28 December 1967

RSIC-740

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by

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No. 1, 33-47 (1967)

Translation Branch
Redstone Scientific Information Center
Research and Development Directorate
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

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The lunar-solar gravitational perturbations for two hypothetical artificial Earth satellites with a sidereal period of revolution of $23^h 56^m 04^s$ are analyzed.

ABSTRACT

Study of the lunar-solar gravitational perturbations of two hypothetical earth satellites having a sidereal period of $23^h 56^m 04^s$. The orbit of the first satellite lies in the equatorial plane while that of the second is perpendicular to the ecliptic plane. At the initial moment (12^h E.T., January 0, 1950), the nodes of the orbits of the two satellites and the moon coincide. The perigee distance of the first satellite is chosen so as to minimize the secular excitations of the orbital eccentricity at fixed values of all the remaining elements. The orbital parameters of the second satellite are chosen according to the condition of maximum secular perturbations of the orbital eccentricity. The orbital eccentricity of the first satellite is 0.1, while that of the second is 0.6. Results show that the orbit of the first satellite is stable and orbital eccentricity decreases at the rate of 0.00127 per year. Rapid increase in the eccentricity of the second satellite limits its lifetime to 5.5 years. Secular and periodic first order perturbations are calculated.

INTRODUCTION

A study is made of the lunar-solar gravitational perturbations of two hypothetical artificial Earth satellites having a sidereal period of revolution of $23^h 56^m 04^s$, i.e., one sidereal day.

The orbit of one of these satellites (we will call it for reference AES-1) lies in the equatorial plane while that of the second (AES-2) lies in a plane perpendicular to the ecliptic plane.

The eccentricity of the orbit of the equatorial satellite is equal to 0.1 and that of the orbit of the second satellite is equal to 0.6, i.e., slightly less than Laplace limit.

At the initial instant (12^{h} E.T., January 0, 1950) the nodes of the orbits of both satellites and of the Moon coincide while the satellites themselves are in the ascending nodes of their orbits. The longitude of the perigee of the AES-1 was chosen so as to minimize the secular perturbations of the eccentricity of its orbit at fixed values of all of the remaining elements.

The orbital parameters of the AES-2 were chosen in accordance with the condition of maximum secular perturbations of the eccentricity of its orbit. Being connected with the perigee distance by the relation

$$p = a(1 - e),$$

this element is the most important one in the evolution of the orbit since the lifetime of the satellite depends on the character of its variation.

As shown by the calculations carried out, the orbit of the equatorial satellite is stable. Because of a decrease in the eccentricity of its orbit by 0.00127 per year attributable to the combined action of the Moon and the Sun, a rise of the perigee altitude takes place at the rate of 53 km per year. In the course of time, as the satellite's orbit approaches a circular orbit, the rate of this rise decreases.

A rapid increase in the eccentricity of the orbit of the AES-2 (by 0.046 per year) leads to the termination of the existence of this satellite in only 5.5 years.

The calculation of the lunar-solar perturbations of the first order relative to the parameter

$$\mu_0 = \frac{m_2}{m_0} \alpha^3$$

(m_0 — the Earth's mass; m_2 — the mass of the perturbing body; α — the ratio of the major half-axes of the orbits of the satellite and the perturbing body) was carried out on the basis of the procedure described in the author's article (Martynenko, 1966) on the assumption that the elements of the Moon's orbit are constant.

The influence of the effects of the noncentricity of the Earth's gravitational field, atmospheric resistance, and solar radiation pressure on the magnitude of the perturbations attributable to the Moon and the Sun was not taken into account. The mutual influence of the lunar and solar perturbations also was not taken into consideration.

The elements of the orbits of the satellites, the Moon, and the Sun are shown in Table 1.

TABLE 1

THE ELEMENTS OF THE ORBITS OF THE AES-1, AES-2, THE MOON AND THE SUN*

	AES-1	AES-2	The Moon	The Sun
a	$6.61066874a_e$	$6.661066874a_e$	$60.323a_e$	$23370.1532a_e$
e	0.1	0.6	0.054900489	0.016751
i	$23^{\circ}.445788$	90°	$5^{\circ}.14538864$	0
Ω	12.139268	12.139268	12.139268	0
π	57.139268	147.139267	208.788281	282.08039
ϵ	12.139268	12.139268	57.786764	279.588317
n	360.985649	360.985649	13.1760685	0.985647344

* Here a_e is the equatorial radius of the Earth.

1. The First-Order Secular Perturbations

On the basis of the differential equations for the osculating elements of a satellite's orbit (Lyakh, 1960):

$$\frac{da}{dt} = 2\mu_0 na \frac{\partial R_1}{\partial L},$$

$$\frac{de}{dt} = -\mu_0 n \left(\cot\varphi \frac{\partial R_1}{\partial \Pi} + \tan\frac{\varphi}{2} \cos\varphi \frac{\partial R_1}{\partial L}\right),$$

$$\frac{di}{dt} = \mu_0 n \sec\varphi \left[\cos(\tau - \Omega) \cot I \left(\frac{\partial R_1}{\partial L} + \frac{\partial R_1}{\partial \Pi}\right)\right.$$

$$\left. + \frac{\cos(\tau - \Omega)}{\sin I} \left(\frac{\partial R_1}{\partial L'} + \frac{\partial R_1}{\partial \Pi'}\right) - \frac{1}{2} \sin I \sin(\tau - \Omega) \left(\frac{\partial R_1}{\partial \nu} - \frac{\partial R_1}{\partial \mu}\right)\right],$$

$$\sin i \, \frac{d\Omega}{dt} = \mu_0 n \sec \varphi \left[\cot I \sin(\tau - \Omega) \left(\frac{\partial R_1}{\partial L} + \frac{\partial R_1}{\partial \Pi} \right) \right.$$

$$+ \frac{\sin(\tau - \Omega)}{\sin I} \left(\frac{\partial R_1}{\partial L'} + \frac{\partial R_1}{\partial \Pi'} \right)$$

$$\left. + \frac{1}{2} \sin I \cos(\tau - \Omega) \left(\frac{\partial R_1}{\partial \nu} - \frac{\partial R_1}{\partial \mu} \right) \right] ,$$

$$\frac{d\pi}{dt} = \tan\frac{i}{2} \sin i \, \frac{d\Omega}{dt} + \mu_0 n \cot \varphi \, \frac{\partial R_1}{\partial e} ,$$

$$\frac{d\epsilon}{dt} = -2\mu_0 n \frac{1}{\alpha} \frac{\partial(\alpha^2 R_1)}{\partial \alpha} + \tan\frac{i}{2} \sin i \, \frac{d\Omega}{dt}$$

$$+ \mu_0 n \tan\frac{\varphi}{2} \cos\varphi \, \frac{\partial R_1}{\partial e} ,$$

$$\frac{dn}{dt} = -3\mu_0 n^2 \frac{\partial R_1}{\partial L} ,$$

where

$$R_1 = \sum_{m=2}^{\infty} \alpha^{m-2} \sum_{s=0}^{\infty} e^{s} \sum_{s'=0}^{\infty} e'^{s'} \sum_{q=0(1)}^{m} \sum_{j=-m}^{+m} \sum_{k=-s'}^{+s'} \sum_{l=-s}^{+s}$$

$$A_{q,j,k,l}^{m,s,s'}(\nu, \mu) \cos[(q+k)L' + (j+l)L - k\Pi' - l\Pi],$$

$$q \equiv |j| \equiv m \pmod 2, \quad |k| \equiv s' \pmod 2, \quad |l| \equiv s \pmod 2,$$

$$L = nt + s - \tau, \qquad L' = n't + \epsilon' - \tau',$$

$$\tau = \Omega + N, \qquad \tau' = \Omega' + N',$$

$$\Pi = \pi - \tau, \qquad \Pi = \pi' - \tau',$$

$$\sin N = \frac{\sin i' \sin(\Omega - \Omega')}{2\sqrt{\nu\mu}} ,$$

$$\sin N' = \frac{\sin i \sin (\Omega - \Omega')}{2\sqrt{\nu\mu}},$$

$$\cos N = \frac{\sin i \cos i' - \cos i \sin i' \cos (\Omega - \Omega')}{2\sqrt{\nu\mu}},$$

$$\cos N' = \frac{-\sin i' \cos i + \cos i' \sin i \cos (\Omega - \Omega')}{2\sqrt{\nu\mu}},$$

$$\nu = \sin^2 \frac{I}{2} = \sin^2 \frac{\Omega - \Omega'}{2} \sin^2 \frac{i + i'}{2} + \cos^2 \frac{\Omega - \Omega'}{2} \sin^2 \frac{i - i'}{2},$$

$$\mu = \cos^2 \frac{I}{2} = \sin^2 \frac{\Omega - \Omega'}{2} \cos^2 \frac{i + i'}{2} + \cos^2 \frac{\Omega - \Omega'}{2} \cos^2 \frac{i - i'}{2},$$

$$\alpha = \frac{a}{a'}, \quad \varphi = \text{arc} \sin e,$$

a, e, i, Ω, π, and ϵ are the elements of the satellite's orbit,

a, e', Ω', π', and ϵ' are elements of the orbit of the perturbing body,

n and n' are the mean motions of the satellite and the perturbing body

and also on the basis of the tables of the coefficients $A^{m,s,s'}_{q,j,k,l}(\nu, \mu)$ and $a^{(m)}_{k',k}(\nu, \mu)$ (Martynenko, 1965) it is easy to obtain expressions for the first-order secular perturbations of the elements of the satellite's orbit relative to the parameter μ_0. Indeed, taking into account that the secular terms in the perturbations of the elements are generated only by those terms of the expansion of the function of R_1 which explicitly do not contain the time, we obtain the following with accuracy to the sixth-order terms relative to the parameters α, e, e'

$$[\delta a] = 0;$$

$$[\delta e] = t \times q_2 \sum_{\substack{q+k=0 \\ j+l=0}} \alpha^{m-2} e^{s} e'^{s'} l A^{m,s,s'}_{q,j,k,l}(\nu, \mu) \times \sin(-kII' - lII)$$

$$= t \times q_2 \{-7.5e^2(1 + 1.5e'^2 + 1.875e'^4)\nu\mu \sin 2II$$

$$- 3.75\alpha ee'(1 + 2.5e'^2)[(1.25\mu^2 + 2.5\nu^2 - 1)\mu \sin(\Pi' - \Pi)$$

$$- (1.25\nu^2 + 2.5\mu^2 - 1)\nu \sin(\Pi' + \Pi)]$$

$$- 2.8125\alpha e^3 e'[8.75\nu\mu^2 \sin(\Pi' - 3\Pi)$$

$$+ (1.25\mu^2 + 2.5\nu^2 - 1)\mu \sin(\Pi' - \Pi)$$

$$- 8.75\nu^2\mu \sin(\Pi' + 3\Pi)$$

$$- (1.25\nu^2 + 2.5\mu^2 - 1)\nu \sin(\Pi' + \Pi)]$$

$$- 19.6875\alpha^2 e^2 [1.75(\nu^2 + \mu^2) - 1]\nu\mu \sin 2\Pi$$

$$- 14.765625\alpha^2 e^2 e'^2 \left[\frac{40}{3}(1.75(\mu^2 + \nu^2) - 1)\nu\mu \sin 2\Pi\right.$$

$$- \left(\frac{105}{90}\mu^2 + 3.5\nu^2 - 1\right)\mu^2 \sin(2\Pi' - 2\Pi)$$

$$\left. + \left(\frac{105}{90}\nu^2 + 3.5\mu^2 - 1\right)\nu^2 \sin(2\Pi' + 2\Pi)\right]$$

$$- 57.6796875\alpha^2 e^4 \nu^2 \mu^2 \sin 4\Pi$$

$$- 13.125\alpha^3 ee' [(2.625\mu^4 + 15.75\nu^2\mu^2$$

$$+ 7.875\nu^4 - 3.5\mu^2 - 7\nu^2 + 1)\mu \sin(\Pi' - \Pi)$$

$$- (2.625\nu^4 + 15.75\nu^2\mu^2 + 7.875\mu^4 - 3.5\nu^2$$

$$- 7\mu^2 + 1)\nu \sin(\Pi' + \Pi)]$$

$$- 118.125\alpha^4 e^2 [4.125(\nu^4 + \mu^4) + 12.375\nu^2\mu^2$$

$$- 4.5(\nu^2 + \mu^2) + 1]\nu\mu \sin 2\Pi\};$$

$$[\delta i] = t \times q_3 \sum_{\substack{q+k=0 \\ j+l=0}} \alpha^{m-2} e^s e'^{s'} [A^{m,s,s'}_{q,j,k,l}(\nu, \mu)(lp_2 + kp_3)\sin(-k\Pi' - l\Pi)$$

$$- p_4(A\nu - A\mu)\cos(-k\Pi' - l\Pi)]$$

$$
\begin{aligned}
= t \times q_3 \{ & -1.5p_4(1 + 1.5e'^2 + 1.875e'^4 - 2.1875e'^6 + 1.5e^2 + 2.25e^2e'^2 \\
& \quad + 2.8125e^2e'^4)(\nu - \mu) \\
& - 3.75e^2(1 + 1.5e'^2 + 1.875e'^4)[2p_2\nu\mu \sin 2\Pi + p_4(\mu - \nu)\cos 2\Pi] \\
& - 3.75\alpha ee'(1 + 2.5e'^2 + 0.75e^2)[(p_2 - p_3)\mu(1.25\mu^2 \\
& \quad + 2.5\nu^2 - 1)\sin(\Pi' - \Pi) \\
& \qquad - p_4(5\nu\mu - 3.75\mu^2 - 2.5\nu^2 + 1)\cos(\Pi' - \Pi) \\
& \qquad - (p_2 + p_3)\nu(1.25\nu^2 + 2.5\mu^2 - 1)\sin(\Pi' + \Pi) \\
& \qquad - p_4(3.75\nu^2 + 2.5\mu^2 - 5\nu\mu - 1)\cos(\Pi' + \Pi)] \\
& - 8.203125\alpha e^3e'[(3p_2 - p_3)\nu\mu^2 \sin(\Pi' - 3\Pi) - p_4\mu(\mu - 2\nu)\cos(\Pi' - 3\Pi) \\
& \quad - (3p_2 + p_3)\nu^2\mu \sin(3\Pi' + \Pi) + p_4\nu(\nu - 2\mu)\cos(3\Pi' + \Pi)] \\
& - 3.75\alpha^2(1 + 5e'^2 + 13.125e'^4 + 5e^2 + 25e^2e'^2 + 1.875e^4)p_4 \\
& \quad \times (1.75\nu^3 + 3.5\nu\mu^2 - 3.5\nu^2\mu - 1.75\mu^3 + \mu - \nu) \\
& - 2.8125\alpha^2e'^2(1 + 3.5e'^2 + 5e^2)[2p_3\nu\mu(1.75\nu^2 + 1.75\mu^2 - 1)\sin 2\Pi' \\
& \quad + p_4(5.25\nu^2\mu + 1.75\mu^3 - \mu - 1.75\nu^3 - 5.25\nu\mu^2 + \nu)\cos 2\Pi'] \\
& - 19.6875\alpha^2e^2(1 + 5e'^2 + 0.5e^2)[2p_2\nu\mu(1.75\nu^2 + 1.75\mu^2 - 1)\sin 2\Pi \\
& \quad + p_4(1.75\mu^3 - 5.25\nu\mu^2 + 5.25\nu^2\mu - \mu - 1.75\nu^3 + \nu)\cos 2\Pi] \\
& + 3.9375\alpha^2e^2e'^2[2(p_2 - p_3)\mu^2(2.1875\mu^2 + 6.5625\nu^2 - 1.875)\sin(2\Pi' - 2\Pi) \\
& \quad - p_4\mu(13.125\nu\mu - 8.75\mu^2 - 13.125\nu^2 + 3.75)\cos(2\Pi' - 2\Pi) \\
& \quad - 2(p_2 + p_3)\nu^2(2.1875\nu^2 + 6.5625\mu^2 - 1.875)\sin(2\Pi' + 2\Pi) \\
& \quad - p_4\nu(8.75\nu^2 + 13.125\mu^2 - 13.125\nu\mu - 3.75)\cos(2\Pi' + 2\Pi)] \\
& - 51.6796875\alpha^2e^4\nu\mu[2p_2\nu\mu \sin 4\Pi + p_4(\mu - \nu)\cos 4\Pi]
\end{aligned}
$$

$$- 13.125\alpha^3 ee'[(p_2 - p_3)\mu(2.625\mu^4 + 15.75\nu^2\mu^2 + 7.875\nu^4$$

$$- 3.5\mu^2 - 7\nu^2 + 1) \sin(\Pi' - \Pi)$$

$$- p_4(31.5\nu\mu^3 + 31.5\nu^3\mu - 13.125\mu^4 - 47.25\nu^2\mu^2$$

$$- 7.875\nu^4 - 14\nu\mu$$

$$+ 10.5\mu^2 + 7\nu^2 - 1) \cos(\Pi' - \Pi)$$

$$- (p_2 + p_3)\nu(2.625\nu^4 + 15.75\nu^2\mu^2 + 7.875\mu^4$$

$$- 3.5\nu^2 - 7\mu^2 + 1) \sin(\Pi' + \Pi)$$

$$- p_4(13.125\nu^4 + 47.25\nu^2\mu^2 + 7.875\mu^4 - 10.5\nu^2 - 7\mu^2 - 31.5\nu^3\mu$$

$$- 31.5\nu\mu^3 + 14\nu\mu + 1) \cos(\Pi' + \Pi)]$$

$$- 6.5625\alpha^4(1 + 10.5e'^2 + 10.5e^2)p_4(4.125\nu^5 + 24.75\nu^3\mu^2$$

$$+ 12.375\nu\mu^4 - 4.5\nu^3$$

$$- 9\nu\mu^2 + \nu - 12.375\nu^4\mu - 24.75\nu^2\mu^3 - 4.125\mu^5$$

$$+ 9\nu^2\mu + 4.5\mu^3 - \mu)$$

$$- 16.40625\alpha^4 e'^2[2p_3\nu\mu(4.125\nu^4 + 12.375\nu^2\mu^2 + 4.125\mu^4 - 4.5\nu^2$$

$$- 4.5\mu^2 + 1) \sin 2\Pi'$$

$$+ p_4(20.625\nu^4\mu + 37.125\nu^2\mu^3 + 4.125\mu^5 - 13.5\nu^2\mu - 4.5\mu^3 + \mu$$

$$- 4.125\nu^5 - 37.125\nu^3\mu^2 - 20.625\nu\mu^4 + 4.5\nu^3$$

$$+ 13.5\nu\mu^2 - \nu) \cos 2\Pi']$$

$$-59.0625\alpha^4 e^2[2p_2\mu\nu(4.125\nu^4 + 12.375\nu^2\mu^2 + 4.125\mu^4 - 4.5\nu^2$$

$$- 4.5\mu^2 + 1) \sin 2\Pi$$

$$+ p_4(20.625\nu^4\mu + 37.125\nu^2\mu^3 + 4.125\mu^5 - 13.5\nu^2\mu$$

$$- 4.5\mu^3 + \mu - 4.125\nu^5$$

$$- 37.125\nu^3\mu^2 - 20.625\nu\mu^4 + 4.5\nu^3 + 13.5\nu\mu^2 - \nu) \cos 2\Pi]$$

$$- 9.84375\alpha^6 p_4(11.171875\nu^7 + 134.0625\nu^5\mu^2 + 201.09375\nu^3\mu^4$$

$$+ 44.6875\nu\mu^6$$

$$- 17.875\nu^5 - 107.25\nu^3\mu^2 - 53.625\nu\mu^4 + 8.25\nu^3 + 16.5\nu\mu^2 - \nu$$

$$- 44.6875\nu^6\mu - 201.09375\nu^4\mu^3 - 134.0625\nu^2\mu^5 - 11.171875\mu^7$$

$$+ 53.625\nu^4\mu + 107.25\nu^2\mu^3 + 17.875\mu^5 - 16.5\nu^2\mu$$

$$- 8.25\mu^3 + \mu)\};$$

$$[\delta\Omega] = t \times q_4 \sum_{\substack{q+k=0 \\ j+l=0}} \alpha^{m-2} e^s e'^{s'} [A^{m,s,s'}_{q,j,k,l}(\nu,\mu)(lp_5 + kp_6) \sin(-k\Pi' - l\Pi)$$

$$+ p_7(A\nu - A\mu) \cos(-k\Pi' - l\Pi)]$$

$$= t \times q_4\{1.5(1 + 1.5e'^2 + 1.875e'^4 - 2.1875e'^6 + 1.5e^2 + 2.25e^2e'^2$$

$$+ 2.8125e^2e'^4) \times p_7(\nu - \mu)$$

$$- 3.75e^2(1 + 1.5e'^2 + 1.875e'^4)[2p_5\nu\mu \sin 2\Pi - p_7(\mu - \nu) \cos 2\Pi]$$

$$-3.75\alpha ee'(1 + 2.5e'^2 + 0.75e^2)[(p_5 - p_6)\mu(1.25\mu^2 + 2.5\nu^2 - 1)\sin(\Pi' - \Pi)$$

$$+ p_7(5\nu\mu - 3.75\mu^2 - 2.5\nu^2 + 1) \cos(\Pi' - \Pi)$$

$$- (p_5 + p_6)\nu(1.25\nu^2 + 2.5\mu^2 - 1) \sin(\Pi' + \Pi)$$

$$+ p_7(3.75\nu^2 + 2.5\mu^2 - 5\nu\mu - 1) \cos(\Pi' + \Pi)]$$

$$- 8.203125\alpha e^3 e'[(3p_5 - p_6)\nu\mu^2 \sin(\Pi' - 3\Pi) + p_7\mu(\mu - 2\nu) \cos(\Pi' - 3\Pi)$$

$$- (3p_5 + p_6)\nu^2\mu \sin(3\Pi' + \Pi) - p_7\nu(\nu - 2\mu) \cos(3\Pi' + \Pi)]$$

$$+ 3.75\alpha^2(1 + 5e'^2 + 13.125e'^4 + 5e^2 + 25e^2e'^2 + 1.875e^4)p_7$$

$$\times (1.75\nu^3 + 3.5\nu\mu^2 - 3.5\nu^2\mu - 1.75\mu^3 + \mu - \nu)$$

$$- 2.8125\alpha^2 e'^2(1 + 3.5e'^2 + 5e^2)[2p_6\nu\mu(1.75\nu^2 + 1.75\mu^2 - 1)\sin 2\Pi'$$

$$- p_7(5.25\nu^2\mu + 1.75\mu^3 - \mu - 1.75\nu^3 - 5.25\nu\mu^2 + \nu)\cos 2\Pi']$$

$$- 19.6875\alpha^2 e^2(1 + 5e'^2 + 0.5e^2)[2p_5\nu\mu(1.75\nu^2 + 1.75\mu^2 - 1)\sin 2\Pi$$

$$- p_7(1.75\mu^3 - 5.25\nu\mu^2 + 5.25\nu^2\mu - \mu - 1.75\nu^3 + \nu)\cos 2\Pi]$$

$$+ 3.9375\alpha^2 e^2 e'^2[2(p_5 - p_6)\mu^2(2.1875\mu^2 + 6.5625\nu^2 - 1.875)\sin(2\Pi' - 2\Pi)$$

$$+ p_7\mu(13.125\nu\mu - 8.75\mu^2 - 13.125\nu^2 + 3.75)\cos(2\Pi' - 2\Pi)$$

$$- 2(p_5 + p_6)\nu^2(2.1875\nu^2 + 6.5625\mu^2 - 1.875)\sin(2\Pi' + 2\Pi)]$$

$$+ p_7\nu(8.75\nu^2 + 13.125\mu^2 - 13.125\nu\mu - 3.75)\cos(2\Pi' + 2\Pi)$$

$$- 51.6796875\alpha^2 e^4\nu\mu[2p_5\nu\mu\sin 4\Pi - p_7(\mu - \nu)\cos 4\Pi]$$

$$- 13.125\alpha^3 ee'[(p_5 - p_6)\mu(2.625\mu^4 + 15.75\nu^2\mu^2 + 7.875\nu^4 - 3.5\mu^2$$

$$- 7\nu^2 + 1)\sin(\Pi' - \Pi)$$

$$+ p_7(31.5\nu\mu^3 + 31.5\nu^3\mu - 13.125\mu^4 - 47.25\nu^2\mu^2$$

$$- 7.875\nu^4 - 14\nu\mu$$

$$+ 10.5\mu^2 + 7\nu^2 - 1)\cos(\Pi' - \Pi)$$

$$- (p_5 + p_6)\nu(2.625\nu^4 + 15.75\nu^2\mu^2 + 7.875\mu^4 - 3.5\nu^2$$

$$- 7\mu^2 + 1)\sin(\Pi' + \Pi)$$

$$+ p_7(13.125\nu^4 + 47.25\nu^2\mu^2 + 7.875\mu^4 - 10.5\nu^2$$

$$- 7\mu^2 - 31.5\nu^3\mu - 31.5\nu\mu^3$$

$$+ 14\nu\mu + 1)\cos(\Pi' + \Pi)]$$

$$+ 6.5625\alpha^4(1 + 10.5e'^2 + 10.5e^2)p_7(4.125\nu^5 + 24.75\nu^3\mu^2$$

$$+ 12.375\nu\mu^4 - 4.5\nu^3$$

$$- 9\nu\mu^2 + \nu - 12.375\nu^4\mu - 24.75\nu^2\mu^3 - 4.125\mu^5$$

$$+ 9\nu^2\mu + 4.5\mu^3 - \mu)$$

$$- 16.40625\,\alpha^4 e'^2 [\,2p_6 \nu\mu (4.125\nu^4 + 12.375\nu^2\mu^2 + 4.125\mu^4 - 4.5\nu^2$$

$$- 4.5\mu^2 + 1)\ \sin 2\mathrm{II}'$$

$$- p_7 (20.625\nu^4\mu + 37.125\nu^2\mu^3 + 4.125\mu^5 - 13.5\nu^2\mu - 4.5\mu^3 + \mu$$

$$- 4.125\nu^5 - 37.125\nu^3\mu^2 - 20.625\nu\mu^4 + 4.5\nu^3$$

$$+ 13.5\nu\mu^2 - \nu)\ \cos 2\mathrm{II}'\,]$$

$$- 59.0625\,\alpha^4 e^2 [\,2p_5 \mu\nu (4.125\nu^4 + 12.375\nu^2\mu^2 + 4.125\mu^4 - 4.5\nu^2$$

$$- 4.5\mu^2 + 1)\ \sin 2\mathrm{II}$$

$$- p_7 (20.625\nu^4\mu + 37.125\nu^2\mu^3 + 4.125\mu^5 - 13.5\nu^2\mu$$

$$- 4.5\mu^3 + \mu$$

$$- 4.125\nu^5 - 37.125\nu^3\mu^2 - 20.625\nu\mu^4 + 4.5\nu^3$$

$$+ 13.5\nu\mu^2 - \nu)\ \cos 2\mathrm{II}\,]$$

$$+ 9.84375\,\alpha^6 p_7 (11.171875\nu^7 + 134.0625\nu^5\mu^2 + 201.09375\nu^3\mu^4$$

$$+ 44.6875\nu\mu^6$$

$$- 17.875\nu^5 - 107.25\nu^3\mu^2 - 53.625\nu\mu^4 + 8.25\nu^3 + 16.5\nu\mu^2 - \nu$$

$$- 44.6875\nu^6\mu - 201.09375\nu^4\mu^3 - 134.0625\nu^2\mu^5 - 11.171875\mu^7$$

$$+ 53.625\nu^4\mu + 107.25\nu^2\mu^3 + 17.875\mu^5 - 16.5\nu^2\mu$$

$$- 8.25\mu^3 + \mu)\,\};$$

$$[\,\delta\pi\,] = q_5 [\,\delta\Omega\,] + t \cdot q_2 \sum_{\substack{q + k = 0 \\ j + l = 0}}{}' \; s\,\alpha^{m-2} e^{s-1} e'^{s'}$$

$$A^{m,s,s'}_{q,j,k,l}(\nu, \mu) \cos (-k\mathrm{II}' - l\mathrm{II})$$

$$
\begin{aligned}
= q_5[\delta\Omega] + t \times q_2\{&1.5e[5(1 + 1.5e'^2 + 1.875e'^4)\,\nu\mu \cos 2\Pi \\
&+ (1 + 1.5e'^2 + 1.875e'^4)(1.5\nu^2 + 1.5\mu^2 - 1)] \\
&- 3.75\alpha e'[(1 + 2.5e'^2 + 2.25e^2)(1.25\mu^2 + 2.5\nu^2 \\
&\quad - 1)\,\mu \cos(\Pi' - \Pi) \\
&\quad + (1 + 2.5e'^2 + 2.25e^2)(1.25\nu^2 + 2.5\mu^2 - 1)\,\nu \cos(\Pi' + \Pi)] \\
&- 24.609375\alpha e^2 e'[\mu \cos(\Pi' - 3\Pi) + \nu \cos(\Pi' + 3\Pi)]\,\nu\mu \\
&+ 3.75\alpha^2 e[(1 + 5e'^2 + 0.75e^2)(4.375\nu^4 + 17.5\nu^2\mu^2 \\
&\quad + 4.375\mu^4 - 5\nu^2 - 5\mu^2 + 1) \\
&\quad + 10.5(1 + 5e'^2 + e^2)(1.75\nu^2 + 1.75\mu^2 - 1)\,\nu\mu \cos 2\Pi] \\
&+ \alpha^2 e e'^2 \left[14.765625 \left(\frac{105}{90}\mu^2 + 3.5\nu^2 - 1\right)\mu^2 \cos(2\Pi' - 2\Pi)\right. \\
&\quad + 28.125(1.75\nu^2 + 1.75\mu^2 - 1)\,\nu\mu \cos 2\Pi' \\
&\quad \left. + 14.765625 \left(\frac{105}{90}\nu^2 + 3.5\mu^2 - 1\right)\nu^2 \cos(2\Pi' + 2\Pi)\right] \\
&+ 103.359375\alpha^2 e^3 \nu^2\mu^2 \cos 4\Pi \\
&- 13.125\alpha^3 e'[(2.625\mu^4 + 15.75\nu^2\mu^2 + 7.875\nu^4 - 3.5\mu^2 \\
&\quad - 7\nu^2 + 1)\,\mu \cos(\Pi' - \Pi) \\
&\quad + (2.625\nu^4 + 15.75\nu^2\mu^2 + 7.875\mu^4 - 3.5\nu^2 \\
&\quad\quad - 7\mu^2 + 1)\,\nu \cos(\Pi' + \Pi)] \\
&+ 6.5625\alpha^4 e[18(4.125\mu^4 + 12.375\nu^2\mu^2 + 4.125\nu^4 - 4.5\mu^2 \\
&\quad - 4.5\nu^2 + 1)\,\nu\mu \cos 2\Pi \\
&\quad + 14.4375\nu^6 + 129.9375\nu^4\mu^2 + 129.9375\nu^2\mu^4 + 14.4375\mu^6 \\
&\quad - 23.625\nu^4 - 94.5\nu^2\mu^2 - 23.625\mu^4 + 10.5\nu^2 + 10.5\mu^2 - 1]\};
\end{aligned}
$$

$$[\delta\epsilon] = q_5[\delta\Omega] + t \times q_6 \sum_{\substack{q+k=0\\ j+l=0}} \alpha^{m-2}(sp_0 - 2me)e^{s-1}e'^{s'}$$

$$A^{m,s,s'}_{q,j,k,l}(\nu,\mu)\cos(-kII' - lII)$$

$$= q_5[\delta\Omega] + t \times q_6\{[1.5e(p_0 - 2e)(1 + 1.5e'^2 + 1.875e'^4) - 2 - 3e'^2$$

$$- 3.75e'^4 - 4.375e'^6]$$

$$\times (1.5\nu^2 + 1.5\mu^2 - 1) + 7.5e(p_0 - 2e)(1 + 1.5e'^2$$

$$+ 1.875e'^4)\nu\mu\cos 2II$$

$$- 3.75\alpha e'[\big((p_0 - 6e)(1 + 2.5e'^2) + 2.25(p_0 - 2e)e^2\big)\big((1.25\mu^2 + 2.5\nu^2$$

$$- 1)\mu\cos(II' - II)$$

$$+ (1.25\nu^2 + 2.5\mu^2 - 1)\nu\cos(II' + II)\big)$$

$$+ 6.5625(p_0 - 2e)e^2\nu\mu\big(\mu\cos(II' - 3II) + \nu\cos(II' + 3II)\big)]$$

$$+ 3.75\alpha^2[\big(-0.8 - 4e'^2 - 10.5e'^4 + e(p_0 - 4e)(1 + 5e'^2)$$

$$+ 0.75e^3(p_0 - 2e)\big)$$

$$\times (4.375\nu^4 + 17.5\nu^2\mu^2 + 4.375\mu^4 - 5\nu^2 - 5\mu^2 + 1)$$

$$+ 10.5e\big((p_0 - 4e)(1 + 5e'^2) + (p_0 - 2e)e^2\big)(1.75\mu^2$$

$$+ 1.75\nu^2 - 1)\nu\mu\cos 2II$$

$$+ e'^2\big(-4.5 - 21e'^2 + 7.5e(p_0 - 4e)\big)(1.75\nu^2 + 1.75\mu^2$$

$$- 1)\nu\mu\cos 2II']$$

$$+ 2.1ee'^2(p_0 - 4e)[(2.1875\mu^2 + 6.5625\nu^2 - 1.875)\mu^2\cos(2II' - 2II)$$

$$+ (2.1875\nu^2 + 6.5625\mu^2 - 1.875)\nu^2\cos(2II' + 2II)]$$

$$+ 27.5625e^3(p_0 - 2e)\nu^2\mu^3\cos 4II]$$

$$- 13.125\alpha^3 e'(p_0 - 10e)[(2.625\mu^4 + 15.75\nu^2\mu^2 + 7.875\nu^4 - 3.5\mu^2$$

$$- 7\nu^2 + 1)\mu \cos(\Pi' - \Pi)$$

$$+ (2.625\nu^4 + 15.75\nu^2\mu^2 + 7.875\mu^4 - 3.5\nu^2$$

$$- 7\mu^2 + 1)\nu \cos(\Pi' + \Pi)]$$

$$- 3.75\alpha^4[\Big(1 + 10.5e'^2 - 1.75e(p_0 - 6e)\Big)(14.4375\nu^6 + 129.9375\nu^4\mu^2$$

$$+ 129.9375\nu^2\mu^4$$

$$+ 14.4375\mu^6 - 23.625\nu^4 - 94.5\nu^2\mu^2 - 23.625\mu^4$$

$$+ 10.5\nu^2 + 10.5\mu^2 - 1)$$

$$+ \Big(52.5e'^2 \cos 2\Pi' - 31.5e(p_0 - 6e)\cos 2\Pi\Big)(4.125\nu^4$$

$$+ 12.375\nu^2\mu^2$$

$$+ 4.125\mu^4 - 4.5\nu^2 - 4.5\mu^2 + 1)\nu\mu]$$

$$- 16\alpha^6(13.746643\nu^8 + 219.946289\nu^6\mu^2 + 494.87915\nu^4\mu^4$$

$$+ 219.946289\nu^2\mu^6$$

$$+ 13.746643\mu^8 - 29.3261718\nu^6 - 263.935546\nu^4\mu^2$$

$$- 263.935546\nu^2\mu^4$$

$$- 29.3261718\mu^6 + 20.3027343\nu^4 + 81.2109375\nu^2\mu^2$$

$$+ 20.3027343\mu^4$$

$$- 4.921875\nu^2 - 4.921875\mu^2 + 0.2734375)\};$$

$$[\delta n] = 0.$$

Here

$$q_2 = \mu_0 n\sqrt{1 - e^2}/e; \qquad p_0 = e\sqrt{1 - e^2}/(1 + \sqrt{1 - e^2});$$

$$q_3 = \mu_0 n/\sqrt{1 - e^2}; \qquad p_2 = \cos N(\mu - \nu)/2\sqrt{\nu\mu};$$

$$q_4 = \mu_0 n / (\sqrt{1 - e^2 \sin i}) ; \qquad p_3 = \cos N / 2\sqrt{\nu\mu};$$

$$q_5 = \tan\frac{i}{2} \sin i; \qquad p_4 = \sqrt{\nu\mu} \sin N;$$

$$q_6 = \mu_0 n; \qquad p_5 = \sin N \, (\mu - \nu) / 2\sqrt{\nu\mu};$$

$$p_6 = \sin N / 2\sqrt{\nu\mu};$$

$$p_7 = \sqrt{\nu\mu} \cos N.$$

The values of the parameters contained in these expressions are shown in Table 2.

TABLE 2

ELEMENTS OF THE RELATIVE POSITION OF THE ORBITS OF THE SATELLITES, THE MOON AND THE SUN

Perturbing Body	AES-1		AES-2	
	The Moon	The Sun	The Moon	The Sun
a	0.109587864	0.000282868	0.109587864	0.000282868
μ_0	0.000016173	0.000007455	0.000016173	0.000007455
ν	0.025288355	0.041281524	0.455158342	0.5
μ	0.974711645	0.958718475	0.544841658	0.5
N	0	0	0	0
N'	0	12°.139268	0	12°.139268
τ, τ'	12°.139268	12°.139268	12°.139268	12°.139268
II	45°	45°	135°	135°
II'	196°.649013	269°.941122	196°.649013	269°.941122
p_0	0.049874371	0.049874371	0.26666666	0.26666666
p_2	3.02364997	2.3058063	0.090046172	0
p_3	3.18472277	2.51331309	1.00404597	1
p_4, p_5, p_6	0	0	0	0
p_7	0.156999536	0.198940594	0.497985166	0.5
q_2	0.0010138	0.000467355	0.000135863	0.000062628
q_3	0.0001024	0.000047208	0.000127372	0.000058714
q_4	0.00025739	0.000118647	0.000127372	0.000058714
q_5	0.082563048	0.082563048	1	1
q_6	0.000101897	0.000046971	0.000101897	0.000046971

Table 3 contains numerical values of the secular changes in the elements of the orbits of the satellites AES-1 and AES-2 attributable to the Moon, the Sun, and the Earth's oblateness after 1 year. The last were calculated by the following formulas (Proskurin, Batrakov, 1960):

$$[\Omega] = -J\left(\frac{a_e}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}\, n \times 365.25,$$

$$[\pi] = \frac{1}{2} J\left(\frac{a_e}{a}\right)^2 \frac{5\cos^2 i - 2\cos i - 1}{(1-e^2)^2}\, \dot{n} \times 365.25,$$

$$[\epsilon] = \frac{t}{2} J\left(\frac{a_e}{a}\right)^2 \frac{(5 + 3\sqrt{1-e^2})\cos^2 i - 2\cos i - 1 - \sqrt{1-e^2}}{(1-e^2)^2}\, n \times 365.25$$

with a_e = 6,378.39 km and J = 0.00164147.

TABLE 3

SECULAR PERTURBATIONS OF THE ELEMENTS OF THE AES-1 AND AES-2 ORBITS ATTRIBUTABLE TO THE MOON, THE SUN, AND THE EARTH'S OBLATENESS

		[a]	[e]	[i]	[Ω]	[π]	[ϵ]
AES-1	The Moon	0	-0.00075	-0.°016	-1.°3	1.°3	-1.°9
	The Sun	0	-0.00052	-0.007	-0.69	0.5	-0.8
	The Oblateness	0	0	0	-5	4.7	10
AES-2	The Moon	0	0.030	0.15	-0.28	-0.9	1
	The Sun	0	0.016	0	0	-0.3	0.7
	The Oblateness	0	0	0	4.7	3.6	1.1

In these formulas "i" is the angle of inclination of the satellite's orbit to the plane of the equator.

As may be seen from Table 3, on the average the secular perturbations of the elements of the satellites' orbits attributable to the Moon are two to three times greater than those attributable to the Sun.

In their turn, the aggregate values of the lunar-solar perturbations of the elements are two to three times smaller than the respective values explained by the Earth's oblateness.

In comparing the secular lunar-solar perturbations of the AES-1 with those of the AES-2, in addition to the difference in the magnitude of the perturbations of the eccentricities already mentioned in the Introduction, attention is drawn by the considerable preponderance of the secular perturbations of the inclination of the AES-2 orbit (because of the Moon) over the respective magnitude for the AES-1. The secular changes in the elements Ω, π, and ϵ caused by the influence of the Moon and the Sun are several times smaller for the AES-2 than for the AES-1.

It is interesting to note that the elements "i" and Ω of the AES-2 orbit undergo practically no secular perturbations from the Sun. This is explained by the fact that $p_2 = p_4 = p_5 = p_6 = \nu - \mu = 0$. Because of this, the greater portion of the addends in the expressions for the secular perturbations of these elements vanish.

2. The First-Order Periodic Perturbations

Graphs shown in Figures 1-24 (Appendix) were constructed using the points with the time intervals of 1 day for the lunar perturbations and 10 days for the solar perturbations in order to secure a general idea of the character of the changes in the elements of the orbits of the satellites being examined, under the effect of the gravitational perturbations attributable to the Moon and the Sun. In addition to evaluating the magnitude of the secular perturbations it was intended to secure information concerning the periodic changes in the elements whose period is comparable in length with the duration of one revolution of the perturbing body. Therefore, the graphs constructed cover the time intervals of 70 days for the lunar perturbations and 700 days for the solar perturbations.

In Figure 25 (Appendix) are shown the secular changes in the elements of the AES-2 orbit during 17 years.

We will examine some of the special characteristics of the periodic perturbations of the elements of the AES-1 and AES-2 attributable to the Moon and the Sun.

AES-1

Perturbations of the major half-axis:

Lunar -- the period is about 30.2 days; $\delta a_{min} = -1.2$ km, $\delta a_{max} = 2.3$ km;

Solar -- the period is 1 year; during this time, two minima of different level are observed: -90 m and -360 m, and two maxima: -40 m and -45m; the minima correspond to the angular departures of the Sun from the perigee by 90 and 270°.

Perturbations of the eccentricity:

Lunar -- the period is equal to the sidereal lunar month (27-1/3 days); the amplitude is about 20×10^{-5};

Solar -- the period is 1 year; the amplitude is 42×10^{-5}; two minima and two maxima during the period set in respectively when the Sun is at 45, 225, 135, and 315° from the perigee.

Perturbations of the inclination:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}003$;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}023$; the minima correspond to the angular departures of the Sun from the perigee by 0 and 180°; the maxima set in when $\upsilon = 90$ and 270° (υ is the true anomaly of the Sun).

Perturbations of the longitude of the ascending node:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}006$;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}05$; the $\delta\Omega$ curve has two inflection points corresponding to $\upsilon = 90$ and 270°.

Perturbations of the longitude of the perigee:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}13$;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}38$; the maxima are at $\upsilon = 0$ and 180°, the minima at $\upsilon = 90$ and 270°.

Perturbations of the mean longitude per epoch:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}0035$;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}015$; the maxima are at $\upsilon = 60$ and 240°, the minima at $\upsilon = 150$ and 330°.

Thus, clearly marked periodic components whose periods coincide with the sidereal periods of the revolution of the perturbing bodies are observed in the osculating elements of the AES-1 orbit in the time intervals examined.

An exception is that of lunar perturbations of the major half-axis whose period is equal to 30.2 days.

And what is more, the extremal values of the solar perturbations of the elements regularly depend on the position of the Sun in relation to the perigee.

The periodic solar perturbations of the elements of the AES-1 orbit are 2 to 10 times greater than the perturbations of the respective elements attributable to the Moon. An exception is that of the perturbations of the major half-axis, which are 6.5 times greater from the Moon than from the Sun.

AES-2

Perturbations of the major half-axis:

Lunar -- the period is 27-1/3 days; $\delta a_{min} = -1.7$ km; $\delta a_{max} = 3$ km;

Solar -- the period is 1 year; during the period, three minima are observed:

at $\upsilon = 0$	$\delta a = 0$,
at $\upsilon = 120°$	$\delta a = 0$,
at $\upsilon = 240°$	$\delta a = -0.5$ km,

and three maxima:

at $\upsilon = 60°$	$\delta a = 1.6$,
at $\upsilon = 160°$	$\delta a = 0.95$,
at $\upsilon = 320°$	$\delta a = 0.4$ km.

Perturbations of the eccentricity:

Lunar -- the period is 27-1/3 days; the amplitude is 0°00025;

Solar -- the period is 1 year; the amplitude is 0°001; the maxima are at $\upsilon = 45$ and $225°$, the minima at $\upsilon = 135$ and $315°$.

Perturbations of the inclination:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}017$;

Solar -- the period is 1 year; during this period two equal minima of the $\delta i = 0$ are observed at $\upsilon = 0$ and $180°$, and two equal maxima of the $\delta i = 0\overset{\circ}{.}23$ at $\upsilon = 90$ and $270°$.

Perturbations of the longitude of the ascending node:

Lunar -- the period is 27-1/3 days; amplitude is $0\overset{\circ}{.}0115$;

Solar -- the period is 1 year; two equal maxima of the $\delta\Omega = 0$ correspond to a $\upsilon = 0$ and $180°$; two equal minima of the $\delta\Omega = 0.135$ at $\upsilon = 90$ and $270°$.

Perturbations of the longitude of the perigee:

Lunar -- the period is 27-1/3 days; the amplitude is $0\overset{\circ}{.}015$;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}095$; the minima are at $\upsilon = 70$ and $250°$, the maxima at $\upsilon = 160$ and $340°$.

Perturbations of the mean longitude per epoch:

Lunar -- the period is 27-1/3 days; the amplitude is 0.025;

Solar -- the period is 1 year; the amplitude is $0\overset{\circ}{.}17$; the maxima are at $\upsilon = 45$ and $225°$, the minima at $\upsilon = 120$ and $300°$.

Thus, for the AES-2, as for the AES-1, in the time intervals of several revolutions of the Moon and the Sun the main role among all periodic perturbations is played by the perturbations whose periods coincide with the sidereal periods of the revolutions of these bodies.

The points corresponding to the extremal values of the solar perturbations of the elements of the AES-2 are regularly located relative to the perigee of the Sun's orbit.

With the exception of the major half-axis all elements of the AES-2 orbit undergo periodic perturbations attributable to the Sun that are several times greater than those attributable to the Moon.

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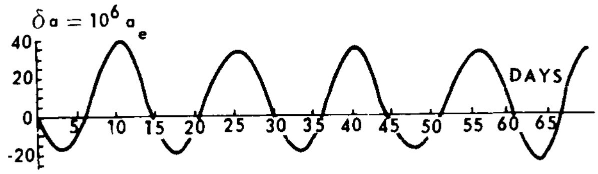
Appendix

THE LUNAR-SOLAR GRAVITATIONAL PERTURBATIONS OF TWO HYPOTHETICAL ARTIFICAL EARTH SATELLITES (THE CURVES)

FIGURE 1. LUNAR PERTURBATIONS OF THE MAJOR HALF-AXIS OF THE AES-2 ORBIT.



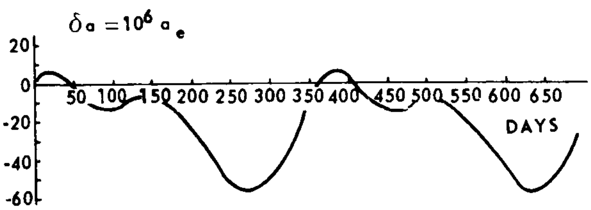
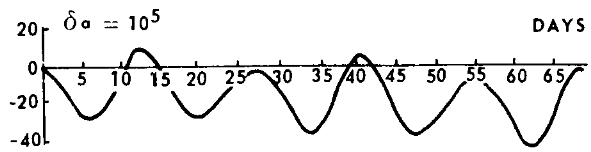


FIGURE 2. SOLAR PERTURBATIONS OF THE MAJOR HALF-AXIS OF THE AES-1 ORBITS.

FIGURE 3. LUNAR PERTURBATIONS OF THE ECCENTRICITY OF THE AES-1 ORBIT.



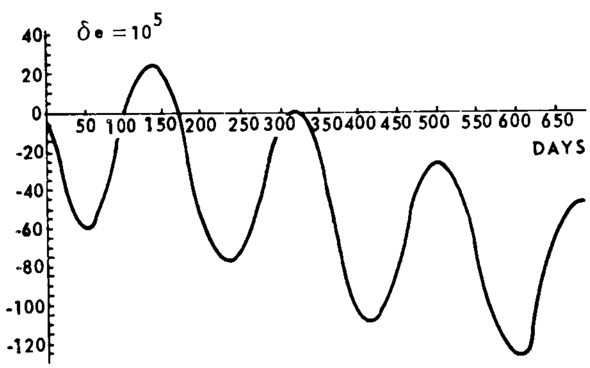
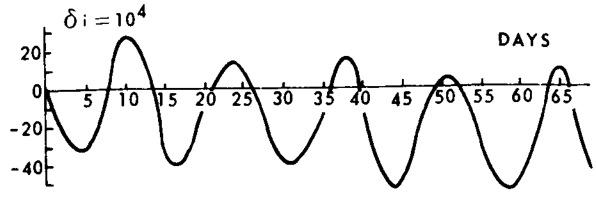


FIGURE 4. SOLAR PERTURBATIONS OF THE ECCENTRICITY OF THE AES-1 ORBIT.

FIGURE 5. LUNAR PERTURBATIONS OF THE INCLINATION OF THE AES-1 ORBIT.



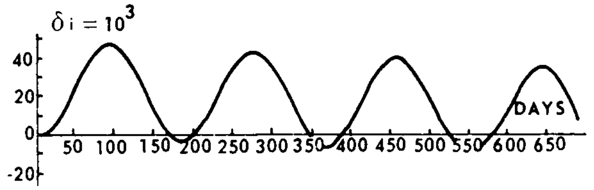


FIGURE 6. SOLAR PERTURBATIONS OF THE INCLINATION OF THE AES-1 ORBIT.

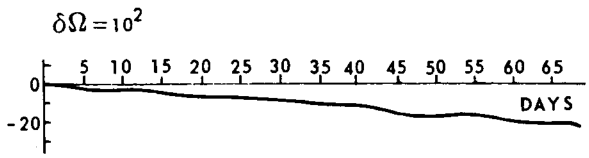


FIGURE 7. LUNAR PERTURBATIONS OF THE LONGITUDE OF THE ASCENDING NODE OF THE AES-1 ORBIT.

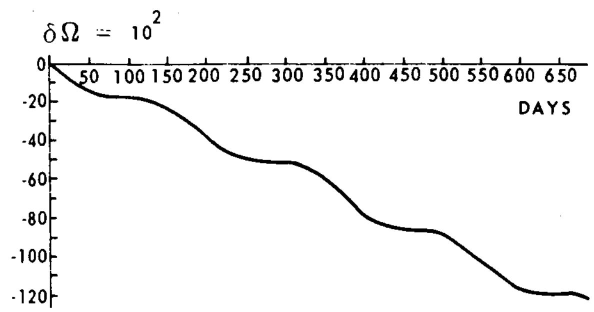


FIGURE 8. SOLAR PERTURBATIONS OF THE LONGITUDE OF THE ASCENDING NODE OF THE AES-1 ORBIT.

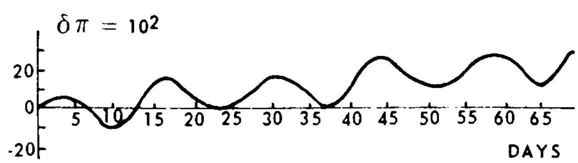


FIGURE 9. LUNAR PERTURBATIONS OF THE LONGITUDE OF THE PERIGEE OF THE AES-1 ORBIT.

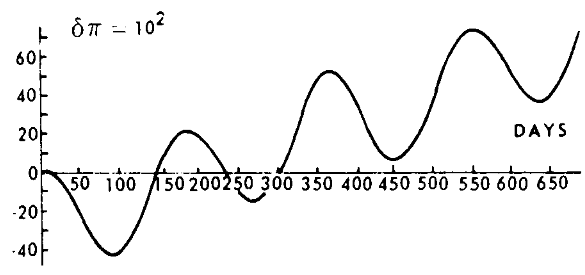


FIGURE 10. SOLAR PERTURBATIONS OF THE LONGITUDE OF THE PERIGEE OF THE AES-1 ORBIT.

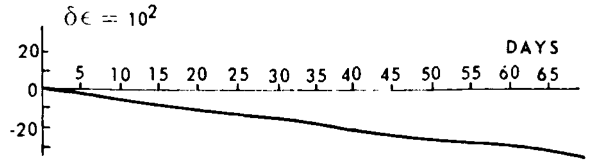


FIGURE 11. LUNAR PERTURBATIONS OF THE MEAN LONGITUDE OF THE AES-1 PER EPOCH.

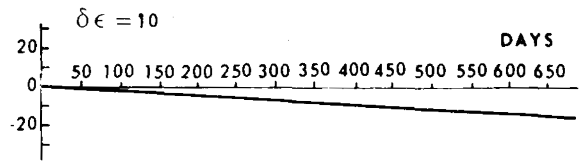


FIGURE 12. SOLAR PERTURBATIONS OF THE MEAN LATITUDE OF THE AES-1 PER EPOCH.

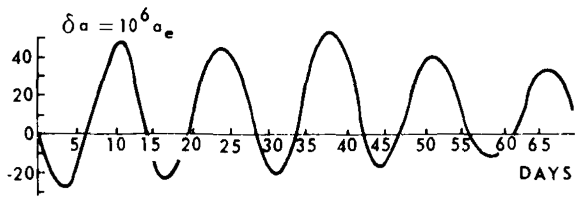


FIGURE 13. LUNAR PERTURBATIONS OF THE MAJOR HALF-AXIS OF THE AES-2 ORBIT.

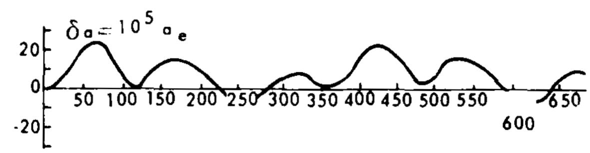


FIGURE 14. SOLAR PERTURBATIONS OF THE MAJOR HALF-AXIS OF THE AES-2 ORBIT.

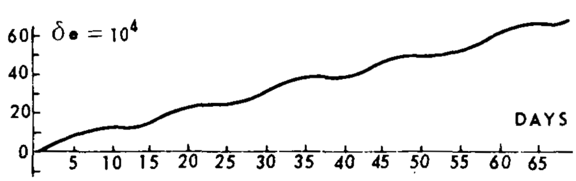


FIGURE 15. LUNAR PERTURBATIONS OF THE ECCENTRICITY OF THE AES-2 ORBIT.

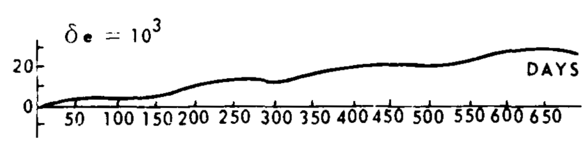


FIGURE 16. SOLAR PERTURBATIONS OF THE ECCENTRICITY OF THE AES-2 ORBIT.

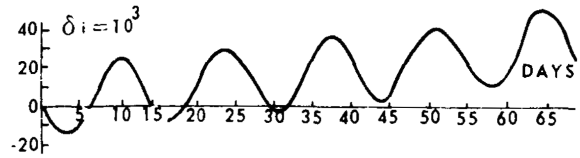


FIGURE 17. LUNAR PERTURBATIONS OF THE INCLINATION OF THE AES-2 ORBIT.

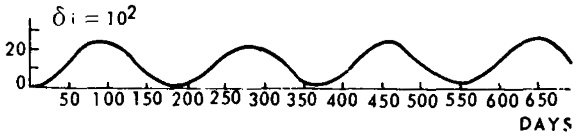
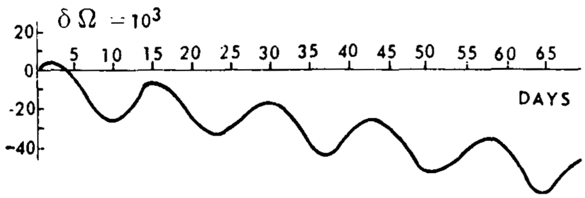


FIGURE 18. SOLAR PERTURBATIONS OF THE INCLINATION OF THE AES-2 ORBIT.

FIGURE 19. LUNAR PERTURBATIONS OF THE ASCENDING NODE OF THE AES-2 ORBIT.



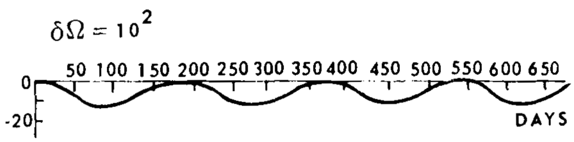


FIGURE 20. SOLAR PERTURBATIONS OF THE LONGITUDE OF THE ASCENDING NODE OF THE AES-2 ORBIT.

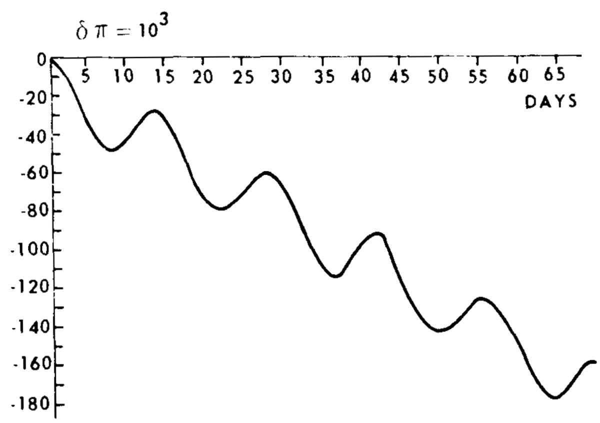


FIGURE 21. LUNAR PERTURBATIONS OF THE LONGITUDE OF THE PERIGEE OF THE AES-2 ORBIT.

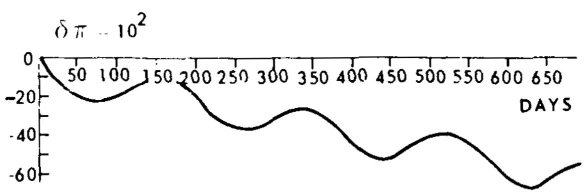
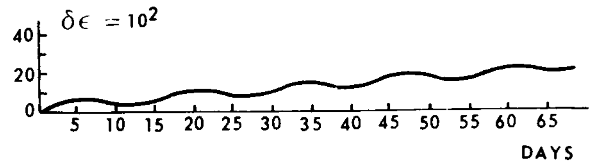


FIGURE 22. SOLAR PERTURBATIONS OF THE LONGITUDE OF THE PERIGEE OF THE AES-2 ORBIT.

FIGURE 23. LUNAR PERTURBATIONS OF THE LONGITUDE OF THE AES-2 PER EPOCH.



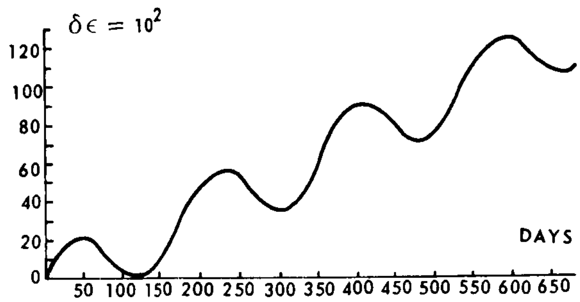
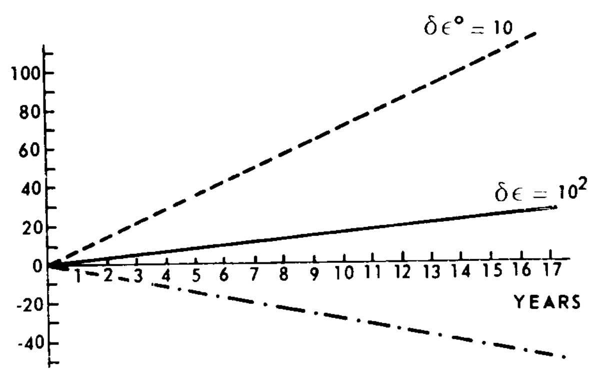


FIGURE 24. SOLAR PERTURBATIONS OF THE MEAN LONGITUDE OF THE AES-2 PER EPOCH.

FIGURE 25. SECULAR SOLAR PERTURBATIONS OF THE ELEMENTS OF THE AES-2 ORBIT: ECCENTRICITY, THE LONGITUDE OF THE PERIGEE AND OF THE MEAN LATITUDE PER EPOCH.

A $\delta\pi^{\circ} \times 10$ IS LAID OFF ON THE LOWER STRAIGHT LINE (THE DOT-AND-DASH LINE).



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1. ORIGINATING ACTIVITY *(Corporate author)* Redstone Scientific Information Center Research and Development Directorate U. S. Army Missile Command Redstone Arsenal, Alabama 35809	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP N/A

3. REPORT TITLE

LUNAR-SOLAR PERTURBATIONS OF A 24-HOUR SATELLITE
Institut Teoreticheskoi Astronomii Biulleten', 11, No. 1, 33-47 (1967)

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Translated from Russian

5. AUTHOR(S) *(First name, middle initial, last name)*

B. K. Martynenko

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
28 December 1967	33	4

8a. CONTRACT OR GRANT NO. N/A b. PROJECT NO. N/A c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) RSIC-740 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
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13. ABSTRACT

Study of the lunar-solar gravitational perturbations of two hypothetical earth satellites having a sidereal period of $23^h 56^m 04^s$. The orbit of the first satellite lies in the equatorial plane while that of the second is perpendicular to the ecliptic plane. At the initial moment (12^h E.T., January 0, 1950), the nodes of the orbits of the two satellites and the moon coincide. The perigee distance of the first satellite is chosen so as to minimize the secular excitations of the orbital eccentricity at fixed values of all the remaining elements. The orbital parameters of the second satellite are chosen according to the condition of maximum secular perturbations of the orbital eccentricity. The orbital eccentricity of the first satellite is 0.1, while that of the second is 0.6. Results show that the orbit of the first satellite is stable and orbital eccentricity decreases at the rate of 0.00127 per year. Rapid increase in the eccentricity of the second satellite limits its lifetime to 5.5 years. Secular and periodic first order perturbations are calculated.

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