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BY TWO-ELECTRON IONS

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ABSTRACT

A one channel model, previously used for He, is extended to allow calculation of elastic scattering by H^- and Li^+ at energies up to three rydbergs. Non-separable ground state wave functions are used, allowance being made for exchange, and dipole and quadrupole polarization potentials. Results for Li^+ are in good agreement with quantum defect method values. Calculated differential cross sections for elastic scattering by H^- at energies of one half and one rydberg are presented. Deviations from Coulomb scattering are marked.

ELASTIC SCATTERING OF SLOW ELECTRONS

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M. R. C. McDowell^{*†}
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I. Introduction

A considerable number of theoretical investigations on elastic scattering of slow electrons by He atoms¹⁻⁴ have been reported recently. They are in substantial agreement with each other, and are consistent (in the sense of dispersion relations³) with experimental data.^{5,6} In view of advances in experimental technique which have allowed measurement^{7,8} of inelastic collision cross sections of slow electrons with H^- (in good agreement with theory⁹) it is of interest to examine elastic scattering by two electron ions.

In this paper the model of Williamson and McDowell² has been extended and applied to elastic scattering by H^- and by Li^+ at energies up to 3 Ry. The theory is outlined in Section II, and the numerical methods employed are discussed in Section III. Phase shifts and scattered intensities are presented in Section IV,

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and in the case of Li^+ compared with the result of a Hartree-Fock calculation¹⁰, and the quantum defect method.

II. Theory

We consider an electron scattered by a two electron system of charge z . The total three electron system is described by the Schrödinger equation,

$$(\mathbf{H} - E) \Psi(1, 2, 3) = 0 \quad (1)$$

with Hamiltonian operator*

$$\mathbf{H} = \sum_{i=1}^3 \left(-\frac{1}{2} \nabla_i^2 - \frac{z}{r_i} \right) + \sum_{i>j} \frac{1}{r_{ij}} \quad (2)$$

In our model the total wave function (123) is represented by an ansatz,

$$\Psi(1, 2, 3) = \sum_{1,2,3} \psi_0(1, 2) F(3) S(123) \quad (3)$$

where $\psi_0(1, 2)$ is a wave function for the ground state of the target when electrons 1 and 2 are bound, $S(123)$ is a spin function, and $F(3)$ the unknown scattering function to be determined, and the sum is over cyclic permutations.

*All quantities will be given in atomic units, the unit of energy being the Rydberg, 13.565 eV.

Taking

$$S(123) = \frac{1}{\sqrt{2}} (\alpha_1 \beta_2 - \alpha_2 \beta_1) \alpha_3 \quad (4)$$

where α, β are the one-electron spin up and spin down functions, we may write, in a usual notation

$$F(3) = \sum_{\ell=0}^{\infty} \frac{f_{\ell}(3)}{r_3} P_{\ell}(\mu_3) \quad (5)$$

the position vector of electron 3 with respect to the target nucleus being

$$\mathbf{r}_3 = (r_3, \theta_3, \phi_3) \text{ and } \mu_3 = \cos \theta_3.$$

The integrodifferential equation for the scattering functions $f_{\ell}(r_3)$ is obtained by projecting out (1) on each partial wave in turn,

$$\int \psi_0^*(1, 2) P_{\ell}^*(\mu_3) S^*(123) [H - E] \Psi(123) d\mathbf{r}_1 d\mathbf{r}_2 d\hat{\mathbf{r}}_3 d\text{Spin} = 0. \quad (6)$$

For simplicity we assume that the ground state function for the target is known exactly:

$$\left(-\frac{1}{2} \nabla_1^2 - \frac{1}{2} \nabla_2^2 - \frac{z}{r_1} - \frac{z}{r_2} + \frac{1}{r_{12}} \right) \psi_0(12) = E_z \psi_0(12) \quad (7)$$

and

$$E = E_z + k^2. \quad (8)$$

We adopt a two-parameter⁽¹¹⁾ variational trial function

$$\psi_0(1, 2) = N (e^{-\alpha r_1 - \beta r_2} + e^{-\beta r_1 - \alpha r_2}) \quad (9)$$

for the target. Then on carrying out the angular and spin integrations (6) yields

$$L_{\ell}^{(0)} f_{\ell}(r_3) = \frac{16 \pi^2 N^2}{(2\ell+1)} \left[\delta_{0\ell} \int_0^{\infty} \int_0^{\infty} f_0(1) Z_{00}(123) dr_1 dr_2 \right. \\ \left. - 2 \int_0^{\infty} \int_0^{\infty} f_{\ell}(1) X_{0\ell}(123) dr_1 dr_2 \right] = R_{\ell}(r_3), \quad (10)$$

where $L_{\ell}^{(0)}$ is the operator

$$L_{\ell}^{(0)} = \frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2} - V_{00}(3). \quad (11)$$

The direct and exchange potentials are

$$V_{00}(r) = -\frac{2z}{r} + \frac{4\pi^2 N^2}{\alpha^3 \beta^3 r} \left[\left[1 - (1 + \alpha r) e^{-\alpha r} \right] + \left[1 - (1 + \beta r) e^{-\beta r} \right] \right. \\ \left. + \frac{128\alpha^3 \beta^3}{(\alpha + \beta)^6} \left[1 - \left(1 + \frac{1}{2} \overline{\alpha + \beta} r \right) e^{-(\alpha + \beta) r} \right] \right],$$

$$Z_{00} = \frac{r_3}{N} \psi_0^*(23) r_2^2 \left\{ (\alpha^2 r_1 - 2\alpha) e^{-\alpha r_1 - \beta r_2} + (\beta^2 r_1 - 2\beta) e^{-\beta r_1 - \alpha r_2} \right. \\ \left. + \frac{1}{N} \psi_0(1, 2) \left[r_1 k^2 + 2z - \frac{2r_1}{r_{12}^7} \right] \right\}, \\ X_{0\ell} = r_1 r_3 r_2^2 \psi_0^*(1, 2) \frac{2}{r_{13,\ell}} \psi_0(2, 3), \quad r_{13,\ell} = \begin{pmatrix} r_{<}^{\ell} \\ r_{>}^{\ell+1} \end{pmatrix}_{13} \quad (12)$$

No account has been taken of the polarizability of the target in this model, so we replace $L_{\ell}^{(0)}$ by

$$L_{\ell} = L_{\ell}^{(0)} - 2 (v_d + v_q) \quad (13)$$

where v_d and v_q are potentials behaving as r^{-4} and r^{-6} respectively at large r . They are chosen in the Bethe-Reeh form¹²

$$v_d(r) = -\frac{9}{2u^4} \left\{ 1 - \frac{1}{3} e^{-2u} \left[1 + 2u + 6u^2 + \frac{20}{3} u^3 + \frac{4}{3} u^4 \right] - \frac{2}{3} e^{-4u} [1 + u]^4 \right\},$$

$$\begin{aligned} v_q(r) = & -\frac{15}{2y^6} \left\{ 1 - 2y e^{-2y} \left[1 + 2y + \frac{4}{3} y^2 + \frac{1}{3} y^3 + \frac{2}{9} y^4 \right. \right. \\ & + \frac{1}{9} y^6 + \frac{1}{18} y^6 - \frac{1}{175} y^7 + \frac{1}{135} y^8 - \frac{2}{135} y^9 \left. \right] \\ & + e^{-4y} \left[1 + 4y + \frac{20}{3} y^2 + 6y^3 + \frac{28}{9} y^4 + \frac{8}{9} y^5 + \frac{1}{9} y^6 \right] \\ & \left. + \frac{8}{135} y^{10} \text{Ei}(-2y) \right\}. \end{aligned}$$

where $Ei(-u) = -E_1(u)$ and $E_1(u)$ is the exponential integral,¹³

$$u = z_1 r, \quad y = z_2 r \quad (15)$$

The parameters z_1, z_2 are chosen so that $(9/z_1)^4 = \alpha_d$, $(15/z_2)^6 = \alpha_q$, where α_d, α_q are the dipole and quadrupole polarizabilities of the target respectively.

III. Numerical Methods

The integrodifferential equation

$$L_\ell f_\ell(r) = R_\ell(r) \quad (10')$$

must be solved subject to the boundary conditions

$$f_\ell(0) = 0, \quad f_\ell(r) \underset{r \rightarrow \infty}{\sim} k^{-1/2} \sin \left(kr + \frac{z_0}{k} \ell \ln 2kr - \frac{1}{2} \ell \pi + \sigma_\ell + \delta_\ell \right) \quad (16)$$

with

$$\sigma_\ell = \arg 1^7 \left(\ell + 1 - \frac{iz_0}{R} \right)$$

and $z_0 = (z - 2)$. The quantity of interest is the non-coulomb part of the phase shift, δ_ℓ . We shall refer to it simply as the phase shift (for the ℓ^{th} partial wave).

The equation (10') was solved by a non-iterative method. Writing

$$f_\ell(r) = P(r) + \mu Q(r) + \nu R(r) \quad (17)$$

where P, Q, R satisfy

$$\begin{aligned} L_{\ell} P &= F(P) \\ L_{\ell} Q &= F(Q) + c_{\ell} r^{\ell+1} e^{-\alpha r} \\ L_{\ell} R &= F(R) + c_{\ell} r^{\ell+1} e^{-\beta r}, \quad c_{\ell} = \frac{16 \pi^2 N^2}{(2\ell+1)} \end{aligned} \quad (18)$$

and $F(\phi)$ involves linear combinations of integrals over the range $(0, r)$ only, the parameters μ, ν may be determined in terms of certain infinite integrals once $P(r), Q(r)$ and $R(r)$ are known. For He this procedure yields phase shifts identical to those obtained previously² by an iterative method. The ordinary differential equations (18) were solved by a Fox-Goodwin predictor-corrector method.

The solutions were normalized by the Strömberg procedure¹⁴ as given by Burgess. His procedure for determining the phase shift when a polarization potential is present may be extended easily to the case of a negative ion ($z_0 = -1$). At large r ,

$$f_{\ell}(r) \underset{r \rightarrow \infty}{\sim} k^{-1/2} \sin [\phi(r) + \delta_{\ell}] \quad (19)$$

where to sufficient accuracy

$$\phi(r) = \phi_1(r) + \frac{1}{2} \Phi(r).$$

Defining

$$\rho = |z_0 \mathbf{r}|, \beta = |z_0 \mathbf{k}|$$

we have (the \pm sign being that of z_0),

$$\begin{aligned} \phi_1^\pm = \chi(\rho)^\pm \pm \frac{1}{\beta} \ell \ln [(\beta^2 \rho + \beta \chi^\pm \pm 1)/\beta] - \frac{1}{2} \rho \pi + \sigma_\ell \mp \frac{1}{\beta} \\ \pm \frac{5(\rho \mp c)}{24 \chi^{(\pm)3}} - \frac{\chi^\pm (3\beta^2 + 4) + \beta \rho (3\beta^2 c + 2) \pm \beta c}{24 \chi (1 + \beta^2 c) (\chi^\pm + \beta \rho)} \\ + \frac{\left(c + \frac{1}{8}\right)}{\sqrt{c}} \cos^{-1} T^\pm(\rho), \end{aligned} \quad (20)$$

with

$$\chi^\pm = +(\beta^2 \rho^2 \pm 2\rho - c)^{1/2}, \quad T^\pm(\rho) = \frac{\beta c \chi^\pm \mp c + \rho}{\rho (1 + \beta^2 c)}, \quad c = \ell(\ell + 1).$$

The polarization term is

$$\begin{aligned} \Phi(\rho) = \frac{z_0^2 a_d}{4c^2} \left[\frac{3(2\rho \mp c)}{\rho (\chi^\pm + \beta \rho)} + \frac{c \chi^\pm}{\rho^2} - \frac{(3 + \beta^2 c)}{\sqrt{c}} \cos^{-1} T^\pm(\rho) \right], \quad c \neq 0 \\ = - \frac{2 z_0^2 a_d (9\beta \rho \chi^\pm + 11\beta^2 \rho^2 \pm 6\rho)}{15 \rho^2 (\chi^\pm + \beta \rho)^3}, \quad c = 0. \end{aligned} \quad (21)$$

When $c = 0$ the \cos^{-1} term in (20) is replaced by $0.25 (\chi^\pm + \beta \rho)^{-1}$. No special treatment of the quadrupole potential is required. Dipole and quadrupole polarizabilities for H^- are given by Stewart.⁵ The value of the quadrupole polarizability used is uncertain by $\pm 20\%$, but in view of its small effect, this is not

significant. The dipole polarizability used for Li^+ is the Hartree-Fock value of Lahiri and Mukkerji¹⁶ and should be accurate; its quadrupole polarizability was taken as zero. The calculated phase shifts for H^- are shown in Table 2. The program actually calculates $\tan \delta_\ell$, and $\delta_\ell \pmod{\pi}$. The calculations indicated that if the phase shift was defined to go to zero in the high energy limit, then the s-wave phase shift must be chosen to be π in the zero-energy limit. This was confirmed by comparing the numerical solution for $k^2 = 0.05$, $\ell = 0$ with the corresponding pure coulomb solution. An extra node is present in the calculated solution at small r ⁽²¹⁾. Physically this occurs because the incoming electron cannot enter the filled 1s-shell, and there is in our model, no bound $(1s)^2 2s$ state of H^{--} . A similar situation occurs for electron scattering by He, but not for Li^+ . In the latter case the $(1s)^2 2s$ state is the ground configuration of Li, and the zero energy s-wave phase shift is found to be 1.261 (see below). The s-wave phase shifts for all these systems are shown in Fig. 1. In Fig. 2 we show $\tan \delta_0$ for H^- both including (d + q) and excluding (d) the quadrupole potential. The local potentials occurring in Eq. (10') are shown in Fig. 3. They are strongly repulsive at large r but the effect of the attractive polarization potentials is to produce a broad positive maximum in $V(r)$ from $r = 2.0$ to $r = 10.0$ with a maximum value of 0.2 Ry. The potential $V(r)$ then becomes strongly attractive for $r \leq 2.0 a_0$. There appears to be a possibility of trapping for low energy electrons. We therefore computed $\tan \delta_0$ for $0.05 \leq k^2 \leq 0.2$ Ry at intervals of 0.01 in k^2 (i.e. 0.136 eV). No evidence for an s-wave resonance

was observed, but in any event no resonance appreciably narrower than 0.1 eV would have been found. Herzenberg and Lau¹⁷ have used our model (with a different form of polarization potential) to investigate s-wave scattering of slow electrons in He, and obtain a resonance by suitably modifying the strength of the exchange terms. Peterkop¹⁸ has also reported an s-wave resonance in He, in a one channel model.

Although the calculated results for He and Li^+ are in excellent agreement with other calculations (and for He, with experiment), it might appear that the model would be less satisfactory for H^- , with its extremely large polarizability. In fact the calculated phase shifts in He and Li^+ are also dominated by the polarizability potential (for small ℓ and k^2) and are substantially in error when polarization is neglected. We would therefore expect our H^- results to be of comparable accuracy.

The effect of introducing a quadrupole polarization is to slightly increase all the phase shifts, in general by less than 10%. The highest value of k^2 at which the p-wave phase shift passes through $\pi/2$ increases from 2.0 to 2.45 when α_q changes from zero to 1300. For given k^2 , the phase shifts decrease slowly with ℓ ($\ell \geq 1$).

Calculated phase shifts for Li^+ are displayed in Table 3. When the polarization potentials are omitted the calculated values are in very close agreement

with these obtained in a Hartree-Fock calculation.¹⁰ This confirms the result obtained for He^{1,2} that provided the target ground state wave function used gives a binding energy at least as good as the Hartree-Fock value, its detailed behavior is not important in determining the low energy phase shifts. In particular it appears that short range $\ell = 0$ correlation in the target is not important. The Li⁺ p-wave phase shift obtained when polarization is included tends to a threshold value (0.170) in close agreement with that (0.188) obtained using the quantum defect method,¹⁹ but rather larger than the no-polarization value of 0.110.

The zero energy s-wave phase shift is 1.261 where the quantum defect method gives 1.232 ± 0.005 . The d-wave phase shift at threshold, is like its quantum defect value, very small (approximately 3.8×10^{-3}).

We have computed angular distributions of elastically scattered electrons for H⁻, where deviations from pure Coulomb scattering would be expected to be significant.

The non-coulomb contribution $f_N(\theta)$ to the scattering amplitude $f(\theta)$ may be written

$$f_N(\theta) = -\frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell+1) e^{2i\sigma_\ell} (e^{2i\delta_\ell} - 1) P_\ell(\cos \theta) \quad (22)$$

and because of the large polarizability of H⁻ a comparatively large number of terms must be retained in the summation over ℓ in (22). We find that for

$k^2 \leq 3.0$, the sum has converged to better than 1% if it is cut off at $\ell = 30$. For $\ell \geq 4$ the exchange terms in (10') may be ignored; phase shifts are then obtained from the suitably normalized solutions of

$$L_\ell(r) f_\ell(r) = 0. \quad (23)$$

In table (2b) we give phase shifts in this approximation for $\ell = 4$ (1) 10 at $k^2 = 0.5, 0.75, 1.0, 3.0$.

Rather than displaying the scattering intensity $I(\theta) = |f(\theta)|^2$ it is convenient first to show $R(\theta)$, the ratio of $I(\theta)$ to the pure coulomb intensity $I_c(\theta)$. Writing $\alpha = 1/k$ we have²⁰

$$R(\theta, k^2) = |1 + N|^2 \quad (24)$$

with (for H^-),

$$N = -2k \sin^2 \frac{\theta}{2} \left\{ \exp -i\alpha \ln \left(\sin^2 \frac{\theta}{2} \right) \right\} \sum_{\ell=0}^{\ell_{\max}} (2\ell+1) e^{2i\delta_\ell} \sin \delta_\ell e^{2i(\sigma_\ell - \sigma_0)} P_\ell(\cos \theta) \quad (25)$$

The calculated values of $R(\theta, k^2)$ for $k^2 = 0.50, 0.75, 1.0, 3.0$ are shown in Fig. 4.

The most significant features are the sharp dip near $\theta = 0.4$, the oscillations in the range $1.0 \leq \theta \leq 2.5$ and the strong enhancement in the backward direction for $k^2 = 0.5$. The fine detail of the calculated oscillations in $R(\theta, k^2)$ at intermediate angles may not be significant.

In Figs. 5a, 5b we show $I(\theta)$ and $I_c(\theta)$ for $R^2 = 0.5$ and 1.0 over the angular range $0.3 \leq \theta \leq \pi$. The calculated intensity is a factor of five lower than the coulomb intensity at 40° , but for $k^2 = 0.5$ it is a factor of four higher in the backward direction.

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Figure Captions

- Fig. 1 Calculated s-wave phase shift for elastic scattering of slow electrons by the two electron systems indicated.
- Fig. 2 $\tan \delta_0(k^2)$ for elastic scattering by H^- . Full curve $\alpha_q = 0$, dashed curve $\alpha_q \neq 0$.
- Fig. 3 The local potentials occurring in Eq. (10') for $Z = 1$ (H^-).
- Fig. 4 Calculated values of $R(\theta, k^2)$, the ratio of $I(\theta)$ to the Coulomb intensity $I_c(\theta)$ for H^- , (1) $k^2 = 0.5$ (2) $k^2 = 0.75$ (3) $k^2 = 1.0$ (4) $k^2 = 3.0$.
- Fig. 5 Calculated differential cross sections for scattering by H^- . The full curve is $I_c(\theta)$, the dashed curve $I(\theta)$. (a) $k^2 = 0.5$, (b) $k^2 = 1.0$.

Table 1

Parameters for H^- , Li^+

	α	β	α_d	α_a	Z	Z_0
H^-	1.04	0.2808	203	1300	1	-1
Li^+	3.295	2.079	0.19	0	3	+1

Table 2a
S-Wave Phase Shifts for H^-

k^2	$\tan \eta_0$		η_0
	d	d + q	d + q
0.05	8.19,-3	8.22,-3	3.150
0.06	1.26,-2	1.27,-2	3.154
0.07	1.84,-2	1.85,-2	3.160
0.08	2.51,-2	2.54,-2	3.167
0.09	3.26,-2	3.31,-2	3.175
0.10	4.06,-2	4.13,-2	3.183
0.25	2.22,-1	2.50,-1	3.389
0.50	4.71,-1	5.59,-1	3.650
0.75	3.49,-1	4.20,-1	3.540
1.0	1.74,-1	2.30,-1	3.369
2.0	-3.49,-1	-3.07,-1	2.844
3.0	-7.43,-1	-6.97,-1	2.510

(d indicates dipole potential only

d + q indicates dipole plus quadrupole potentials)

Table 2b

Calculated H^- Phase Shifts, Notation as in Table 2a

k^2	$\ell = 1$		$\ell = 2$	
	d	d+q	d	d+q
0.10	4.32,-2	4.38,-2	2.95,-2	2.99,-2
0.55	1.49,-1	1.53,-1	7.41,-2	8.08,-2
0.20	3.72,-1	4.00,-1	1.59,-1	1.63,-1
0.25	7.28,-1	8.12,-1	2.64,-1	2.76,-1
0.50	1.71	1.82	8.13,-1	8.78,-1
0.75	1.78	1.86	1.04	1.11
1.0	1.75	1.82	1.11	1.18
1.5	1.65	1.70	1.12	1.17
2.0	1.57	1.61	1.09	1.14
3.0	1.44	1.46	1.03	1.07

Table 2c

Dipole Plus Quadrupole Potentials Only

k^2	$\ell = 3$	$\ell = 4$	$\ell = 4$ no exchange
0.25	0.139	0.0798	-
0.5	0.400	0.221	-
0.75	0.667	0.403	0.390
1.0	0.768	0.503	0.497
1.5	0.858	0.624	-
2.0	0.869	0.669	-
3.0	0.846	0.687	-

Table 2d

Phase Shifts for H^- , $\ell \geq 4$, no exchange

(dipole plus quadrupole potential only)

$k_0^2 \backslash \ell$	4	5	6	7	8	9	10
0.50	0.230	0.147	0.0944	0.0637	0.0449	0.0327	0.0244
0.75	0.390	0.244	0.158	0.107	0.075	0.0540	0.0403
1.0	0.497	0.329	0.221	0.152	0.107	0.0775	0.0577
3.0	0.687	0.552	0.447	0.360	0.288	0.231	0.185

Table 3

Phase Shifts for Li^+ , Dipole Potential Only

k^2	$\ell = 0$	$\ell = 1$	$\ell = 1, \text{H.F.}$		$\ell = 1, \alpha_d = 0$	$\ell = 2$
0	(1.261)	(0.170)	(0.110)	-	-	(0.0038)
0.04	1.258	0.173	0.113	-	-	0.0044
0.09	1.255	0.177	0.117	-	-	0.0052
0.16	1.248	0.182	0.120	0.122	-	0.0063
0.25	1.241	0.189	0.126	-	-	0.0074
0.36	1.230	0.195	0.132	0.133	-	0.0088
0.49	1.221	0.202	0.140	0.140	-	0.0108
0.64	1.209	0.210	0.147	0.146	-	0.0135
0.81	1.194	0.217	0.153	-	-	0.0157
1.0	1.178	0.224	0.161	-	-	0.0182

- (1) Values for $k^2 = 0$ are obtained by graphical extrapolation.
(2) For the p-wave phase shift the successive columns show the results (i) in our model with $\alpha_d = 0.19$, (ii) in a Hartree-Fock calculation,¹⁰ (iii) in our model with $\alpha_d = 0$.

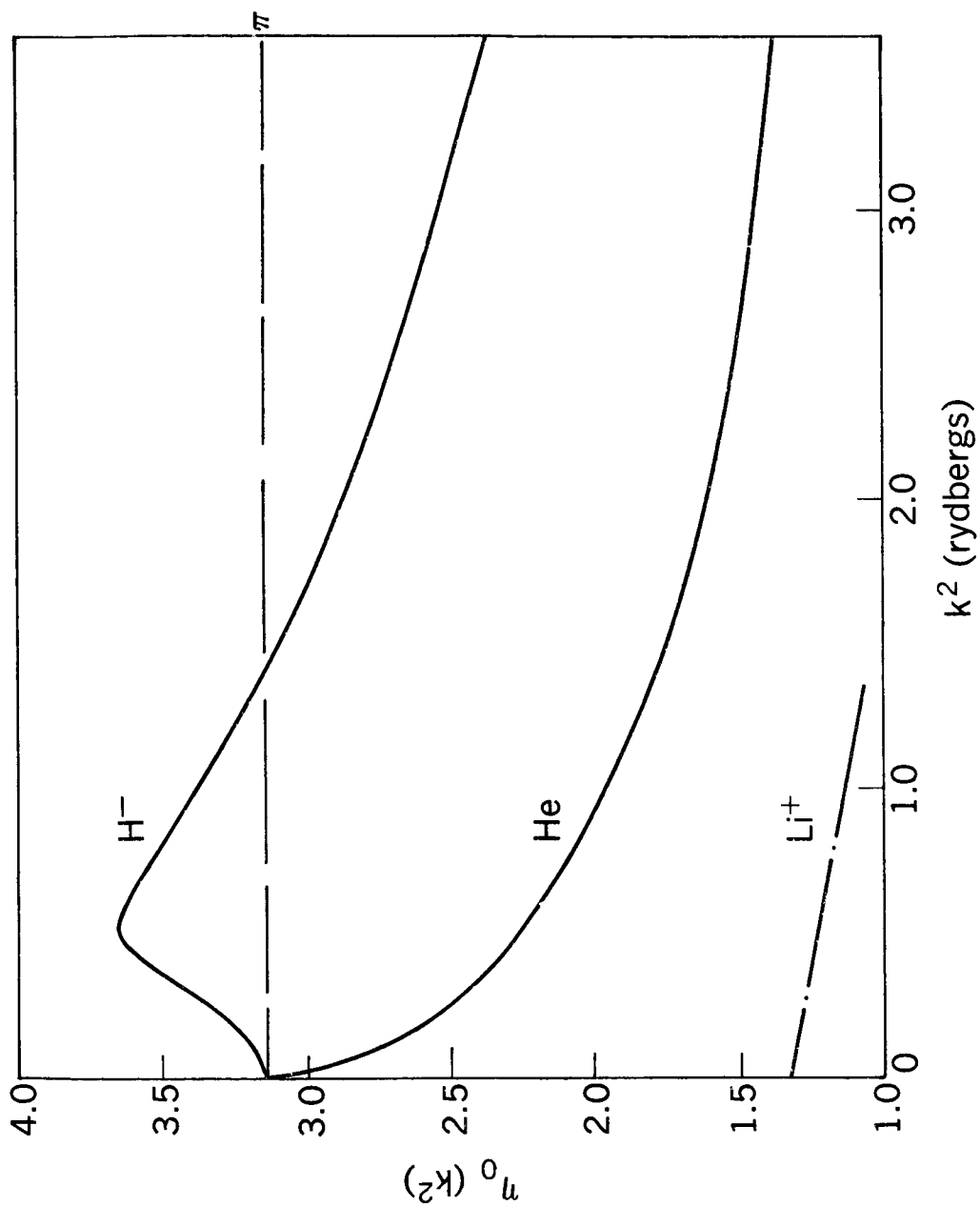


Figure 1. Calculated s-wave phase shift for elastic scattering of slow electrons by the two electron systems indicated.

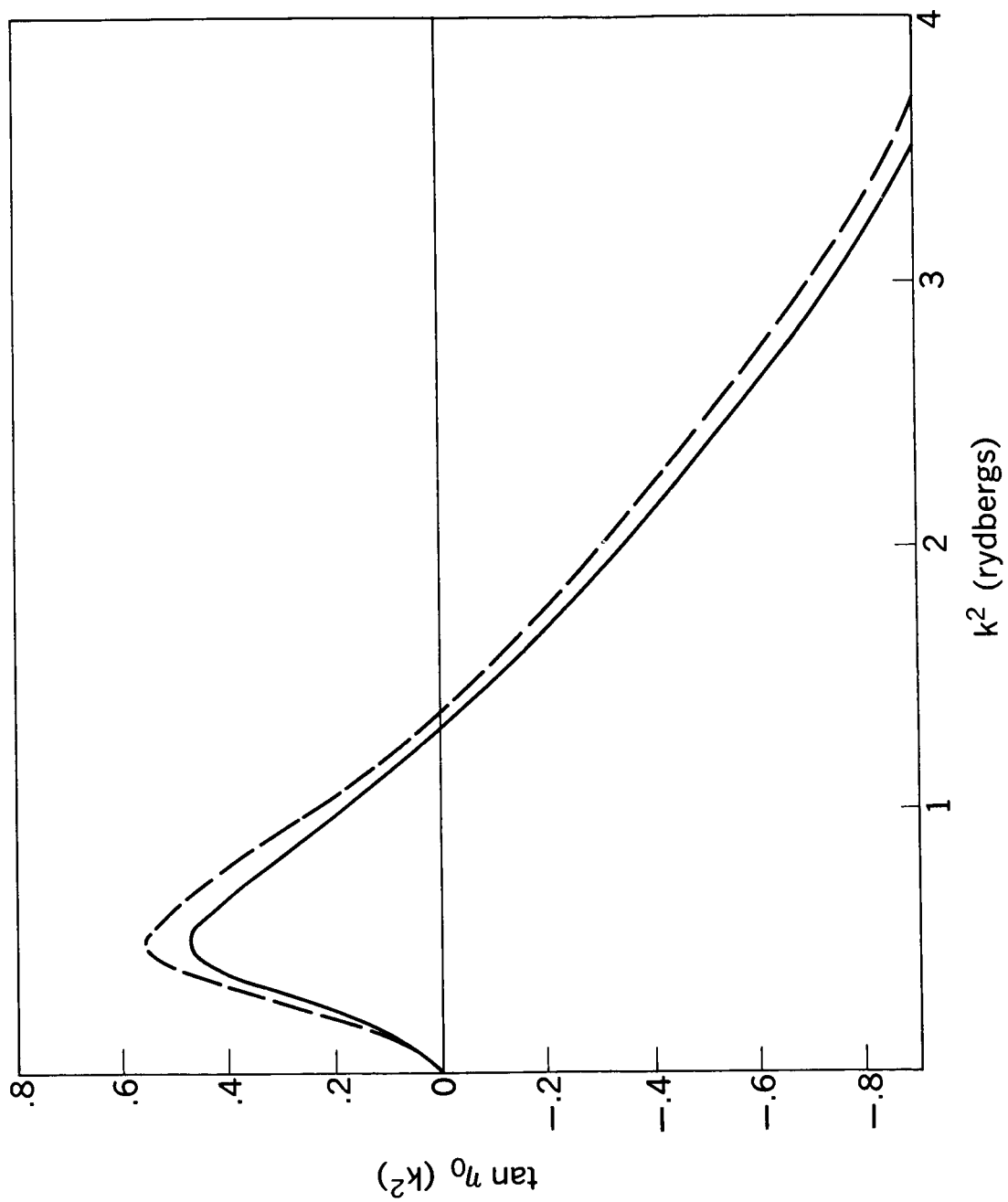


Figure 2. $\tan \delta_0(k^2)$ for elastic scattering by H^- . Full curve $\alpha_q = 0$, dashed curve $\alpha_q \neq 0$.

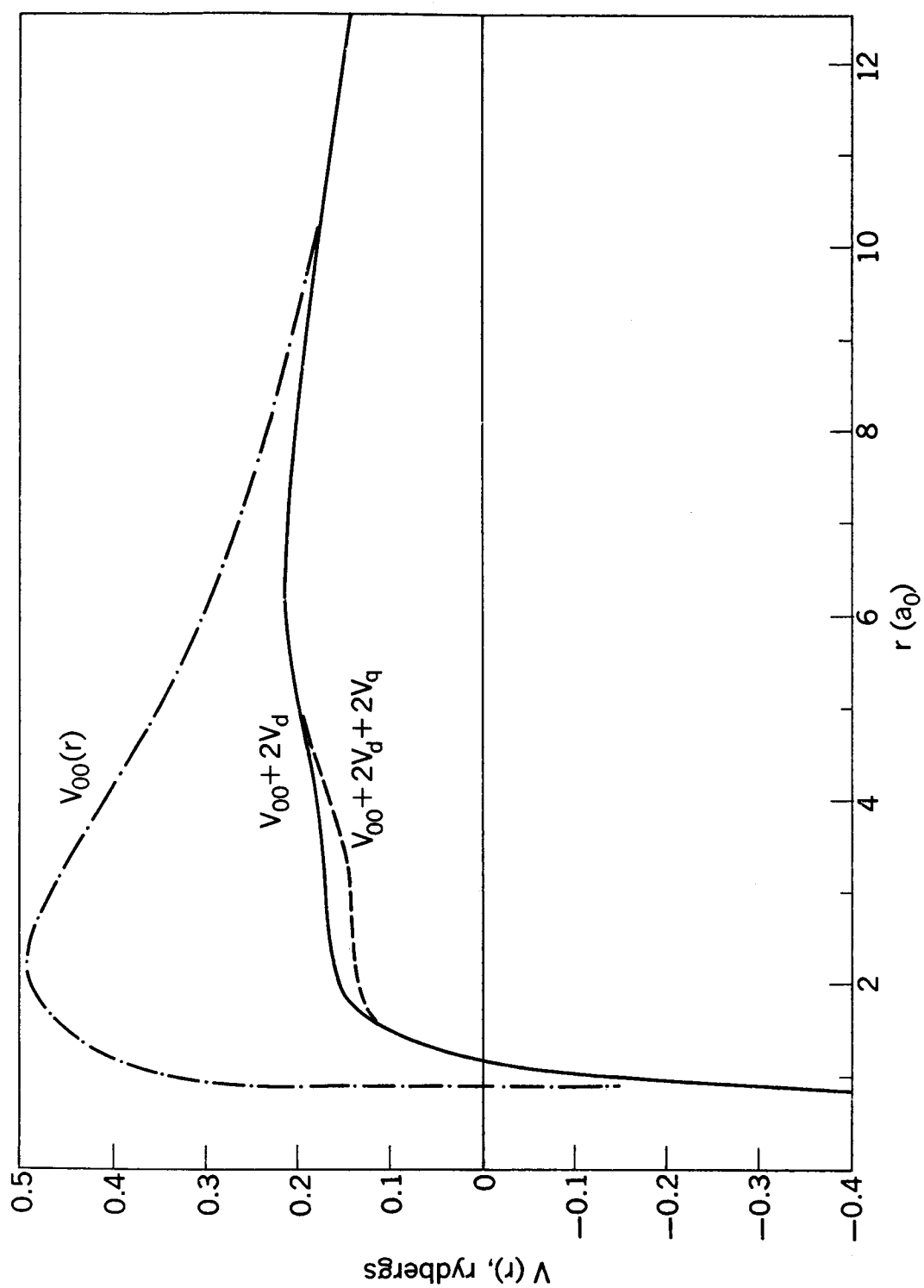


Figure 3. The local potentials occurring in Eq. (10') for $Z = 1$ (H^-).

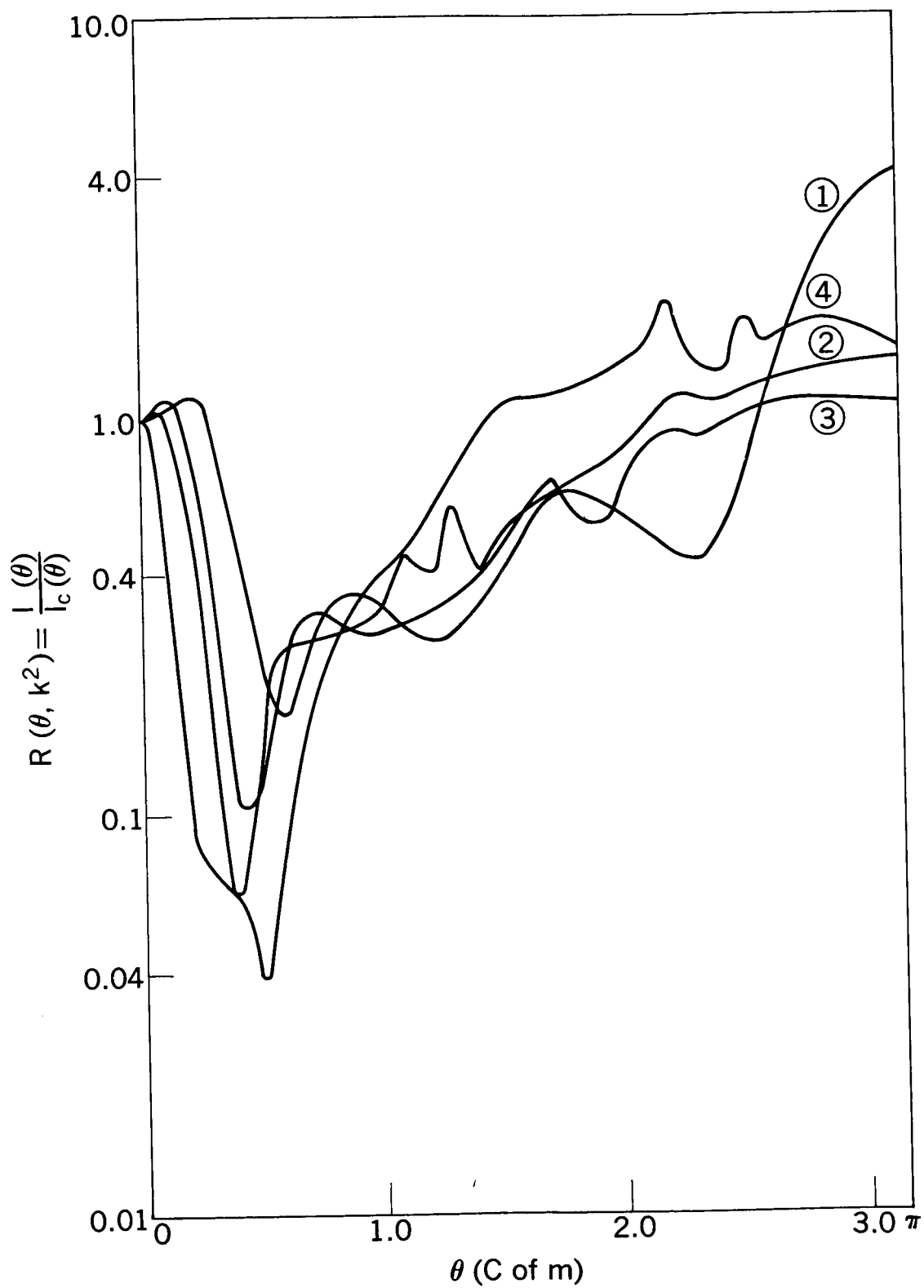


Figure 4. Calculated values of $R(\theta, k^2)$, the ratio of $I(\theta)$ to the Coulomb intensity $I_c(\theta)$ for H^- , (1) $k^2 = 0.5$ (2) $k^2 = 0.75$ (3) $k^2 = 1.0$ (4) $k^2 = 3.0$.

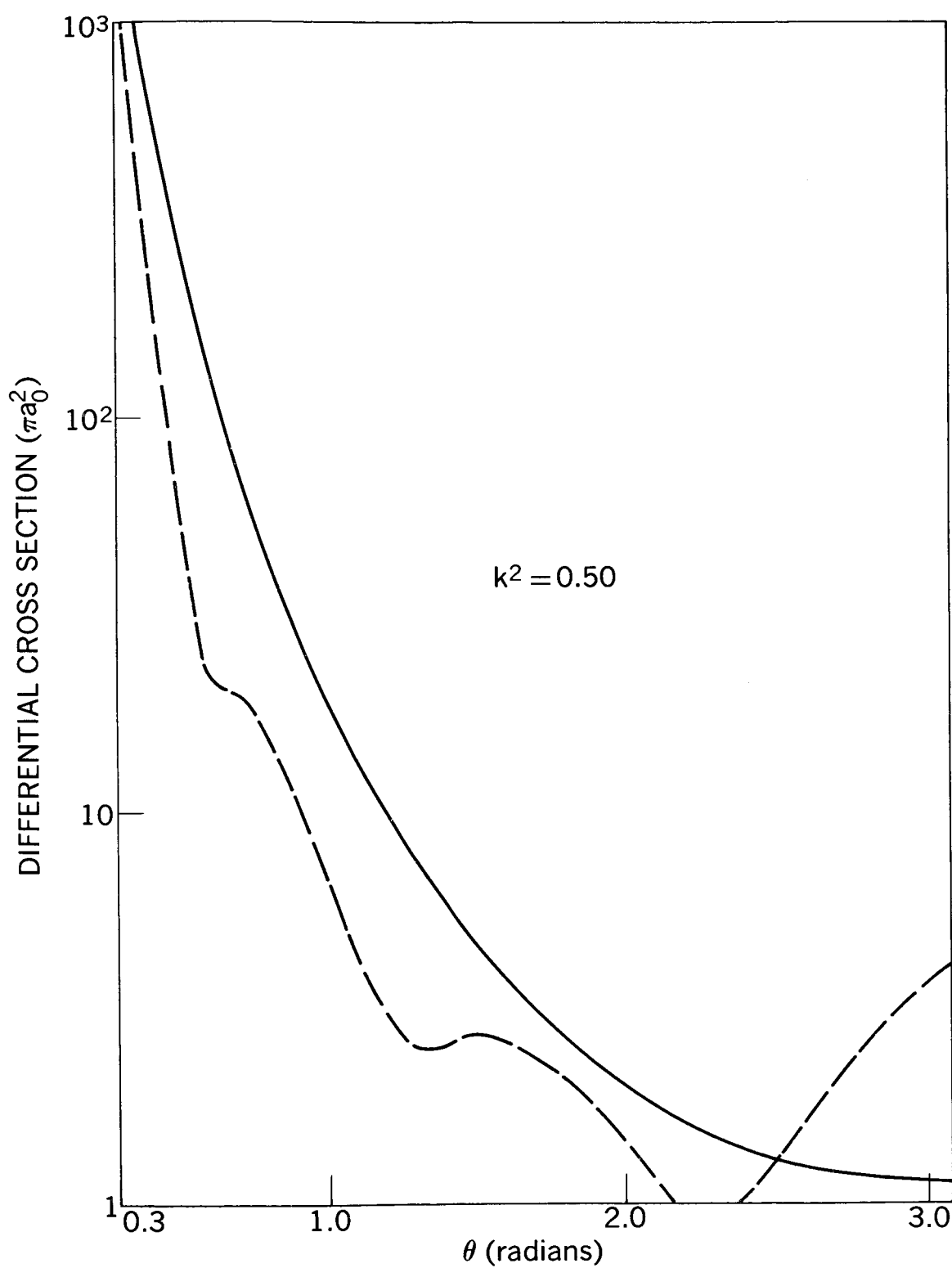


Figure 5a. Calculated differential cross sections for scattering by H^- . The full curve is $l_c(\theta)$, the dashed curve $l(\theta)$. (a) $k^2 = 0.5$, (b) $k^2 = 1.0$.

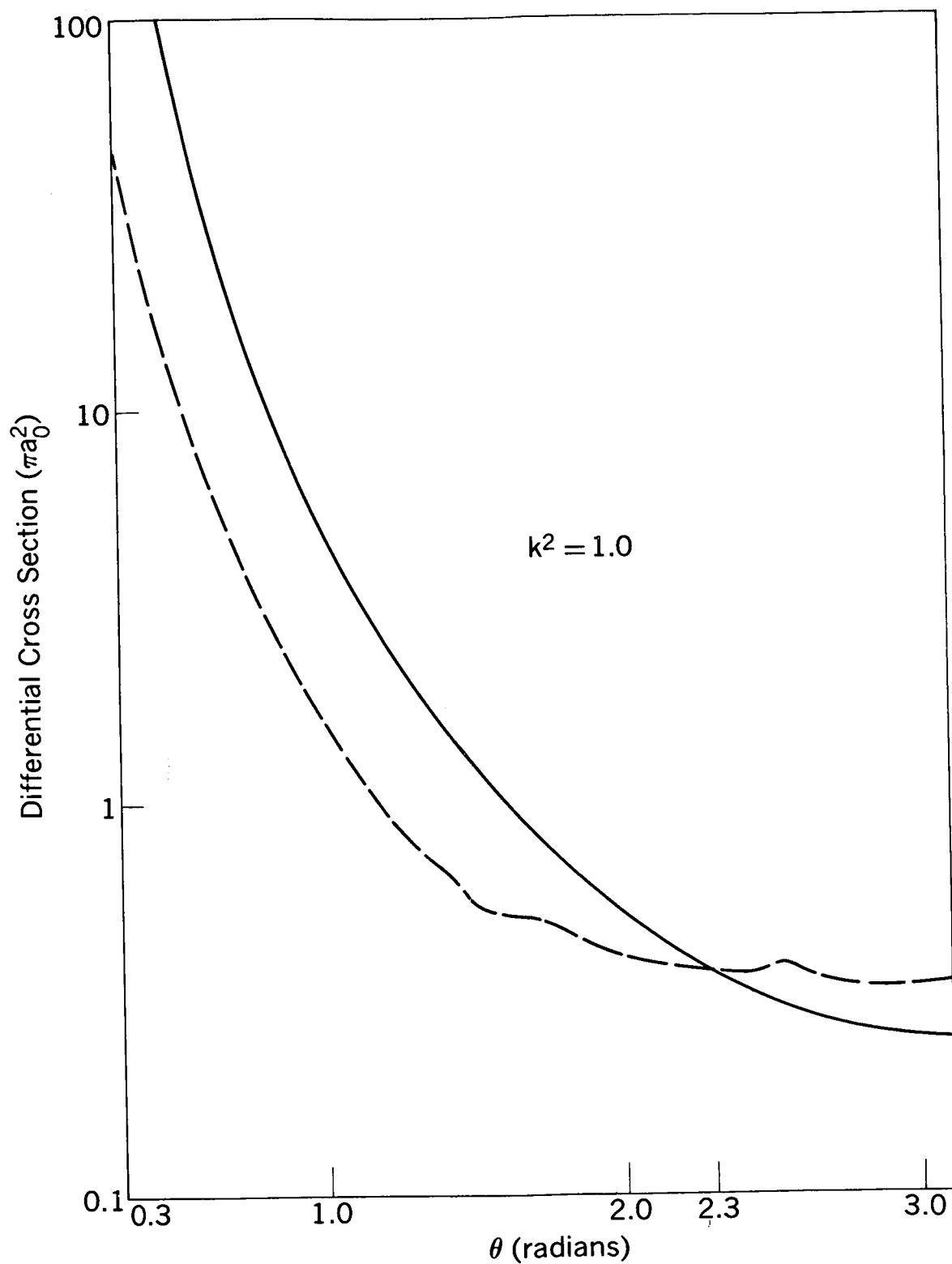


Figure 5b. Calculated differential cross sections for scattering by H^- . The full curve is $l_c(\theta)$, the dashed curve $l(\theta)$. (a) $k^2 = 0.5$, (b) $k^2 = 1.0$.