8$ angular radius at 35800km altitude). The angular distance from the sun to which we may point is probably about 20°, using proper baffling tubes extending beyond the aperature. For a near-earth orbit, the occultation by the earth is the major obstacle to observation. This paper shows that long observing intervals are possible from near-earth orbits, and that with a suitably designed observing program and several satellite observatories, the entire celestial sphere can be covered in unocculted fashion several times per year.

DISCUSSION

The total period of time which observations may be made in any given direction before occultation depends upon the orbital elements and the direction of viewing. It is assumed that any astronomical observations will be made from a circular orbit, so it is adequate to define the orbit in terms of

    H = the height above the earth's surface

    i = the orbital inclination

    ω = the angle from the first point of Aries to ascending node.

The primary effect of an eccentric orbit is that H is a function of time, so that the apparent size of the earth varies throughout the orbit. Thus if the H used here is taken as the minimum (perigee) of the elliptical orbit, the resulting occultation will be equal to or less than that calculated here.

The viewing direction, with respect to the orbit, is given by (ϕ, θ) where

ϕ = the angle from the ascending node to the viewing direction (0° to 360°)

θ = the angle from the orbital plane to the viewing direction (-90° to +90°)

The coordinates (ϕ, θ) are related to the astronomical coordinates right ascension and declination (α, δ) through the standard matrix transformation

$$\begin{pmatrix} \cos\phi\cos\theta \\ \sin\phi\cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \cdot \begin{pmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix} \quad (1)$$

which is expanded to

$$\left.\begin{aligned} \cos\phi\cos\theta &= \cos\Omega\cos\alpha\cos\delta + \sin\Omega\sin\alpha\cos\delta \\ \sin\phi\cos\theta &= -\cos i \sin\Omega\cos\alpha\cos\delta \\ &\quad + \cos i \cos\Omega\sin\alpha\cos\delta + \sin i \sin\delta \\ \sin\theta &= \sin i \sin\Omega\cos\alpha\cos\delta \\ &\quad - \sin i \cos\Omega\sin\alpha\cos\delta + \cos i \sin\delta \end{aligned}\right\} \quad (2)$$

If we define the angle ψ as the angle between the tangent to the orbit and the tangent to the "edge" of the earth, then the earth can be said to fill the sky by a plane angle of $180° - 2\psi$. If we look in a direction normal to the orbital plane, any stars in the cone with half-apex-angle ψ are unoccluted by the earth during the orbital rotation of the satellite. That is, viewing is unobstructed by the earth as long as $|\theta| \geqslant 90° - \psi$. This is illustrated in Figure 1 where we see that

$$\cos\psi = \frac{R+h}{R+H} \quad (3)$$

where R = earth radius

> h = height of atmosphere to point where it is no longer
> detrimental to astronomical observations tangent
> to that height. (Assumed hereafter to be 100km).

The absorption and scattering produced by the earth's atmosphere increases the effective radius of the earth as an occulter, and the acceptable effective radius depends upon the background acceptable for the observations to be performed. In this paper a value of 100km has been assumed for h. Scattering of sunlight is the predominant problem, and preliminary calculations indicate that viewing tangent to 100km the scattered sunlight is about 13-14 magnitude/arc sec^2. Observations of extremely faint objects would require a larger value for h. The background would be equal to the zodiacal background level (near 110° from the sun) if h were increased from 100 to 250km. At H=450km this would reduce the unocculted cone by 4°; i.e. the half angle of this cone would be reduced from 18° to 14°.

If the telescope were at height h, then for exactly 1/2 the orbital period each point in the sky would be observable, regardless of direction (except that the orbital poles represent singular points). That is, as the telescope moved through an orbital longitude of π radians it could look in any given direction without occultation. When the orbit is at a height greater than h, then the telescope may move through an angle 2γ radians in addition to π radians. Figure 2 shows the relationship between the orbital height, the viewing angle θ, and the angle γ. In terms of angle ψ, as defined above,

$$\cos \gamma = \cos \psi \cos [\sin^{-1} (\tan \psi \tan \theta)], \qquad (4)$$

for $|\theta| \leq \frac{\pi}{2} - \psi$. If $|\theta| > \frac{\pi}{2} - \psi$, then the earth does not occult at all, and we may think of γ as infinite (except as affected by the precession of the nodes).

The total time the telescope may point in any given direction without occultation is just

$$\tau = \frac{P}{2\pi} (\pi + 2 \gamma) \qquad (5)$$

where P is the orbital period. Figure 3 shows τ as a function of viewing angle θ for several representative low earth orbits. We see that the viewing time τ increases steadily as θ increases, and rises sharply as θ approaches the orbital poles. Low altitude orbits are of primary consideration because of logistics and radiation problems. In a synchronous orbit, the angular radius of the earth is only 8°8, the shortest observing time in any

direction is 22.8 hours, and 92.5% of the celestial sphere is visible without occultation by the earth during each 24 hour period.

Figure 4 shows the celestial sphere as it would appear from any orbit with $\omega=0°$ (the ascending node oriented toward the vernal equinox) and $i=28°5$ (inclination $28°5$ is planned for the Apollo missions.) In this Figure we see 1) the size of the Earth as seen from synchronous orbit (35,800 km), 2) the size of the Earth as seen from an orbital height of 450km, and 3) contours of equal θ (i.e. equal viewing time per orbit as shown in Figure 3). Figure 5 shows equal θ contours for $\omega=0°$ and $i=60°$. As i decreases, the orbital poles approach the celestial poles. At i=0, (equatorial orbit) the equal-θ contours correspond to declination contours, (the nearly horizontal contours in Figures 4 and 5). As Ω decreases, the orbital poles and equal-θ contours move horizontally to the left in Figures 4 and 5. A computer program is available to investigate any values of h, H, i and Ω.

Because of the oblateness of the earth, the orbit of an earth satellite is not fixed in space, but the apsidal and nodal lines precess. For the assumed circular orbit, the line of nodes regresses (precesses opposite to the direction of orbital motion) at the rate

$$\dot{\Omega} = -\frac{3}{2} J_2 \frac{R^2}{(R+H)^2} \dot{n} \cos i \tag{6}$$

where

$$\dot{n} = n_0 \left[1+\frac{3}{2} J_2 \frac{R^2}{(R+H)^2} (1- \frac{3}{2} \sin^2 i)\right] \approx n_0 \tag{7}$$

and J_2 is the 2nd harmonic coefficient of a Fourier expansion of the gravitational potential of the earth (1.083×10^{-3}). n_0 is the mean daily motion ($2\pi/P$).

The variation of $\dot{\Omega}$ as a function of i for various values of H is shown in Figure 6, where $\dot{\Omega}$ is expressed in arcmin/min. The period of the regression is also indicated.

As the node regresses, the θ contours shown in Figures 4 and 5 move to the left. In Figure 7 is shown the

swaths of the celestial sphere swept out by the unocculted circles in one regression period (52 days) for orbital altitude 450km and inclination 28°5. Any celestial object within these swaths will be observable continuously over several orbital periods, and for some parameters the time will be several days. The viewing time τ for objects with $|\theta| > \frac{\pi}{2} - \psi$ is

$$\tau = \frac{2}{|\dot{\Omega}|} \cos^{-1}(\frac{\cos\ \psi\ -\ \cos\ i\ \sin\ \delta}{\sin\ i\ \cos\ \delta}) \qquad (8)$$

For a star at the declination of the pole of the orbit, τ is about 266 hours. Obviously celestial objects at the orbital pole can be viewed the longest, so the "most available" declination zones are $\pm\ \delta = \frac{\pi}{2} - i$.

The problem, then, is to determine the optimal orbit and observing program to accomplish the best astronomy under the constraints imposed by radiation and by the launch vehicle. It is assumed that the launch date and time would be chosen to place the sun and the ascending node in the most desirable initial position. The node regresses at about 7° per day, (at 28°5; 450km) and the sun advances 1° per day, so the sun moves past the orbital polar regions at the rate of about 8° per day. In the worst case (i ≳ 90° - 23 1/2°) the orbital pole moves through the area of the sky near the sun (determined by the baffling design of the telescope and probably ≈40° in diameter) in five days. Thus, this is roughly the period of inaccessibility for one unocculted pole. During this time, however, the opposite pole is accessible. If the object of interest were the sun itself, then the most desirable orbit would be inclined at an angle greater than 90° - 23°5. In fact the orbit can be tailored to be "sun-synchronous."

For observations not of the sun, the observing program is best "declination-oriented." That is, the orbital inclination is chosen so that the unocculted belt is at the declination of interest, and the objects to be observed lie in that belt. Obviously, one orbital inclination cannot allow extended observations over the whole celestial sphere. Figure 8 is intended to show just a few objects of possible interest, such as nearby galaxies and star clusters and associations. Generally, stars and nebulae of interest are near the galactic plane; galaxies are far from the galactic plane. It would be difficult, if not impossible, to devise a comprehensive unocculted observing program at one declination. However, a number of interesting celestial objects do lie between $\delta=-43°$ and $\delta=-80°$, which is the south polar belt of the Apollo orbit.

To allow long term (unocculted) observations of all the celestial sphere, several orbiting observatories are required at various orbital inclinations between about 15° and 75°. As a specific example, since a 450km orbit has a polar belt about 36° wide, then two orbits of i=72° and i=36° would cover all but to within ±18° of the celestial poles. Three orbits at i=75°, i=45° and i=15° would cover the entire celestial sphere.
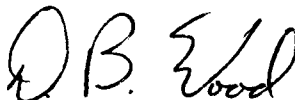
Naturally the entire celestial sphere can be covered from any orbit if we do not mind occultation by the earth once every revolution. An orbit near 90° inclination is of limited use because it precesses very slowly. An orbit near 0° inclination is undesirable because as it precesses the orbital poles hardly move.

CONCLUSIONS

Earth occultation in a low orbit (in excess of a few hundred km) should present no appreciable handicap to observing. A family of three or more orbiting observatories can cover the entire celestial sphere such that sometime within one nodal period of about two months, any celestial object is observable continuously for tens of hours. Data reduced after one pass could dictate the observations to be made on the next pass. In effect, the celestial "year" is about two months long, and each "night" is about ten days long.

Unocculted observing programs must be designed in declination belts, and most certainly cannot be classified as to type of object to observe. Galaxies, galactic cap stars and quasars might be observed for one node orientation, and a week or so later galactic clusters, emission nebulae and T-associations would be observed, as the orbital pole crossed the galactic plane.

In particular if a low energy orbit (28°5; 450km) is the only one available, it offers the opportunity to make extensive observations of the Magellanic clouds, the two nearest globular clusters, and interesting star associations and nebulae in the constellations Scorpio, Centaurus, and Monoceros, without concern for earth occultation every 47 minutes.
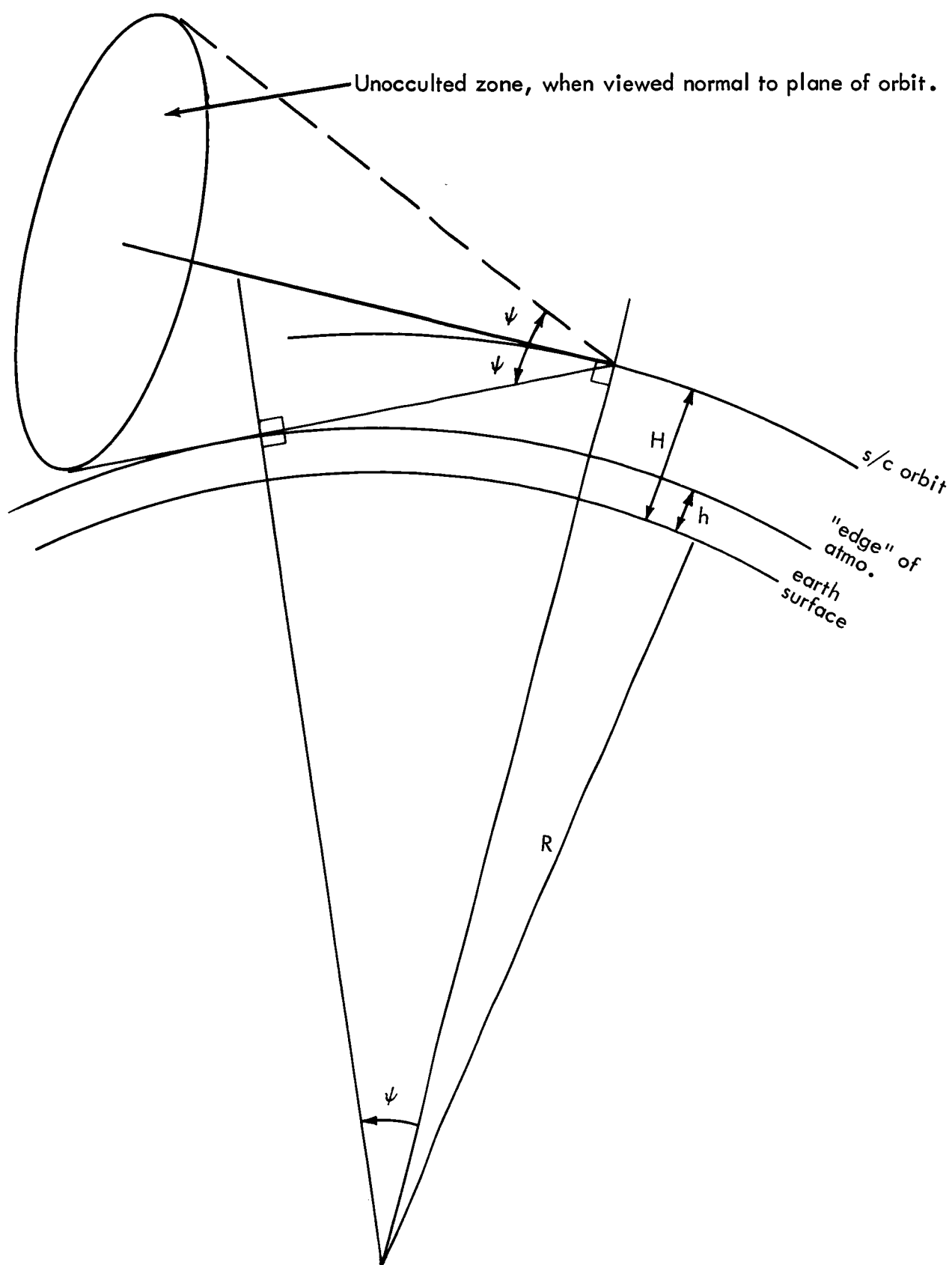
D. B. Wood

1015-DBW-mat

Attachments
  Figures 1-8

Unocculted zone, when viewed normal to plane of orbit.

ψ

ψ

H

h

s/c orbit

"edge" of atmo.

earth surface

R

ψ

FIGURE 1 - DERIVATION OF ANGLE FROM ORBITAL TANGENT TO ATMOSPHERIC EDGE

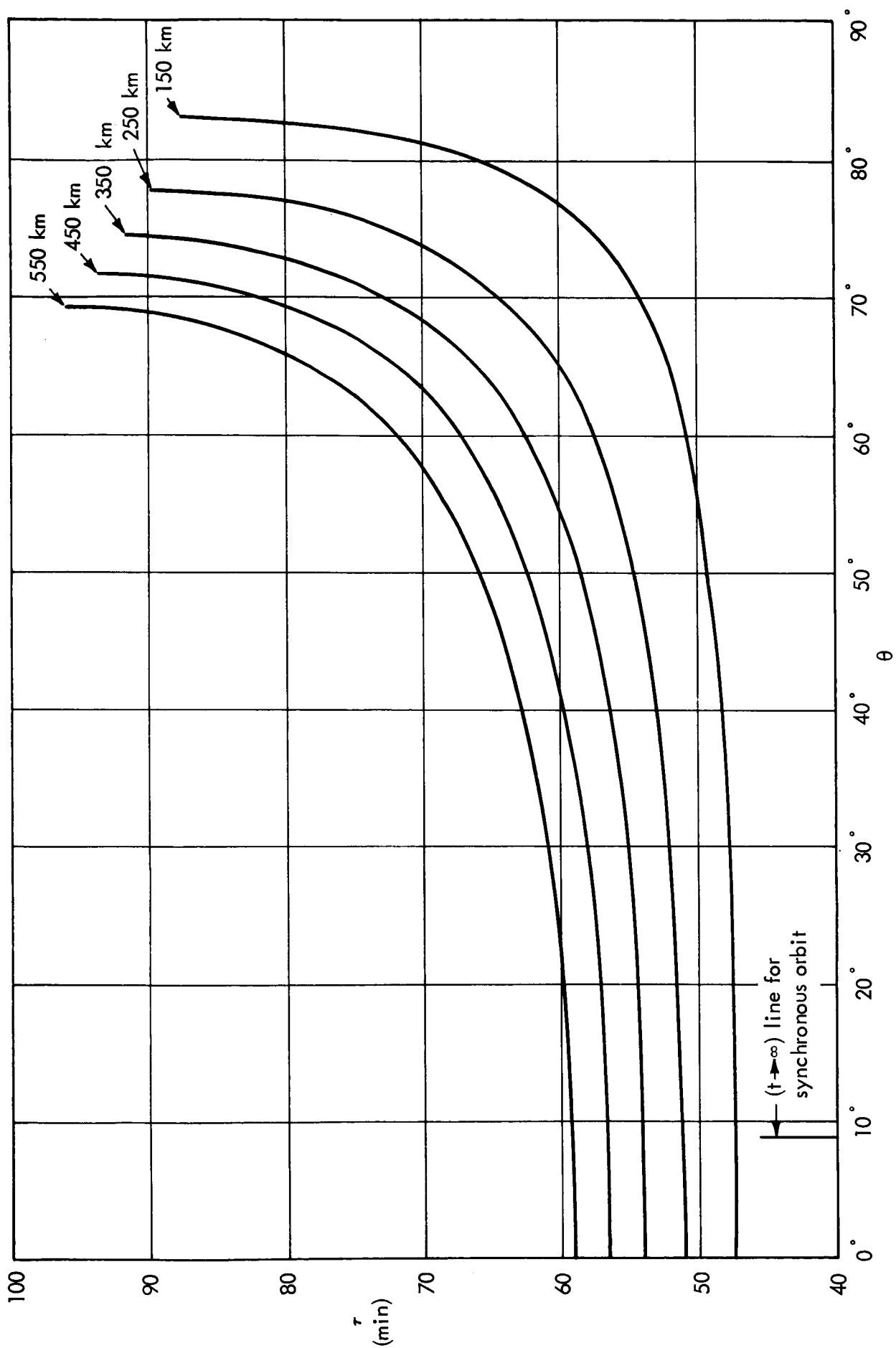FIGURE 2 – DERIVATION OF OBSERVATION TIME AS A FUNCTION OF LOOK ANGLE

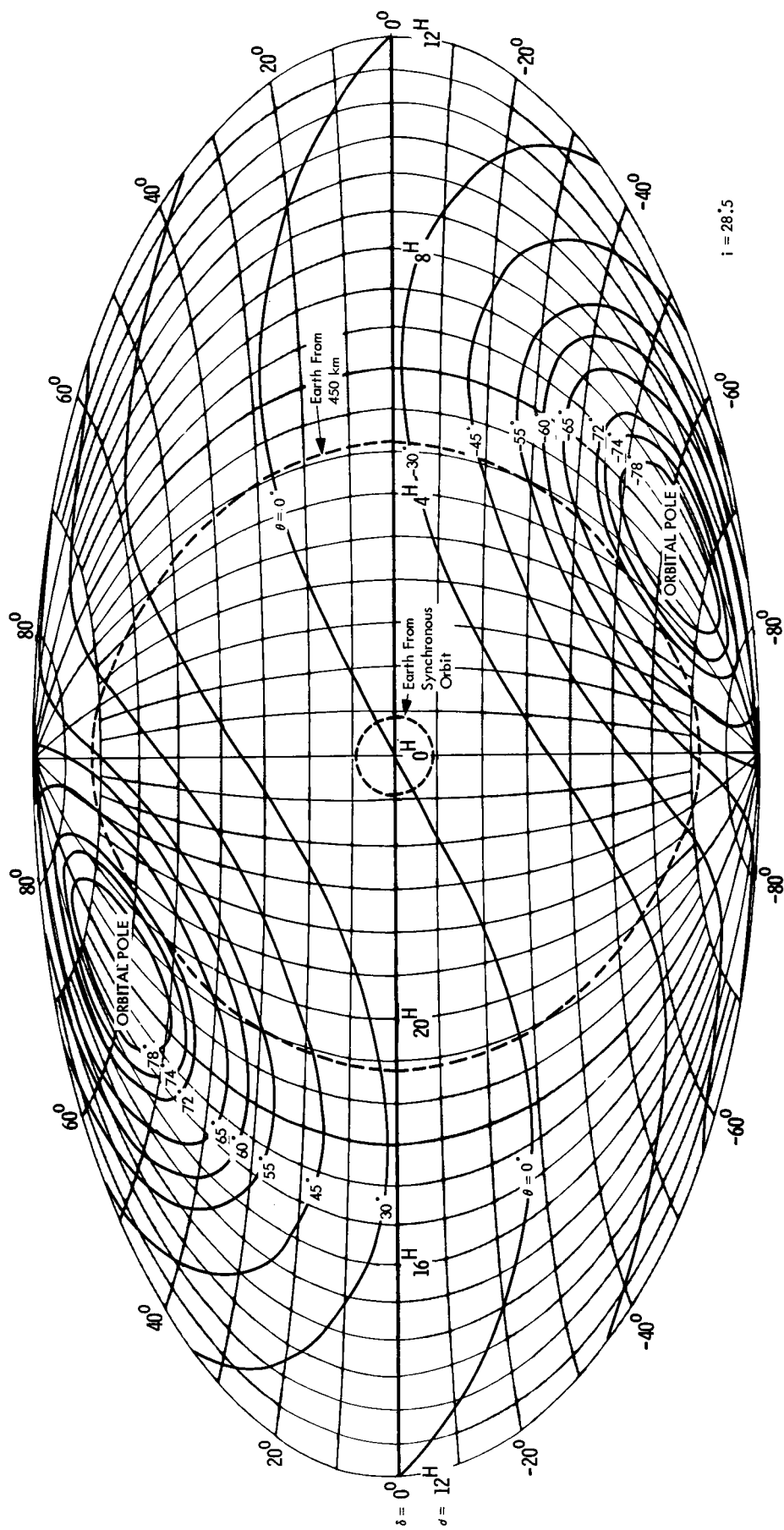FIGURE 3 — OBSERVATION TIME AS A FUNCTION OF LOOK ANGLE AND ORBITAL ALTITUDE
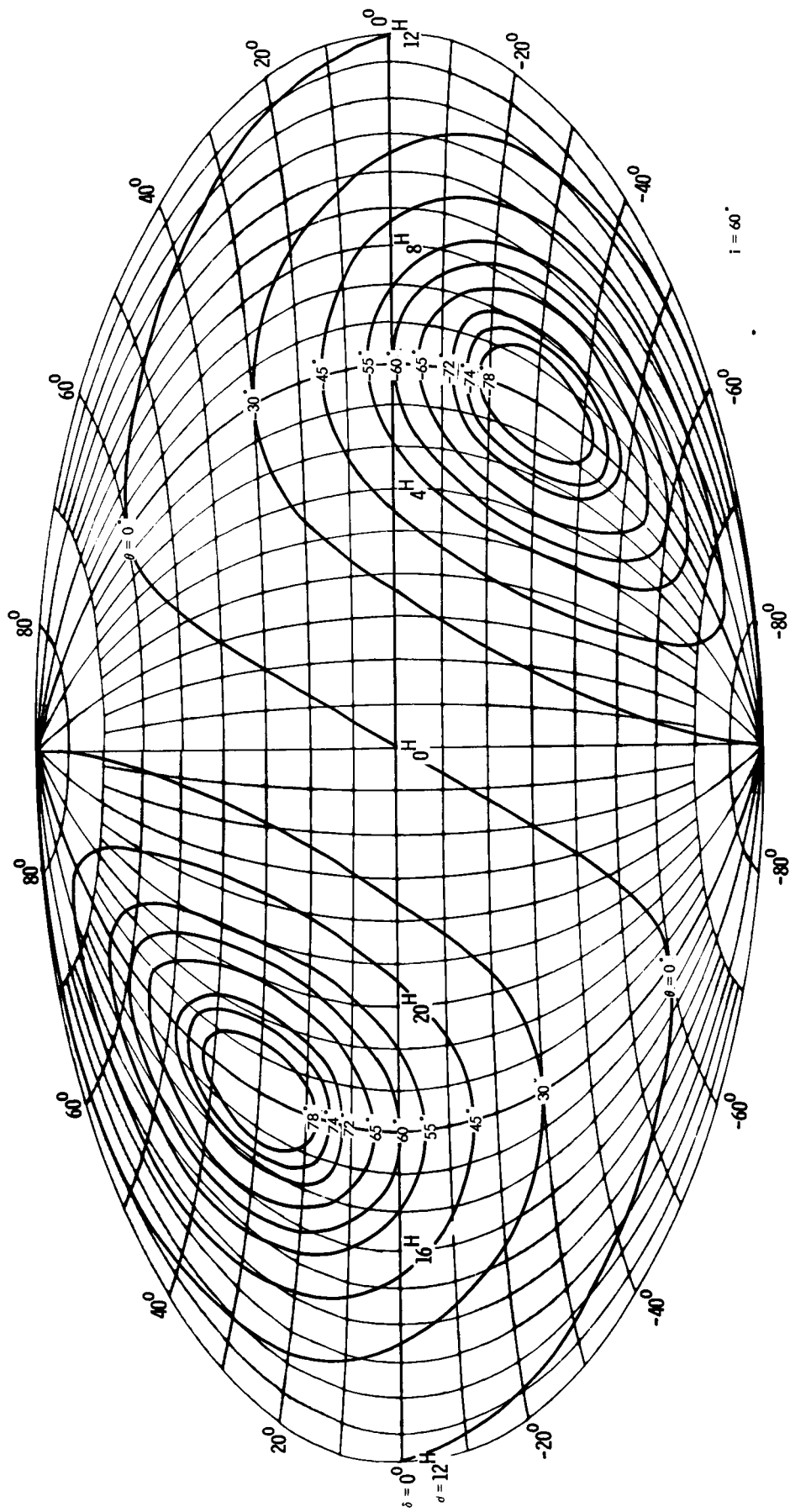
FIGURE 4 – CELESTIAL SPHERE AS SEEN FROM ORBIT WITH i = 28°.5

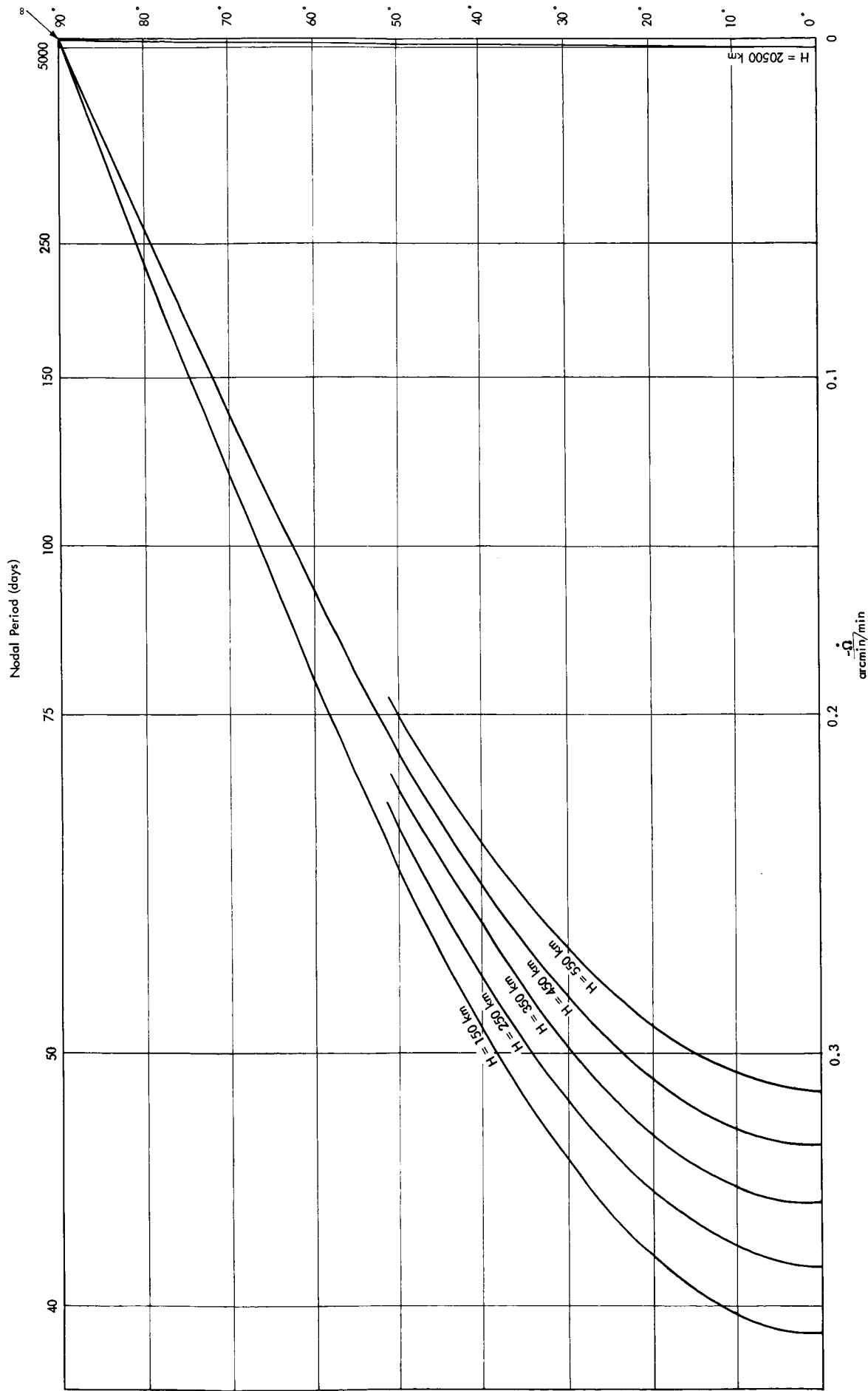FIGURE 5 – CELESTIAL SPHERE AS SEEN FROM ORBIT WITH i = 60°

FIGURE 6 – REGRESSION OF NODE AS A FUNCTION OF INCLINATION AND ORBITAL ALTITUDE
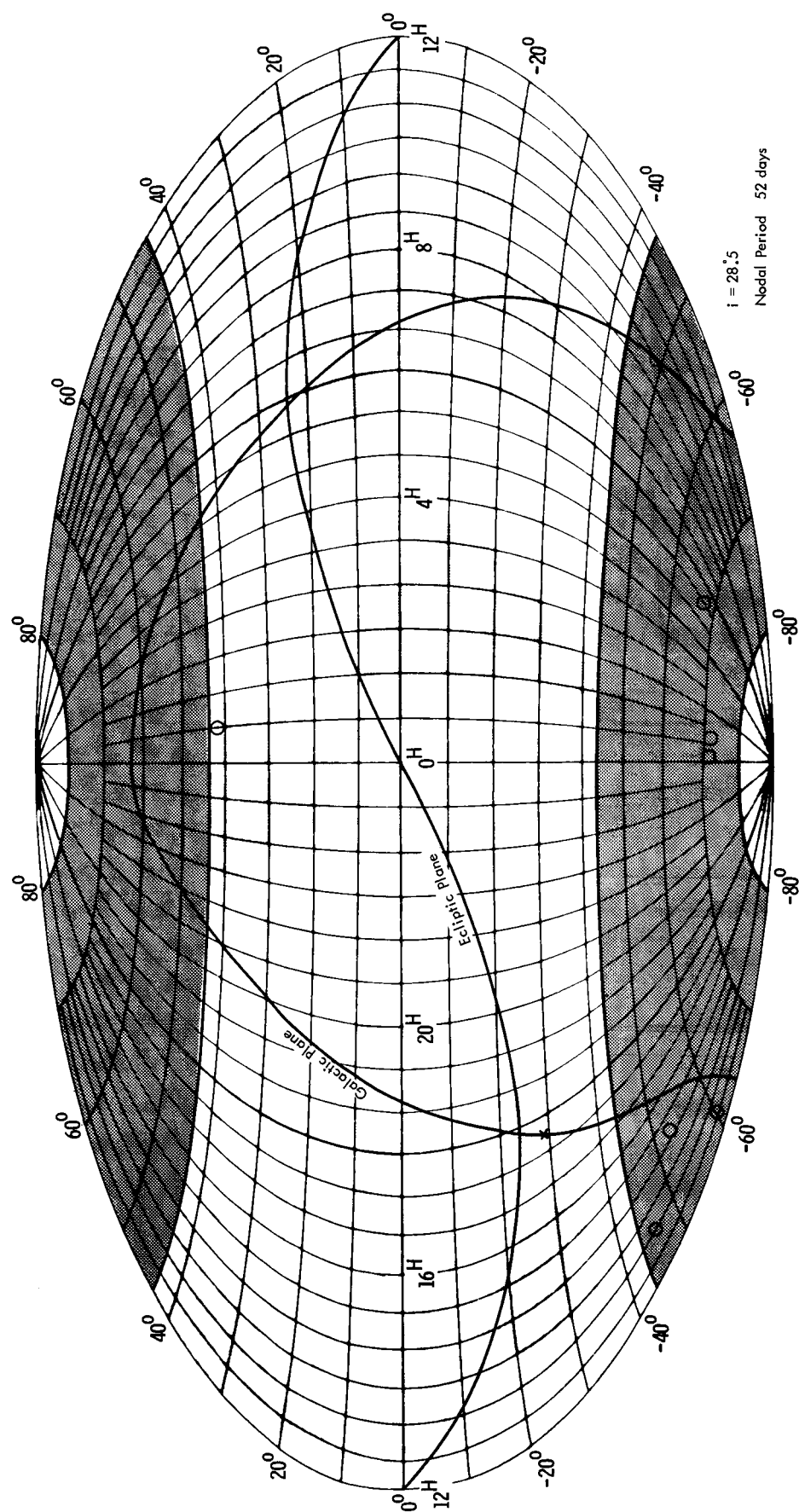
FIGURE 7 - AREA OF CELESTIAL SPHERE OBSERVABLE AT UNOCCULTED POLES FROM APOLLO APPLICATIONS ORBIT (i = 28°.5, H = 450 KM)

i = 28°.5

Nodal Period 52 days

FIGURE 8 – CELESTIAL SPHERE, SHOWING SOME AREAS OF INTEREST

• BRIGHT STARS

⊗ GALACTIC CENTER

○ GALAXIES: CLUSTERS

BELLCOMM, INC.

Subject: Effect of Earth Occultation on   From: D. B. Wood
Astronomical Observations from
Earth Orbit - Case 710.

DISTRIBUTION LIST

NASA Headquarters

Messrs. W. O. Armstrong/MTX
         M. J. Aucremanne/SG
         R. H. Chase/SG
         P. E. Culbertson/MLA
         J. H. Disher/MLD
         F. P. Dixon/MTY
         P. Grosz/MTL
         E. W. Hall/MTG
         T. A. Keegan/MA-2
         D. R. Lord/MTD
         J. L. Mitchell/SG
         B. G. Noblitt/MTY
         E. J. Ott/SG
         M. J. Raffensperger/MTE
         L. Reiffel/MA-6
         L. Roberts/OART-M (2)
         N. G. Roman/SG

Goddard Space Flight Center

         K. L. Hallam/613
         A. E. Hedin/621
         W. G. Stroud/110

Manned Spacecraft Center

         B. G. Jackson/TE
         E. H. Olling/ET4
         R. O. Piland/AH

Marshall Space Flight Center

         J. A. Downey/R-RP-J
         G. von Tiesenhausen-R-AS-VO

Bellcomm, Inc.

Messrs. F. G. Allen
         G. M. Anderson
         A. P. Boysen, Jr.
         B. F. Brown
         J. P. Downs
         D. R. Hagner
         P. L. Havenstein
         N. W. Hinners
         W. C. Hittinger
         B. T. Howard
         D. B. James
         J. Kranton
         K. E. Martersteck
         R. K. McFarland
         J. Z. Menard
         I. D. Nehama
         G. T. Orrok
         I. M. Ross
         F. N. Schmidt
         W. Strack
         J. M. Tschirgi
         R. L. Wagner
         J. E. Waldo
All Members Division 101
All Members Division 102
Department 1023
Central File
Library